# Algorithmen zur Visualisierung von Graphen

## Kompaktierung von orthogonalen Layouts

Vorlesung im Sommersemester 2009

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# Flussnetzwerk Längenzuweisung

**Definition Flussnetzwerk $N_{\text{hor}} = ((W_{\text{hor}}, A_{\text{hor}}); l; u; b; \text{cost})$**

- $W_{\text{hor}} = \mathcal{F}$
- $A_{\text{hor}} = \{(f, g) \mid f, g$ besitzen gemeinsames horizontales Kantensegment und $f$ liegt unterhalb von $g\}$
- $l(a) = 1 \quad \forall a \in A_{\text{hor}}$
- $u(a) = \infty \quad \forall a \in A_{\text{hor}}$
- $\text{cost}(a) = 1 \quad \forall a \in A_{\text{hor}}$
- $b(f) = 0 \quad \forall f \in W_{\text{hor}}$

# Flussnetzwerk Längenzuweisung

**Definition Flussnetzwerk $N_{\text{vert}} = ((W_{\text{vert}}, A_{\text{vert}}); l; u; b; \text{cost})$**

- $W_{\text{vert}} = \mathcal{F}$
- $A_{\text{vert}} = \{(f, g) \mid f, g$ besitzen gemeinsames vertikales Kantensegment und $f$ liegt links von $g\}$
- $l(a) = 1 \quad \forall a \in A_{\text{vert}}$
- $u(a) = \infty \quad \forall a \in A_{\text{vert}}$
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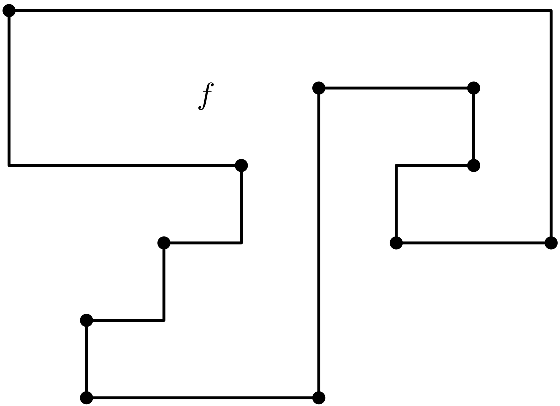
# Verfeinerung von $(G, H)$ – innere Facette



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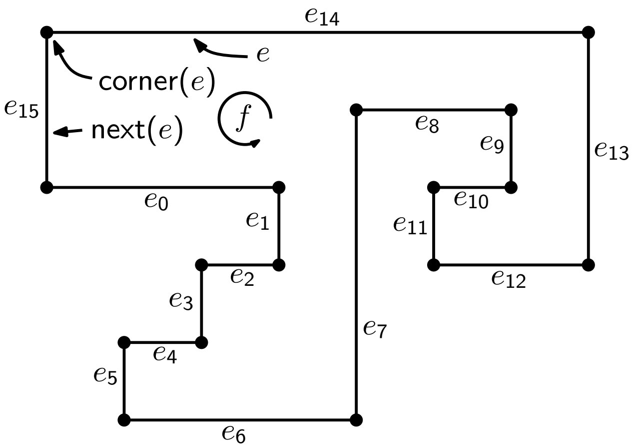

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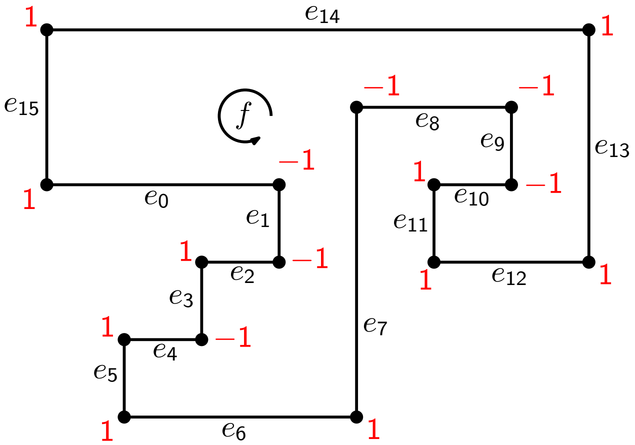

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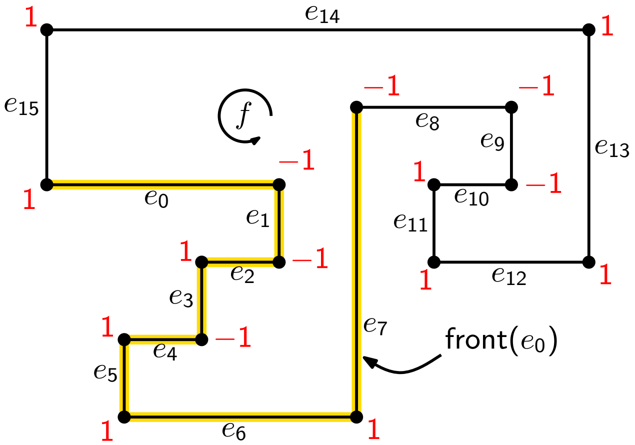

