Algorithmen zur Visualisierung von Graphen

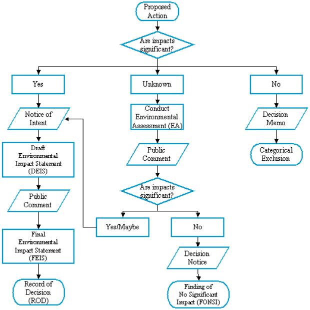
Symmetrien in Serienparallelen Graphen
Geradlinige planare Gitterzeichnungen

Vorlesung im Sommersemester 2009

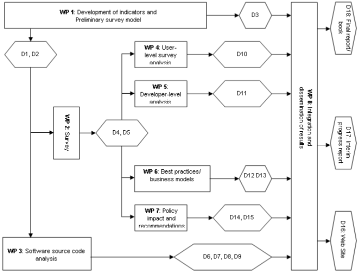
Martin Nöllenburg

23.07.2009

SP-Graphen in Anwendungen



Ablaufdiagramme



PERT-Diagramme

(Program Evaluation and Review Technique)

Außerdem: Linearzeitalgorithmen für sonst NP-vollständige Probleme (z.B. Maximum Independent Set)

Symmetrien in SP-Zeichnungen

Definition: Automorphismen eines DAG

Ein Automorphismus eines DAG $G = (V, E)$ ist eine Knotenpermutation $\pi : V \to V$, die Adjazenzen respektiert und alle Kantenrichtungen erhält oder alle Kantenrichtungen umdreht:

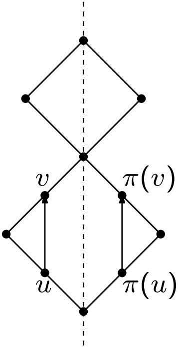
$$(u, v) \in E \Leftrightarrow (\pi(u), \pi(v)) \in E$$

oder

$$(u, v) \in E \Leftrightarrow (\pi(v), \pi(u)) \in E.$$

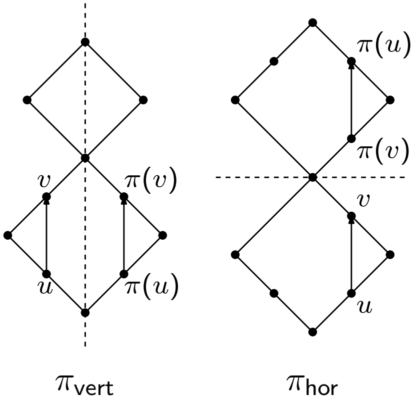
Die Automorphismen von G bilden mit der Hintereinanderausführung eine Gruppe.

Symmetrien in SP-Graphen

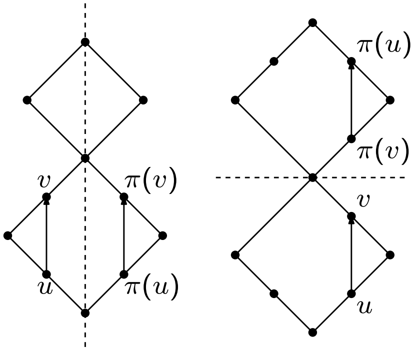


π_{vert}

Symmetrien in SP-Graphen

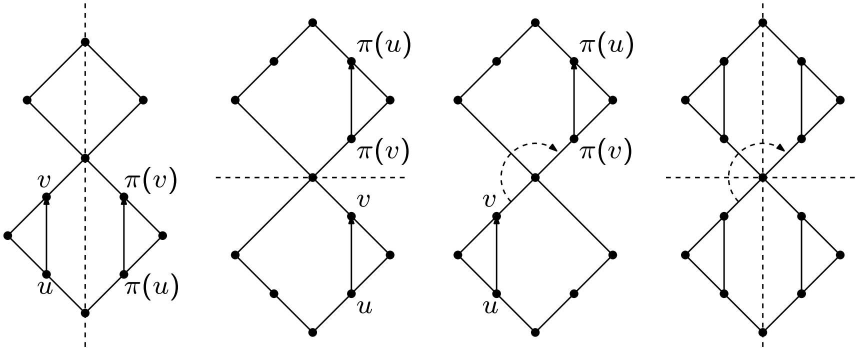


Symmetrien in SP-Graphen



π_{vert} π_{hor} π_{rot}

Symmetrien in SP-Graphen



π_{vert} π_{hor} π_{rot} $\{\pi_{\text{vert}}, \pi_{\text{hor}}, \pi_{\text{rot}}\}$

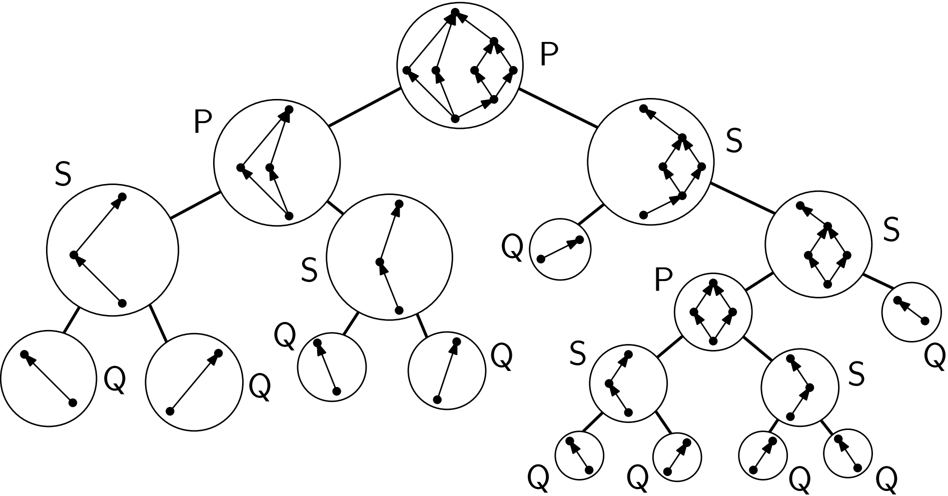
Symmetrien in SP-Zeichnungen

Satz (Hong, Eades, Lee ’00)

