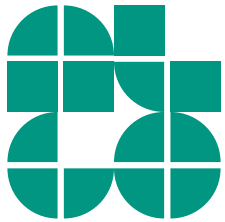
Vorlesung Algorithmische Kartografie

Beschriftung in dynamischen Karten: Rotieren

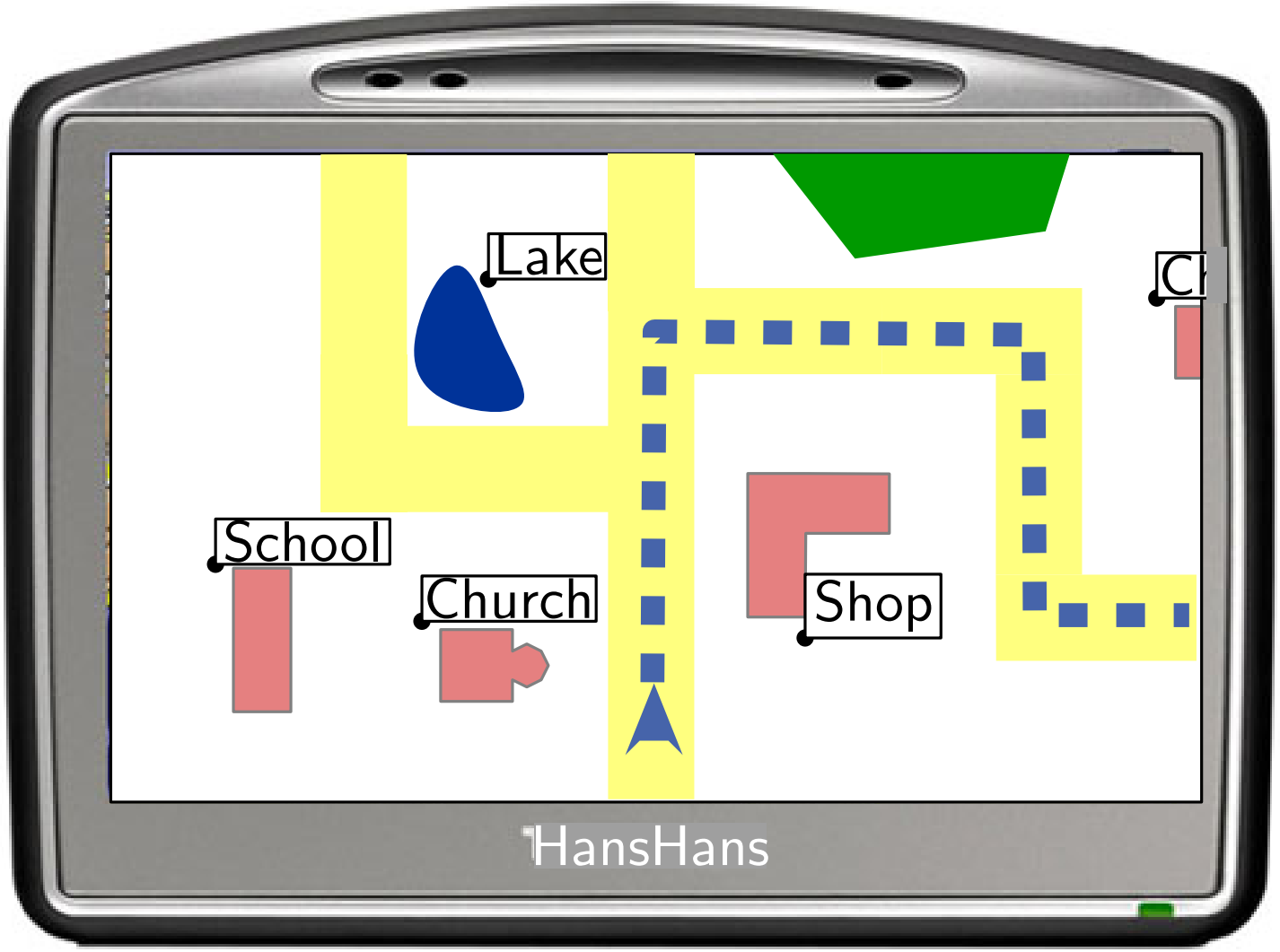
LEHRSTUHL FÜR ALGORITHMIK I · INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

Andreas Gemsa

18.06.2013



Motivation



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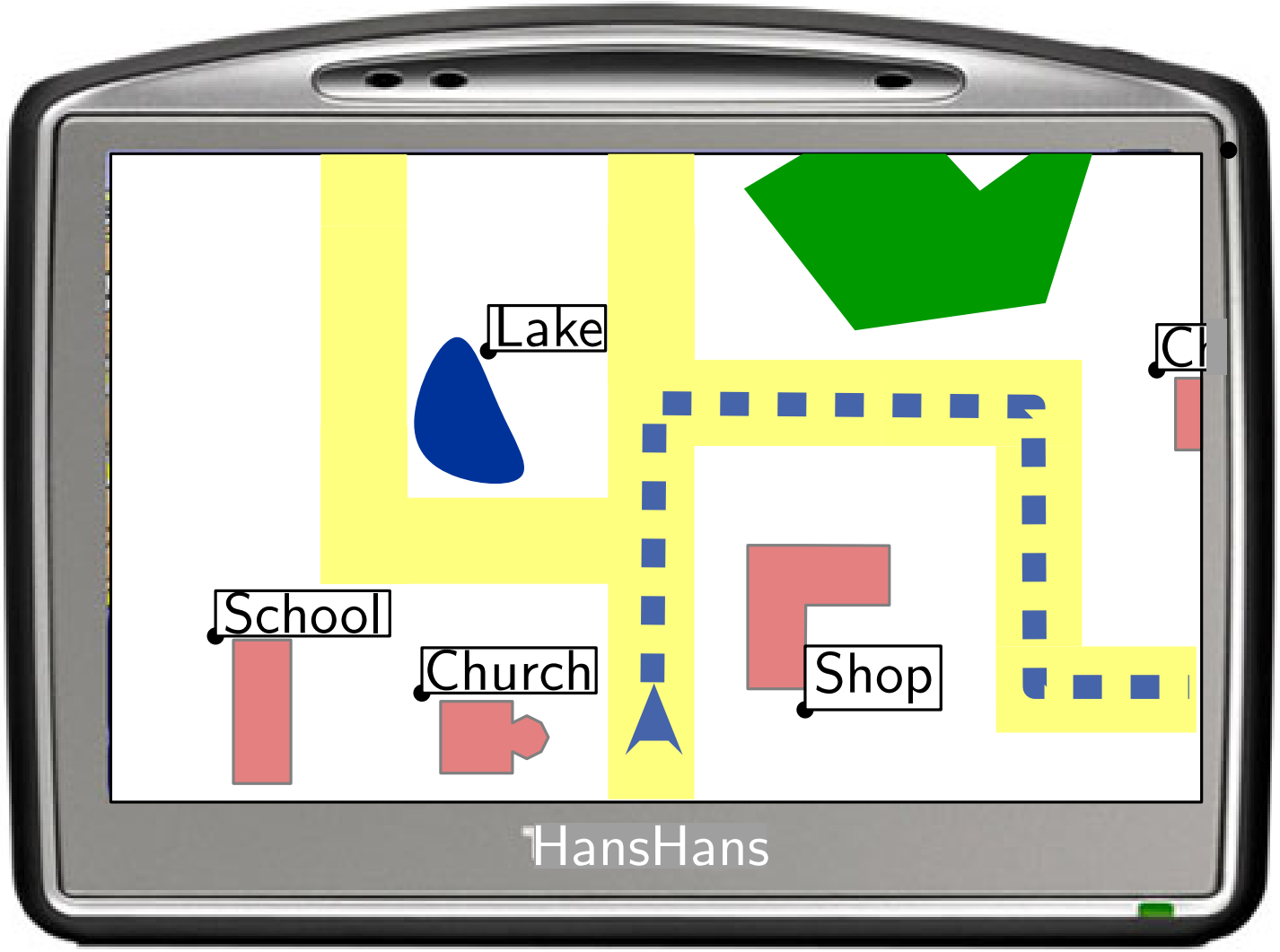

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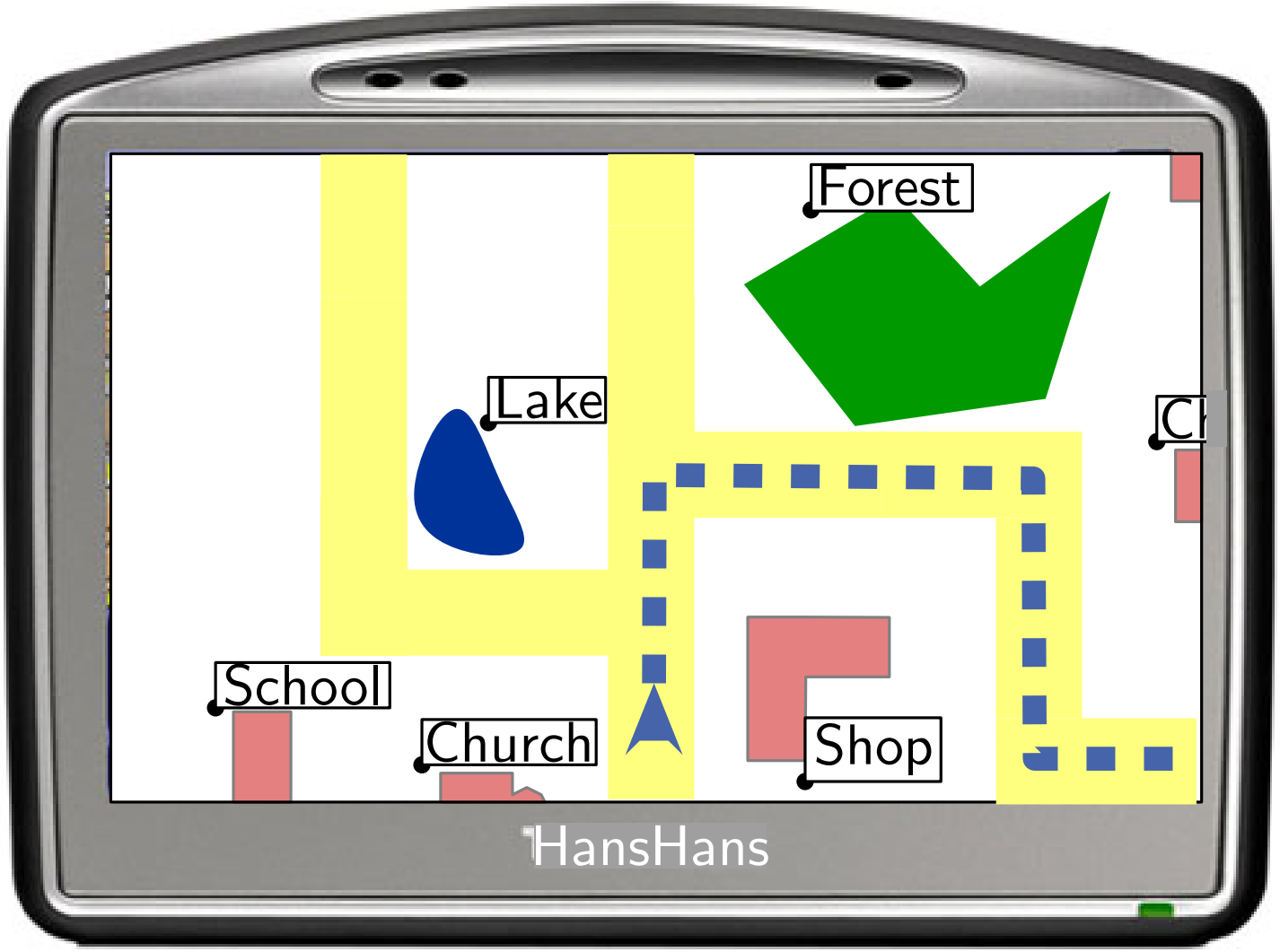

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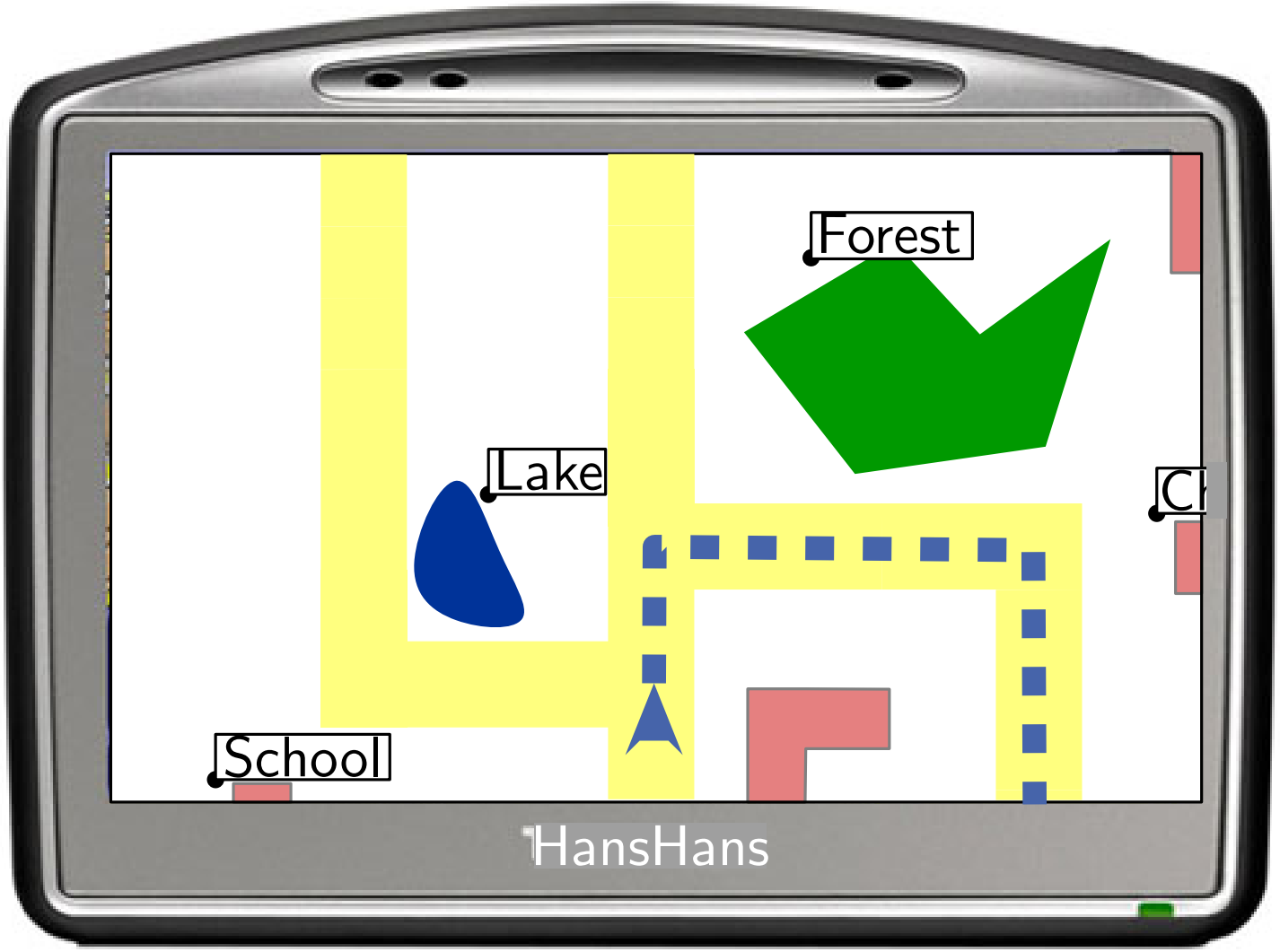

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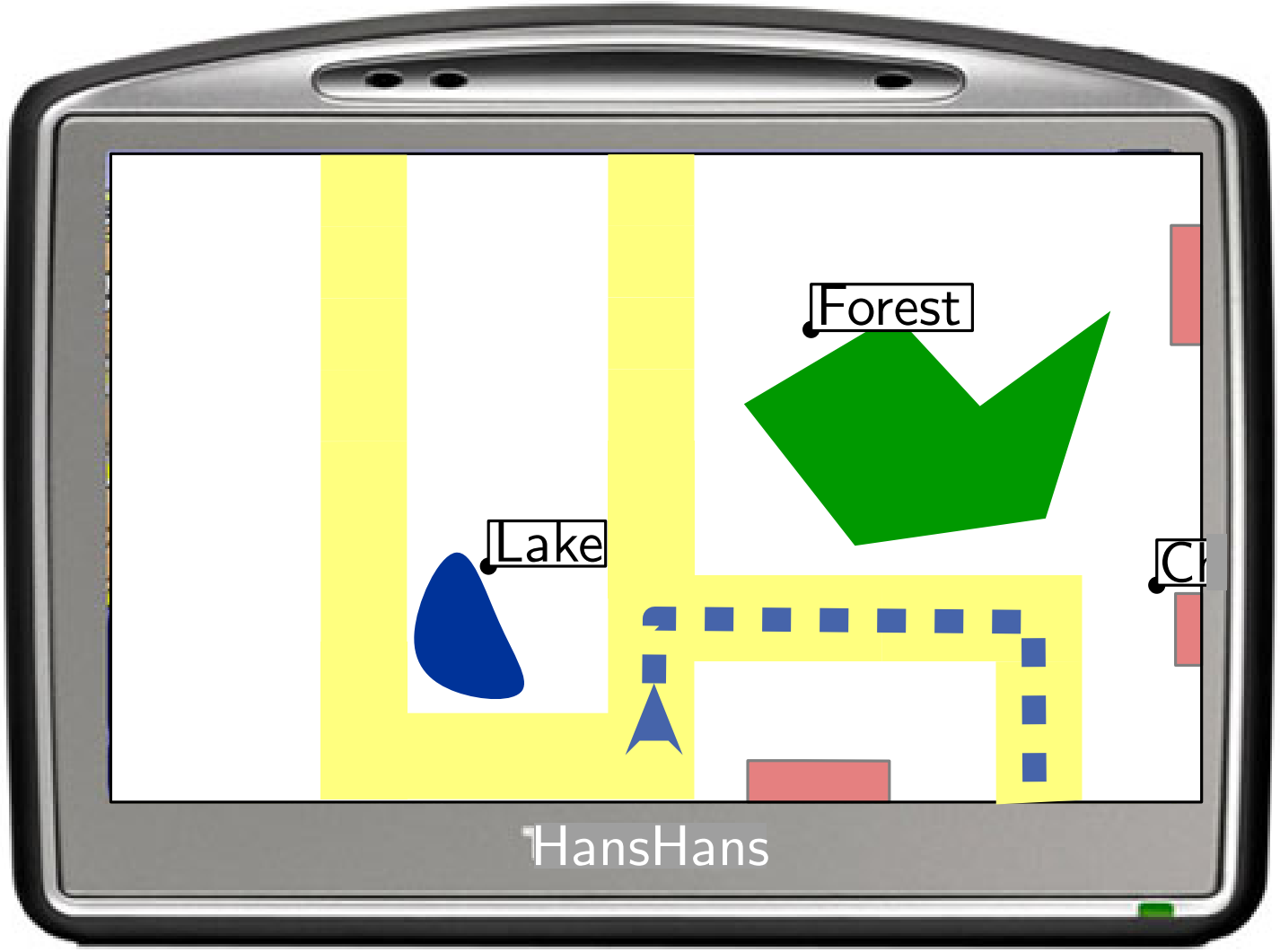

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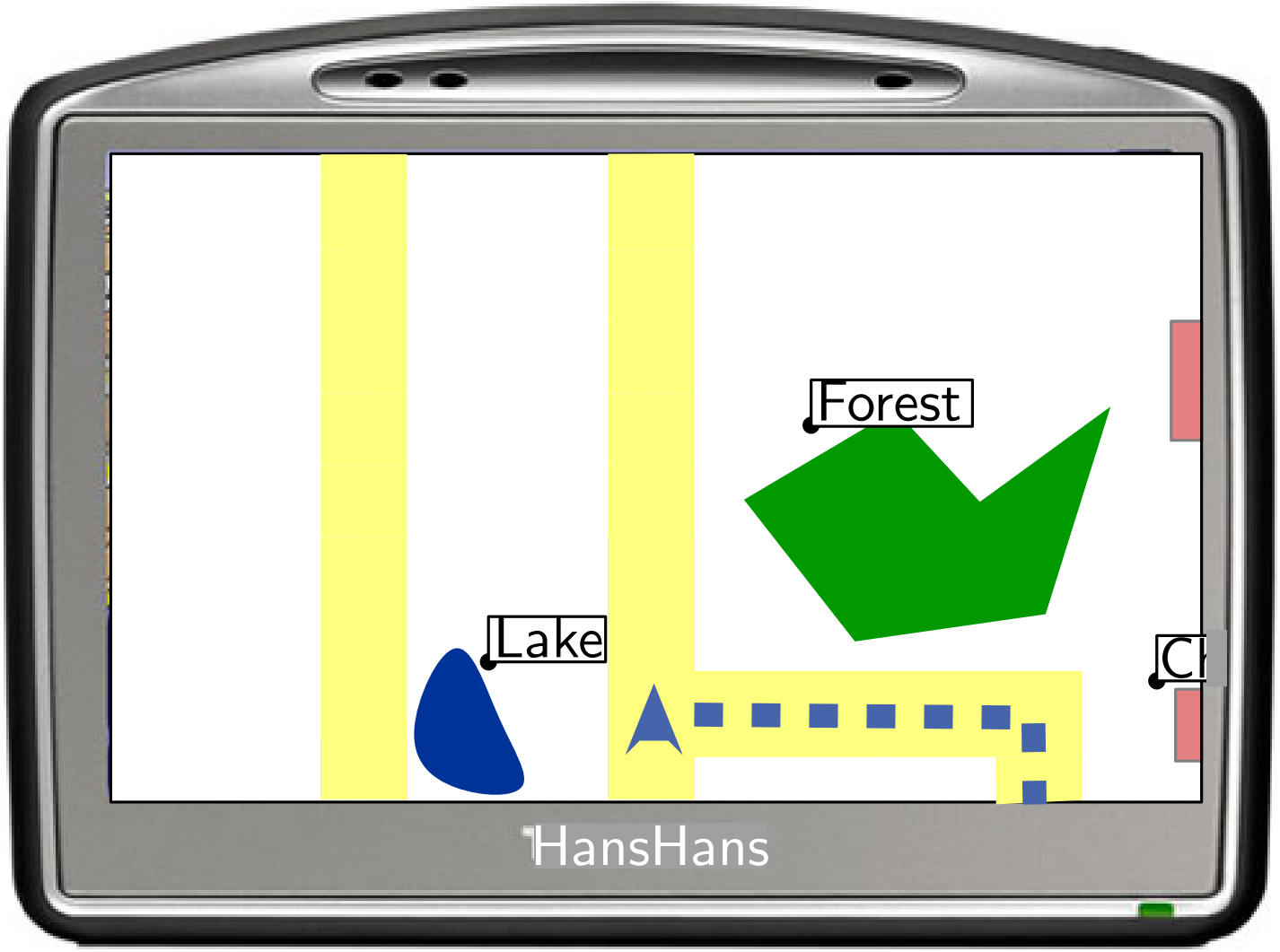

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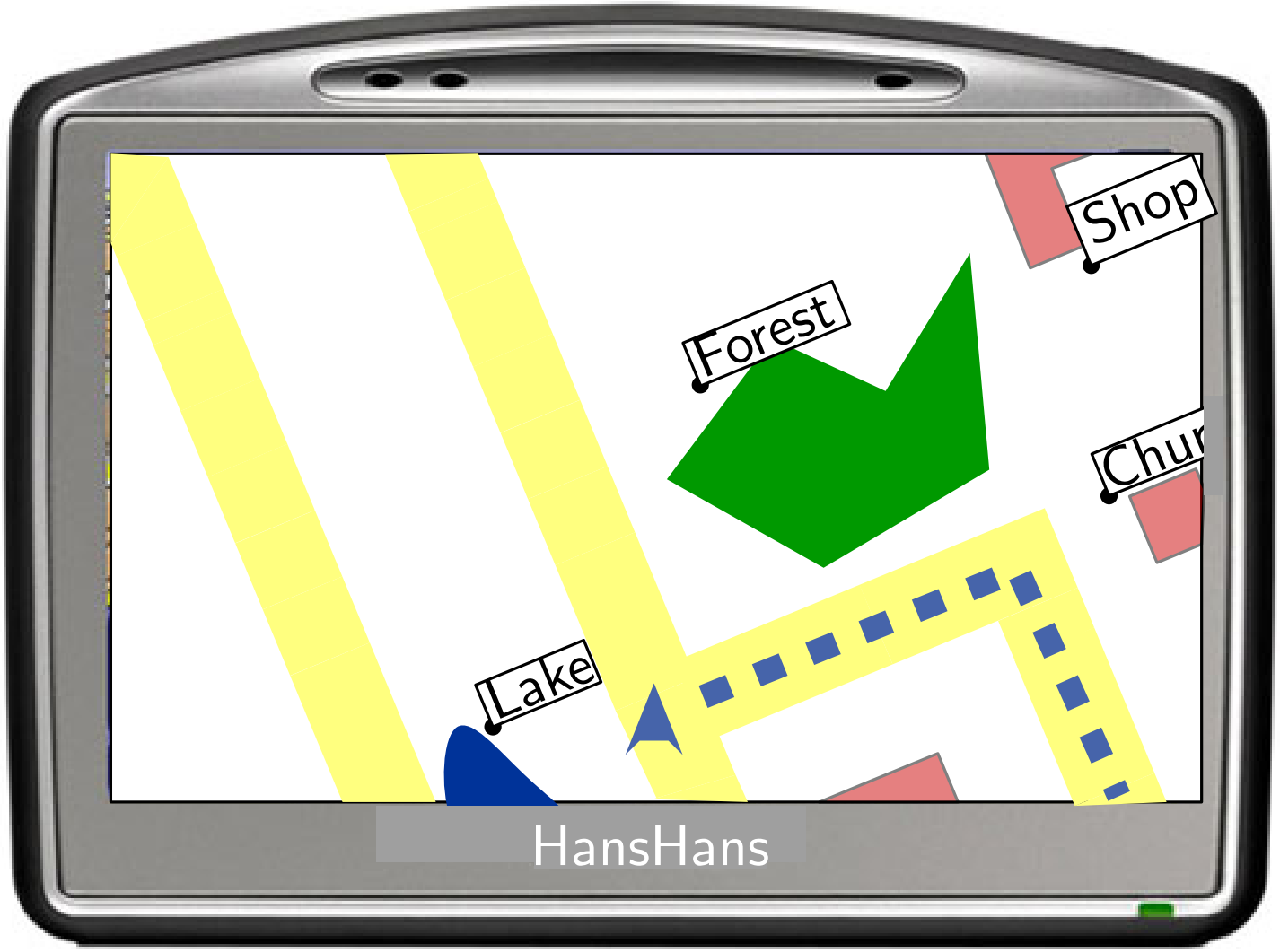

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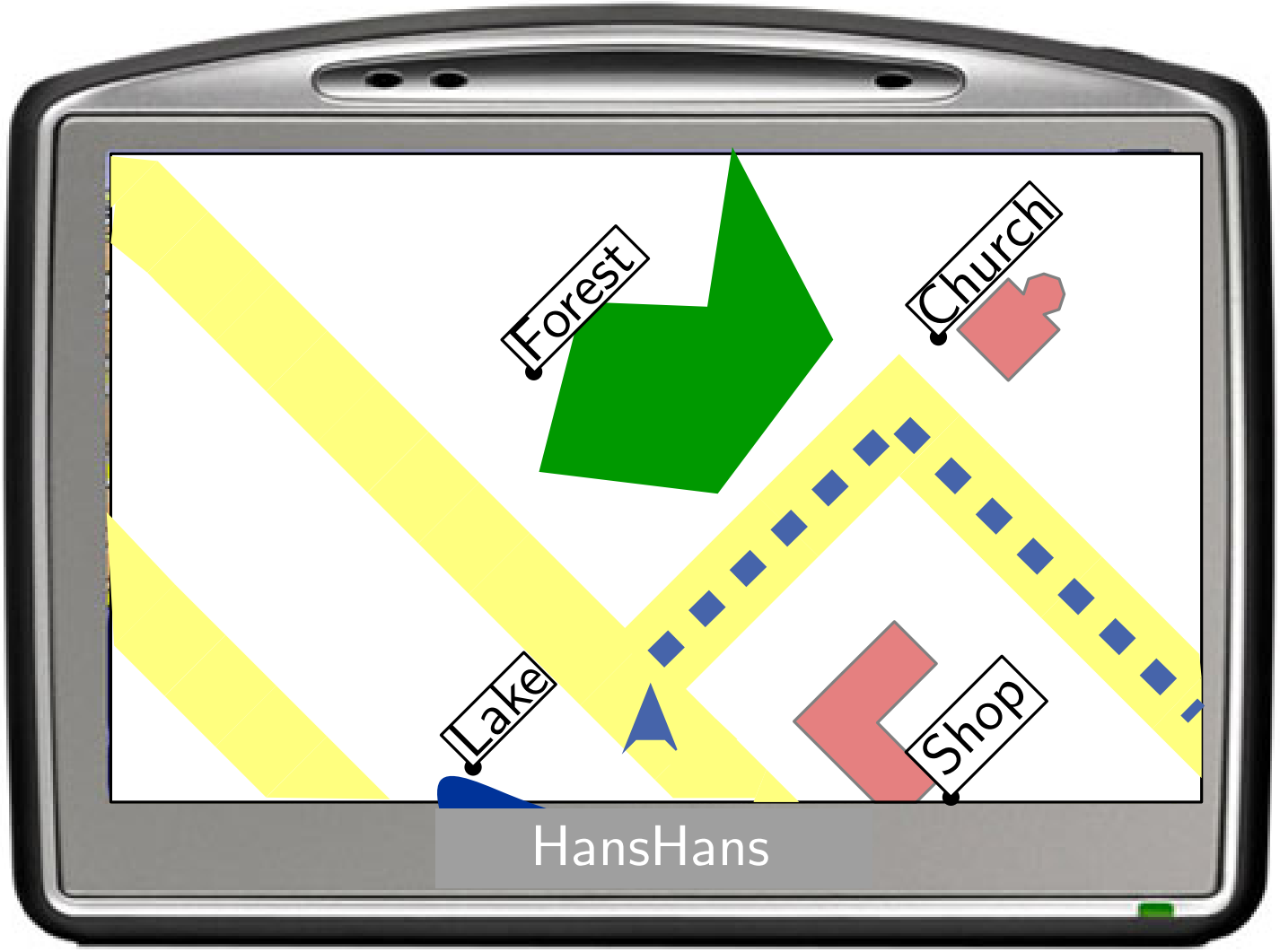

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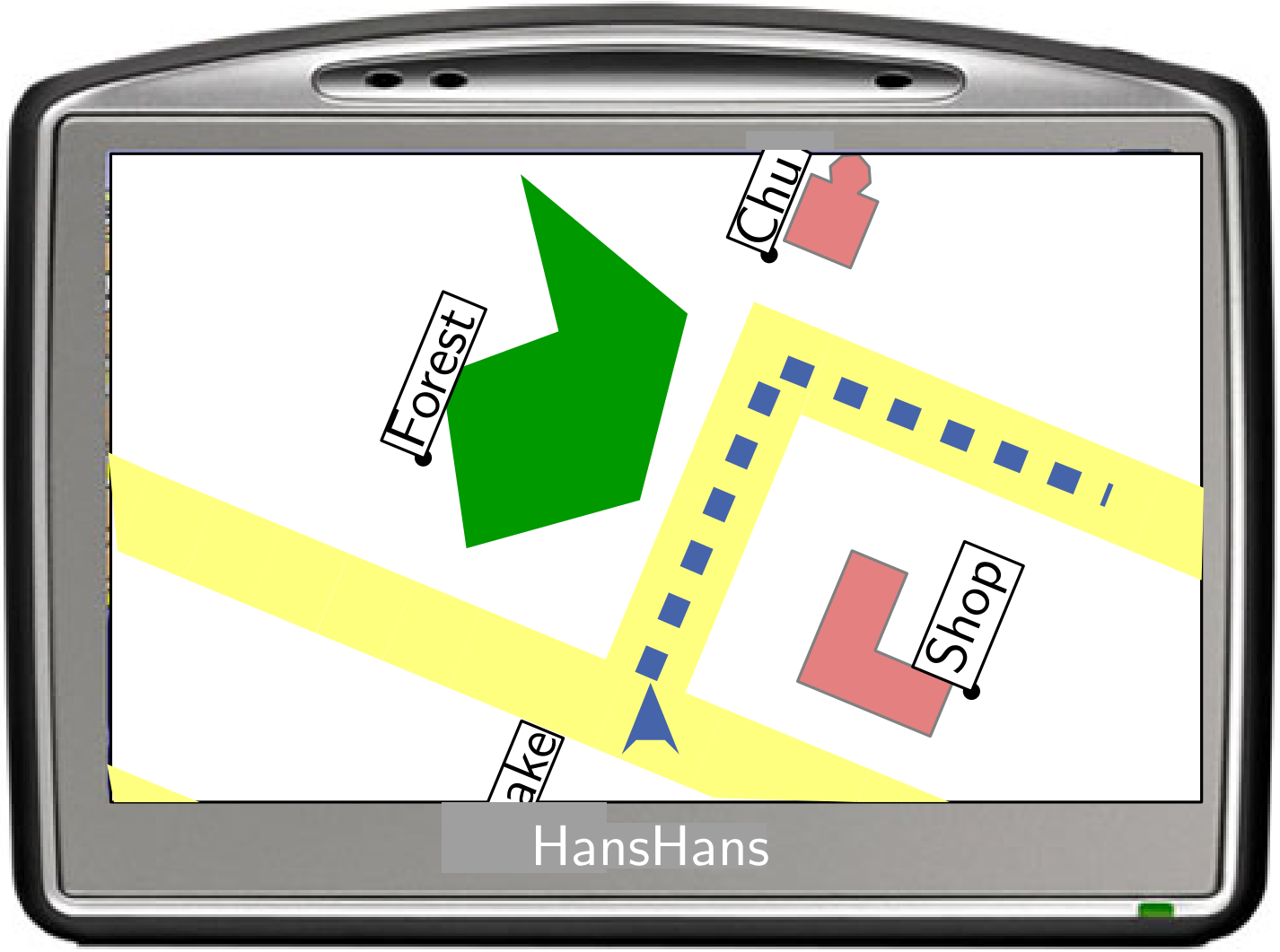

