Theta-Graph

Strukturen

- Motivation
- Definition von Theta-Graph
- Beispiel nach der Definition
- Eingenschafen von Theta-Graph
- Implemetierungsalgorithmus von Theta-Graph
- Beispiel nach des Algorithmus
- Definition des Spanners
- Theta-Graph ist Spanner
- Beweis
- Folgerung

Motivation

- Realisierung Spanner (approximativ vollständiger Graph)

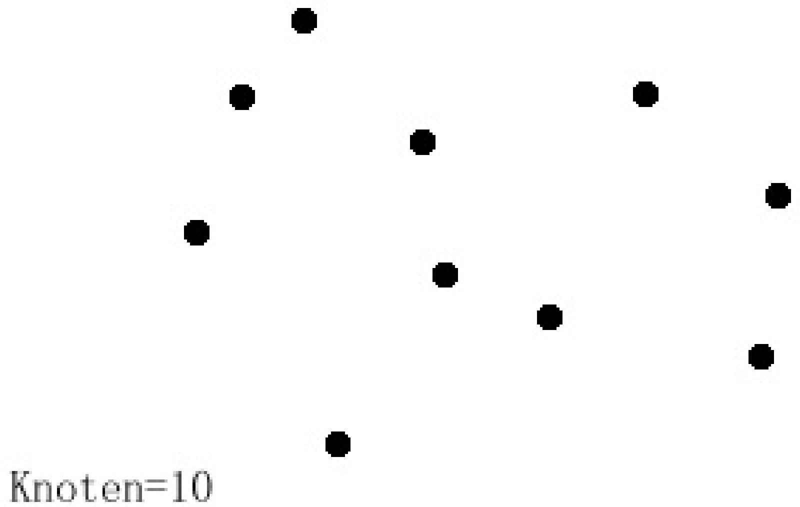
- Entwicklung der Approximationsalgorithmen oder in der Heuristik

Definition von Theta Graph

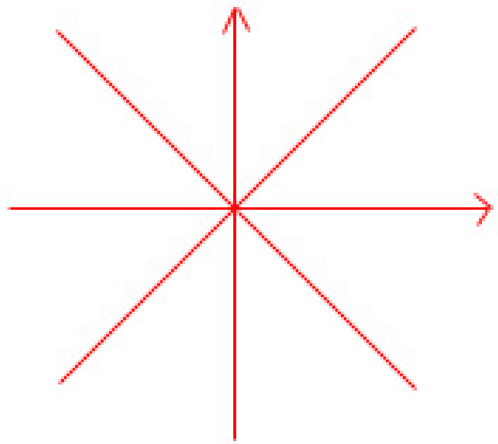
- Theta-Graph

In Fläche um jeden Knote in Sektoren von Koordinatesystem mit dem örtlichfestgelegten Winkel und schließen den Knote an den nächsten Nachbar in jedem Sektor

