Cops and Robber: When Capturing is not Surrounding

WG 2023 · 29.6.2023

Paul Jungeblut, Samuel Schneider, Torsten Ueckerdt
Cops and Robber

2-Players: $k$ Cops, 1 Robber

Rules:
Cops and Robber

2-Players: $k$ Cops $\cdots$ 1 Robber

Rules:
- Cops go first.
Cops and Robber

2-Players: \(k\) Cops \(\cdots\) 1 Robber

Rules:
- Cops go first.
- Robber is second.
Cops and Robber

2-Players: \( k \) Cops, 1 Robber

Rules:

- Cops go first.
- Robber is second.
- Moves are between adjacent vertices.
Cops and Robber

2-Players: \( k \) Cops
1 Robber

Rules:
- Cops go first.
- Robber is second.
- Moves are between adjacent vertices.
- Robber tries to flee indefinitely.
Cops and Robber

2-Players: \( k \) Cops, 1 Robber

Rules:
- Cops go first.
- Robber is second.
- Moves are between adjacent vertices.
- Robber tries to flee indefinitely.
- Cops win by **capturing** the robber.
Cops and Robber

2-Players: \( k \) Cops \( \cdots \) 1 Robber

Rules:
- Cops go first.
- Robber is second.
- Moves are between adjacent vertices.
- Robber tries to flee indefinitely.
- Cops win by capturing the robber.

\[ \text{Cop number } c(G) : \]
How many cops are necessary to capture the robber?
Surrounding Variants

vertex surround
Surrounding Variants

vertex surround

edge surround
Surrounding Variants

vertex surround

edge surround

\[c_V(G)\] \[c_E(G)\]

How many cops are necessary to vertex/edge surround the robber?
Surrounding Variants

Restrictive variants:

\[ c_{V,r}(G) : \]
Robber must not end his move on a cop.

\[ c_{E,r}(G) : \]
Robber must not move through a cop.

How many cops are necessary to vertex/edge surround the robber?
A Little Bit of History

\( c(G) : \)

- Quilliot '78
- Nowakowski, Winkler '83
- Aigner, Fromme '84
A Little Bit of History

\[ c(G) : \]
- Quilliot ’78
- Nowakowski, Winkler ’83
- Aigner, Fromme ’84

\[ c_{V,r}(G) : \]
- Burgess et al. ’20
A Little Bit of History

\[ c(G) : \]
- Quilliot '78
- Nowakowski, Winkler '83
- Aigner, Fromme '84

\[ c_{V,r}(G) : \]
- Burgess et al. '20

\[ c_{E,r}(G) : \]
- Crytser et al. '20
A Little Bit of History

\[ c(G) : \]
- Quilliot '78
- Nowakowski, Winkler '83
- Aigner, Fromme '84

\[ c_{V,r}(G) : \]
- Burgess et al. '20

\[ c_{V}(G) : \]
- new

\[ c_{E,r}(G) : \]
- Crytser et al. '20

\[ c_{E}(G) : \]
- new
Our Results

Unification
of the four surrounding variants

**Theorem:** (informal)
Strategy for \( k \) cops in some variant
gives a strategy for \( \leq k \cdot 2\Delta \) cops
in any other variant.

max. degree
Our Results

Unification

of the four surrounding variants

**Theorem:** (informal)
策略 for \( k \) cops in some variant gives a strategy for \( \leq k \cdot 2\Delta \) cops in any other variant.

Separation

from classical variant

**Conjecture:** (Crytser et al.)
\[ c_{E,r}(G) \leq c(G) \cdot \Delta(G) \]
Our Results

Unification
of the four surrounding variants

**Theorem:** (informal)
Strategy for \( k \) cops in some variant gives a strategy for \( \leq k \cdot 2\Delta \) cops in any other variant.

Separation
from classical variant

**Conjecture:** (Crytser et al.)
\[ c_{E,r}(G) \leq c(G) \cdot \Delta(G) \]

**Theorem:**
There are infinitely many \( G \) with:
- \( c(G) = 2 \) and \( \Delta(G) = 3 \)
- \( c_{V}(G), c_{V,r}(G), c_{E}(G) \) and \( c_{E,r}(G) \) are unbounded
### Unifying the Surrounding Variants

#### Upper bounds: Simulation

<table>
<thead>
<tr>
<th></th>
<th>not restricted</th>
<th>restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertices</td>
<td>$c_V(G)$</td>
<td>$c_{V,r}(G)$</td>
</tr>
<tr>
<td>edges</td>
<td>$c_E(G)$</td>
<td>$c_{E,r}(G)$</td>
</tr>
</tbody>
</table>
# Unifying the Surrounding Variants

## Upper bounds: Simulation

<table>
<thead>
<tr>
<th></th>
<th>not restricted</th>
<th>restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertices</td>
<td>$c_V(G)$</td>
<td>$c_{V,r}(G)$</td>
</tr>
<tr>
<td>edges</td>
<td>$c_E(G)$</td>
<td>$c_{E,r}(G)$</td>
</tr>
</tbody>
</table>

A restricted robber is a weaker robber.
## Upper bounds: Simulation

<table>
<thead>
<tr>
<th></th>
<th>not restricted</th>
<th>restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vertices</strong></td>
<td>$c_V(G)$</td>
<td>$c_{V,r}(G)$</td>
</tr>
<tr>
<td><strong>edges</strong></td>
<td>$c_E(G)$</td>
<td>$c_{E,r}(G)$</td>
</tr>
</tbody>
</table>

Replace each cop by a group of $\Delta(G)$ cops.
## Unifying the Surrounding Variants

### Upper bounds: Simulation

<table>
<thead>
<tr>
<th></th>
<th>not restricted</th>
<th>restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vertices</strong></td>
<td>$c_V(G)$</td>
<td>$c_{V,r}(G)$</td>
</tr>
<tr>
<td></td>
<td>$\times \Delta$</td>
<td>$\times 2$</td>
</tr>
<tr>
<td><strong>edges</strong></td>
<td>$c_E(G)$</td>
<td>$c_{E,r}(G)$</td>
</tr>
<tr>
<td></td>
<td>$\times \Delta$</td>
<td>$\times 2$</td>
</tr>
</tbody>
</table>

Replace edge cop by two vertex cops.

