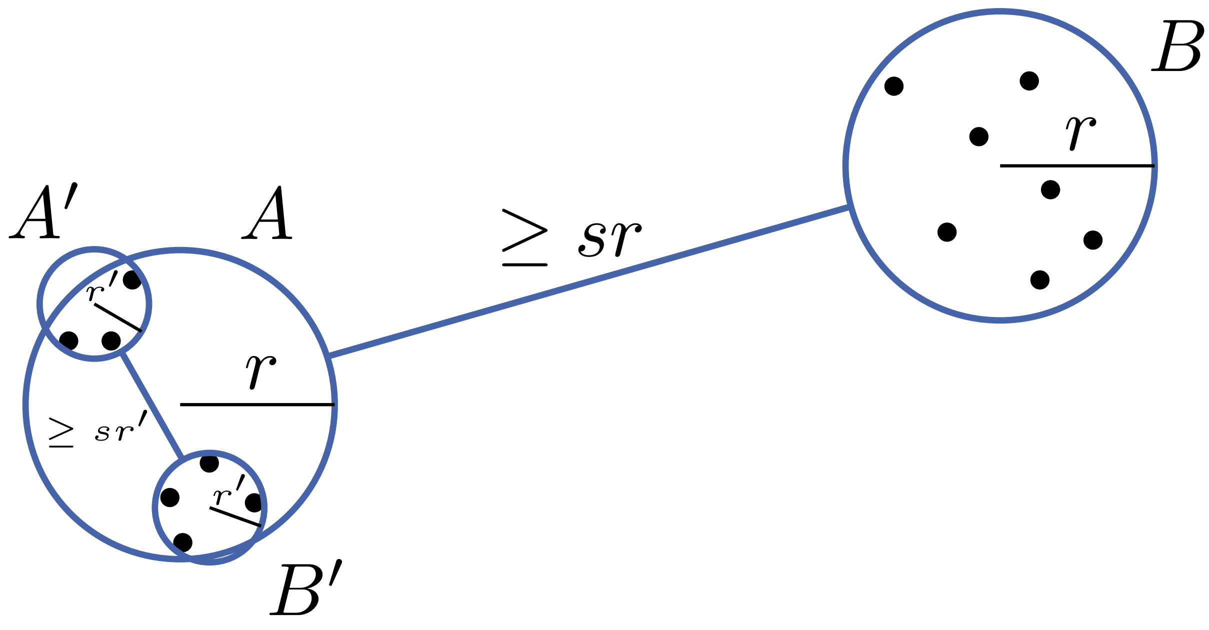
Computational Geometry Lecture
Applications of WSPD & Visibility Graphs

INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

Tamara Mchedlidze
25.05.2018

Recall: Well-Separated Pair Decomposition

Def: A pair of disjoint point sets A and B in $\mathbb{R}^d$ is called s-**well separated** for some $s > 0$, if A and B can each be covered by a ball of radius r whose distance is at least sr.



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- $A_i, B_i \subset P$ for all i
- $A_i \cap B_i = \emptyset$ for all i
- $\bigcup_{i=1}^{m} A_i \otimes B_i = P \otimes P$
- $\{A_i, B_i\}$ s-well separated for all i

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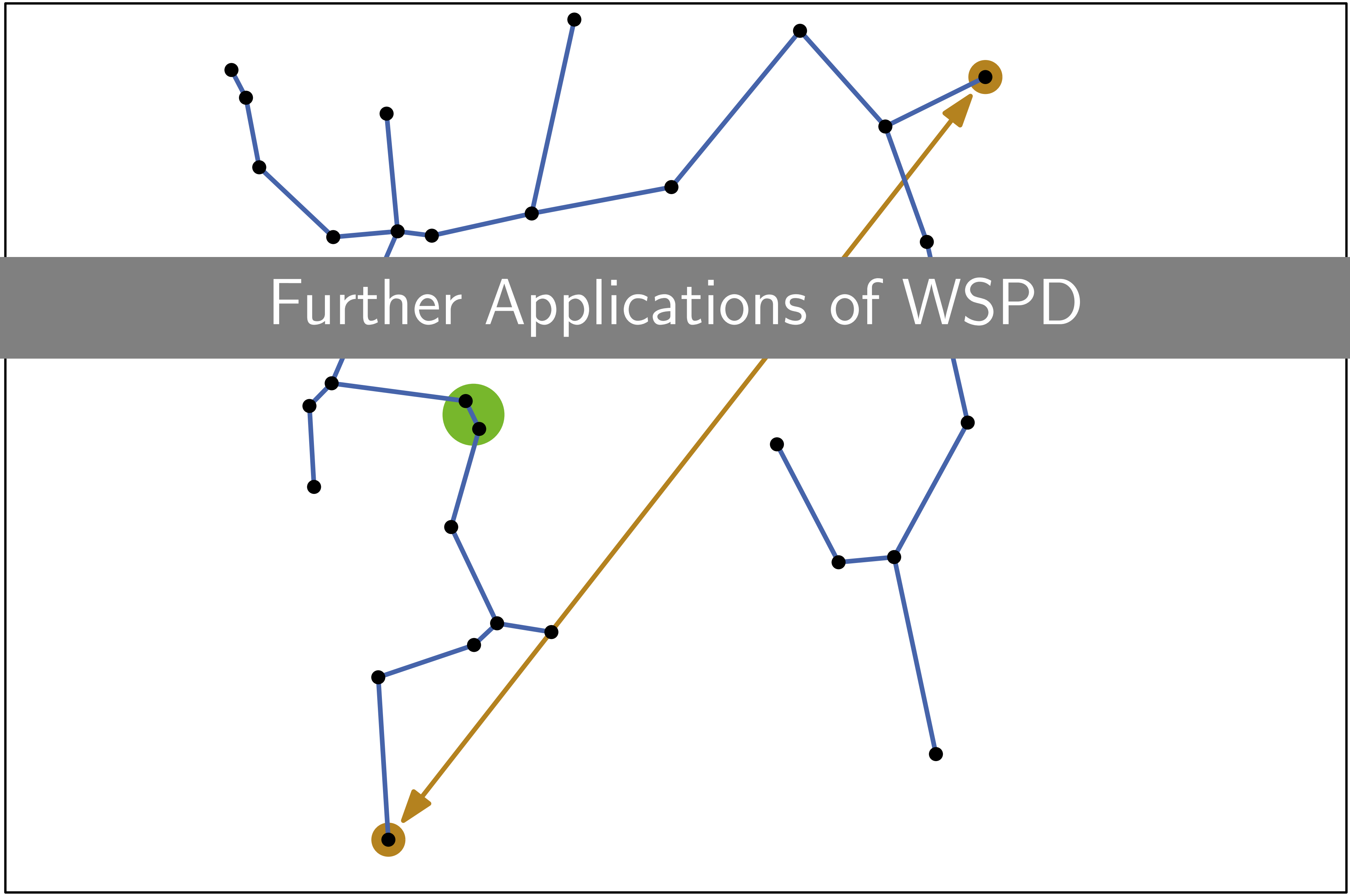
Thm 3: Given a point set P in $\mathbb{R}^d$ and $s \geq 1$ we can construct an s-WSPD with $O(s^d n)$ pairs in time $O(n \log n + s^d n)$.



Further Applications of WSPD

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Thm 5: The MST obtained from a $(1+\varepsilon)$-spanner of P is a $(1+\varepsilon)$-approximation of the EMST of P.

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Thm 6: The diameter obtained from an s-WSPD of P for $s = 4/\varepsilon$ is a $(1 + \varepsilon)$-approximation of the diameter of P.

Closest Pair of Points

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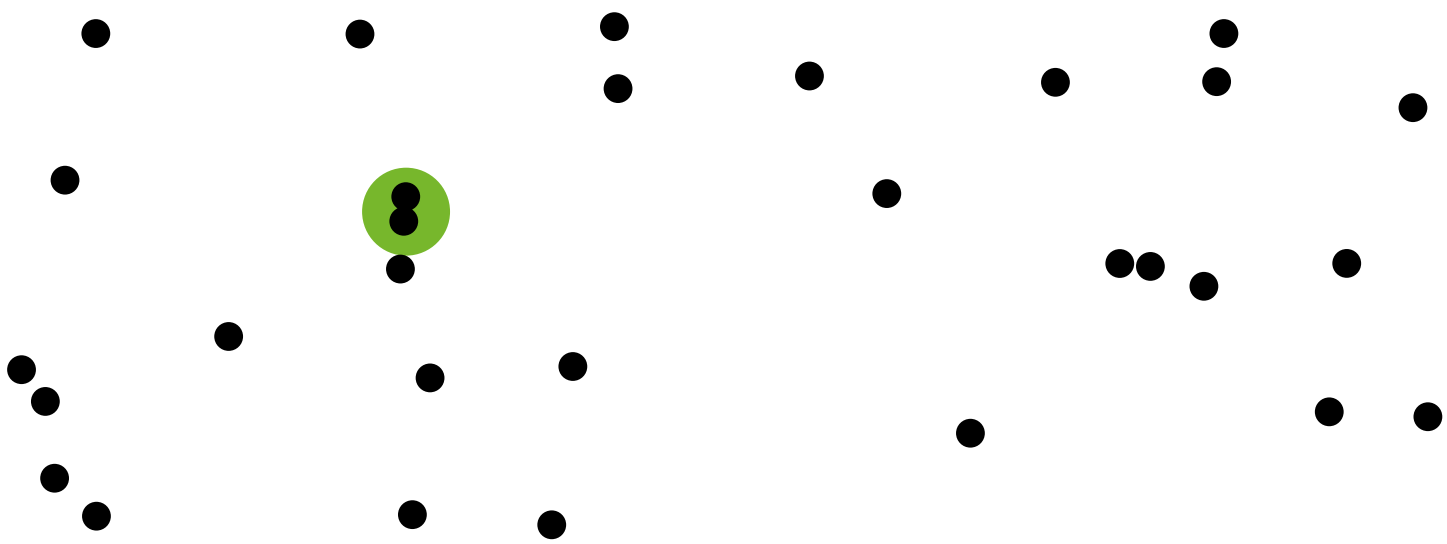

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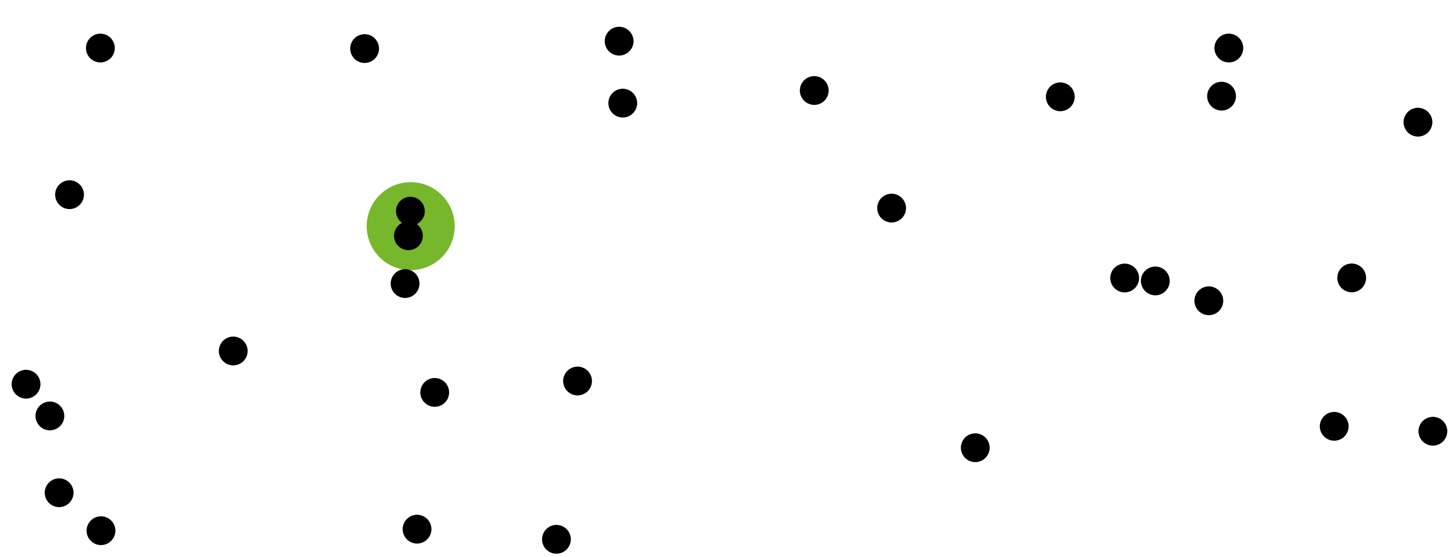

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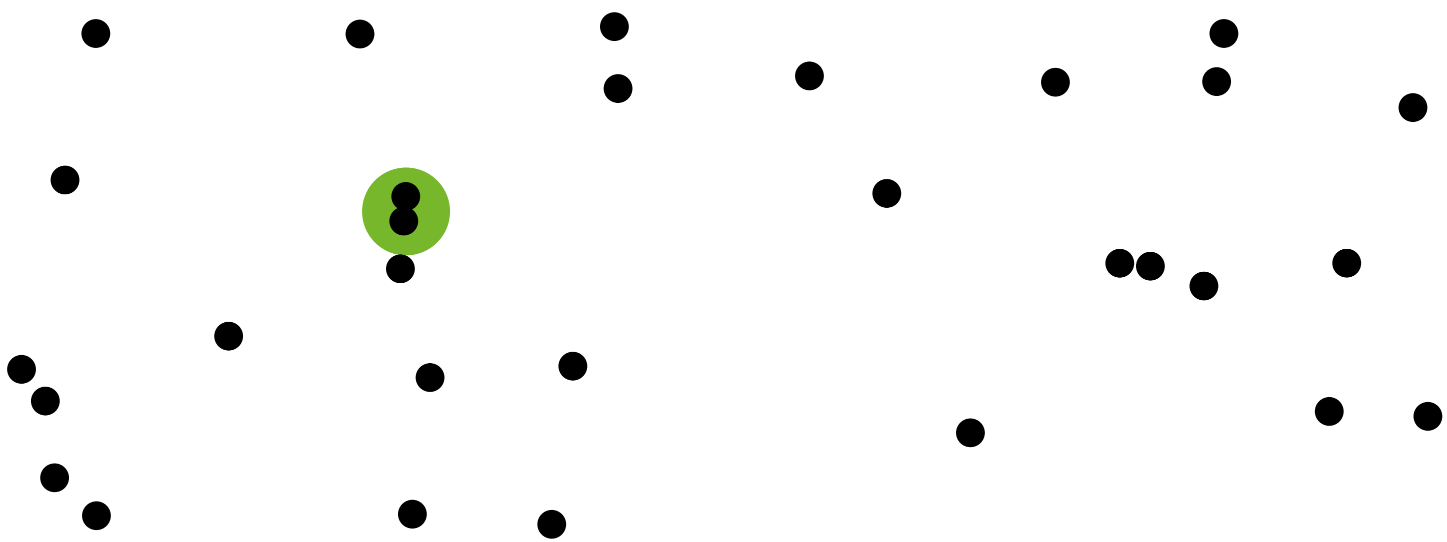
Exercise: For $s > 2$ this actually yields the closest pair.



Discussion

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WSPD is useful whenever one can do without knowing all $\Theta(n^2)$ exact distances in a point set and approximate them instead. One example are force-based layout algorithms in graph drawing, where pairwise repulsive forces of n points need to be calculated.

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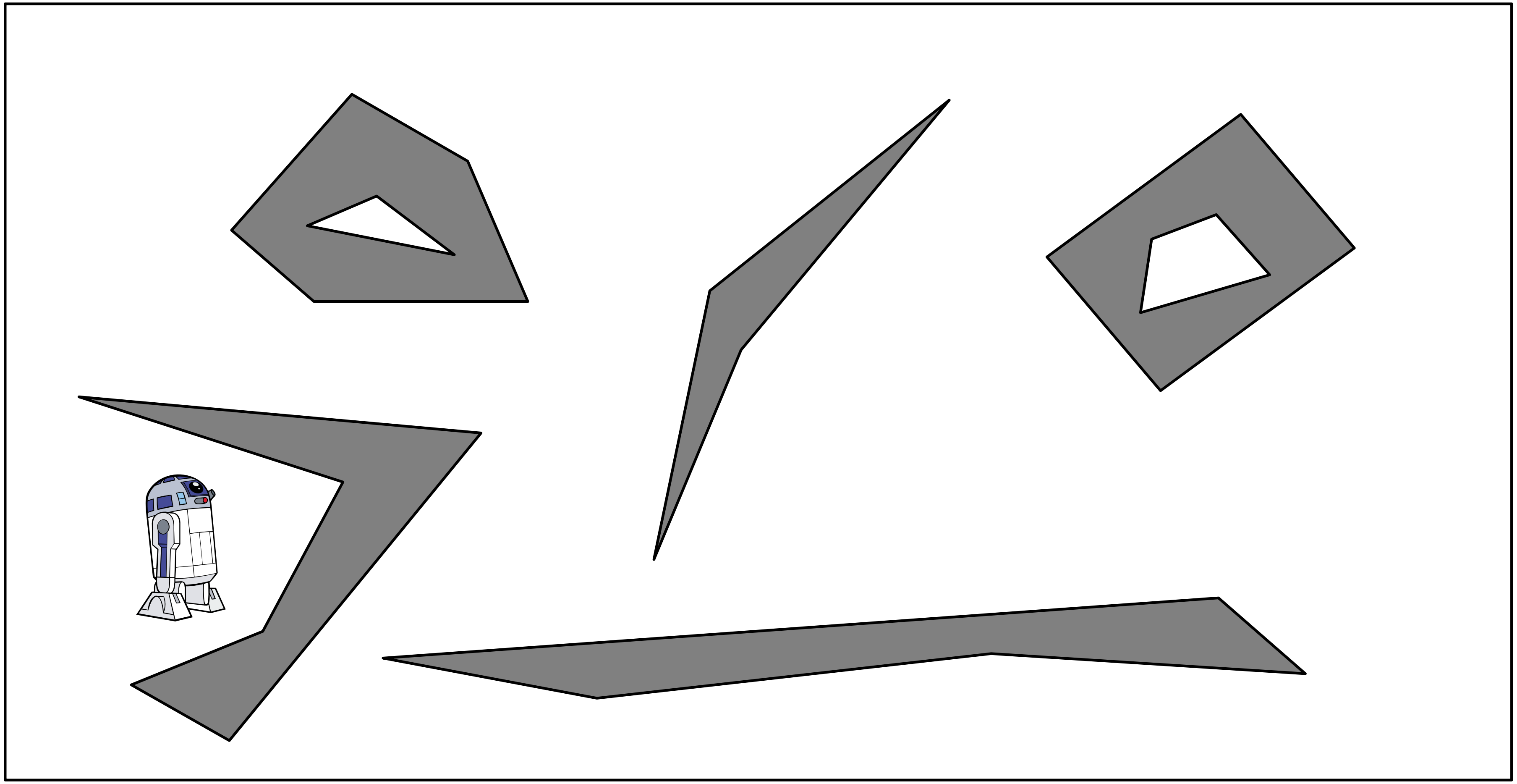
Can we achieve the same time bounds with exact computations?