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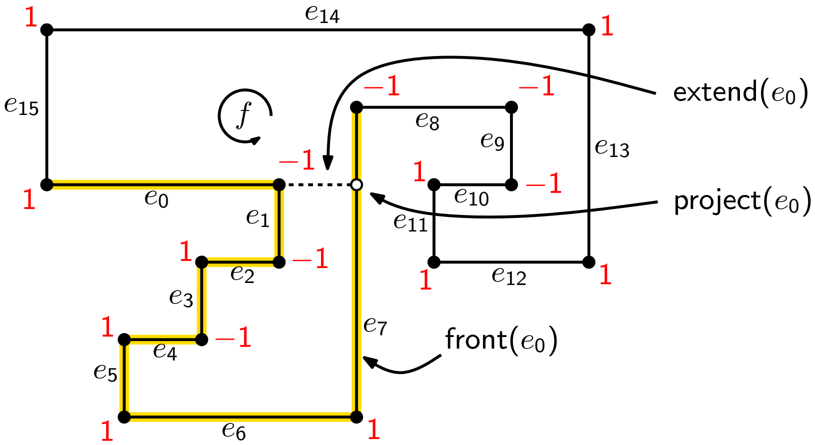

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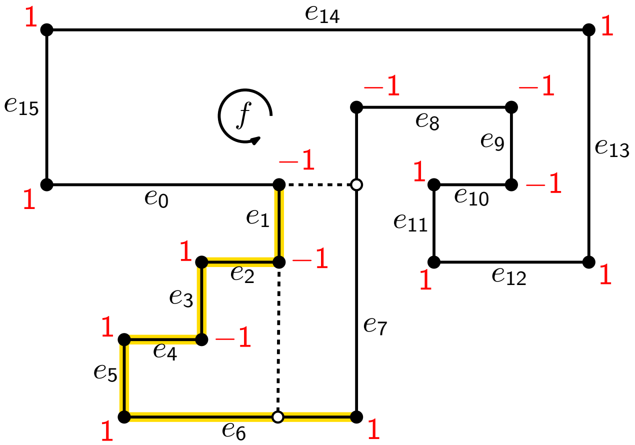

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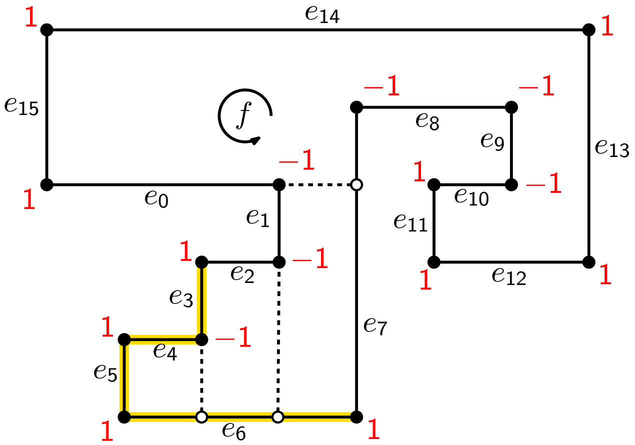

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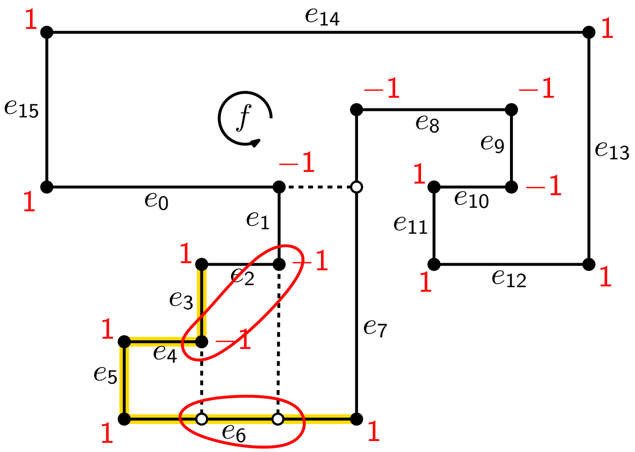

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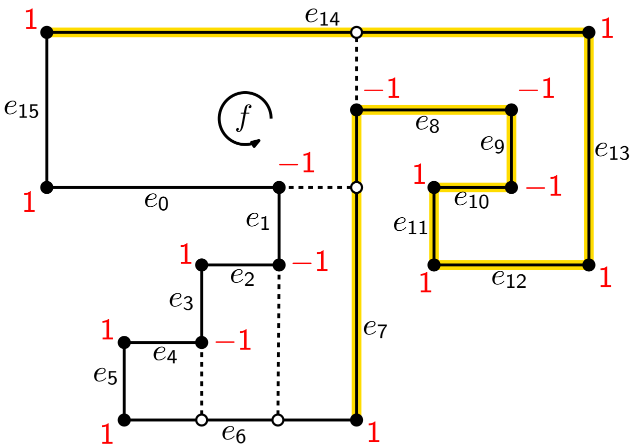

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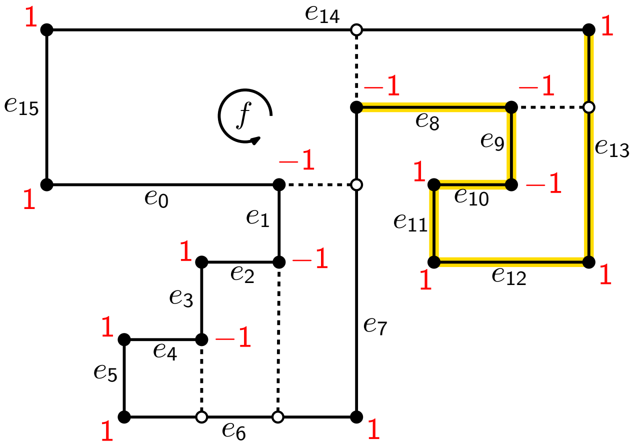