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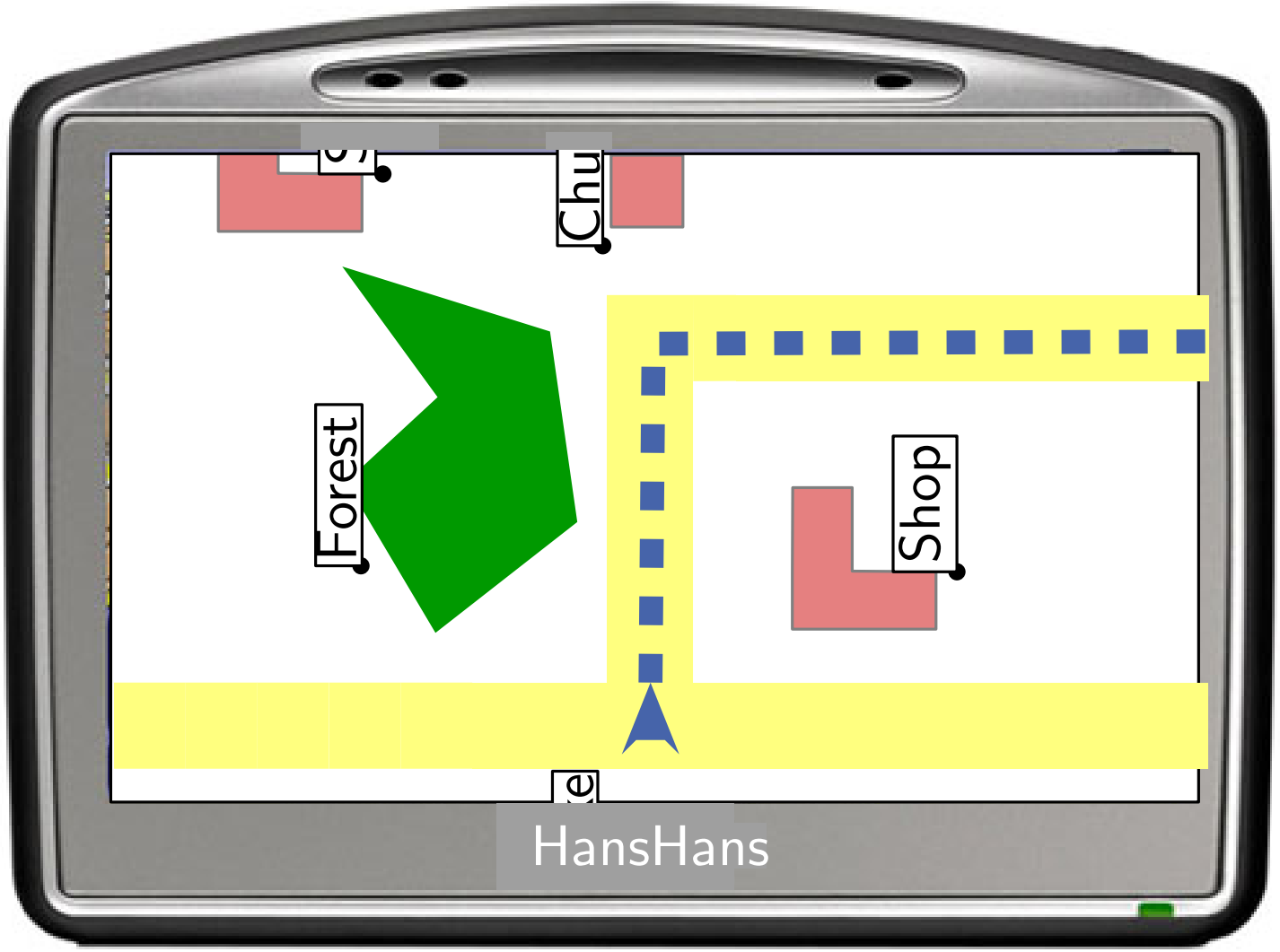

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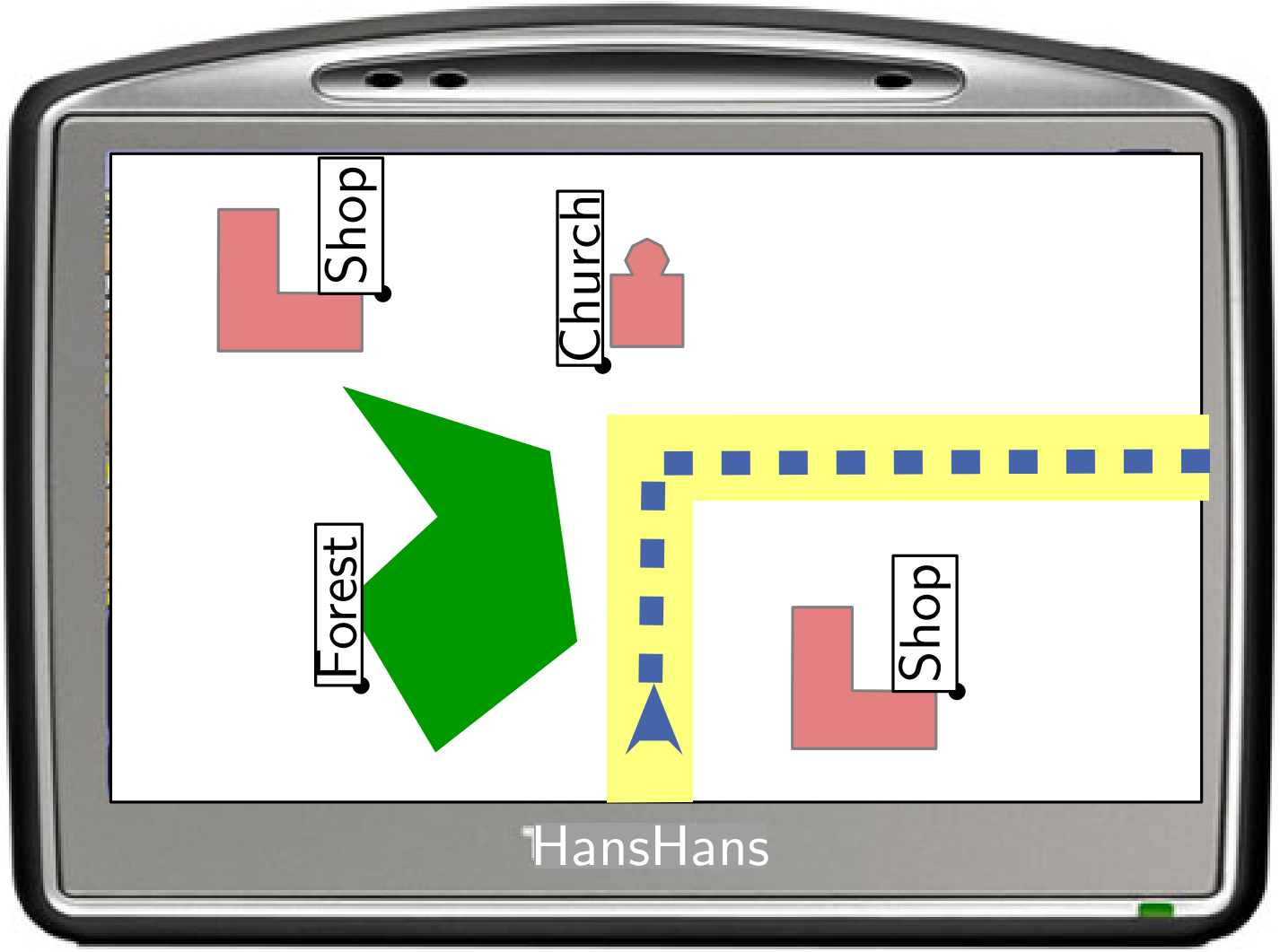

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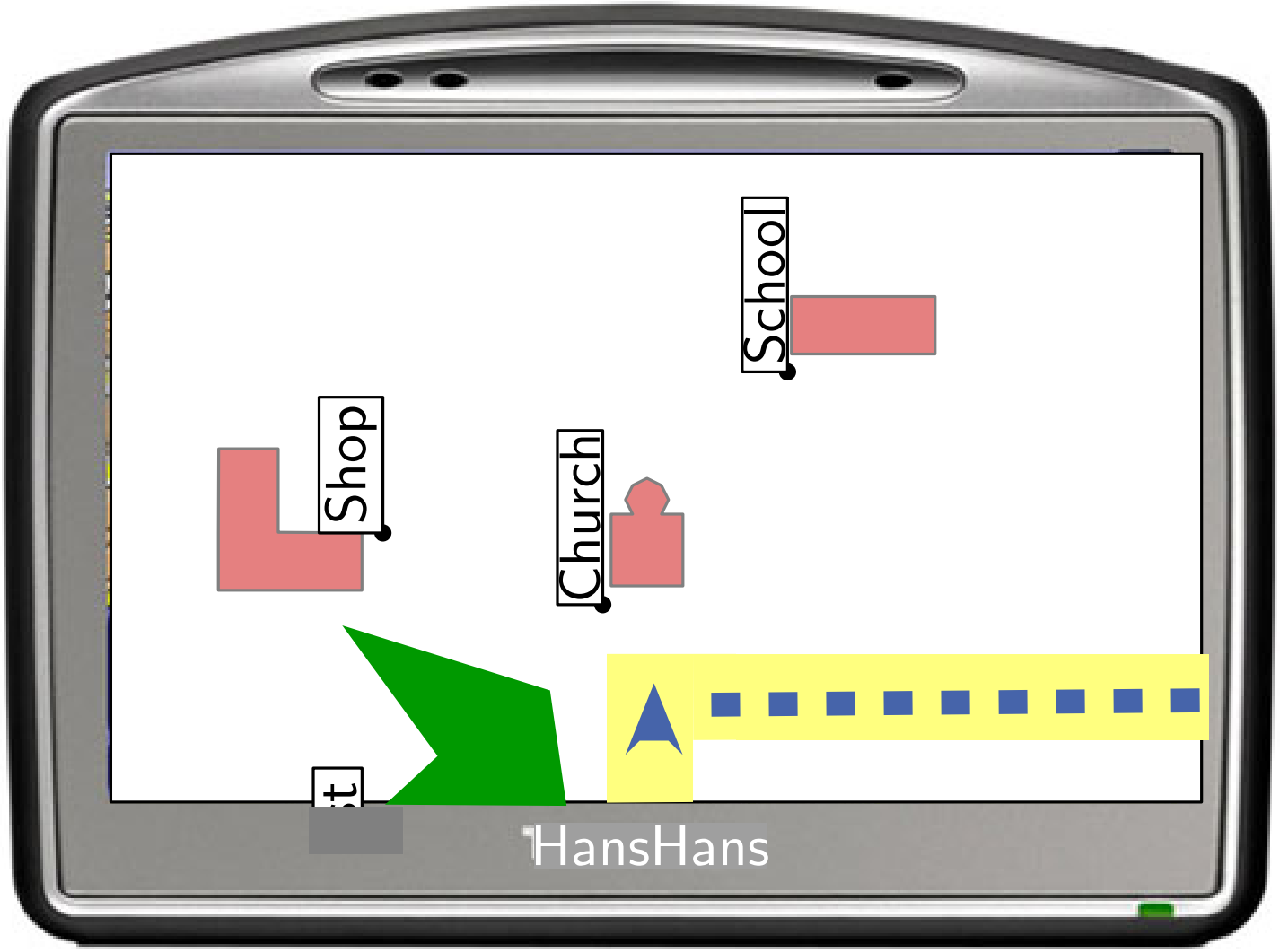

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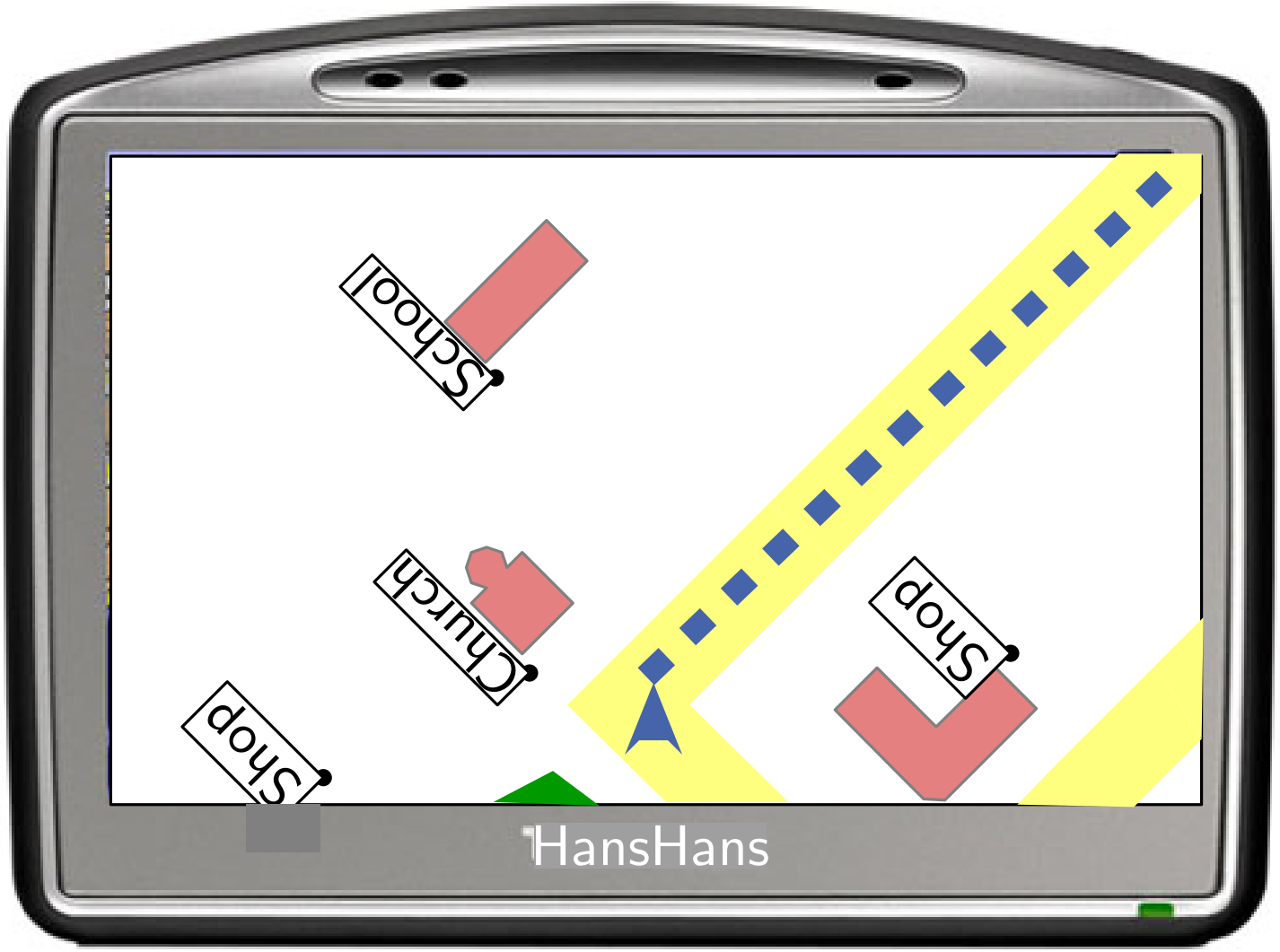

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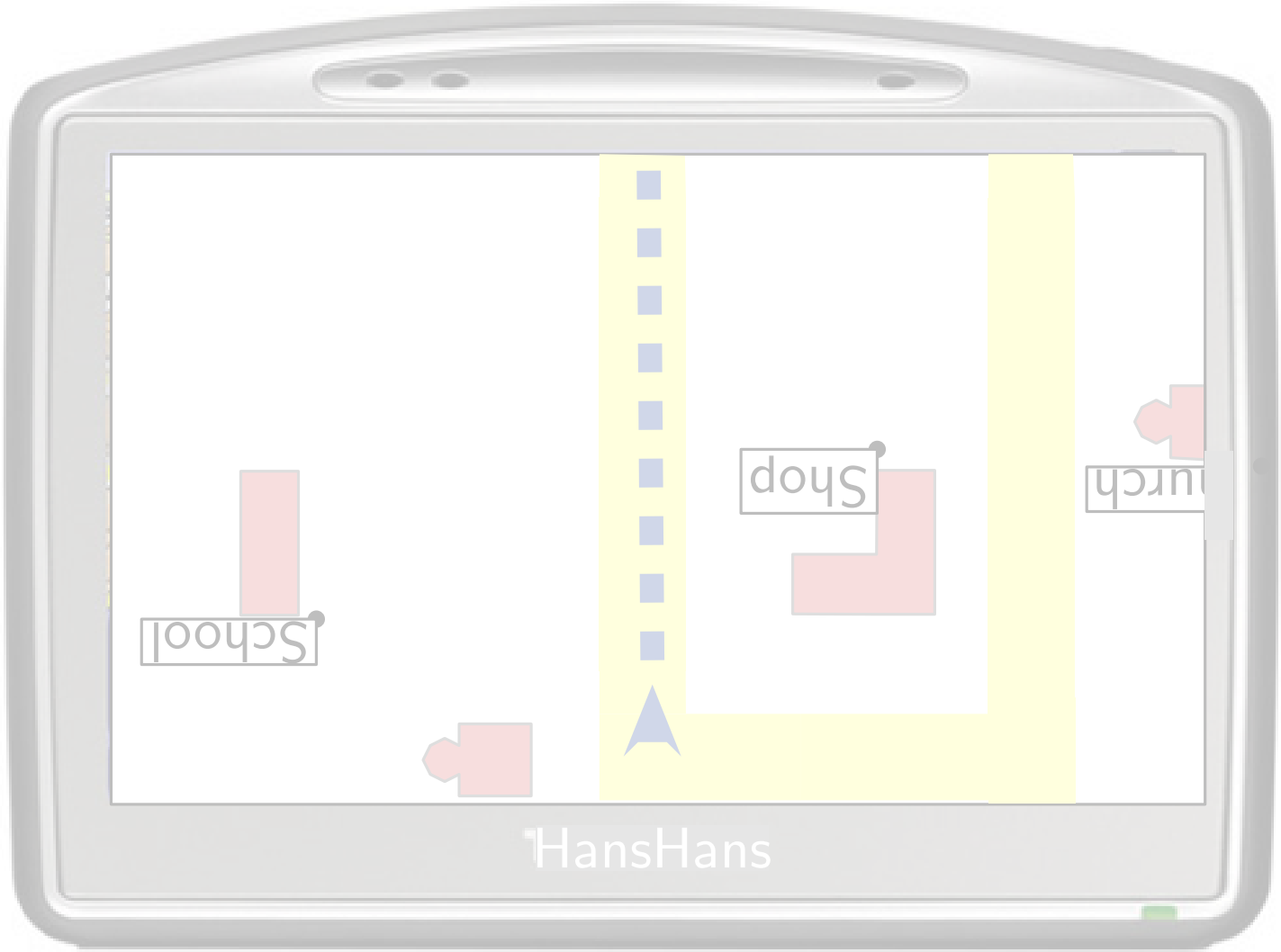

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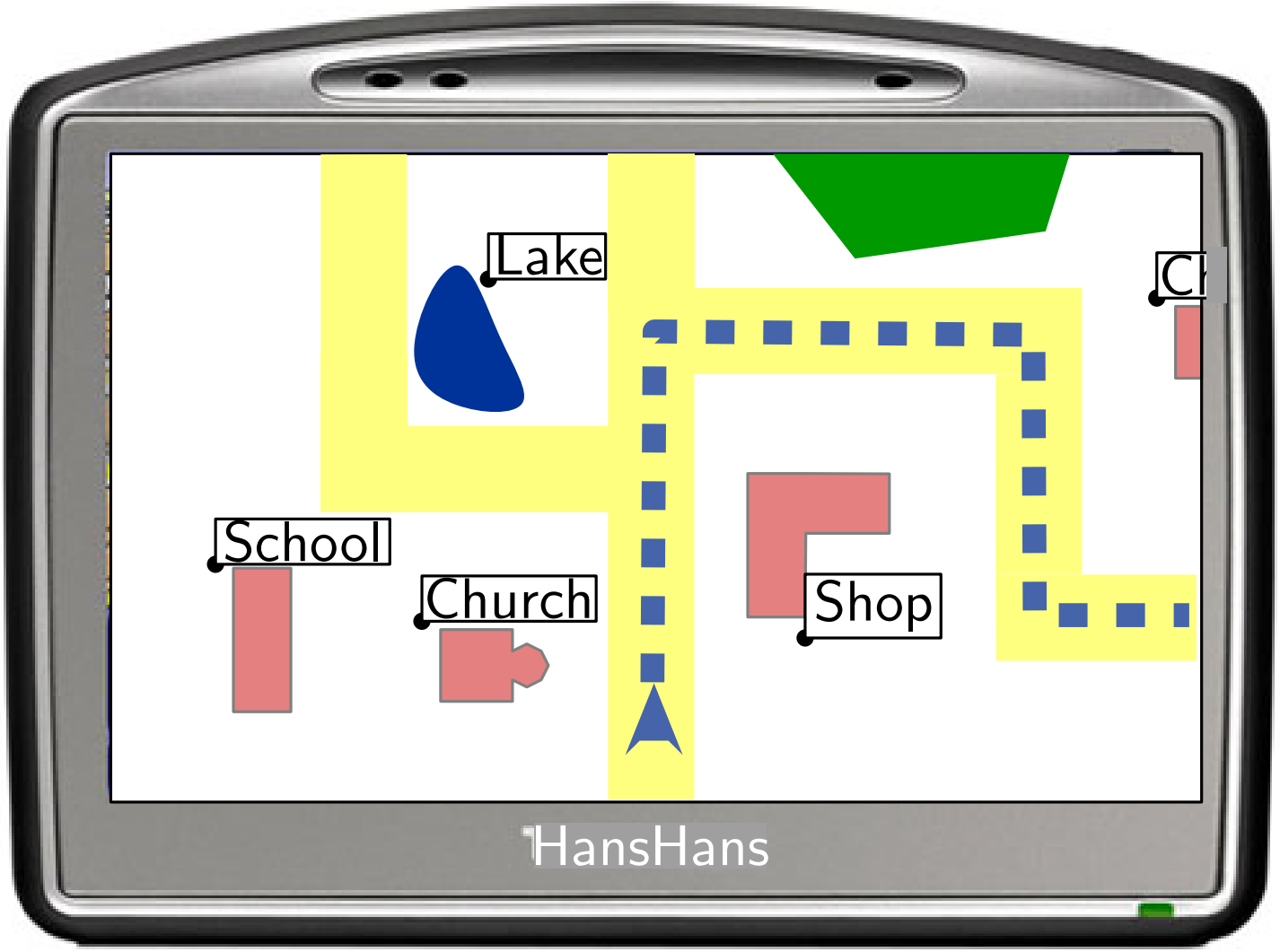

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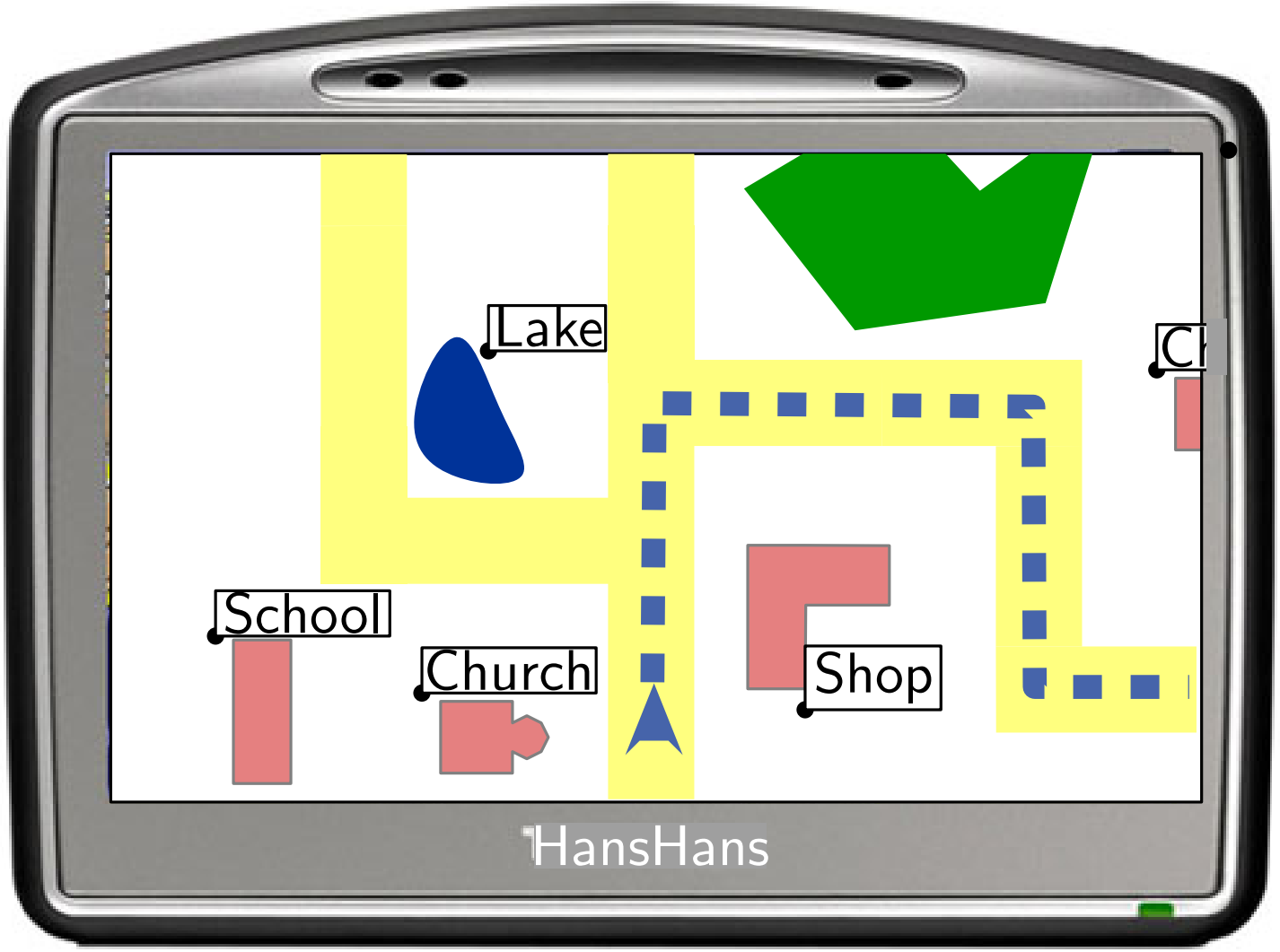

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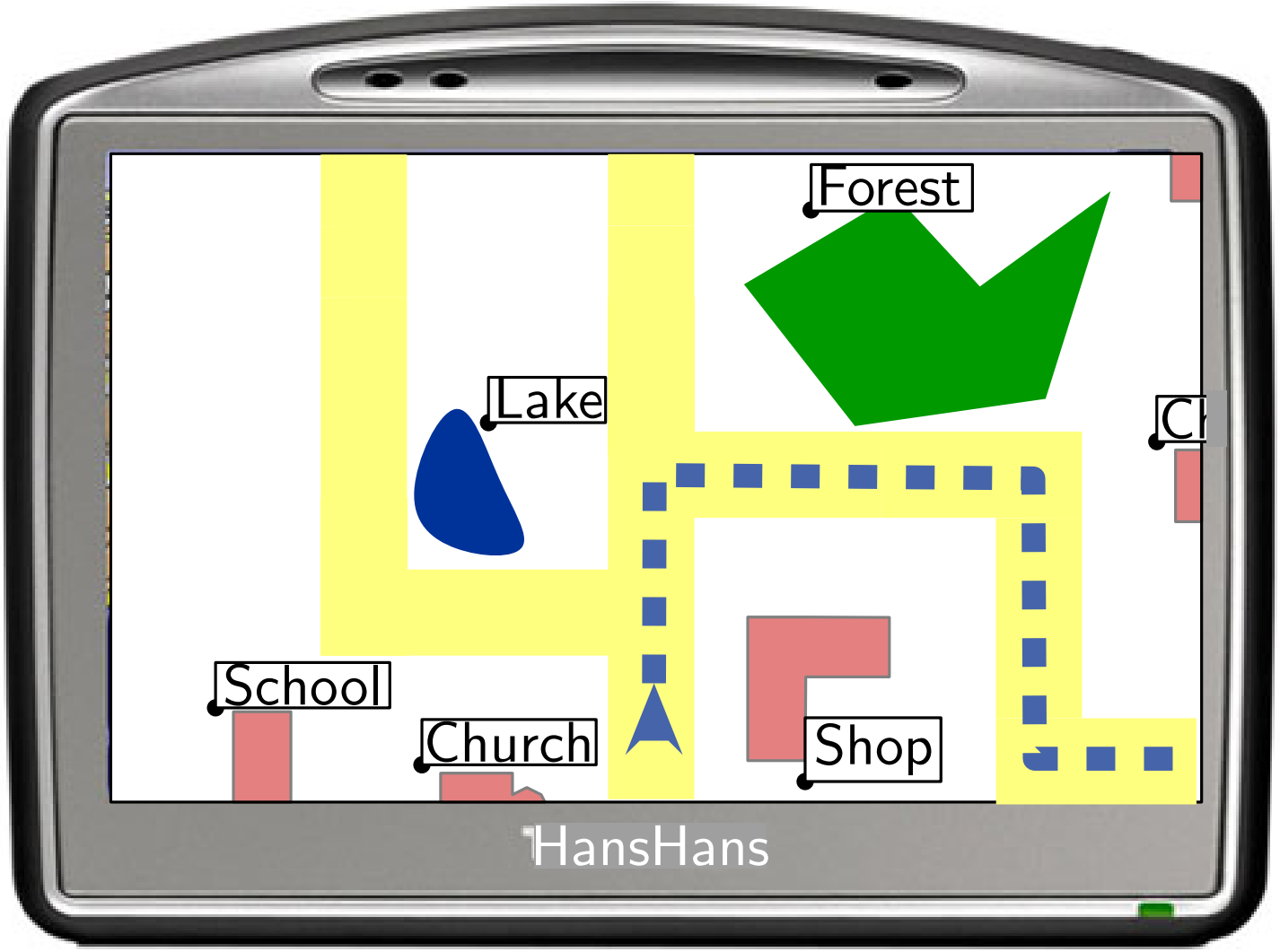

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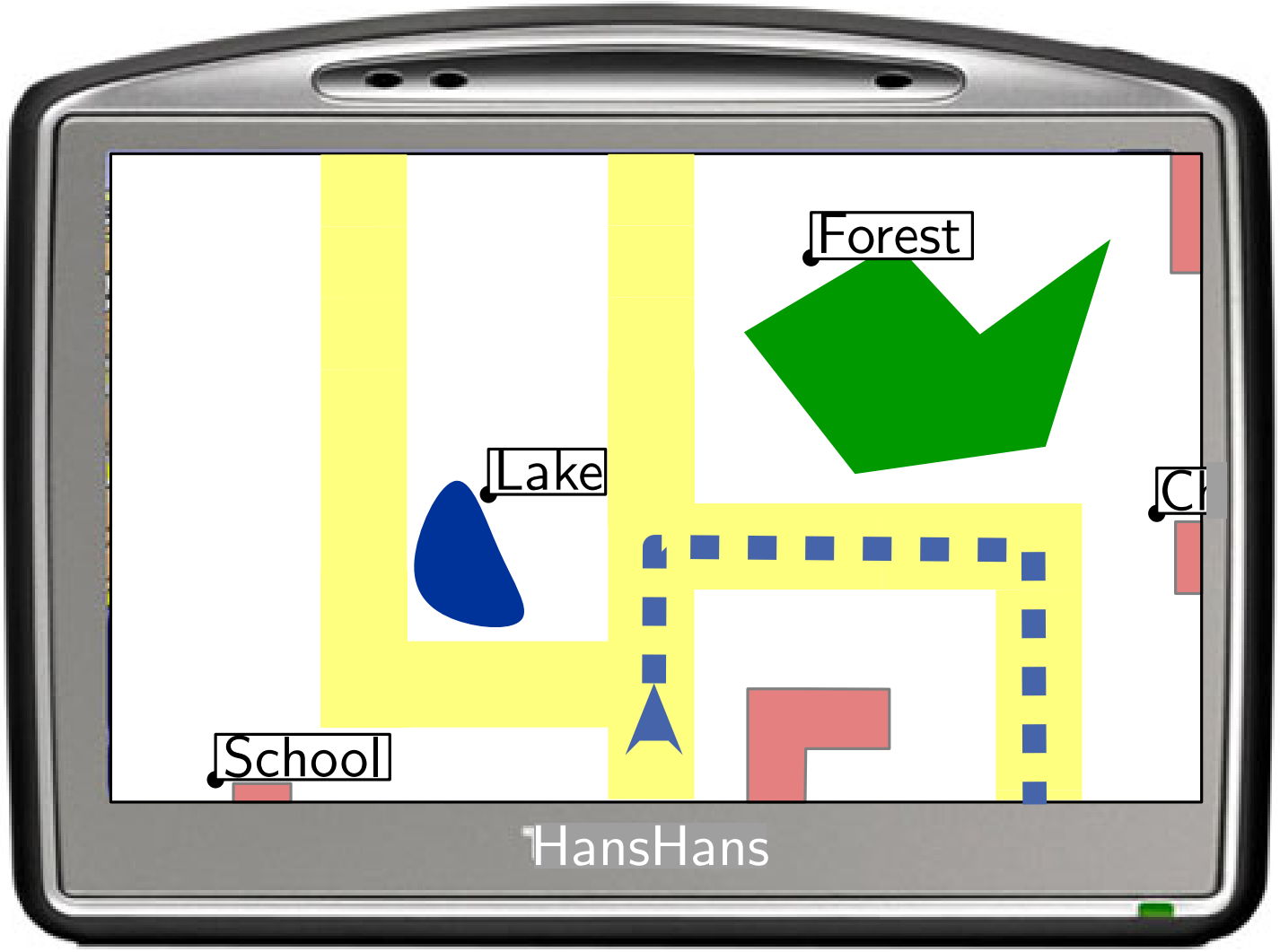

