PHAST - Hardware Accelerated Shortest path Trees

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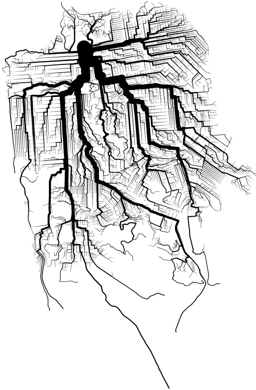
March 25, 2011

Single-Source Shortest Paths

request:

- given a (positively) weighted directed graph $G = (V, E, w)$ and a source node s
- compute distances from s to all other nodes in the graph
- applications: compute many trees for map services (sometimes even all-pairs shortest paths)



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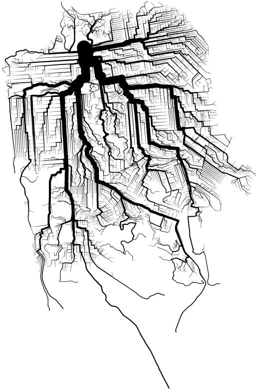

solution:

- Dijkstra [Dij59]

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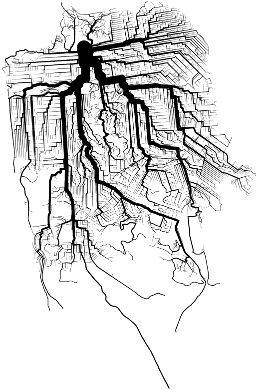

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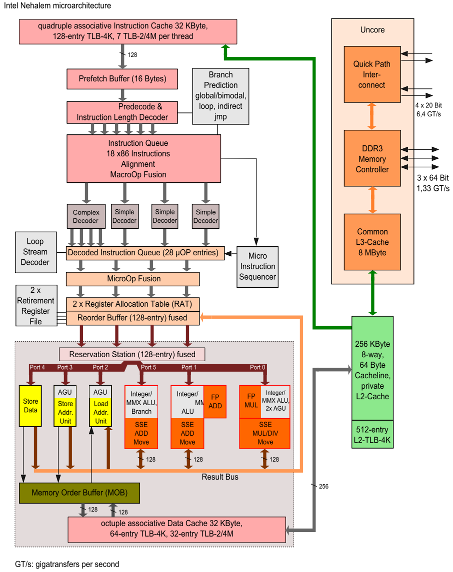
some facts:

- $O(m + n \log n)$ with Fibonacci Heaps [FT87]
- linear (with a small constant) in practice [Gol01]
- exploiting modern hardware architecture is complicated

Modern CPU architecture

some facts:

- multiple cores
- more cores than memory channels
- hyperthreading
- multi-socket systems
- steep memory hierarchy
- cache coherency
- no register coherency



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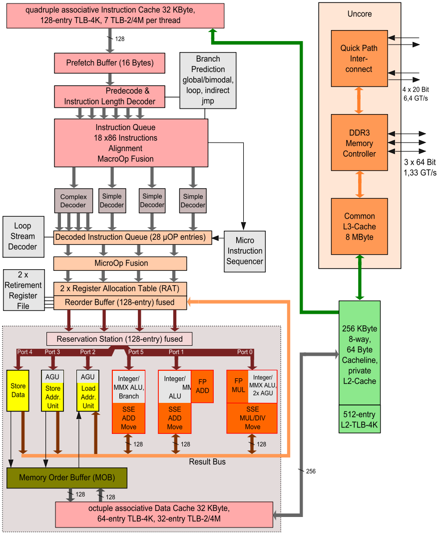
⇒ **algorithms need to be tailored**

⇒ **speedups of 100x possible**

Intel Nehalem microarchitecture



GT/s: gigatransfers per second

GPU Architecture

some facts:

- many cores (up to 512)
- high memory bandwidth (5x faster than CPU)
- but main $\rightarrow$ GPU memory transfer slow ($\approx$ 20x)
- no cache coherency
- Single Instruction Multiple Threads model
  (thread groups follow same instruction flow)
- barrel processing used to hide DRAM latency
  $\Rightarrow$ need to keep thousands of independent (!) threads busy
- access of a thread group to memory only efficient for certain patterns

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Parallelizing Dijkstra's Algorithm

multiple trees:

- multi-core by source
- instruction-level parallelism exploitable [Yan10]
- approach not applicable for a GPU implementation
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- speculation
- Δ-stepping [MS03],[MBBC09]
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other problem:

- data locality

$\Rightarrow$ memory bandwidth bound

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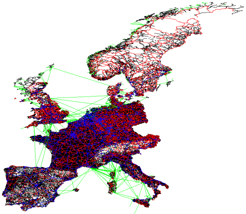
experiments:

- input: Western European road network
- 18M nodes, 23M road segments

Dijkstra:	≈ 3.0 s	$\Rightarrow$ not real-time
BFS:	≈ 2.0 s	



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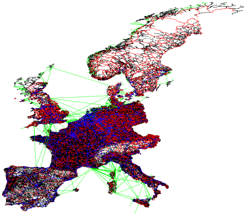

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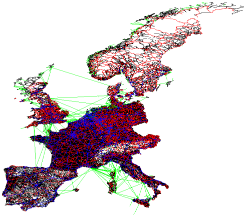

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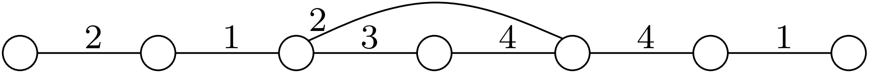
a new 2-phase algorithm for computing shortest path trees: [DGNW11]

- preprocessing:
  - a few minutes
  - works well in graphs with low highway dimension, e.g., road networks
- faster shortest path tree computation:
  - without optimization as fast as BFS
  - allows to exploit hardware architecture on all levels
  - $\Rightarrow$ up to 3 orders of magnitude faster than Dijkstra

Outline

[GSSD08]

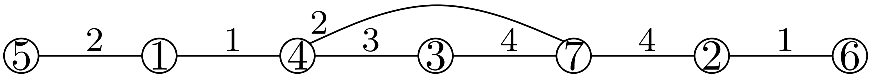
preprocessing:



[GSS**D**08]

preprocessing:

- order nodes by importance (heuristic)



Contraction Hierarchies: A 2-phase algorithm for exact route planning

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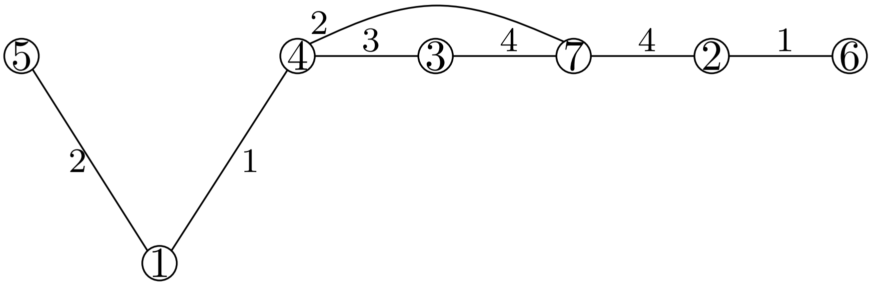

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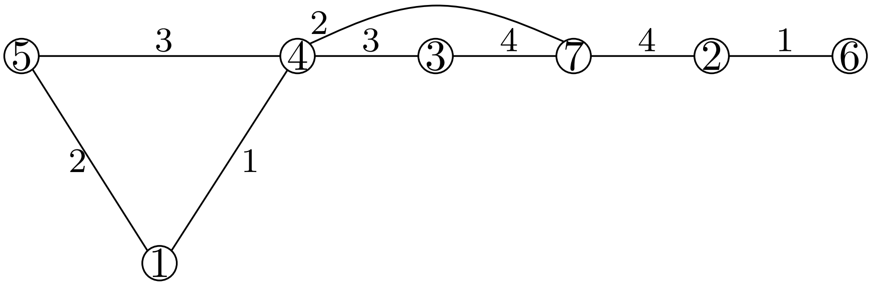

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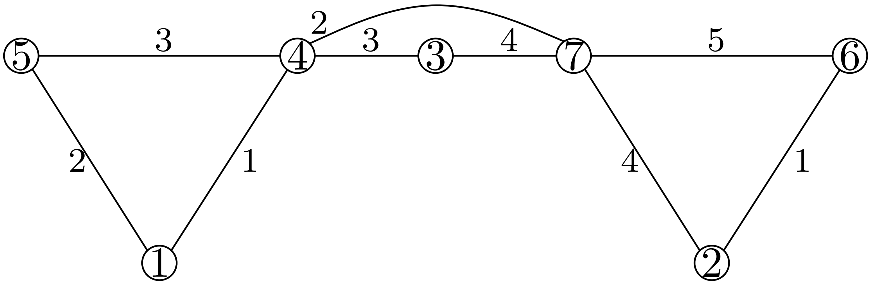