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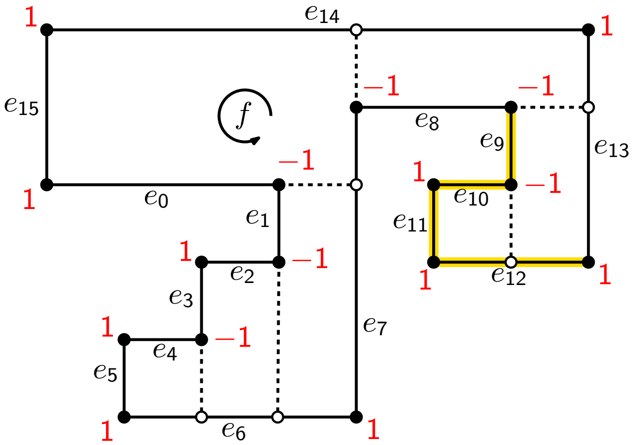

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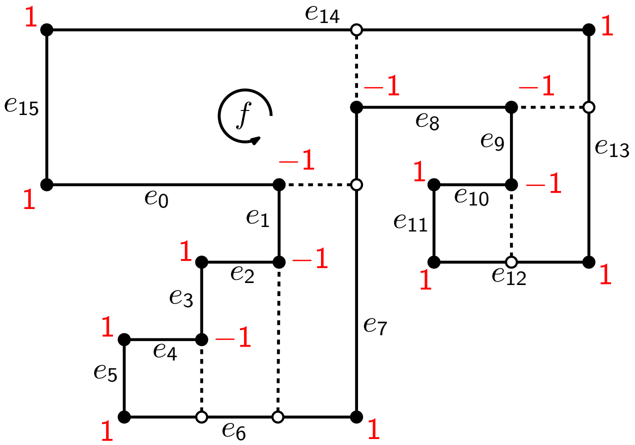

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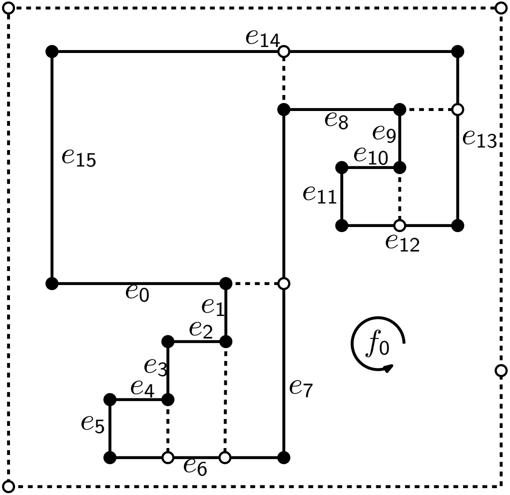

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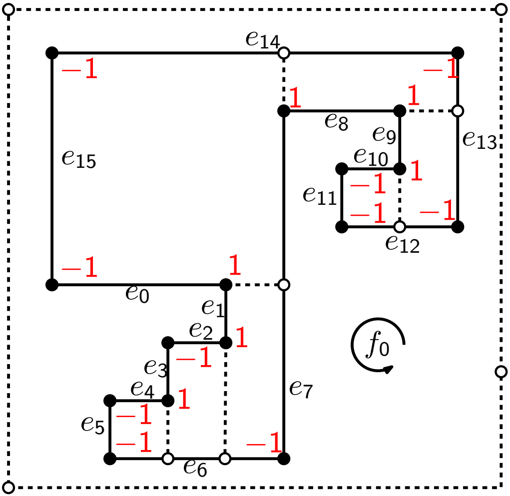

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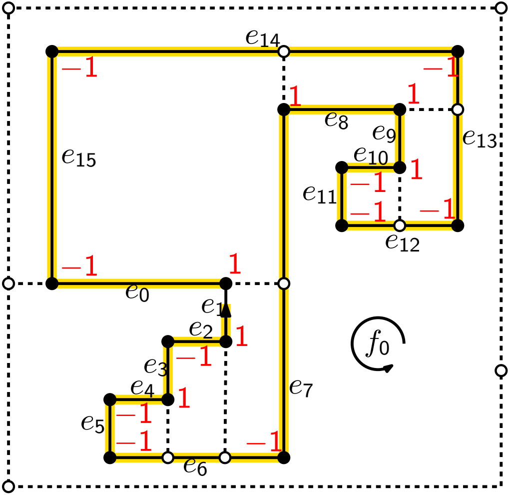

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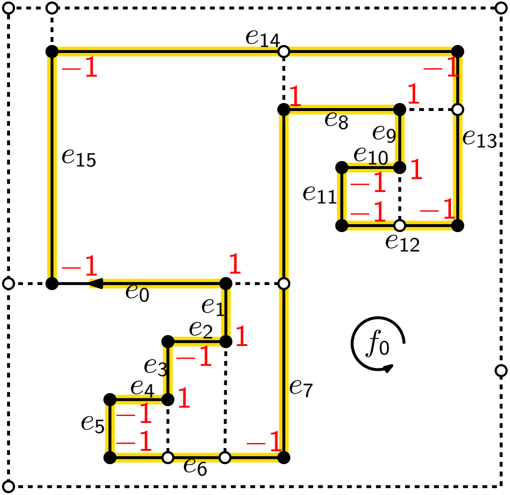