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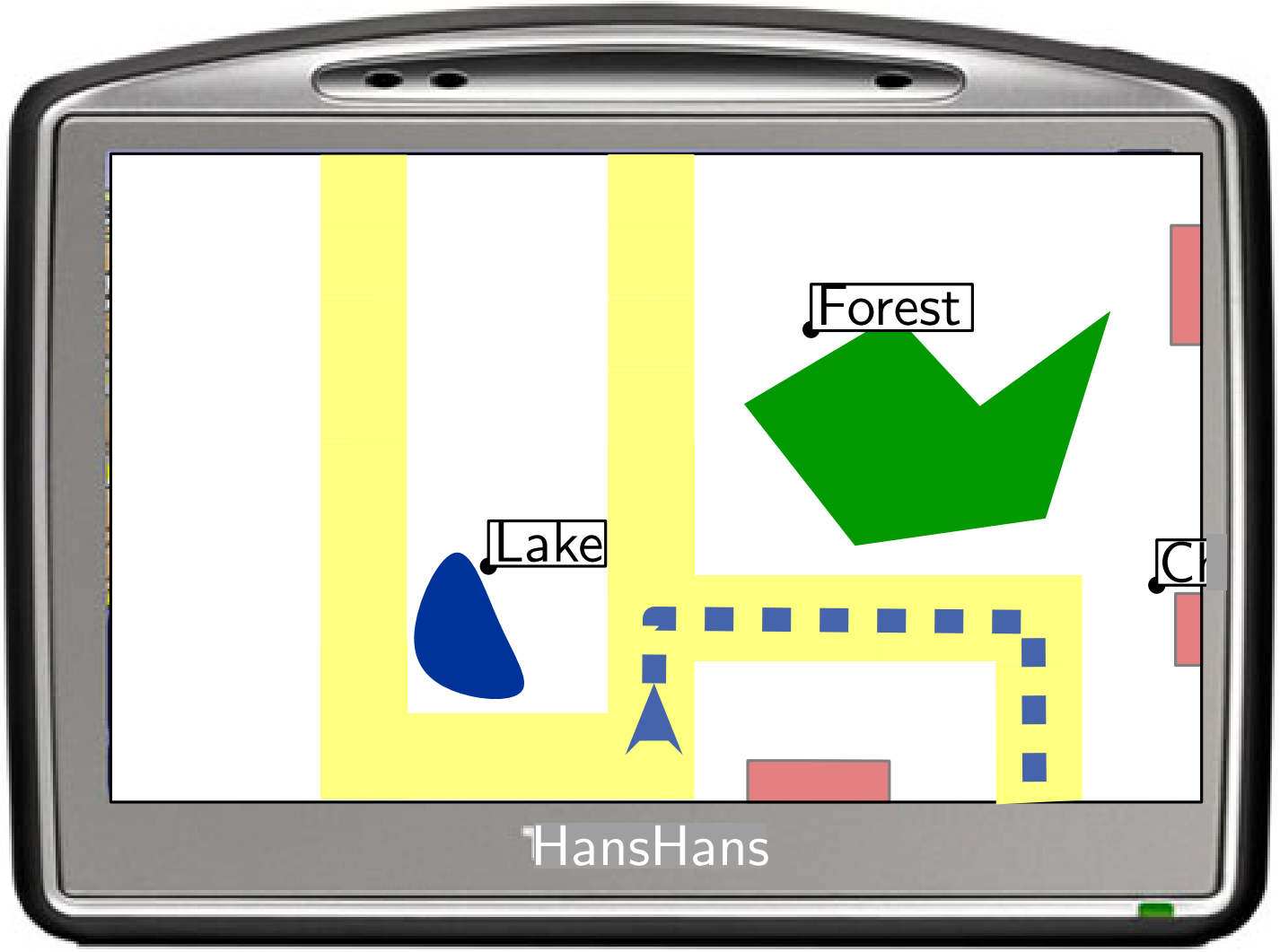

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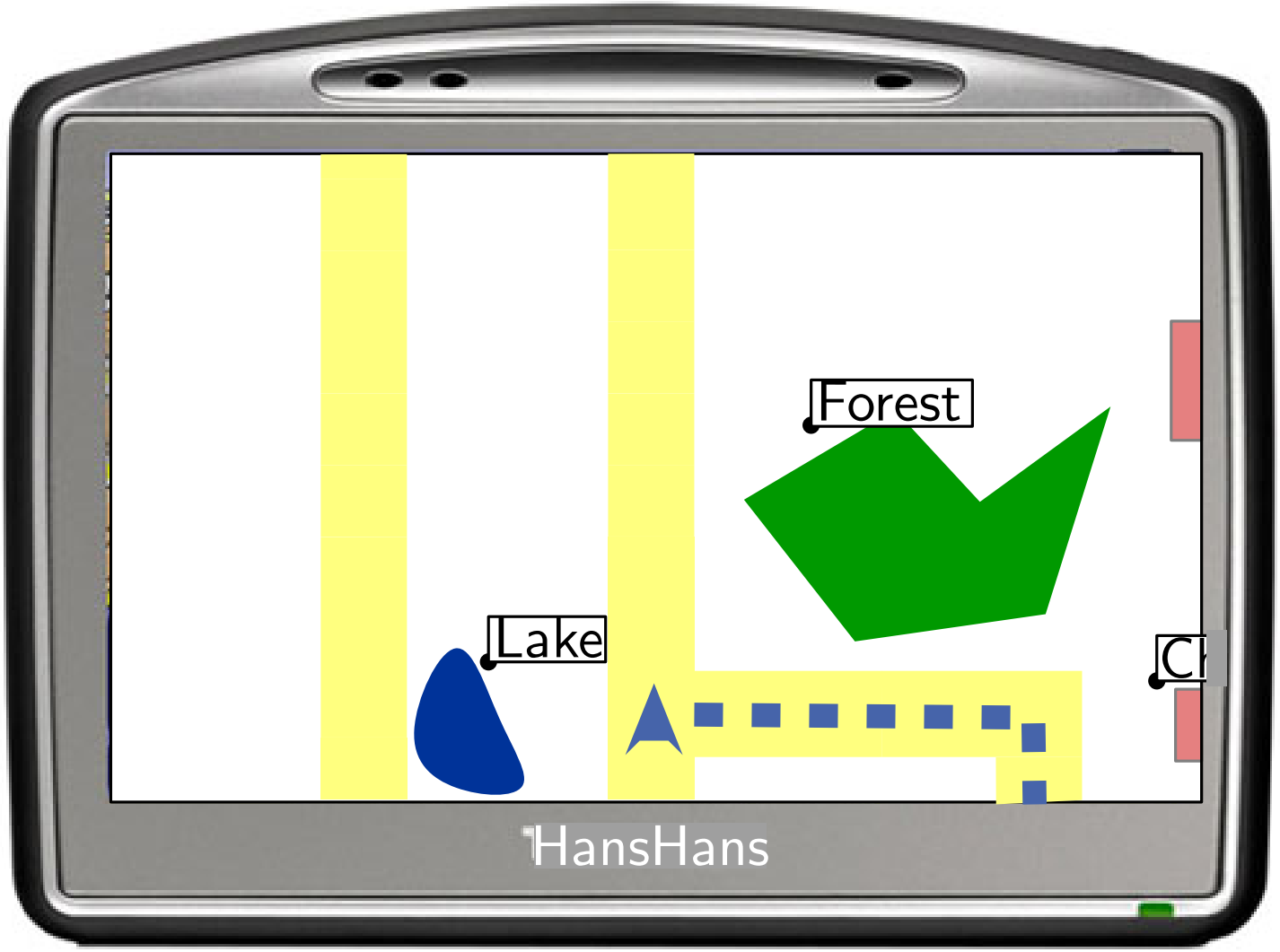

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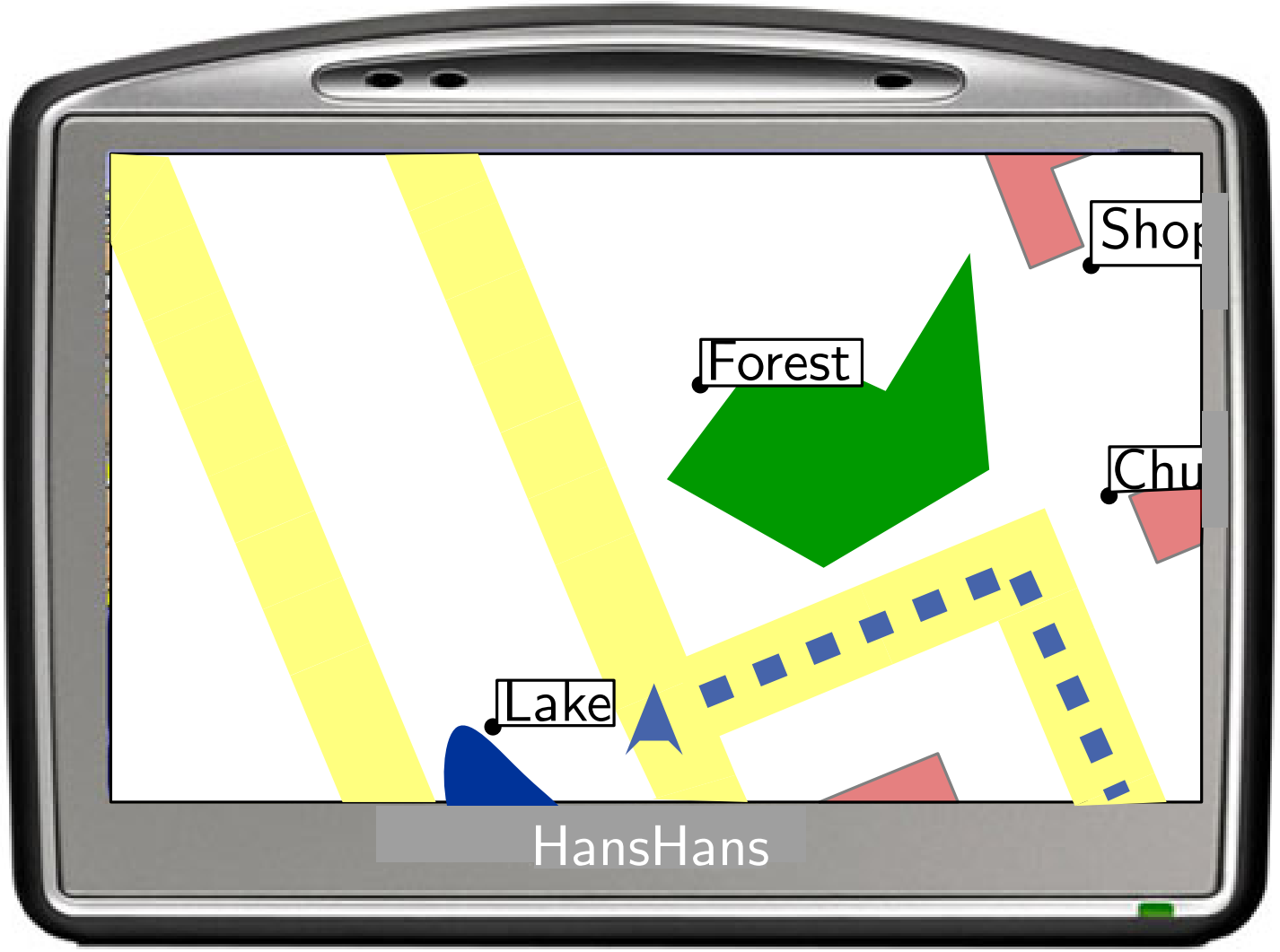

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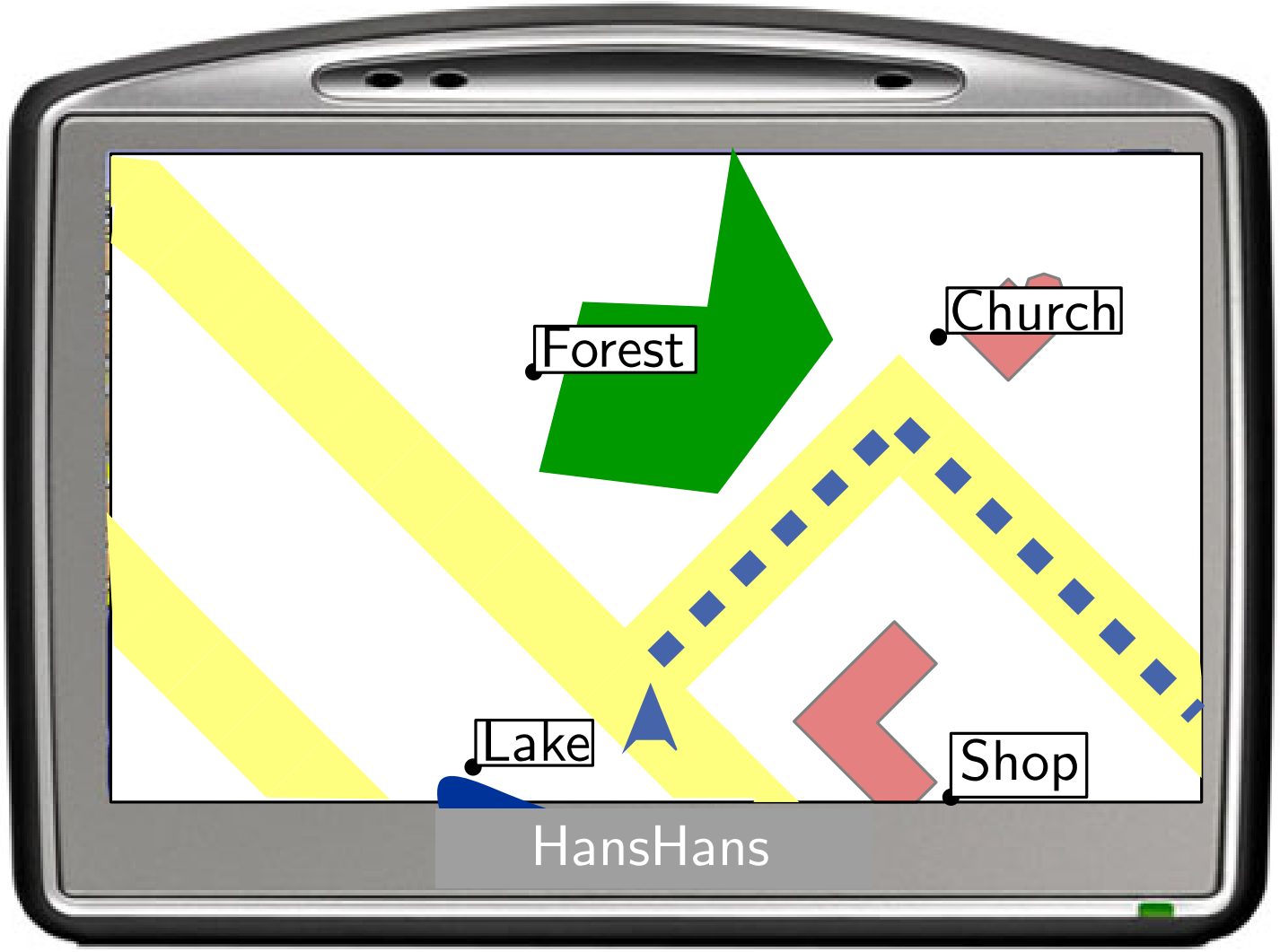

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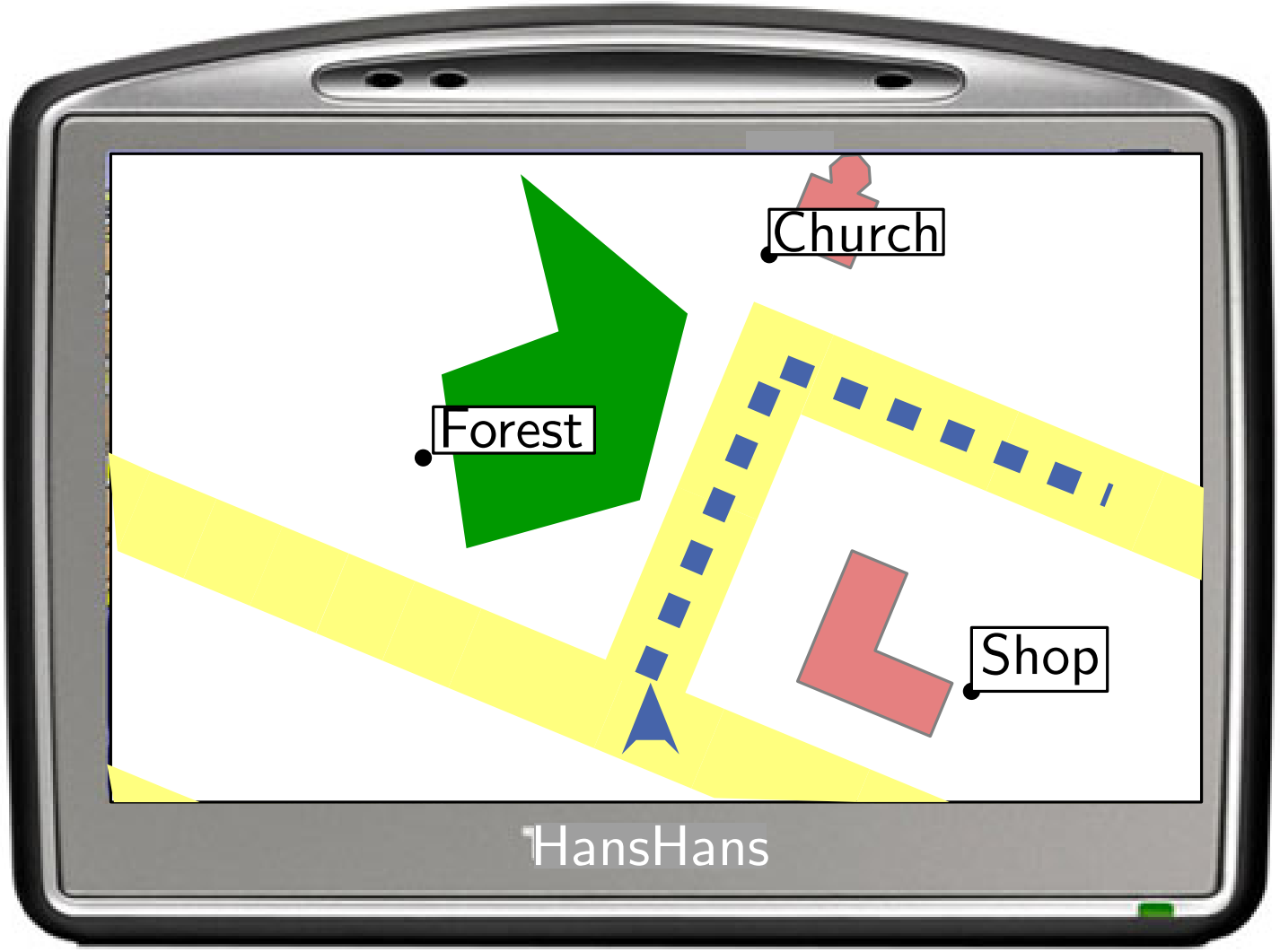

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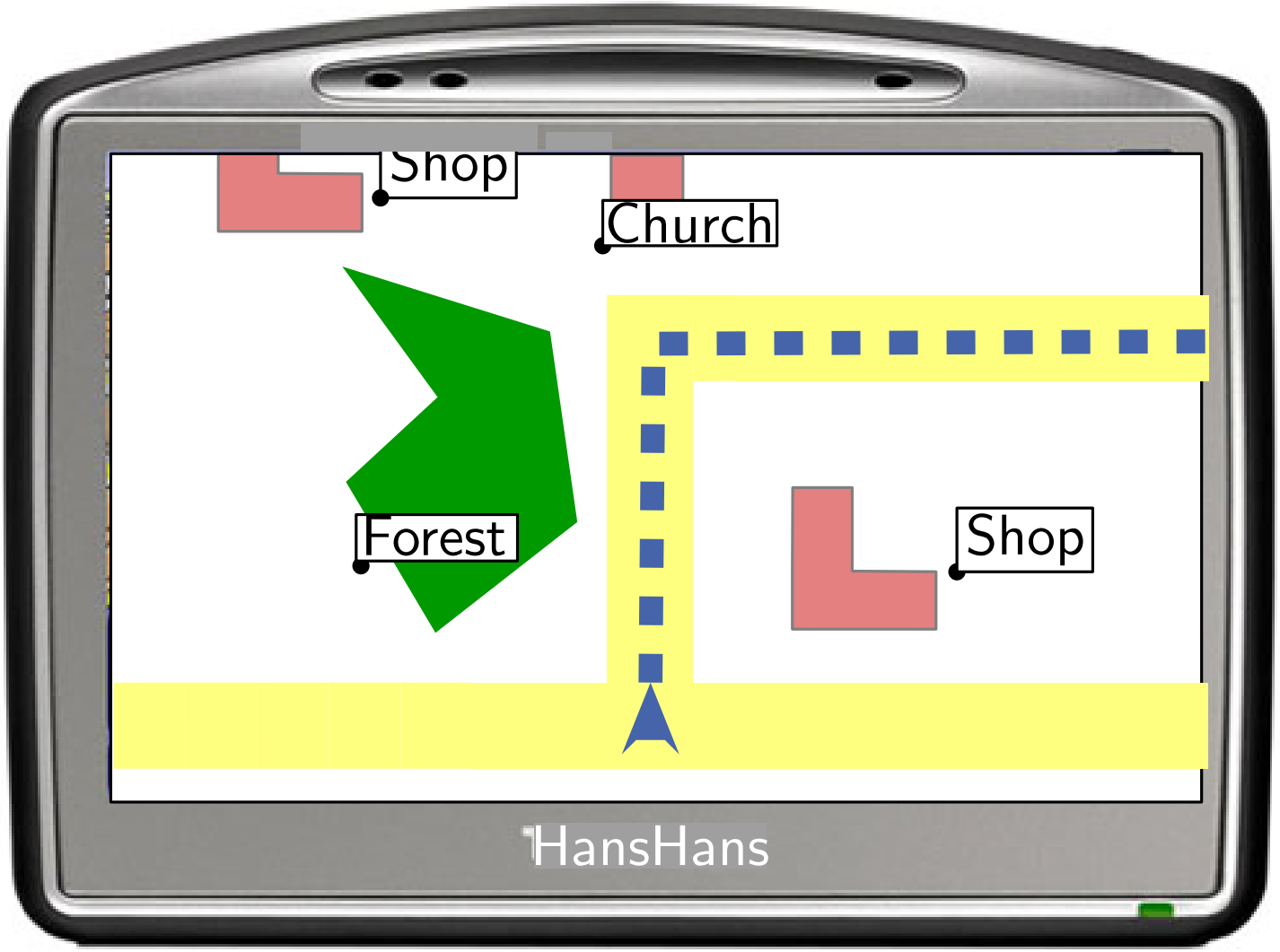

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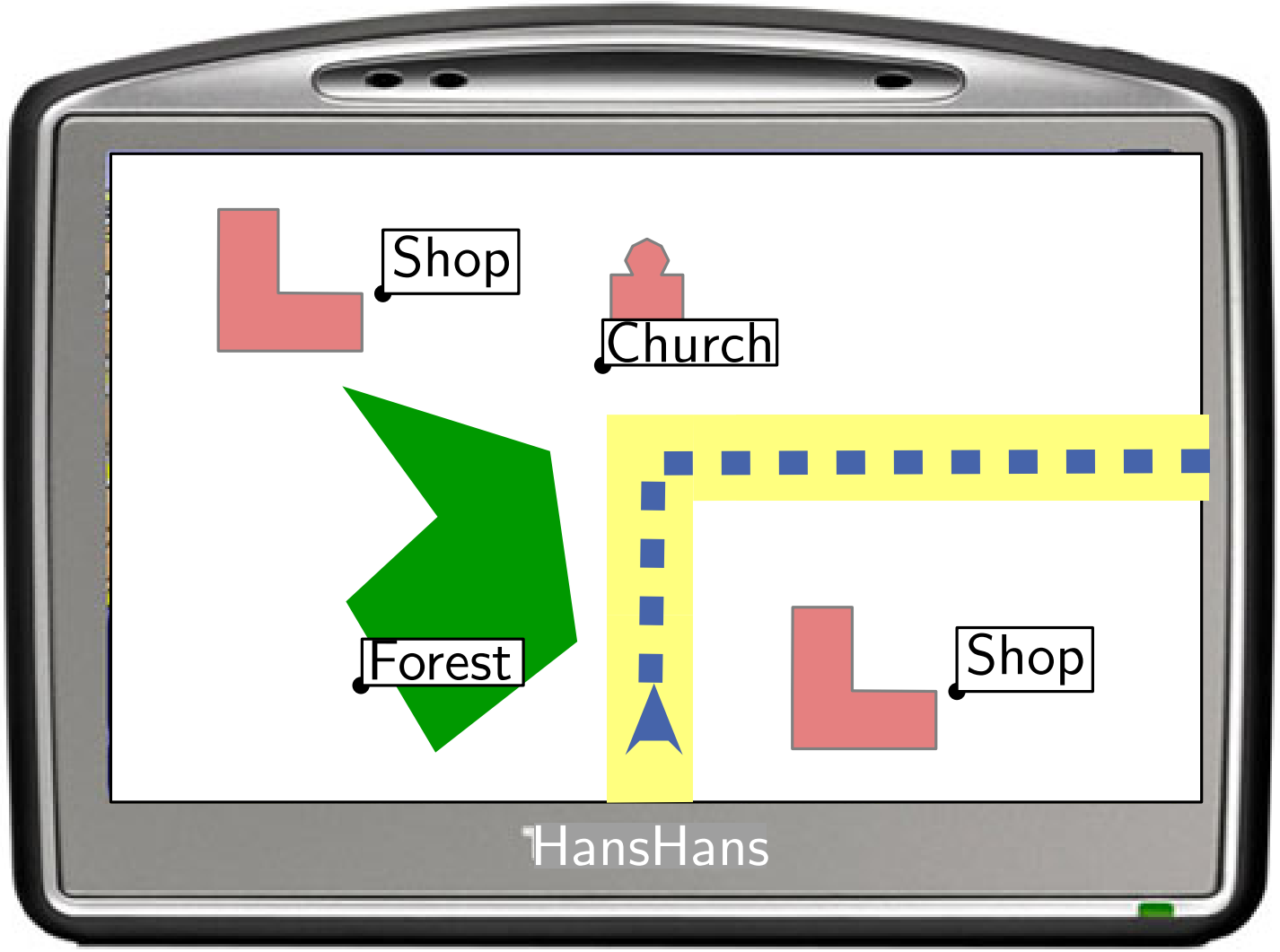

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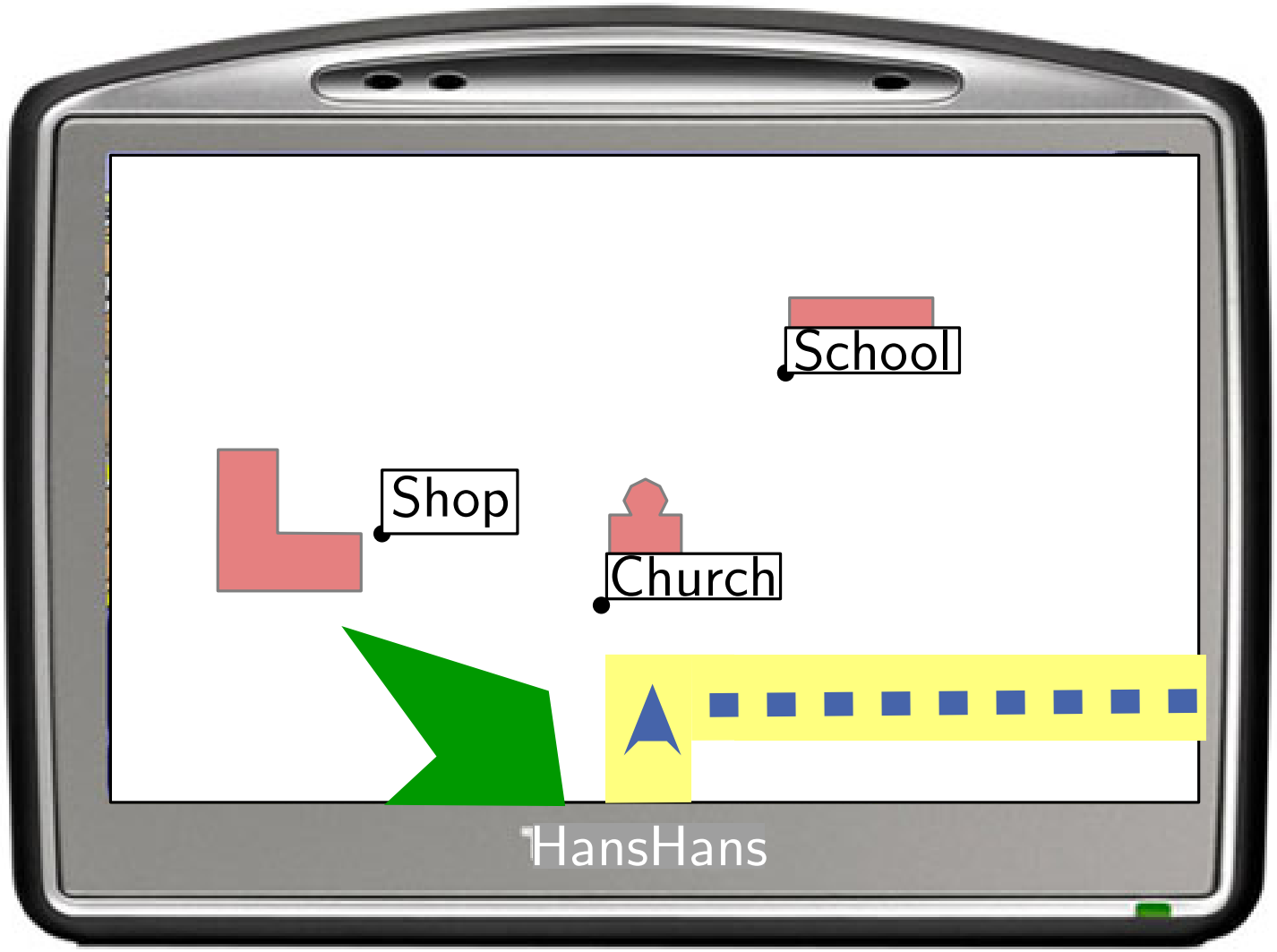

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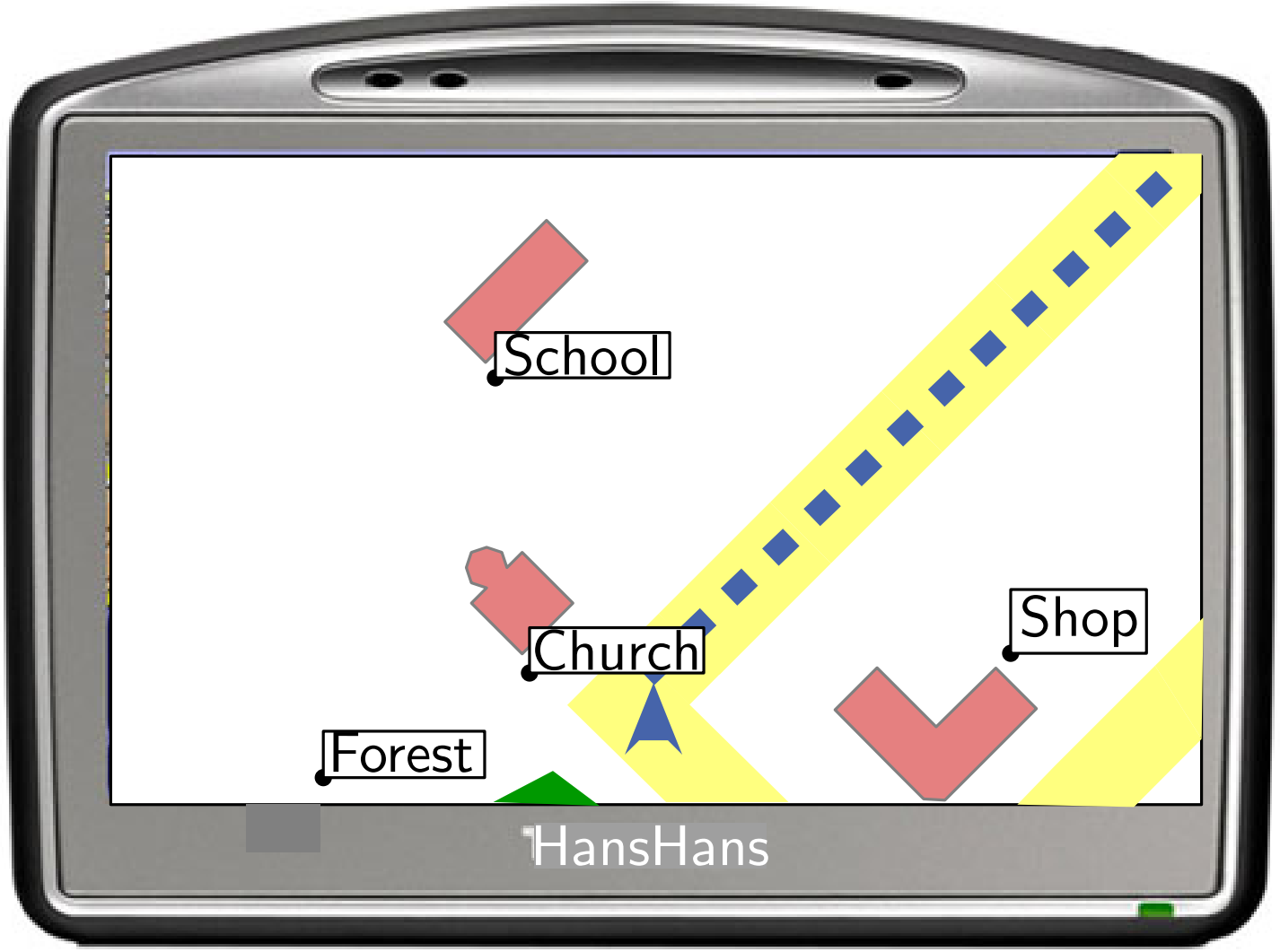