In $\mathbb{R}^2$ this is often true, but not in $\mathbb{R}^d$ for $d > 2$. (e.g. EMST, diameter)

Motion planning and Visibility Graphs

Robot Motion Planning



Problem: Given a (point) robot at position p_{start} in a area with polygonal obstacles, find a shortest path to p_{goal} avoiding obstacles.

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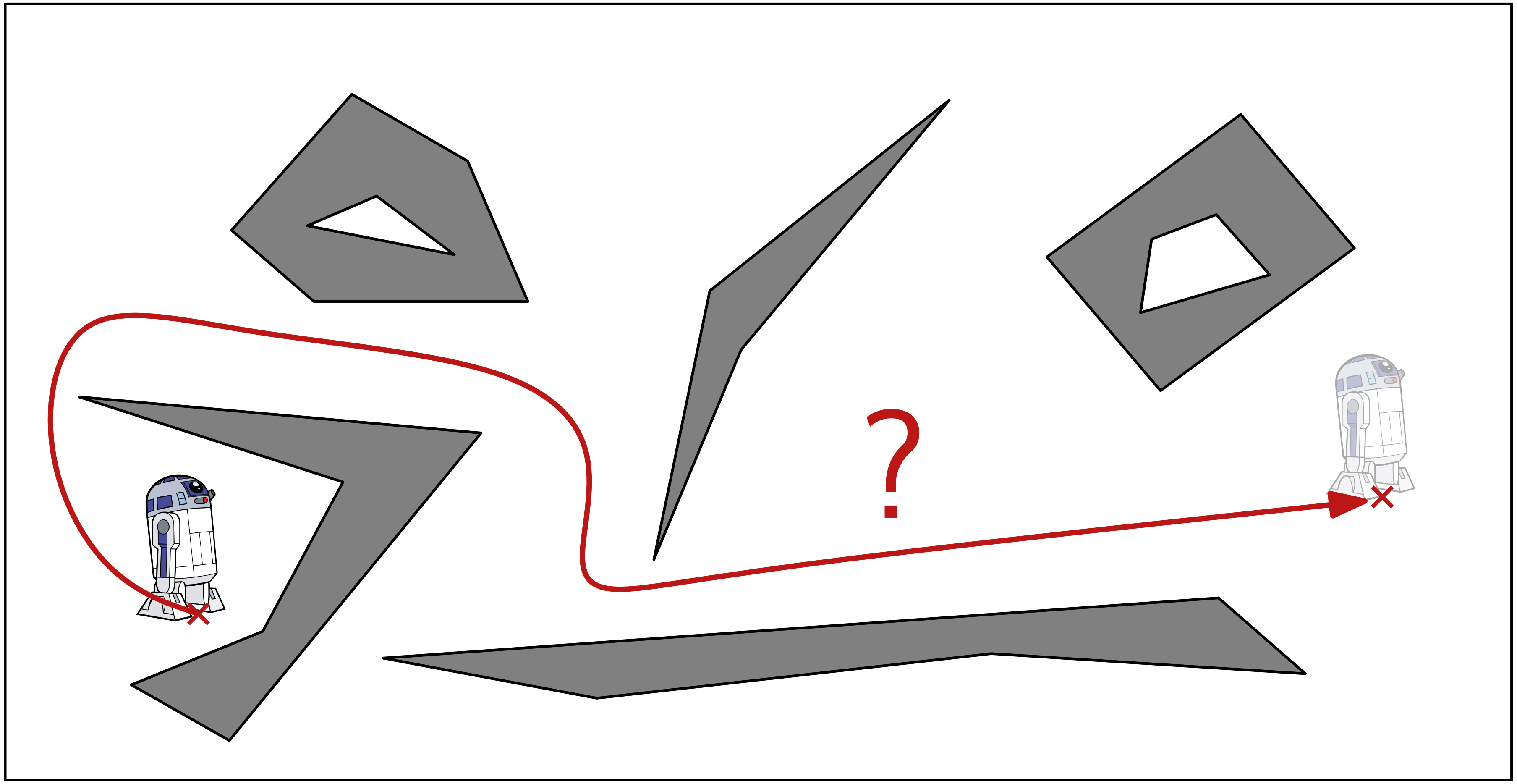

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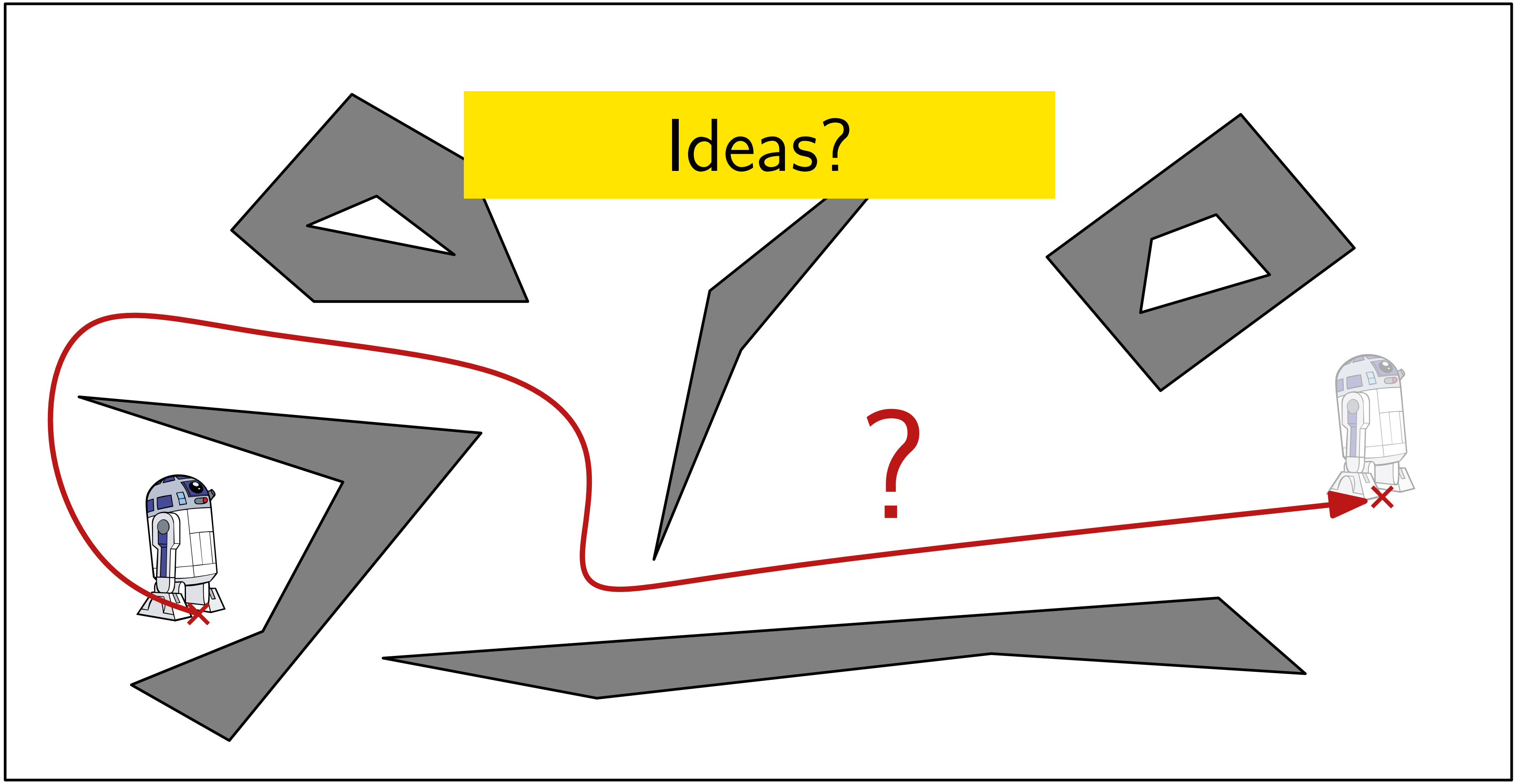

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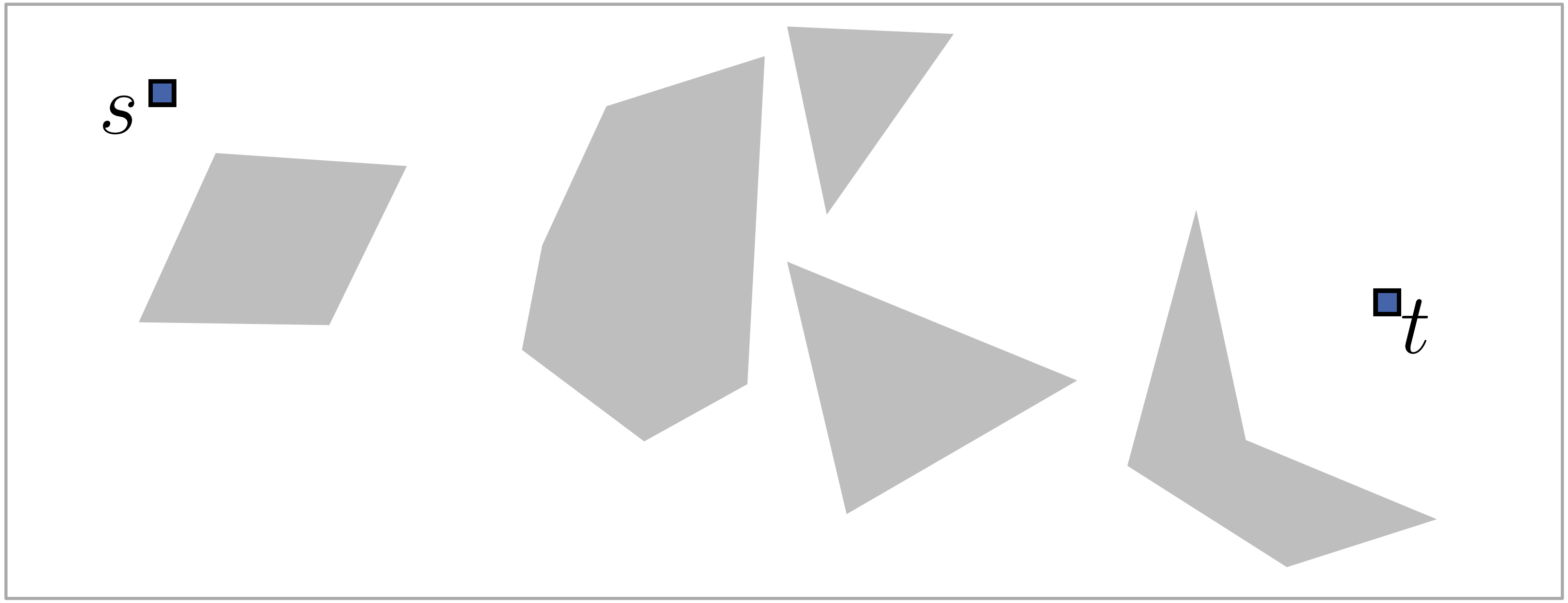
Shortest Paths in Polygonal Areas

Lemma 1: For a set S of disjoint open polygons in $\mathbb{R}^2$ and two points s and t not in S



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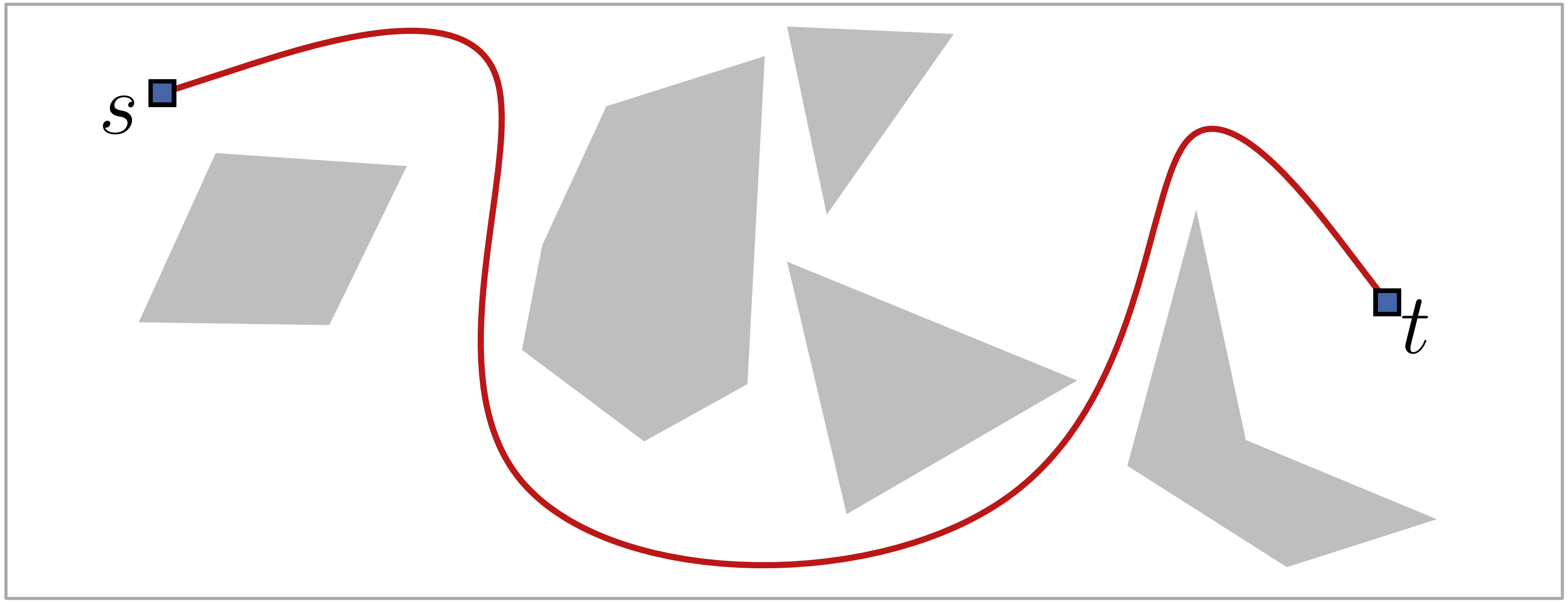

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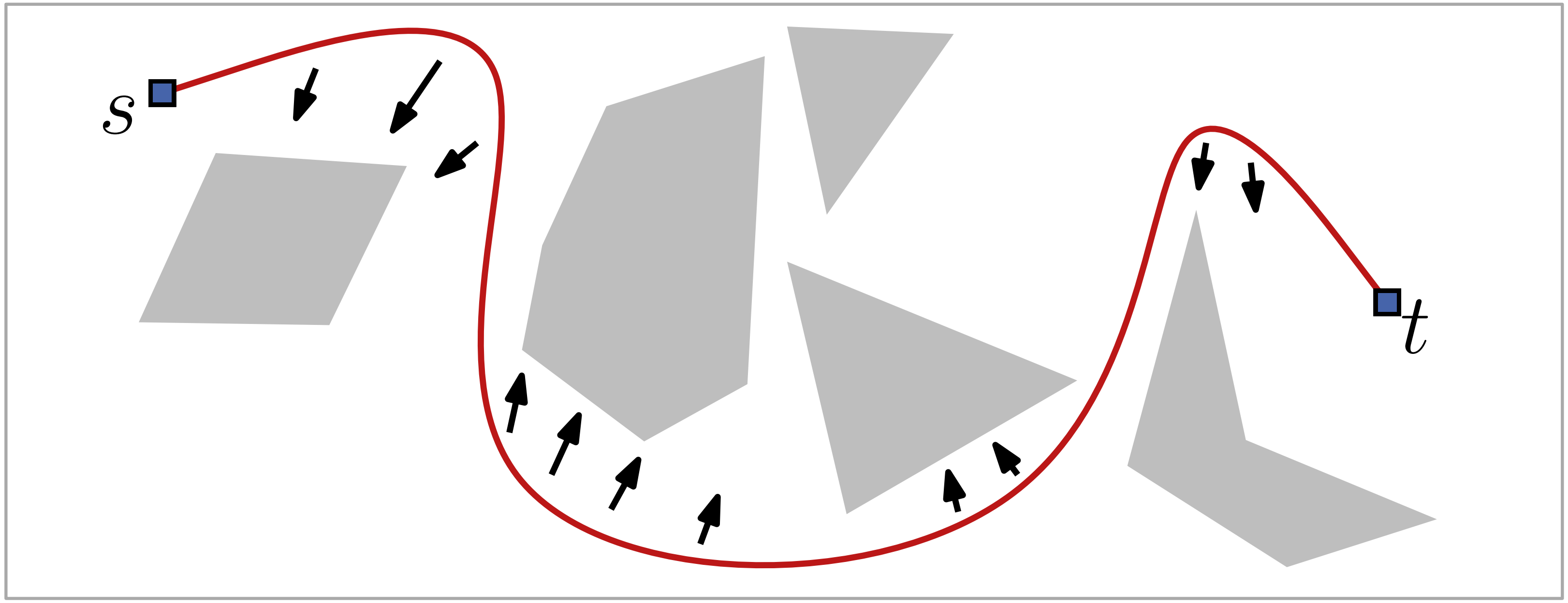

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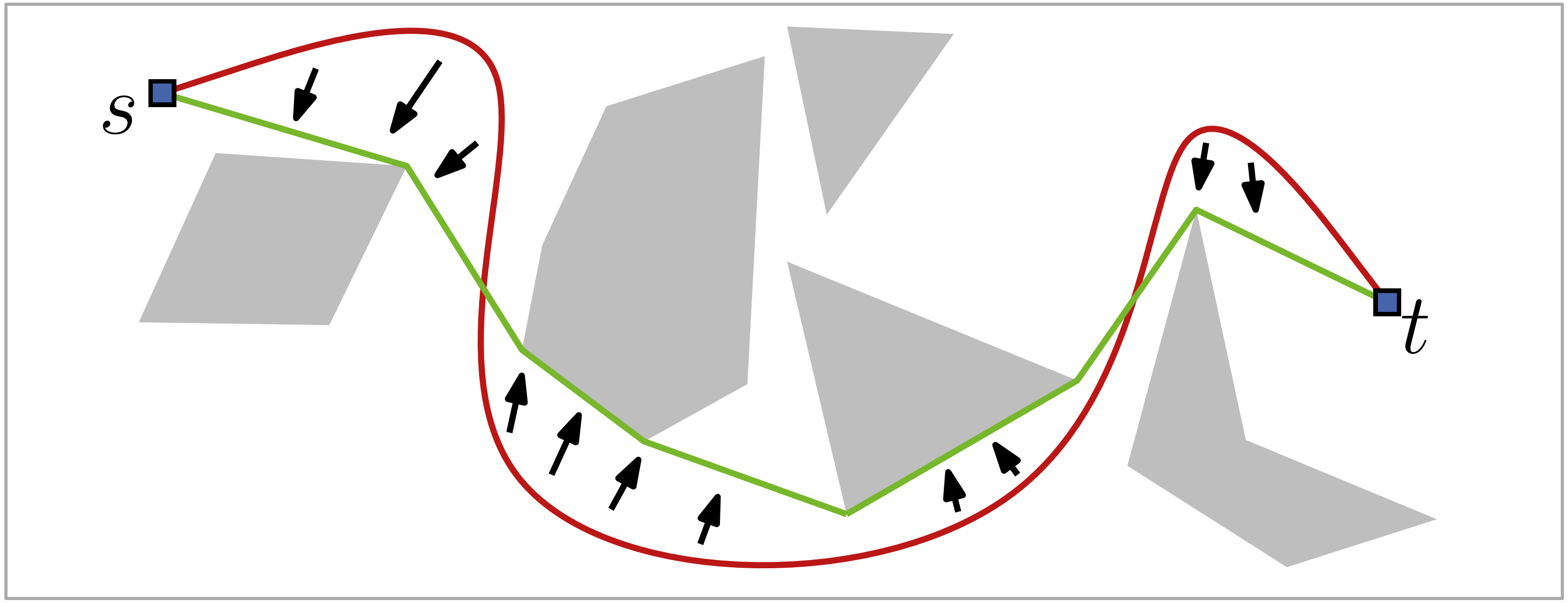