Die in einem kreuzungsfreien Aufwärtslayout eines SP-Graphen darstellbaren Symmetrien sind entweder

1. $\{\text{id}\}$
2. $\{\text{id}, \pi\}$ mit $\pi \in \{\pi_{\text{vert}}, \pi_{\text{hor}}, \pi_{\text{rot}}\}$
3. $\{\text{id}, \pi_{\text{vert}}, \pi_{\text{hor}}, \pi_{\text{rot}}\}$.

Knotenkodierung im Dekompositionsbaum



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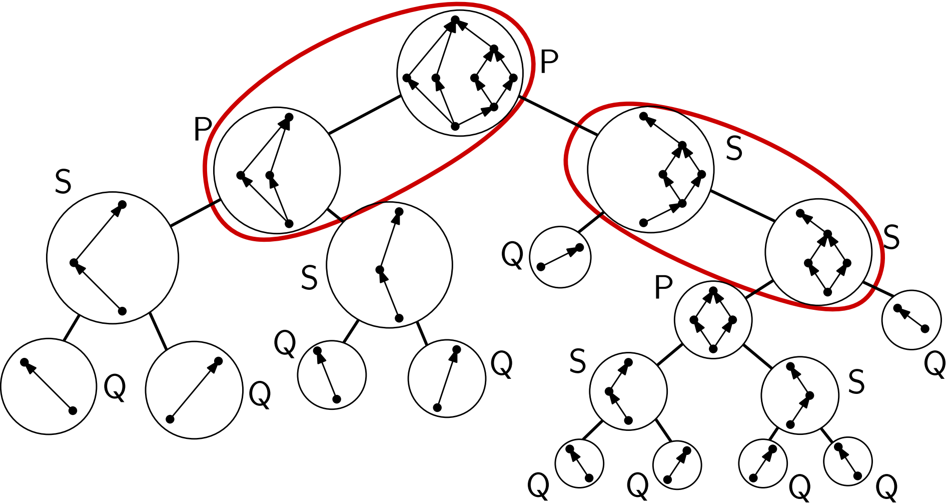

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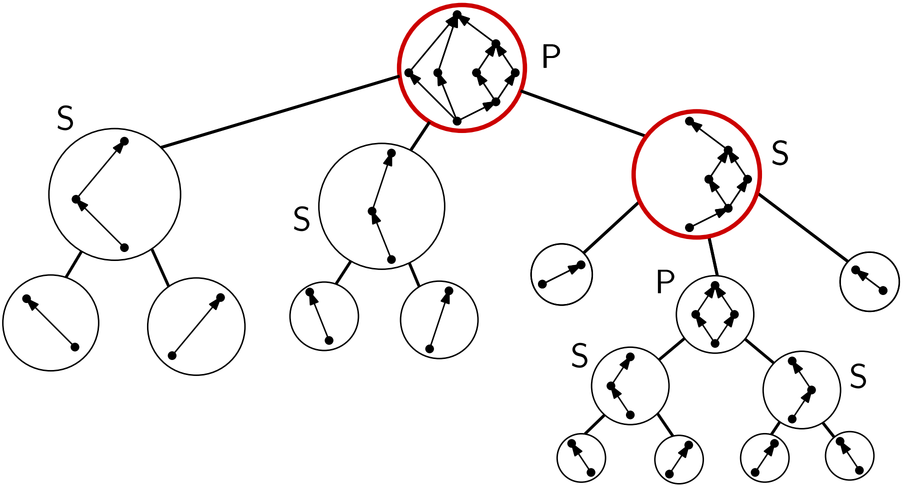

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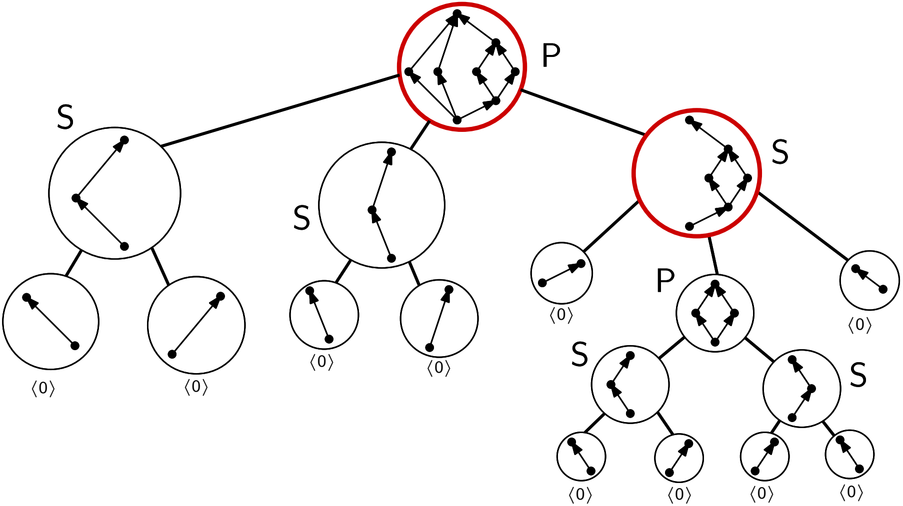