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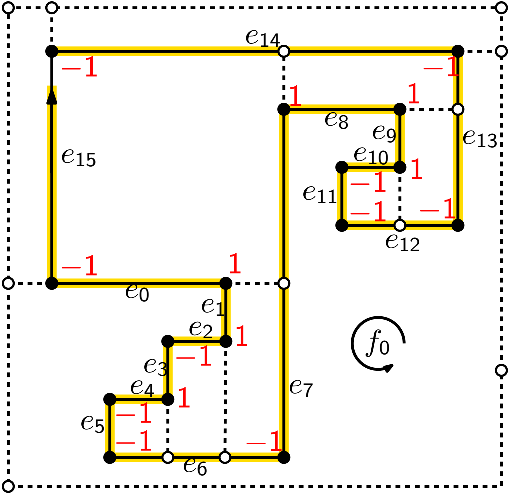

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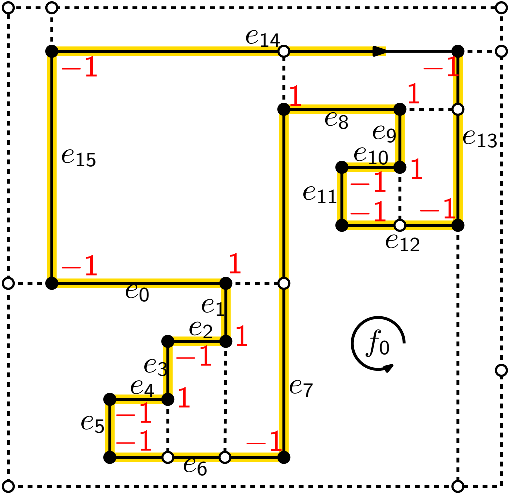

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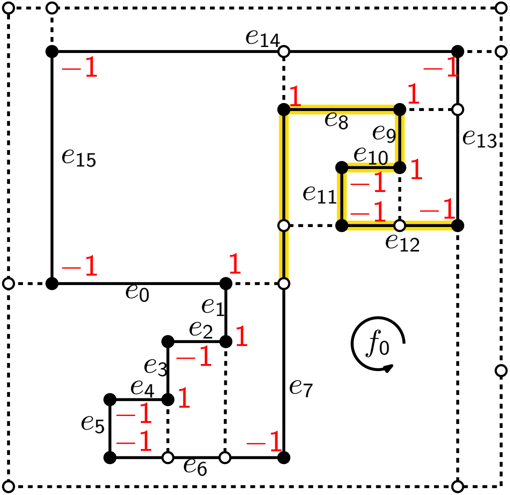

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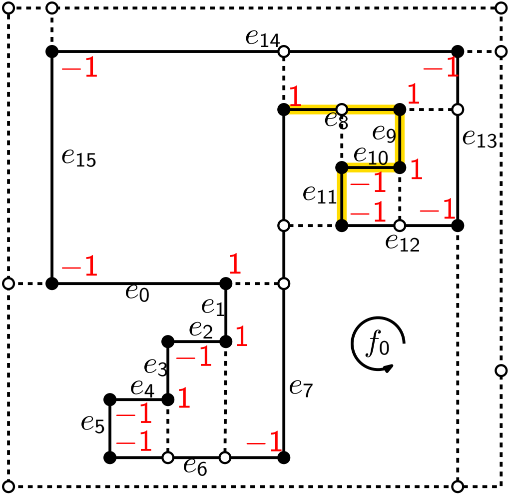

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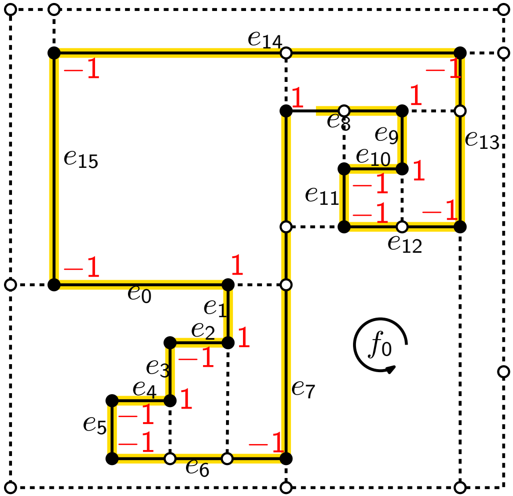

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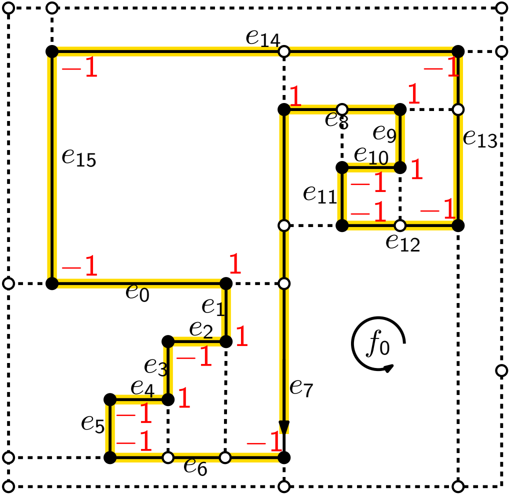