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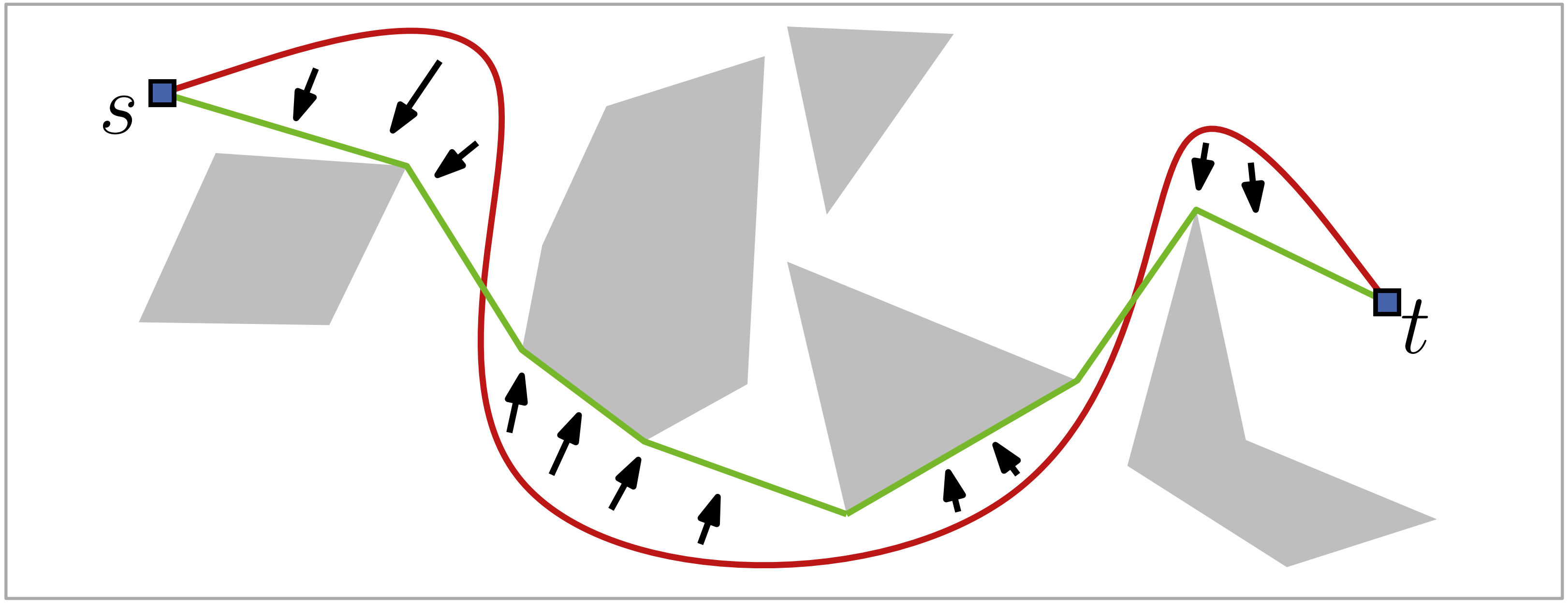

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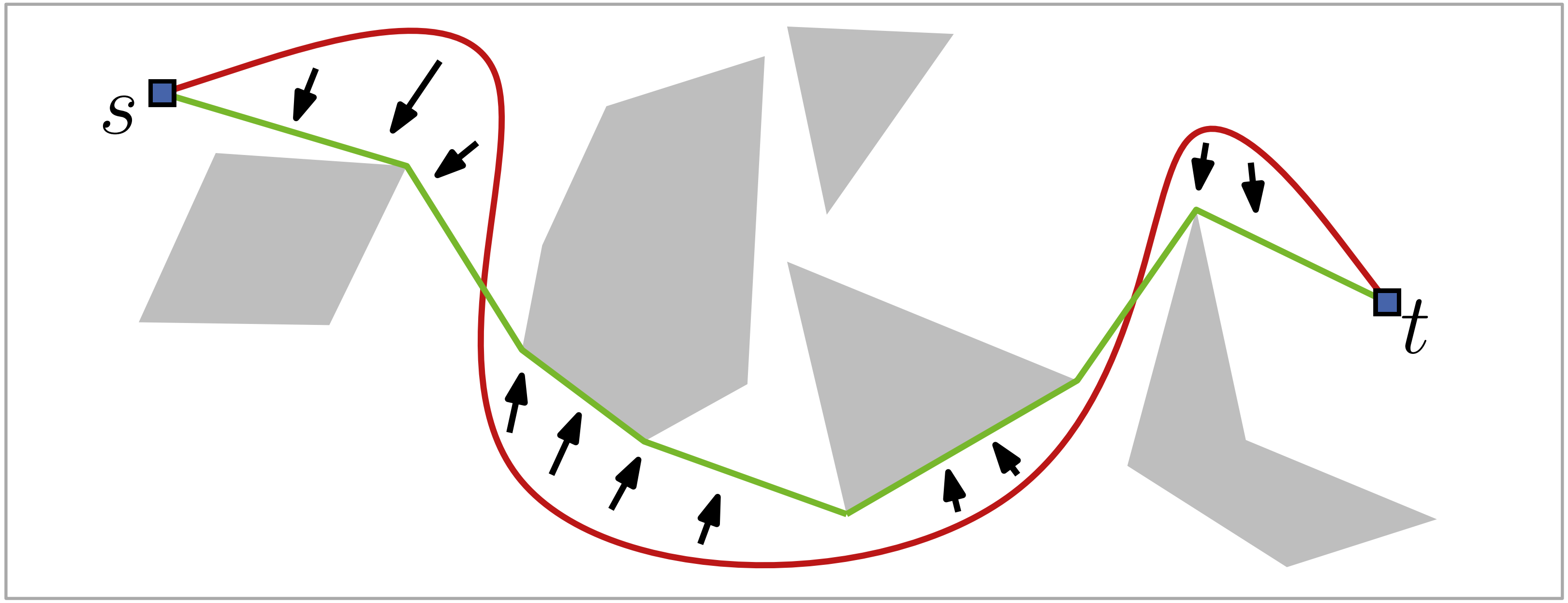

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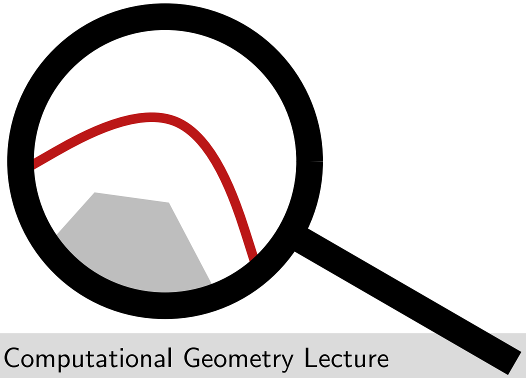

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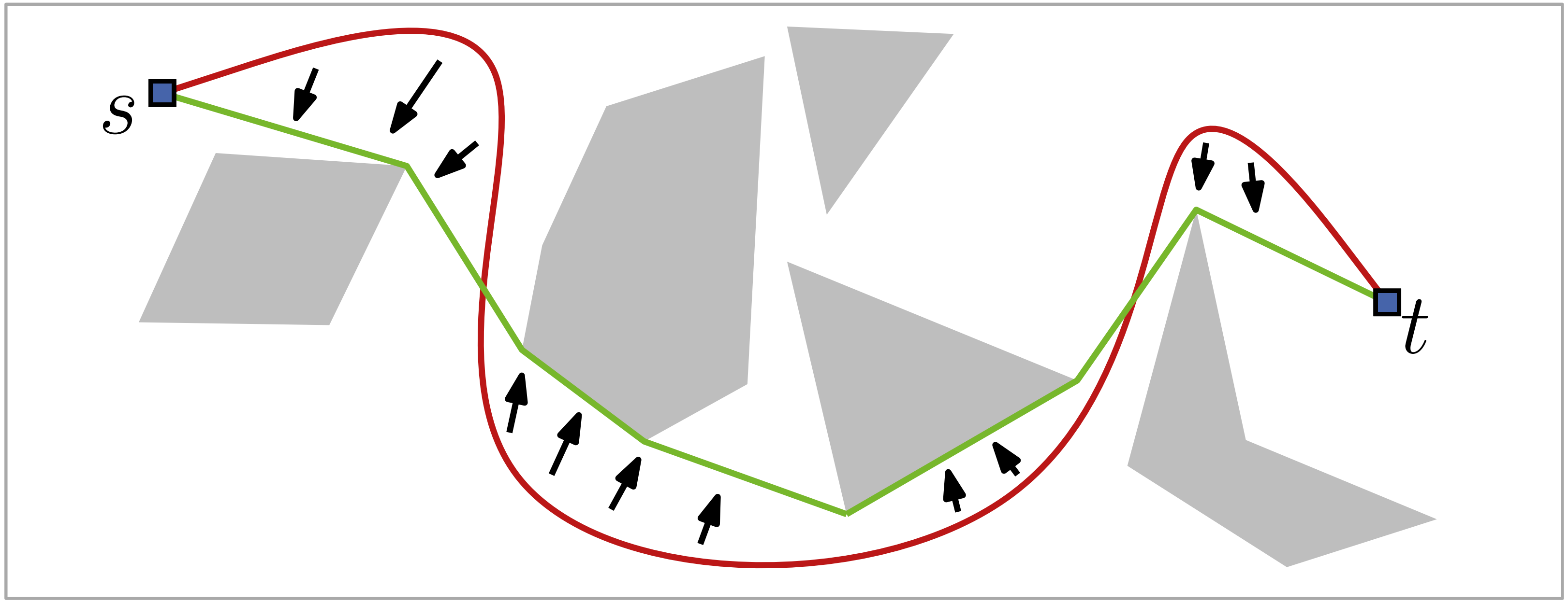

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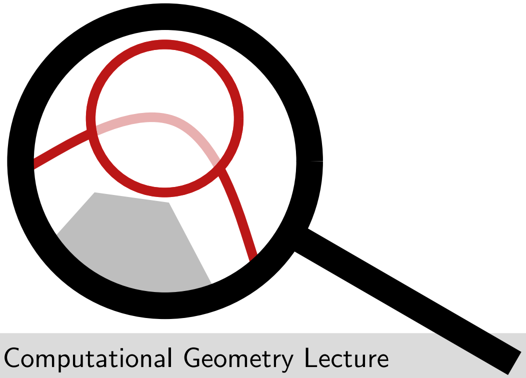

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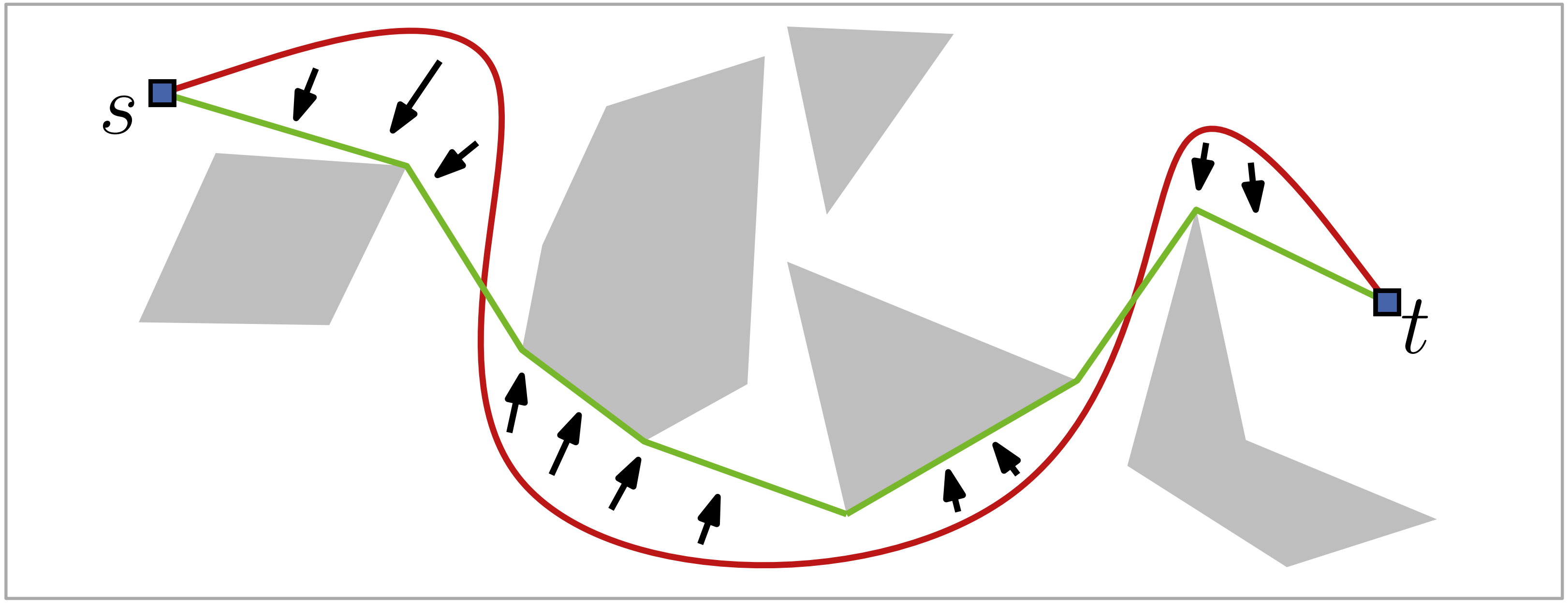

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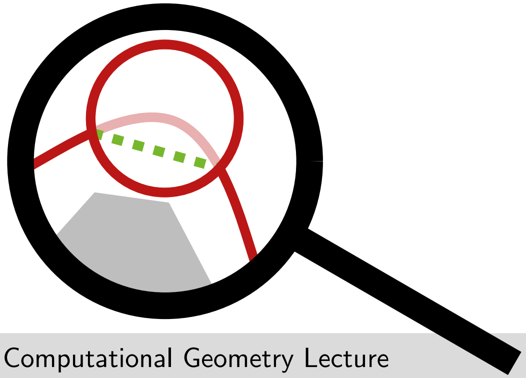

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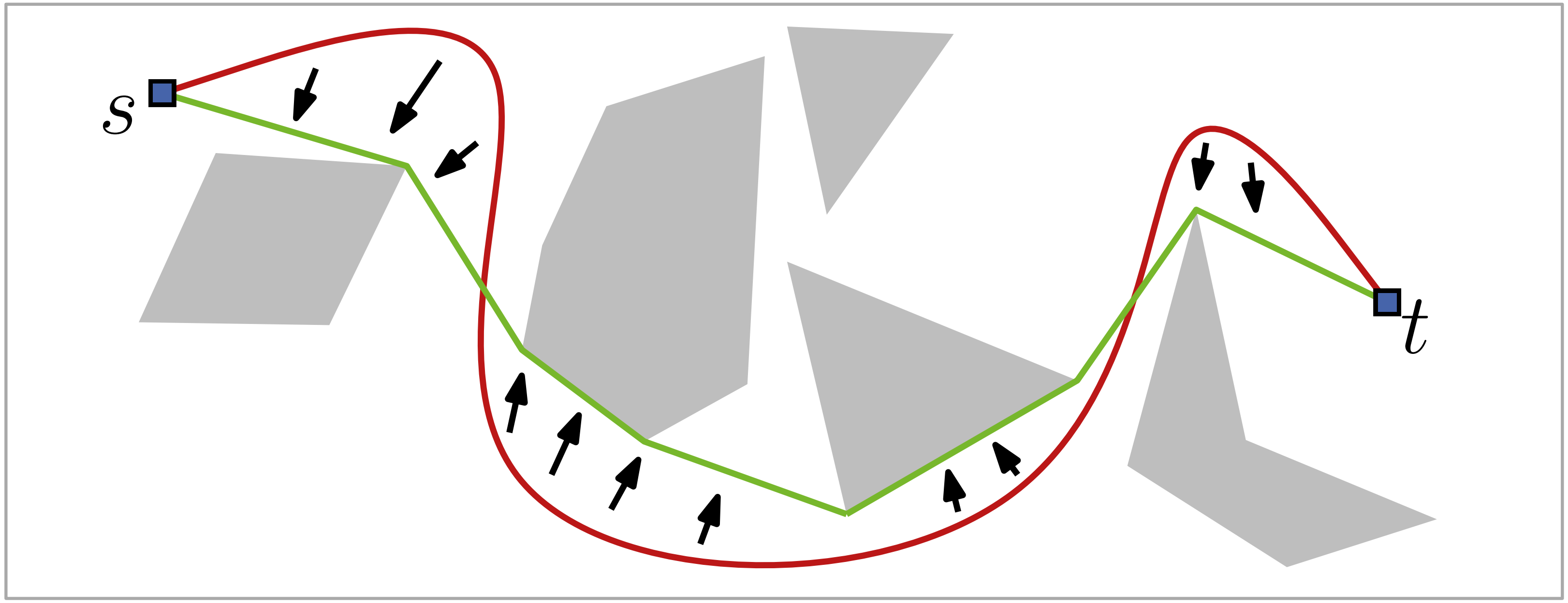

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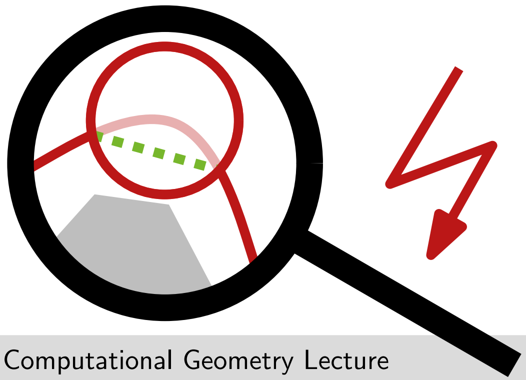