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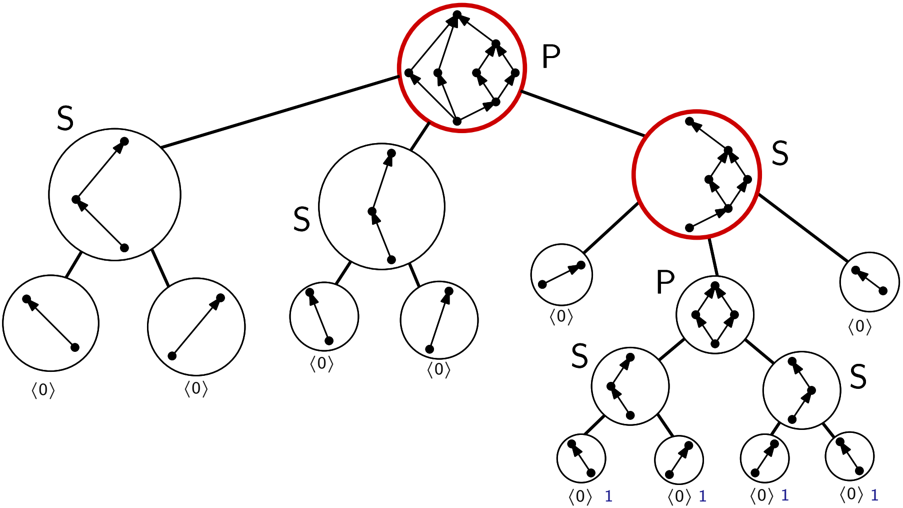

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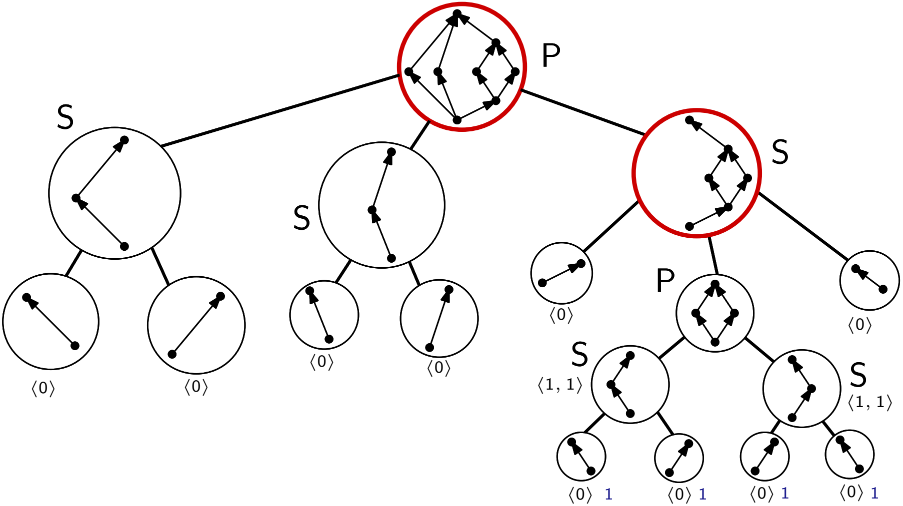

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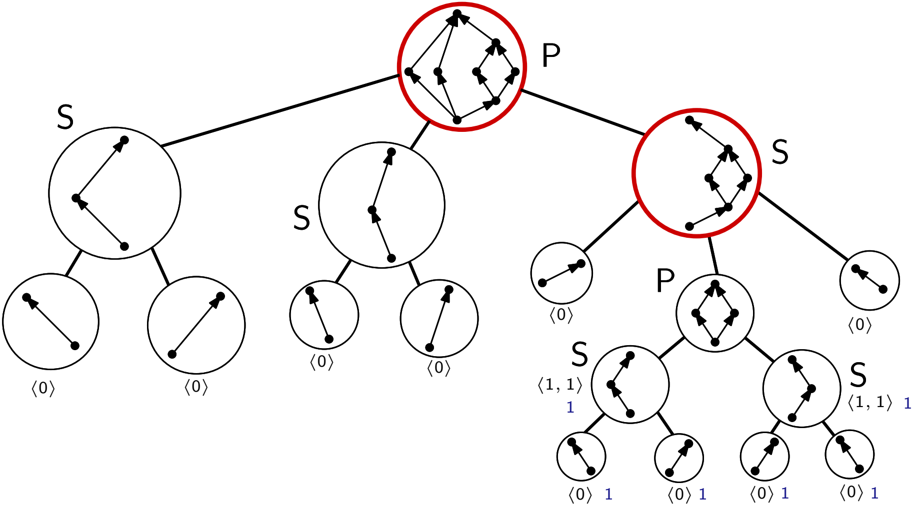