**Simulation**

$\times \Delta \times \Delta \times 2 \times 2$
### Unifying the Surrounding Variants

#### Upper bounds: Simulation

<table>
<thead>
<tr>
<th></th>
<th>not restricted</th>
<th>restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vertices</strong></td>
<td>$c_V(G)$</td>
<td>$c_{V,r}(G)$</td>
</tr>
<tr>
<td></td>
<td>$\times 2$  $\times \Delta$</td>
<td>$\times 2$  $\times \Delta$</td>
</tr>
<tr>
<td><strong>edges</strong></td>
<td>$c_E(G)$</td>
<td>$c_{E,r}(G)$</td>
</tr>
<tr>
<td></td>
<td>$\times \Delta$</td>
<td>$\times \Delta$</td>
</tr>
</tbody>
</table>

Replace vertex cop by a group of $\Delta(G)$ edge cops.

Follow original cop.
Unifying the Surrounding Variants

Upper bounds: Simulation

<table>
<thead>
<tr>
<th></th>
<th>not restricted</th>
<th>restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td>vertices</td>
<td>$c_V(G)$</td>
<td>$c_{V,r}(G)$</td>
</tr>
<tr>
<td></td>
<td>$\times 2 \times \Delta$</td>
<td>$\times 2 \times \Delta$</td>
</tr>
</tbody>
</table>

|                | $c_E(G)$      | $c_{E,r}(G)$ |
|                | $\times \Delta$ | $\times \Delta$ |

Lower bounds: Constructions

Tight examples for all claimed inequalities:
- complete (bipartite) graphs
- regular graphs (with “leaves”)
- based on “MOLS” (mutually orthogonal Latin squares)
- line graphs of complete graphs
Observation:
surrounding $\implies$ capturing
$\sim c_X(r) (G) \geq c(G)$

Question: other direction?
Observation:
surrounding $\Rightarrow$ capturing
\[ c_X(\cdot, r)(G) \geq c(G) \]

Question: other direction?

Conjecture: (Crytser et al.)
\[ c_{E,r}(G) \leq c(G) \cdot \Delta(G) \]
Separating “Capturing” and “Surrounding”

**Observation:**
surrounding $\Rightarrow$ capturing
\[ c_X(r)(G) \geq c(G) \]

**Question:** other direction?

**Conjecture:** (Crytser et al.)
\[ c_{E,r}(G) \leq c(G) \cdot \Delta(G) \]

**Theorem:**
There are infinitely many $G$ with:
- $c(G) = 2$ and $\Delta(G) = 3$
- $c_V(G), c_{V,r}(G), c_E(G)$ and $c_{E,r}(G)$ are unbounded
Separating “Capturing” and “Surrounding”
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: $c(G) = 2$
Separating “Capturing” and “Surrounding”

2 cops can capture the robber:
$c(G) = 2$
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: $c(G) = 2$
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: \( c(G) = 2 \)
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: $c(G) = 2$
Separating “Capturing” and “Surrounding”

2 cops can capture the robber:
\[ c(G) = 2 \]
2 cops can capture the robber: $c(G) = 2$
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: \( c(G) = 2 \)
Separating “Capturing” and “Surrounding”

2 cops can capture
the robber:
\( c(G) = 2 \)
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: $c(G) = 2$
Separating “Capturing” and “Surrounding”

2 cops can capture the robber:
\[ c(G) = 2 \]

\[ k = 8 \]

\[ k - 1 \] cops cannot surround robber:
\[ c_V(G) \geq k \]
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: $c(G) = 2$

$k - 1$ cops cannot surround robber: $c_V(G) \geq k$
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: \( c(G) = 2 \)

\( k = 8 \)

\( k - 1 \) cops cannot surround robber: \( c_V(G) \geq k \)

\(< \frac{k}{2} \)
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: \( c(G) = 2 \)

\( k = 8 \)

\( k - 1 \) cops cannot surround robber: \( c_V(G) \geq k \)

Cops and Robber – When Capturing is not Surrounding
Paul Jungeblut, Samuel Schneider, Torsten Ueckerdt
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: \( c(G) = 2 \)

\( k = 8 \)

\( k - 1 \) cops cannot surround robber: \( c_V(G) \geq k \)

< k/4
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: $c(G) = 2$

$k - 1$ cops cannot surround robber: $c_V(G) \geq k$
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: \( c(G) = 2 \)

\( k - 1 \) cops cannot surround robber: \( c_V(G) \geq k \)

\( k = 8 \)
Separating “Capturing” and “Surrounding”

2 cops can capture the robber: \( c(G) = 2 \)

\( k - 1 \) cops cannot surround robber: \( c_V(G) \geq k \)