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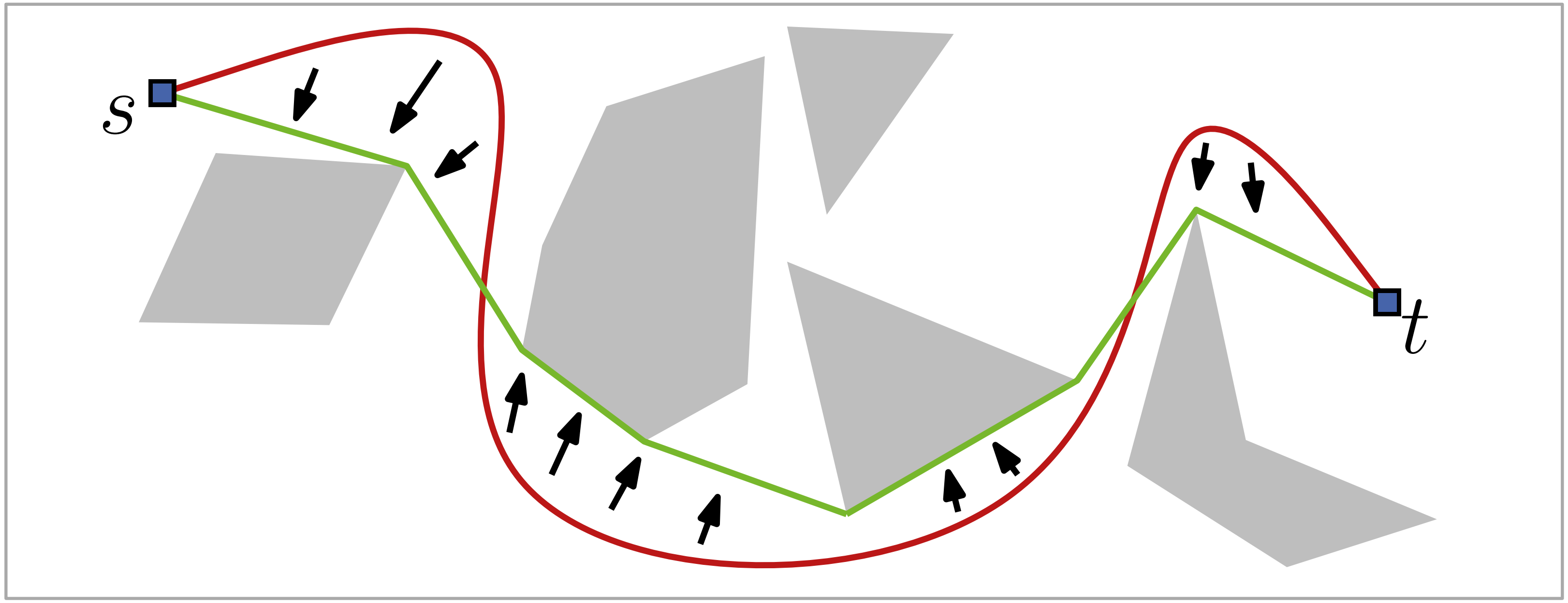

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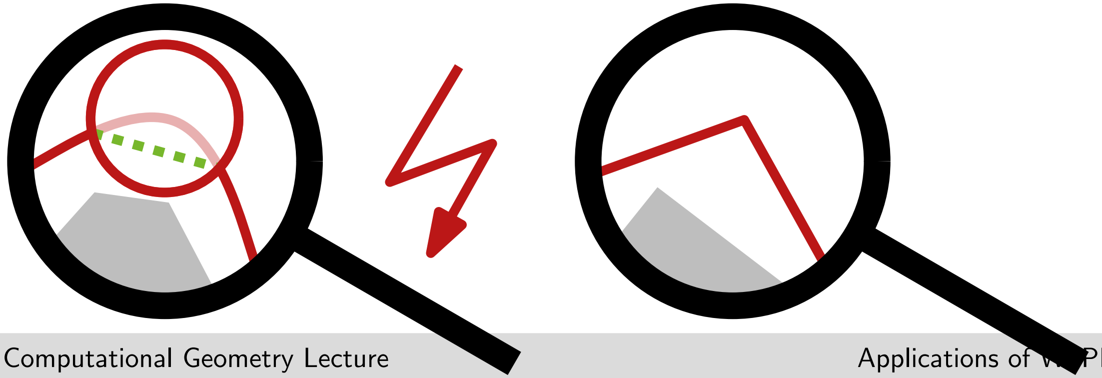

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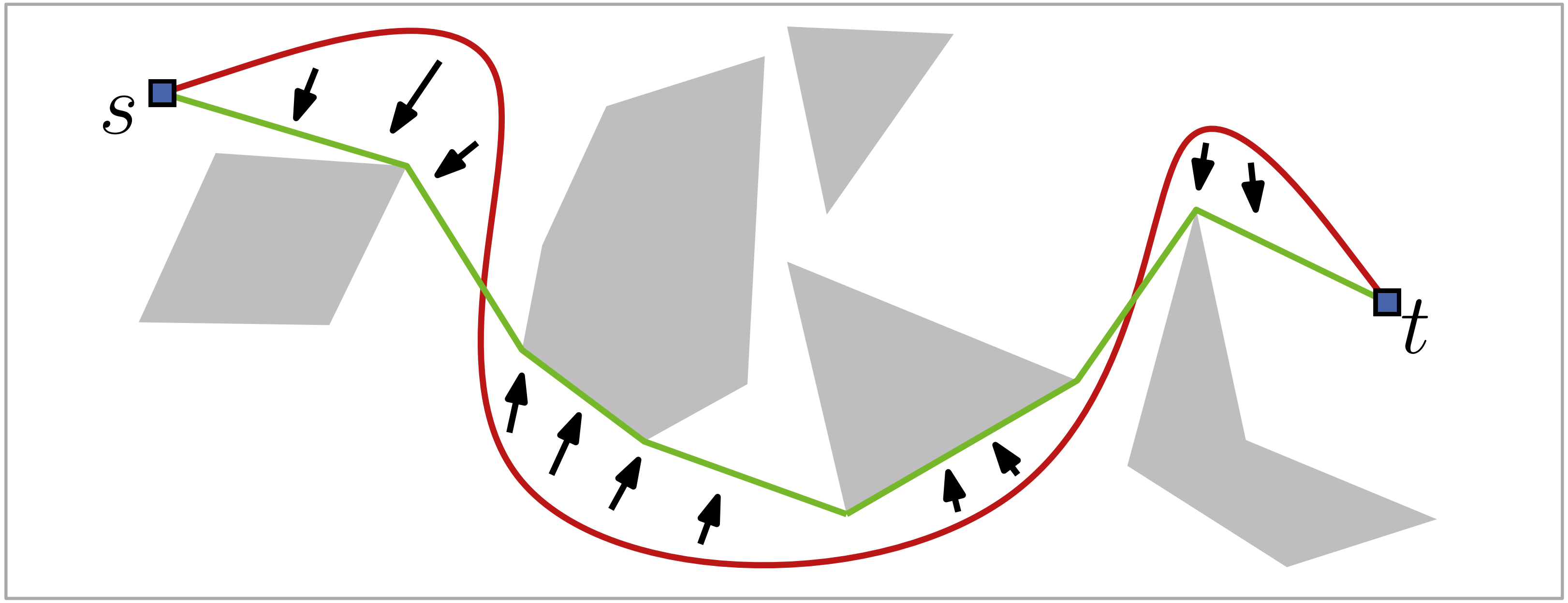

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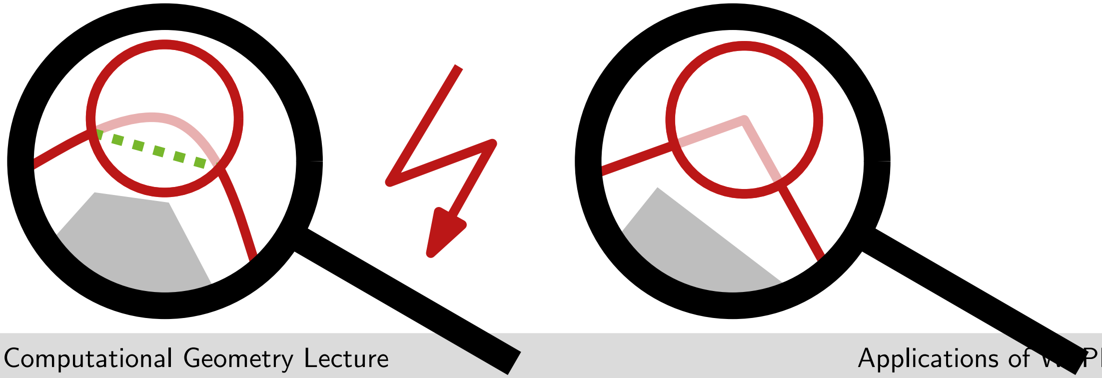

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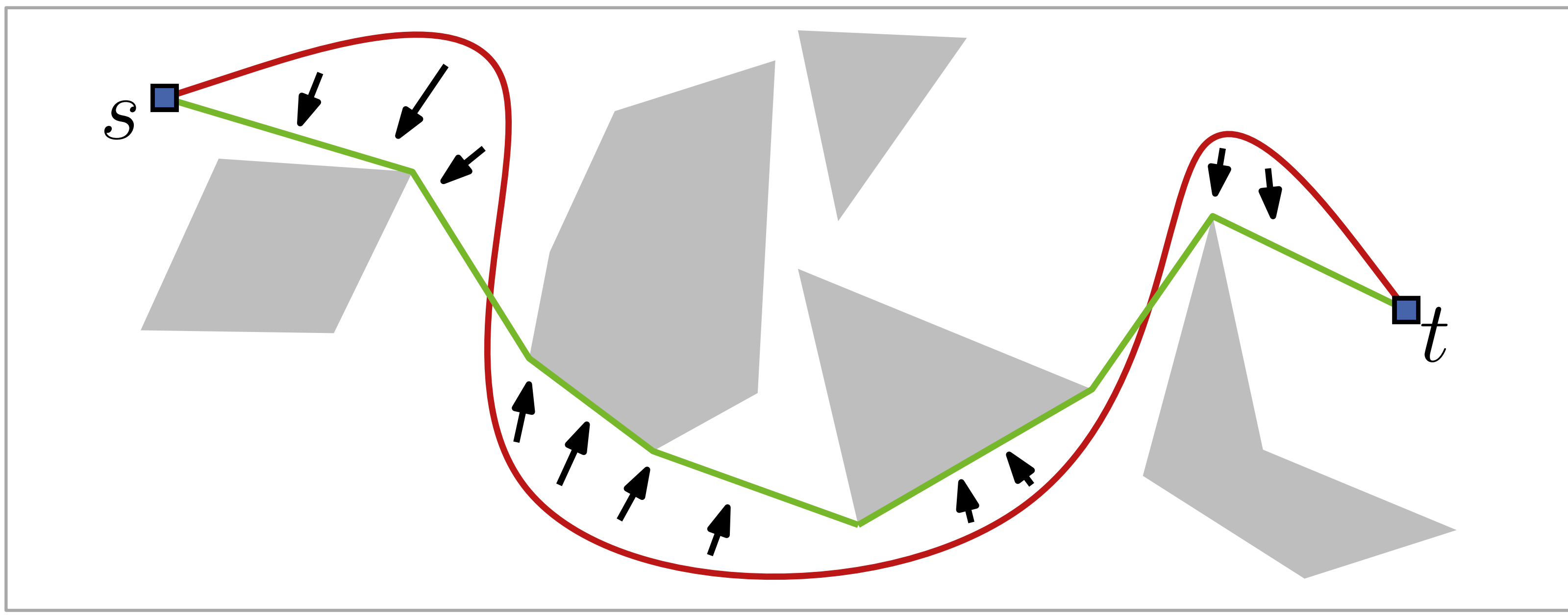

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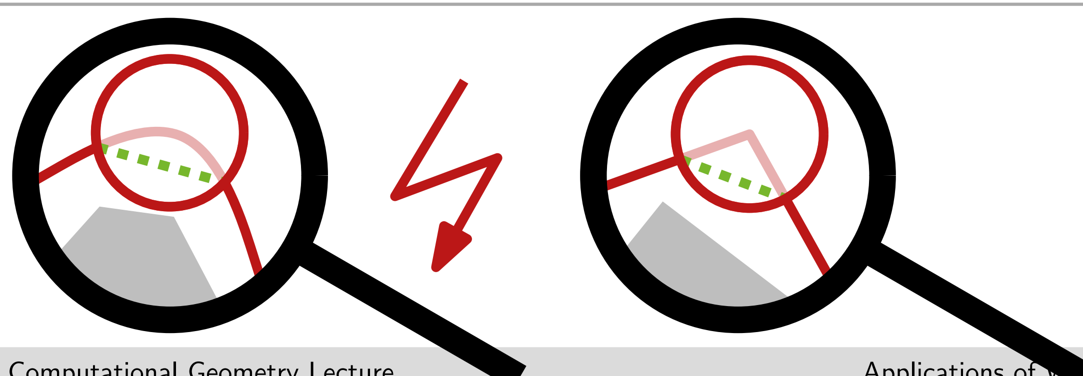

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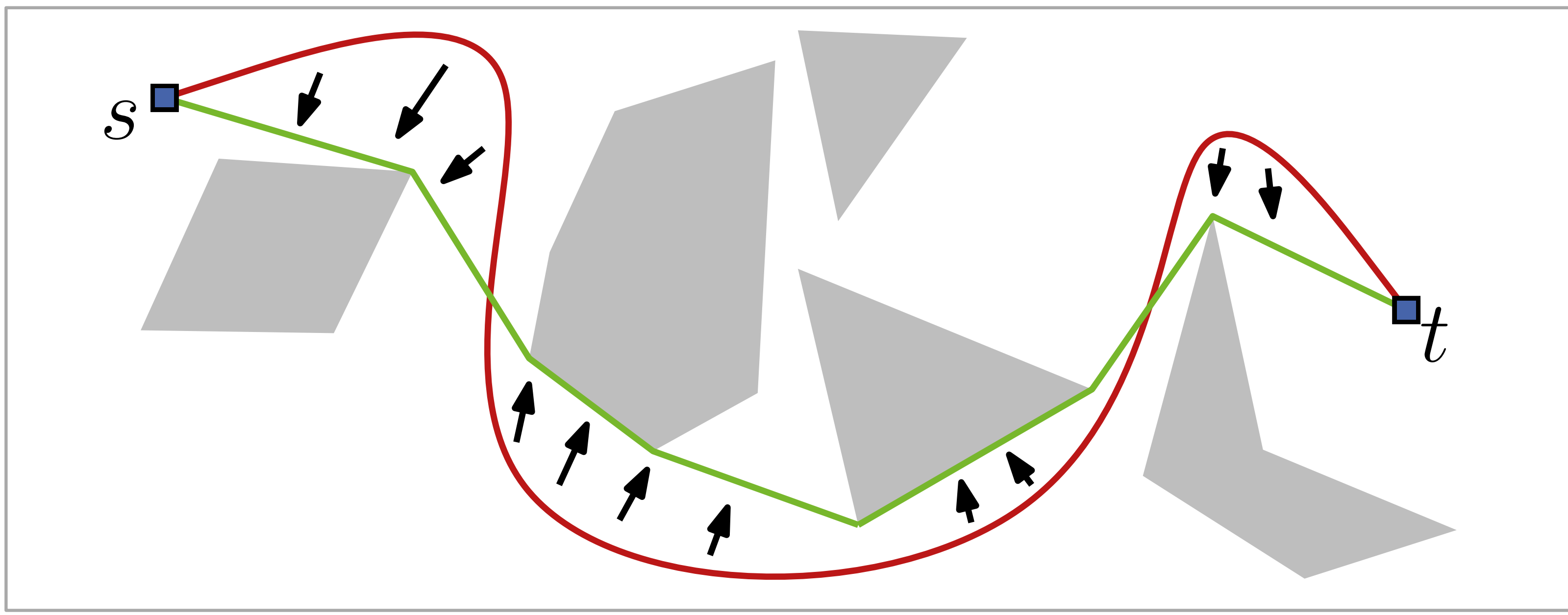

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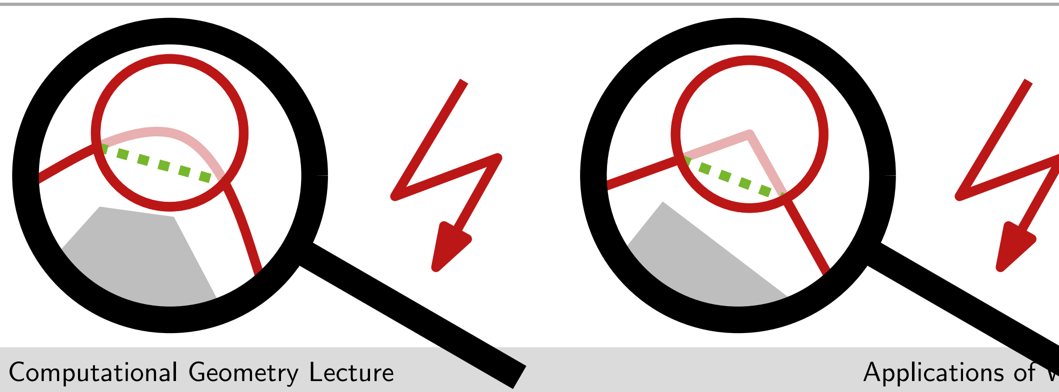

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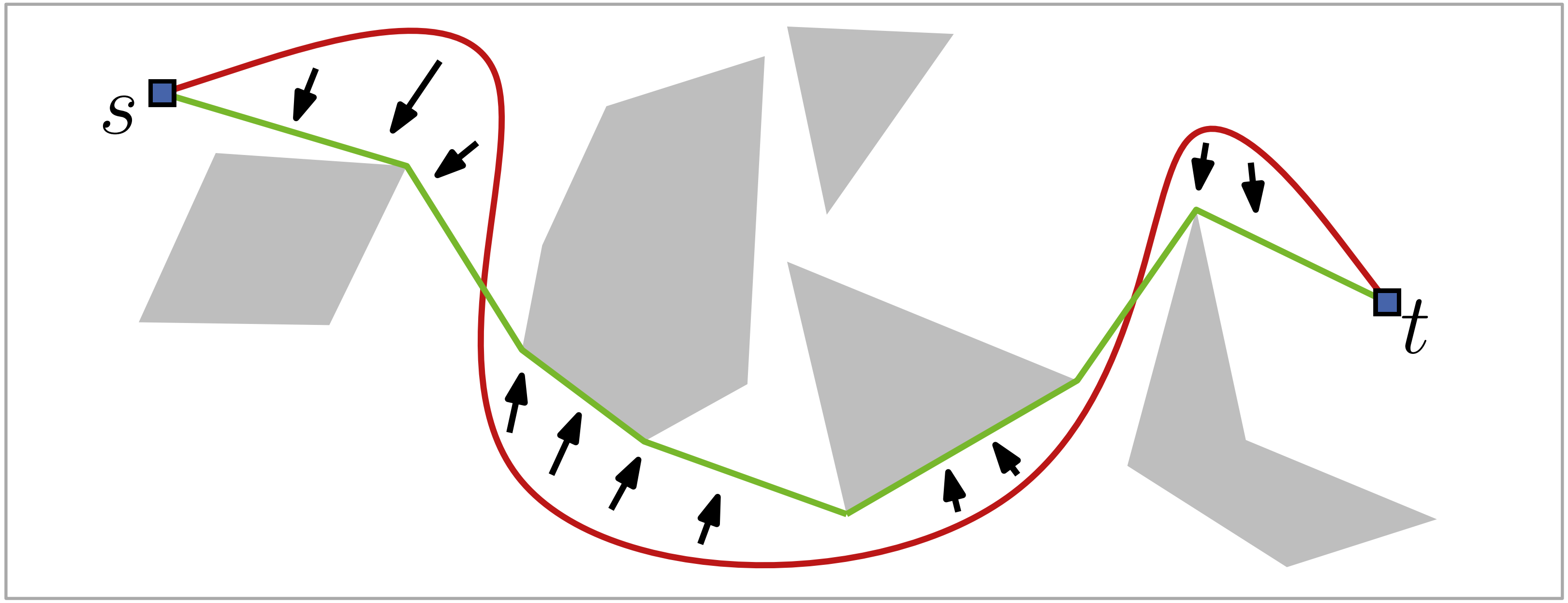