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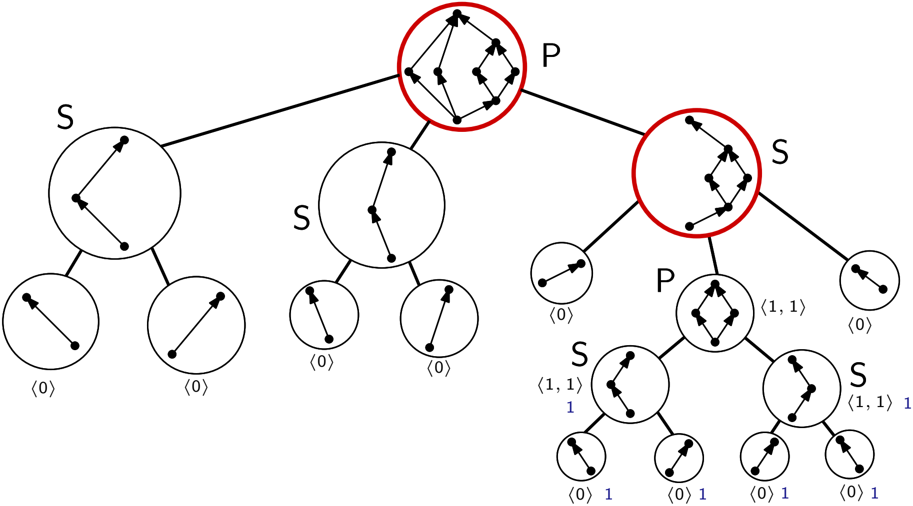

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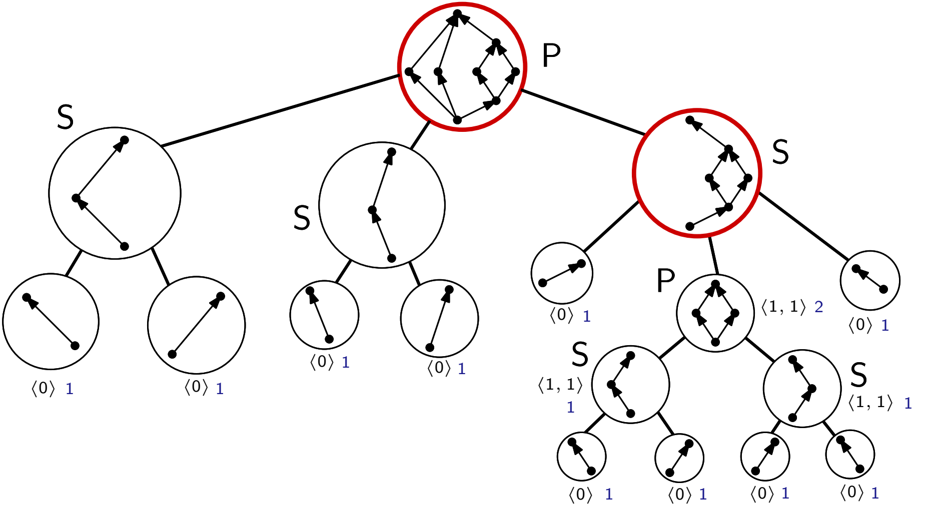

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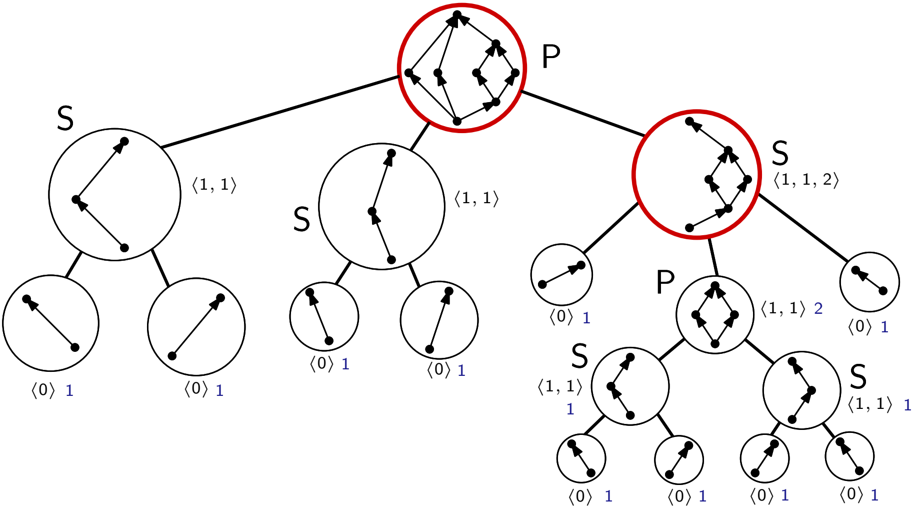