Beispiel nach der Definition

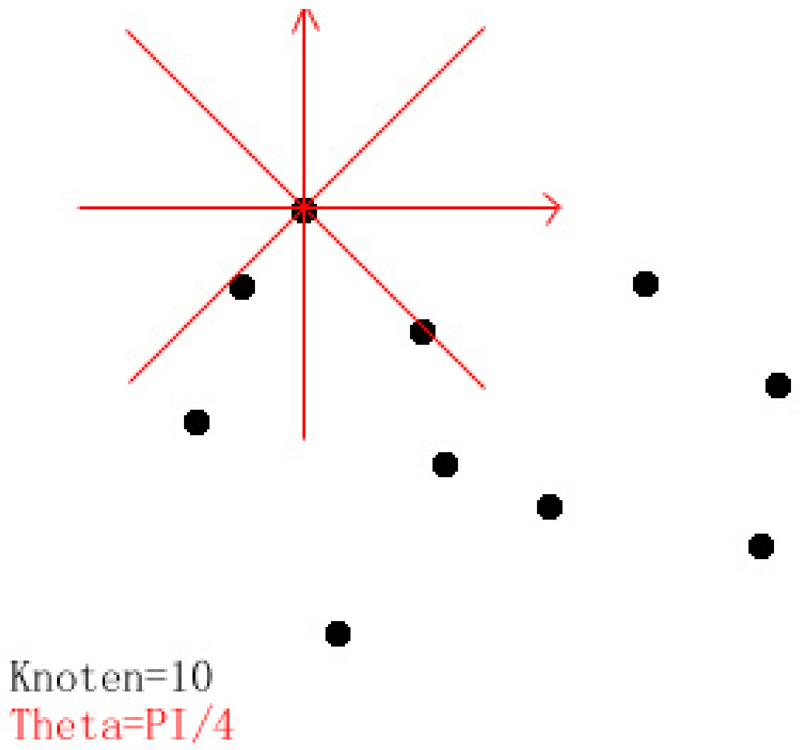


Beispiel nach der Definition

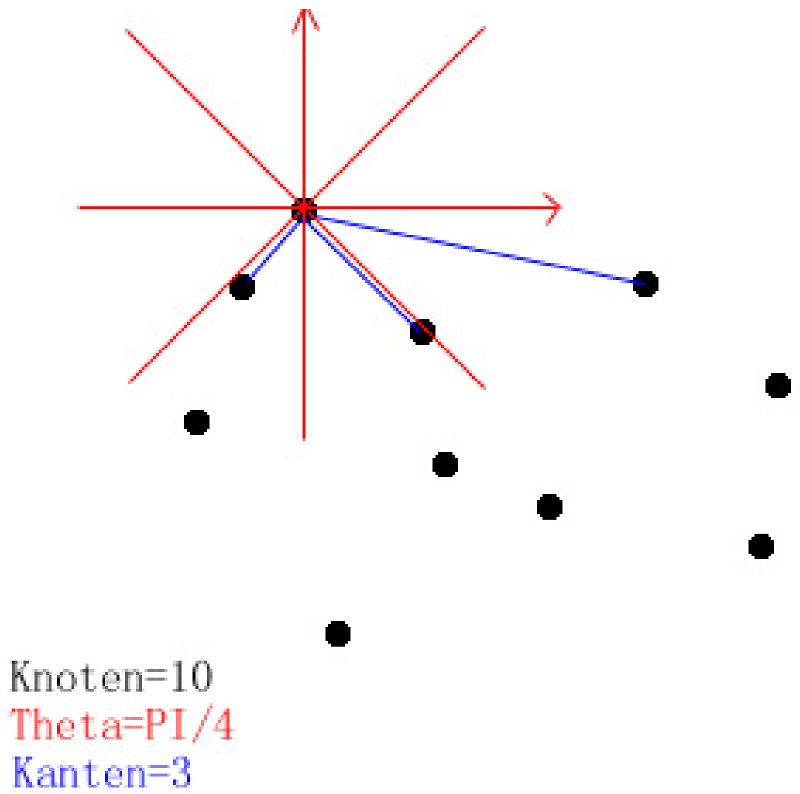


Theta=PI/4

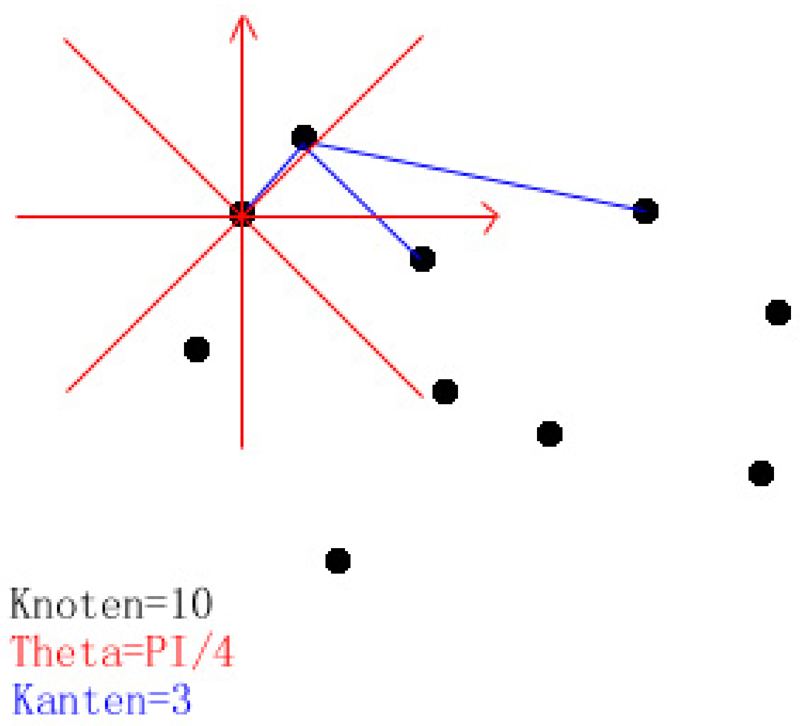
Beispiel nach der Definition



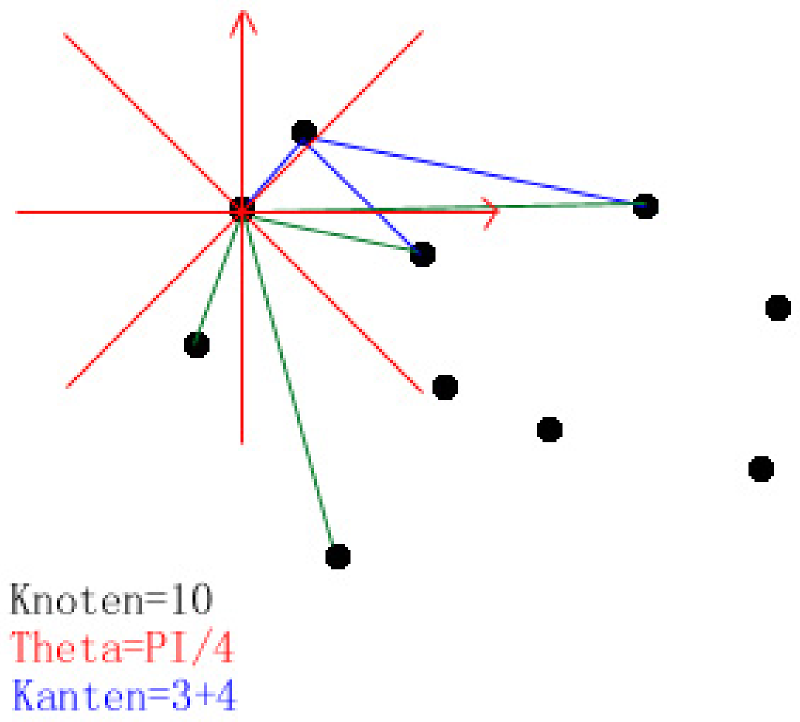
Beispiel nach der Definition



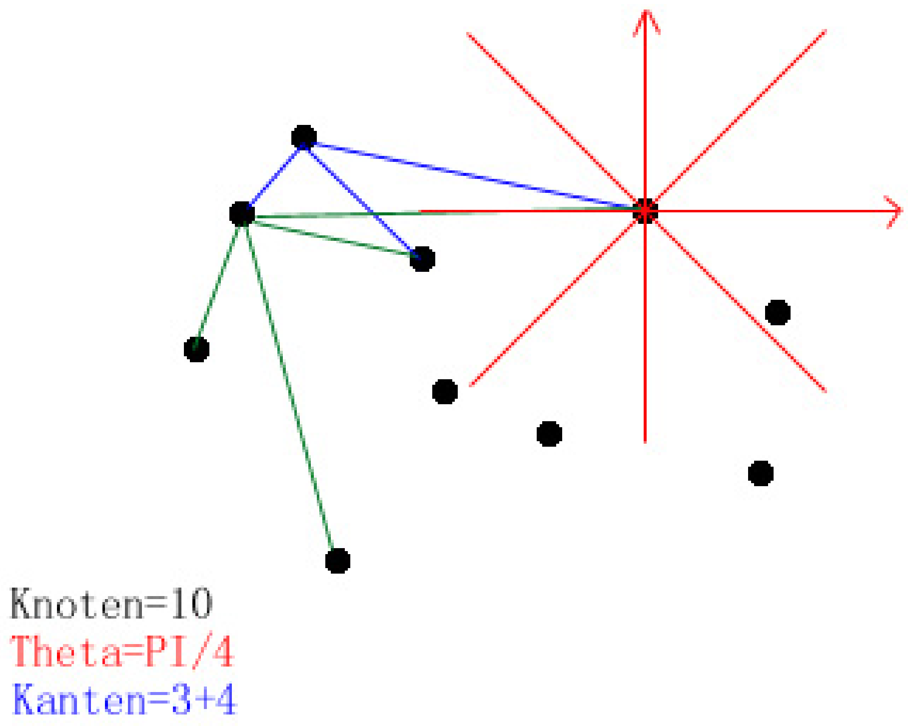
Beispiel nach der Definition



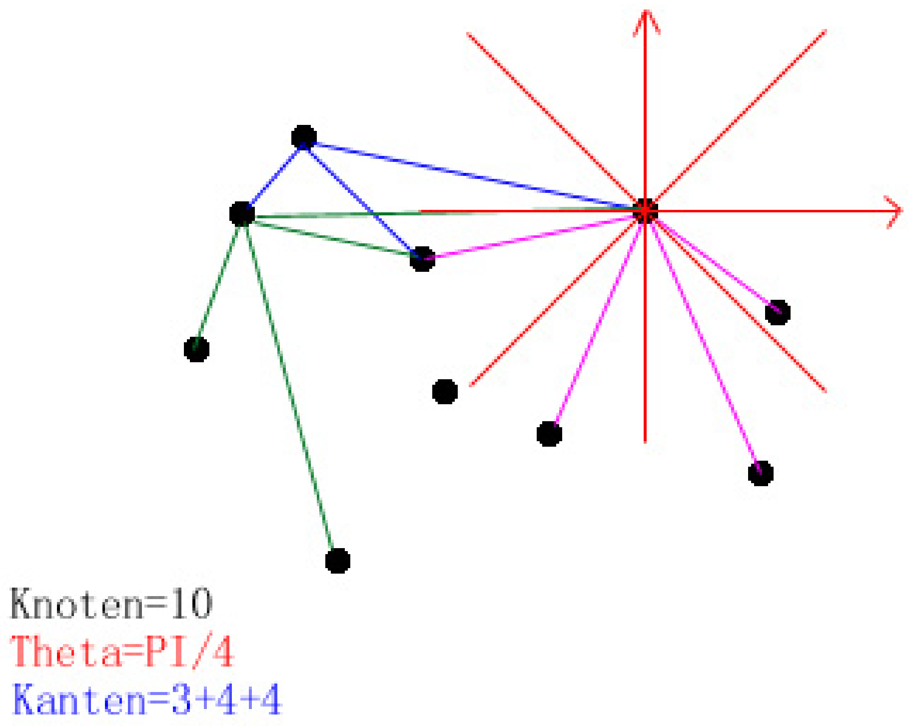
Beispiel nach der Definition



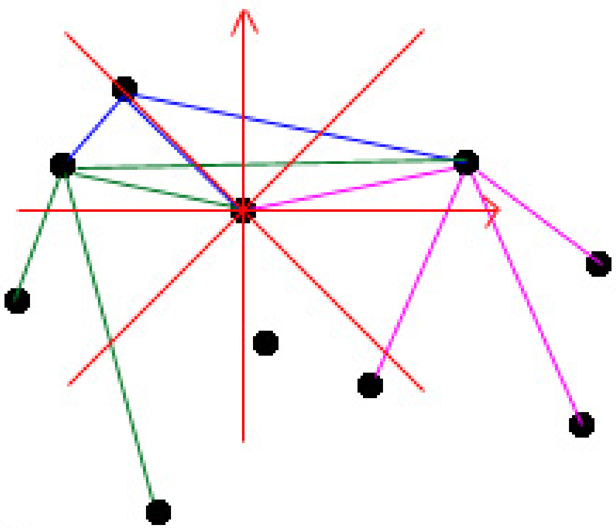
Beispiel nach der Definition



Beispiel nach der Definition

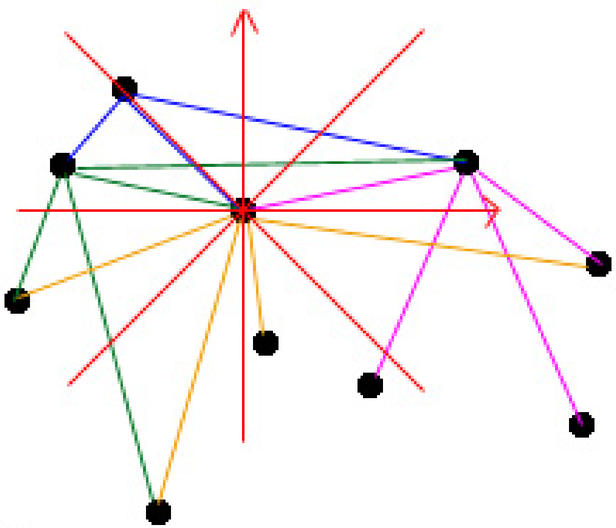


Beispiel nach der Definition



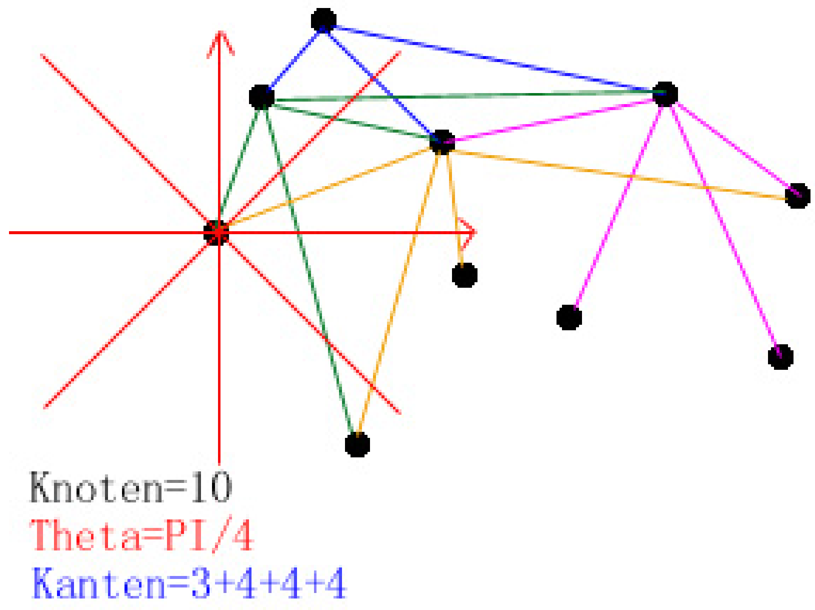
Knoten=10
Theta=PI/4
Kanten=3+4+4

Beispiel nach der Definition

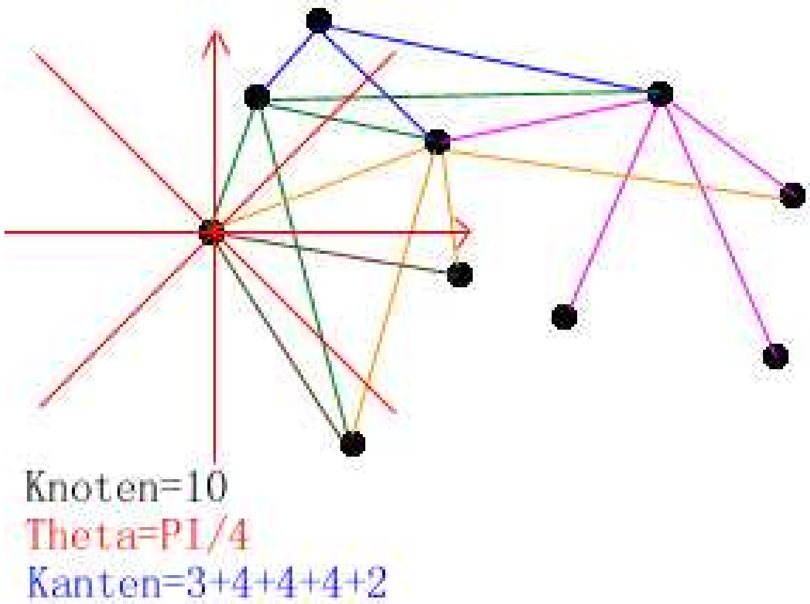


Knoten=10
Theta=PI/4
Kanten=3+4+4+4

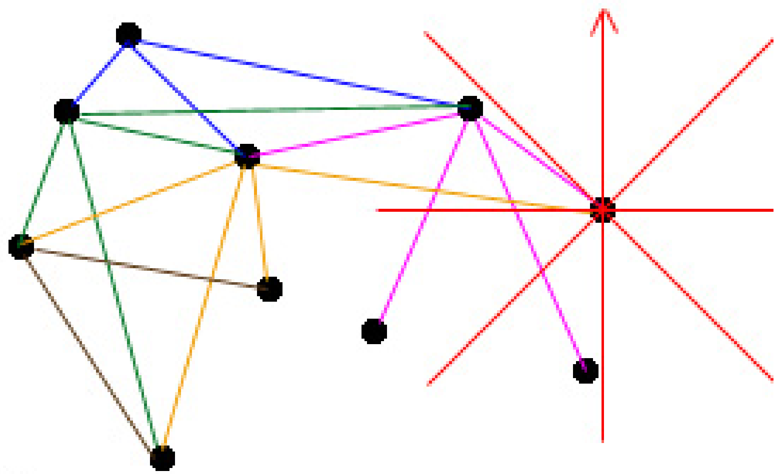
Beispiel nach der Definition



Beispiel nach der Definition

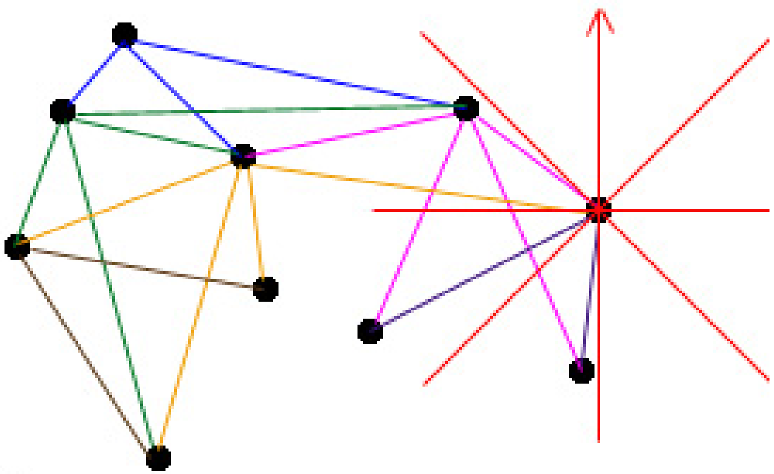


Beispiel nach der Definition



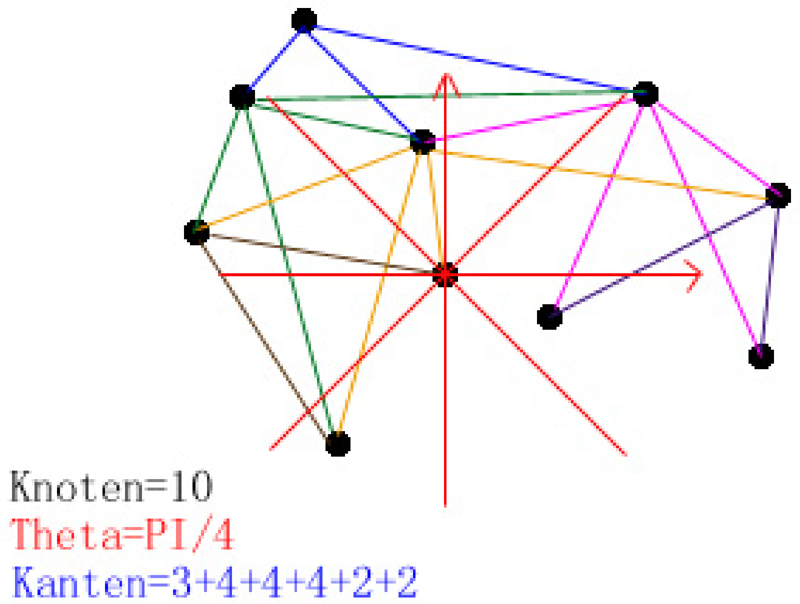
Knoten=10
Theta=PI/4
Kanten=3+4+4+4+2

Beispiel nach der Definition

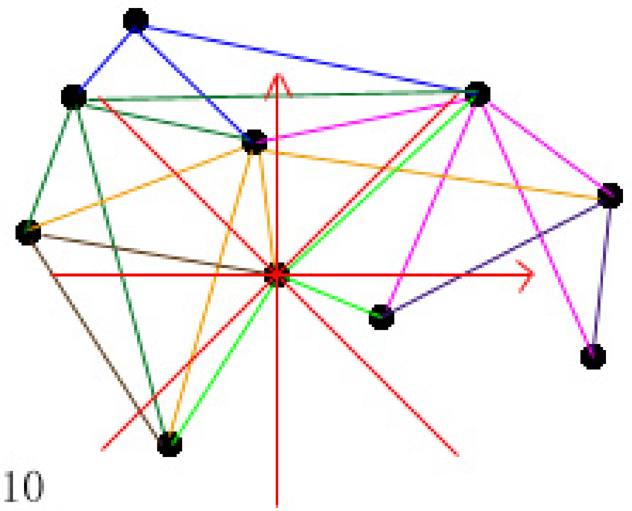


Knoten=10
Theta=PI/4
Kanten=3+4+4+4+2+2

Beispiel nach der Definition

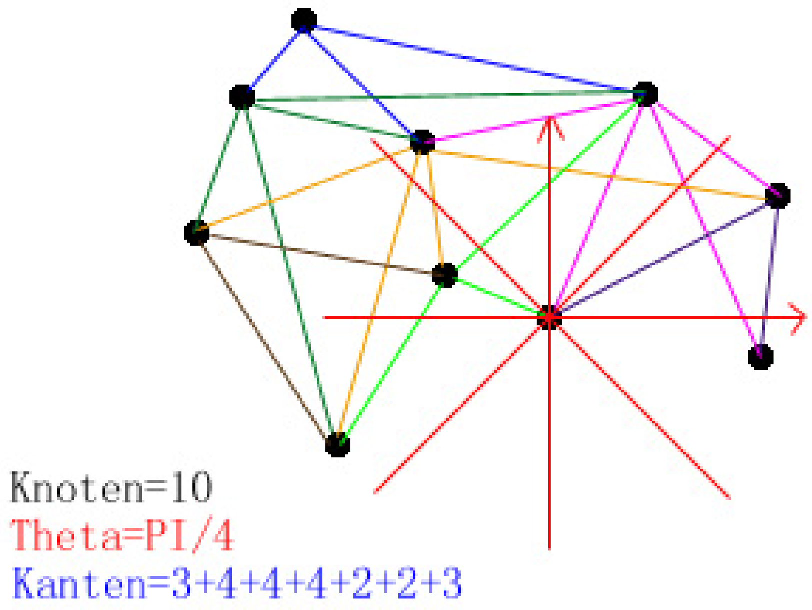


Beispiel nach der Definition

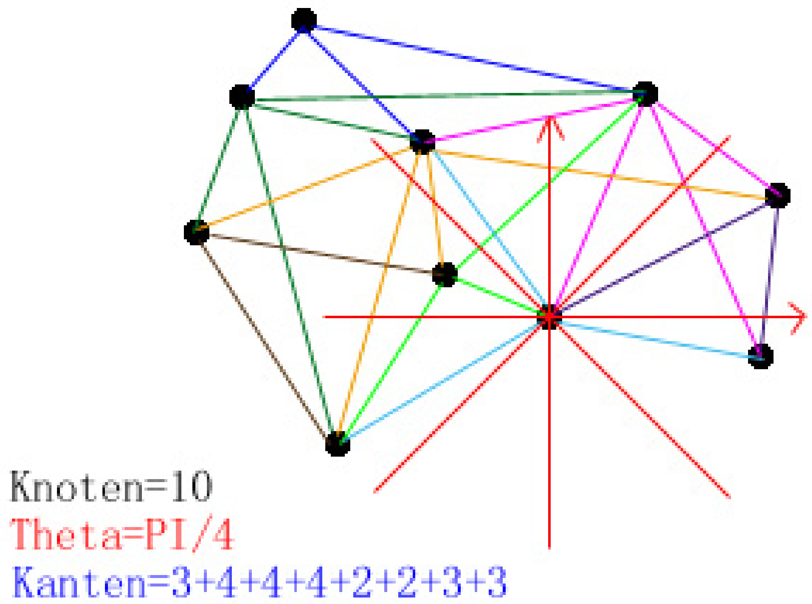


Knoten=10
Theta=PI/4
Kanten=3+4+4+4+2+2+3

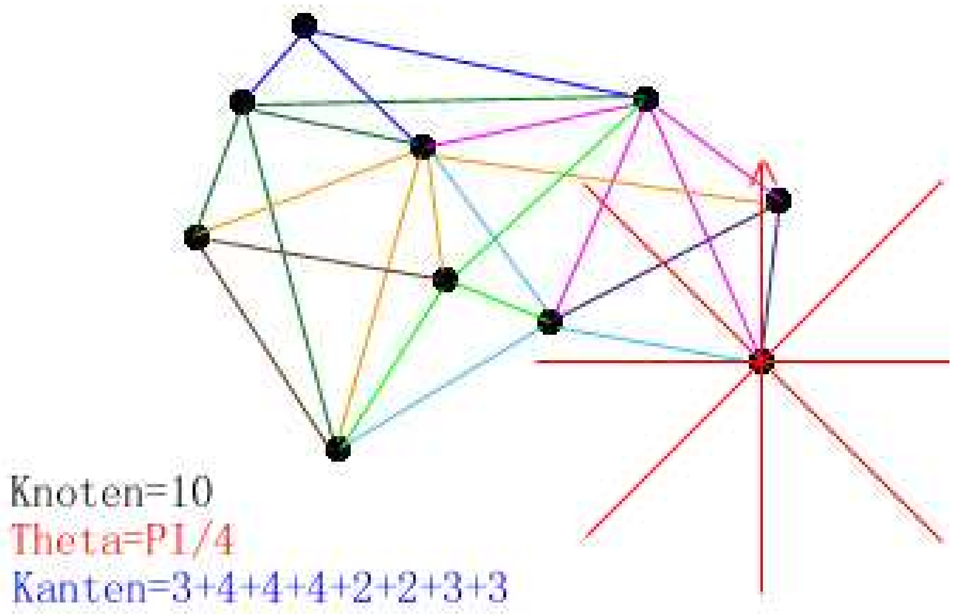
Beispiel nach der Definition



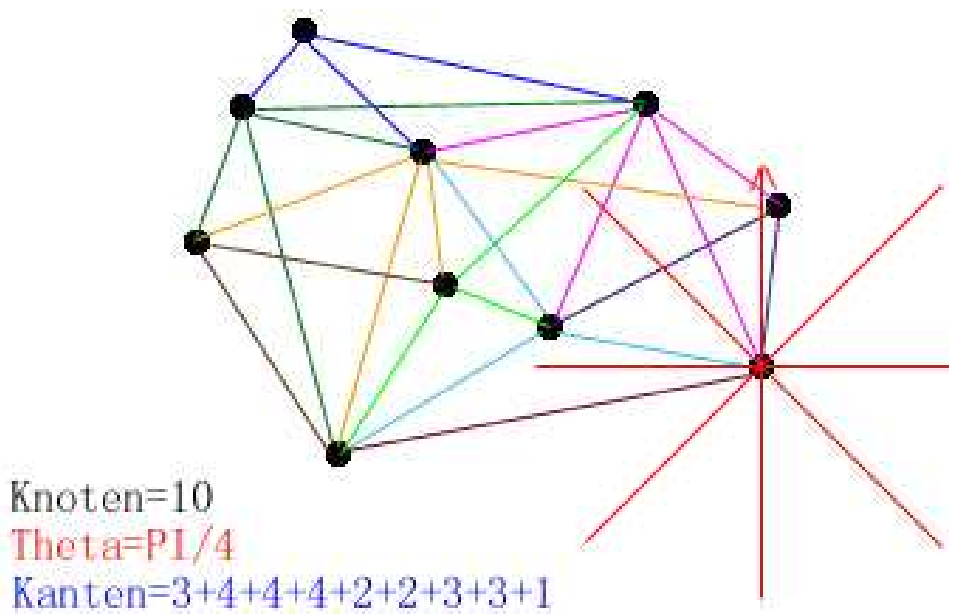
Beispiel nach der Definition



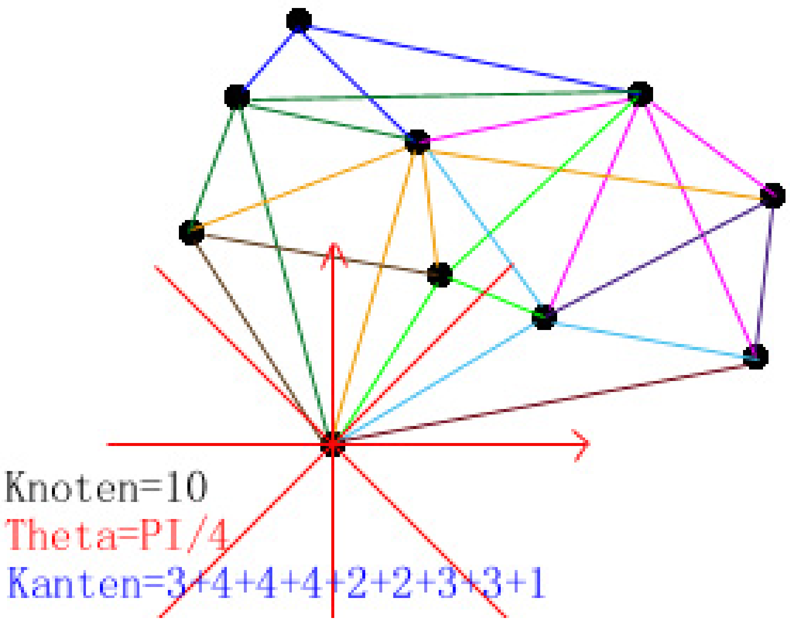
Beispiel nach der Definition



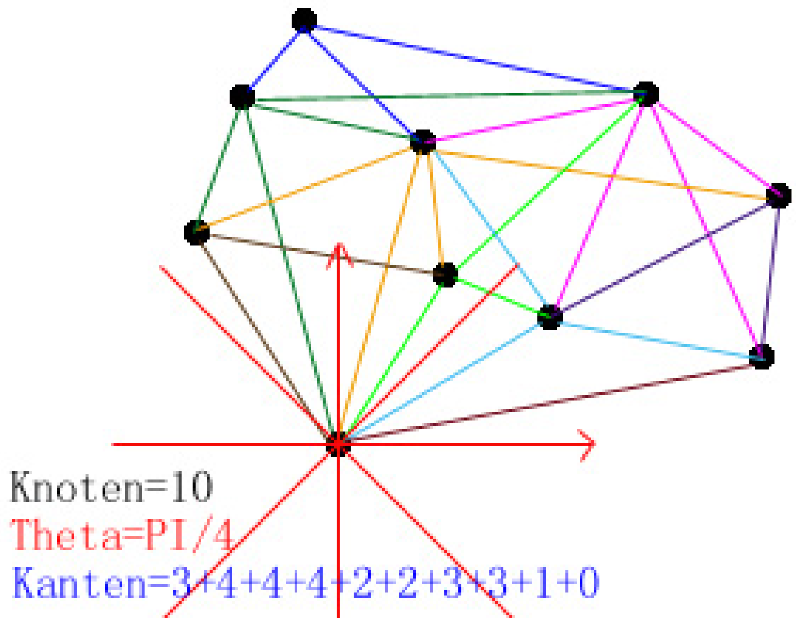
Beispiel nach der Definition



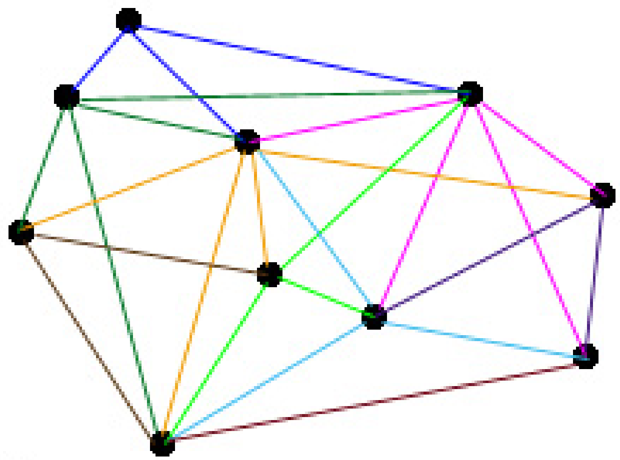
Beispiel nach der Definition



Beispiel nach der Definition