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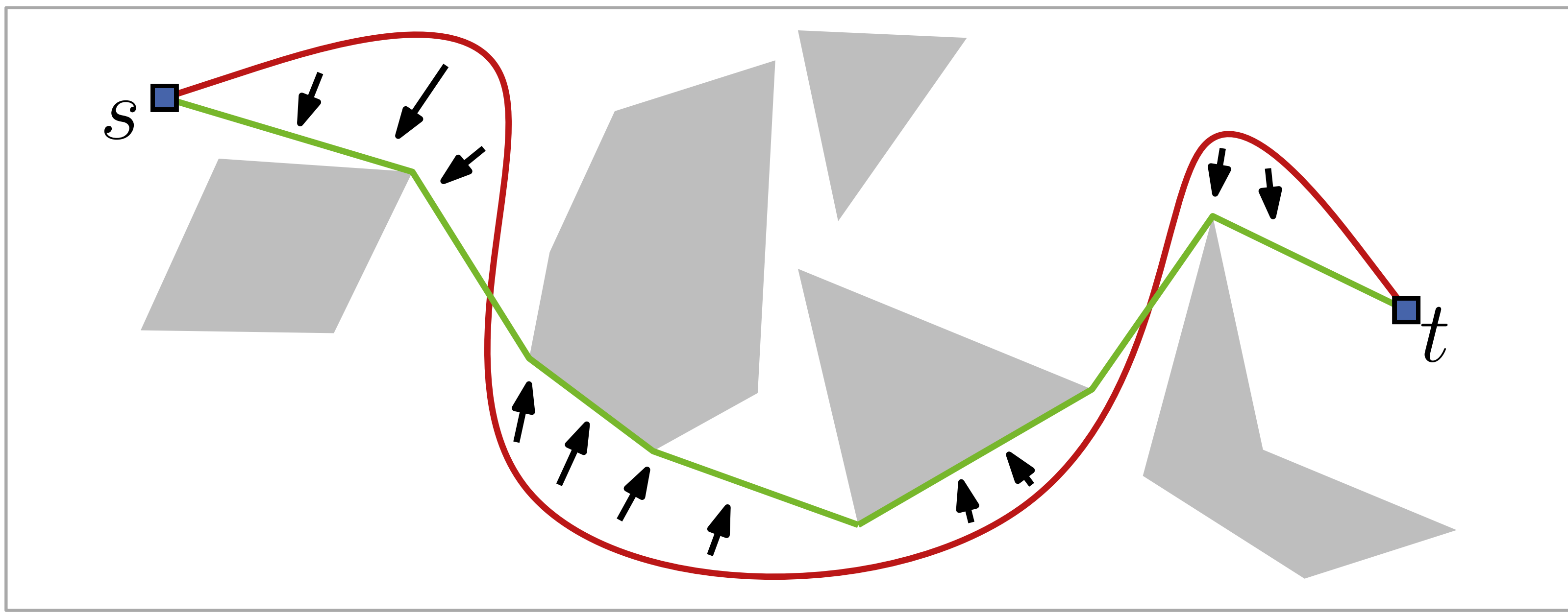

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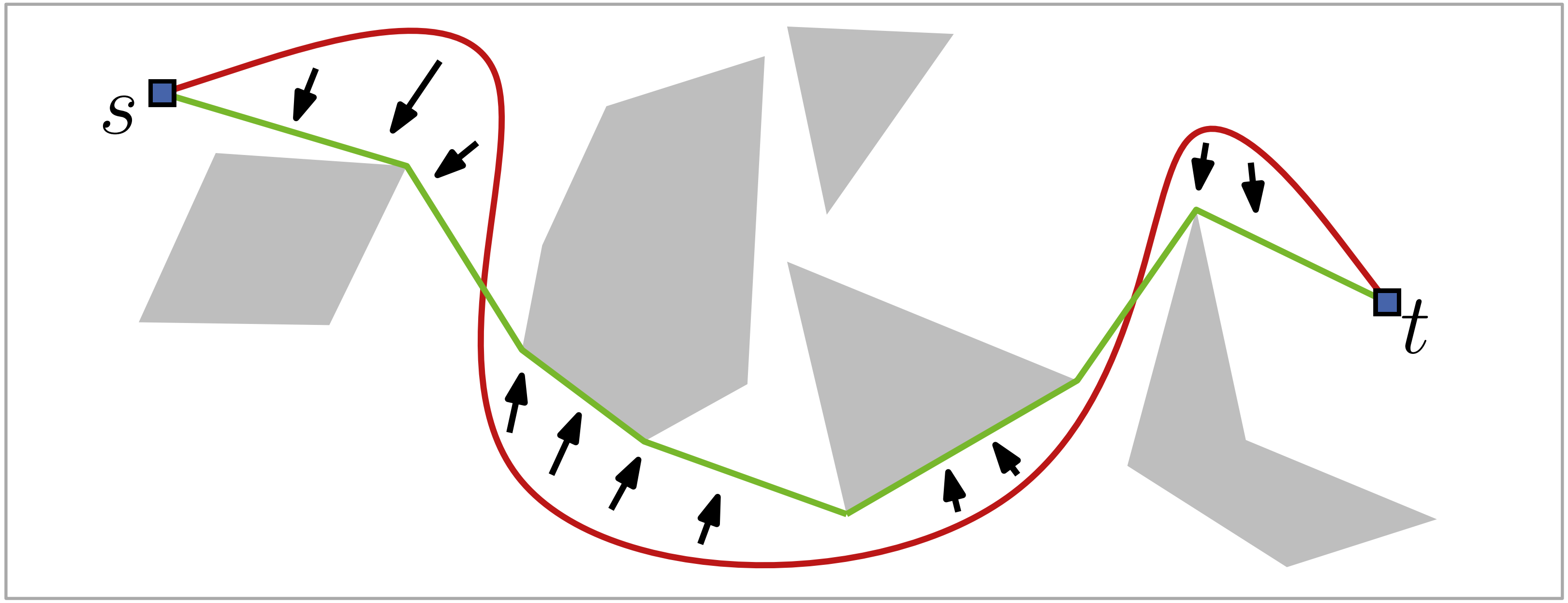

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Visibility Graph

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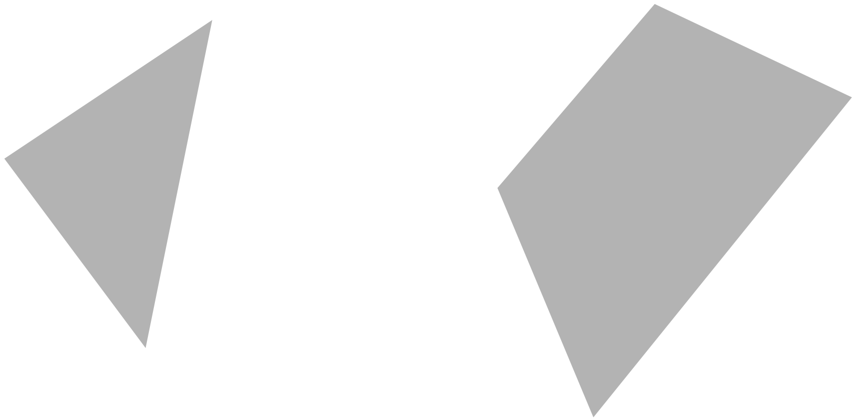

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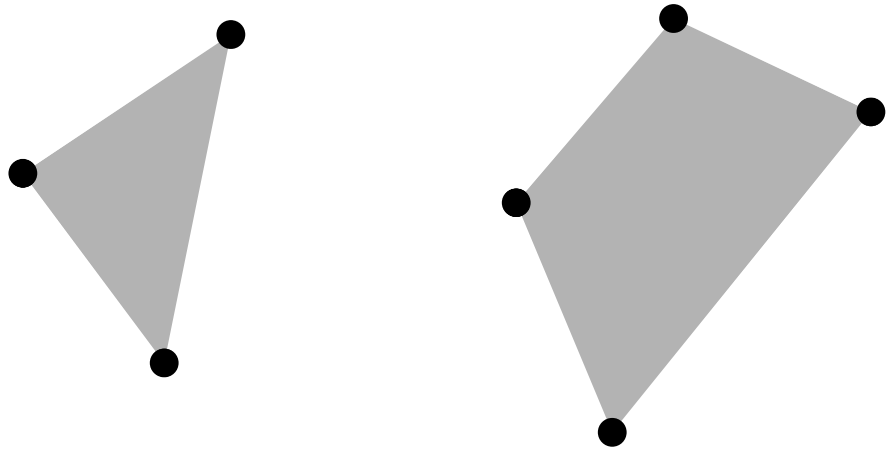

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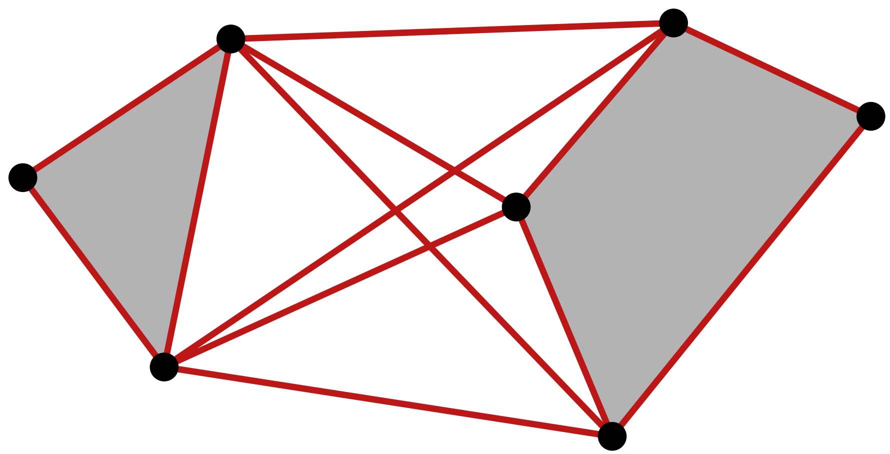

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Def.: Then $G_{\text{vis}}(S) = (V(S), E_{\text{vis}}(S))$ is the **visibility graph** of S with $E_{\text{vis}}(S) = \{uv \mid u, v \in V(S) \text{ and } u \textit{ sees } v\}$ und $w(uv) = |uv|$.

Visibility Graph

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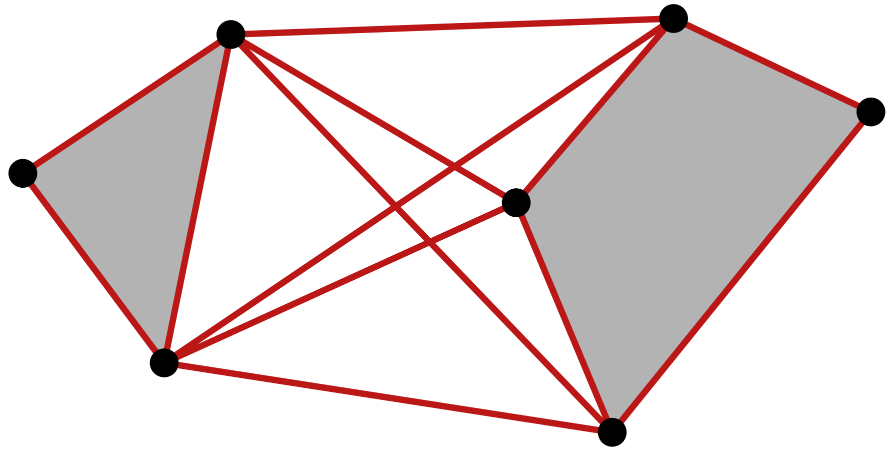

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Where u **sees** v $:\Leftrightarrow \overline{uv} \cap \bigcup S = \emptyset$

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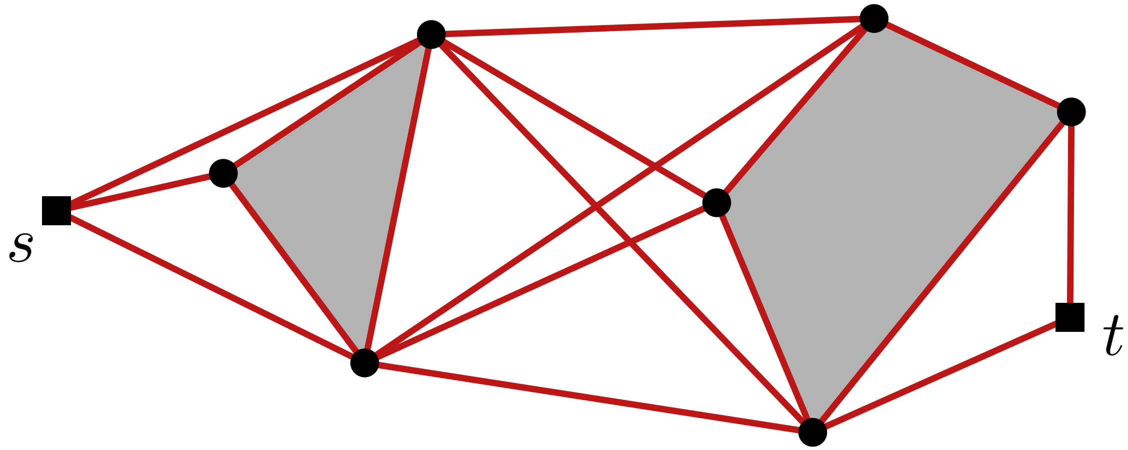


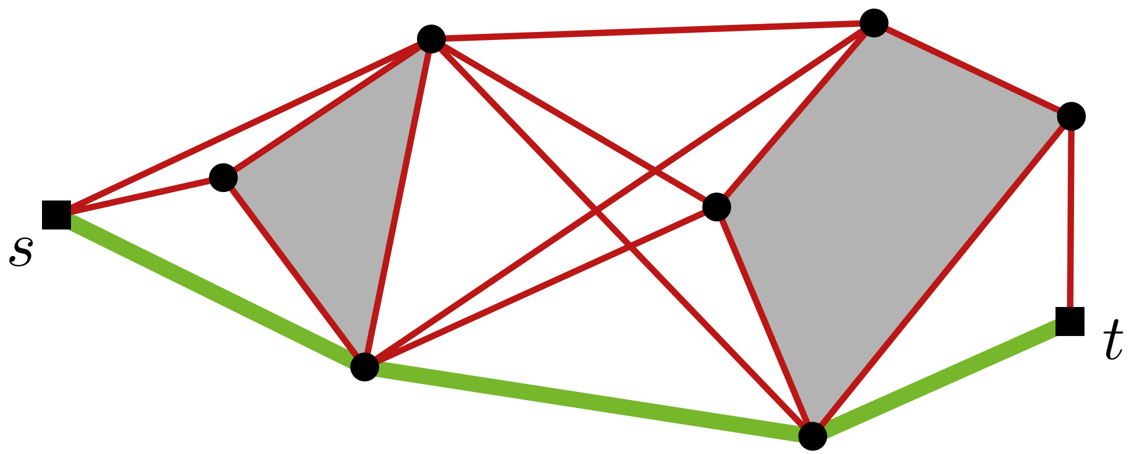
...with point set $V(S)$.

Def.: Then $G_{\text{vis}}(S) = (V(S), E_{\text{vis}}(S))$ is the **visibility graph** of S with $E_{\text{vis}}(S) = \{uv \mid u, v \in V(S)$ and u *sees* $v\}$ und $w(uv) = |uv|$.
Where u **sees** $v :\Leftrightarrow \overline{uv} \cap \bigcup S = \emptyset$

Define $S^\star = S \cup \{s, t\}$ and $G_{\text{vis}}(S^\star)$ analogously.

Visibility Graph

Given a set S of disjoint open polygons...



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Define $S^\star = S \cup \{s, t\}$ and $G_{\text{vis}}(S^\star)$ analogously.

Lemma 1
$\Rightarrow$

A shortest st-path in $\mathbb{R}^2$ avoiding obstacles in S is equivalent to a shortest st-path in $G_{\text{vis}}(S^\star)$.

Algorithm

ShortestPath(S, s, t)

Input: Obstacles S, points $s, t \in \mathbb{R}^2 \setminus \bigcup S$
Output: Shortest collision-free st-path in S

1 $G_{\mathsf{vis}} \leftarrow$ VisibilityGraph$(S \cup \{s, t\})$
2 **foreach** $uv \in E_{\mathsf{vis}}$ **do** $w(uv) \leftarrow |uv|$
3 **return** Dijkstra$(G_{\mathsf{vis}}, w, s, t)$

Algorithm

ShortestPath(S, s, t) $n = |V(S)|, m = |E_{\text{vis}}(S)|$

Input: Obstacles S, points $s, t \in \mathbb{R}^2 \setminus \bigcup S$
Output: Shortest collision-free st-path in S

```
G_vis ← VisibilityGraph(S ∪ {s, t})                      ?
foreach uv ∈ E_vis do w(uv) ← |uv|                       O(m)
return Dijkstra(G_vis, w, s, t)                          O(n log n + m)
```

Algorithm

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ShortestPath(S, s, t) $\quad$ $n = |V(S)|, m = |E_{\text{vis}}(S)|$

Input: Obstacles S, points $s, t \in \mathbb{R}^2 \setminus \bigcup S$

Output: Shortest collision-free st-path in S

1 $G_{\text{vis}} \leftarrow$ VisibilityGraph$(S \cup \{s, t\})$ $\quad O(n^2 \log n)$

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$O(n^2 \log n)$

Thm 1: A shortest st-path in an area with polygonal obstacles with n edges can be computed in $O(n^2 \log n)$ time.