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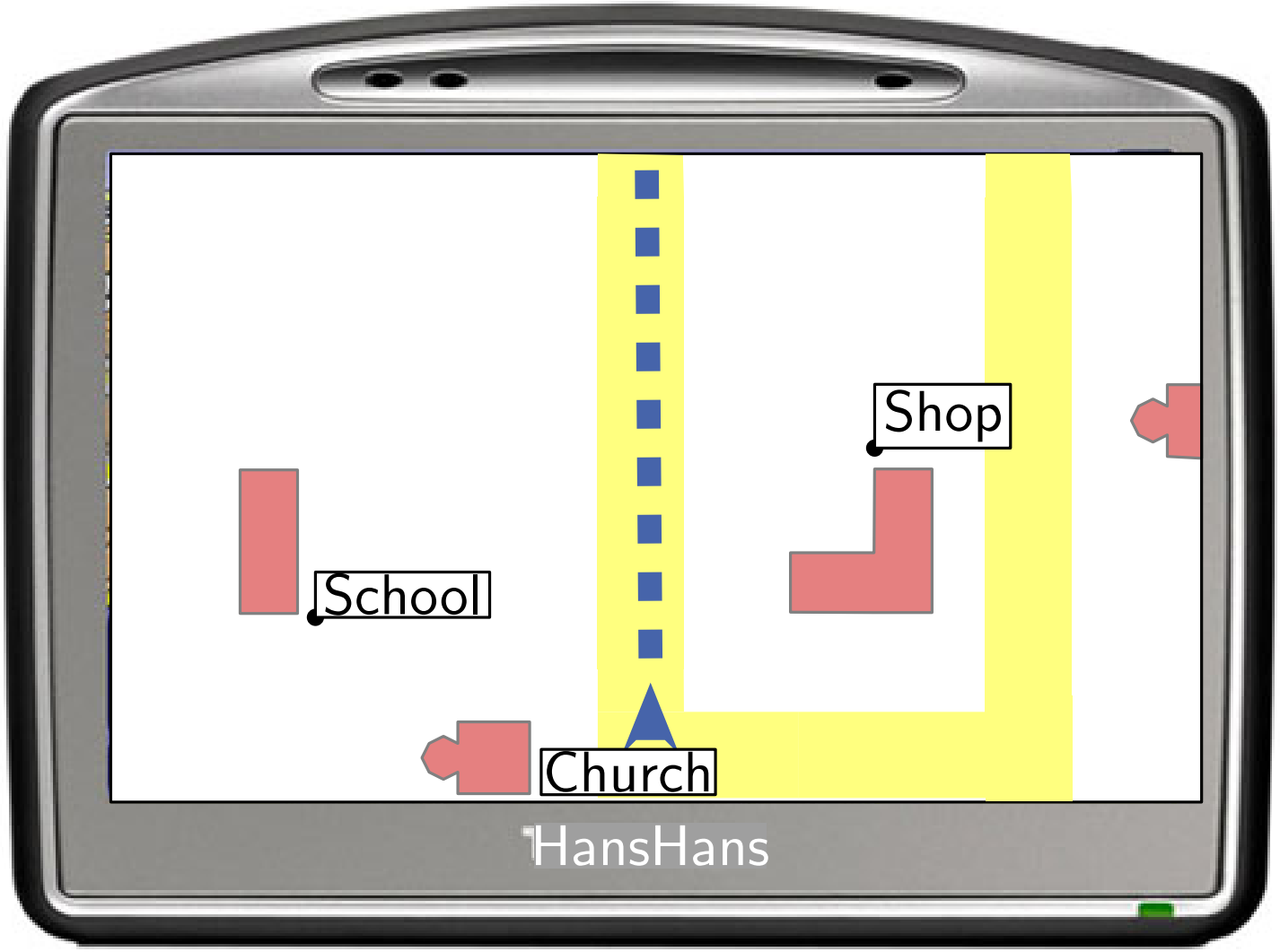

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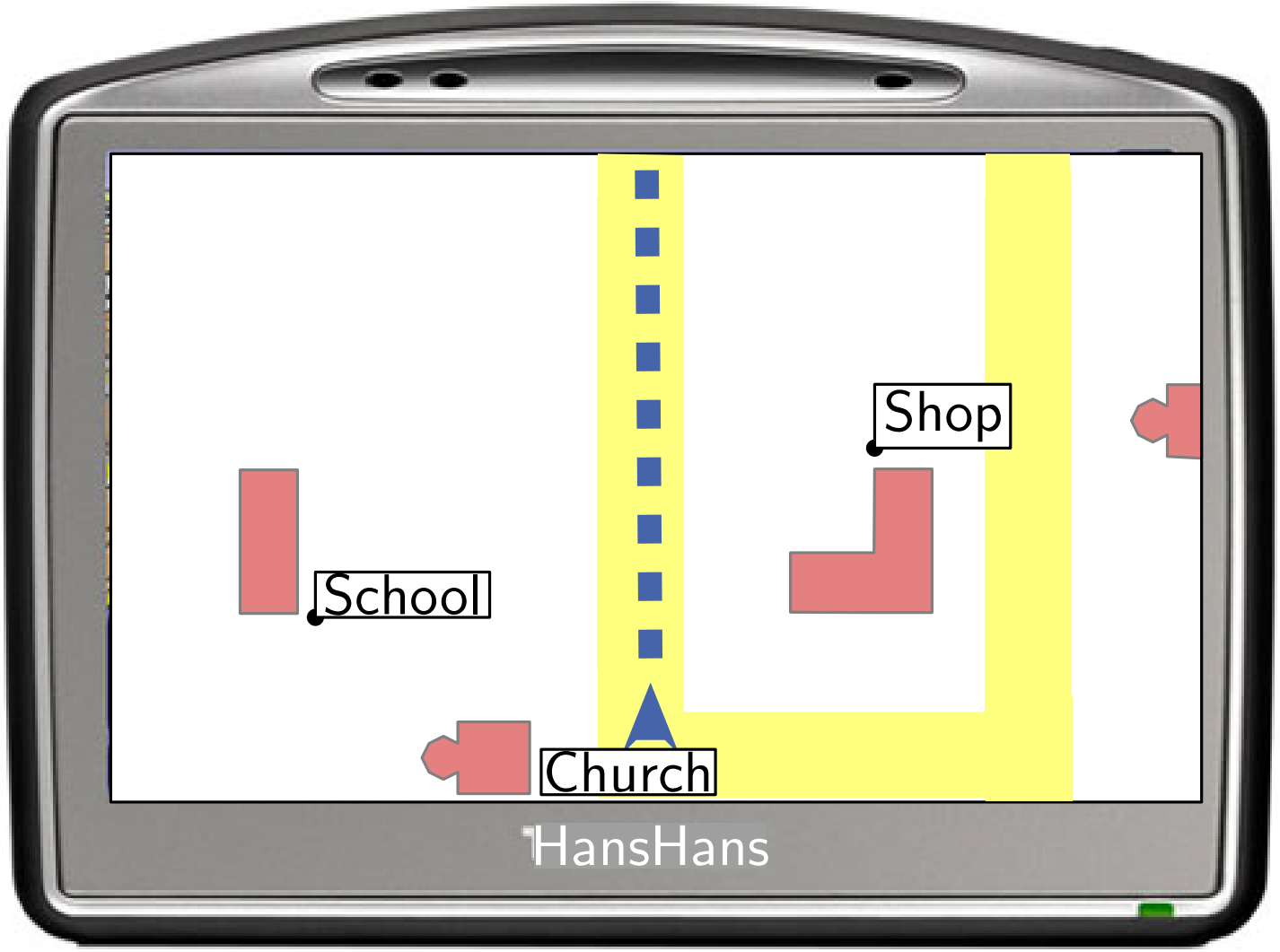

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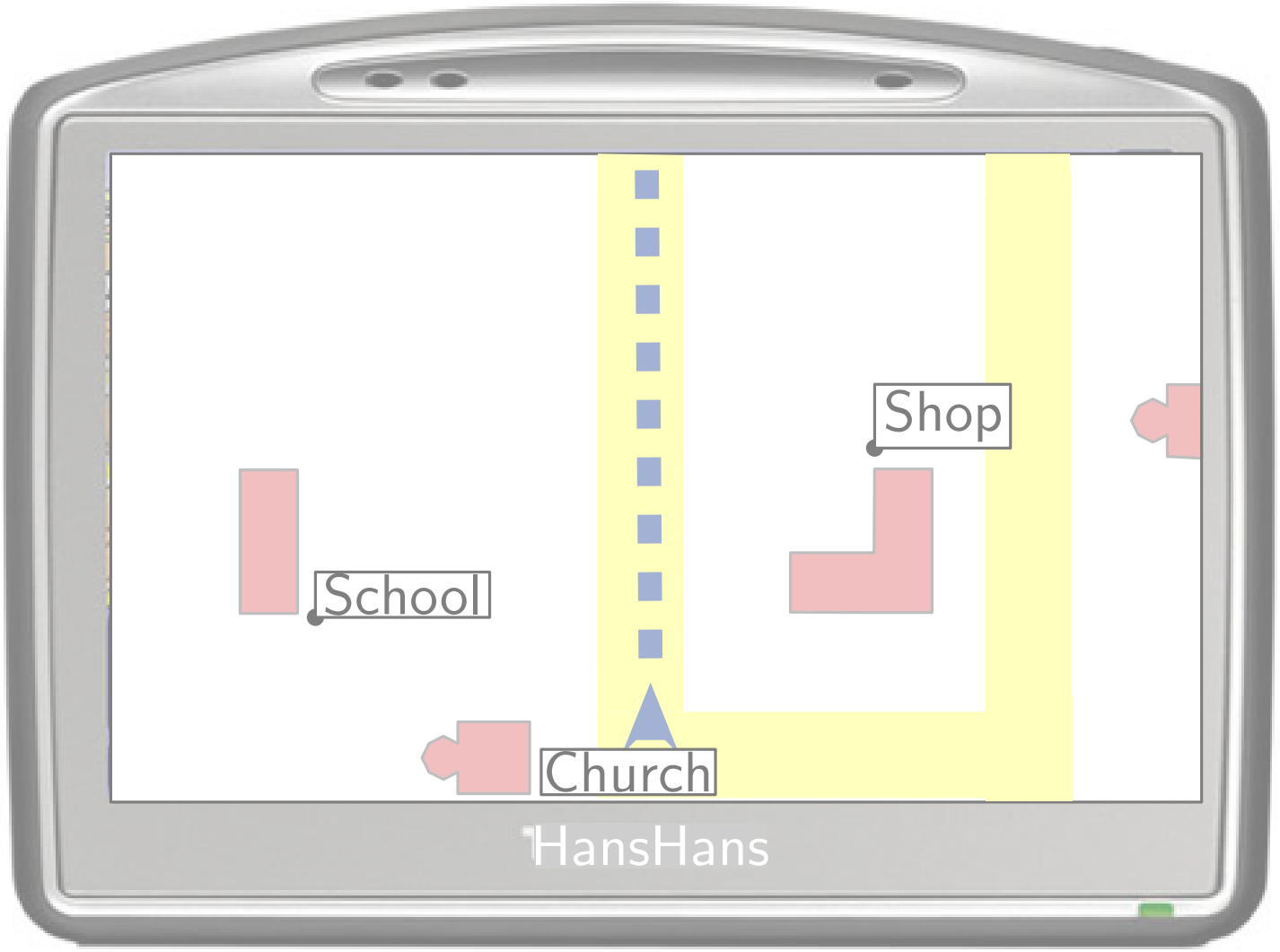

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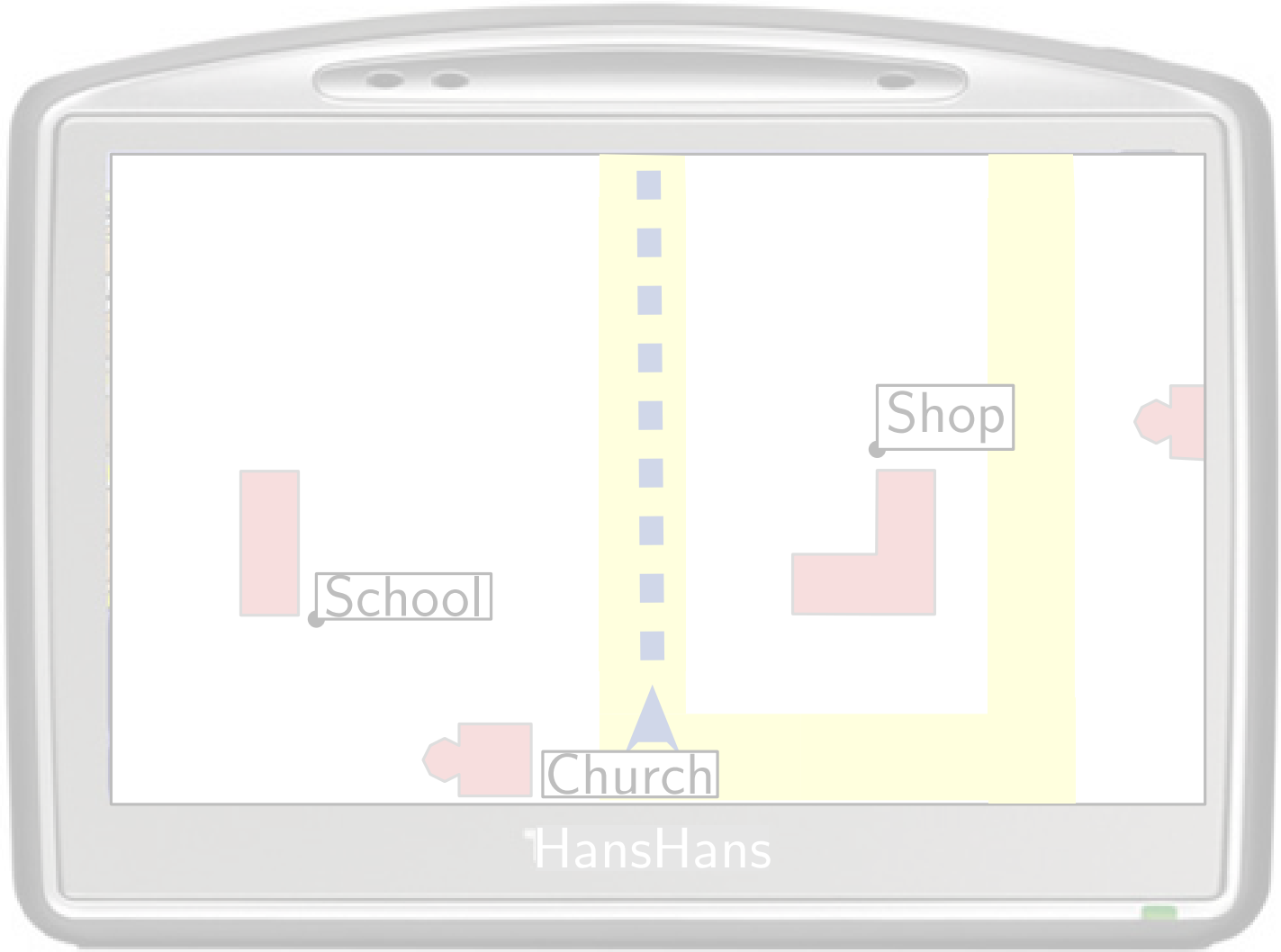
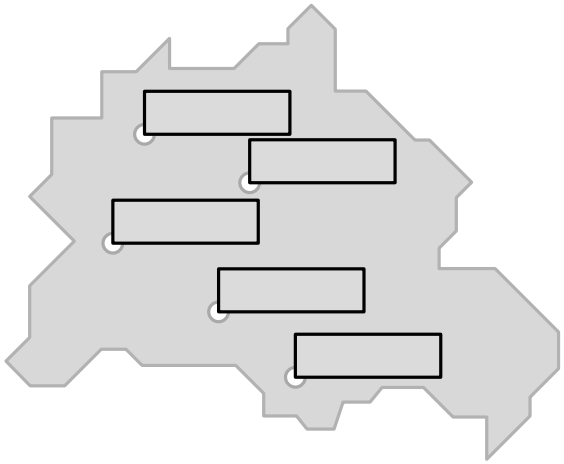


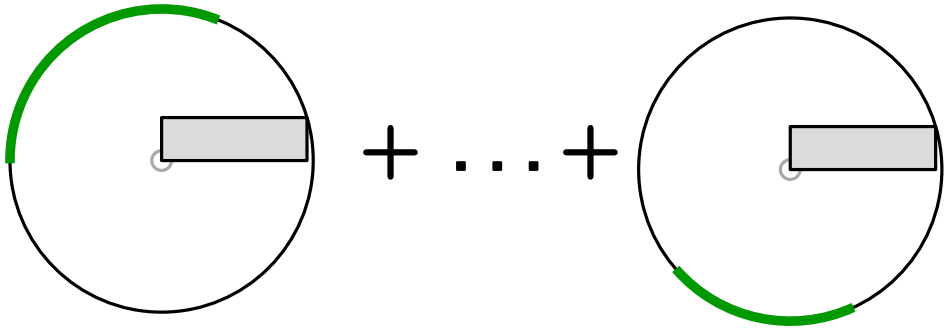
Motivation

Problem Statement

Input: A map M, points P, **valid** labeling $L(P)$



MaxTotal



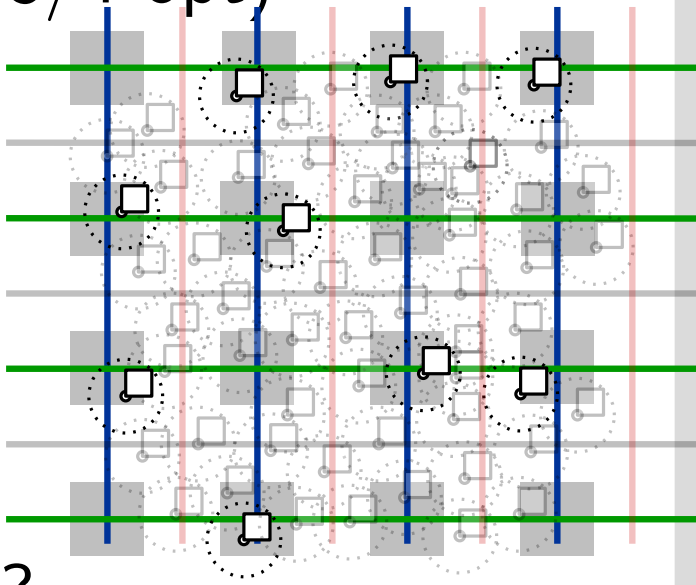
MaxTotal

Output: consistent rotation labeling that maximizes **the sum of all active ranges**

Summary I

Results:

- MaxTotal is NP-complete, MaxMin is NP-hard
- There is an EPTAS for MaxTotal
- EPTAS uses **line stabbing** (geometric problems)
- MaxMin is hard to approximate (within 3/4 opt)



Open Problems:

- Approximation algorithm for MaxMin?
- Simpler/Better algorithms for MaxTotal?

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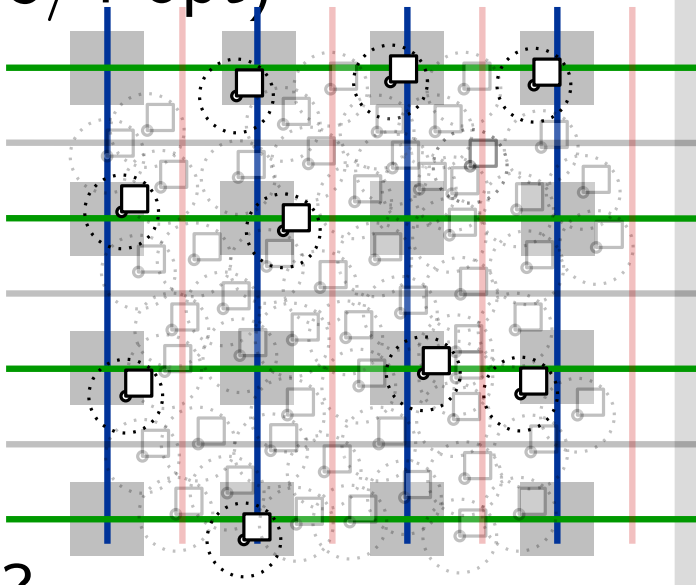

Open Problems:

- Approximation algorithm for MaxMin?
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Did we actually solve the problem?

Trajectory-Based Dynamic Map Labeling

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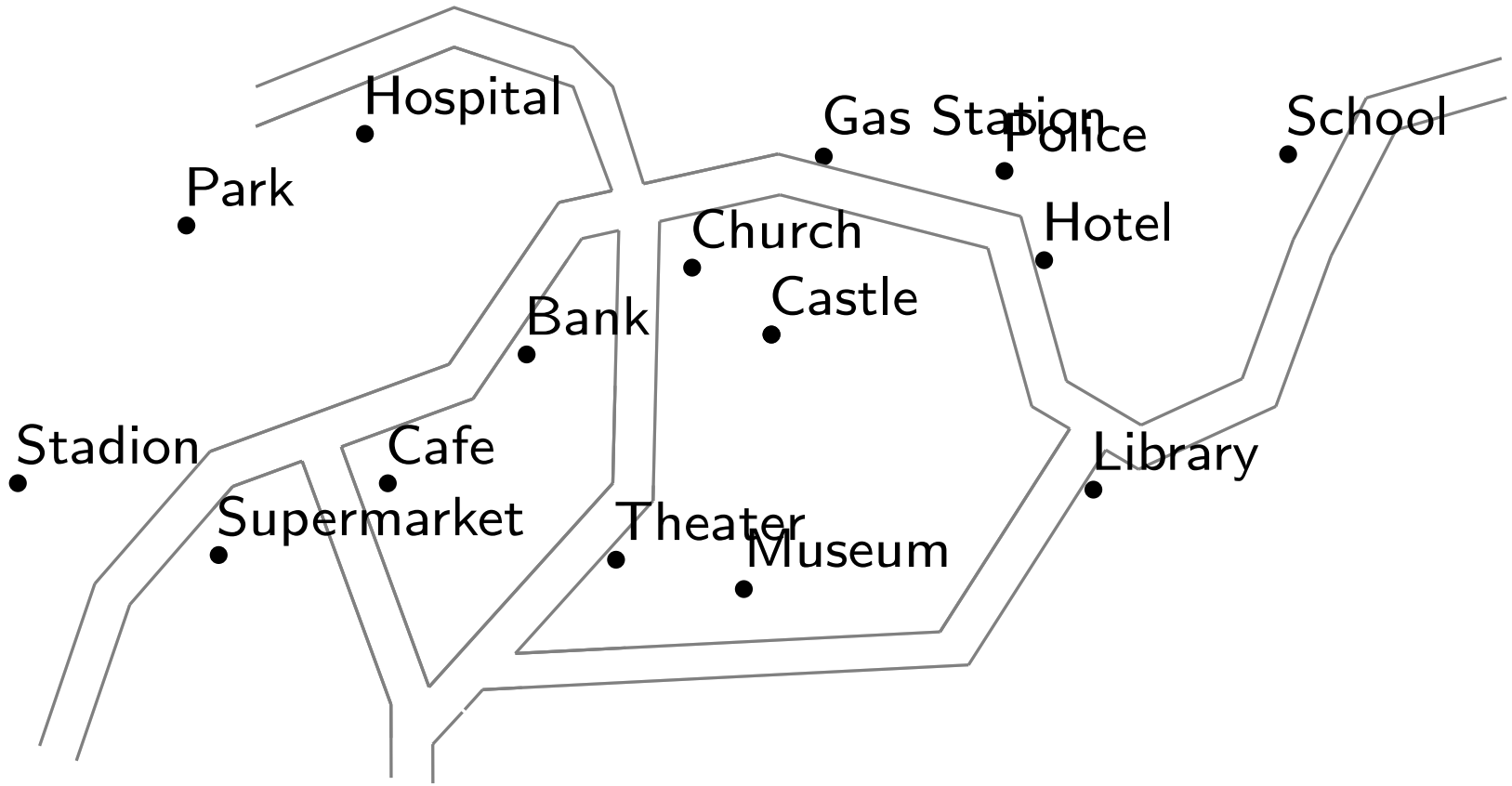

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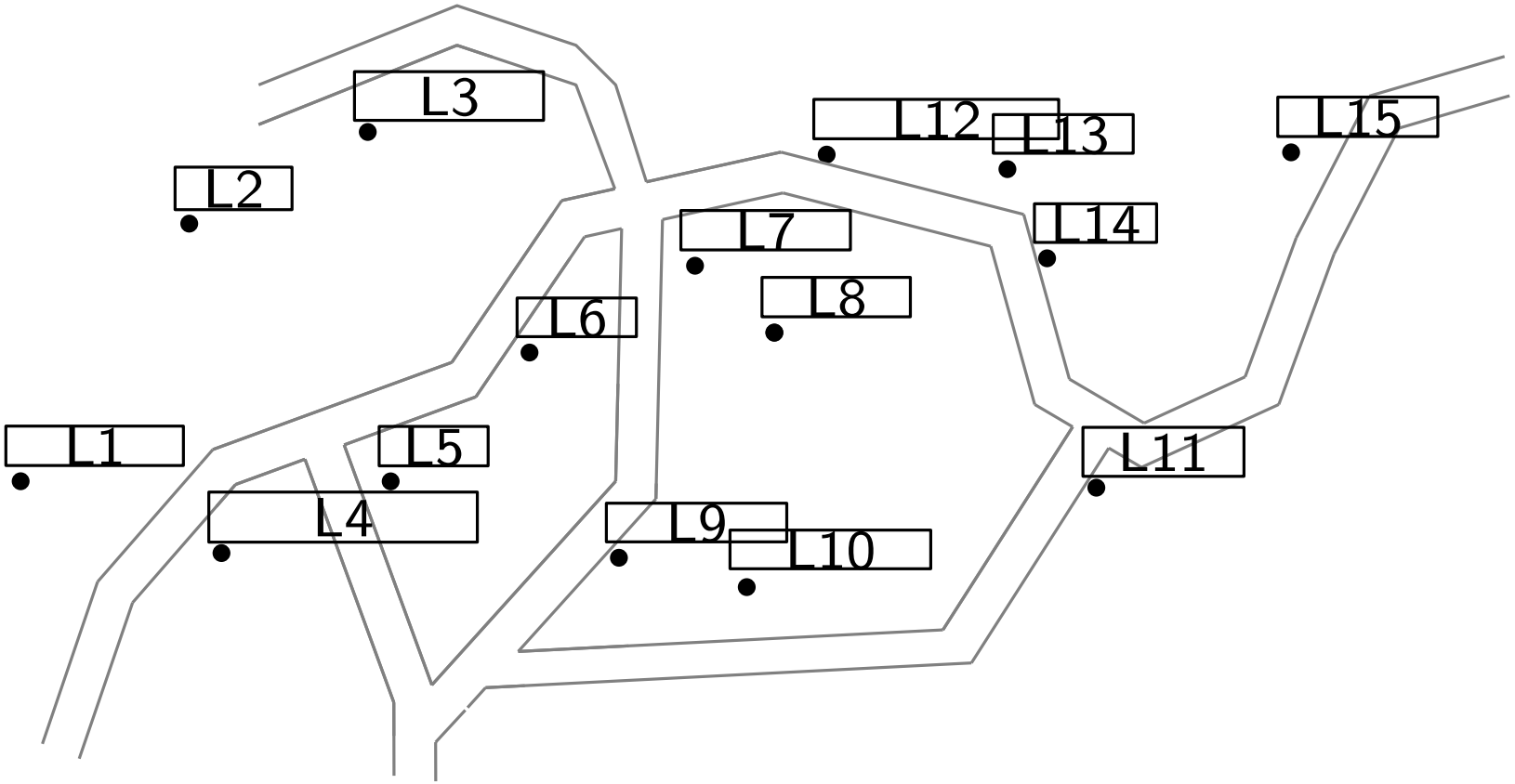

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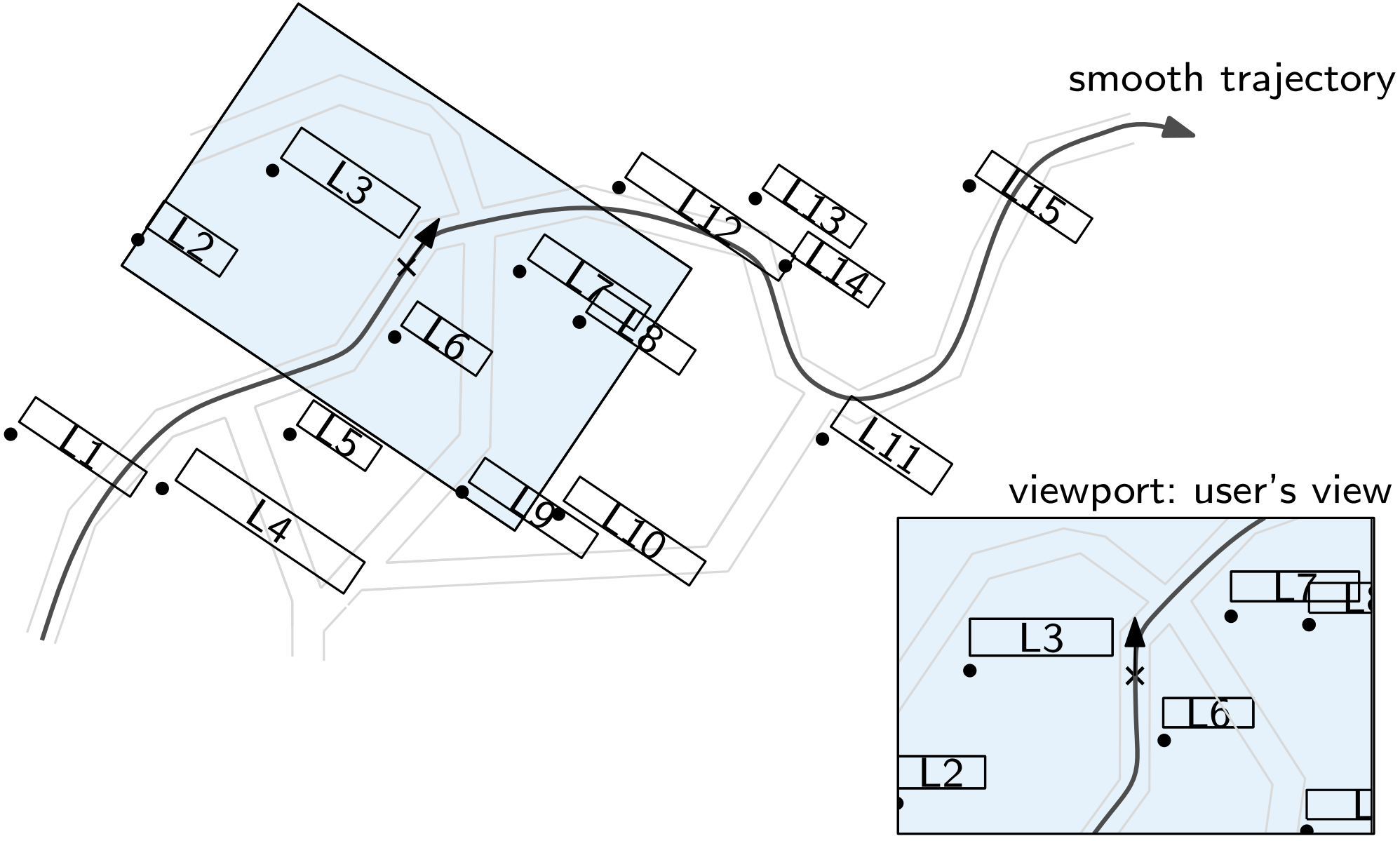

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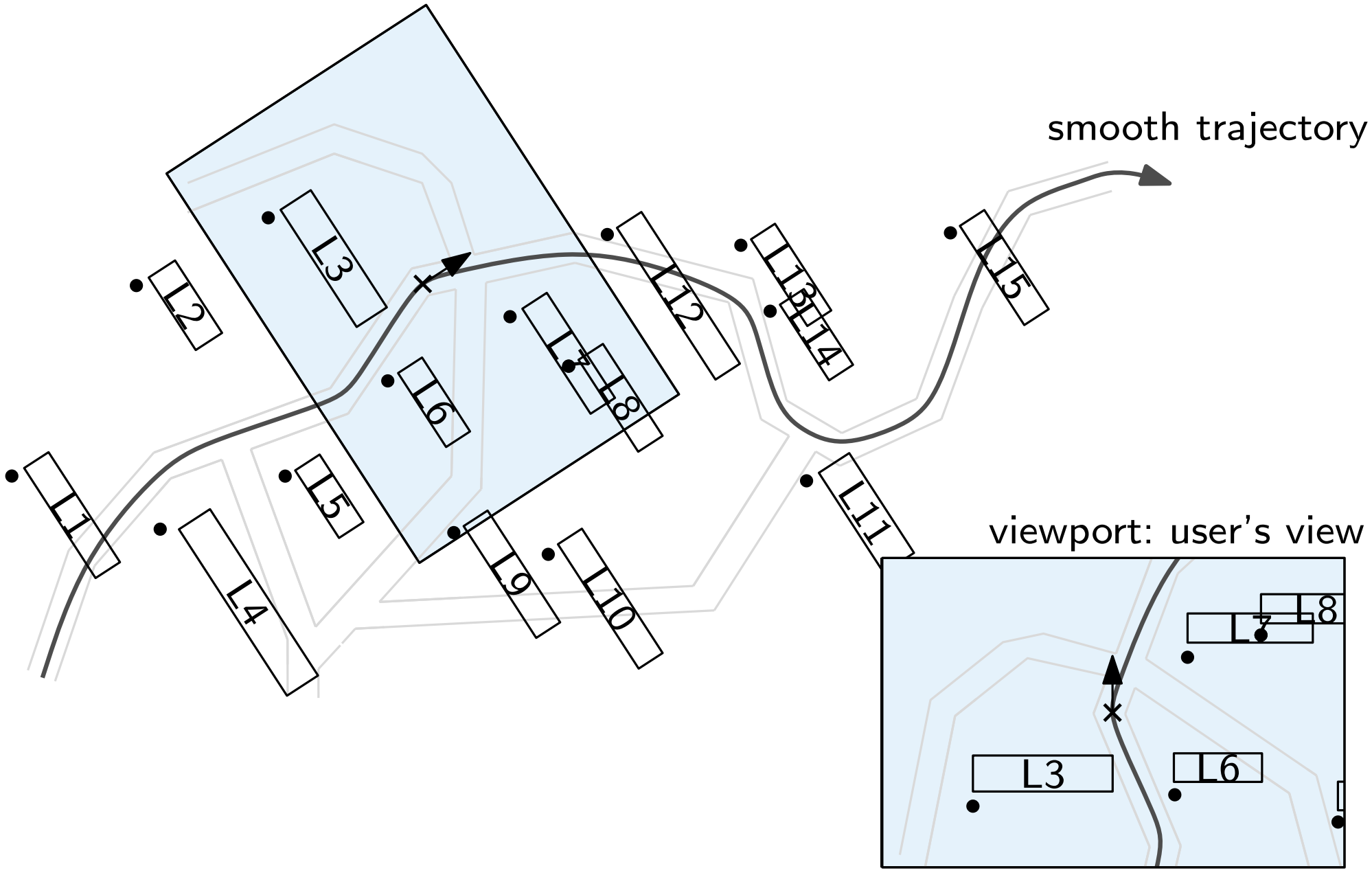