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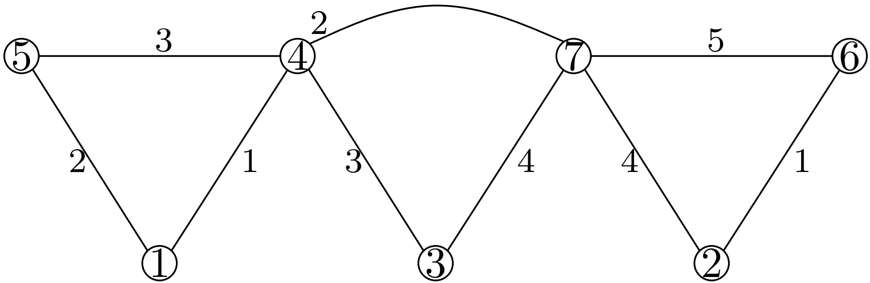

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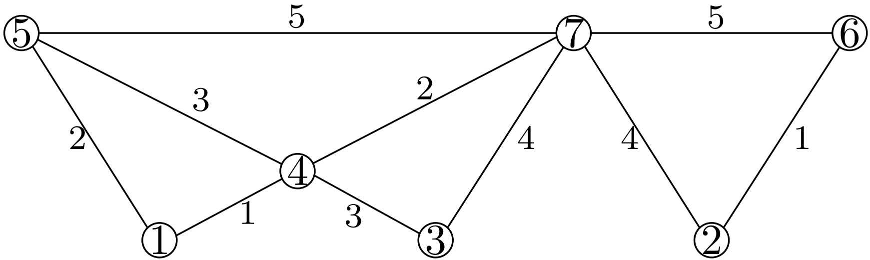

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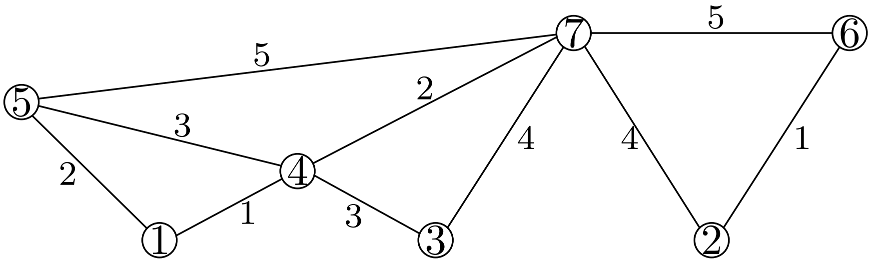

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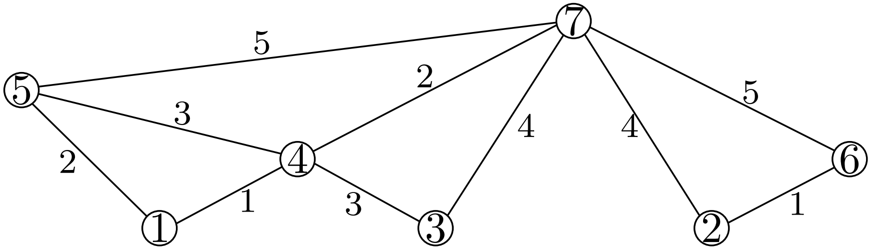

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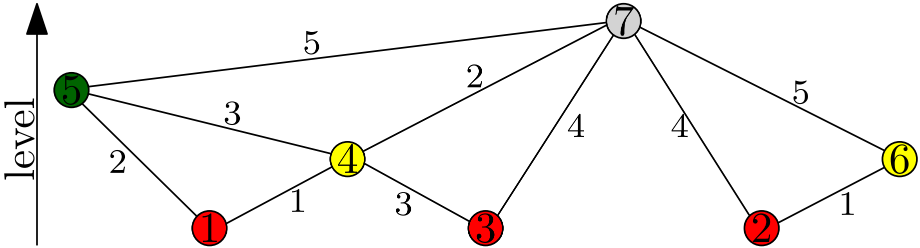

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preprocessing:

- order nodes by importance (heuristic)
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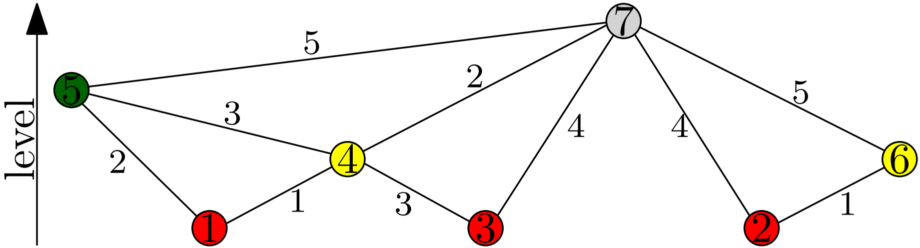
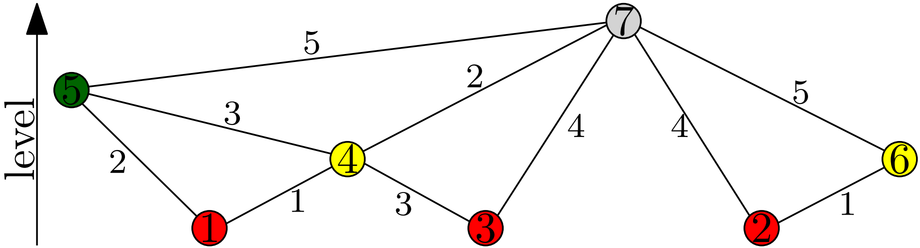
- assign levels (ca. 150 in road networks)
- $\approx$ 5 minutes, 75% increase in number of edges
- heavily relies on the metric (assumes a strong hierarchy)



point-to-point query [GSSD08]

- modified bidirectional Dijkstra
- only follow edges to more important nodes



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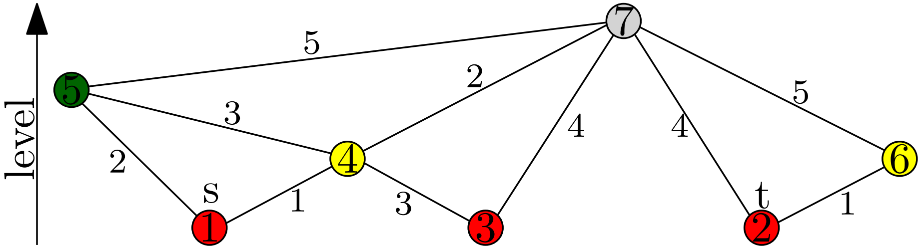

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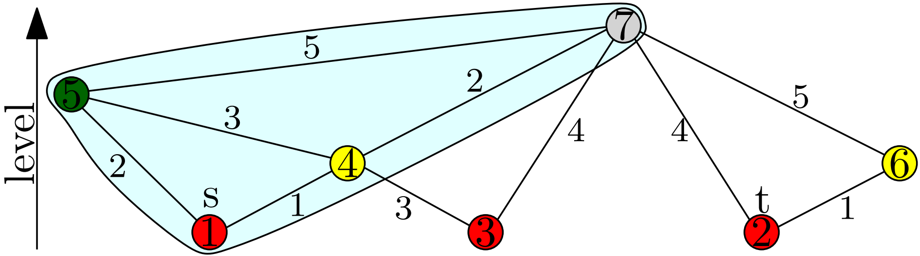

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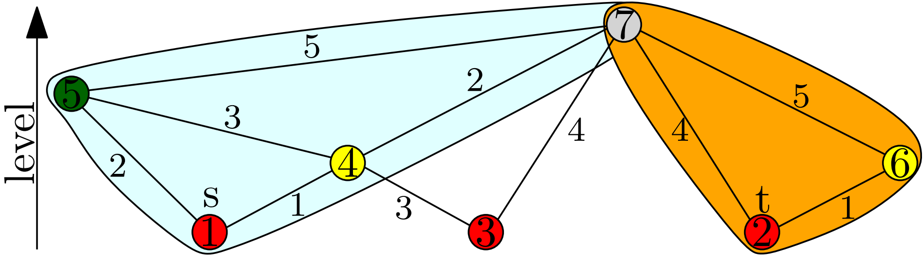