Beispiel nach der Definition



Knoten=10
Theta=PI/4
Kanten=3+4+4+4+2+2+3+3+1+0=26

Eingenschafen von Theta-Graph

- $\theta = \frac{2\pi}{k}$,wobei k ist eine Ganzzahl(k > 8)
- Jeder Knoten hat höchsten k Kanten
- Im allgemeinen hat ein Knoten Typ i Kanten

wobei 1<=i<=k und $\frac{2\pi(i-1)}{k} \leq \phi \leq \frac{2\pi i}{k}$

der winkel ϕ von eine kante und x-coordinate ist.

Implemetierungsalgorithmus von Theta-Graph

- Methode

  Man findet alle Kanten nach Typ i (zuerste ist Typ1 , dann ist Typ2 , usw.) für jede Knote.

- Defintion

  Drei Ordnungen α, β, γ : In jeder Ordnung werden die Knoten durch die Einrichtung ihrer Projektionen auf die orientierte Linie georgernet,die durch den Nullpunkt einen Winkel von ϕ mit der X-Achse bildet.

Implemetierungsalgorithmus von Theta-Graph

Ordnung α : $\phi = \dfrac{2\pi(i-1)}{k}$

Ordnung β : $\phi = \dfrac{2\pi(i-1)}{k} + \dfrac{\pi}{2}$

Ordnung γ : $\phi = \dfrac{2\pi i}{k} + \dfrac{\pi}{2}$

Implemetierungsalgorithmus von Theta-Graph

- ImplemetierungsAlgrorithmus

1) Insert point p into table T.
2) If p has a predecessor q in T then report that pq is a type i edge.
3) Repeat Forver

If p has a successor r in T then

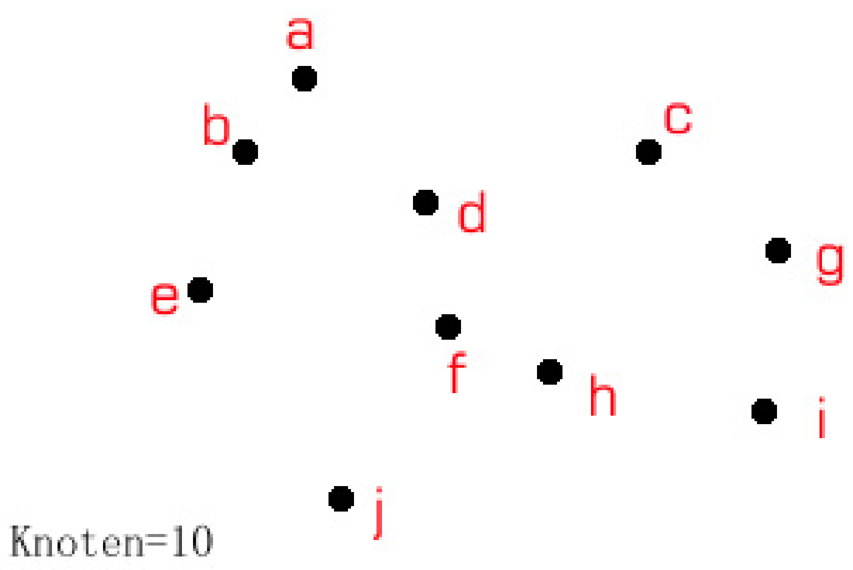
If α(r) >α (p) then delete r from T

else exit loop

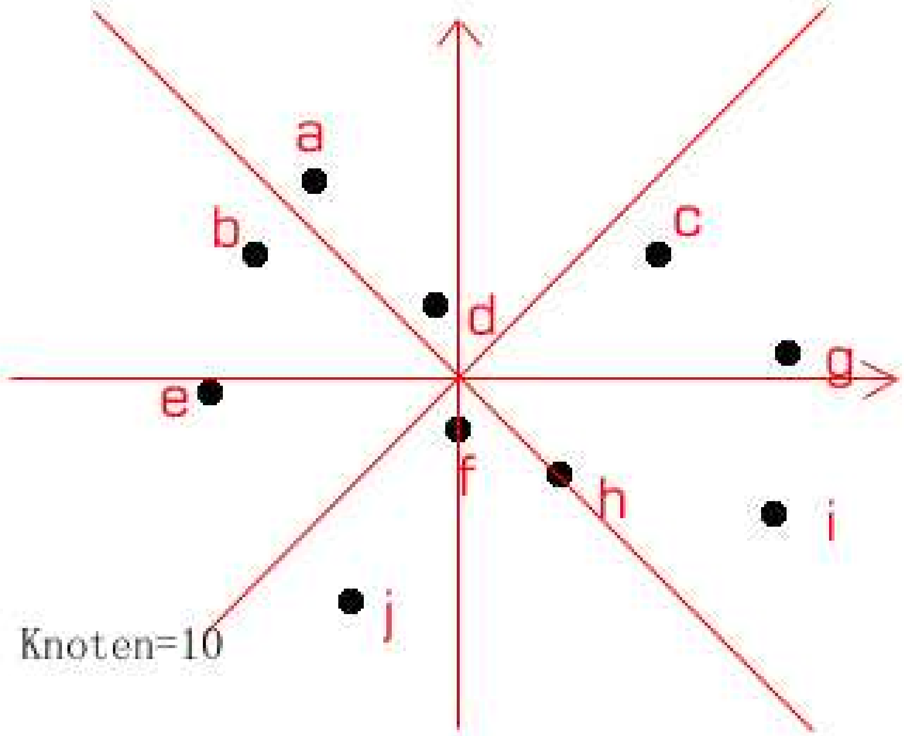
Bemerkung:

1) Abtastung in unaufsteigender Ordnung β
2) Am Anfang ist Tablle T leer
3) Nach Ordnung γ stecketet die neue Knote ein

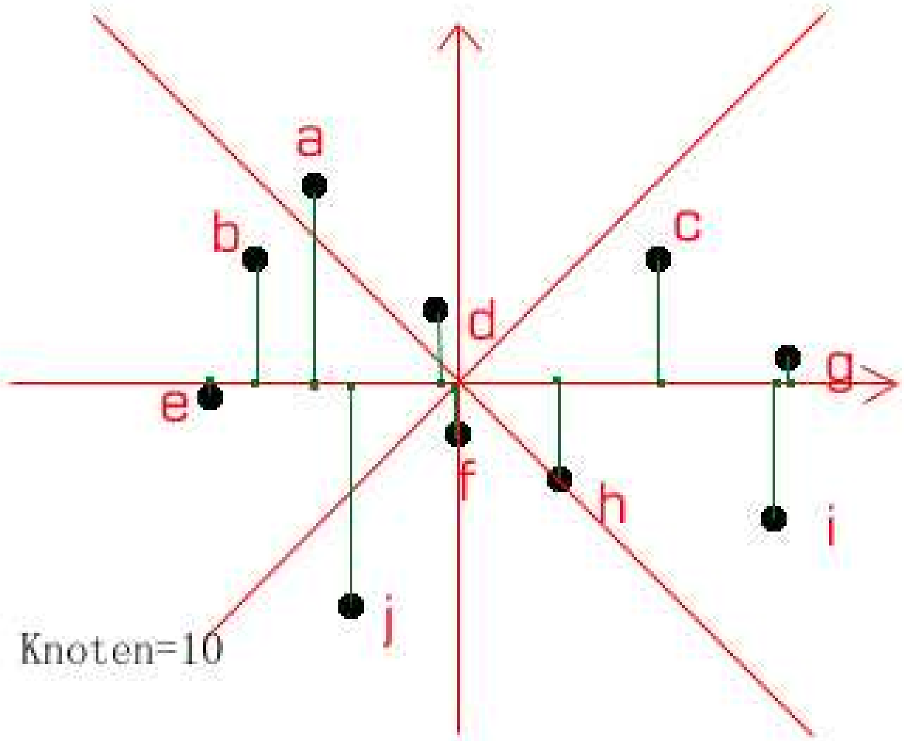
Beispiel Nach Implemetierungsalgorithmus



Beispiel Nach Implemetierungsalgorithmus

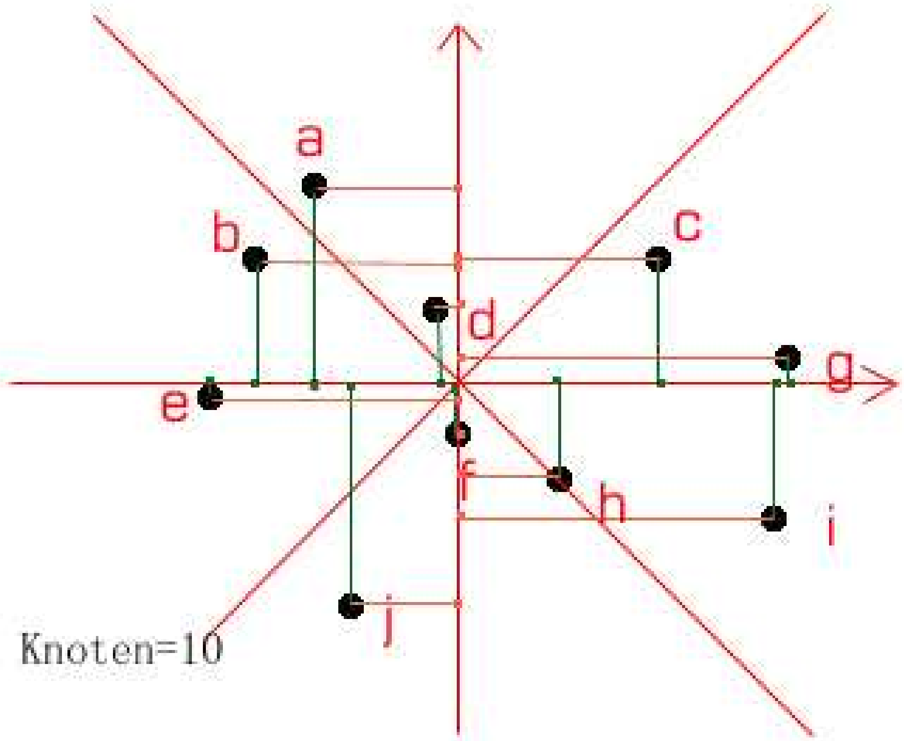


Beispiel Nach Implemetierungsalgorithmus



α = {e,b,a,j,d,f,h,c,i,g}

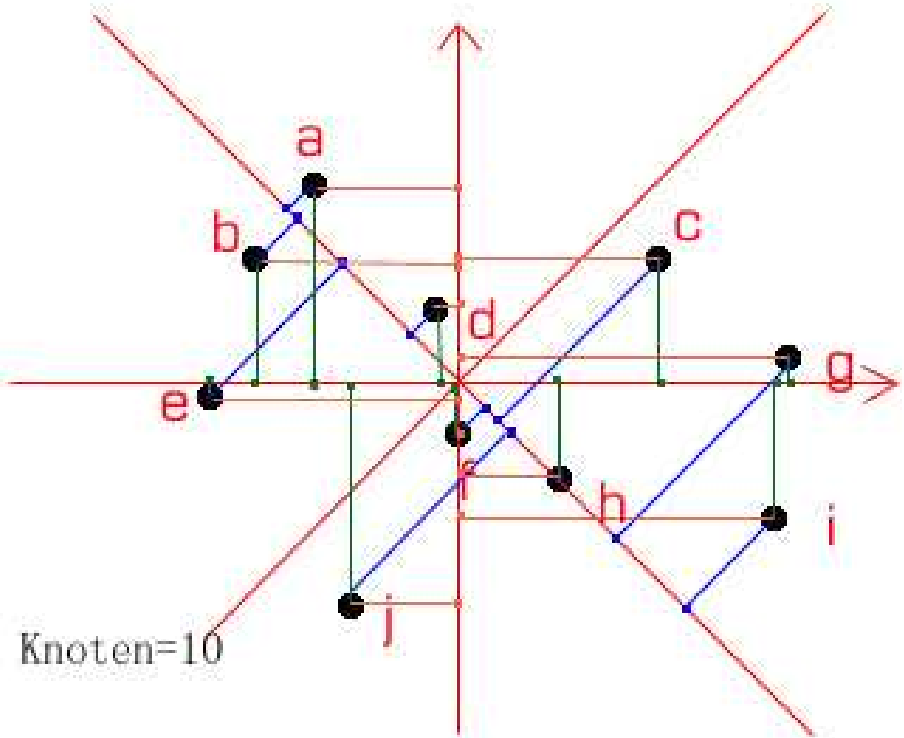
Beispiel Nach Implemetierungsalgorithmus