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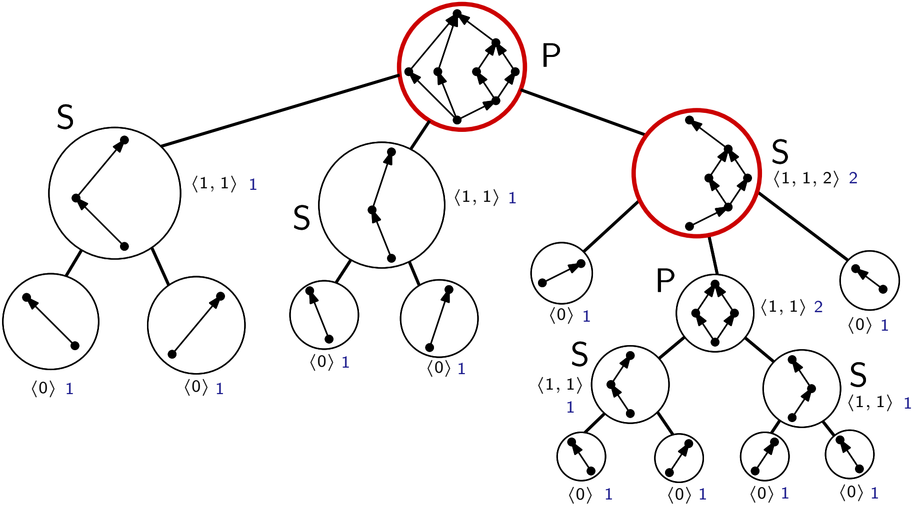

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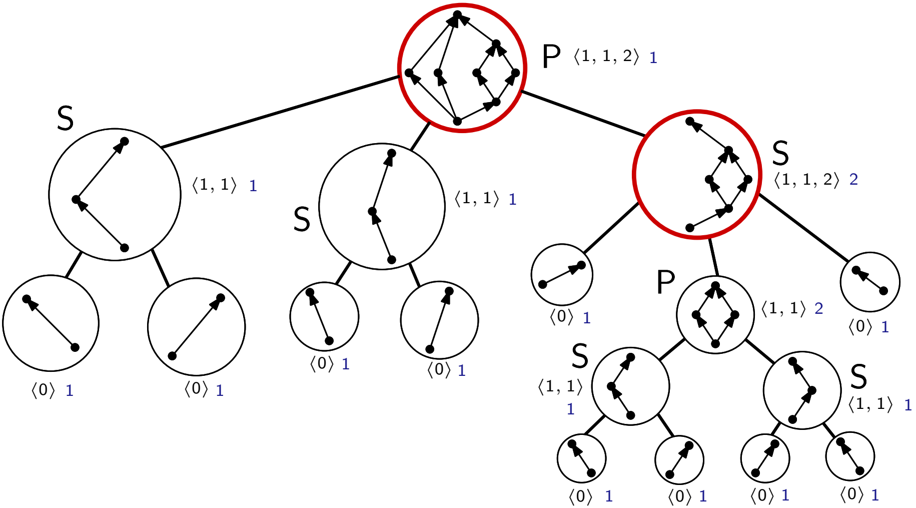
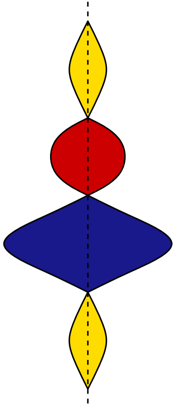
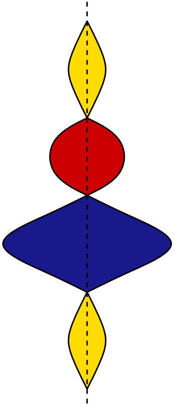


Beweisidee vertikale Symmetrie

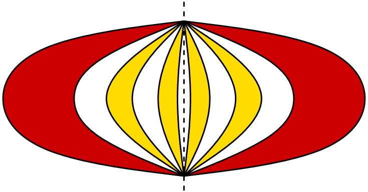


S-Knoten

Beweisidee vertikale Symmetrie

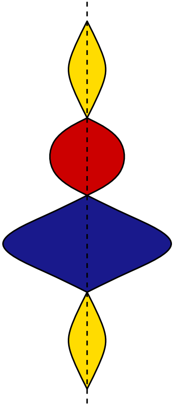


S-Knoten

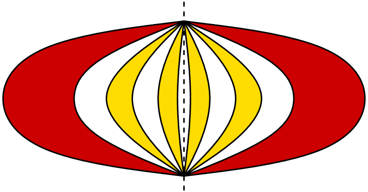


P-Knoten, alle Klassen gerade Anzahl

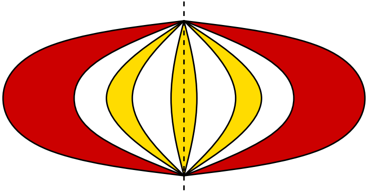
Beweisidee vertikale Symmetrie



S-Knoten
alle Knoten v-symm.

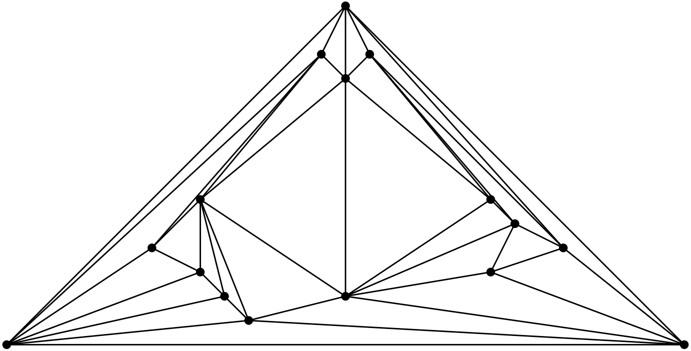


P-Knoten, alle Klassen gerade Anzahl



P-Knoten, eine Klasse ungerade Anzahl & v-symm.