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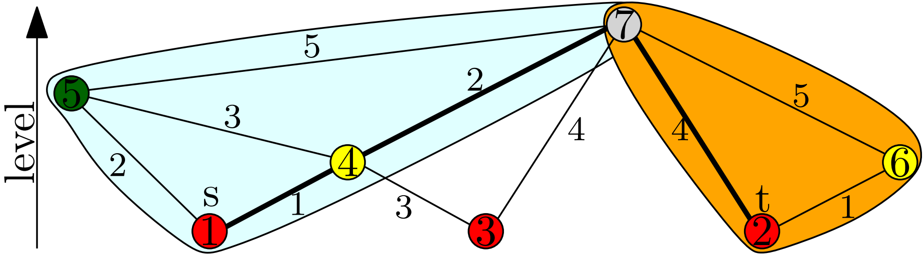

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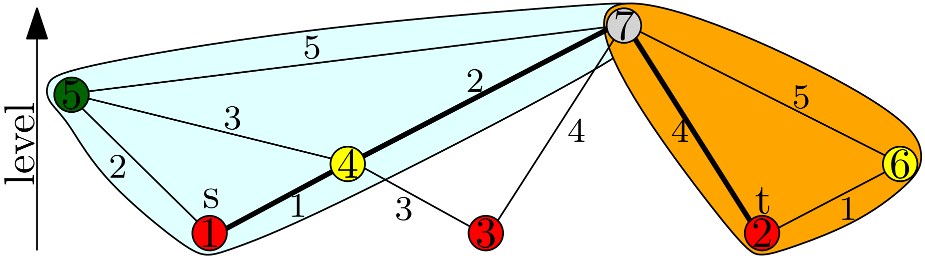
good performance on road networks:

- each upward search scans about 500 nodes
- 10000x faster than bidirectional Dijkstra (point-to-point)



Outline

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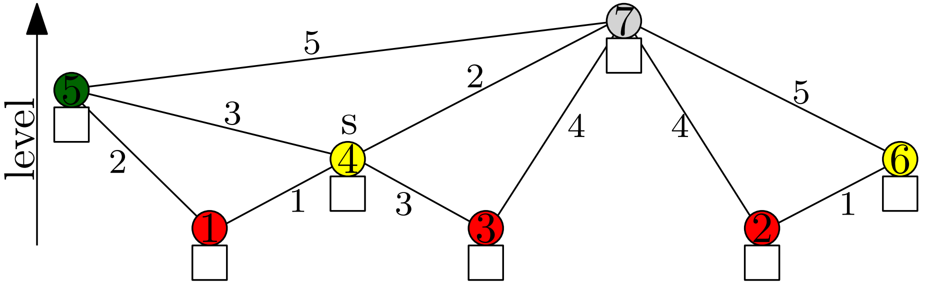
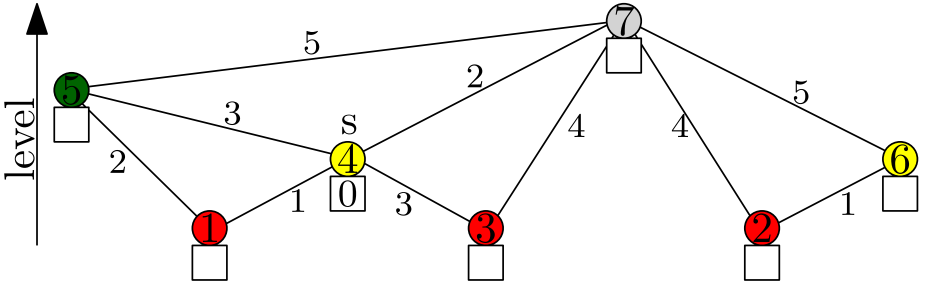


Replacing Dijkstra

one-to-all search from source *s*:

- run CH forward search from s (≈ 0.05 ms)



one-to-all search from source *s*:

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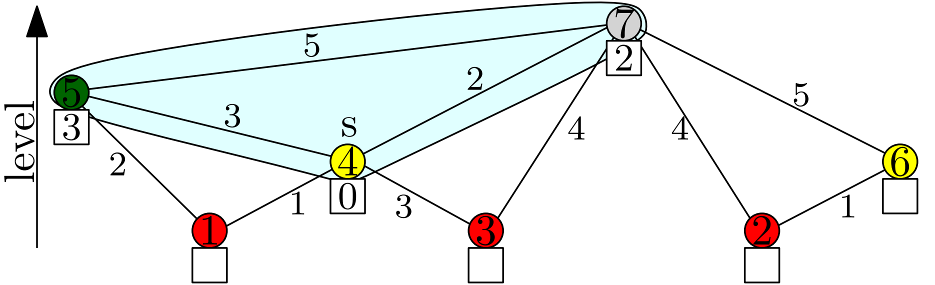
- set distance labels *d* of reached nodes



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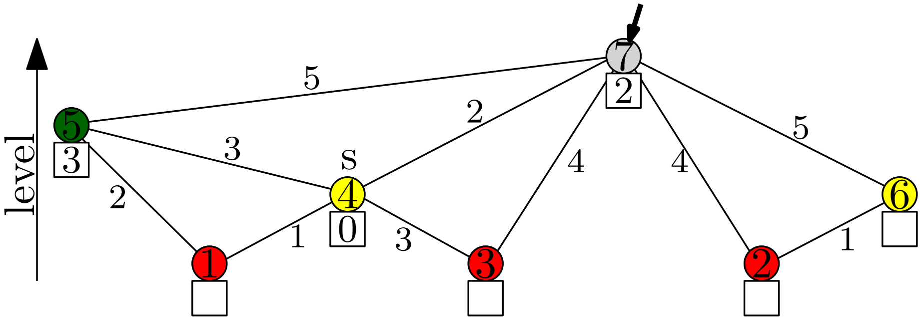

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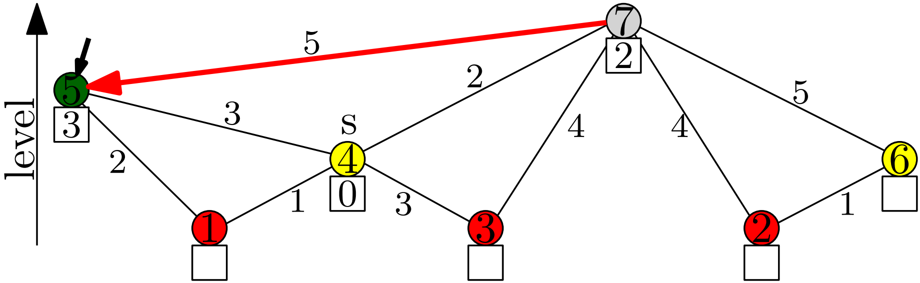

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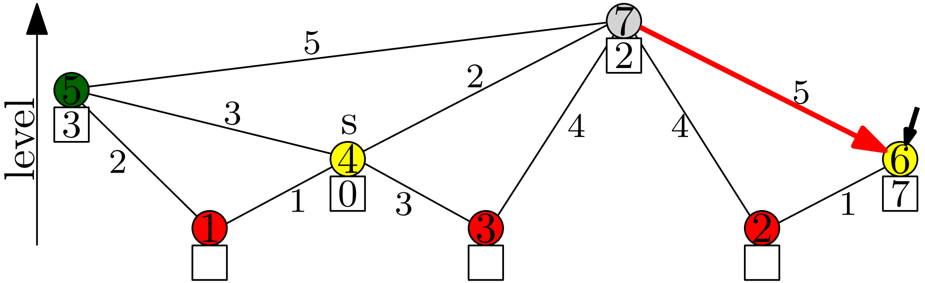

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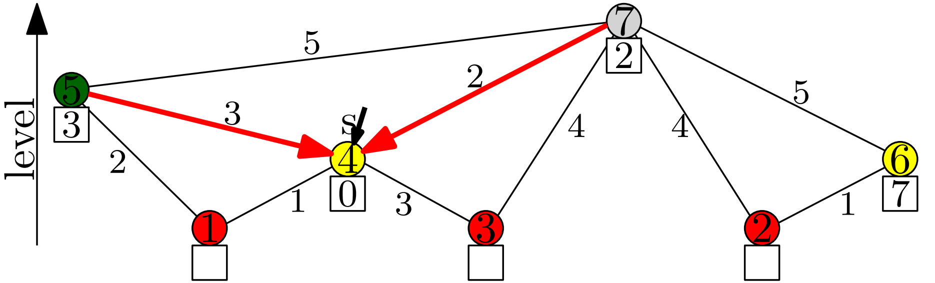

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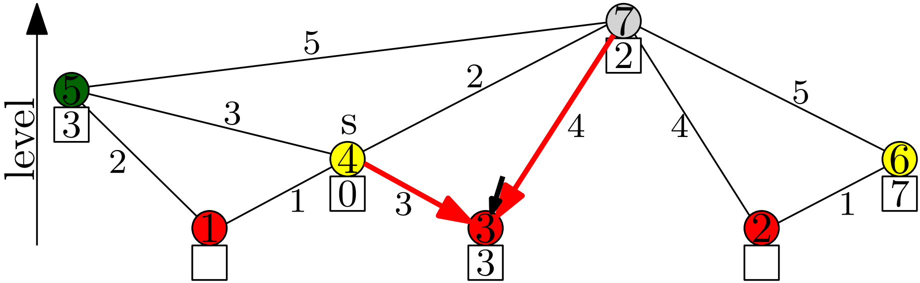