α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

Beispiel Nach Implemetierungsalgorithmus

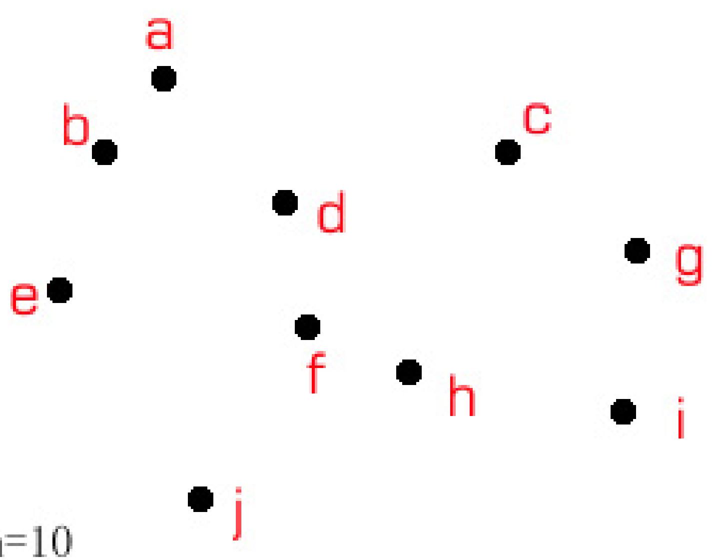


α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



Knoten=10

1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α (r) > α(p) then delete r from T
   else exit loop

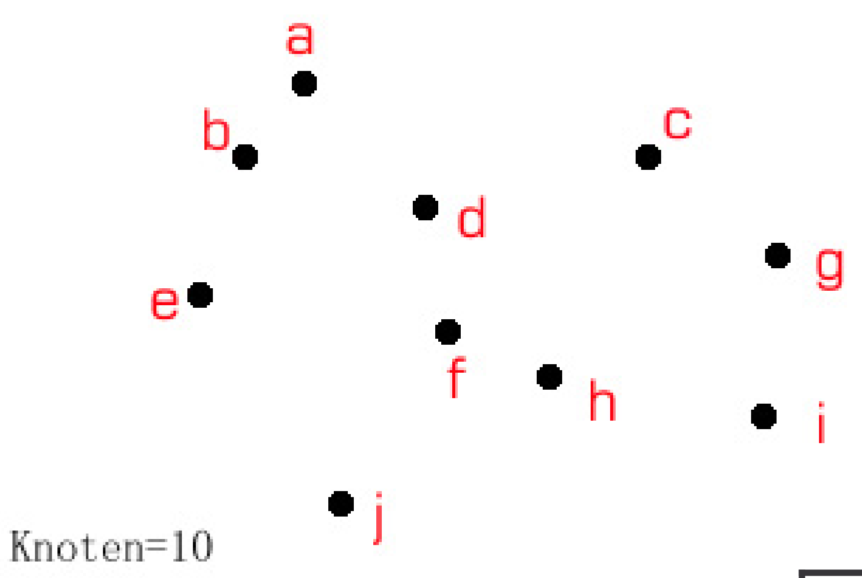
Knoten	Kanten(Typ 1)
a	

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α (r) > α(p) then delete r from T
else exit loop

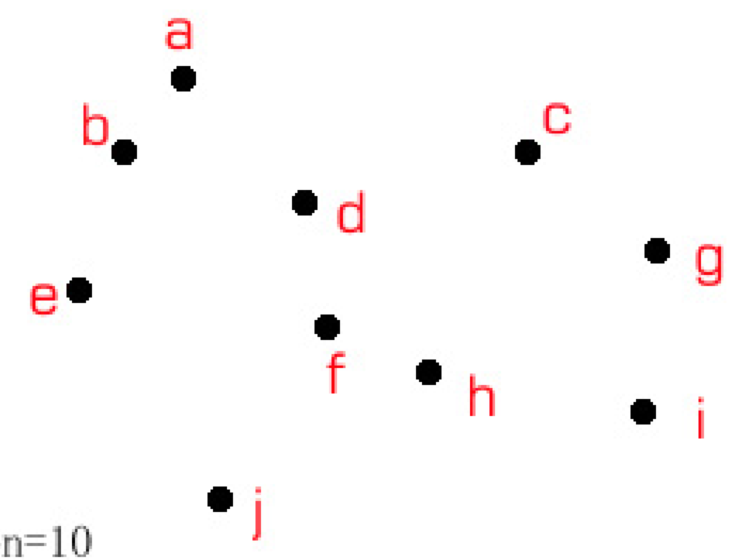
Knoten	Kanten(Typ 1)
a	

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



Knoten=10

1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

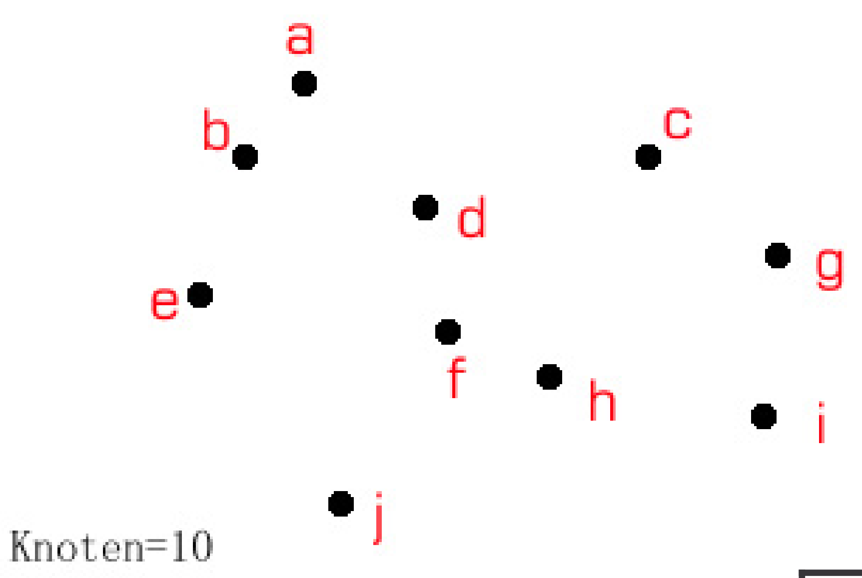
Knoten	Kanten(Typ 1)
a	

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α (r) > α(p) then delete r from T
else exit loop

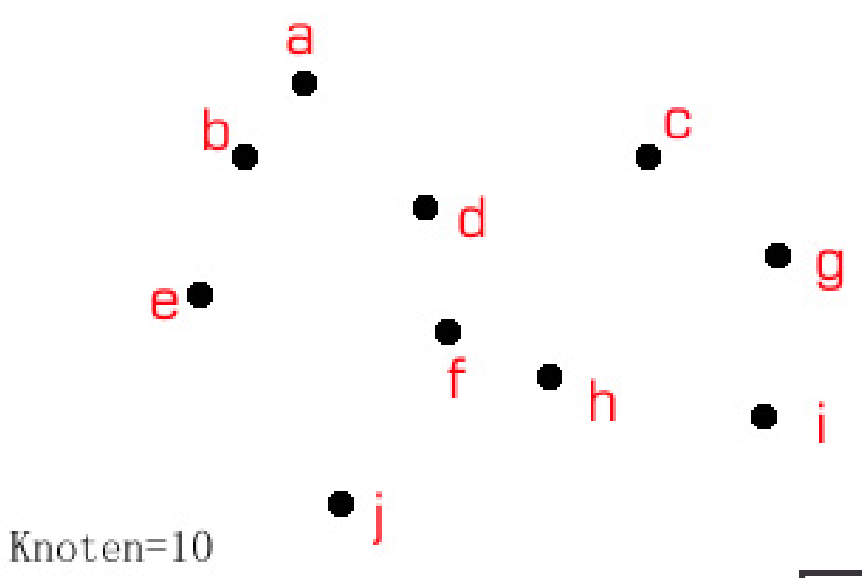
Knoten	Kanten(Typ 1)
c a	

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

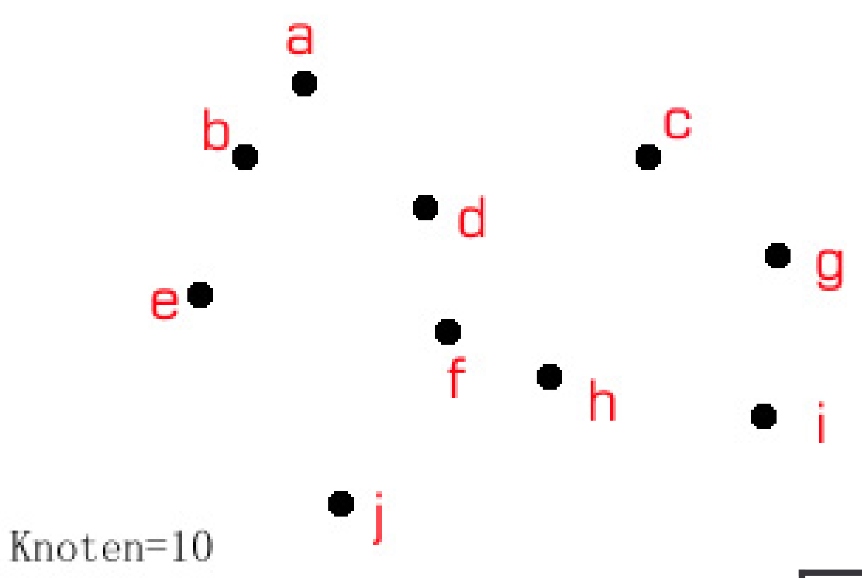
Knoten	Kanten(Typ 1)
c a	

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
      If α(r) > α(p) then delete r from T
   else exit loop

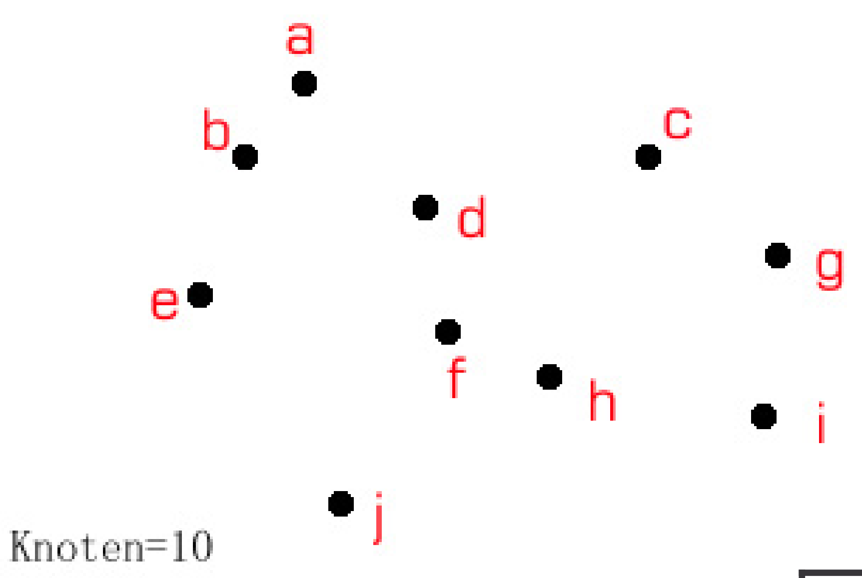
Knoten	Kanten(Typ 1)
c a	

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α (r) > α(p) then delete r from T
   else exit loop

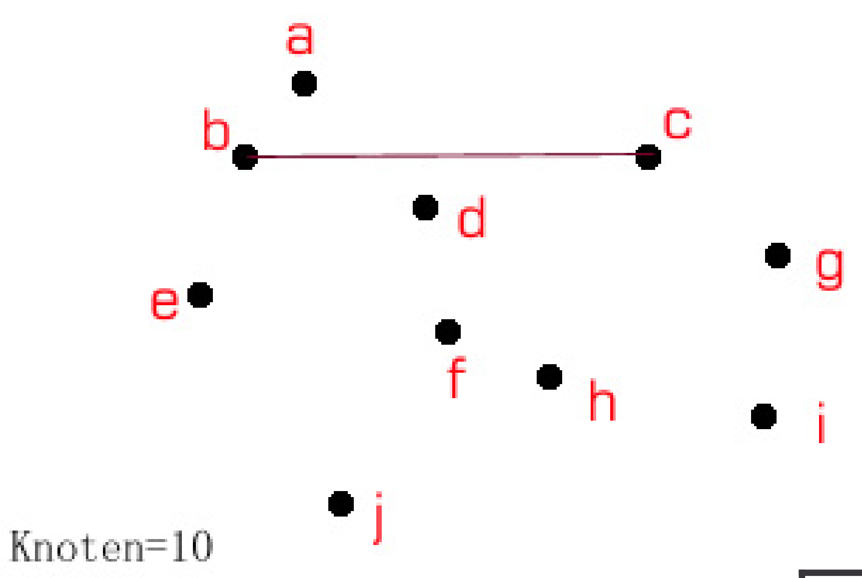
Knoten	Kanten(Typ 1)
c b a	

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α (r) > α(p) then delete r from T
   else exit loop