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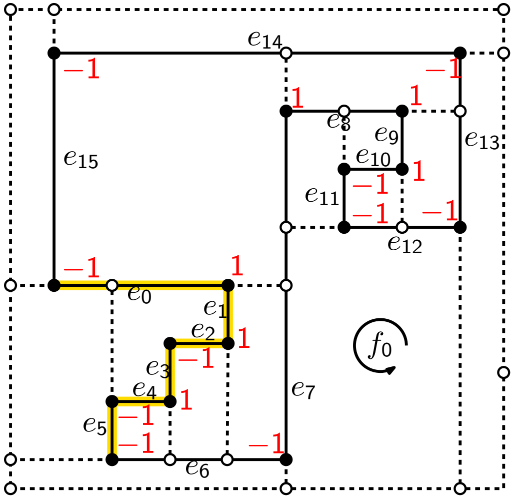

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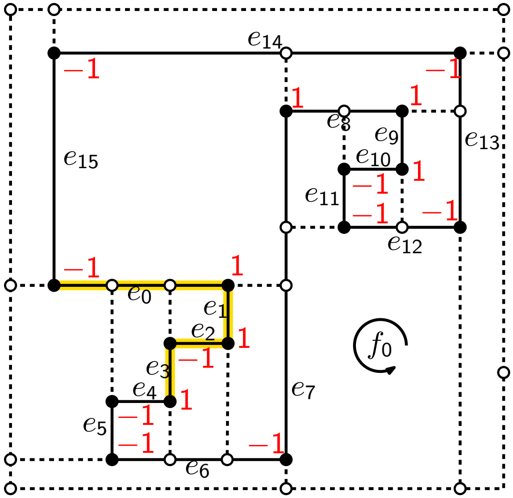

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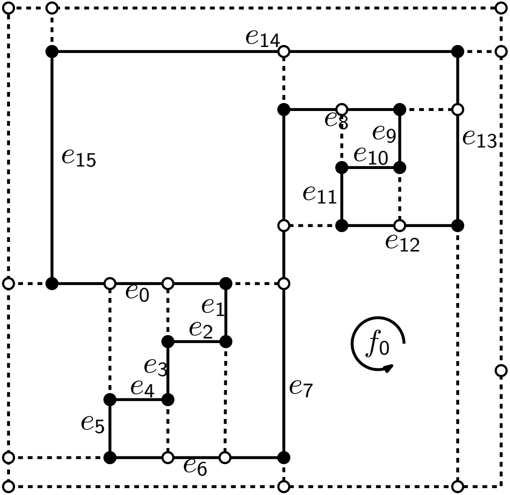

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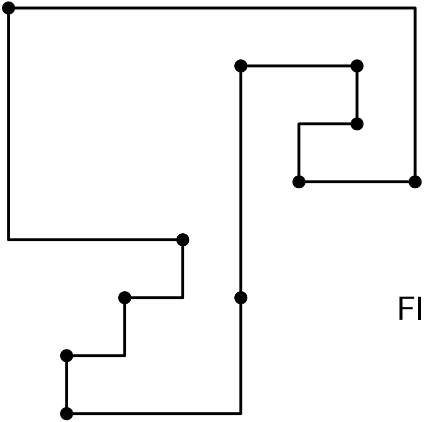

Flächenminimal?

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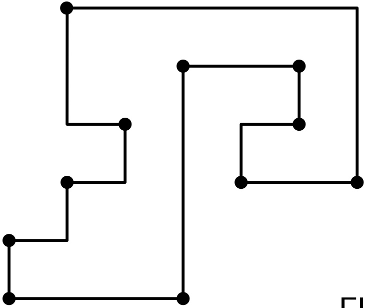


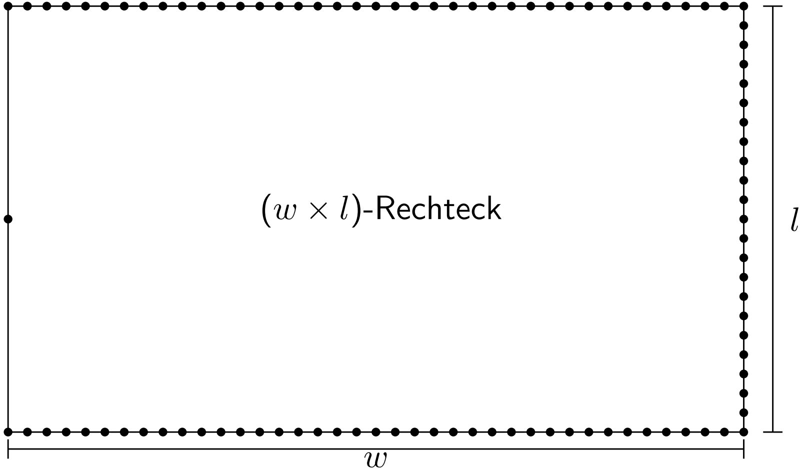
Flächenminimal?

Nein!