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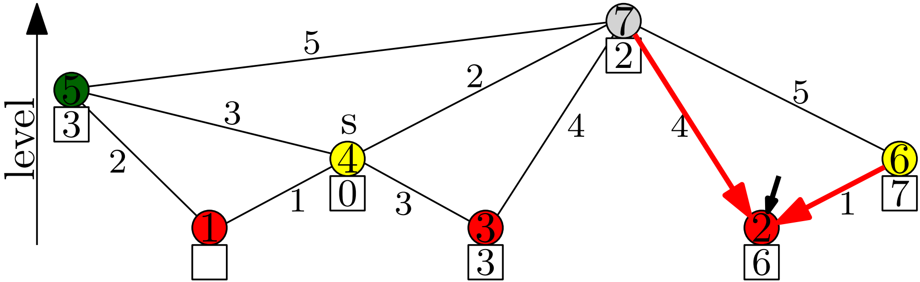

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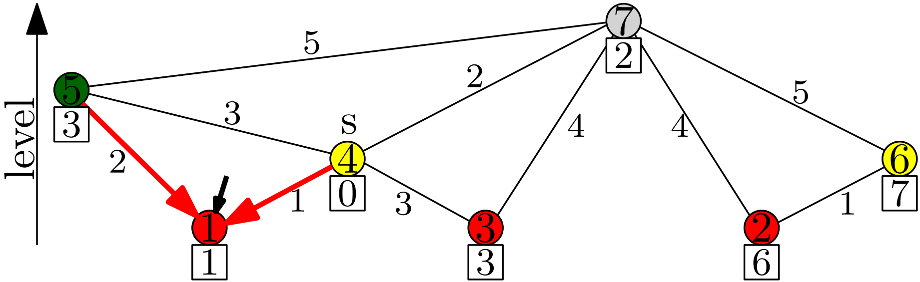

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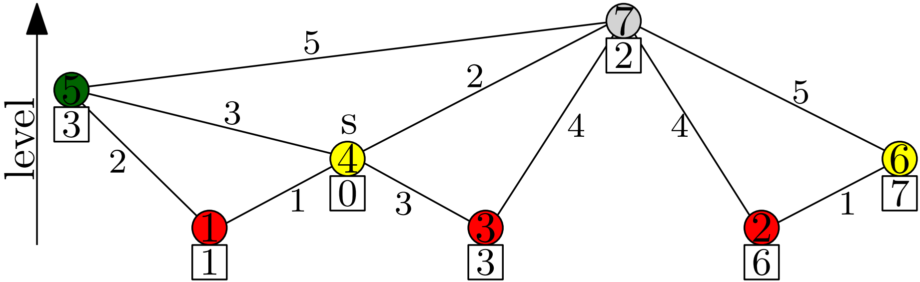
- top-down processing without priority queue (ca. 2.0 s)



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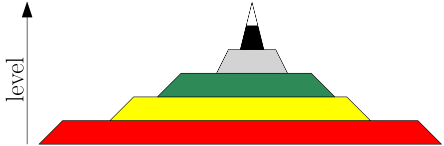
observation:

- top-down process is the bottleneck



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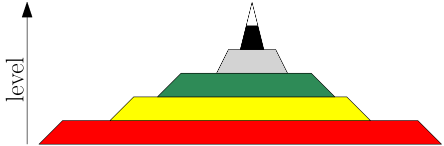

dist(u): 

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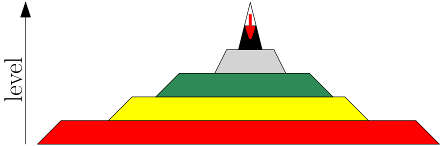

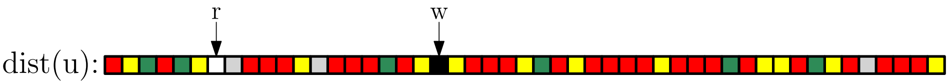

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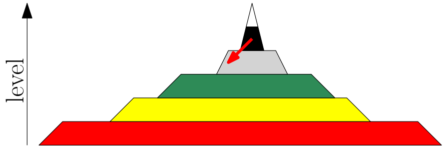

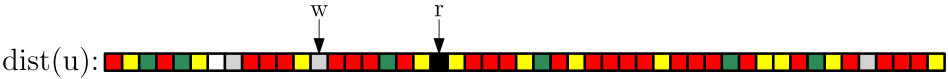

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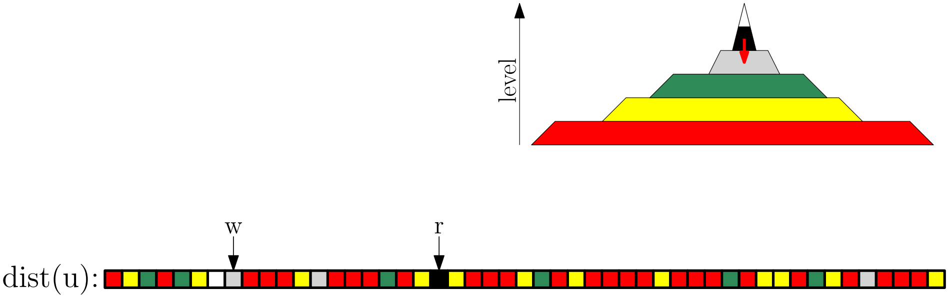

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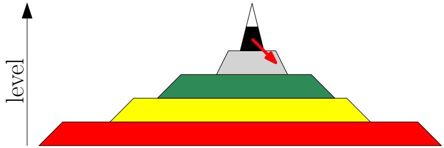

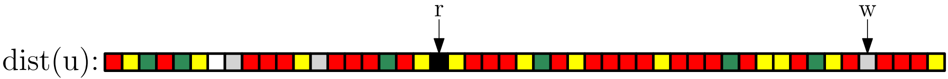

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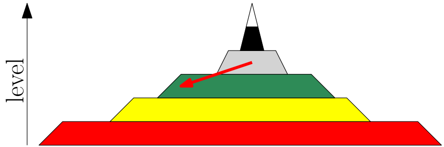

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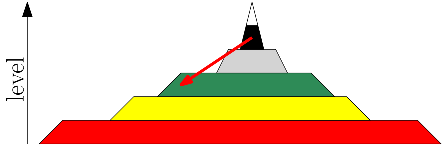

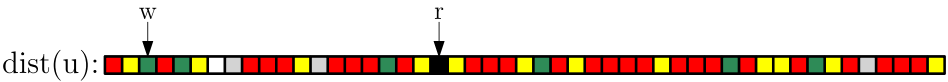

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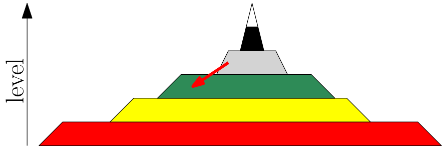

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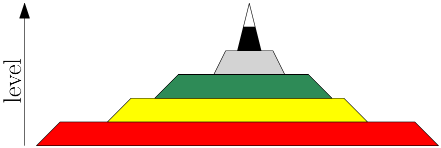

dist(u):

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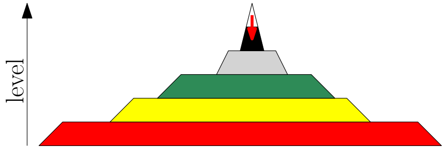

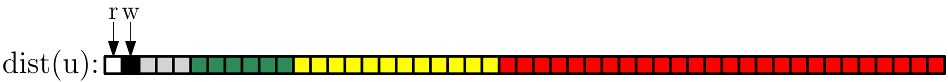

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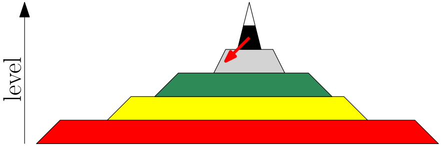

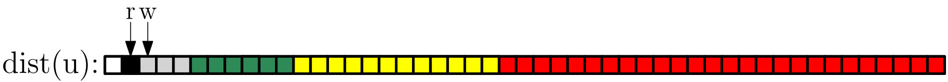

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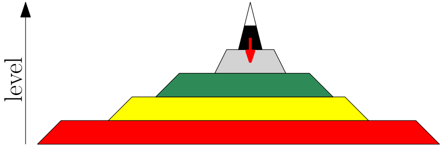

