



# Shortest Feasible Paths with Charging Stops for Battery Electric Vehicles

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# **Route Planning for Electric Vehicles**



#### **Electric vehicles:**

- Future means of transportation
- Run on regenerative energy sources

But:

- Restricted battery capacity
- Long recharging times

**Therefore :** Route planning applications have to consider:

- Energy consumption
- Charging stops





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**Therefore :** Route planning applications have to consider:

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... but we would still like to travel fast







**Task:** Given some source *s* and target *t* in a road network

- Find the fastest route from *s* to *t*
- Such that the battery does not deplete





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  - Battery can neither be depleted nor overcharged
- 3. Energy efficient driving vs. time consuming charging stops
  - Detour for reaching a charging station
- 4. Charging is not uniform
  - Interrupt charging and take another station





Find the fastest route from *s* to *t*:









Find the fastest route from *s* to *t*:



# Reachable area Horizon Charging station







#### Find the fastest route from *s* to *t*:



# Reachable area That the station





















#### Find the fastest route from *s* to *t*:









#### Find the fastest route from *s* to *t*:



# Reachable area Charging station





#### Find the fastest route from *s* to *t*:







#### Find the fastest route from s to t:



Fast charging station / swapping station





#### Find the fastest route from *s* to *t*:













# Find the fastest route from s to t: (t)

# Reachable area

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#### Find the fastest route from *s* to *t*:













#### Find the fastest route from *s* to *t*:









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• Larger battery  $\Rightarrow$  simpler problem ?







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# **Related Work**



#### Quick and Energy-Efficient Routes: [Storandt '12] Computing Constrained Shortest Paths for Electric Vehicles

- Supports only battery swapping stations (BSS)
- Computes overlay graph of directly reachable BSS
- **BSS** fully recharges in constant time  $\Rightarrow$  simple scalar overlay graph
- $\Rightarrow$  Not applicable / sub-optimal in our scenario



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#### Routing of Electric Vehicles: [Merting et al. '15] Constrained Shortest Path with Resource Recovering Nodes

- Theoretical analysis of the problem
- Show missing sub-path property
- Optimal solutions can visit a charging station several times





- Incorporate battery constraints in one single function
  - Maps initial State of Charge (SoC) onto resulting SoC
  - Value  $-\infty$  indicates an infeasible SoC





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For a single edge or path:

Representable using maximal 3 values





#### Formally:

- A function cf:  $[0, M] \times \mathbb{R}_{\geq 0} \rightarrow [0, M]$ , which maps
  - Initial SoC  $\beta_s$  and
  - Desired charging time  $\tau_c$  onto
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# **Observation:**

- Realisitc charging functions are simpler
- Reducable to univariate function:

 $\widetilde{c}f: \mathbb{R}_{\geq 0} \to [0, M]$  $cf(\beta, \tau_c) := \widetilde{c}f(\tau_c + \widetilde{c}f^{-1}(\beta))$ 







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# Example:

Charging an 60% full battery for 1 time unit cf(60%, 1)







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# Karlsruhe Institute of Technology

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#### Example:

Charging an 60% full battery for 1 time unit  $cf(60\%, 1) = \widetilde{c}f(1 + 2) = 80\%$ 





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# **Properties:**

- Monotonically increasing (Spending time will not decrease SoC)
- Concave (As SoC rises, the charging rate may only decline)







#### Algorithm:

- Based on multi-criteria Dijkstra
- If no charging station has been used: label = tuple (trip time, SoC)
- Per vertex: Maintain a set of Pareto-optimal labels





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# Solution:

- Delay this decision!
- Keep track of the last passed charging station





**Label:** A label  $\ell$  at vertex v is a quadruple  $(\tau_t, u, \beta_u, f_{[u,...,v]})$  with:

- Trip time  $\tau_t$  from s to v (including charging times except at u)
- The last seen charging station u (initially  $\perp$ )
- **SoC**  $\beta_u$  by which the last charging station (u) was reached
- Consumption profile  $f_{[u,...,v]}$  of the path from u to v (initially  $\perp$ )





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#### Edge relaxation:

- Label propagation along an edge: Constant time operation
- Given a label  $\ell_v = (\tau_t, \upsilon, \beta_u, f_{[\upsilon, ..., v]})$  at v and an edge e = (v, w):

 $\ell_w := (\tau_t + \tau_d(e), \, {\scriptstyle \textit{U}}, \, \beta_u, \, f_{[u, \ldots, v]} \circ f_e)$ 





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## Label Pareto-domination:

- Linear in complexity of charging function
- **Our solution:** Check Pareto-domination rarely

(Only for the next label to be settled)





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# **Reaching another charging station:**

- Our labels can store at most one charging station
- Have to specify the charging time for the second last station
- Theorem 1 in the paper proves that this is easy



# **Speed Up Techniques**



#### **CFP & Contraction Hierarchies:**

- Shortcut-based technique
- Shortcuts have to maintain Pareto-sets (w.r.t. travel time & energy consumption)

**Problem:** Shortcut size grows exponentially  $\Rightarrow$  uncontracted core



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- Goal directed technique
- Potential function  $\pi(v)$  estimates time from v to target t

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# CHArge = CH & A\* & CFP:

- Stop contraction as mean core degree gets to big
- Combine CH-query with A\*-search on core





#### Road Networks:

Instance	# Vertices	# Arcs	# Arcs with $f_e < 0$	#CS
Ger (PTV)	4 692 091	10805429	1 119 710 (10.36%)	1 966
Eur (PTV)	22 198 628	51 088 095	6 060 648 (11.86%)	13810
Osg (OSM)	5 588 146	11711088	1 142 391 (9.75%)	643

- Elevation data: SRTM, v4.1 (srtm.csi.cgiar.org)
- Energy consumption: [Hausberger et al. 09]
   Micro-scale emission model (PHEM), calibrated to Peugeot iOn
- Charging stations: ChargeMap (chargemap.com) random distributions
- Station Types: Battery swapping stations (BSS)
  - Superchargers (50 % in 20 min, 80 % in 40 min)
  - Regular stations (44 kW; 22 kW; 11 kW)





			Prepro.	Query		[Storandt '12]
Instance	#CS	М	[mm:ss]	Feas.	[ms]	[ms]
တ္ Osg	1 000	100 km	11:37	100 %	122	539
о В Osg	100	150 km	11:10	99%	206	1 150
<u></u> ≥ Osg	643	100 km	11:21	98%	326	
O Osg	643	150 km	11:28	99%	308	
တ္တ Ger	1 966	16 kWh	5:03	100%	1 398	
Ger	1 966	85 kWh	4:59	100%	1013	
<u> </u>	13810	16 kWh	30:32	63%	10786	
Õ Eur	13810	85 kWh	30:16	100%	47 921	
တ္ Ger	1 966	16 kWh	5:03	100%	8629	
Ger	1 966	85 kWh	4:59	100%	2614	
Eur	13810	16 kWh	30:32	63%	24 148	
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# **Final Remarks & Conclusion**



- Route planning for EVs raises new challenges
  - Considering energy consumption is essential
  - Charging stops should be planned in advance
  - Results in a (weakly) NP-hard problem
- Our approach **CHArge**:
  - Can handle arbitrary charging station types
  - Moderate preprocessing times
  - Fast queries on continental sized networks: Europe ~1 min; Germany ~1 sec
  - Even better results possible, using heuristics: Europe 0.1 – 1 sec; Germany 20 – 100 ms often optimal solutions, mean error ~1%