# Kompaktierung ist NP-schwer [Patrignani '01]

- Grobstruktur von $(G, H)$
  - Begrenzung
  - Gürtel
  - Klauselgadgets
  - Variablengadgets
- bestimme geeigneten Wert $K$
- $(G, H)$ lässt sich in Fläche $K$ zeichnen gdw. $\Phi$ erfüllbar

# Begrenzung, Gürtel, Kolbengadget



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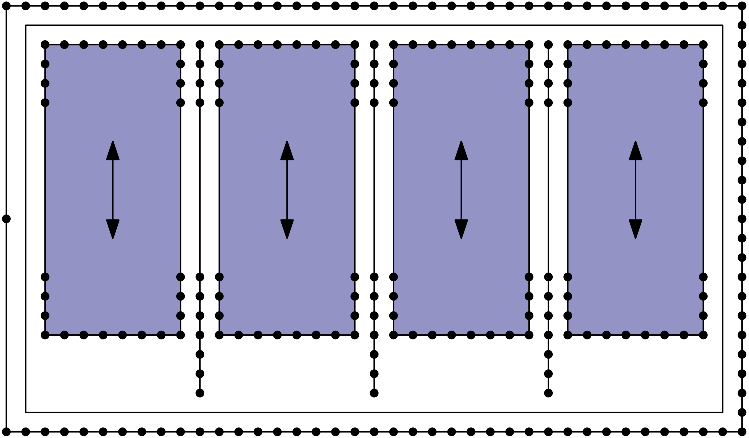

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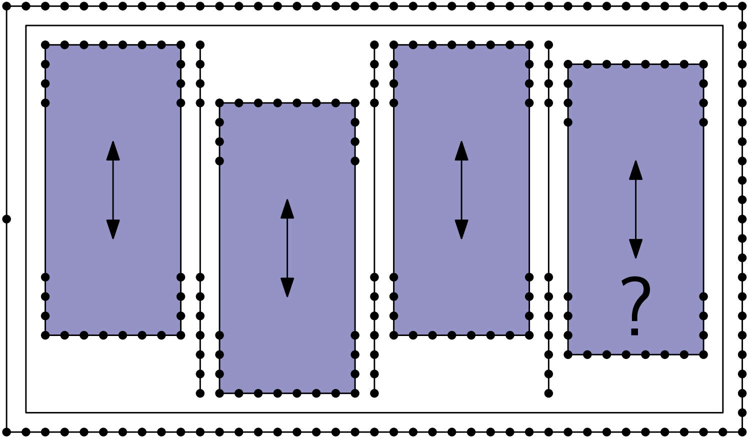

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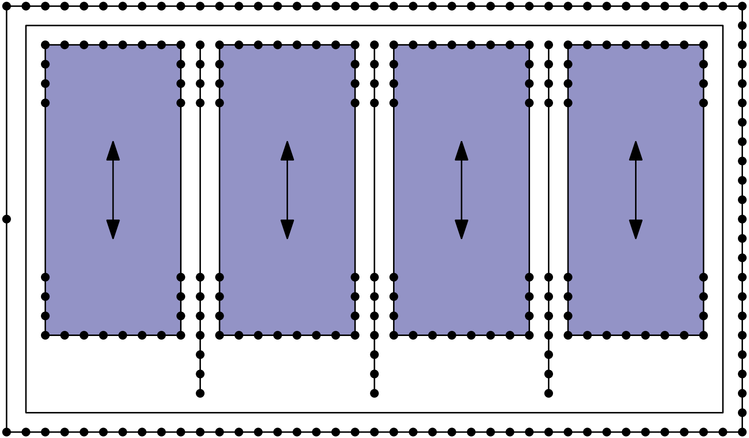

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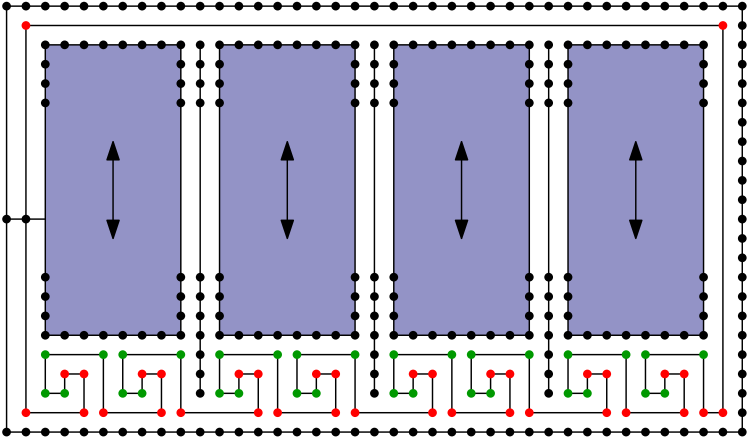

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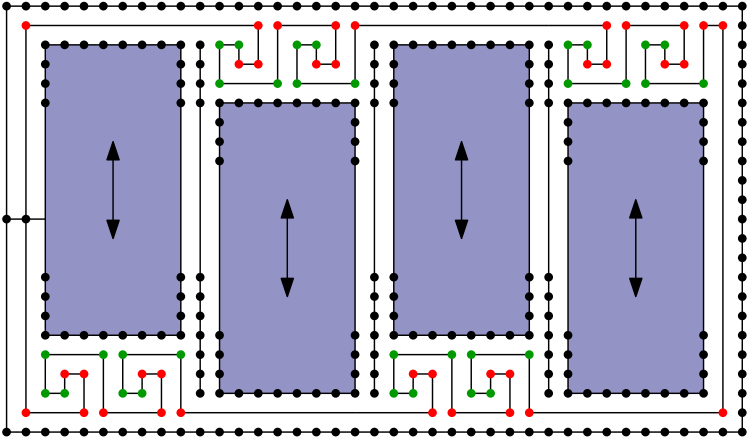