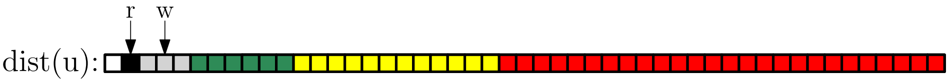

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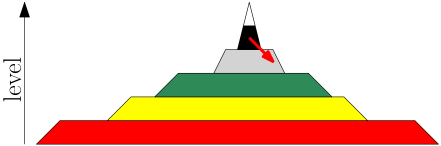

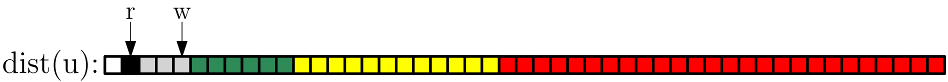

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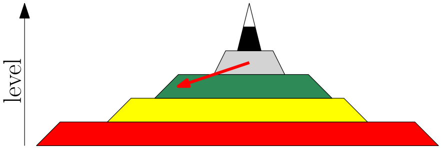

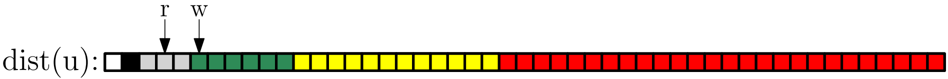

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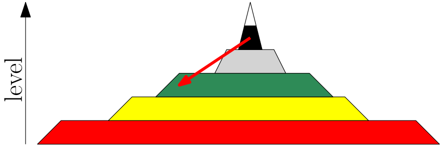

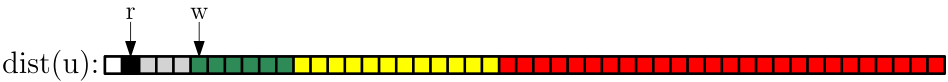

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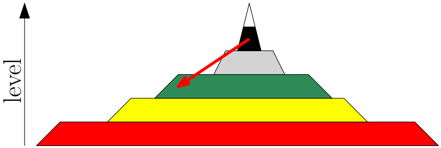

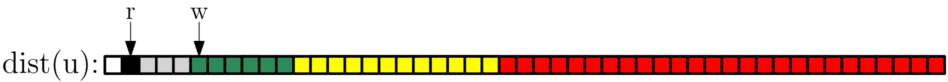

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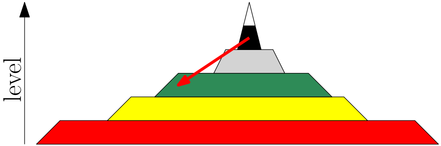

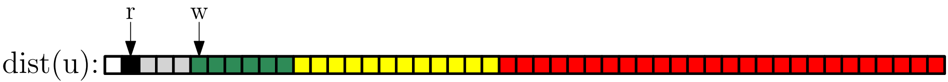

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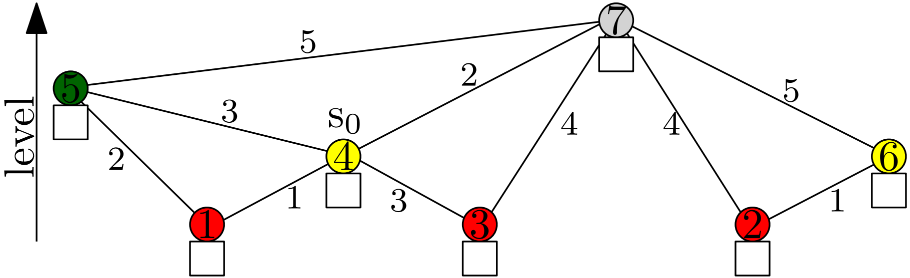

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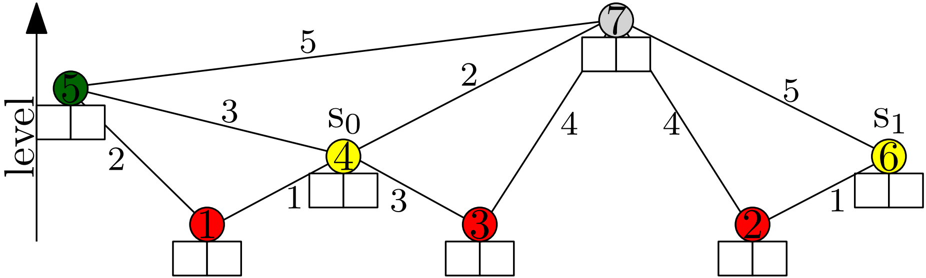

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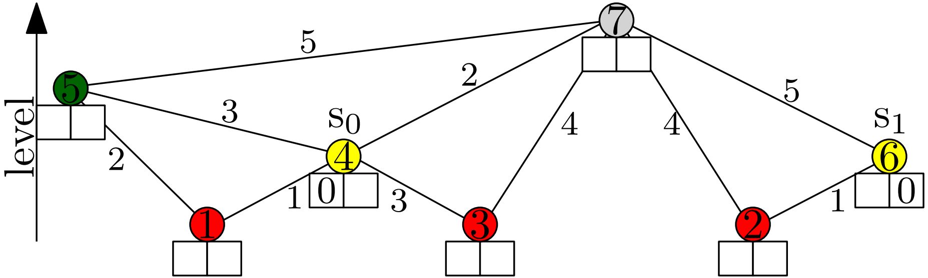

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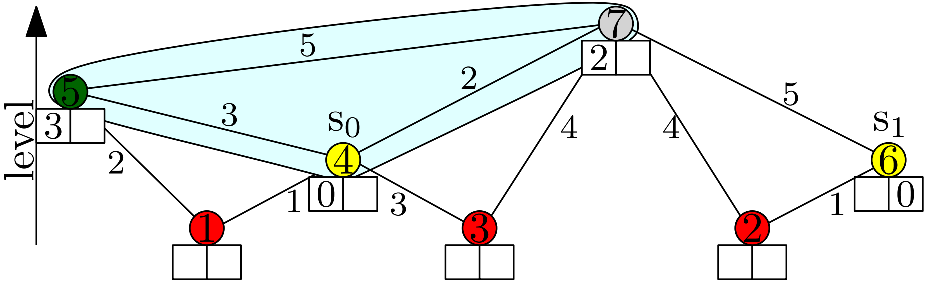

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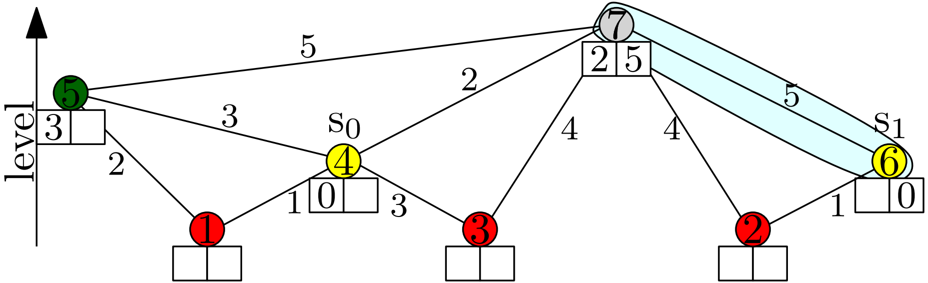

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- run k forward searches
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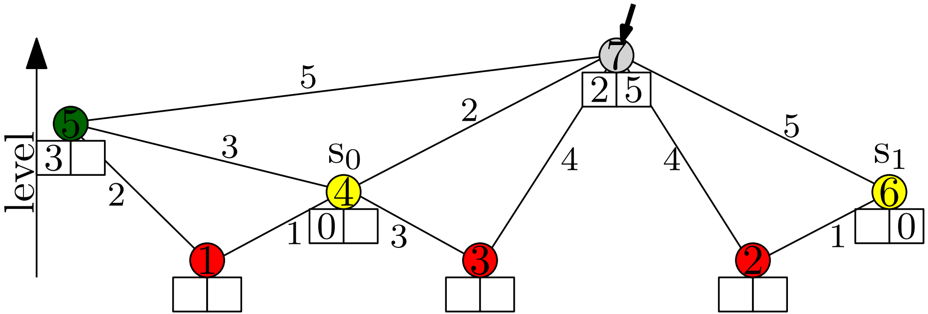