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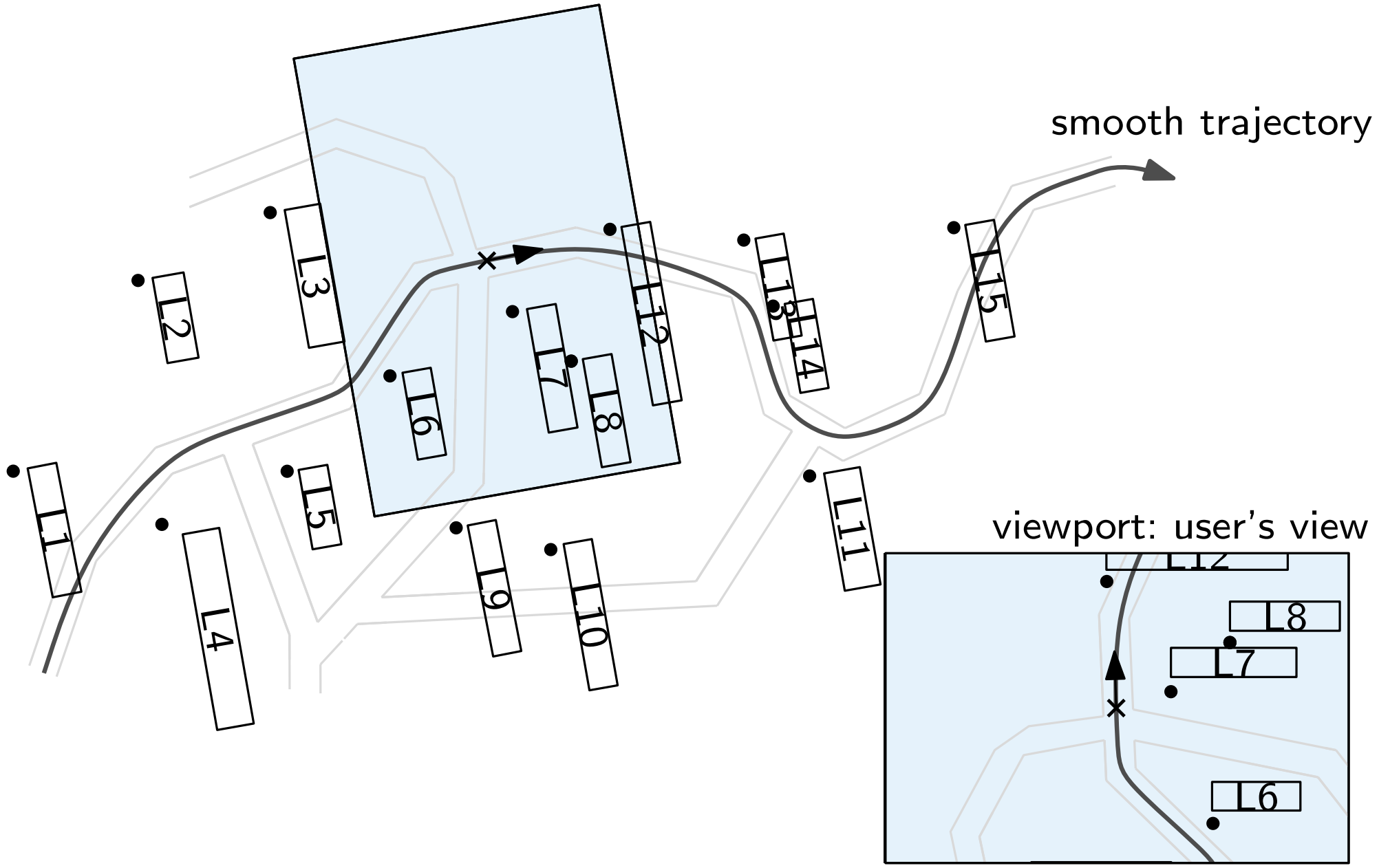

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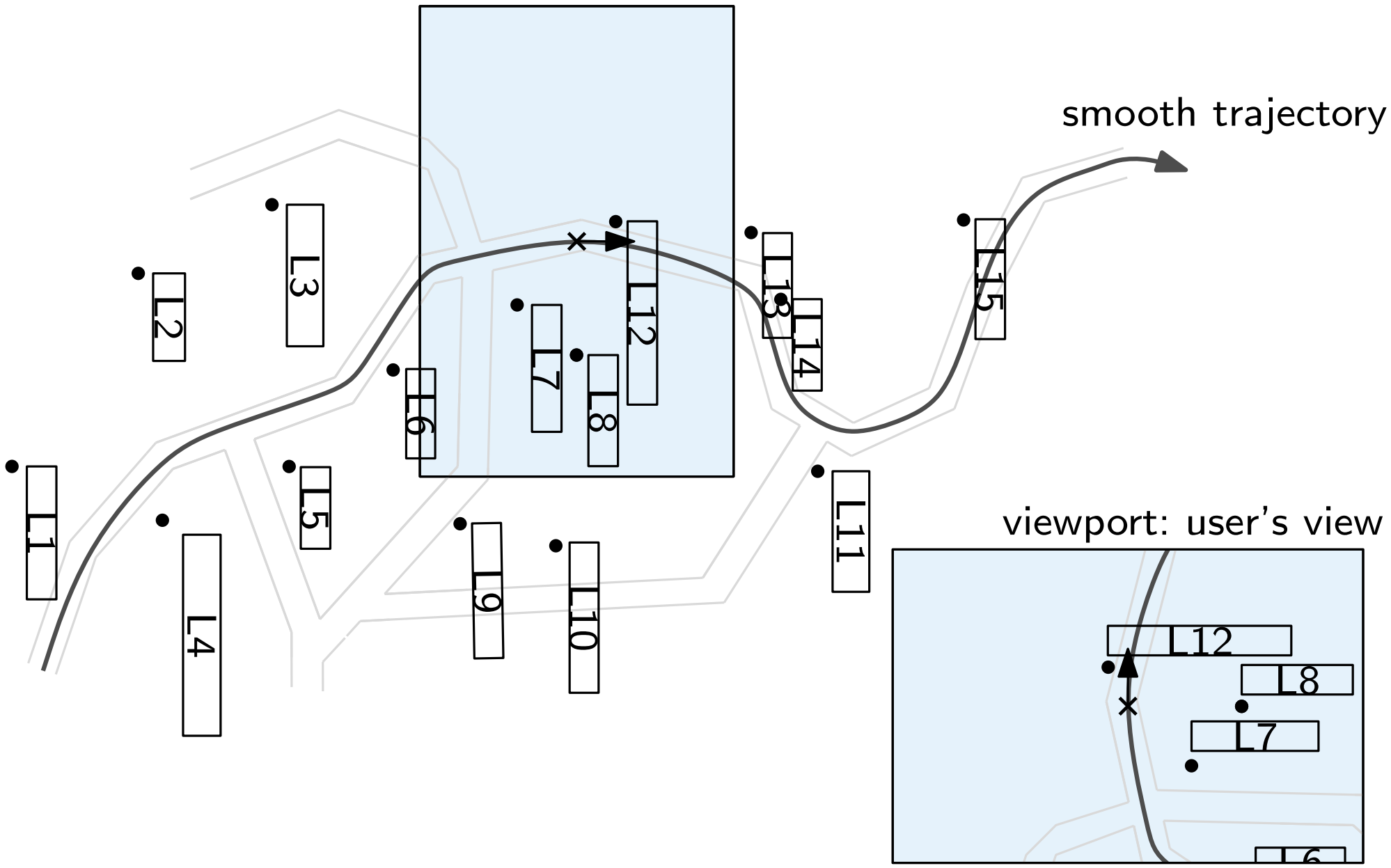

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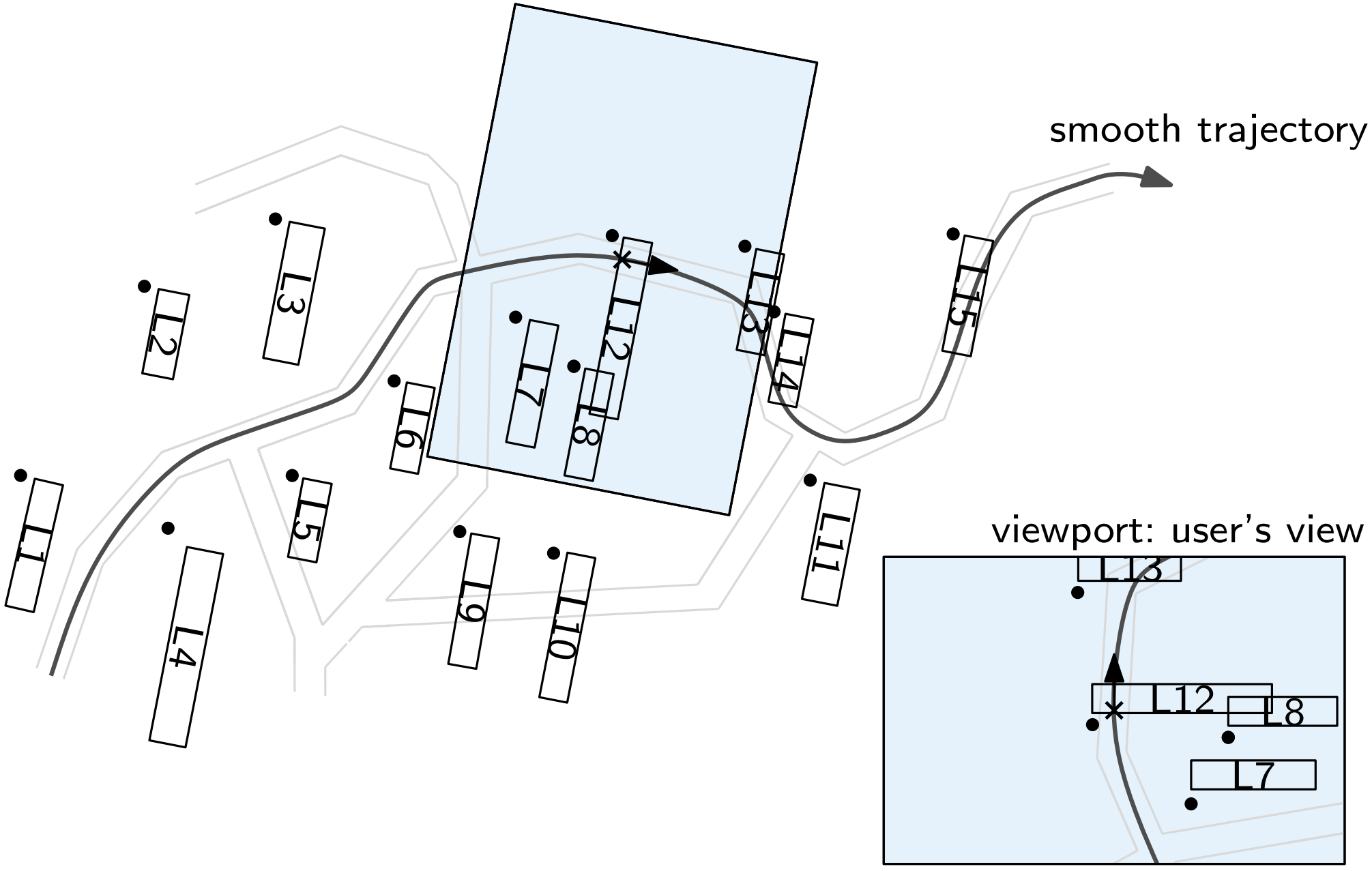

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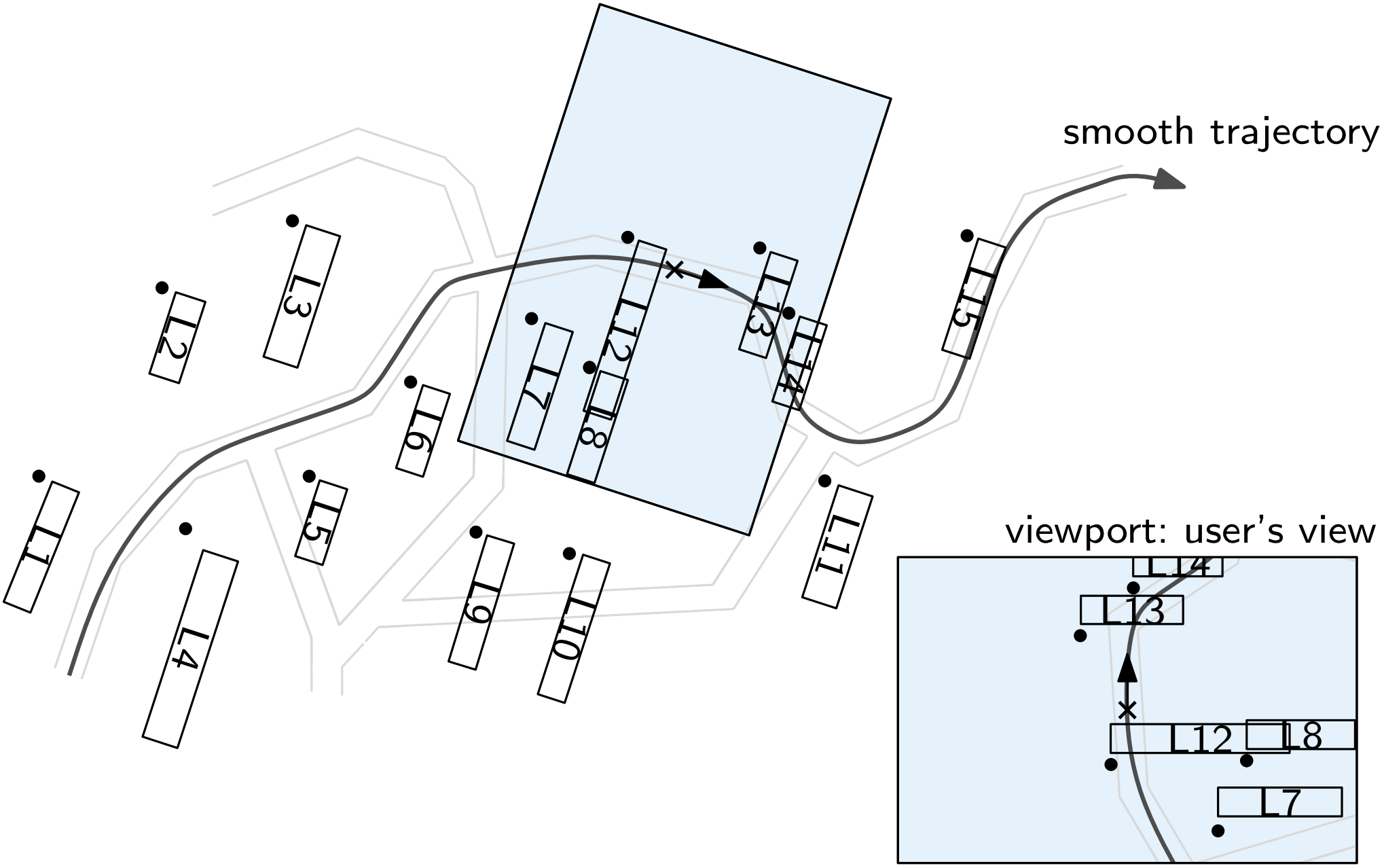

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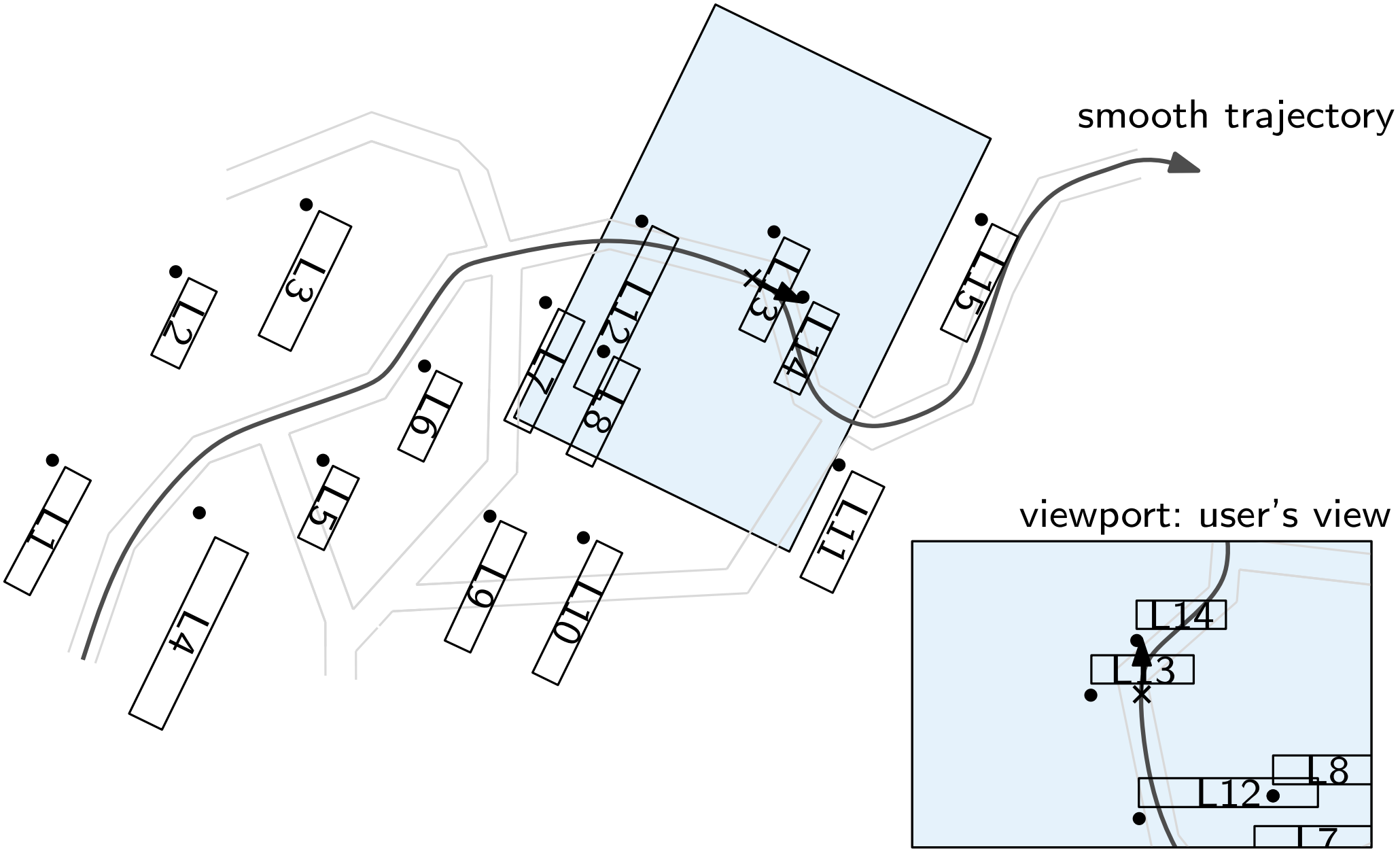

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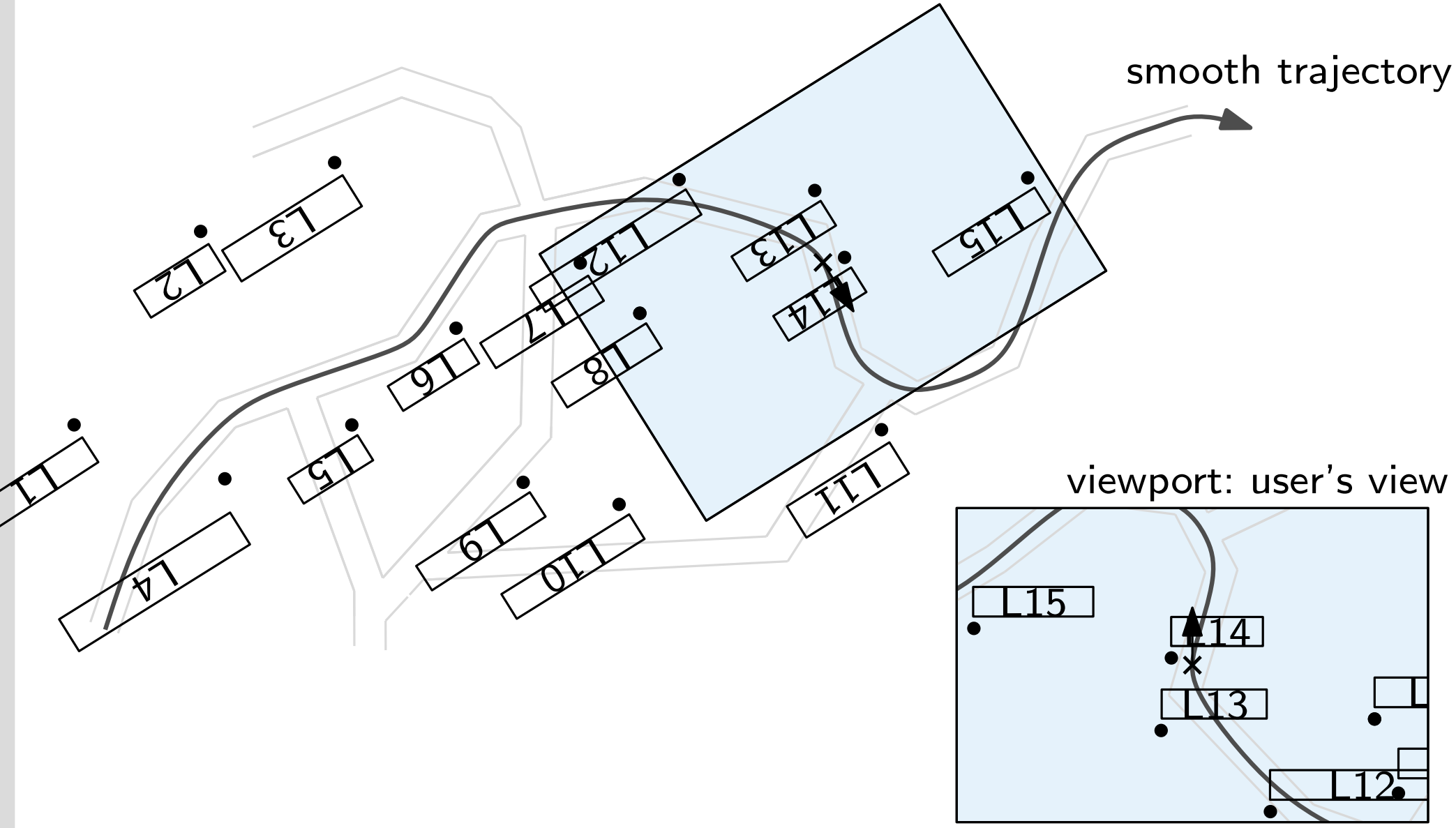

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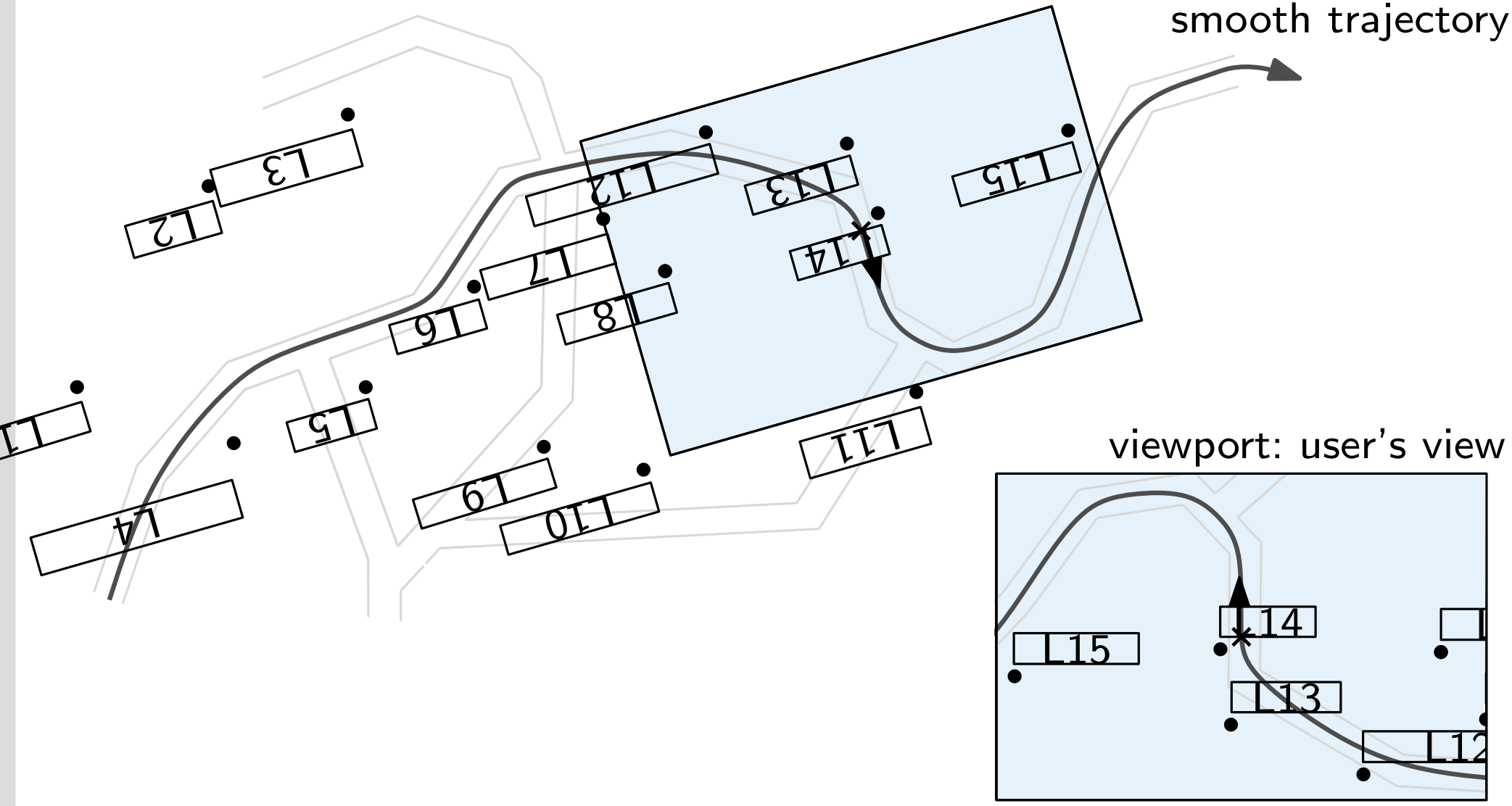

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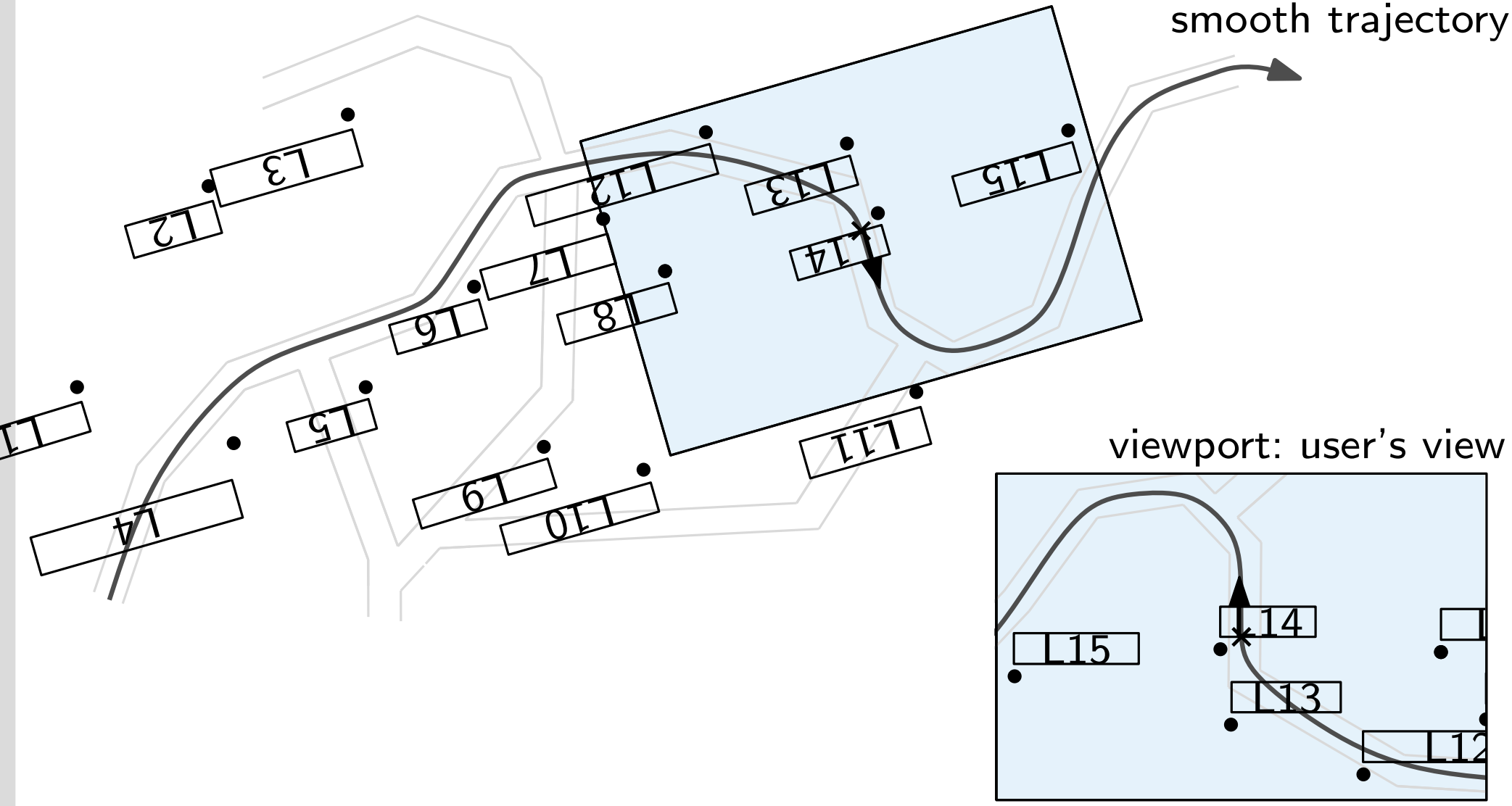
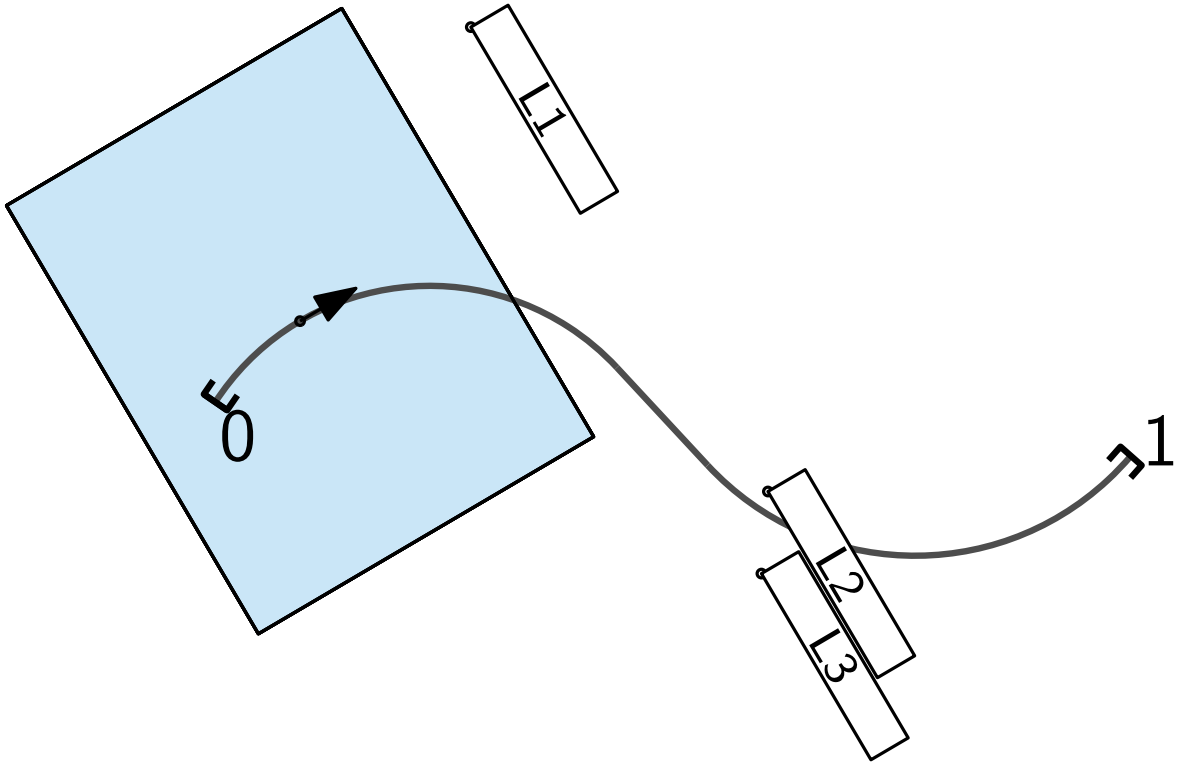
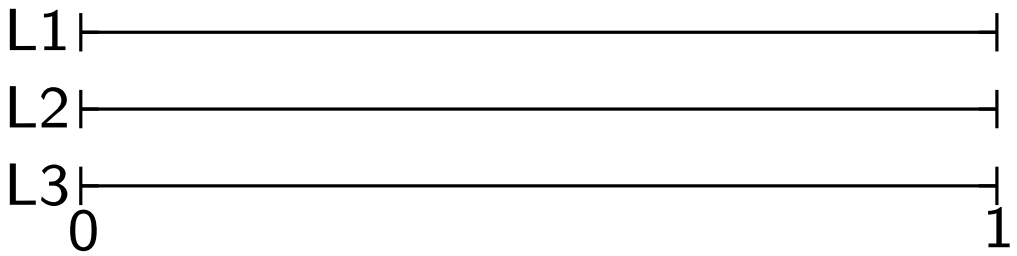


Mapping to Intervals

Label is **present** at time t if it intersects the viewport at time t



Time:



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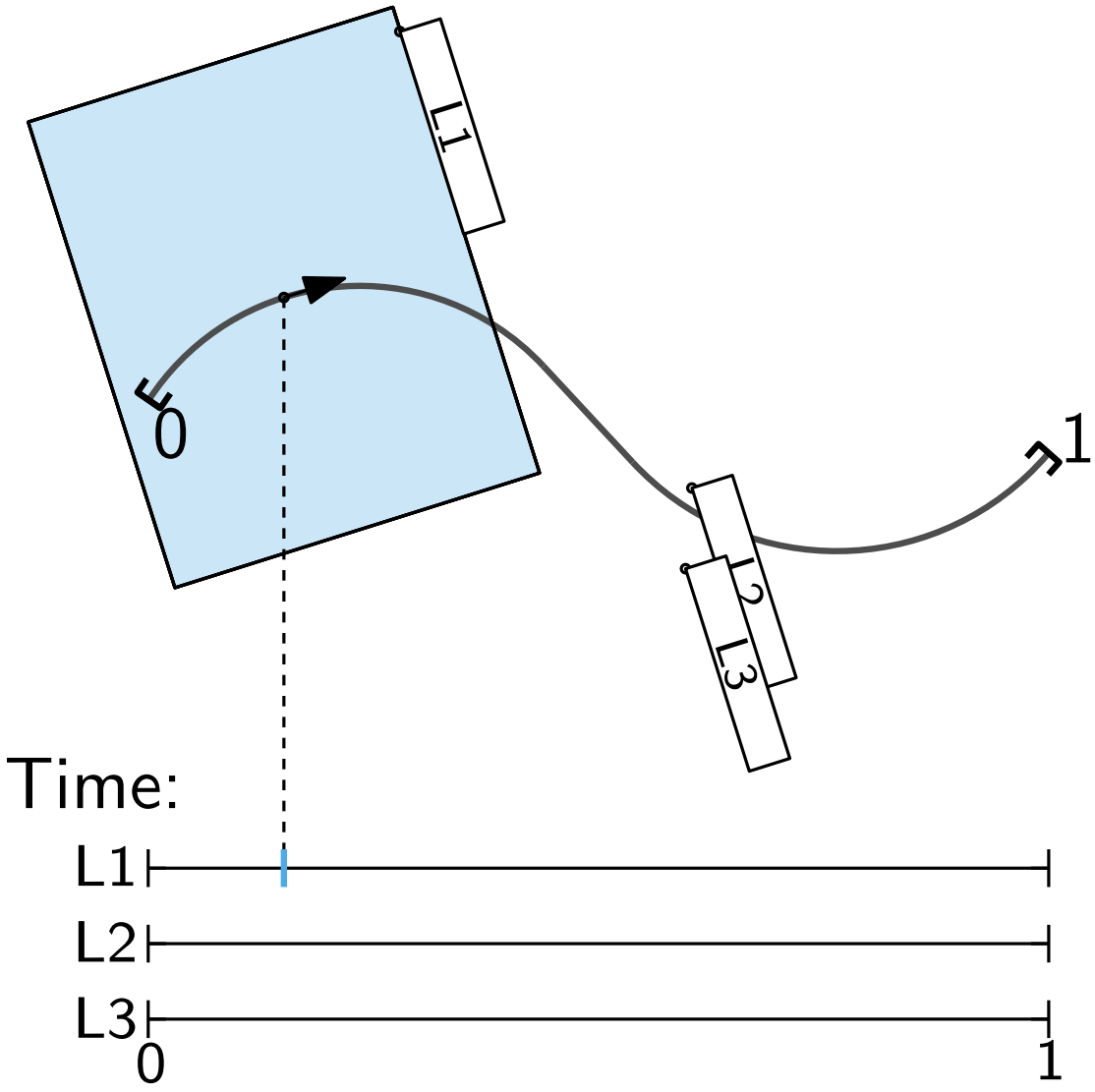