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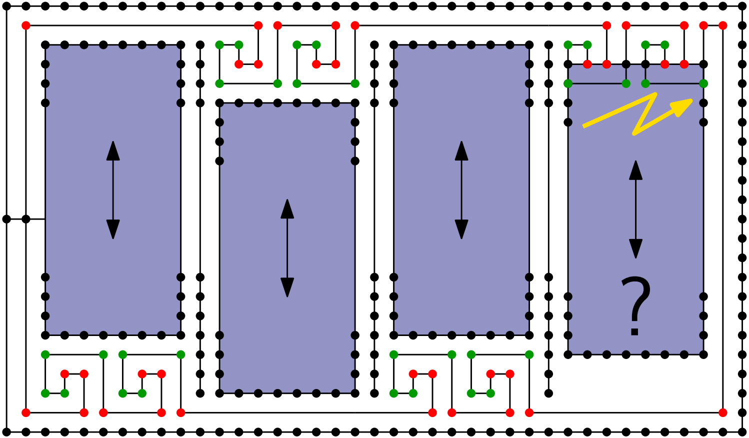

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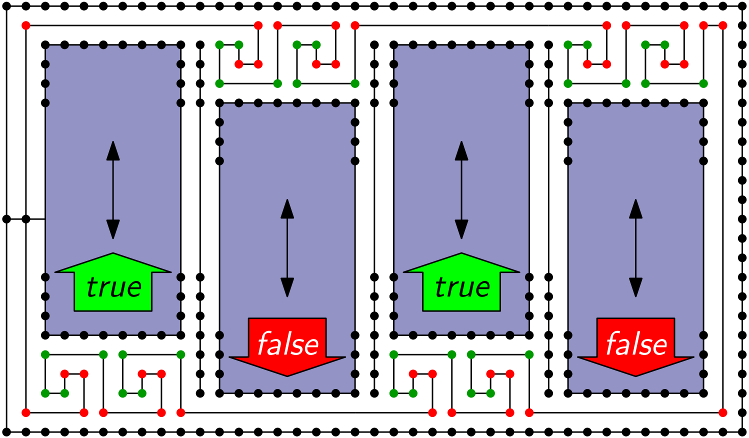
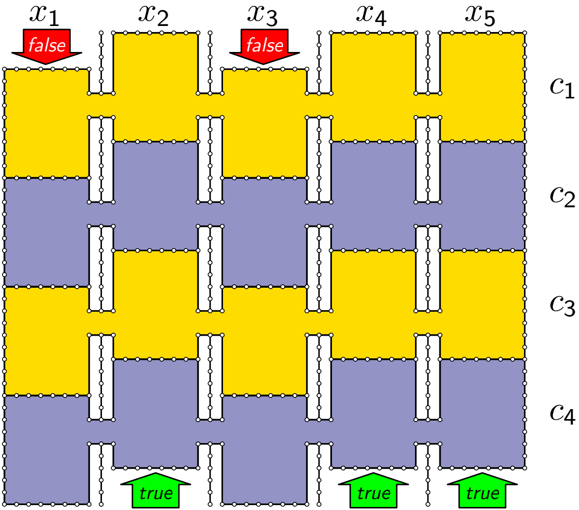


# Klauselgadgets



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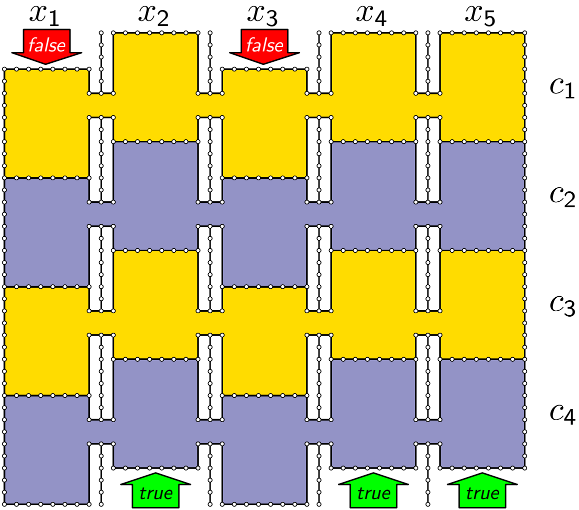


Beispiel:

$c_1 = x_2 \vee \overline{x_4}$

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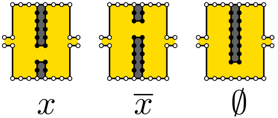
$c_3 = x_5$

$c_4 = x_4 \vee \overline{x_5}$



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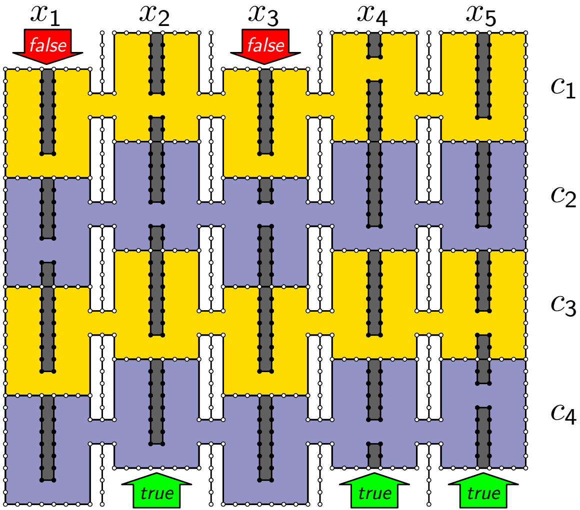