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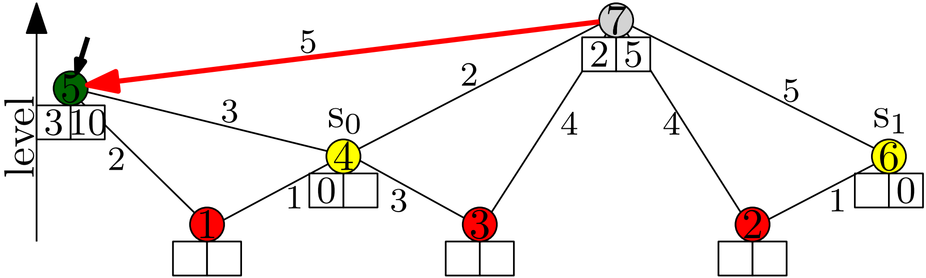

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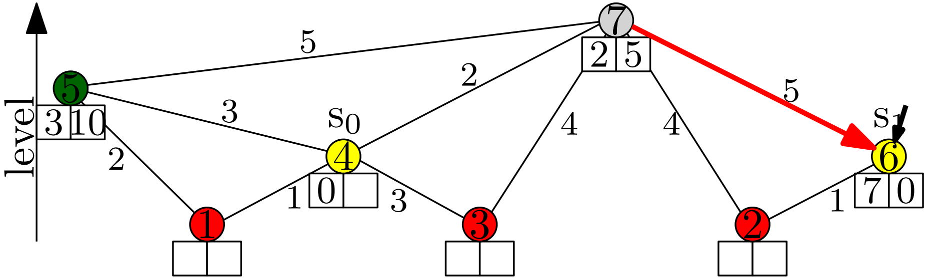

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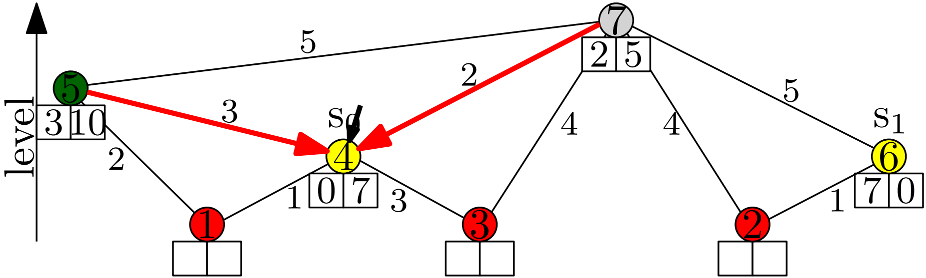

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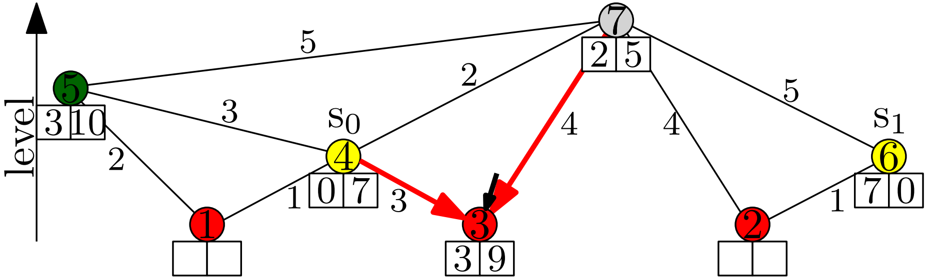

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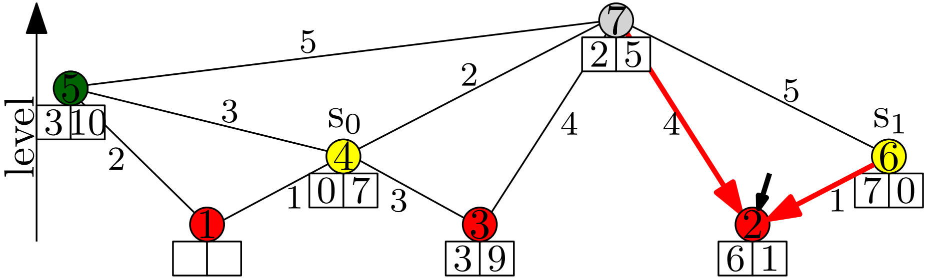

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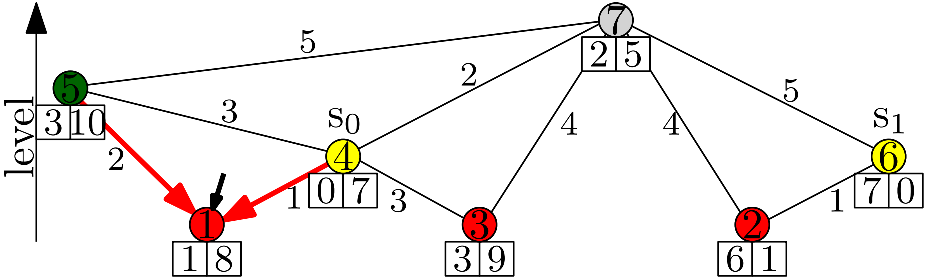

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idea:

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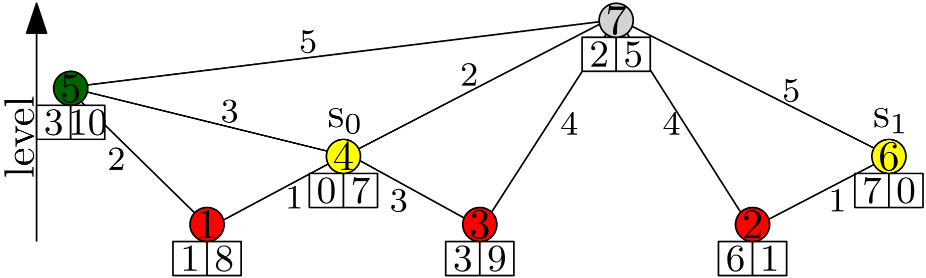
- 96.8 ms per tree ($k = 16$)



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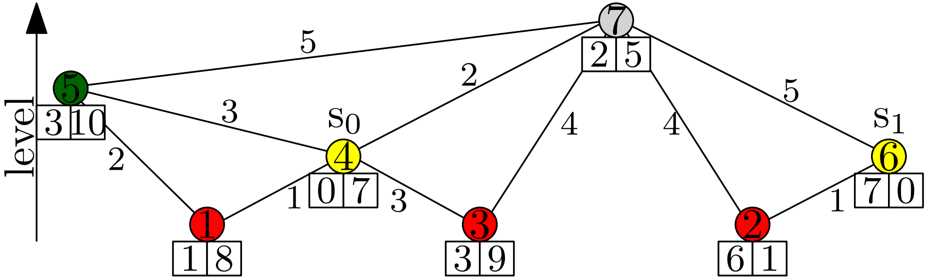
- 128-bit registers
- basic operations (min, add) on four 32-bit integers in parallel
- scan 4 sources at once



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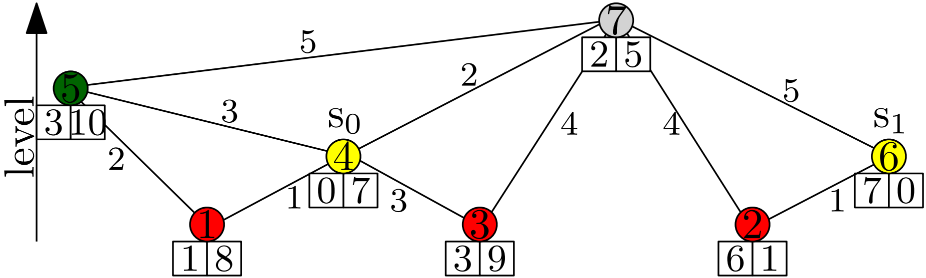
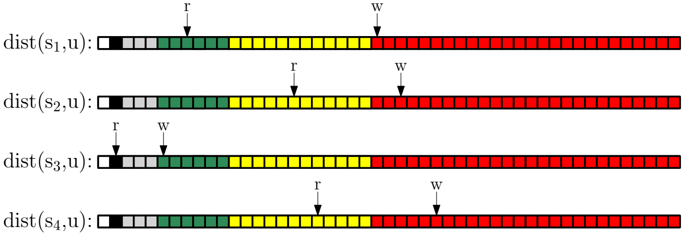


Outline

Parallelization

obvious way of parallelization

- by sources



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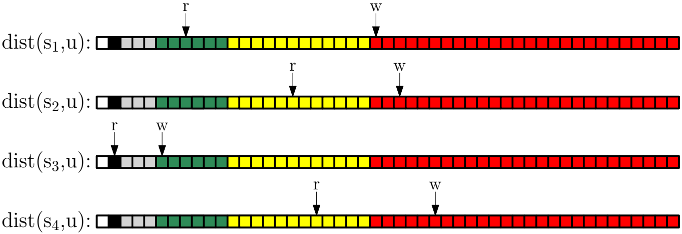

results:

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- multi-core by source nodes

$\Rightarrow$ 64 sources in parallel (4 cores)