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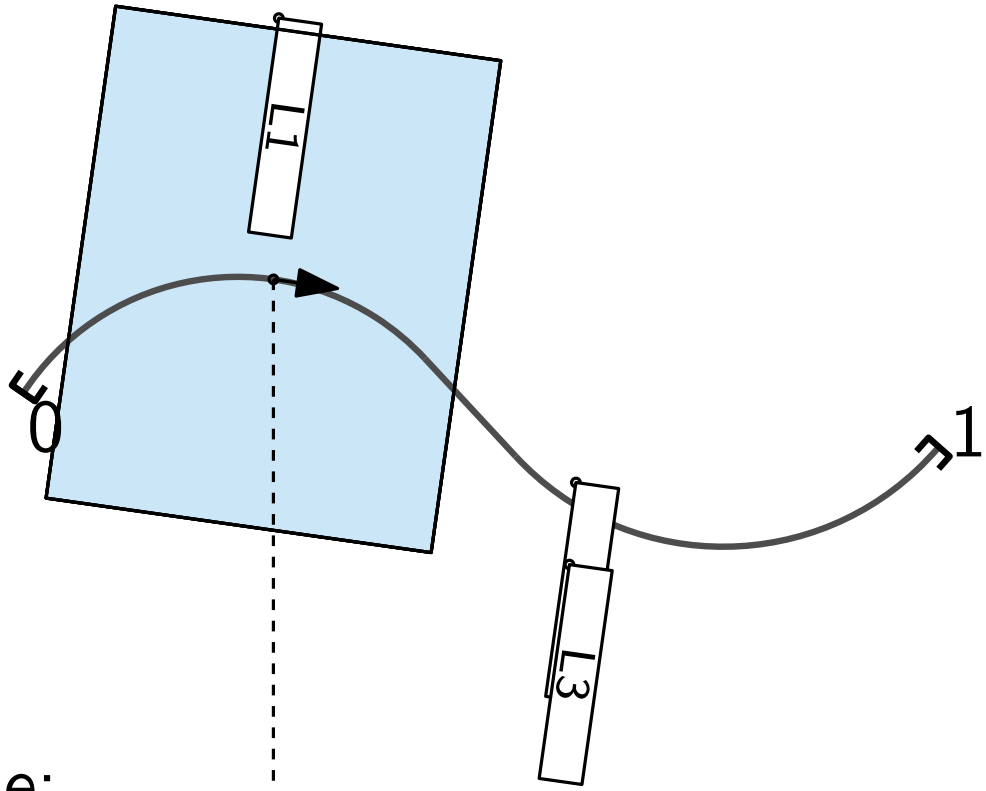

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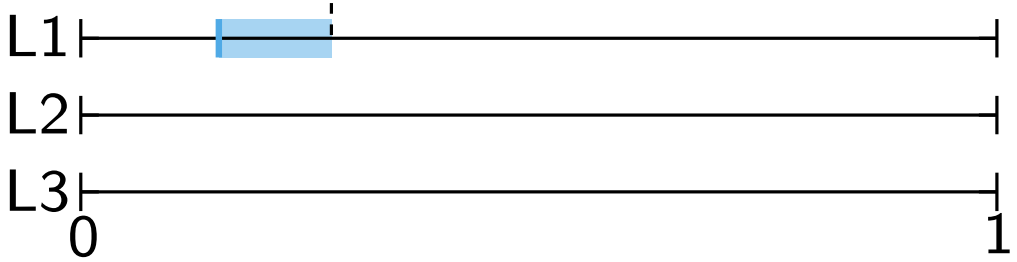

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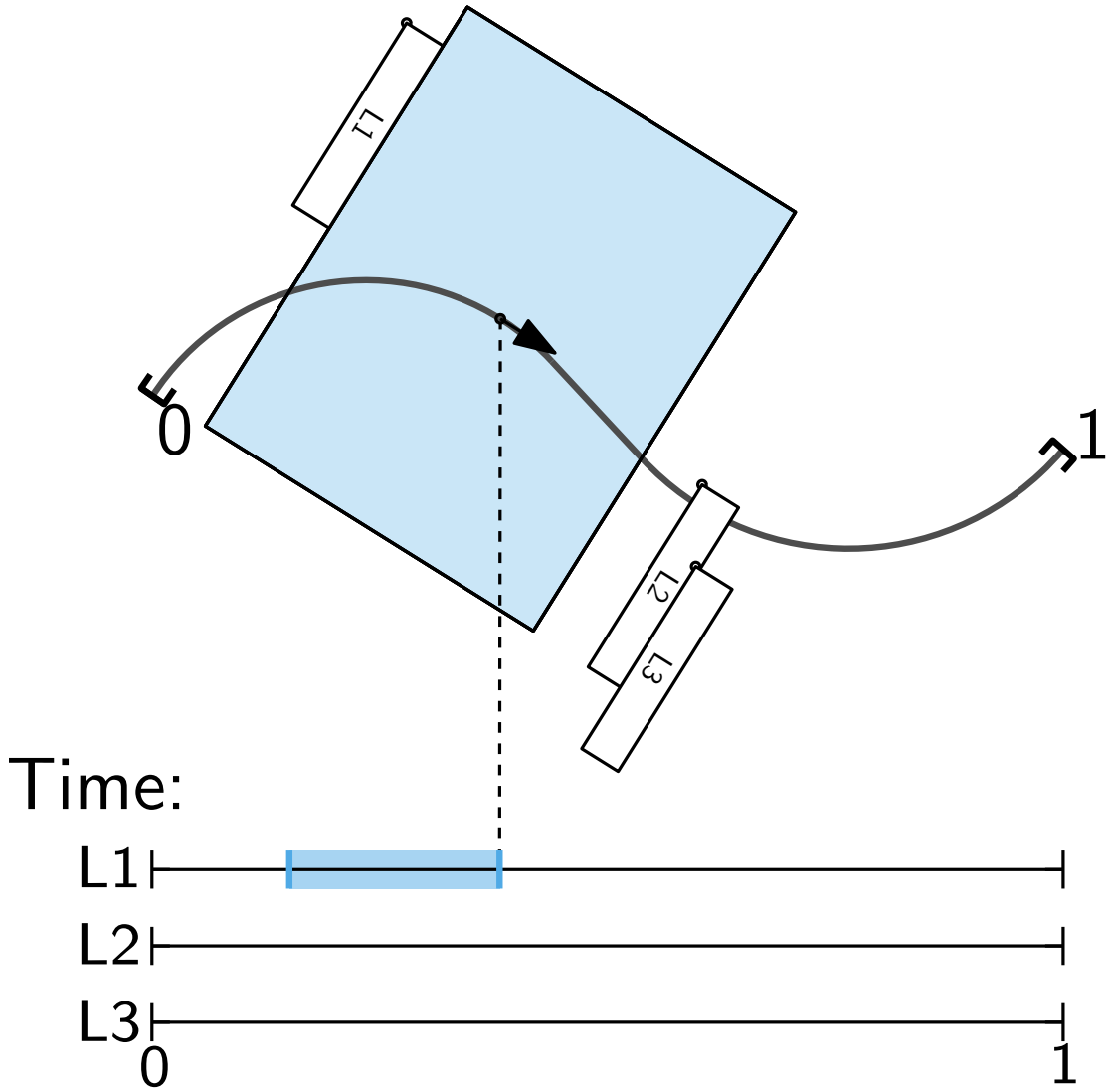

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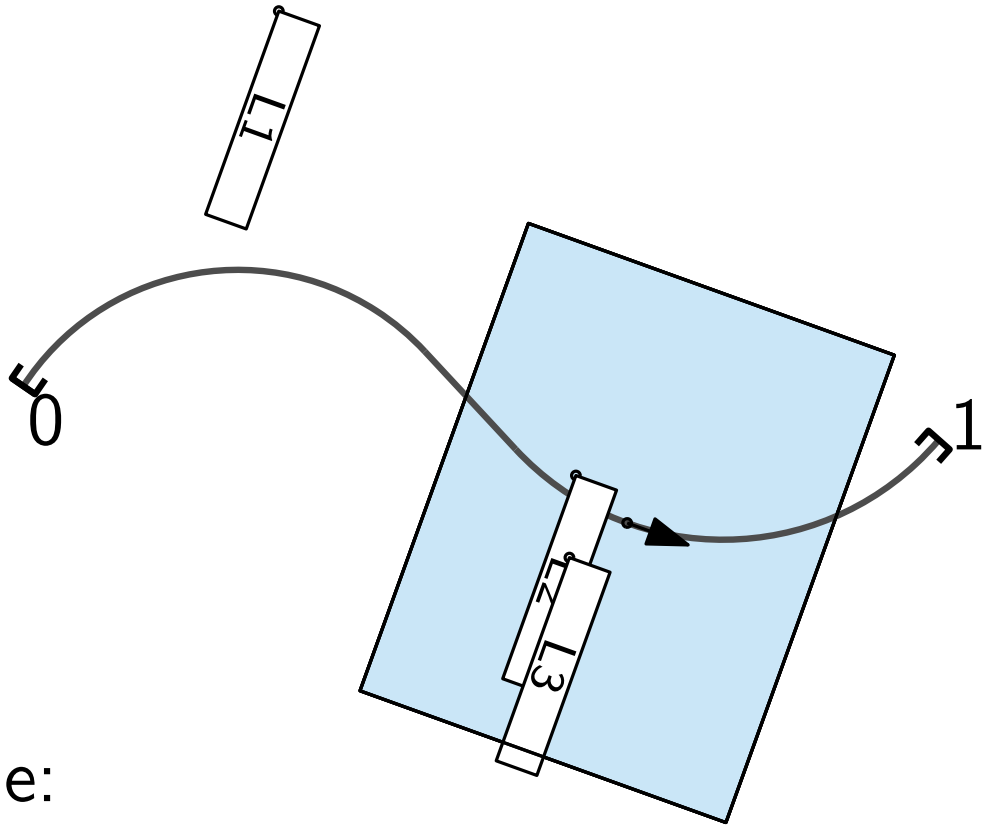

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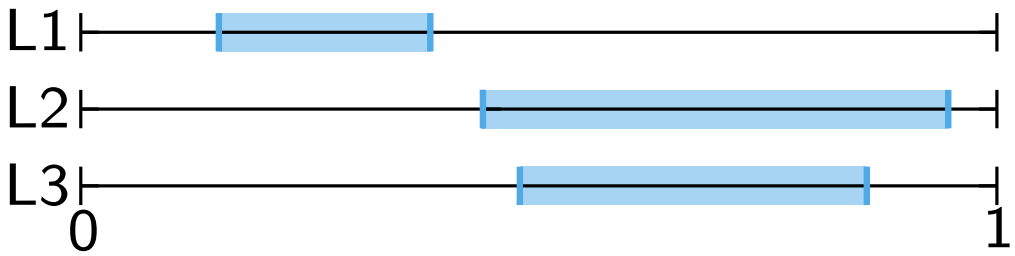
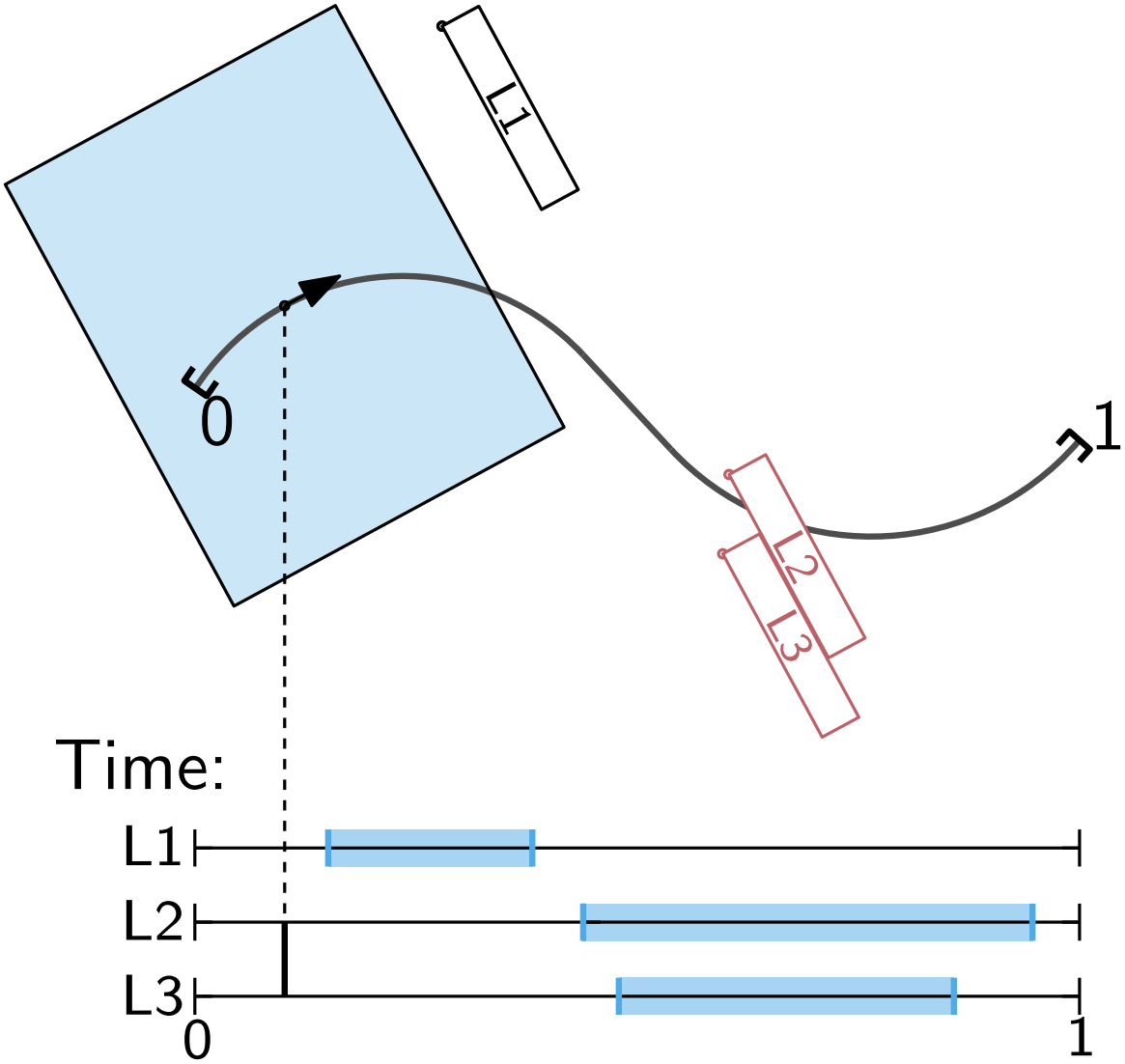


set of *presence intervals*

Mapping to Intervals

Two labels are in **conflict** at time t if they intersect at time t



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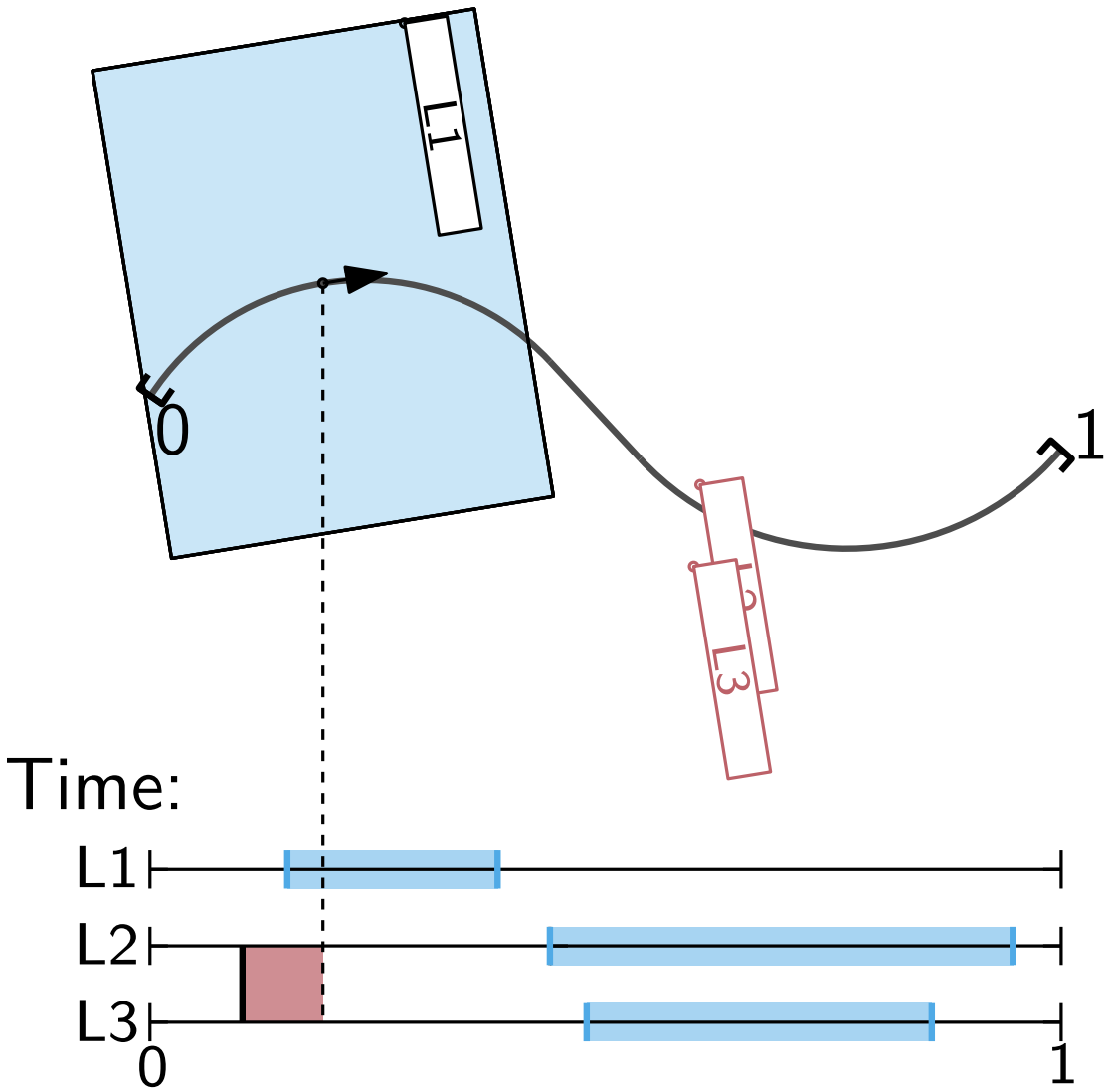

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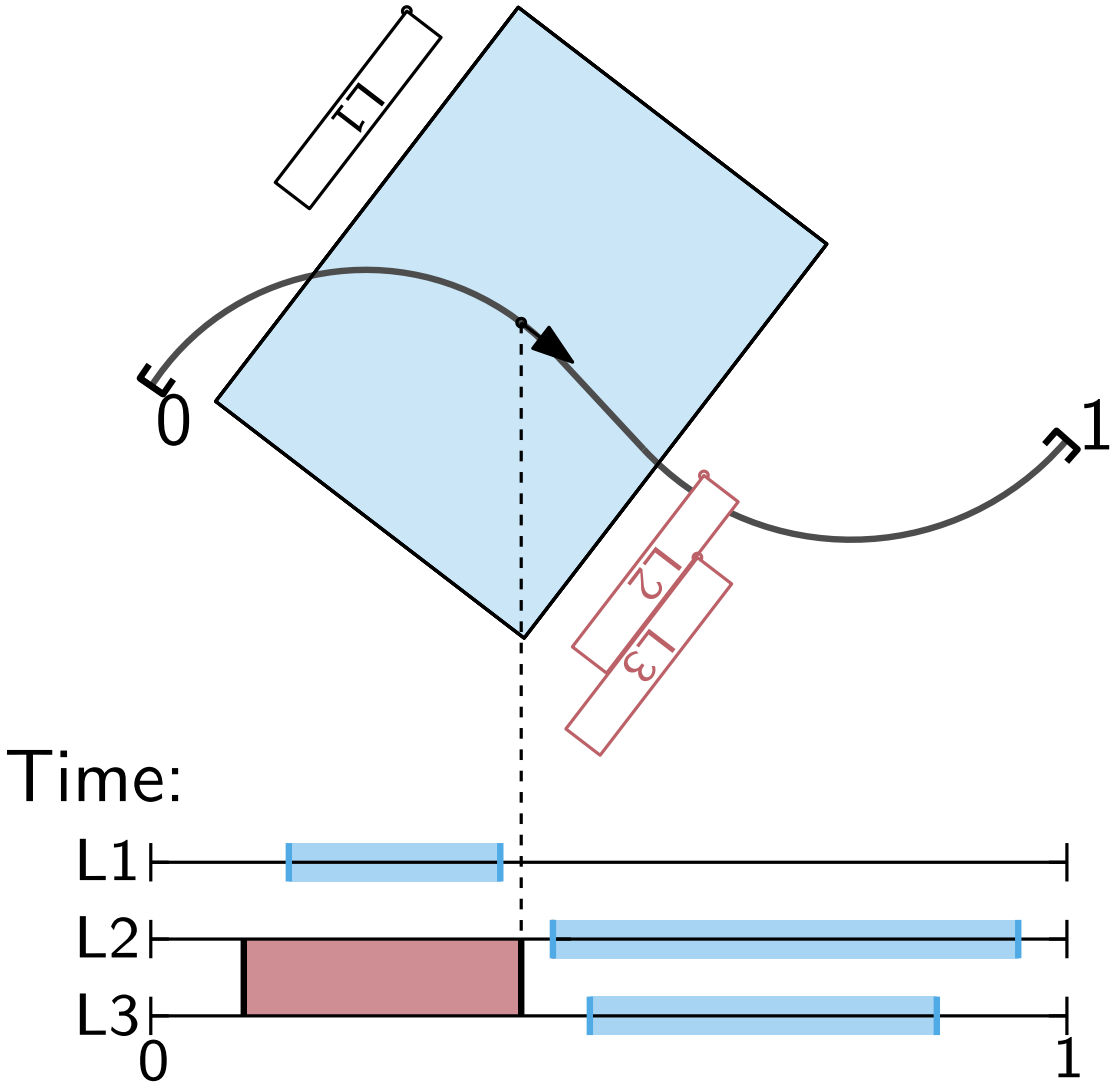

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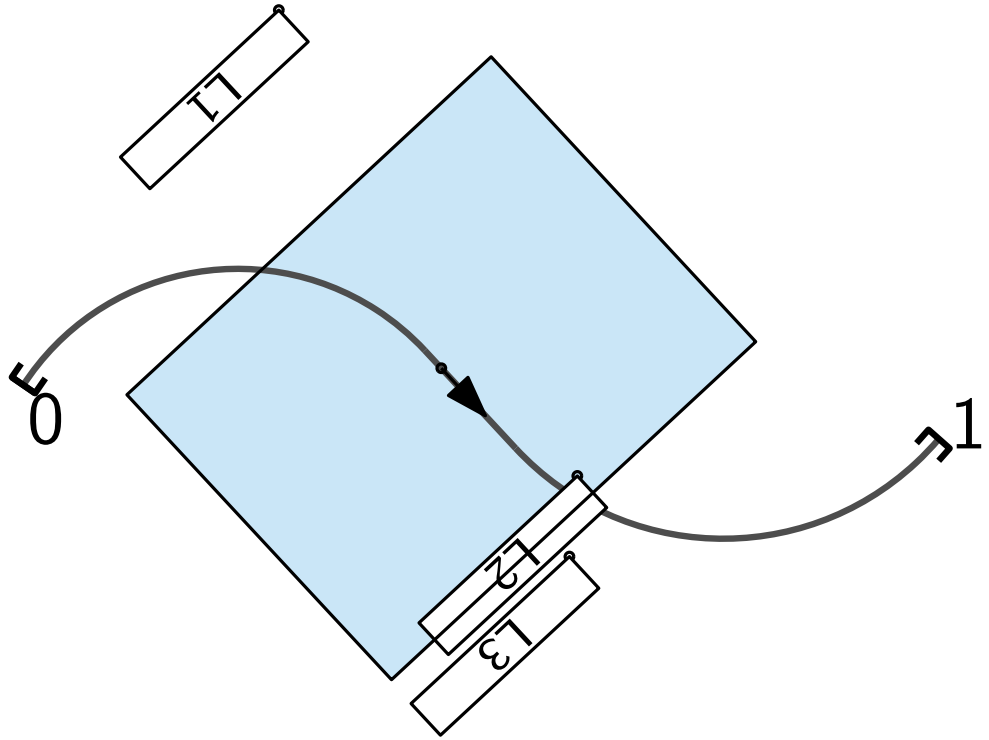
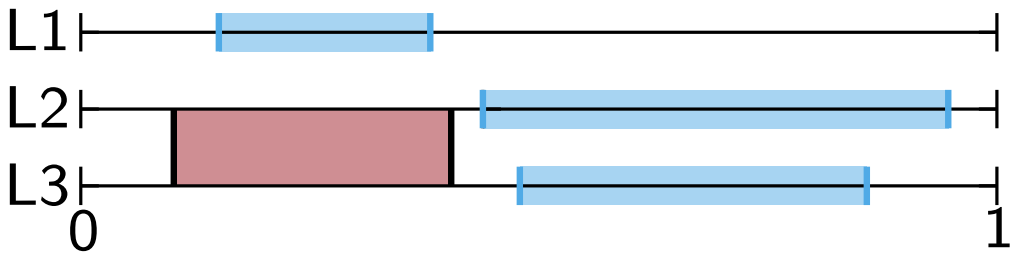


Time:



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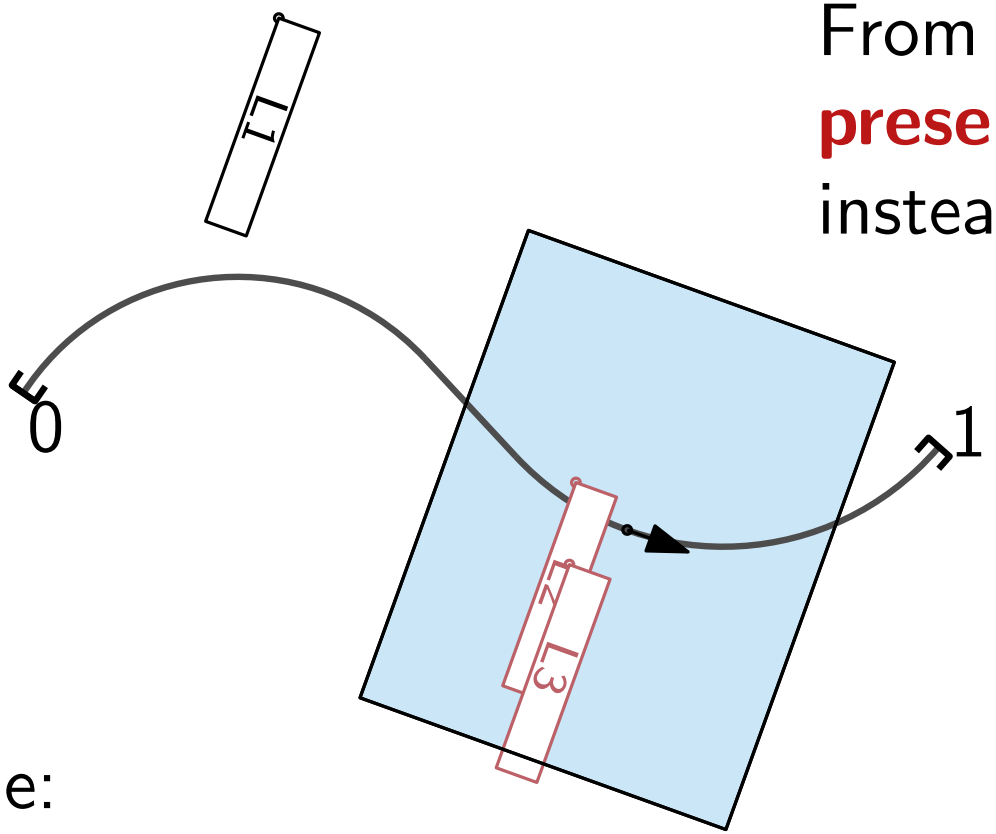
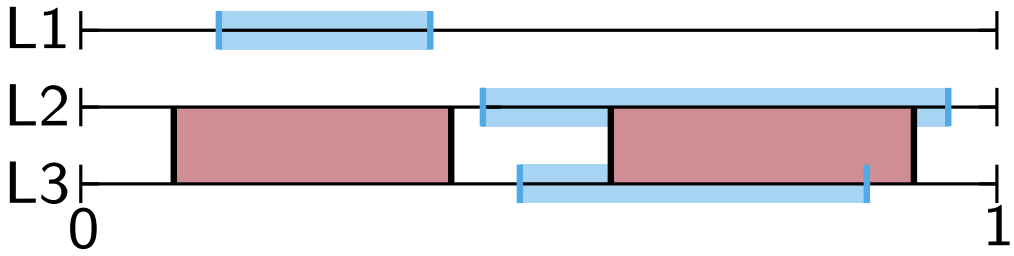


From now on:
presence and conflict intervals
instead of labels.

Time:



set of **conflict intervals**

Problem Definition

Overview

Input:
map m, set P of points, set L of corresponding labels

Output:
consistent rotation labeling that maximizes
the sum of all active ranges

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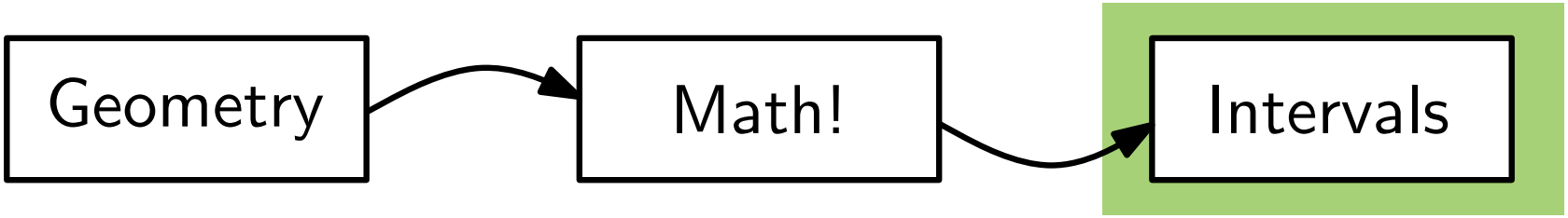

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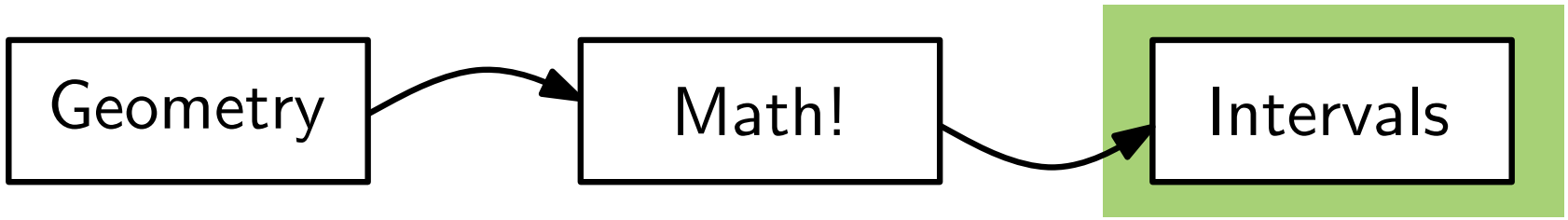
Overview

Input:
map m, set P of points, set L of corresponding labels

Input':
set of presence intervals, conflicts



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Activity of Labels

Label is **active** at time t if it is displayed at time t

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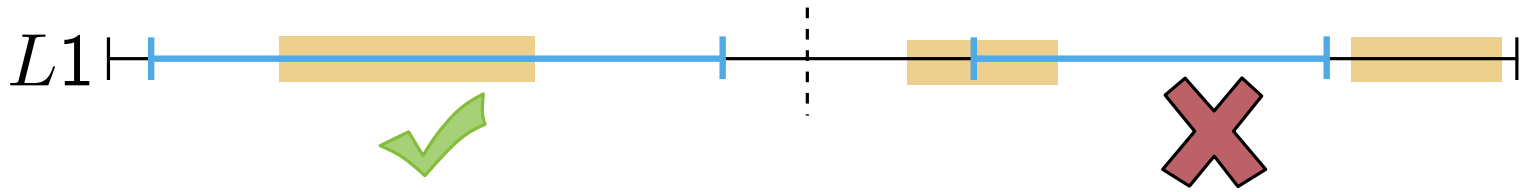
Only active if present



Label is present. Label is active. Labels are in conflict.