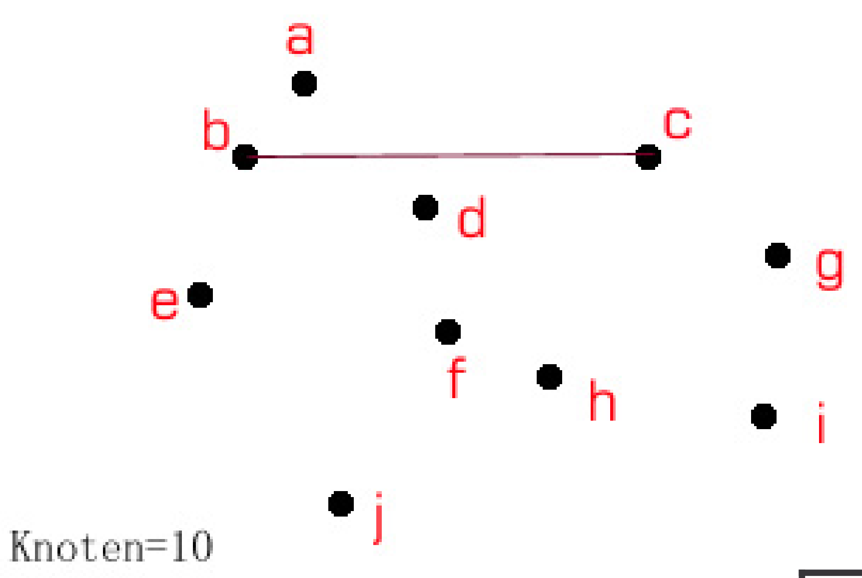
Knoten	Kanten(Typ 1)
c b a	bc,

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

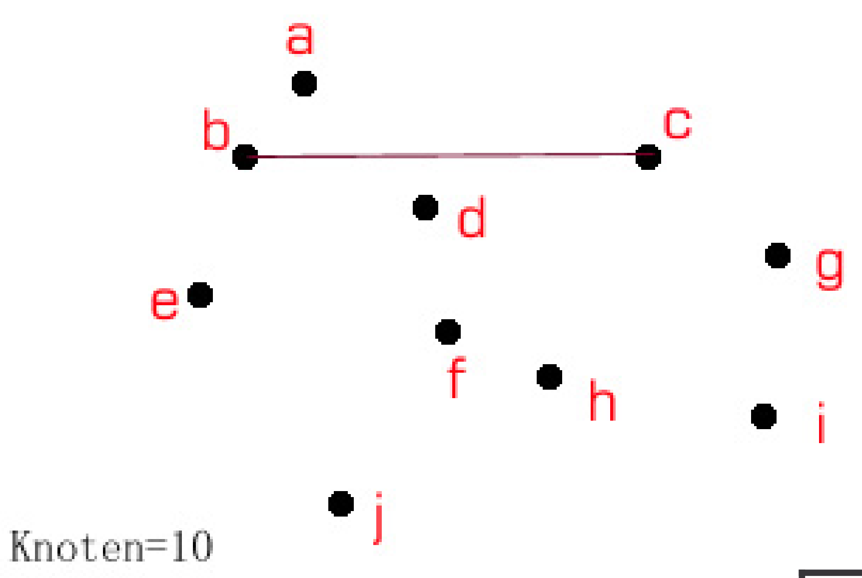
Knoten	Kanten(Typ 1)
c b (a)	bc,

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α (r) > α(p) then delete r from T
else exit loop

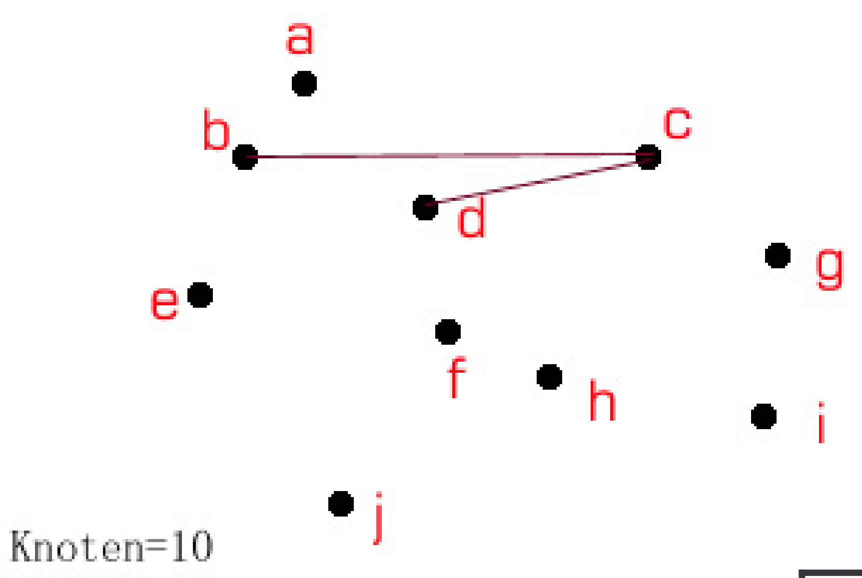
Knoten	Kanten(Typ 1)
c d b (a)	bc,

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α (r) > α(p) then delete r from T
   else exit loop

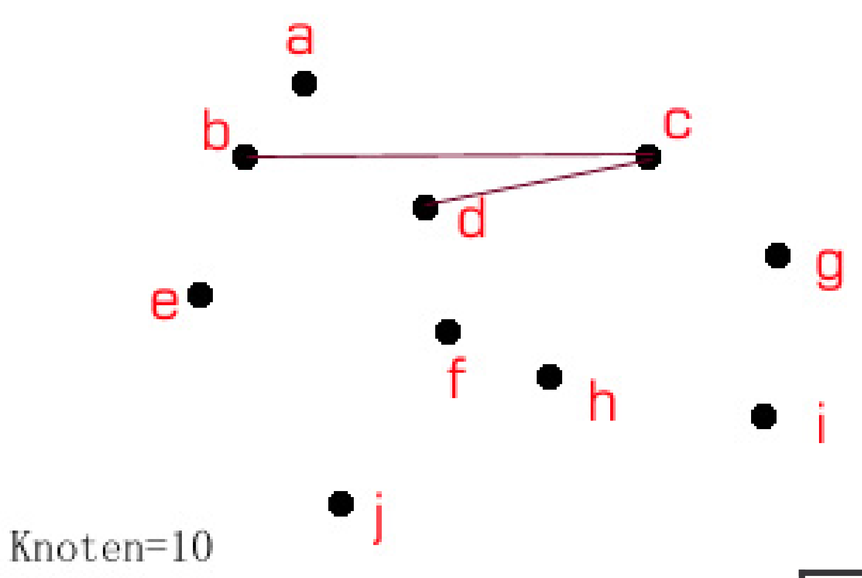
Knoten	Kanten(Typ 1)
c d b (a)	bc,dc,

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

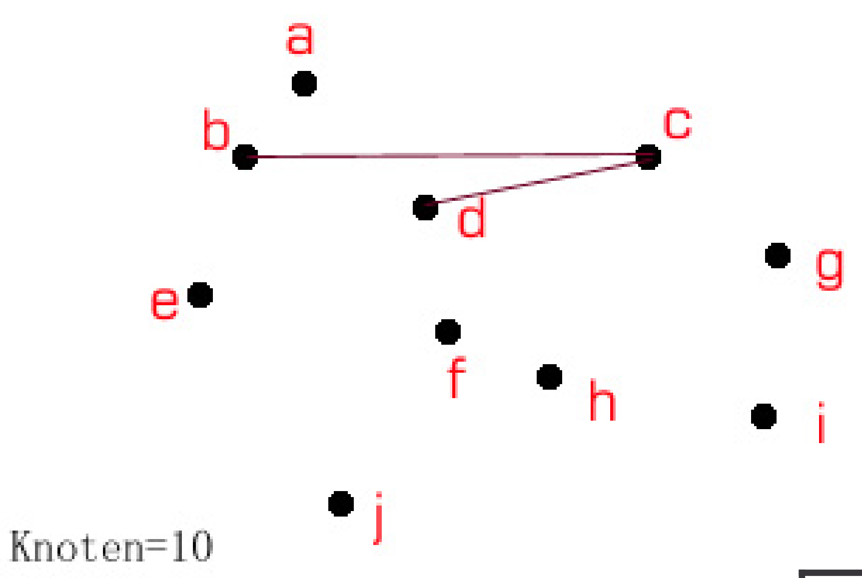
Knoten	Kanten(Typ 1)
c d b (a)	bc,dc,

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If $\alpha(r) > \alpha(p)$ then delete r from T
   else exit loop

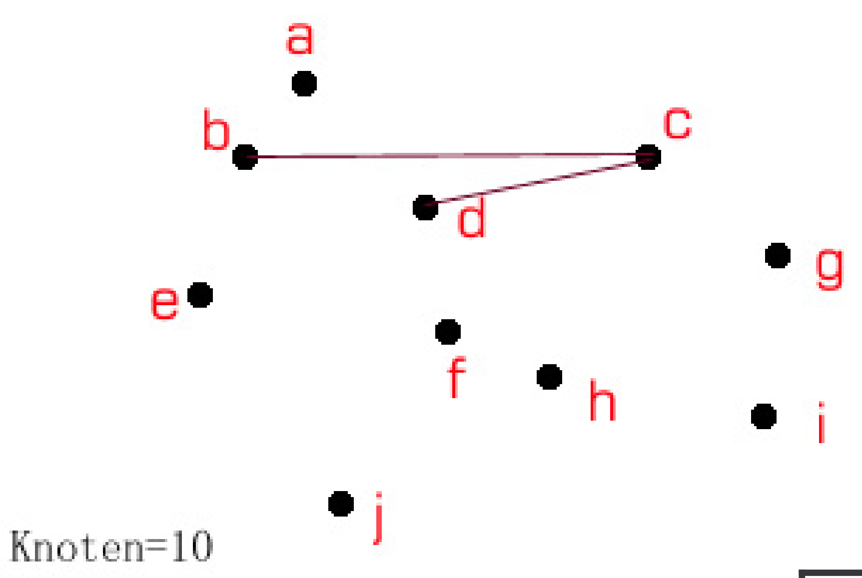
Knoten	Kanten(Typ 1)
g c d b (a)	bc,dc,

$\alpha = \{e,b,a,j,d,f,h,c,i,g\}$

$\beta = \{a,c,b,d,g,e,f,h,i,j\}$

$\gamma = \{i,g,h,j,c,f,d,e,b,a\}$

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α (r) > α(p) then delete r from T
else exit loop

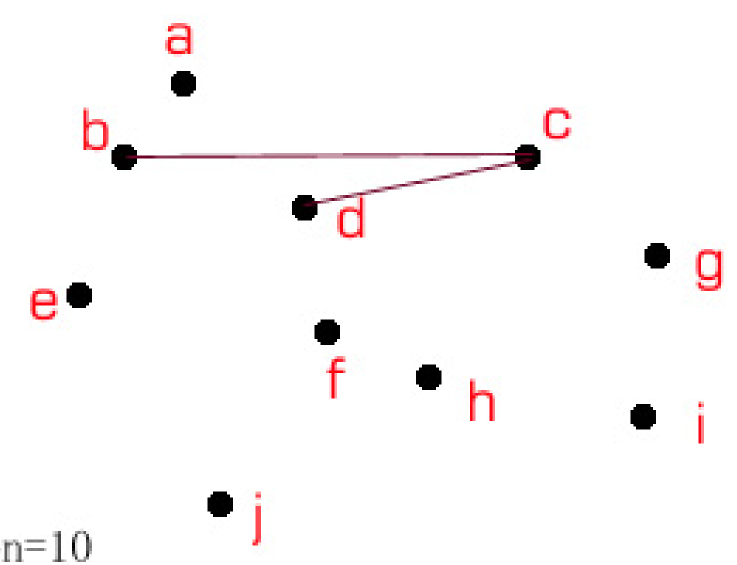
Knoten	Kanten(Typ 1)
g c d b (a)	bc,dc,

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α(r) > α(p) then delete r from T
else exit loop

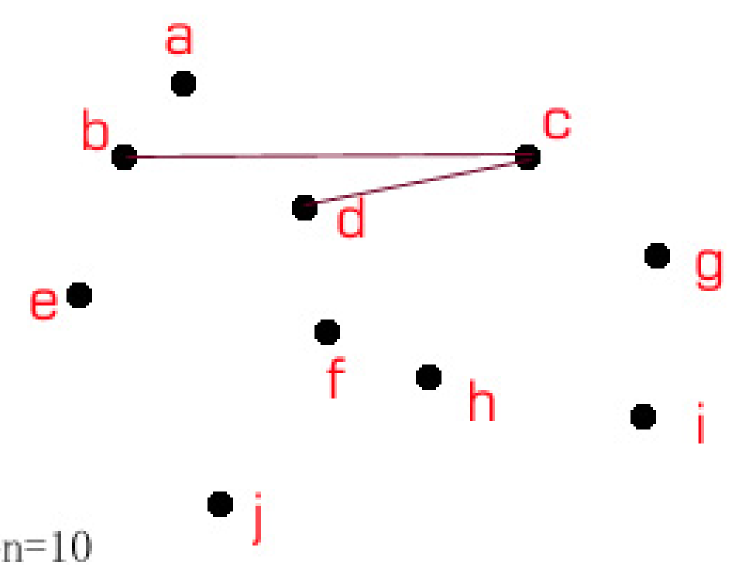
Knoten	Kanten(Typ 1)
g c d b (a)	bc,dc,

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



Knoten=10

1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α (r) > α(p) then delete r from T
   else exit loop

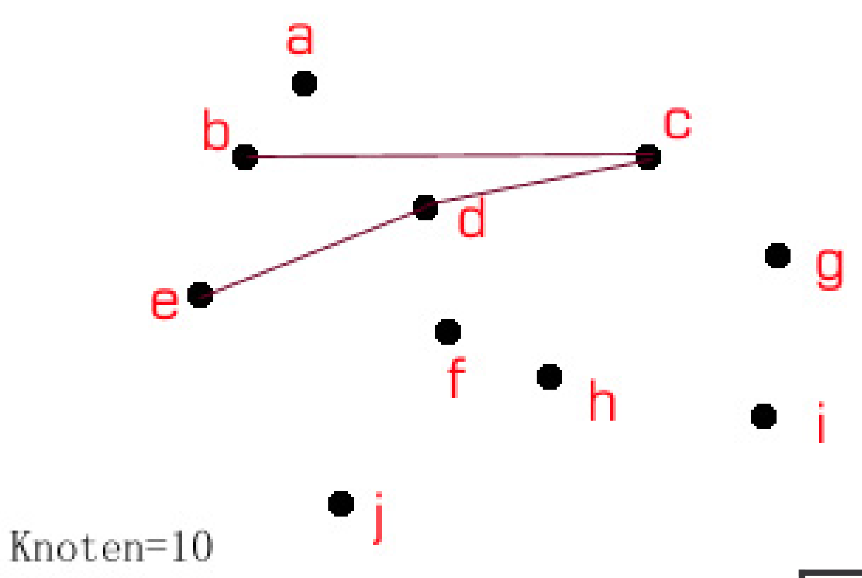
Knoten	Kanten(Typ 1)
g c d e b (a)	bc,dc,

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α (r) > α(p) then delete r from T
else exit loop

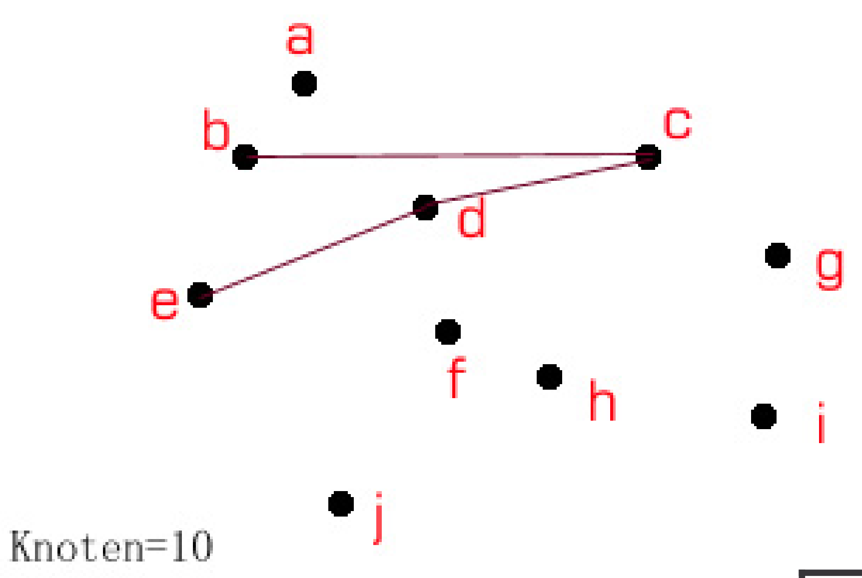
Knoten	Kanten(Typ 1)
g c d e b (a)	bc,dc,ed