- 18.8 ms per tree on average

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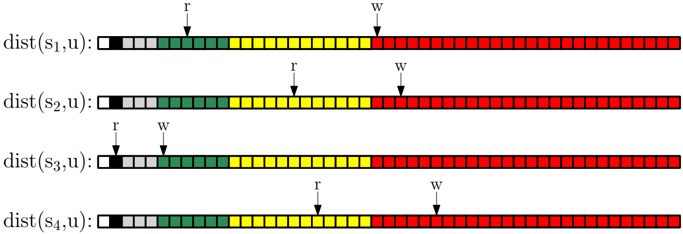

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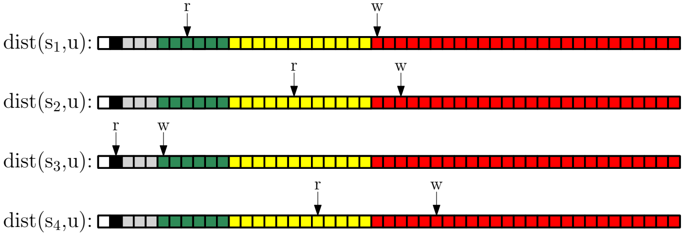

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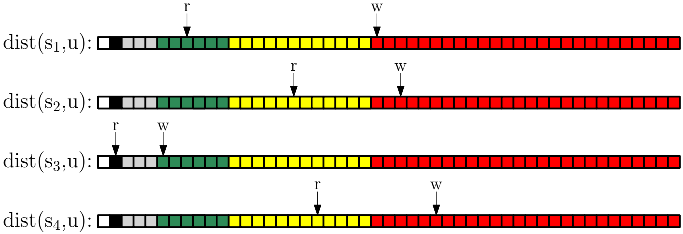

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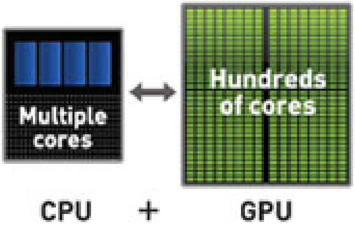
- can a GPU help?

Outline

GPU Architecture



Intel Xeon X5680:

- 3.33 GHz
- 32 GB/s memory bandwidth
- 6 cores

NVIDIA GTX 580:

- 772 MHz, 1.5 GB RAM
- 192 GB/s memory bandwidth
- 16 cores, 32 parallel threads (a warp) per core $\Rightarrow$ 512 threads in parallel

GPHAST - Basic Ideas

observation:

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- limited by memory bandwidth

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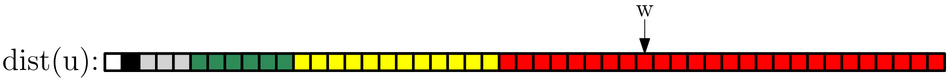
problem:

- not enough memory on GPU to compute thousands of trees in parallel
- we need to parallelize a single tree computation

Parallel Linear Sweep

observation:

- when scanning level i:
  - only incoming arcs from level $> i$ are relevant
  - writing distance labels in level i, read from level $> i$
  - distance labels for level $> i$ are correct
- scanning a level-i node is independent from other level-i nodes



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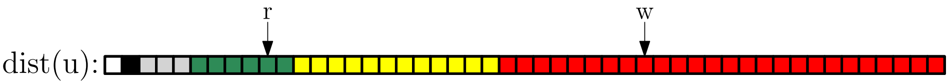

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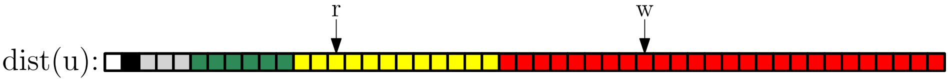

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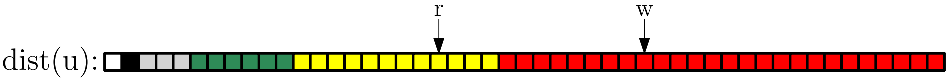

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- synchronization after each level
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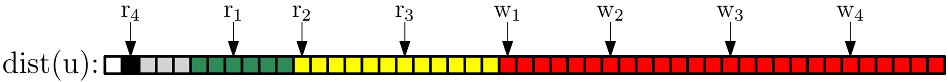

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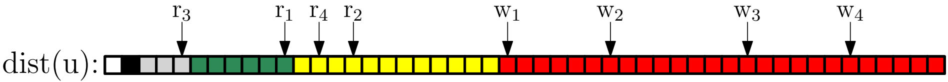

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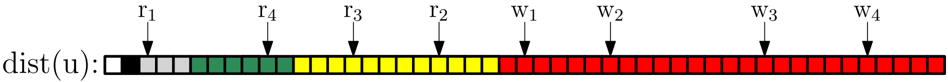
- 511 speedup over Dijkstra



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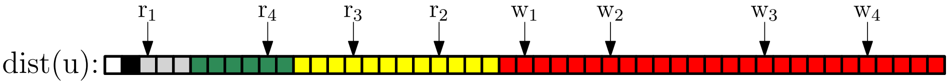

All-Pairs Shortest Paths

algorithm	device	time	energy [MJ]
Dijkstra	4-core workstation	197d	
	12-core server	60d	
	48-core server	35d	
PHAST	4-core workstation	94h	
	12-core server	36h	
	48-core server	20h	
GPHAST	GTX 580		

All-Pairs Shortest Paths

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	12-core server	60d	
	48-core server	35d	
PHAST	4-core workstation	94h	
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4-core workstation without GPU: 163 watts
4-core workstation with GPU: 375 watts
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	12-core server	60d	1725.9
	48-core server	35d	2265.5
PHAST	4-core workstation	94h	55.2
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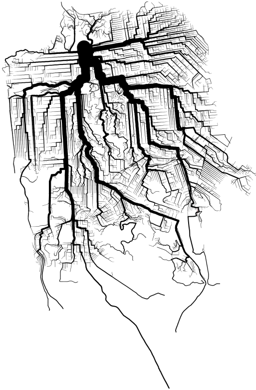
Outline

Conclusion

summary:

- one tree on a GPU: 5.5 ms
  (about 0.31 ns per entry)
- real-time computation of shortest path trees
- 16 trees on a GPU at once: 2.2 ms per tree
  (about 0.13 ns per entry)
- APSP in 11 hours (on a workstation with one GPU),
  instead of half a year (on 4 cores)
- APSP-based computation becomes practical
- 150 times more energy-efficient than Dijkstra's algorithm



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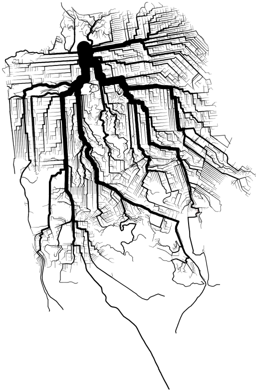

other recent results:

- point-to-point shortest paths with a few memory accesses
- refinement of highway dimension
- graph partitioning
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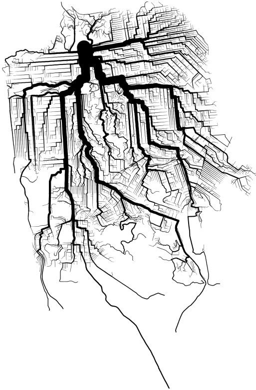

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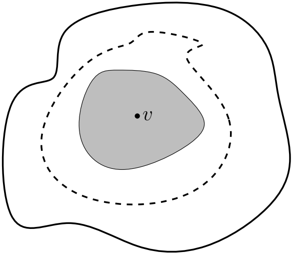
Thank you for your attention!

Appendix

Graph Partitioning I: Filtering

[DGRW11]

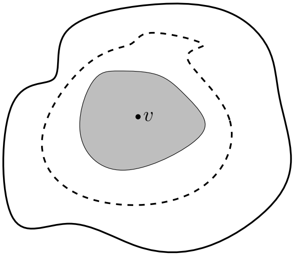


1. natural cut detection

- pick a random center
- use BFS to define a core and a ring
- find minimum cut between them
- repeat multiple times

Graph Partitioning I: Filtering

[DGRW11]



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- pick a random center
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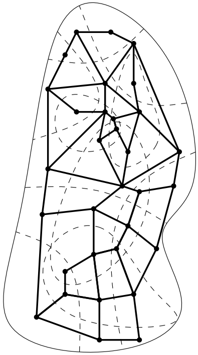
2. contraction

- keep only edges that appeared in some cut
- contract the rest into fragments
- reduces graph by several orders of magnitude
- preserves natural cuts between dense regions
  (e.g., bridges, national borders, mountain passes...)