Inkrementelles Gitterlayout für planare Graphen



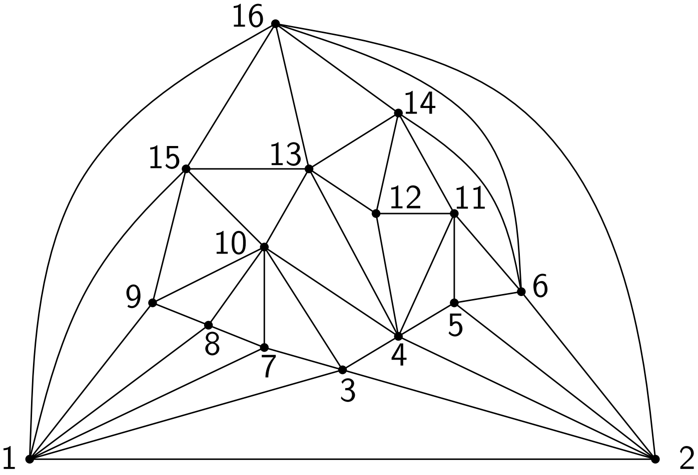
Kanonische Knotenordnung

Definition: Kanonische Knotenordnung

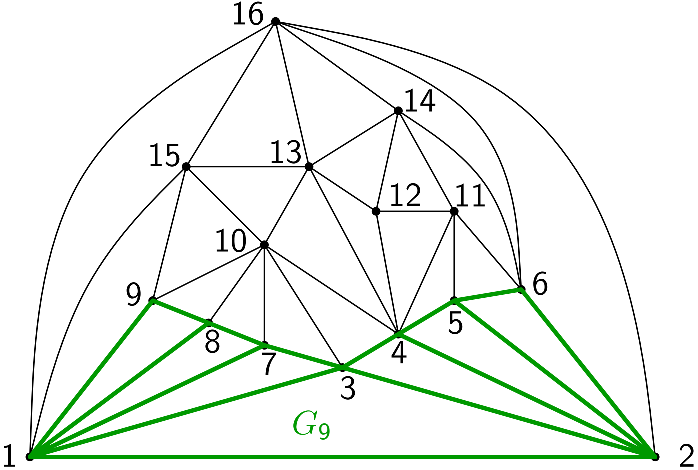
Sei $G = (V, E)$ ein triangulierter, planar eingebetteter Graph mit $n \geq 3$ Knoten. Eine Knotenordnung $\pi = (v_1, v_2, \dots, v_n)$ heißt *kanonische Ordnung*, falls gilt

1. der von $\{v_1, \dots v_k\}$ induzierte Teilgraph G_k ist 2-zusammenhängend und intern trianguliert
2. (v_1, v_2) ist Außenkante von G_k
3. für $k < n$ liegt v_{k+1} in der äußeren Facette von G_k und alle Nachbarn von v_{k+1} in G_k bilden ein Intervall auf dem Rand $C_o(G_k)$ der äußeren Facette von G_k

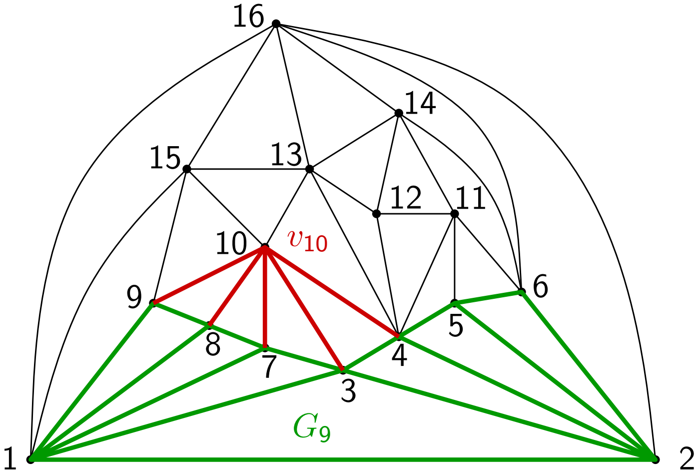
Beispiel kanonische Ordnung



Beispiel kanonische Ordnung



Beispiel kanonische Ordnung

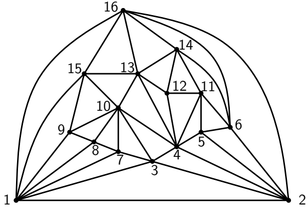


Algorithmus kanonische Ordnung

Algorithm 1: Canonical-Ordering

forall $v \in V$ **do**
  chords(v) $\leftarrow$ 0; out(v) $\leftarrow$ false; mark(v) $\leftarrow$ false;
out(v_1), out(v_2), out(v_n) $\leftarrow$ true;
for $k = n$ **to** 3 **do**
  wähle $v \neq v_1, v_2$ mit mark(v) $=$ false, out(v) $=$ true, chords(v) $=$ 0;
  $v_k \leftarrow v$; mark(v) $\leftarrow$ true;
  $(w_1 = v_1, w_2, \ldots, w_{t-1}, w_t = v_2) \leftarrow C_o(G_{k-1})$;
  $(w_p, \ldots, w_q) \leftarrow$ unmarkierte Nachbarn von v_k;
  out(w_i) $\leftarrow$ true for all $p < i < q$;
  aktualisiere chords($\cdot$) für diese w_i und ihre Nachbarn;