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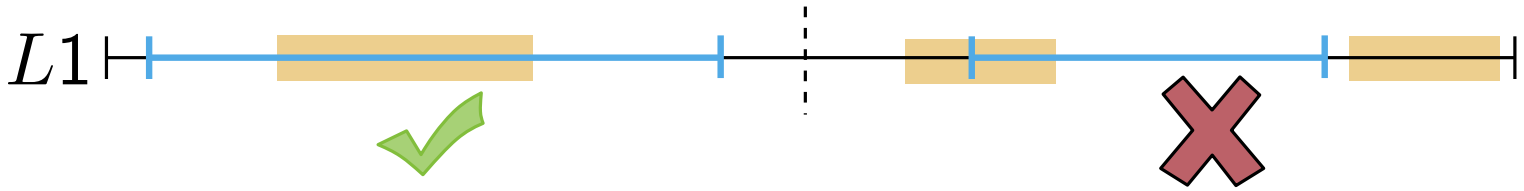
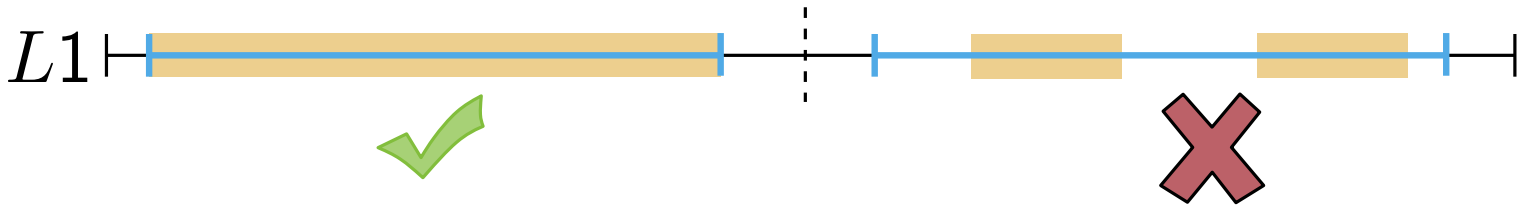


Either active for the **full presence** intervall **or not** at all



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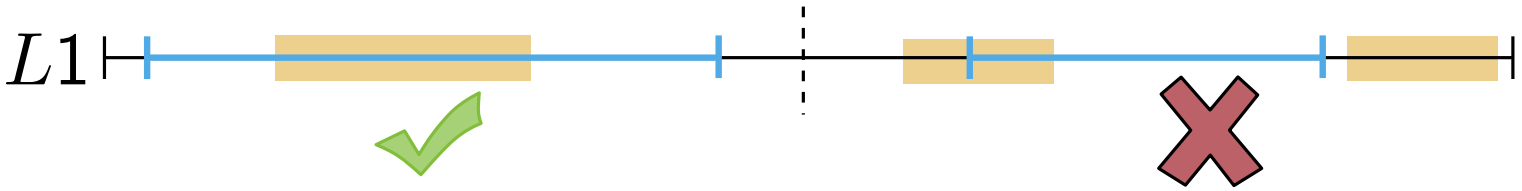

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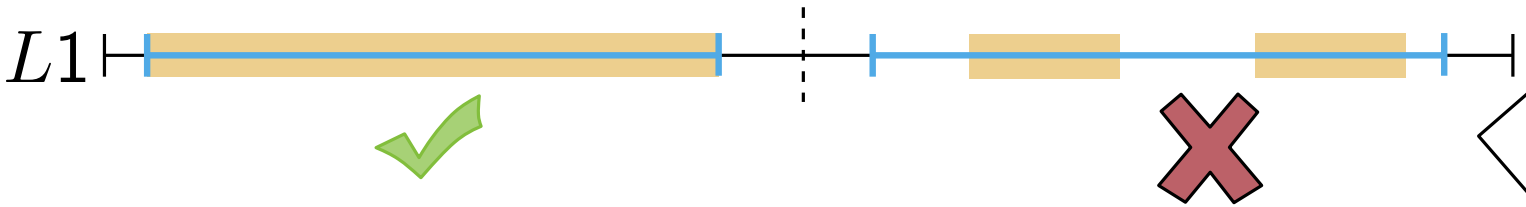

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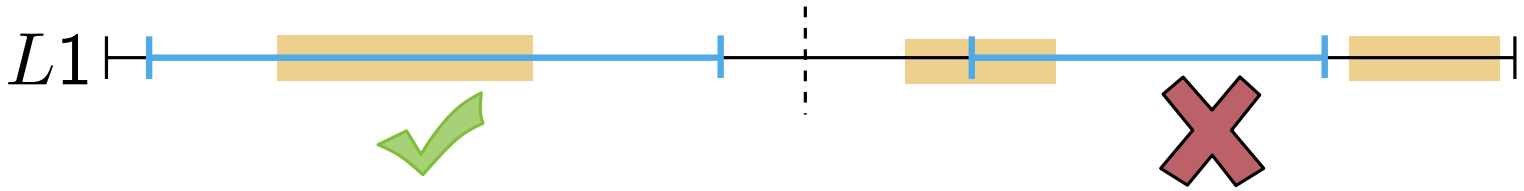

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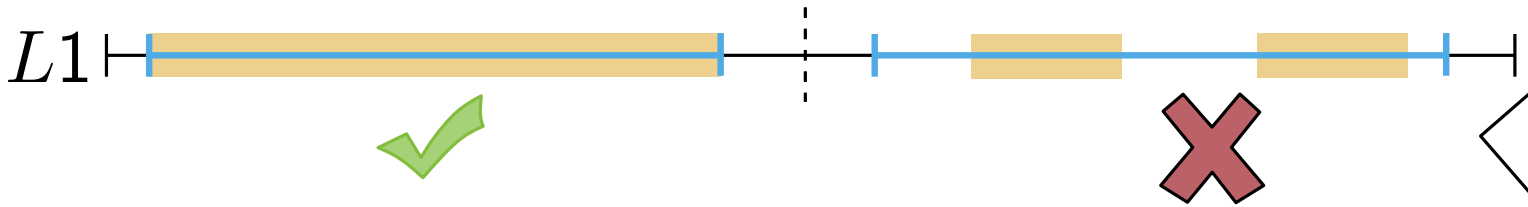

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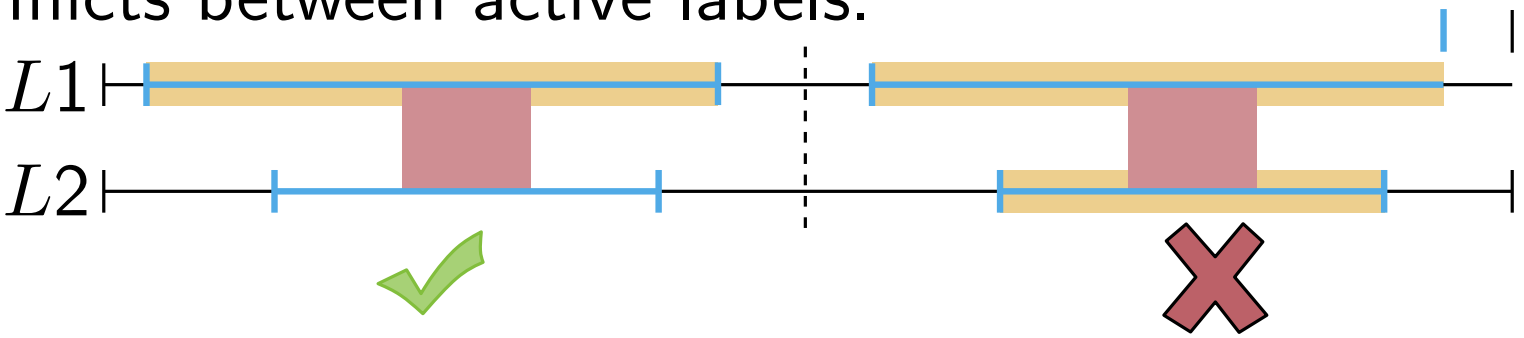
No conflicts between active labels.



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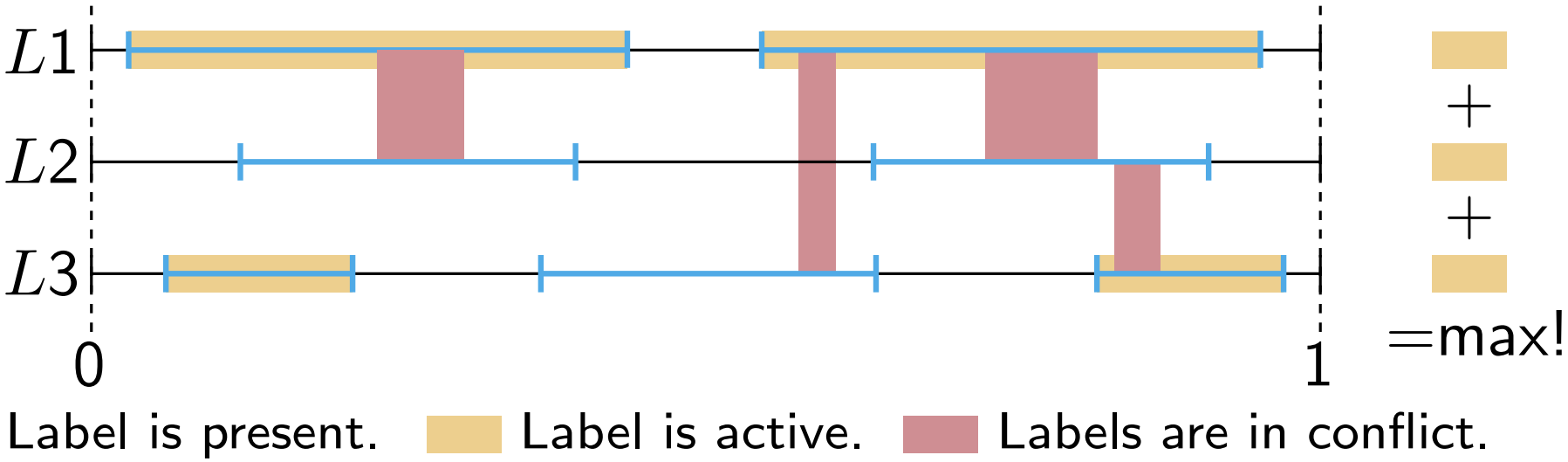
Problem Definition

Problem GeneralMaxTotal:
Given: Set I of presence intervals, conflicts
Find: Subset $J \subseteq I$ such that $\sum_{j \in J} \text{length}(j)$ is maximal, and J is conflict free

Set J of intervals is **conflict free** if no two in J intervals are in conflict



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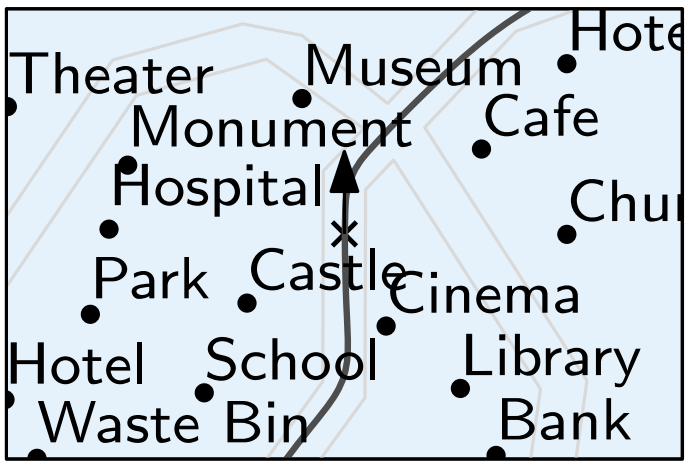
Theorem 5

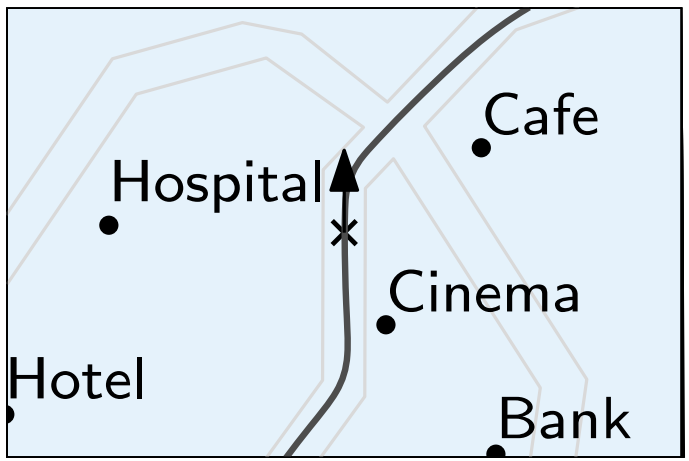
GeneralMaxTotal is NP-complete

Problem Definition

Problem k-RestrictedMaxTotal:
Given: Set I of presence intervals, conflicts
Find: Subset $J \subseteq I$ such that $\sum_{j \in J} \text{length}(j)$ is maximal, and J is conflict free, and **there is no t that is contained in more than k intervals in J**.

Set J of intervals is **conflict free** if no two in J intervals are in conflict





$k = 5$

k-RestrictedMaxTotal

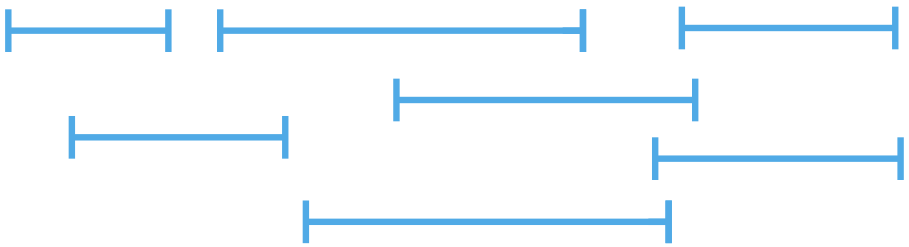
Theorem 4

k-RestrictedMaxTotal can be solved in polynomial time

Case $\mathbf{k = 1}$: Problem is equivalent to computing a maximum weighted independent set on an **interval graph**

$\Rightarrow O(n)$ time if intervals are sorted

[Hsiao et al., 1992]



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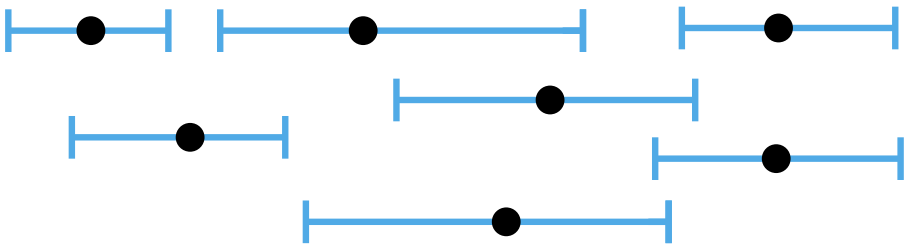

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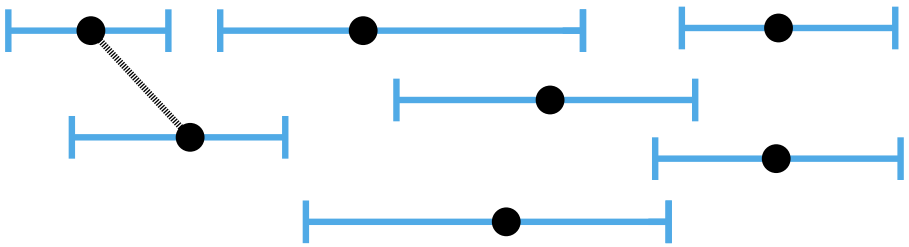

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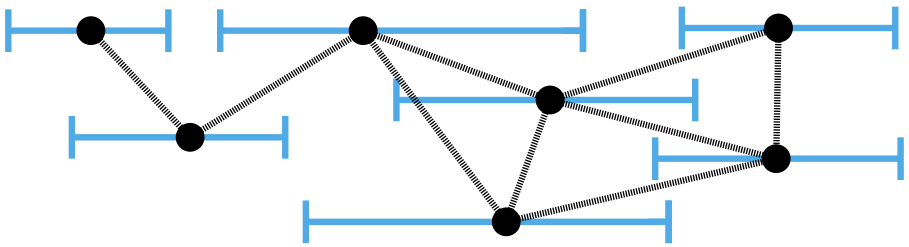

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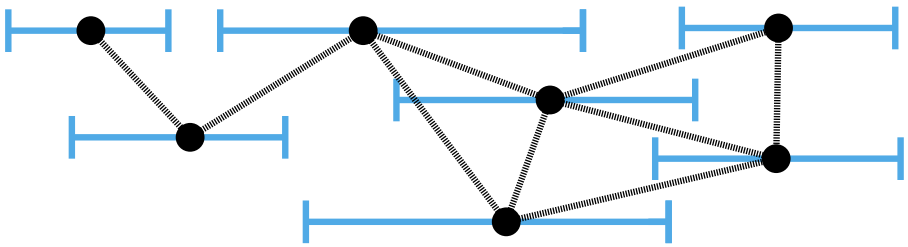

weight of a vertex = length of its interval

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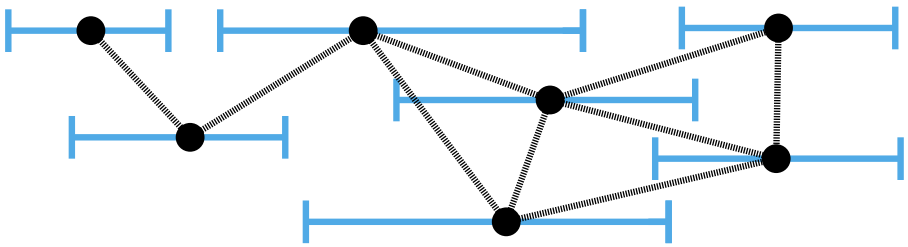

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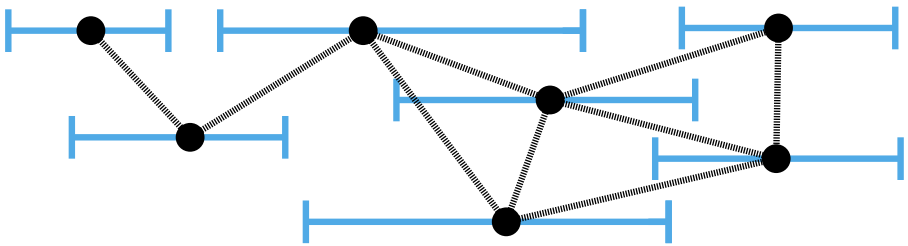

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Problem is related to **k-coloring** of interval graphs

There are $O(n + k)/O(kS(n))$ algorithms

[Carlisle, Lloyd '93]

k-RestrictedMaxTotal

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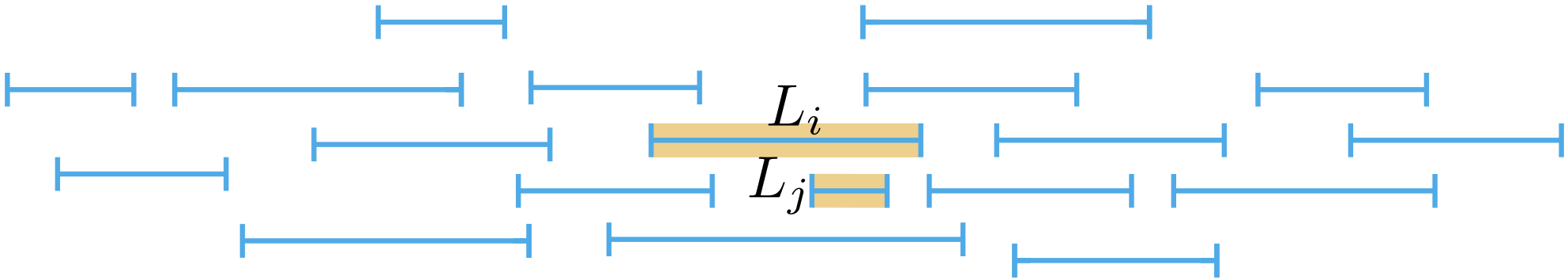
Case $\mathbf{k = 2}$: Because of **conflicts** more difficult than $k = 1$



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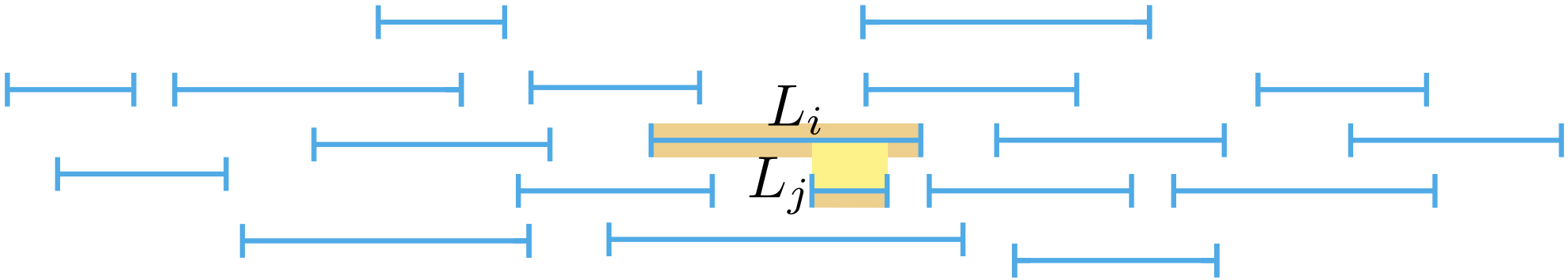

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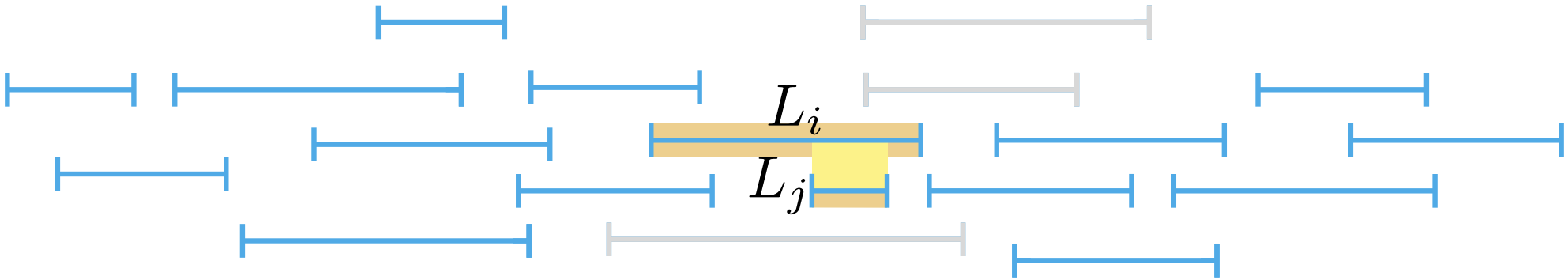

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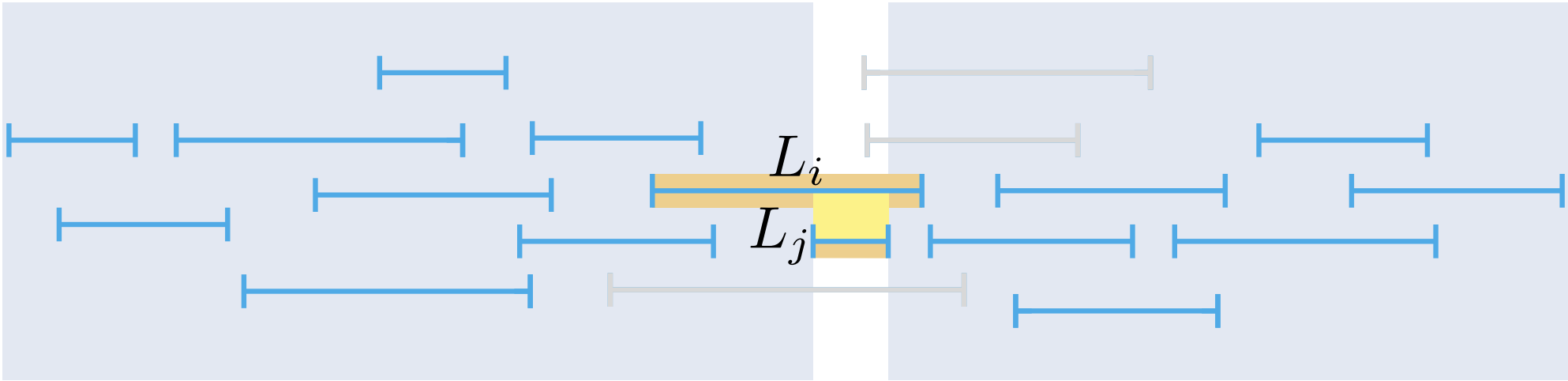

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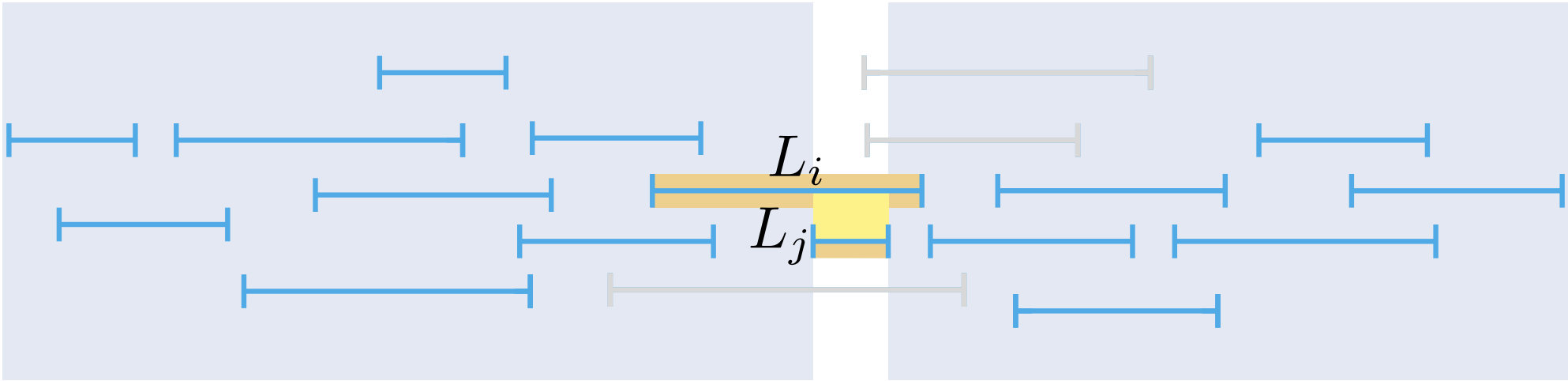
A **separating tuple** are two non-conflicting, but overlapping intervals



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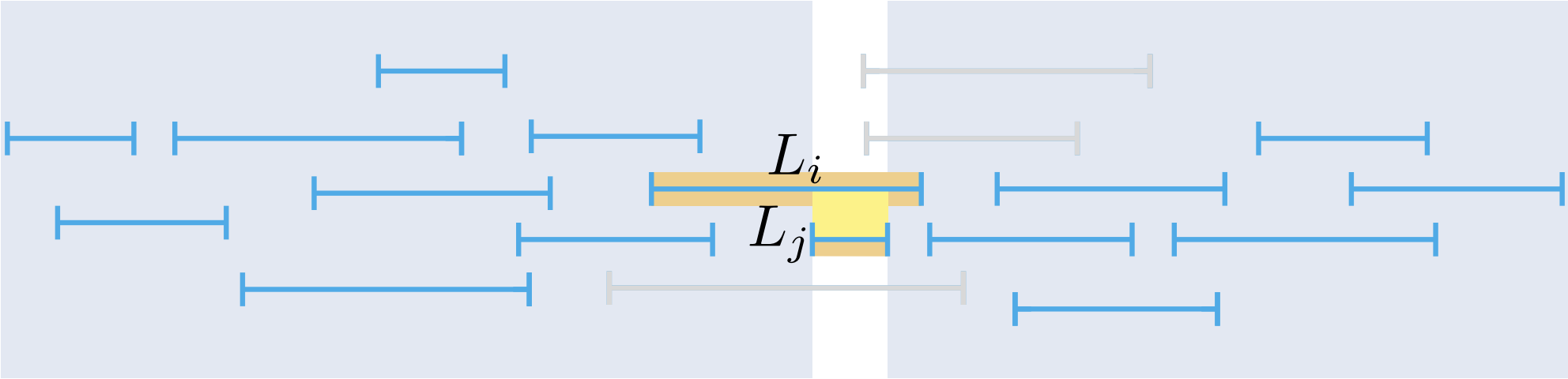

Observation:
a separating tuple splits the instance into two sub-instances

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Theorem 4

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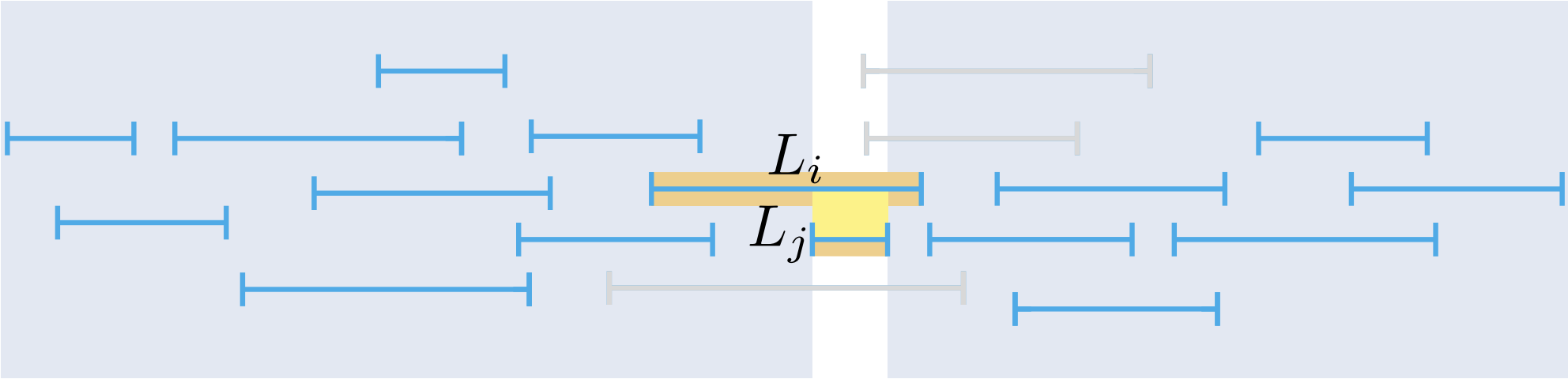

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Dynamic Programming

k-RestrictedMaxTotal

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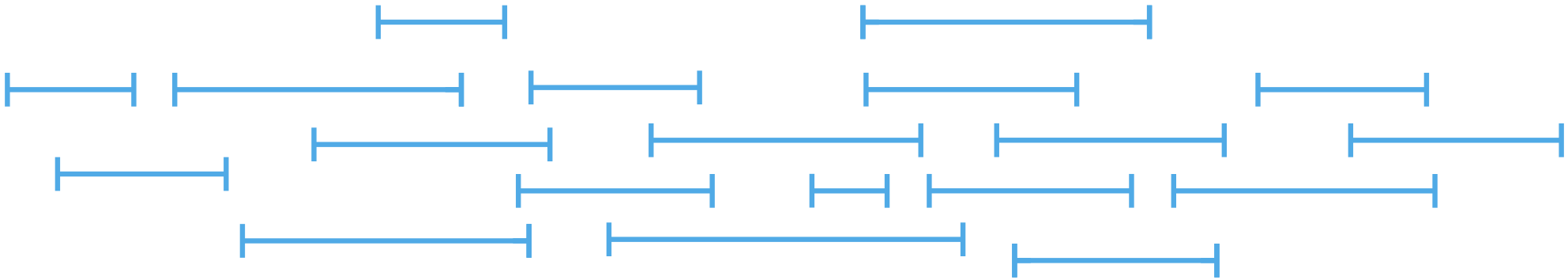

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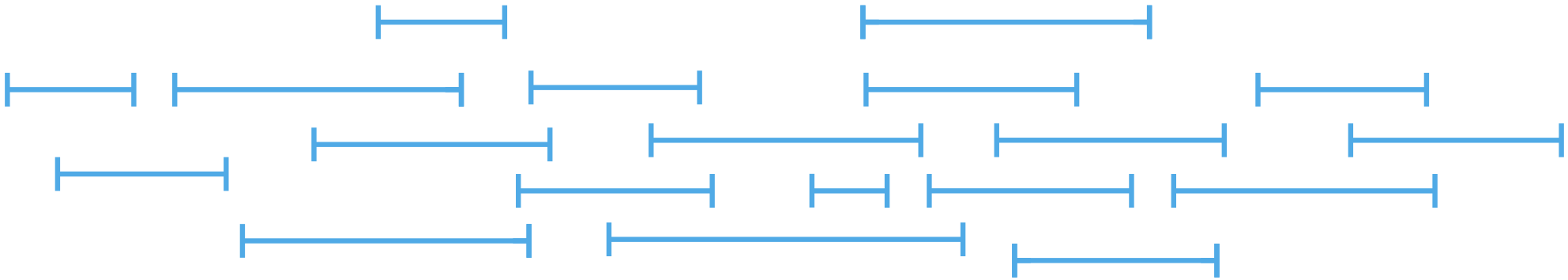

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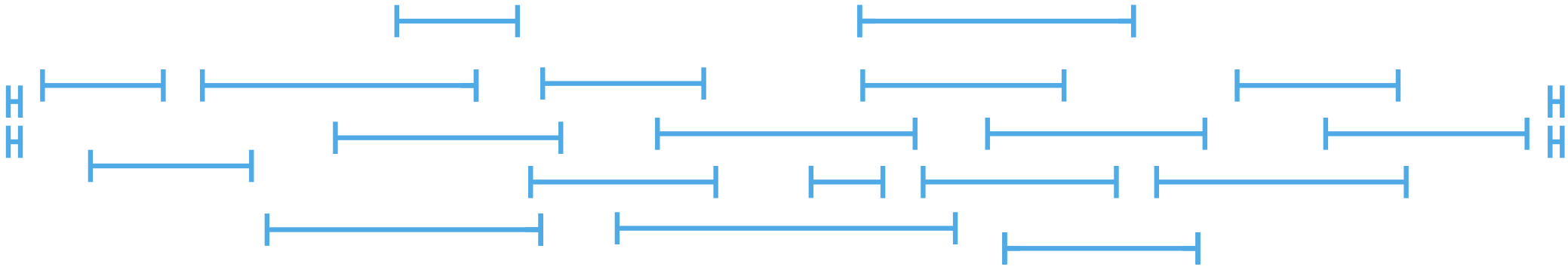


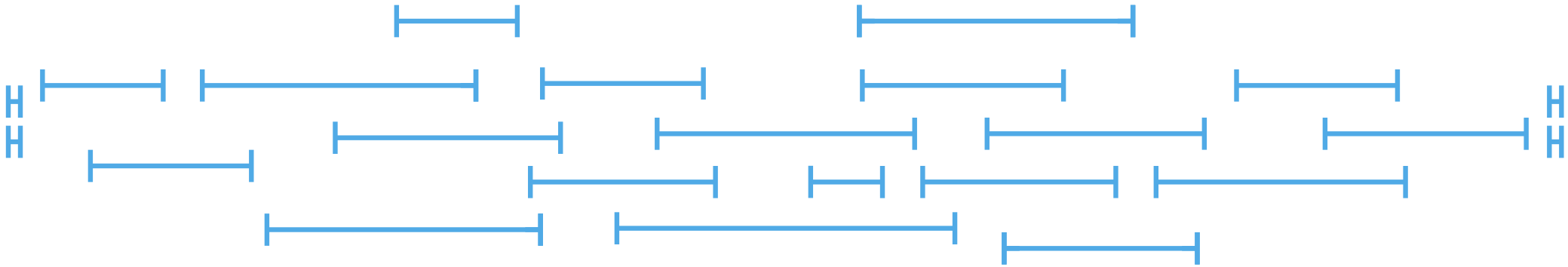
k-RestrictedMaxTotal

Theorem 4

k-RestrictedMaxTotal can be solved in polynomial time

Case $\mathbf{k = 2}$:

- add two dummy separating tuple
- enumerate all separating tuples s_i and sort them
- maintain one-dimensional table $\mathcal{T}$



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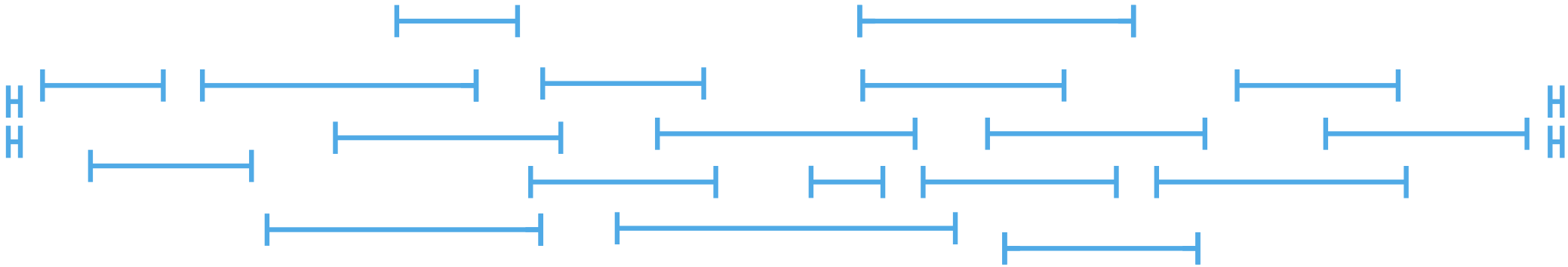

$\mathcal{T}[i]$: optimal solution for all $s_j, j < i$ that contains s_i.