Graph Partitioning II: Assembly

[DGRW11]

1. run greedy algorithm

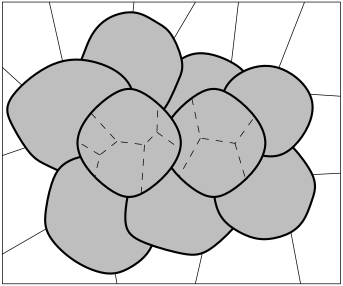
- join well-connected fragments
- find maximal solution

2. run local search

- reoptimize pairs of adjacent cells
- fragments can move to neighboring cells

3. enhanced optimizations (optional)

- multistart, recombination, branch-and-bound



$\Rightarrow$ **yields best known solutions for road networks**

Case Study: Point-to-Point Shortest Paths

two phase approach:

- preprocess network to compute auxillary data
- use data to speed up queries
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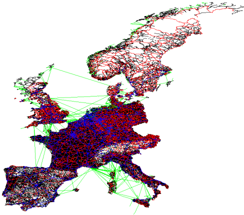

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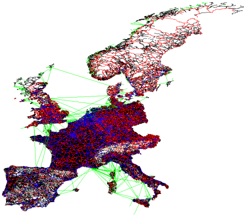


observation:

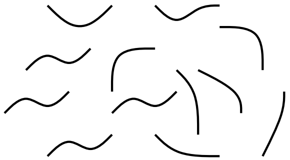
- excellent performance in practice
- used in production
- prime example for algorithm engineering
- but for a long time: no theoretical justification

A Theoretical Justification: Highway Dimension

[AFGW10]

(r, k) **shortest path cover**

- all shortest paths with length between r and $2r$ are hit



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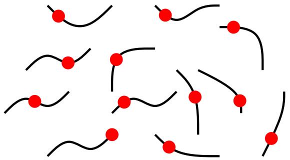

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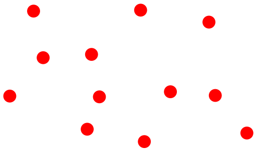

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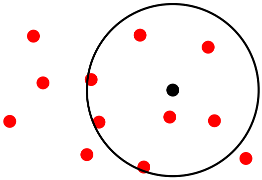

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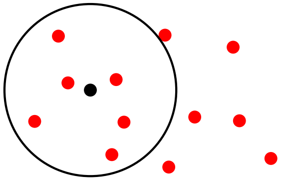

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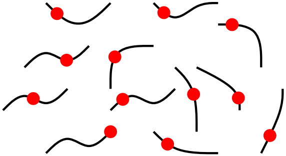

Highway Dimension

A graph with highway dimension h has an (r, h)-SPC for all r.

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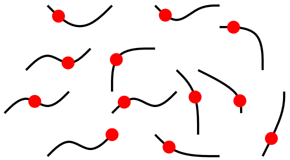

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A graph with highway dimension h has an (r, h)-SPC for all r.

results:

- sublinear query bounds for many algorithms
- best query bound: a labeling algorithm
- has not been considered in practical implementations

A Labeling Algorithm

[CPPR04]

preprocessing:

- compute a label $L(v)$ for each vertex v
- compute $\mathrm{dist}(v, w)$ for each vertex $w \in L(v)$
- obey the label property:
  for all s, t a shortest s–t path intersects $L(s) \cap L(t)$

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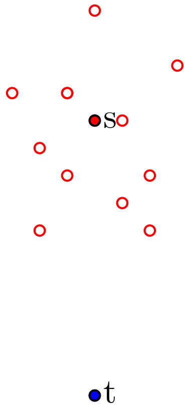

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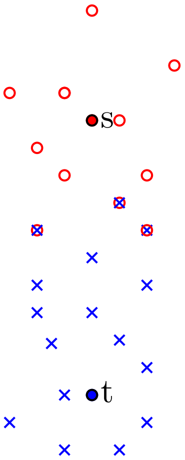

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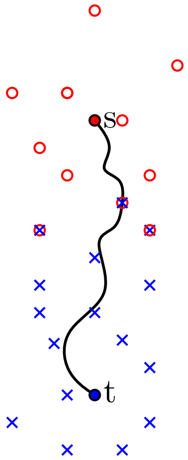

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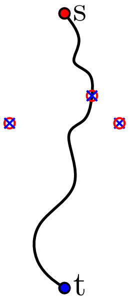

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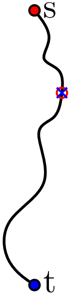

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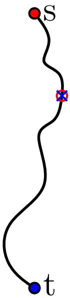

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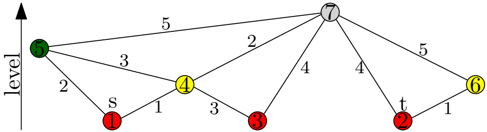
observation:

- practical if labels are small
- how to compute labels efficiently?
- SPC algorithms currently are too slow
  (maybe PHAST can help)

Practical Implementation: HubLabels

[ADGW11]

idea:



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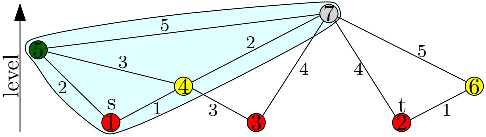

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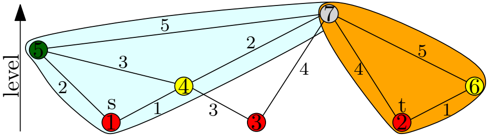

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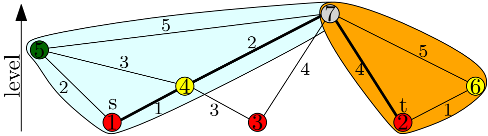

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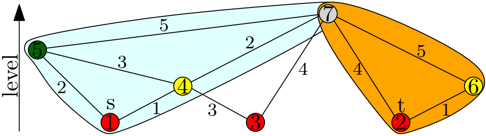

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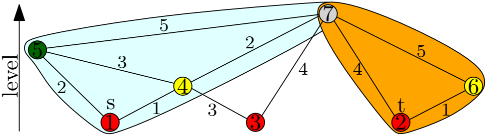

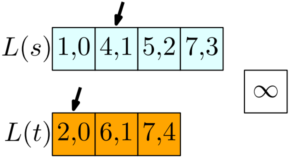

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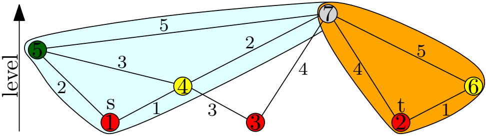

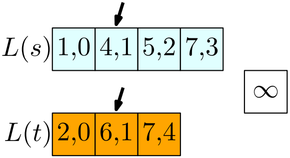

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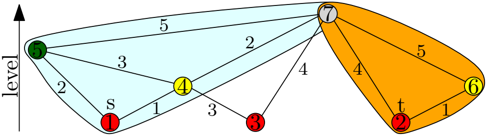

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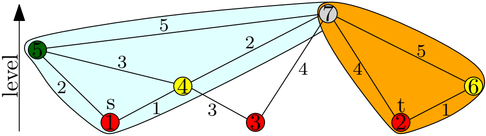

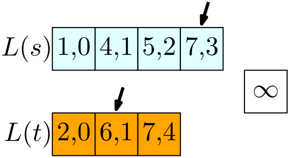

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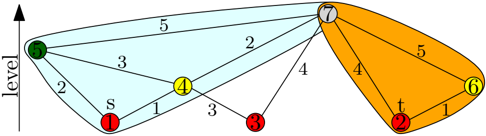

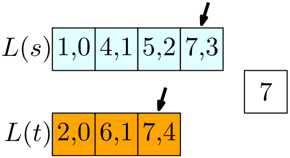

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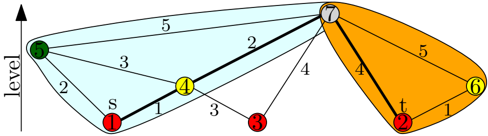

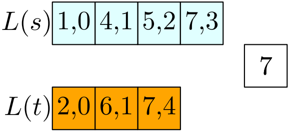

query:

- process like merge sort
- update whenever the ids match
- very cache-efficient

problem:

- average label sizes of around 500 ⇒ 150 GB of data

Optimizations

[ADGW11]

label sizes:

- 80% of the nodes in search spaces unnecessary
- prune by bootstrapping
- SPC algorithms on small important subgraph

$\Rightarrow$ average label size shrinks to 85 ($\rightarrow$ 24 GB)

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reduce number of cache lines read:

- use compression ($\rightarrow$ 6 GB)
- define partition oracle to accelerate long-range queries
- many algorithmic low-level optimizations

$\Rightarrow$ we fetch only a few cache lines from memory

Results

	preprocessing		query
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Results

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$\Rightarrow$ **new running time record**