Computing a Visibility Graph

VisibilityGraph(S)

Input: Set of disjoint polygons S

Output: Visibility graph $G_{\mathsf{vis}}(S)$

```
E ← ∅
foreach v ∈ V(S) do
    W ← VisibleVertices(v, S)
    E ← E ∪ {vw | w ∈ W}
return (V(S), E)
```

$E \leftarrow \emptyset$

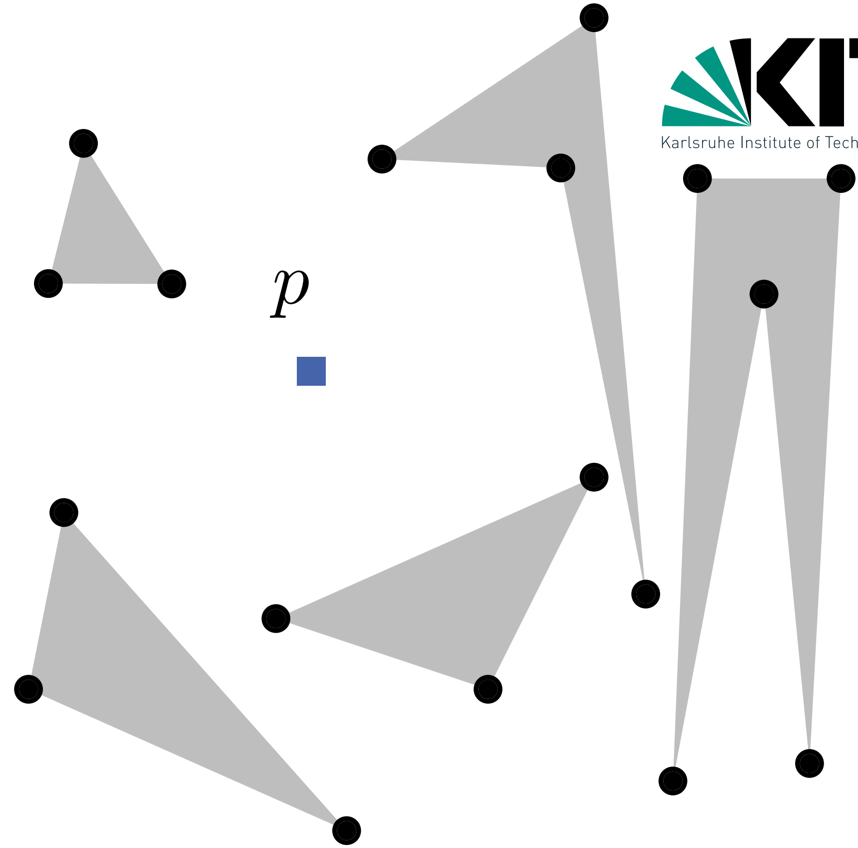
foreach $v \in V(S)$ **do**

- $W \leftarrow \text{VisibleVertices}(v, S)$
- $E \leftarrow E \cup \{vw \mid w \in W\}$

return $(V(S), E)$

Computing Visible Nodes

VisibleVertices(p, S)



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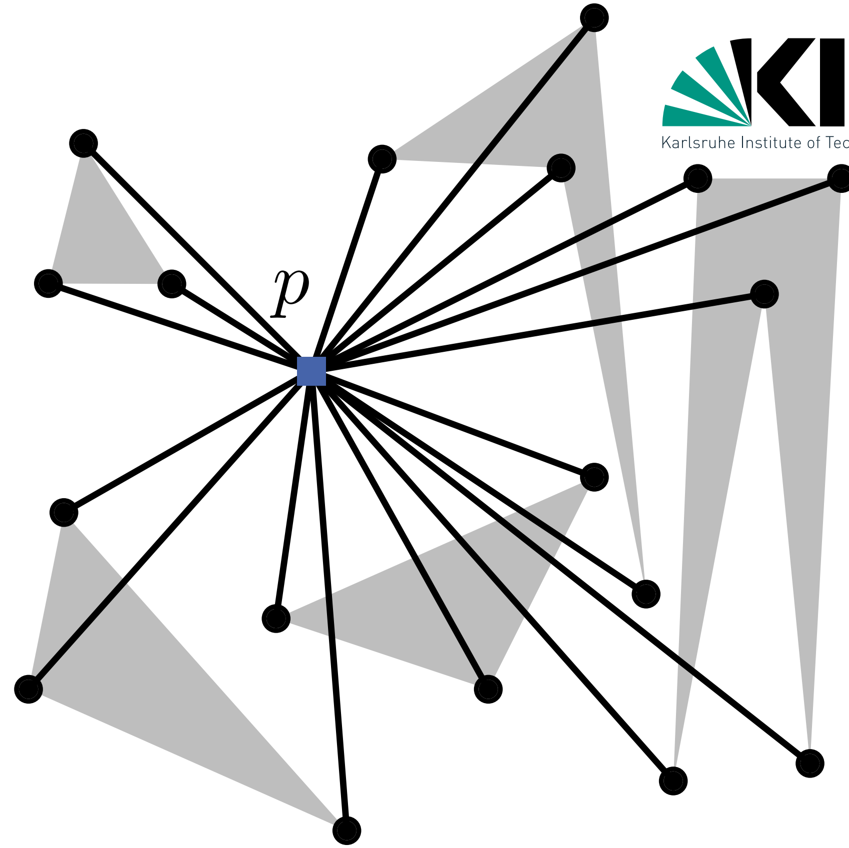

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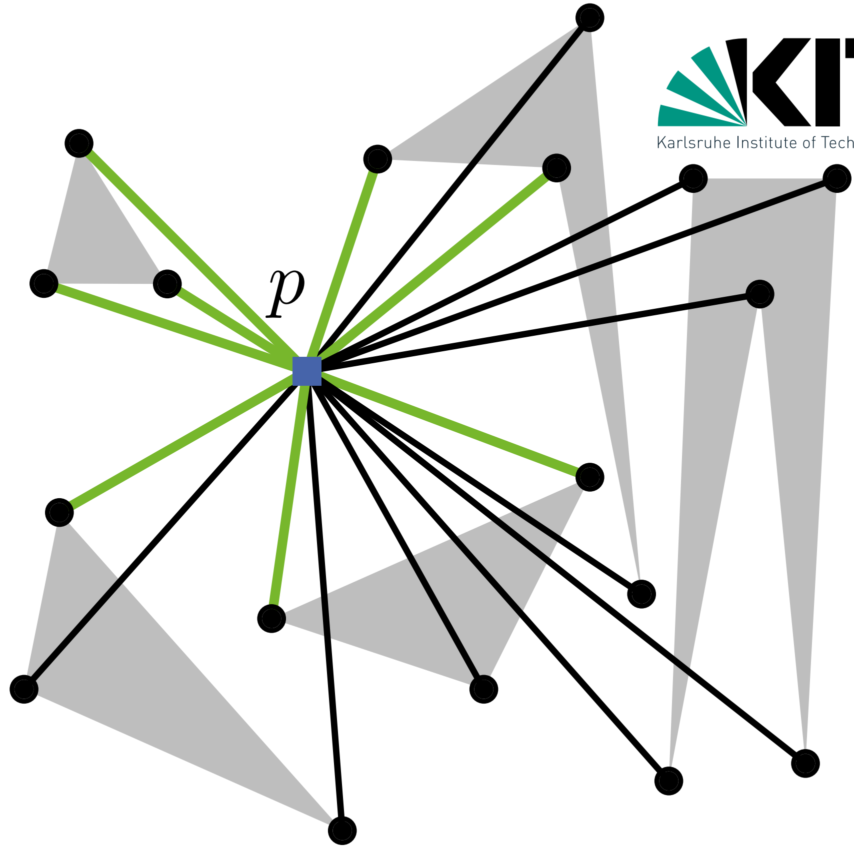
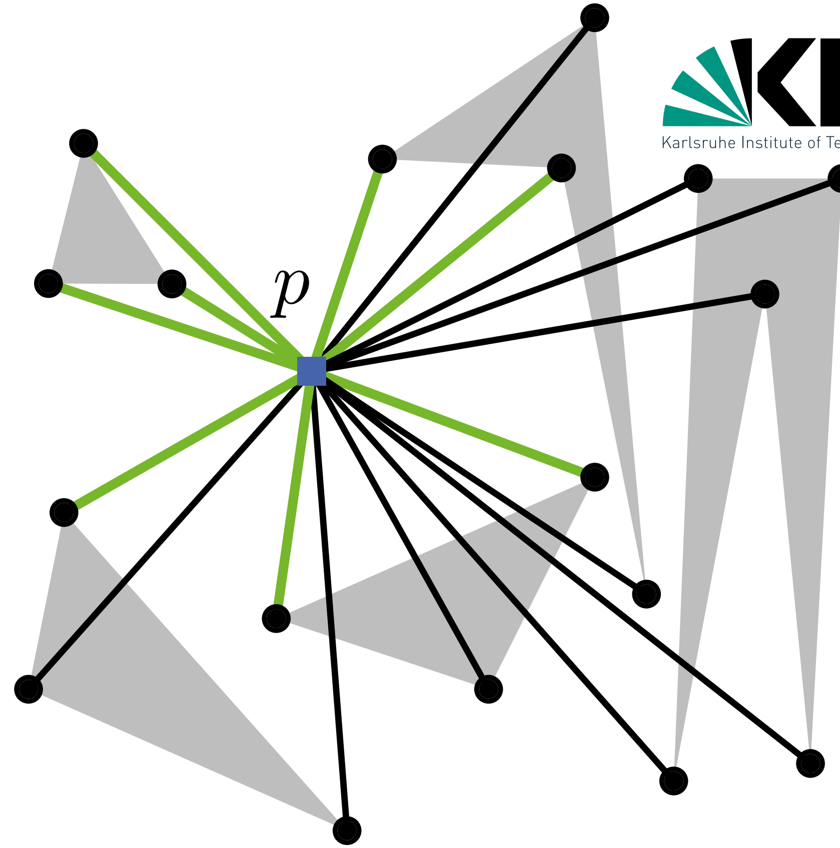


Computing Visible Nodes

VisibleVertices(p, S)

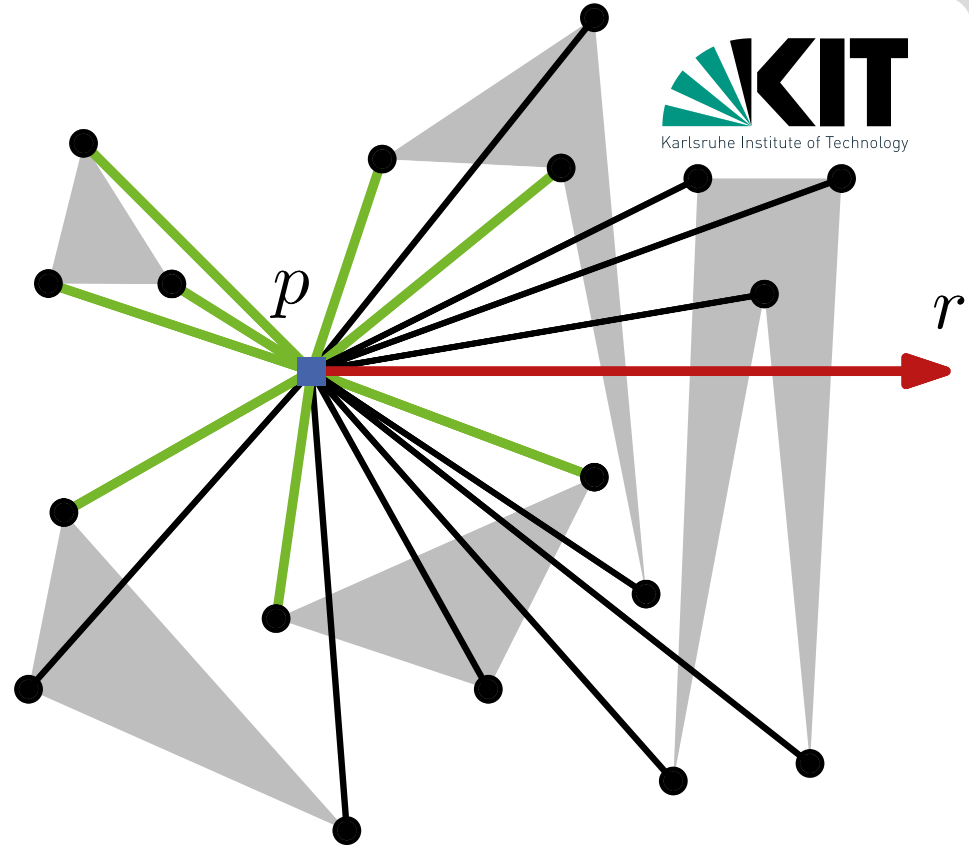
Problem: Given p and S, find in $O(n \log n)$ time all nodes that p sees in $V(S)$!



Computing Visible Nodes

VisibleVertices(p, S)

$r \leftarrow \{p + (k, 0) \mid k \in \mathbb{R}_0^+\}$



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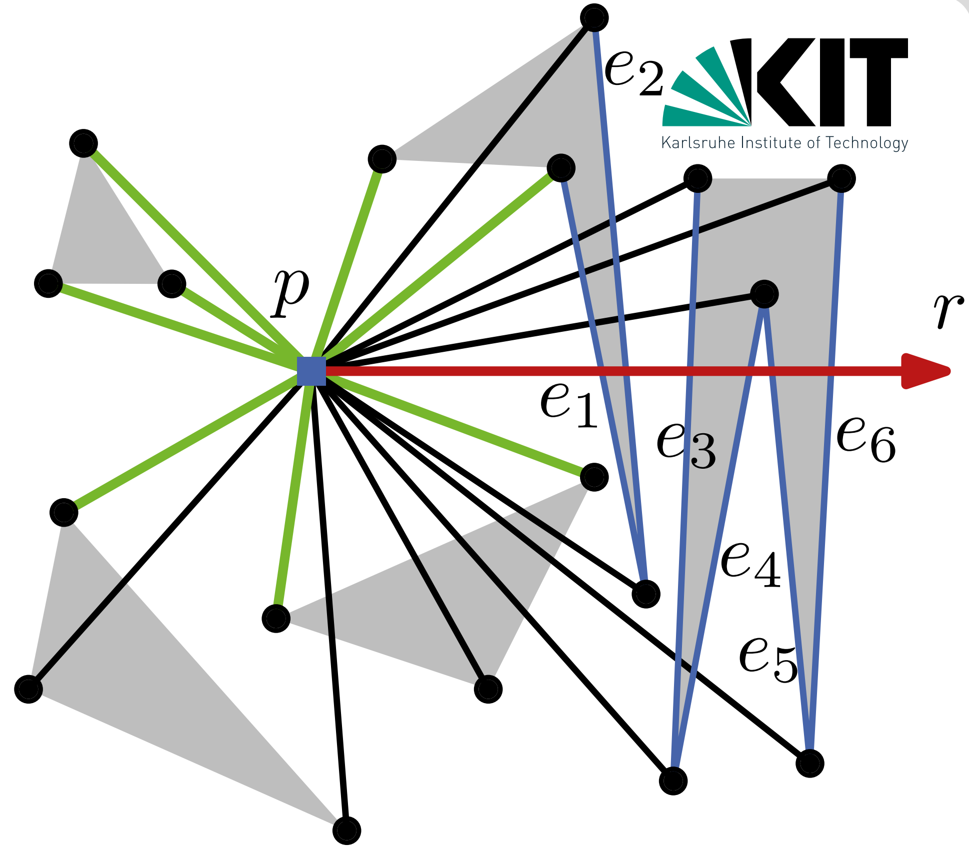

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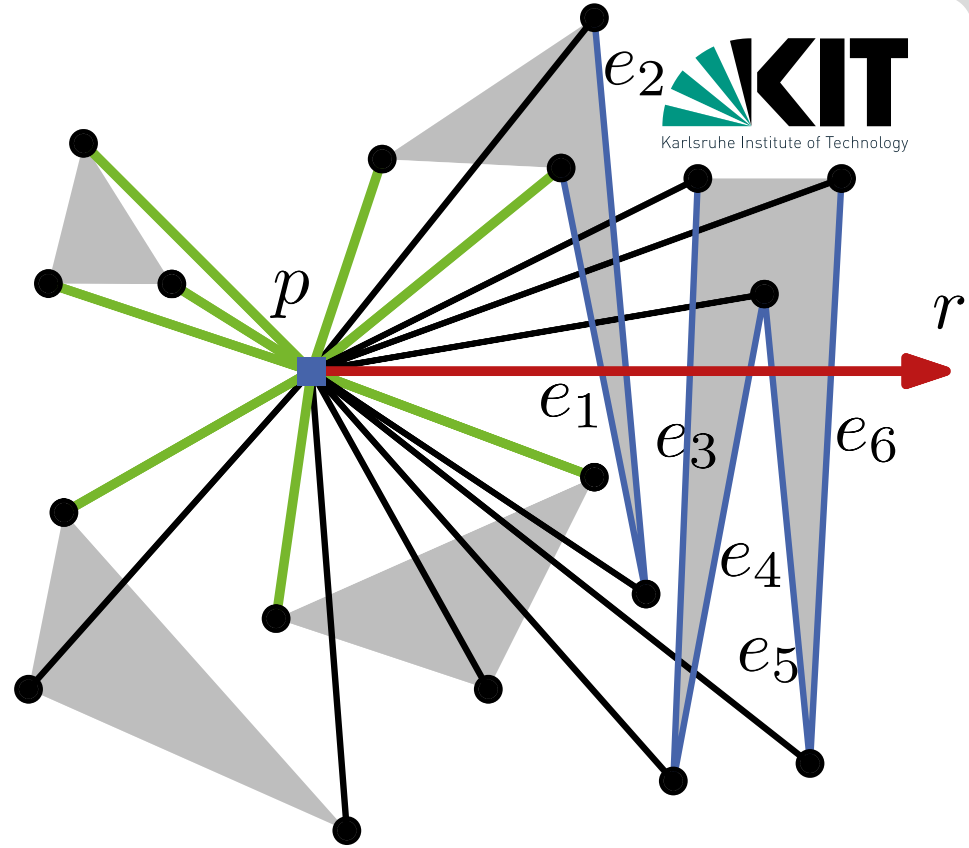