Beispiel



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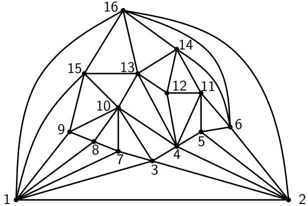

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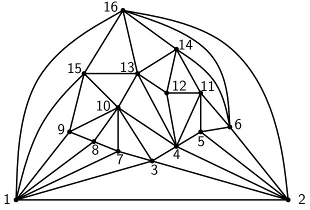

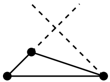

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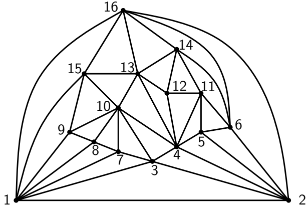

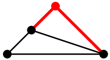

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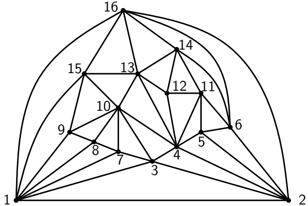

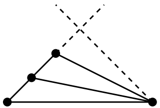

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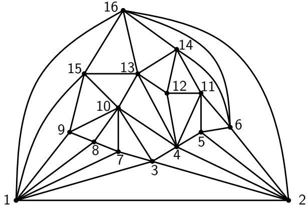

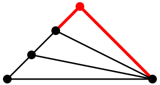

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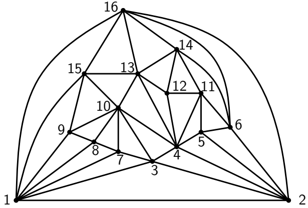

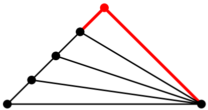

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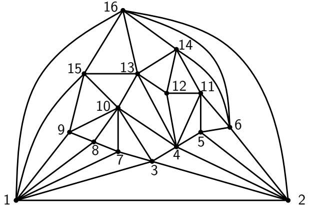

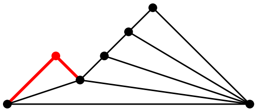

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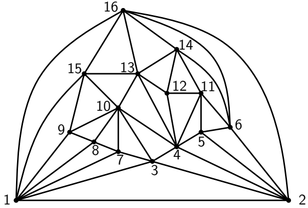

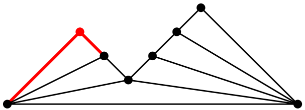

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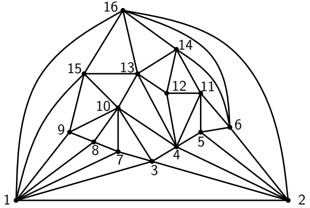

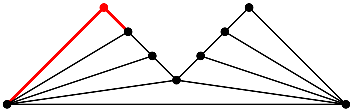

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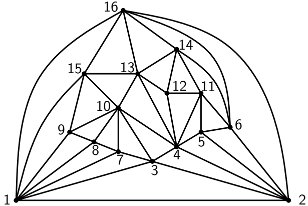

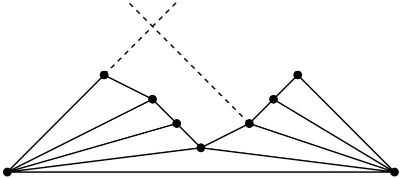

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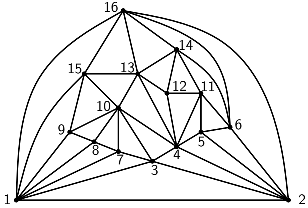

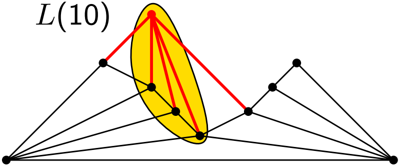

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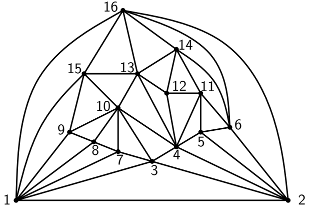

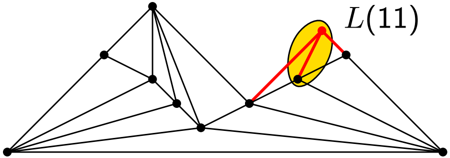

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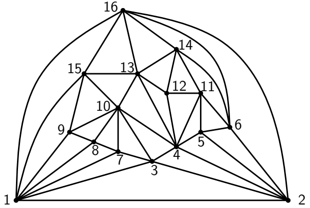

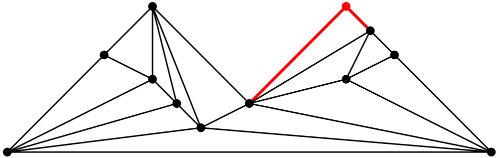

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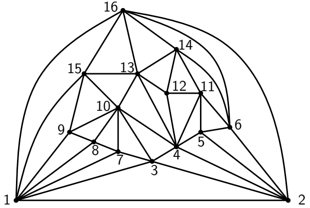