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

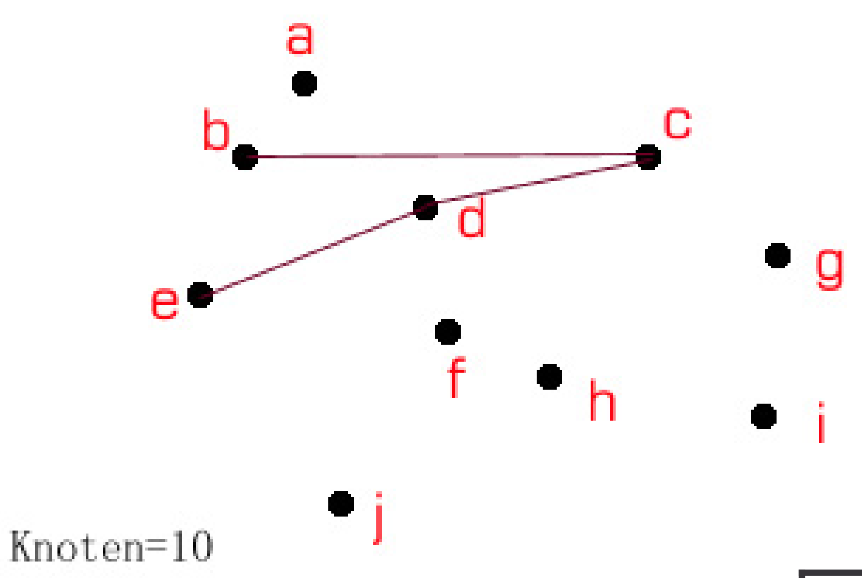
Knoten	Kanten(Typ 1)
g c d e (b) (a)	bc,dc,ed

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α (r) > α(p) then delete r from T
   else exit loop

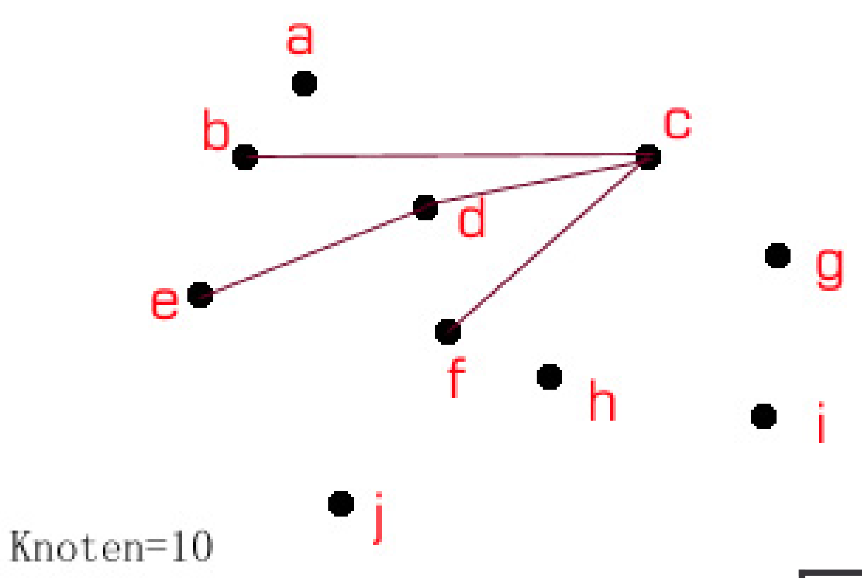
Knoten	Kanten(Typ 1)
g c f d e (b) (a)	bc,dc,ed

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α (r) > α(p) then delete r from T
   else exit loop

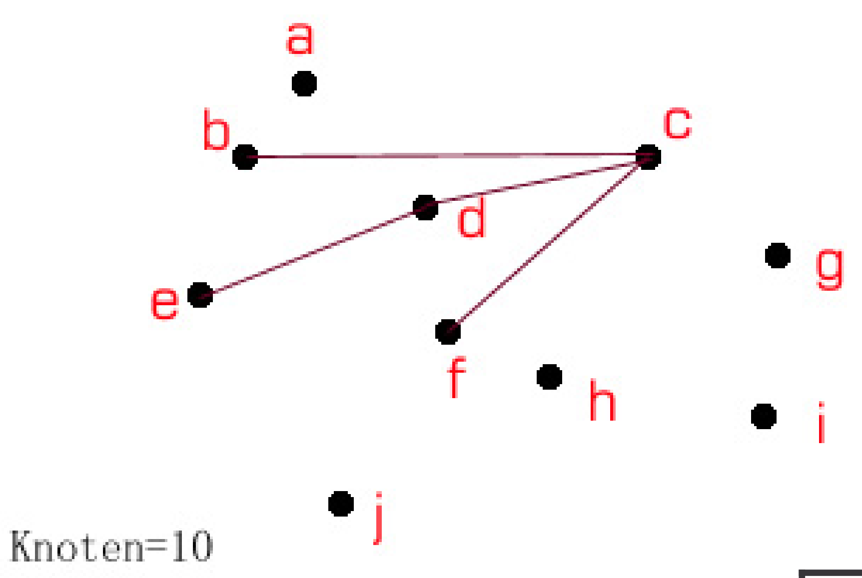
Knoten	Kanten(Typ 1)
g c f d e (b) (a)	bc,dc,ed,fc

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

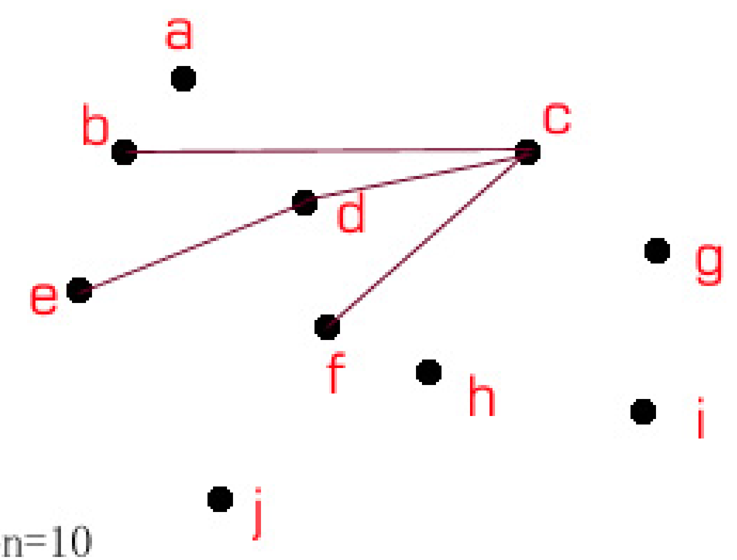
Knoten	Kanten(Typ 1)
g c f d e (b) (a)	bc,dc,ed,fc

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



Knoten=10

1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α (r) > α(p) then delete r from T
   else exit loop

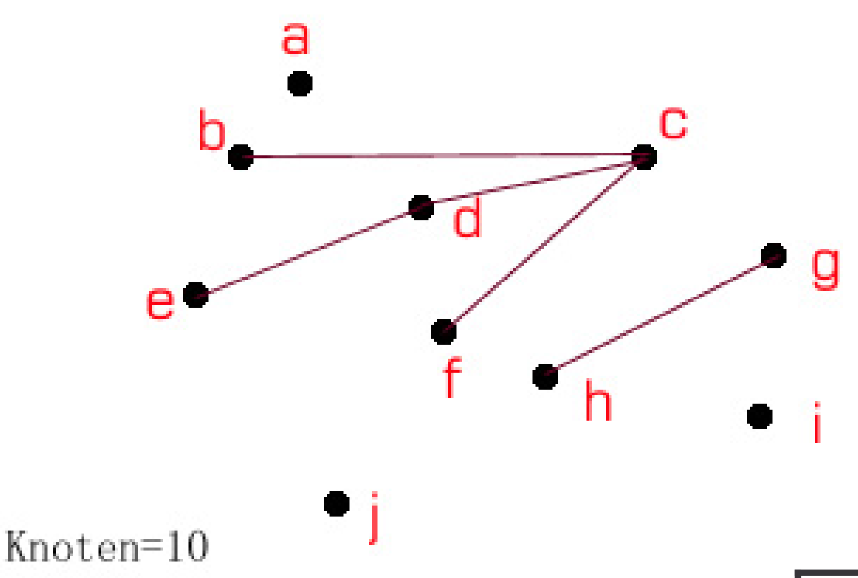
Knoten	Kanten(Typ 1)
g h c f d e (b) (a)	bc,dc,ed,fc

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

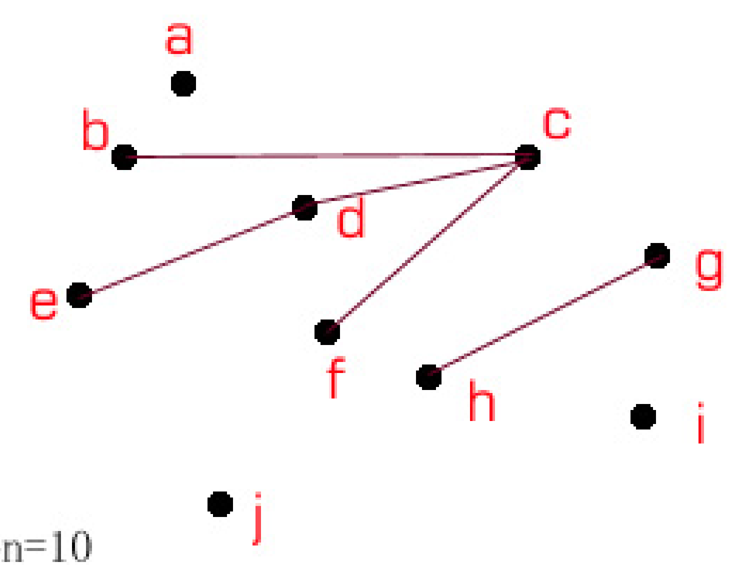
Knoten	Kanten(Typ 1)
g h c f d e (b) (a)	bc,dc,ed,fc,hg

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

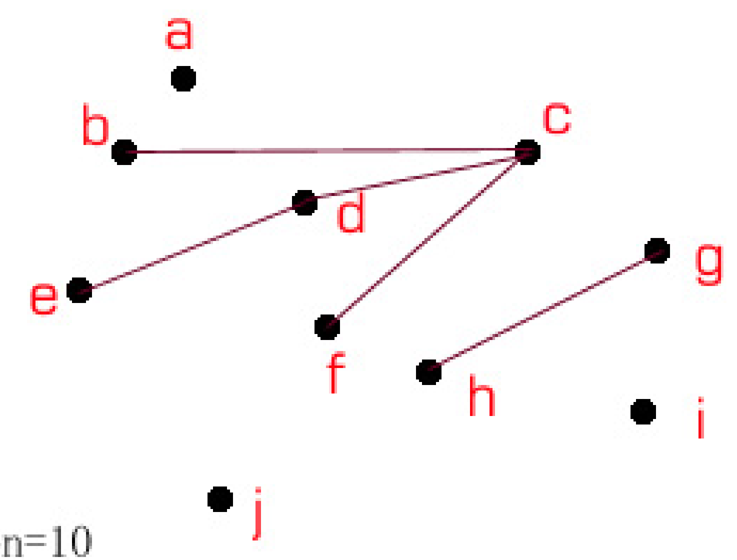
Knoten	Kanten(Typ 1)
g h (c) f d e (b) (a)	bc,dc,ed,fc,hg

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



Knoten=10

1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α (r) > α(p) then delete r from T
else exit loop

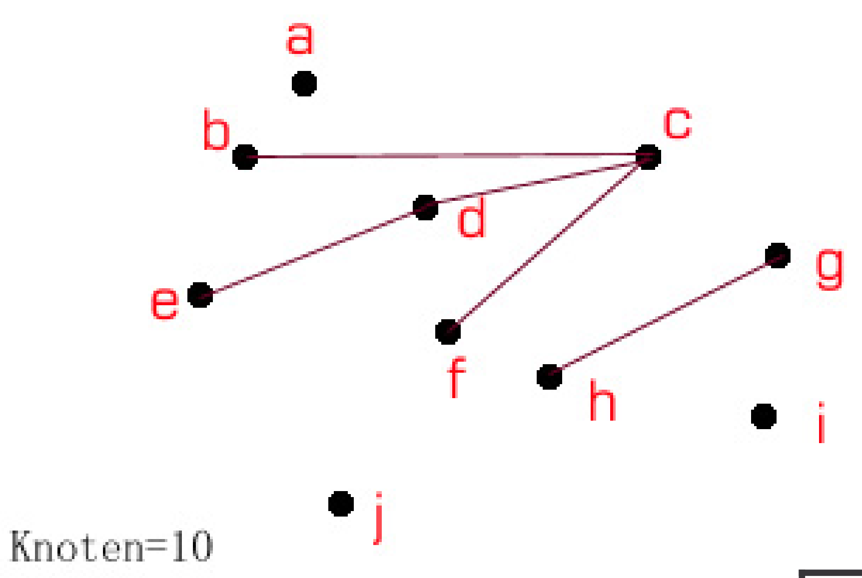
Knoten	Kanten(Typ 1)
i g h (c) f d e (b) (a)	bc,dc,ed,fc,hg

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α(r) > α(p) then delete r from T
else exit loop

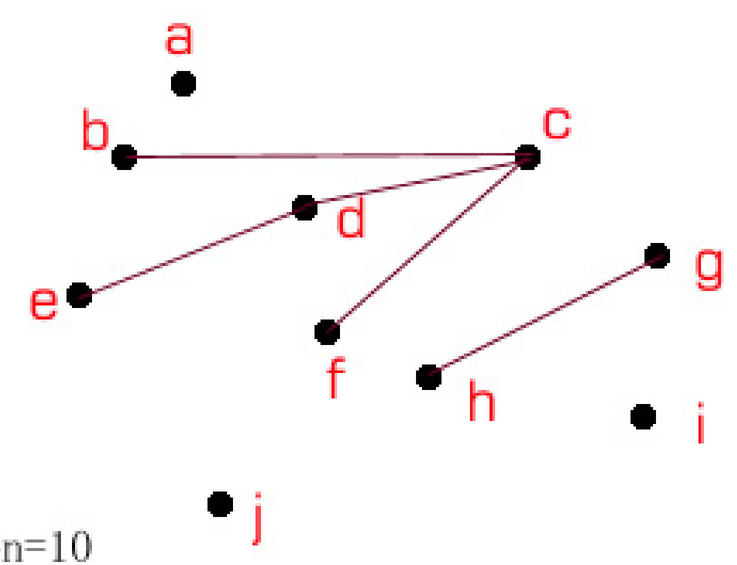
Knoten	Kanten(Typ 1)
i g h (c) f d e (b) (a)	bc,dc,ed,fc,hg

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



Knoten=10

1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

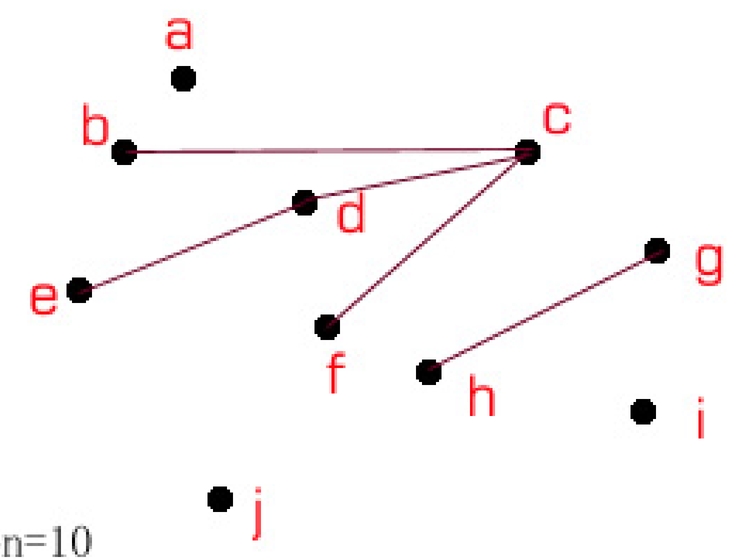
Knoten	Kanten(Typ 1)
i (g) h (c) f d e (b) (a)	bc,dc,ed,fc,hg

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



Knoten=10

1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α (r) > α(p) then delete r from T
else exit loop

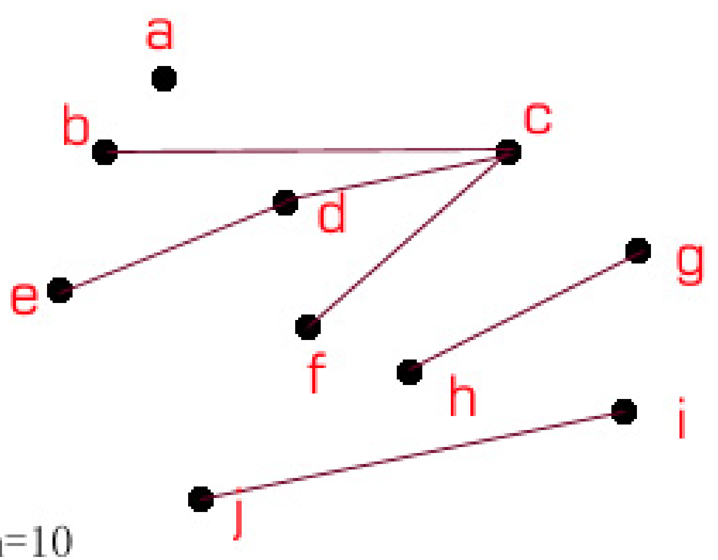
Knoten	Kanten(Typ 1)
i (g) h j (c) f d e (b) (a)	bc,dc,ed,fc,hg

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



Knoten=10

1) Insert point p into table T.