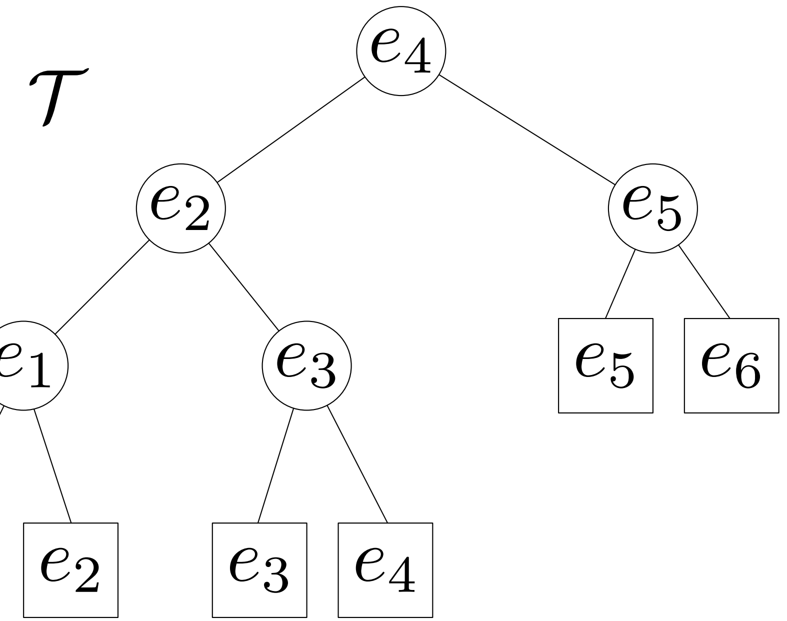

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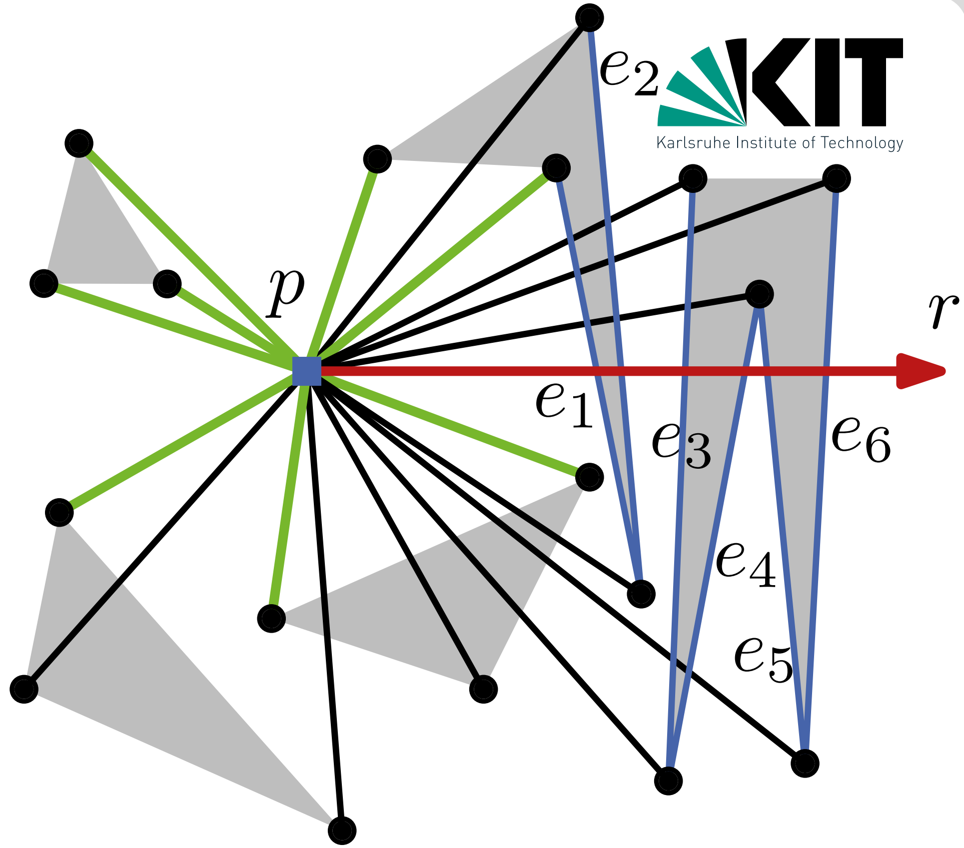

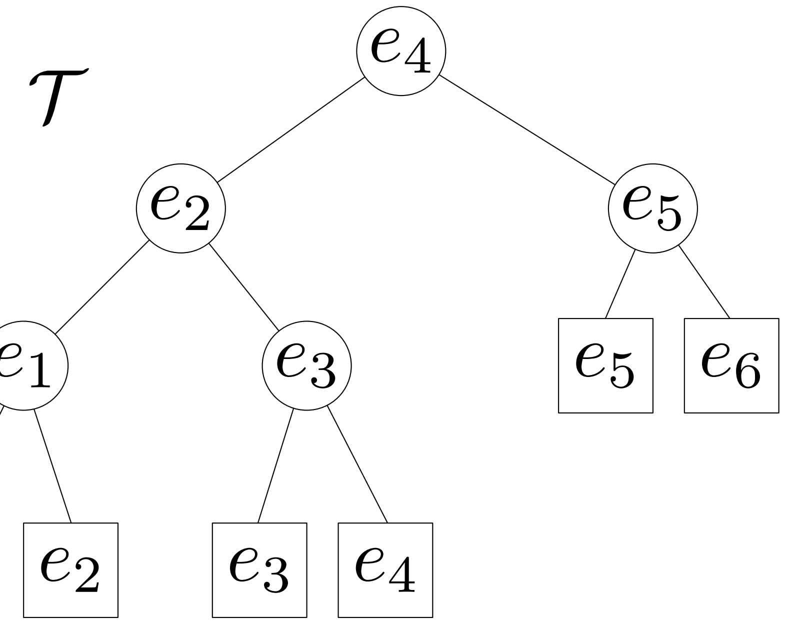

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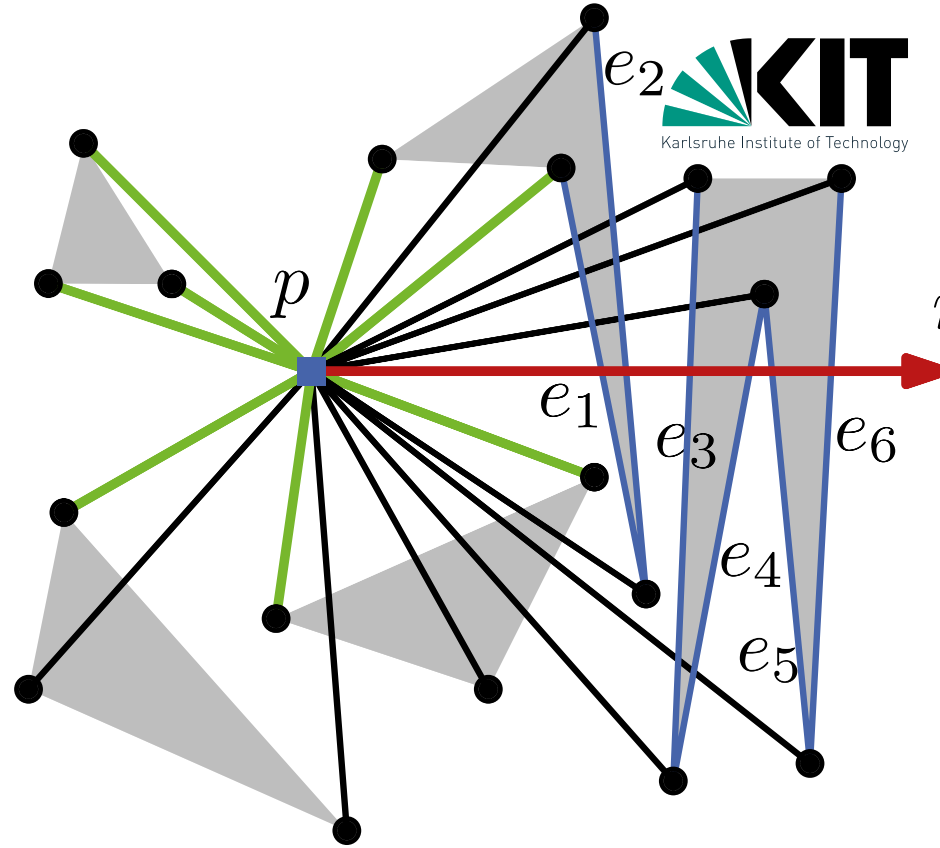
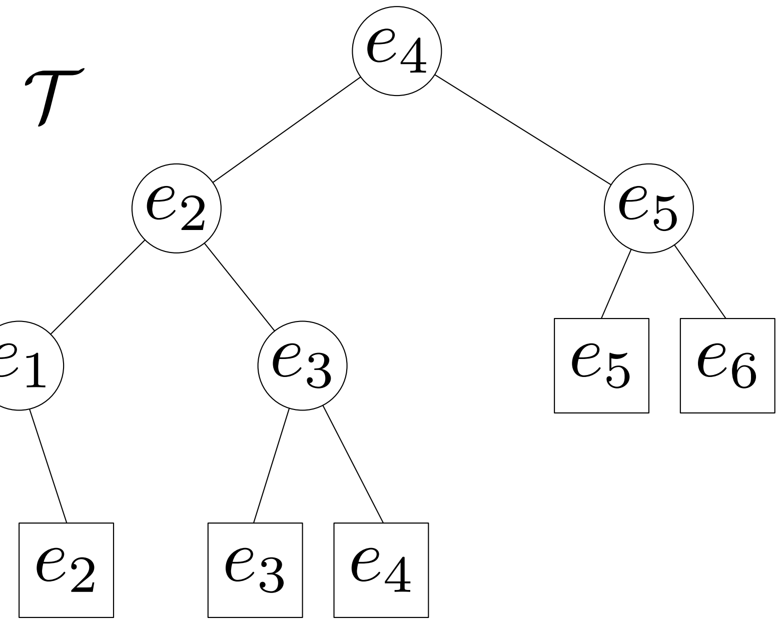
$\angle v < \angle v'$ or
$(\angle v = \angle v'$ and $|pv| < |pv'|)$





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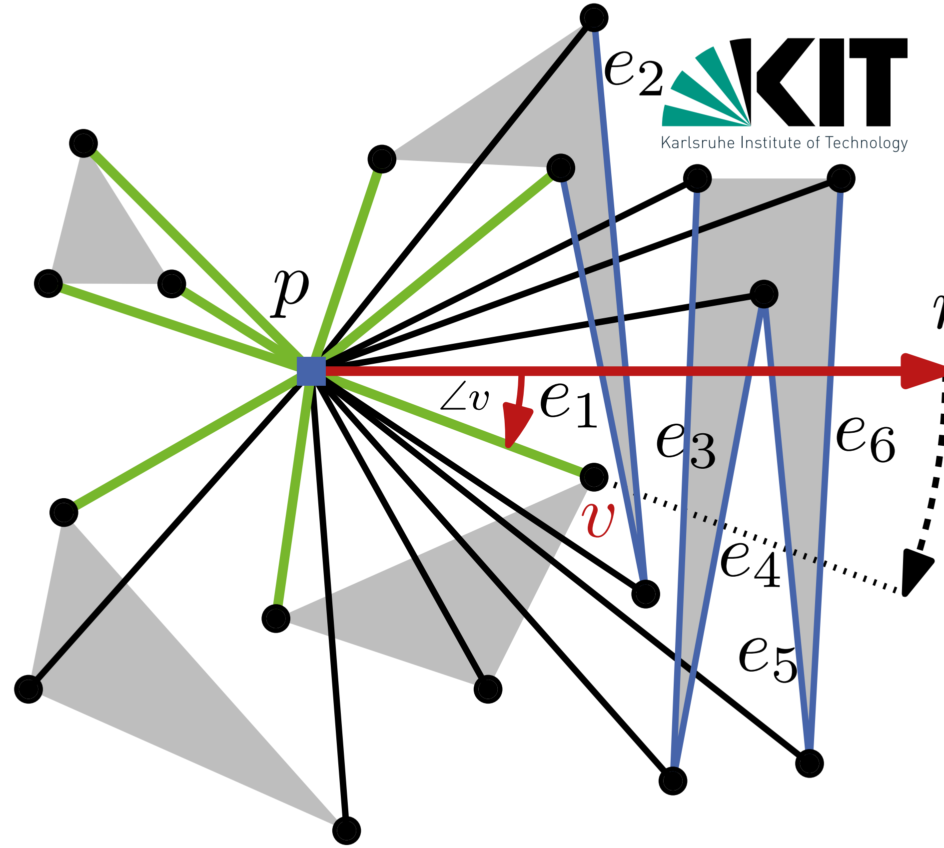

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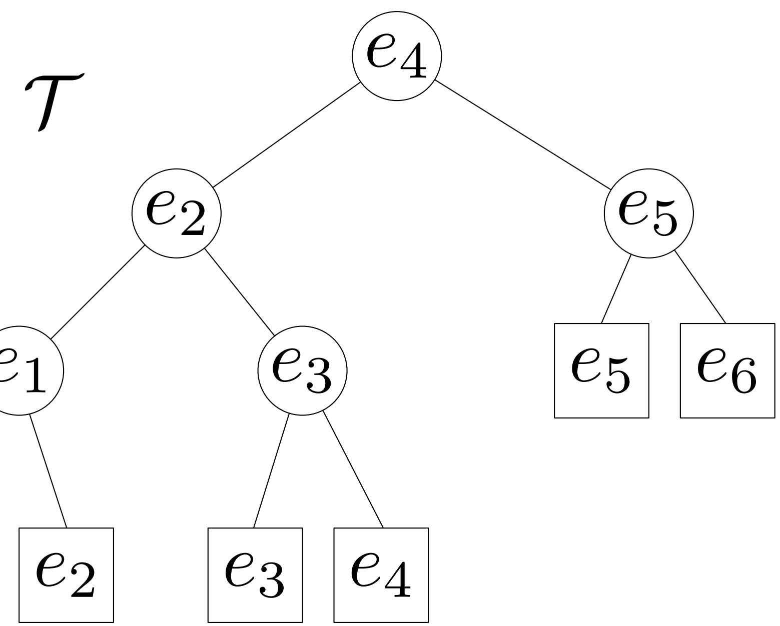

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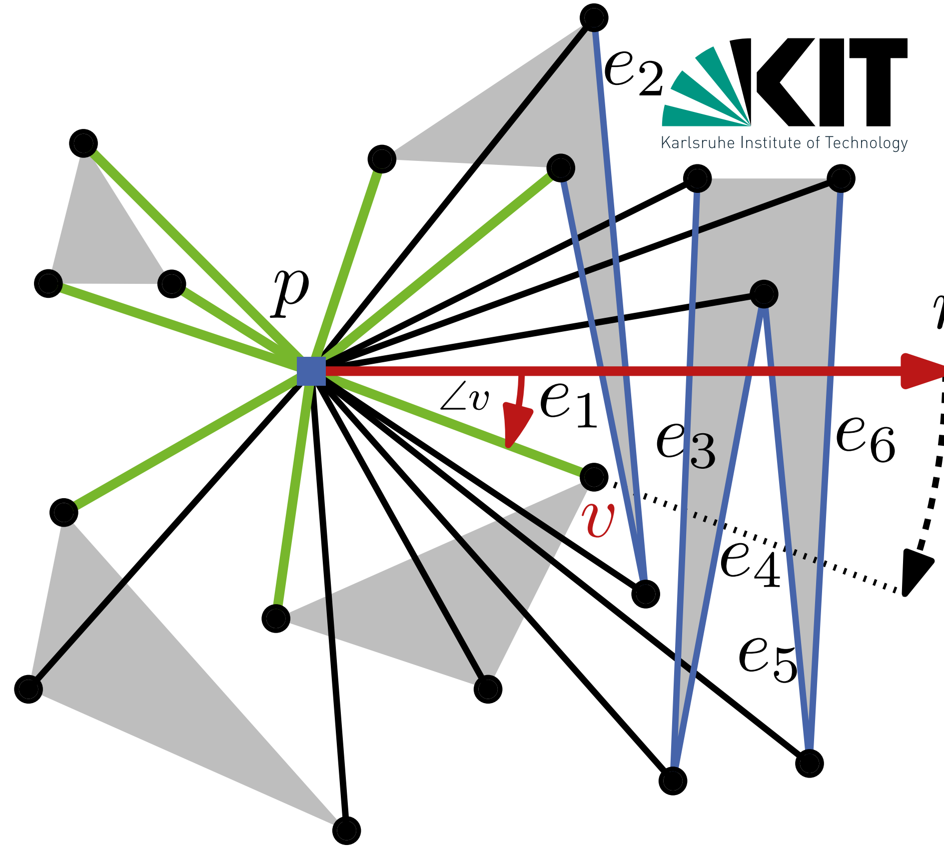

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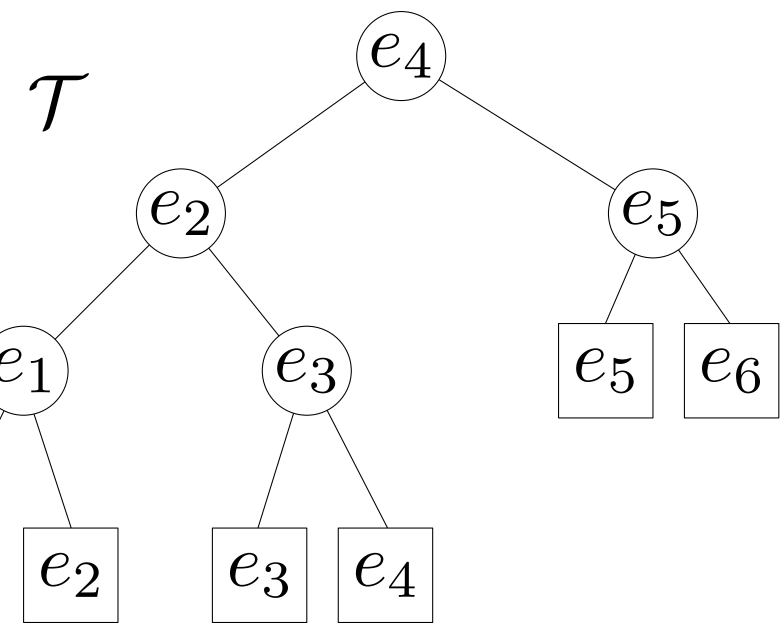

Sweep method with rotation

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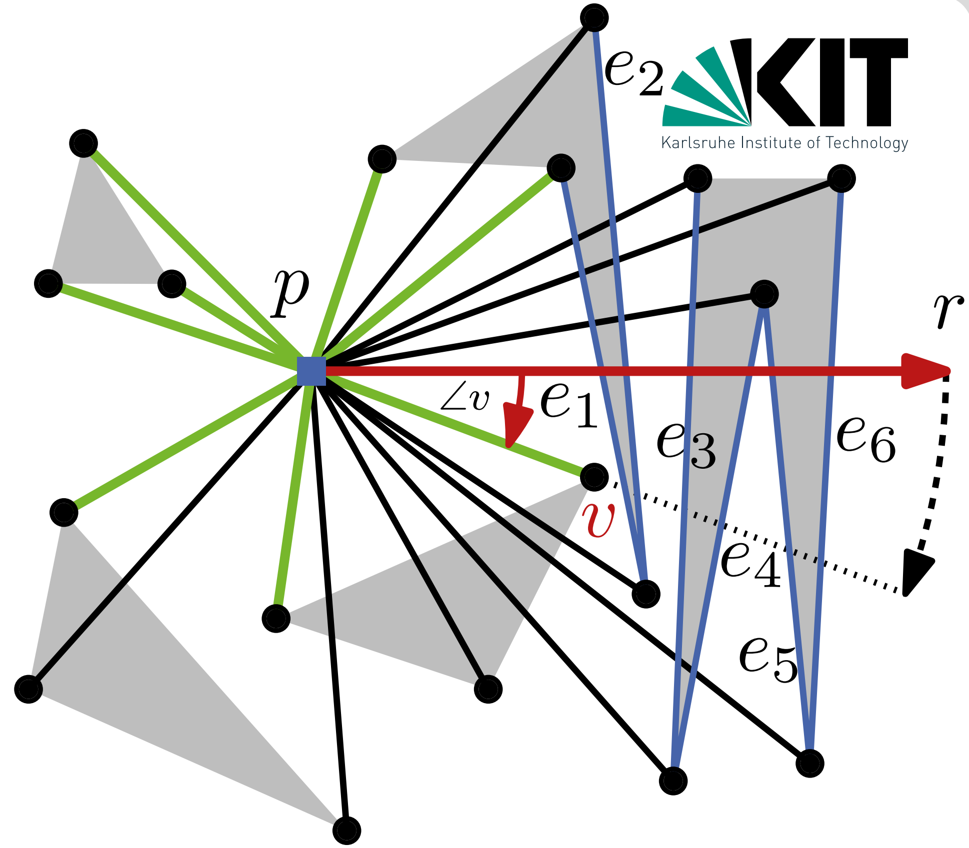