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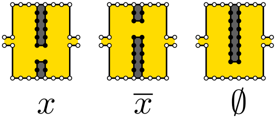

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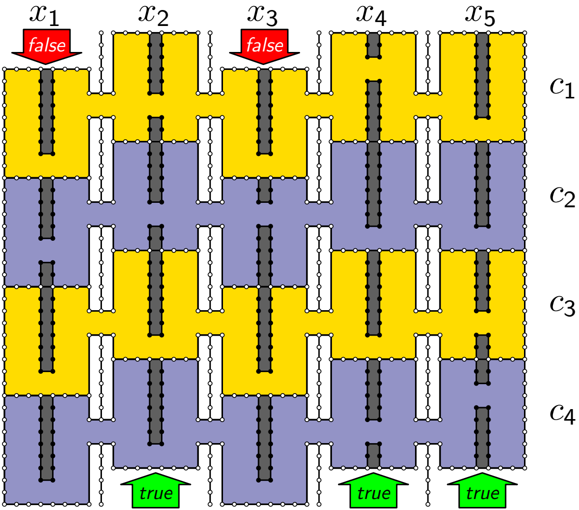


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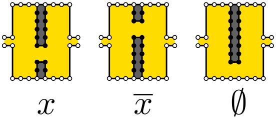
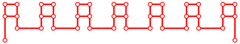
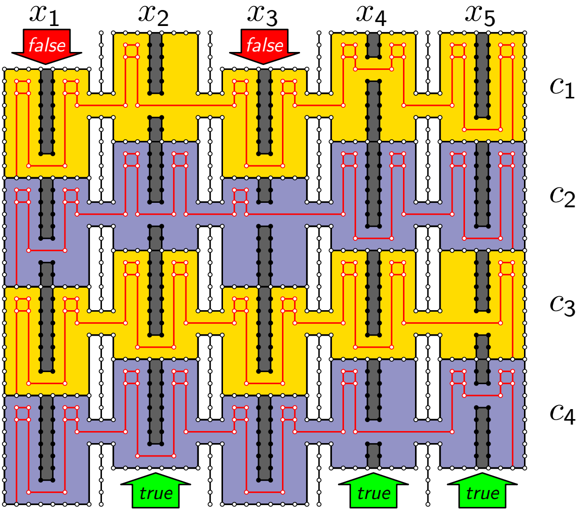


lege $(2n-1)$-A-Kette durch jede Klausel

# Klauselgadgets

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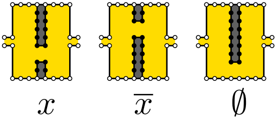


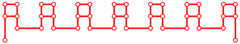

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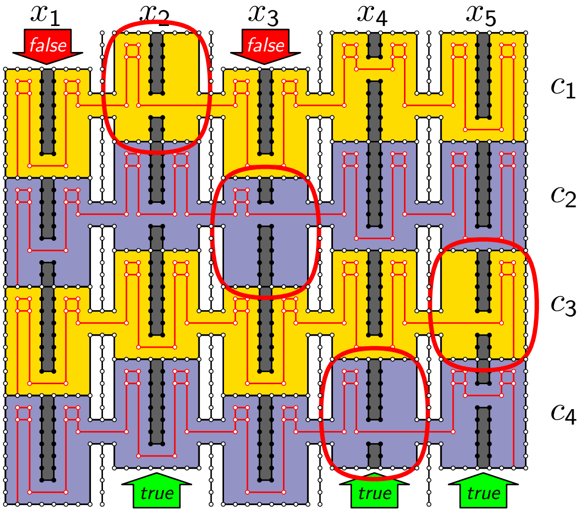


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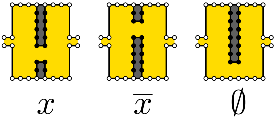


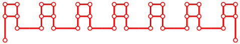

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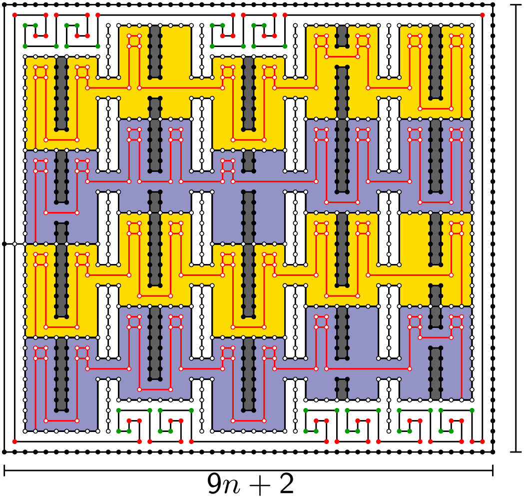
# Komplette Reduktion




Setze

$K = (9n + 2) \cdot (9m + 7)$

Es gilt:

$(G, H)$ auf Fläche $K$ zeichenbar

$\Leftrightarrow$

$\Phi$ erfüllbar