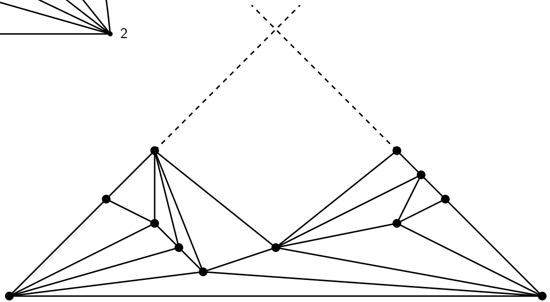

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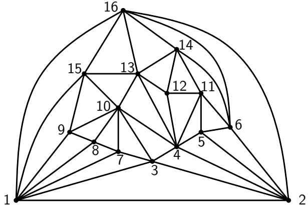

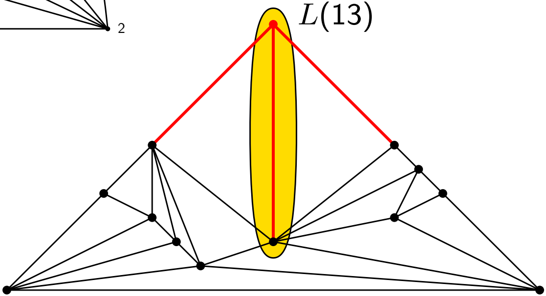

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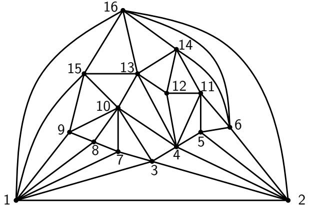

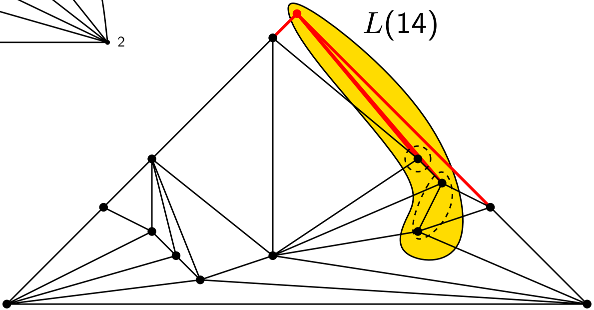

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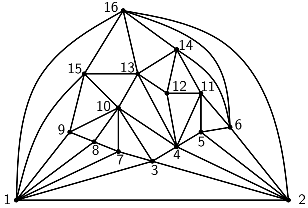

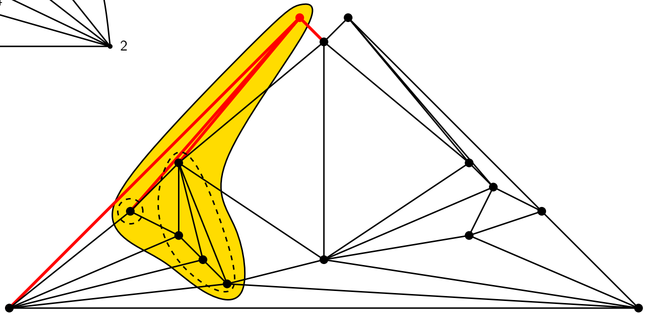

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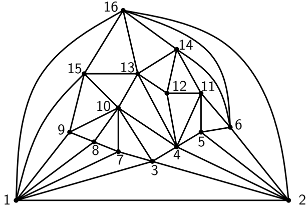
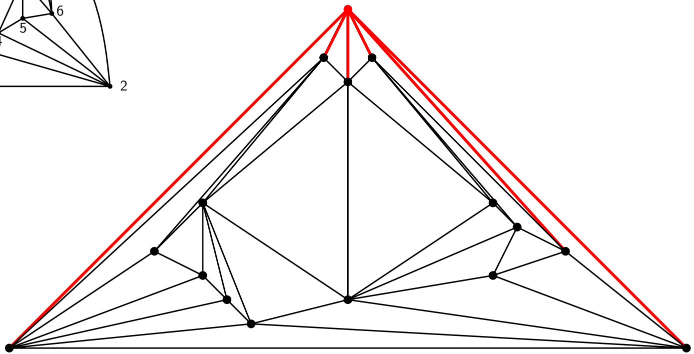
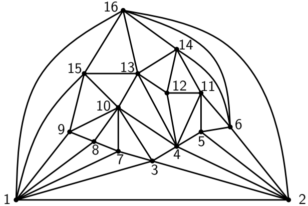
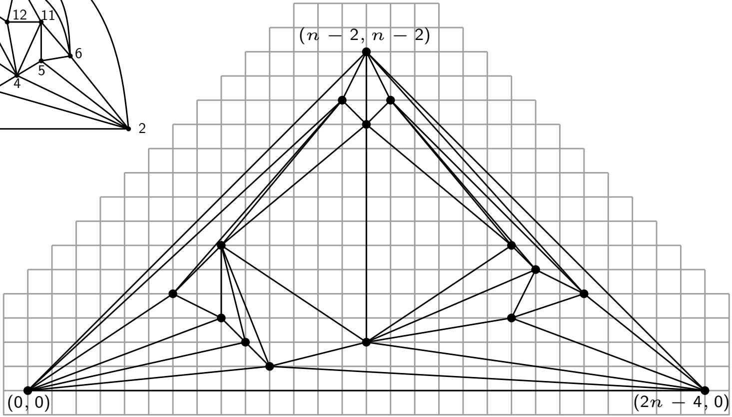




Beispiel





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Viel Erfolg in der Prüfung!