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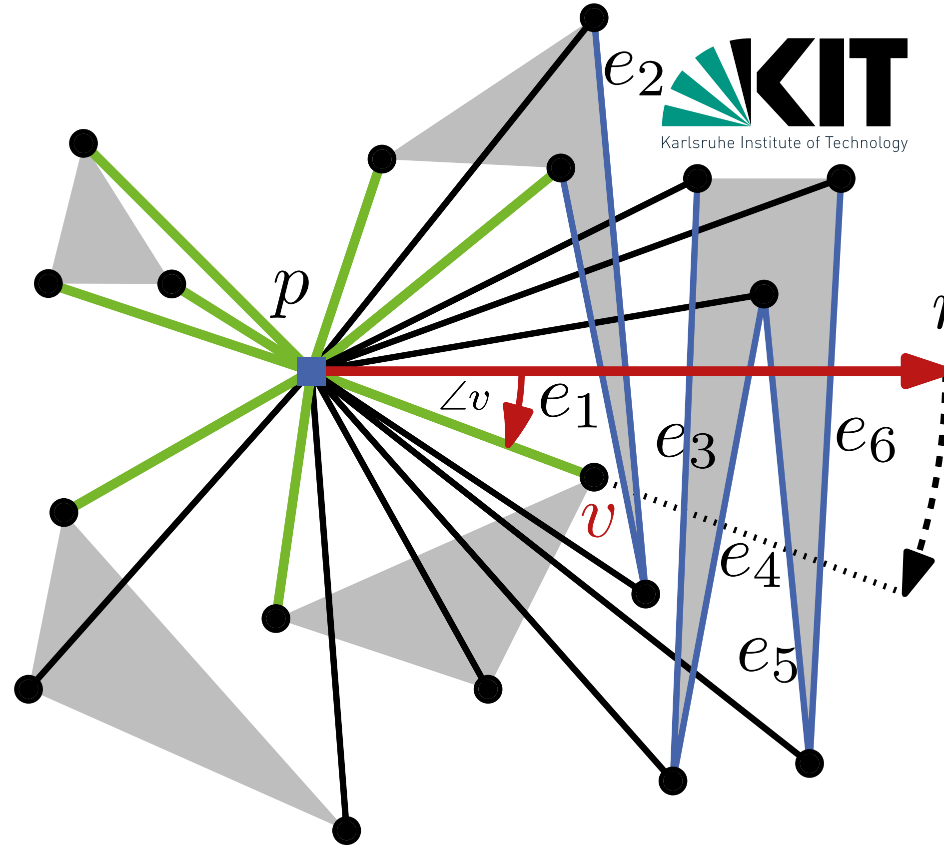

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- $\mathcal{T} \leftarrow$ balancedBinaryTree(I)
- $w_1, \ldots, w_n \leftarrow$ sort $V(S)$ in cyclic order around p
- $W \leftarrow \emptyset$
- **for** $i = 1$ **to** n **do**
  - **if** Visible(p, w_i) **then**
    - $W \leftarrow W \cup \{w_i\}$
  - Add to $\mathcal{T}$ edges incident to w_i: CW from $\overrightarrow{pw_i}^+$
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- **return** W

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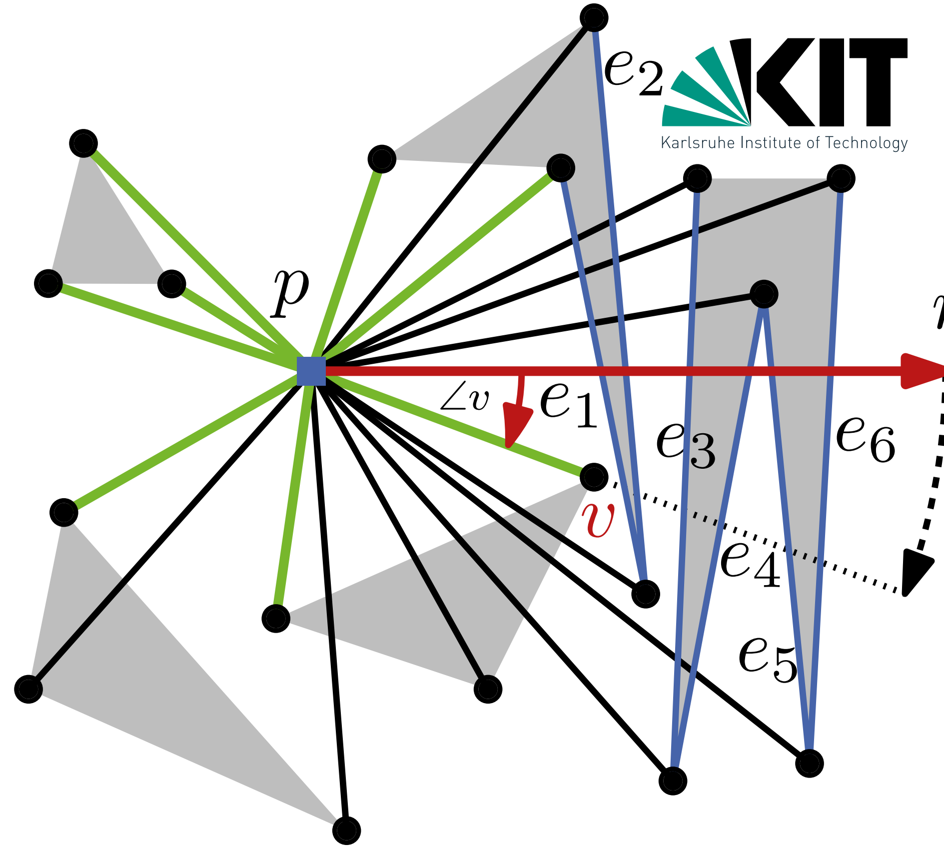

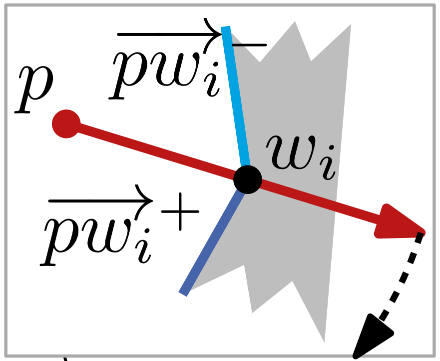

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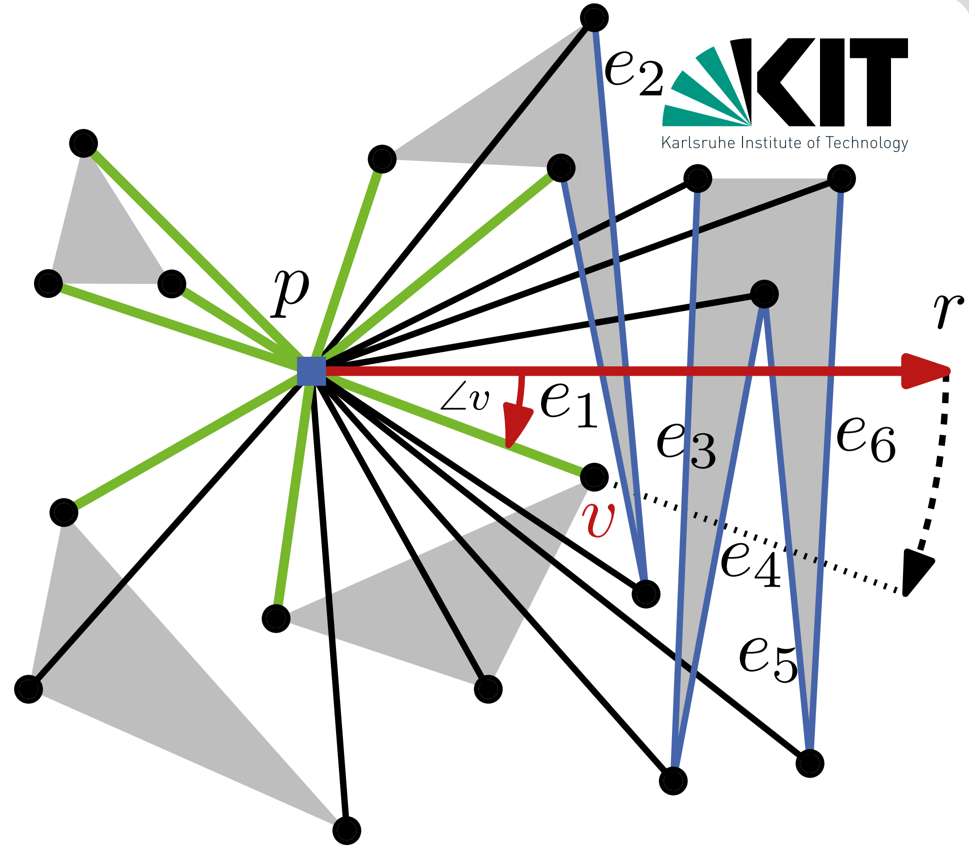
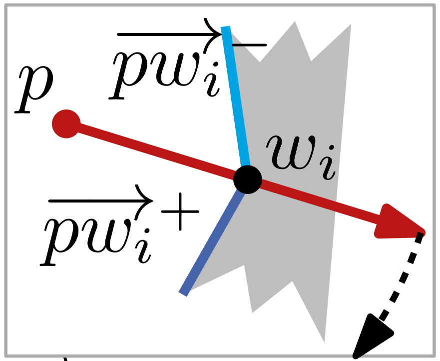
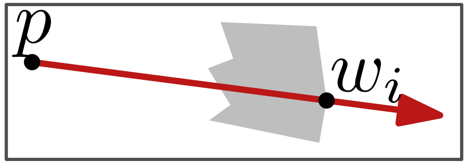




Visibility Case Analysis

Visible(p, w_i)

if $\overline{pw_i}$ intersects polygon of w_i **then**
 ⌊ **return** false



nil

Visibility Case Analysis

Visible(p, w_i)

if $\overline{pw_i}$ intersects polygon of w_i **then**
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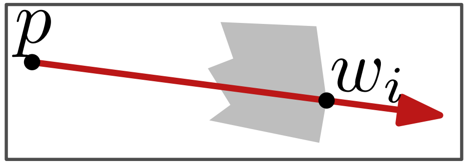
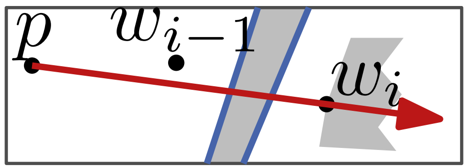
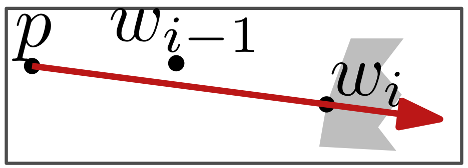
if $i = 1$ or $w_{i-1} \notin \overline{pw_i}$ **then**
    $e \leftarrow$ edge of leftmost leaf of $\mathcal{T}$
    if $e \neq$ nil and $\overline{pw_i} \cap e \neq \emptyset$ **then**
        return false
    else return true







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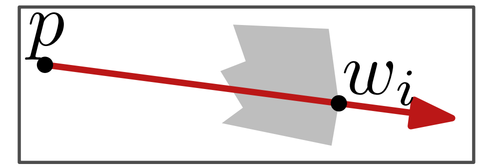

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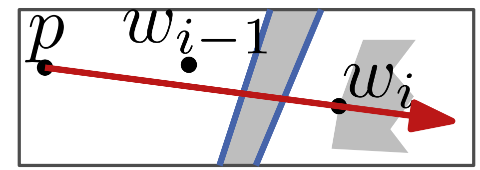
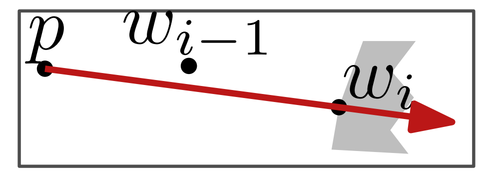
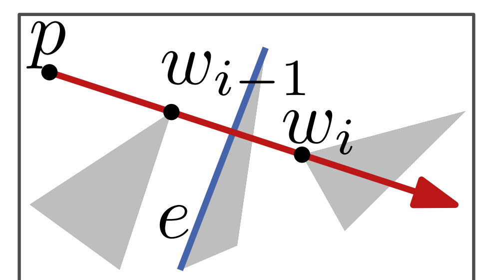




else
  if w_{i-1} is not visible **then**
    return false
  else
    $e \leftarrow$ find edge in $\mathcal{T}$, that $\overline{w_{i-1}w_i}$ cuts; **if** $e \neq$ nil
     then return false
    else return true



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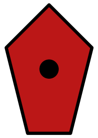

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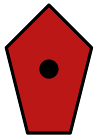

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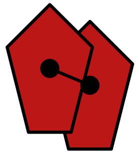

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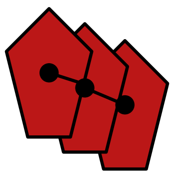

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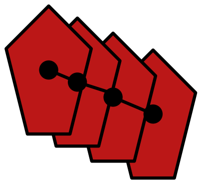

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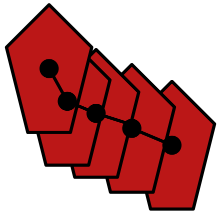

Discussion

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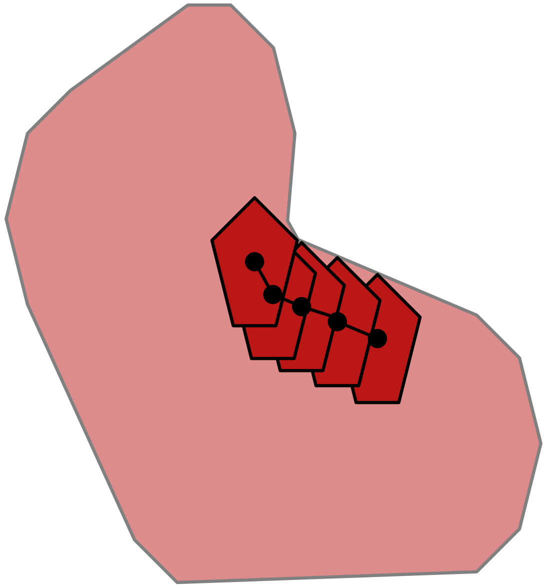

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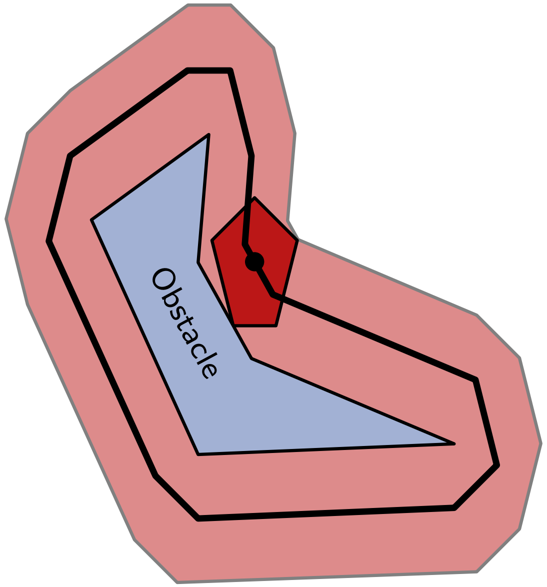

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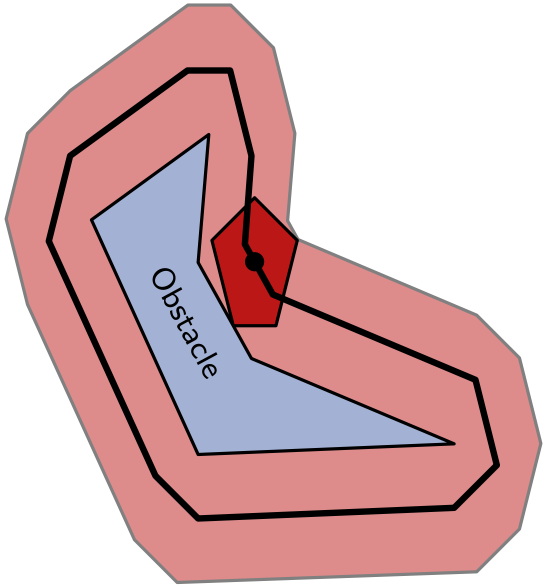

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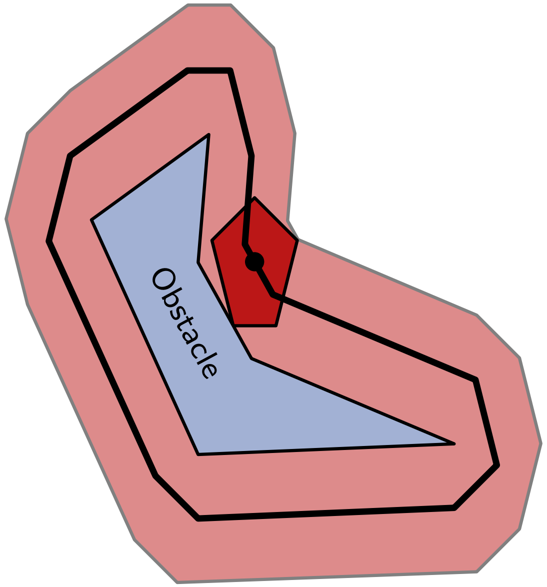

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[Ghosh, Mount 1987]

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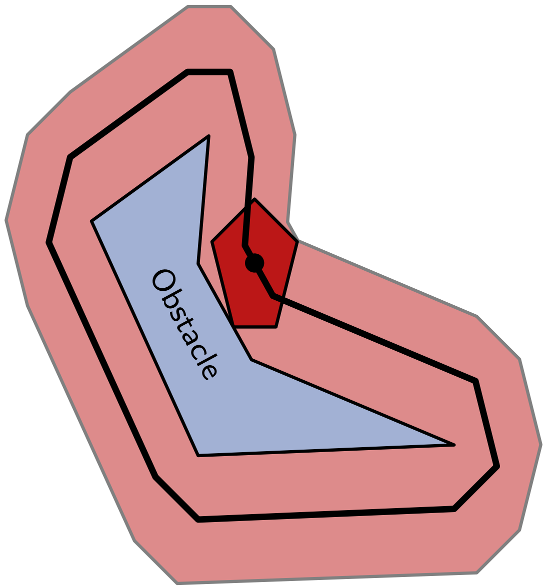

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If you search only for *one* shortest Euclidean st-path, there is an algorithm with optimal $O(n \log n)$ time. [Hershberger, Suri 1999]