2) If p has a predecessor q in T
then report that pq is a type i edge.

3) Repeat Forver
If p has a successor r in T then
If α (r) > α(p) then delete r from T
else exit loop

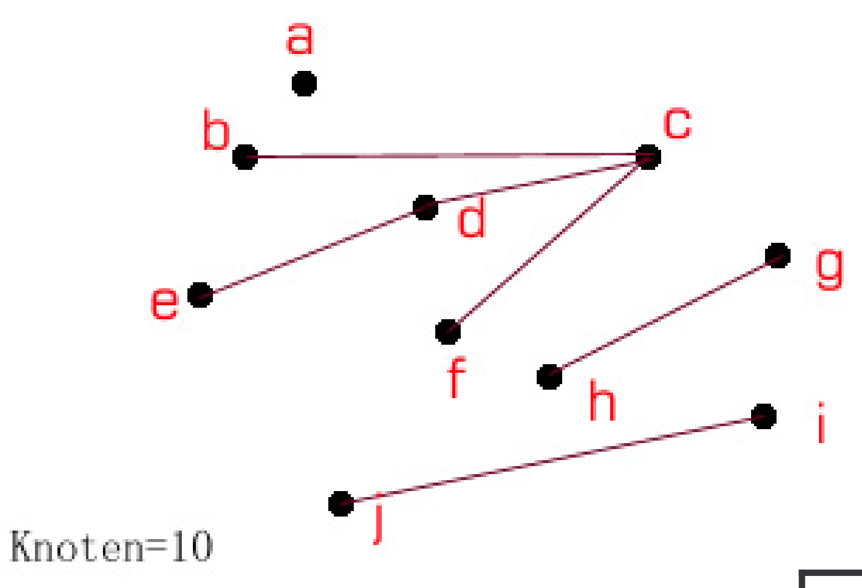
Knoten	Kanten(Typ 1)
i (g) h j (c) f d e (b) (a)	bc,dc,ed,fc,hg, jh

α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus



1) Insert point p into table T.

2) If p has a predecessor q in T
   then report that pq is a type i edge.

3) Repeat Forver
   If p has a successor r in T then
   If α(r) > α(p) then delete r from T
   else exit loop

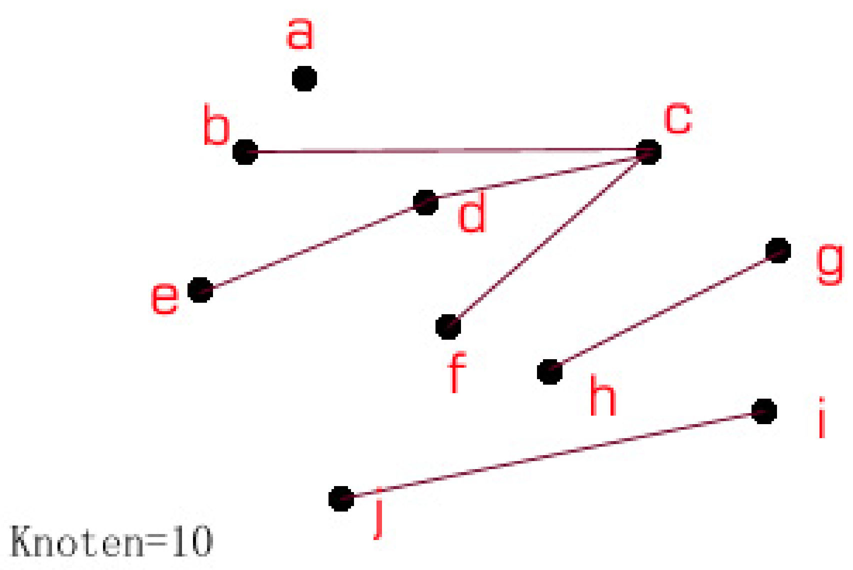
Knoten	Kanten(Typ 1)
i (g) h j (c) (f) (d) e (b) (a)	bc,dc,ed,fc,hg, jh

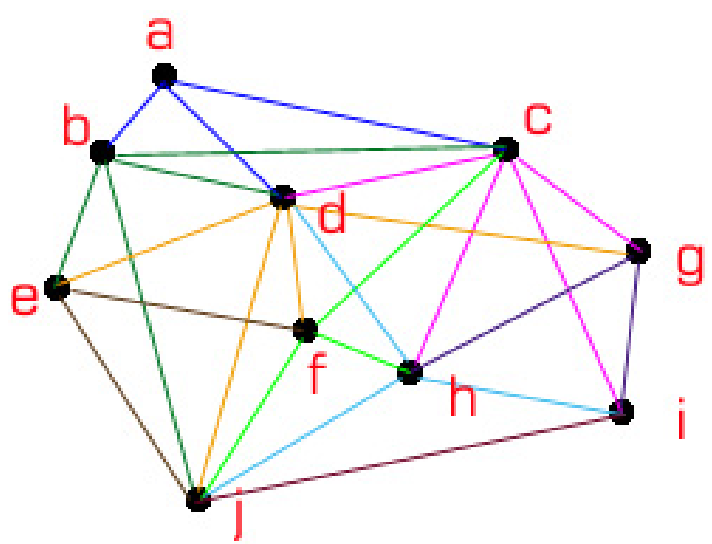
α = {e,b,a,j,d,f,h,c,i,g}

β = {a,c,b,d,g,e,f,h,i,j}

γ = {i,g,h,j,c,f,d,e,b,a}

Beispiel Nach Implemetierungsalgorithmus





Vergleichen

Definition des Spanners

- Difintion

  p,q= zwei beliebige konten in graph G

  d(p,q)=euklidische distanz

  G(p,q)=länge eines kürzesten wegs in graph G

- Spanner

  Spanner ist ein Subgraph von G und im Spanner ist der Maximalwert des Verhältnisse G(p,q) / d(p,q) in konstantem Rahmen.

Theta-Graph ist Spanner

- Definiton

  $\theta(p,q)$: Länge eines kürzesten Wegs von p bis q in theta-graph

  m: die Anzahl von Knoten in $\theta(p,q)$

  d(p,q) : euklidische distanz

- **Lemma**

  Wenn es ein kürzesten Weg von p bis q in Theta-graph , $\theta = \frac{2\pi}{k}$(k ist eine Ganzzahl und k > 8) gibt, und der Weg fährt über m Knoten durch,dann

$$\frac{\theta(p,q)}{d(p,q)} \leq \frac{1}{\cos\theta}\left(\frac{\tan^m\theta - 1}{\tan\theta - 1}\right) + \tan^m\theta$$

Beweis des Lemmas

- Methode

  durch Induktionsbeweis

- Definition

  s_n : n-te Knoten im Weg von p bis q in theta-graph (0<=i<=m),davon $s_0 = p$

  $\theta(p,q)$: Länge eines kürzesten Wegs von p bis q in theta-graph

  $\theta'(p,q)$: Länge eines Wegs p bis q in theta-graph mit solche Beschränkung:

Beweis des Lemmas

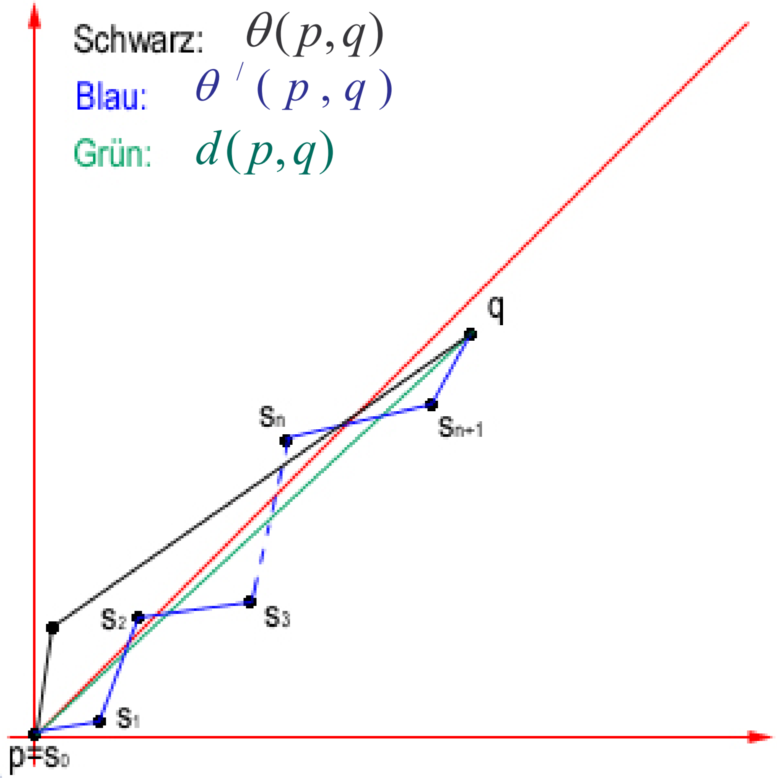
Beschränkung für $\theta'(p,q)$:

Wenn der Winkel ϕ von $\overrightarrow{s_n q}$ und x-achse ist $\frac{2\pi(i-1)}{k} \leq \phi \leq \frac{2\pi i}{k}$

d.h $\overrightarrow{s_n q}$ ist ein Kanten von Typ i ,dann ist die Kanten

von $\overrightarrow{s_n s_{n+1}}$ auch Typ i

Offenbar: $\theta(p,q) \leq \theta'(p,q)$

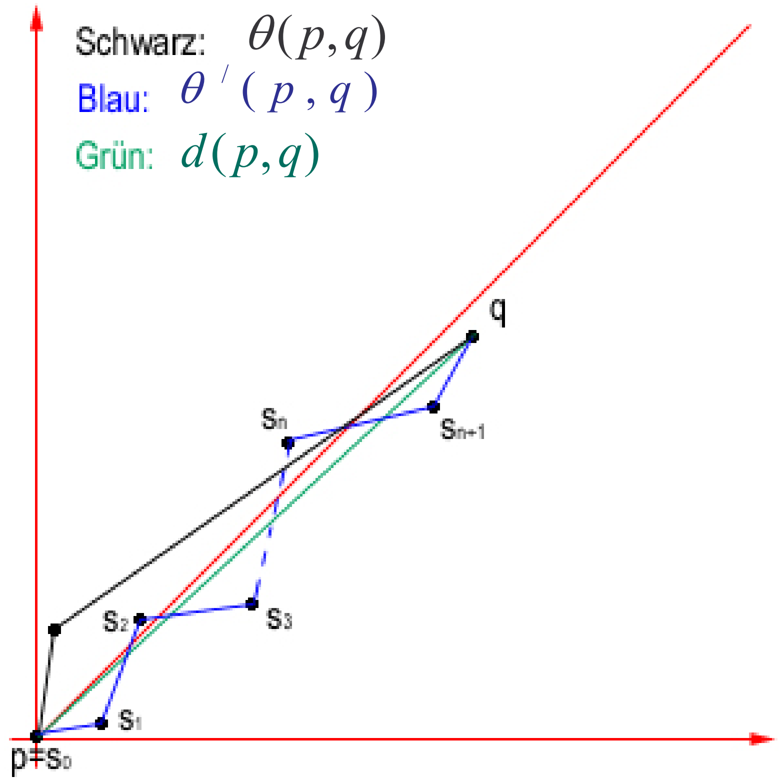


Beweis des Lemmas

Falls wir $$\frac{\theta^{/}(p,q)}{d(p,q)} \le \frac{1}{\cos\theta}\left(\frac{\tan^{m}\theta - 1}{\tan\theta - 1}\right) + \tan^{m}\theta$$

bewiesen haben , dann gilt auch

$$\frac{\theta\ (p,q)}{d(p,q)} \le \frac{1}{\cos\theta}\left(\frac{\tan^{m}\theta - 1}{\tan\theta - 1}\right) + \tan^{m}\theta$$



Beweis des Lemmas

1) Wenn m=0 ist , dann ist $\theta'(p,q) = d(p,q)$;

2)Annahme: $\dfrac{\theta'(p,q)}{d(p,q)} \leq \dfrac{1}{\cos\theta}\left(\dfrac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta$ ist richtig

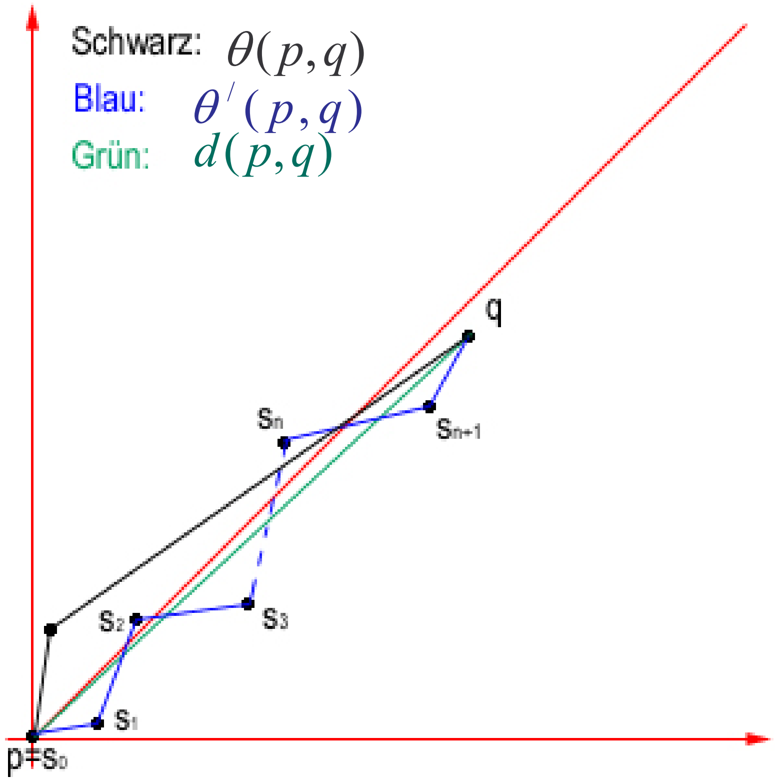
Claim: Verhältnis $\dfrac{\theta'(p,q)}{d(p,q)}$ hat Maximumswert,genau dann wenn

(a) q im x-achse ist

(b) der Winkel von $\overrightarrow{ps_1}$ und x-achse maximal α ($0 \leq \alpha < \dfrac{2\pi}{k}$) hat

Beweis des Claims

$$\frac{\theta'(p,q)}{d(p,q)} \le \frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta$$