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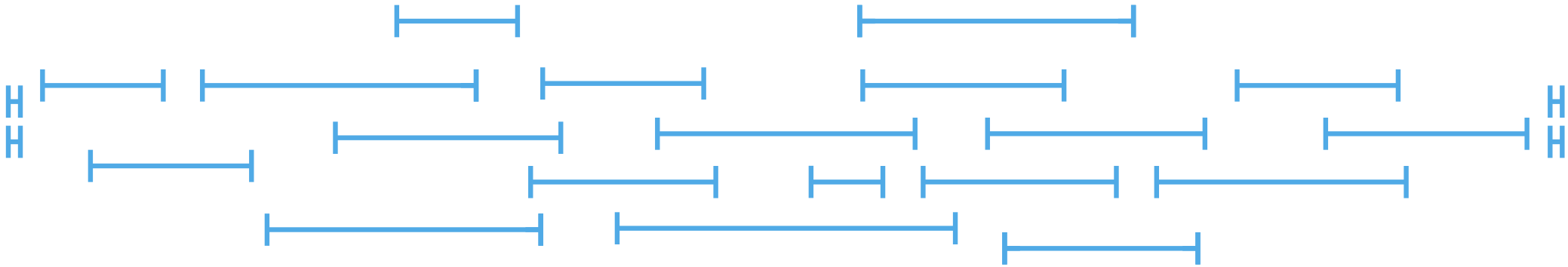

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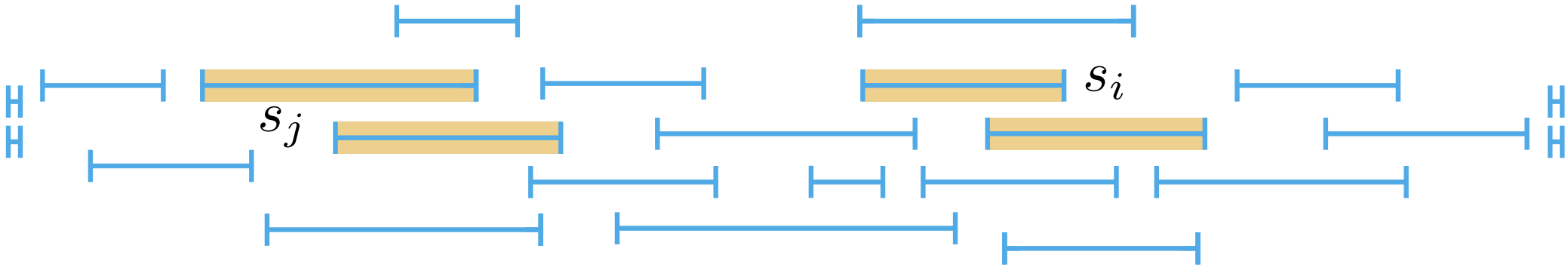

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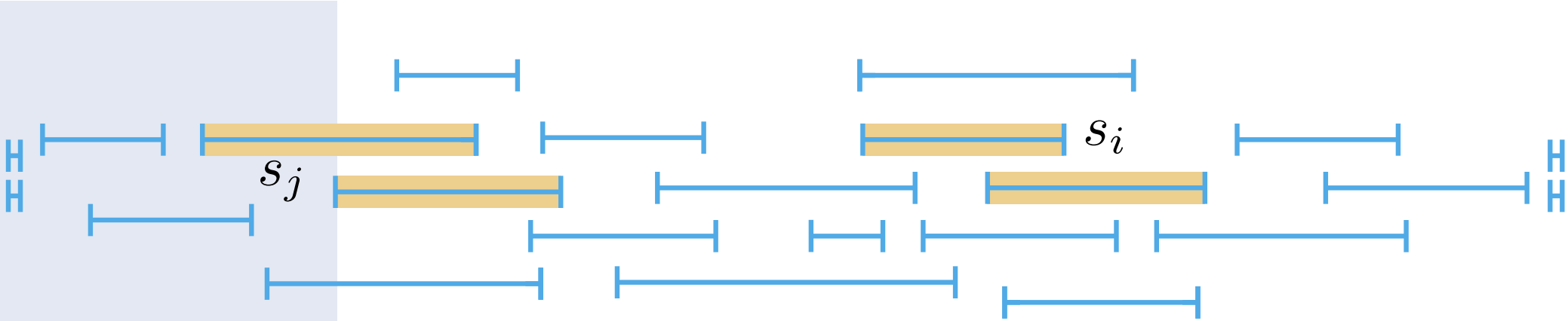

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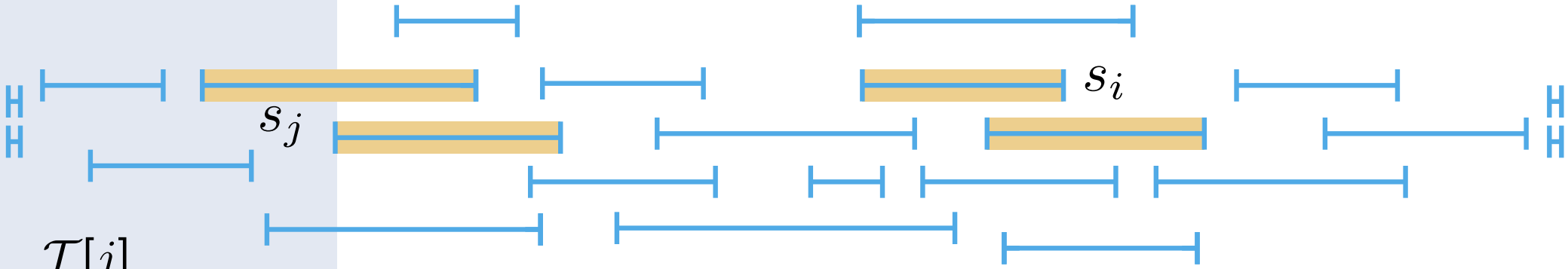


$\mathcal{T}[i]$: optimal solution for all $s_j, j < i$ that contains s_i.

construct $\mathcal{T}$ from left to right by guessing **preceding** separating tuple

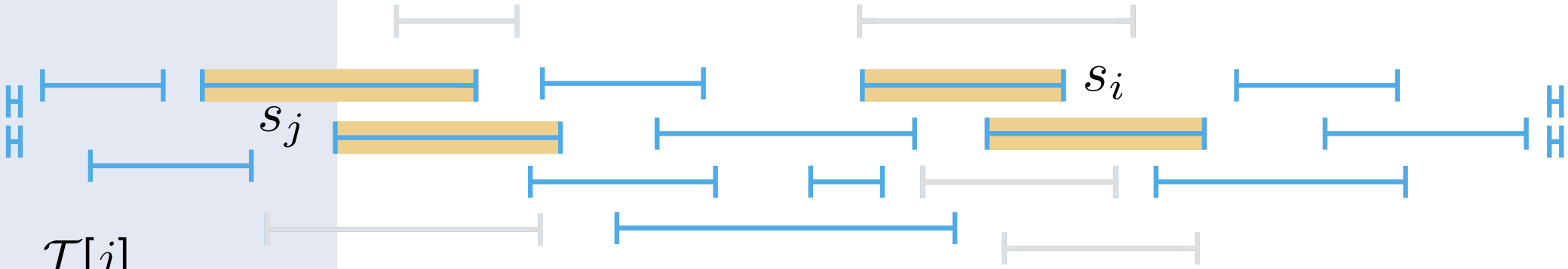
k-RestrictedMaxTotal

Theorem 4

k-RestrictedMaxTotal can be solved in polynomial time

Case $\mathbf{k = 2}$:

- add two dummy separating tuple
- enumerate all separating tuples s_i and sort them
- maintain one-dimensional table $\mathcal{T}$



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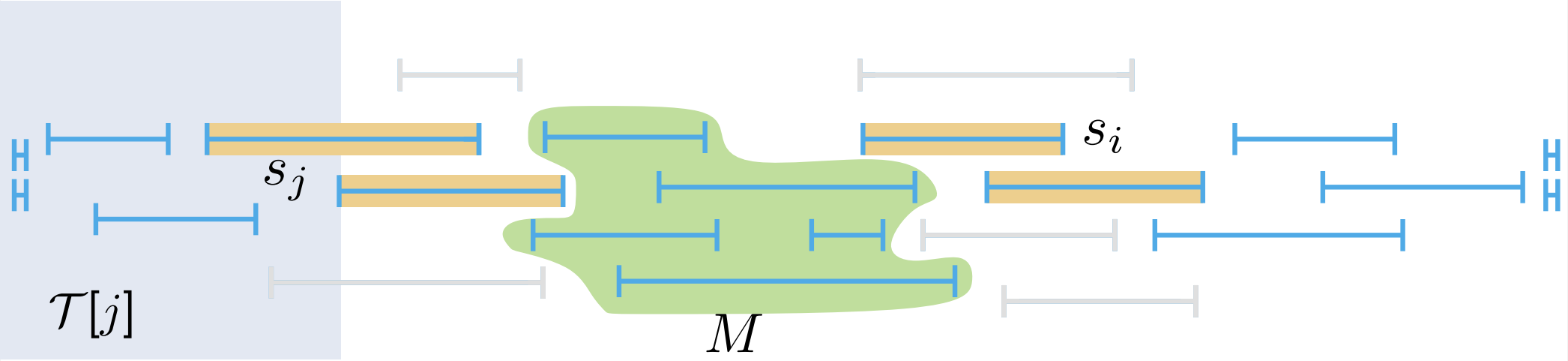

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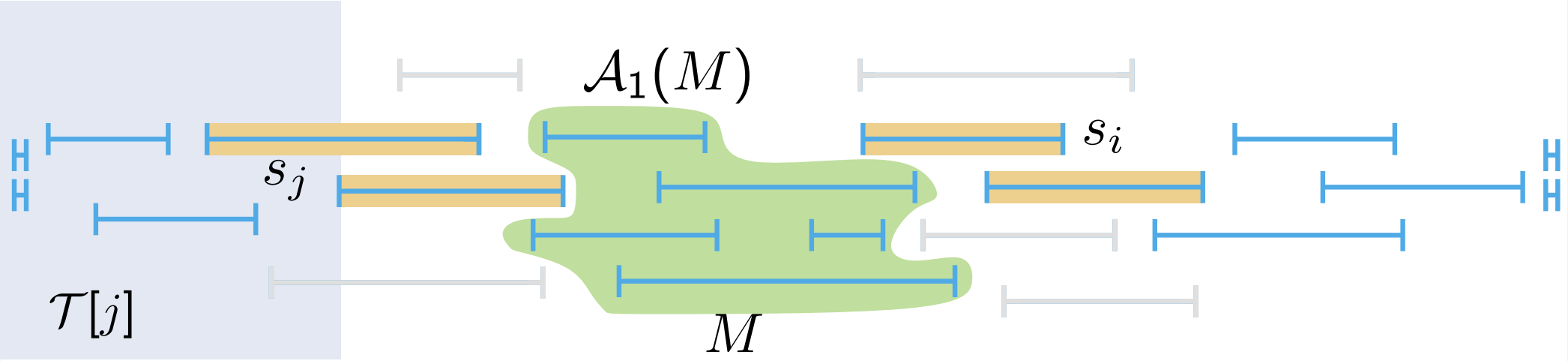

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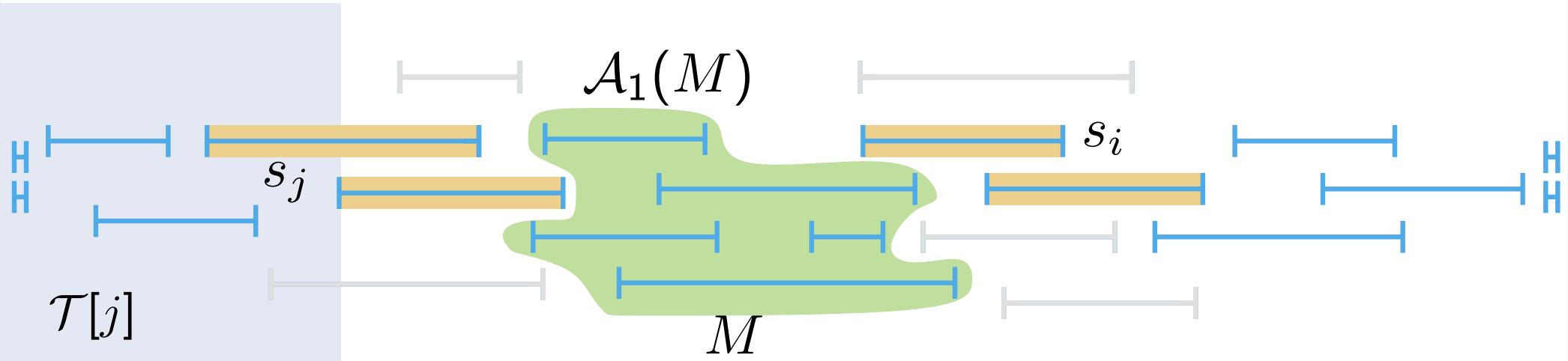

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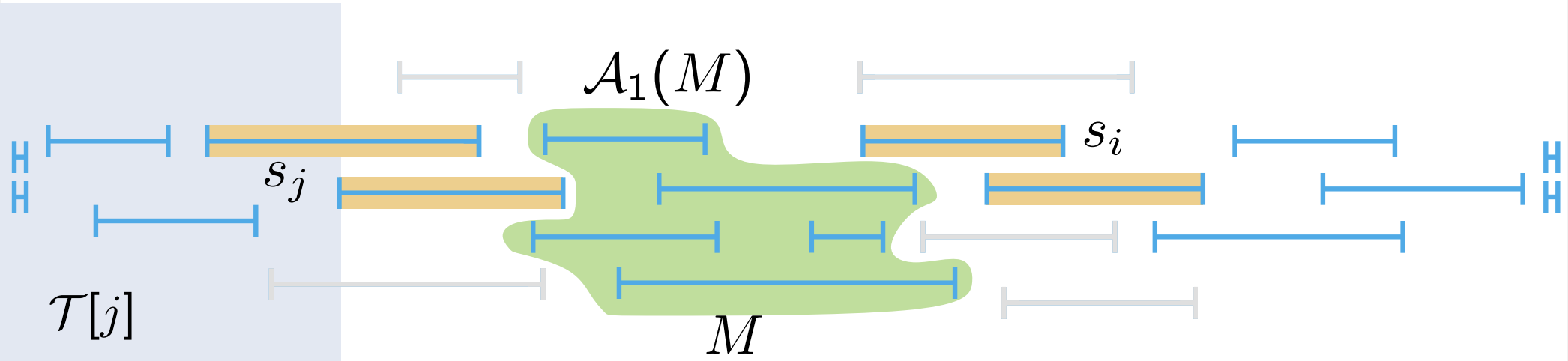

$$T[i] = \max_{j<i}\{T[j] + \mathcal{A}_1(M) + \text{length}(s_i) \mid s_i, s_j \text{ compatible } \}$$

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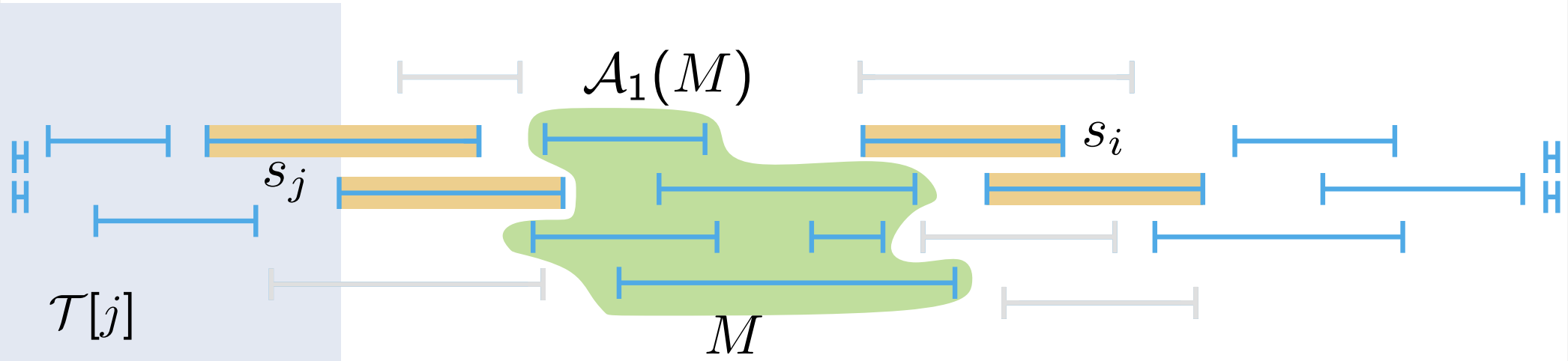

$$T[i] = \max_{j<i}\{\underbrace{T[j]}_{O(1)} + \underbrace{\mathcal{A}_1(M)}_{O(n)} + \underbrace{\text{length}(s_i)}_{O(1)} \mid s_i, s_j \text{ compatible } \}$$

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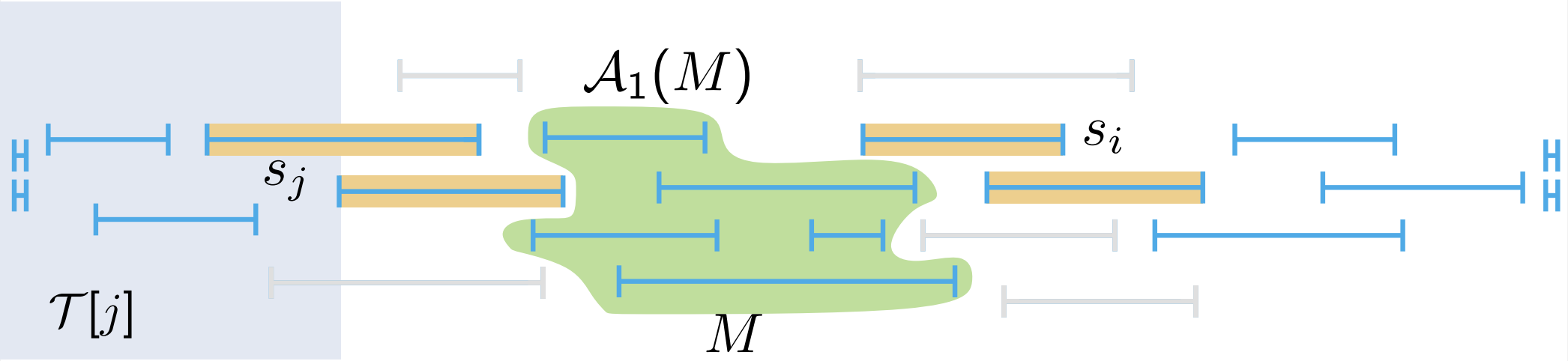

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$O(\#\text{sep. tu.})$

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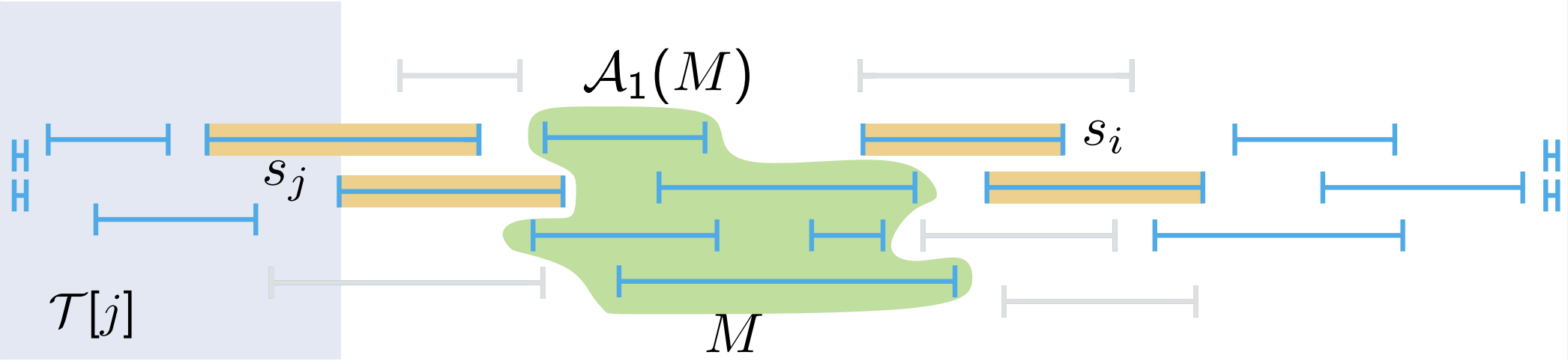

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$O(1)$ $O(n)$ $O(1)$

$O(\#\text{sep. tu.})$

k-RestrictedMaxTotal

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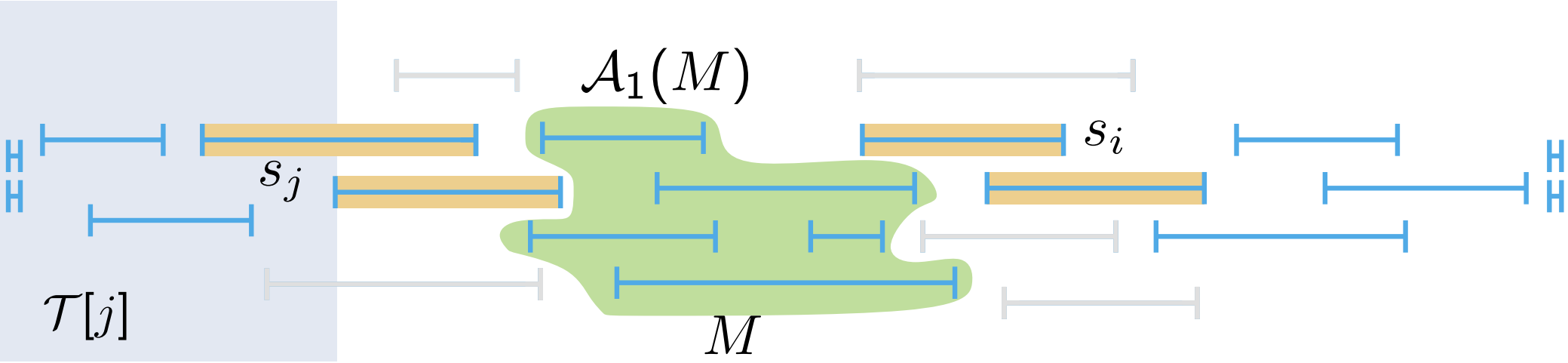
Case $\mathbf{k = 2}$:

Time complexity: $O(n \cdot (\#\text{sep. tu.})^2)$



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$O(\#\text{sep. tu.})$

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k-RestrictedMaxTotal

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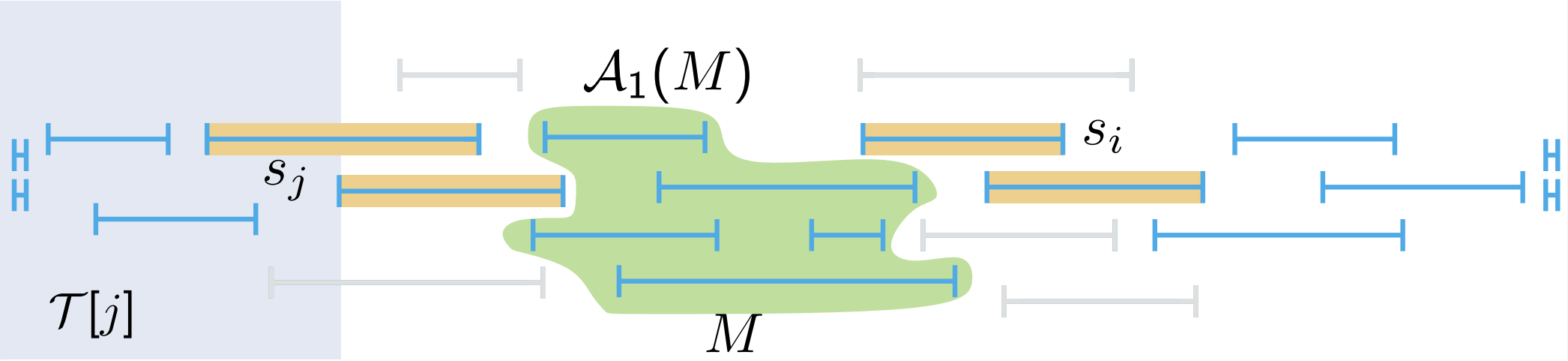
Case $\mathbf{k = 2}$:

Time complexity: $O(n \cdot (\#\text{sep. tu.})^2) \quad = O(n^5)$



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$O(\#\text{sep. tu.})$

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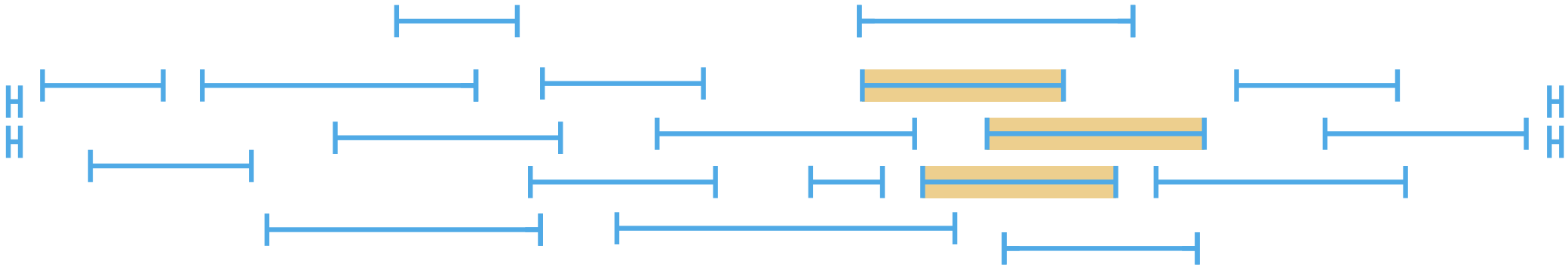
Case $\mathbf{k = 3}$:



k-RestrictedMaxTotal

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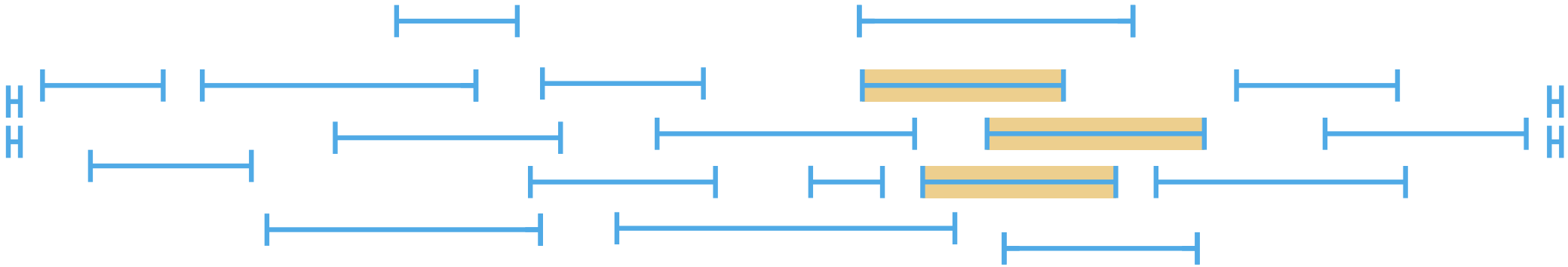
Case $\mathbf{k = 3}$:



$$T[i] = \max_{j<i}\{T[j] + \mathcal{A}_2(M) + \text{length}(s_i) \mid s_i, s_j \text{ compatible } \}$$

k-RestrictedMaxTotal

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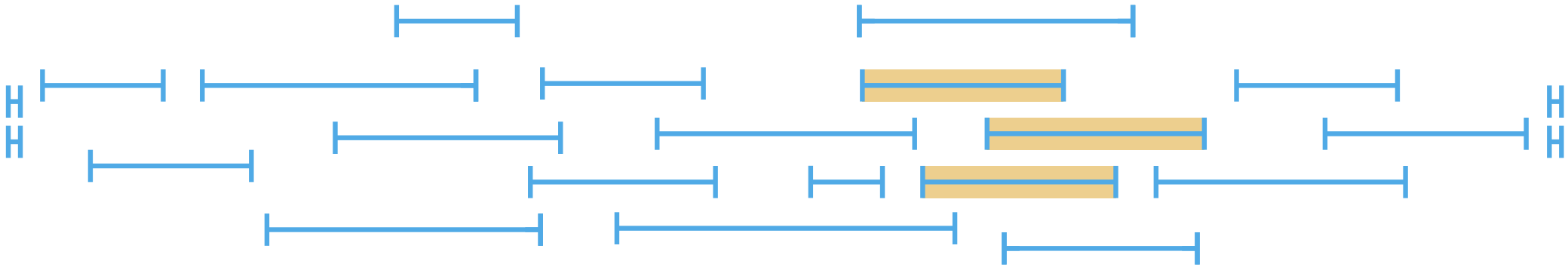
Case $\mathbf{k = 3}$:



Can be generalized to:

$T[i] = \max_{j<i}\{T[j] + \mathcal{A}_2(M) + \text{length}(s_i) \mid s_i, s_j \text{ compatible } \}$

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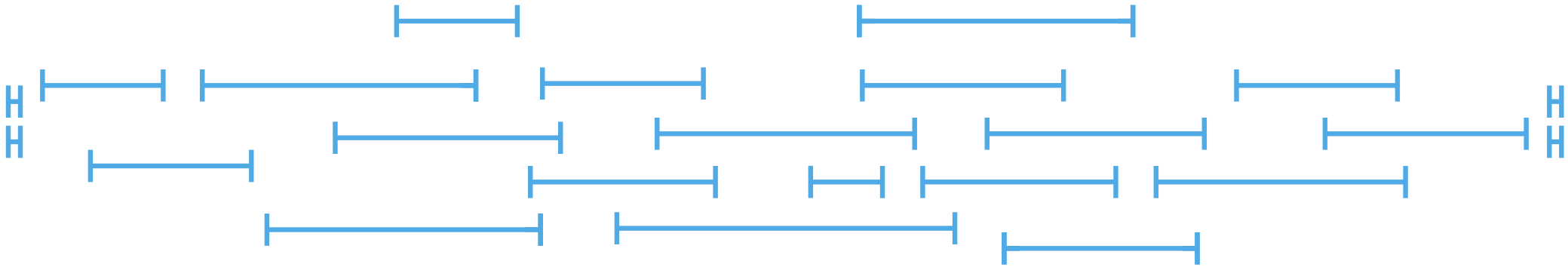
Case $\mathbf{k > 1}$:



Can be generalized to:

$T[i] = \max_{j<i}\{T[j] + \mathcal{A}_{k-1}(M) + \text{length}(s_i) \mid s_i, s_j \text{ compatible } \}$

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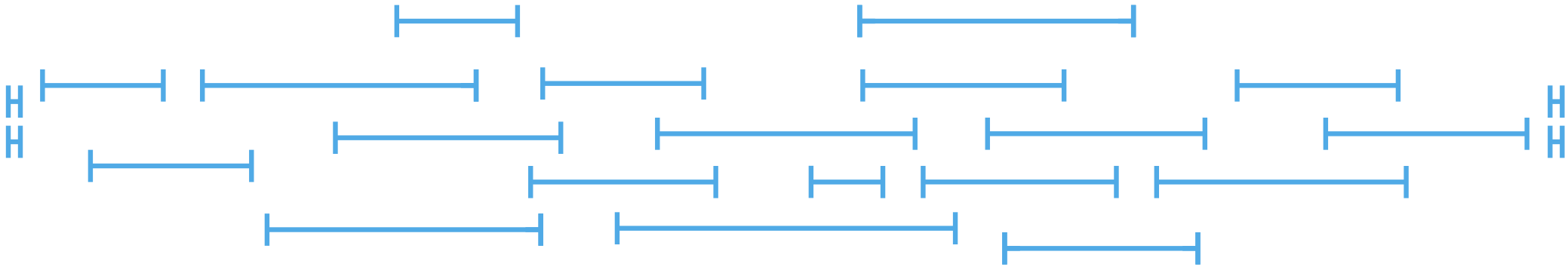
Case $\mathbf{k > 1}$:



Can be generalized to:

$T[i] = \max_{j<i}\{T[j] + \mathcal{A}_{k-1}(M) + \text{length}(s_i) \mid s_i, s_j \text{ compatible } \}$

Time complexity: $O(n^{k^2+k-1})$

Summary II

Our results:

- GeneralMaxTotal is $\mathcal{NP}$-complete
- k-RestrictedMaxTotal is polynomial solvable
- new labeling model is versatile
- algorithms **do not** rely on geometry

Open Problems:

- k-RestrictedMaxTotal is polynomial solvable but slow (how bad for real-world data sets?)
- trajectory-based labeling with **zooming**?