Setzt s_1 in p ein

=> $$\frac{\theta'(s_1,q)}{d(s_1,q)} \le \frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta$$

=> $$\theta'(s_1,q) \le d(s_1,q)\left[\frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta\right]$$

$\theta'(p,q) = \theta'(p,s_1) + \theta'(s_1,q)$ und $\theta'(p,s_1) = d(p,s_1)$

=> $$\theta'(p,q) \le d(p,s_1) + d(s_1,q)\left[\frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta\right]$$

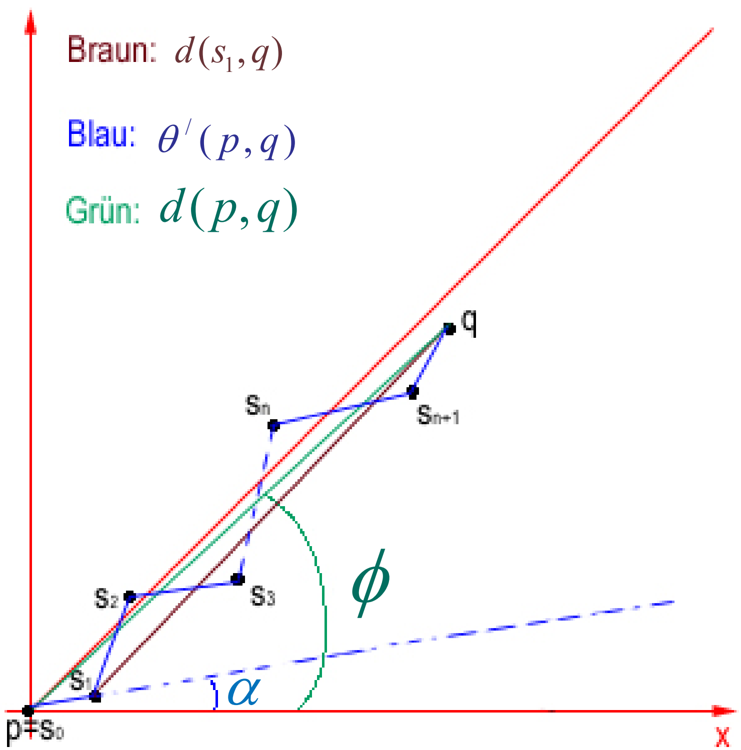
Beweis des Claims

$$\theta^{/}(p,q) \le d(p,s_1) + d(s_1,q)\left[\frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta\right]$$

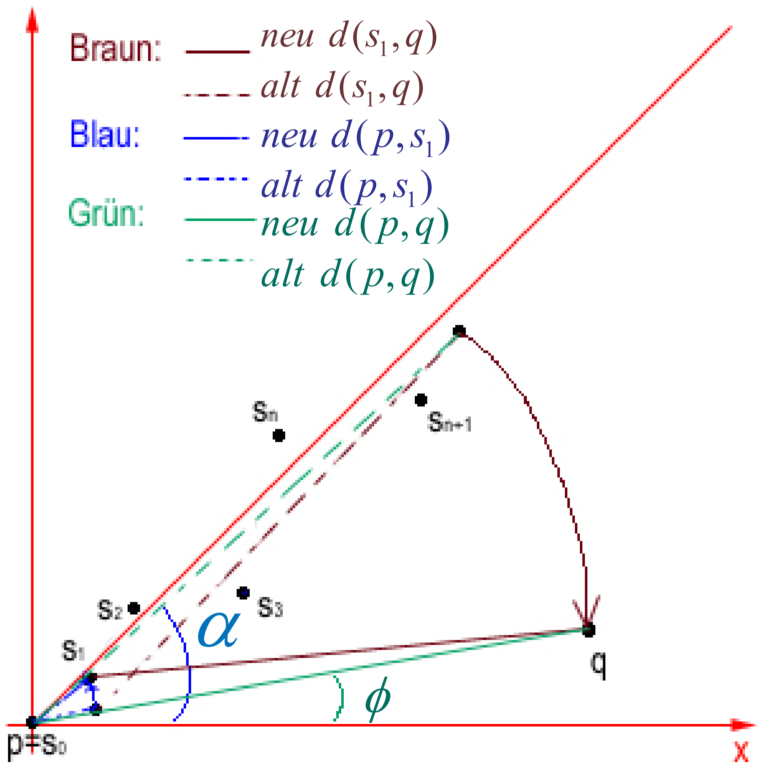
q und s_1 werden sich im beschränkte Rahmen ($0 \le \alpha < \frac{2\pi}{k}$)beweget,aber die Verhältnis $\frac{\theta^{/}(p,q)}{d(p,q)}$ wird nicht weniger ,d.h. $d(p,s_1)$ und $d(s_1,q)$ werden nicht weniger ,und $d(p,q)$ wird sich nicht erhöht.

Beweis des Claims

Falls $\alpha < \phi$ ist ,dann müssen wir α und ϕ umtauschen ,dann werden $d(p, s_1)$ und $d(s_1, q)$ nicht weniger ,und $d(p,q)$ wird sich nicht erhöht,wenn q sich um s_1 ins x-achse dreht.

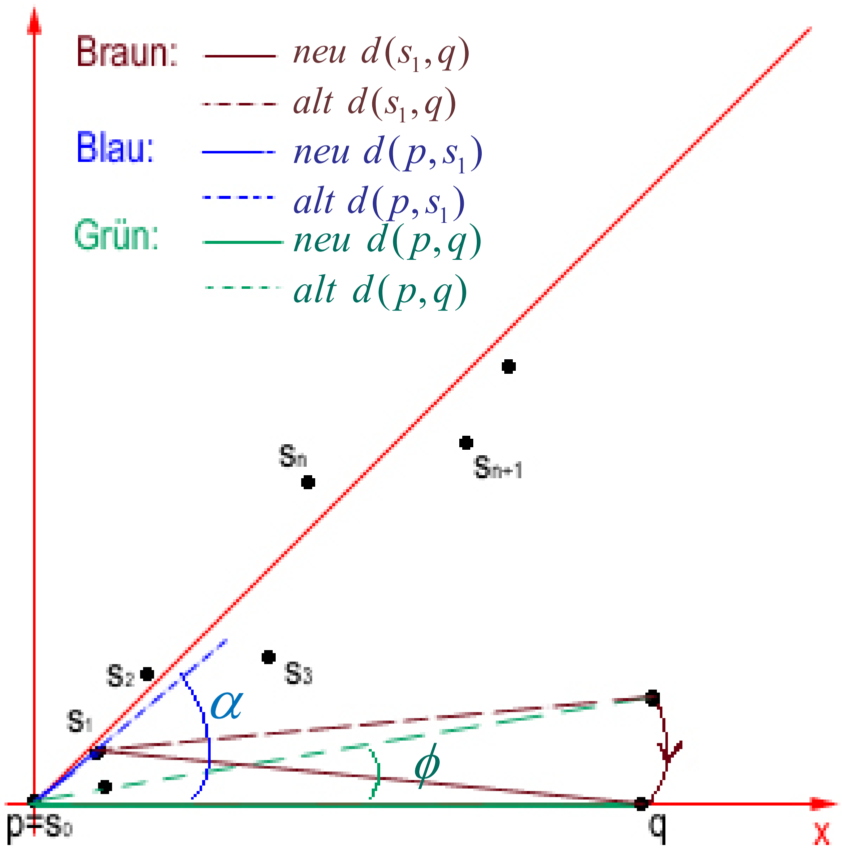


=>



Beweis des Claims

(a) q ist im x-achse



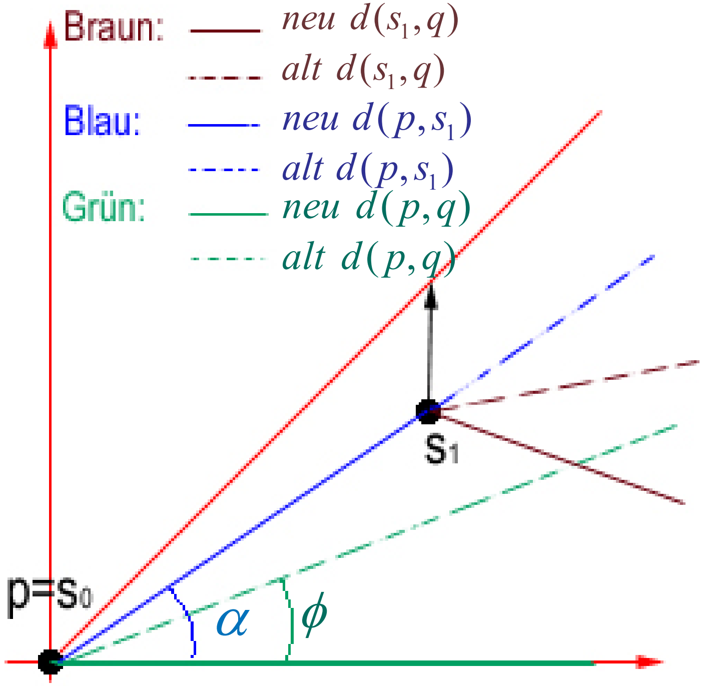
wenn q sich um s_1 ins x-achse dreht, ändern $d(p,s_1)$ und $d(s_1,q)$ nicht,und

$\alpha > \phi \Rightarrow d(p,q)$ wird sich nicht erhöht.

D.h wenn q im x-achse ist,dann ist $d(p,q)$ minimal .

Beweis des Claims

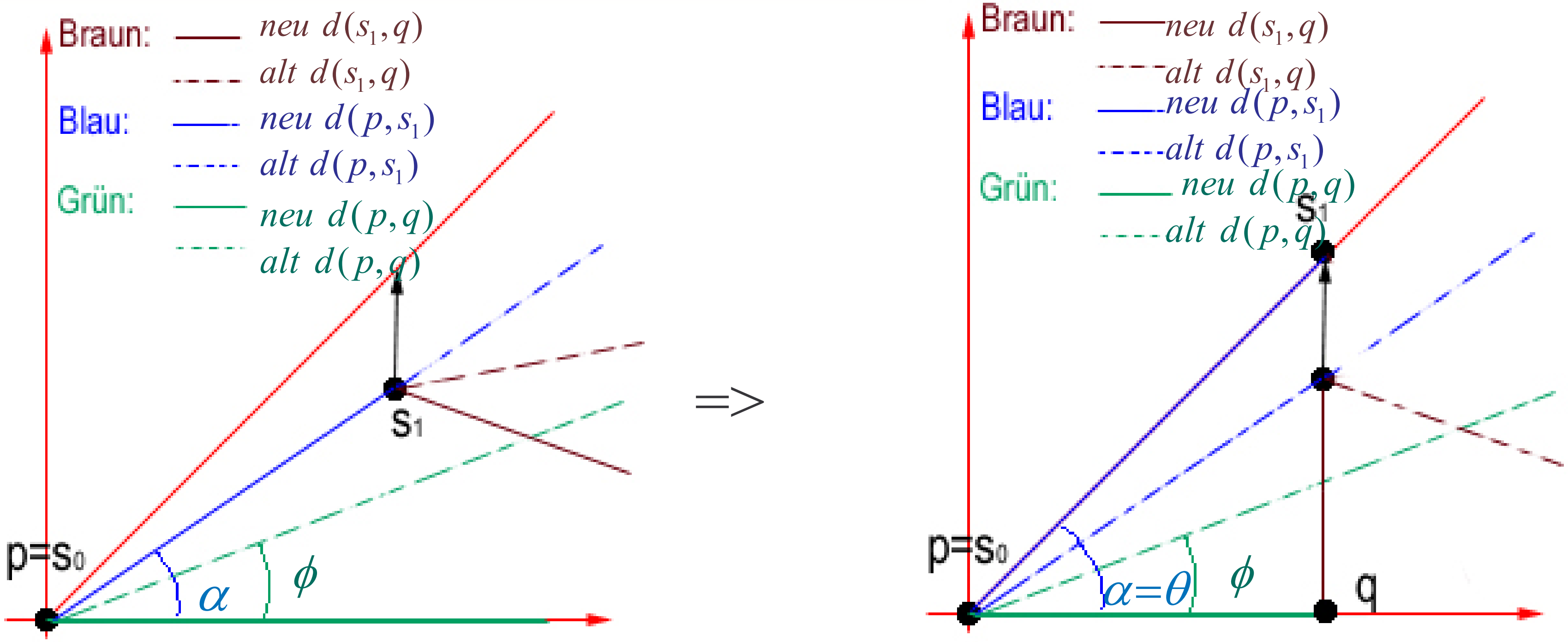
(b) der Winkel α von und x-achse maximal $(0 \le \alpha < \frac{2\pi}{k})$ hat



Wenn s_1 sich entlang y-coordinate beweget ,dann α wird vergrössert aber $0 \le \alpha < \frac{2\pi}{k}$, $d(p,s_1)$ und $d(s_1,q)$ werden sich erhöht,aber $d(p,q)$ ändert nicht

D.h.wenn $\alpha \to \frac{2\pi}{k}$ ist ,dann ist $d(p,s_1)$ maximal. W

Beweis des Lemmas



Nach Claim,ist $0 \le d(p,s_1) < \dfrac{d(p,q)}{\cos\theta}$ und $d(p,q)$ ist minimal und $d(p,s_1)$ ist maximal.

=> $d(s_1,q) = d(p,q)\tan\theta$

Beweis des Lemmas

3) $\theta'(p,q) \le d(p,s_1) + d(s_1,q)\left[\frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta\right]$

Setzt $0 \le d(p,s_1) < \frac{d(p,q)}{\cos\theta}$ und $d(s_1,q) = d(p,q)\tan\theta$ ein

$$=> \theta'(p,q) \le \frac{d(p,q)}{\cos\theta} + d(p,q)\tan\theta\left[\frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta\right]$$

$$=> \quad \frac{\theta'(p,q)}{d(p,q)} \le \frac{1}{\cos\theta} + \tan\theta\left[\frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta\right]$$

$$=> \quad \frac{\theta'(p,q)}{d(p,q)} \le \frac{1}{\cos\theta} + \tan\theta\left[\frac{1}{\cos\theta}\left(\frac{\tan^{m-1}\theta - 1}{\tan\theta - 1}\right) + \tan^{m-1}\theta\right]$$

Beweis des Lemmas

$$=> \quad \frac{\theta^{/}(p,q)}{d(p,q)} \le \frac{1}{\cos\theta} + \left(\frac{\tan^{m}\theta - \tan\theta}{\cos\theta(\tan\theta - 1)}\right) + \tan^{m}\theta$$

$$=> \frac{\theta^{/}(p,q)}{d(p,q)} \le \frac{(\tan\theta - 1)}{\cos\theta(\tan\theta - 1)} + \left(\frac{\tan^{m}\theta - \tan\theta}{\cos\theta(\tan\theta - 1)}\right) + \tan^{m}\theta$$

$$=> \quad \frac{\theta^{/}(p,q)}{d(p,q)} \le \frac{\tan\theta - 1 + \tan^{m}\theta - \tan\theta}{\cos\theta(\tan\theta - 1)} + \tan^{m}\theta$$

$$=> \quad \frac{\theta^{/}(p,q)}{d(p,q)} \le \frac{\tan^{m}\theta - 1}{\cos\theta(\tan\theta - 1)} + \tan^{m}\theta$$

$$=> \quad \frac{\theta^{/}(p,q)}{d(p,q)} \le \frac{1}{\cos\theta}\left(\frac{\tan^{m}\theta - 1}{\tan\theta - 1}\right) + \tan^{m}\theta \qquad \text{W}$$

Folgerung

Wenn im theta-graph ist $m \to \infty$ und k>8 , dann ist $\tan\theta < 1$,deshalb ist Verhältnisse

$$\frac{\theta(p,q)}{d(p,q)} = \frac{1}{\cos\theta}\left(\frac{1}{1-\tan\theta}\right) = B$$

k	B
10	4.52
15	1.97
20	1.56
25	1.39
30	1.30
35	1.24
40	1.20

Schönen Dank