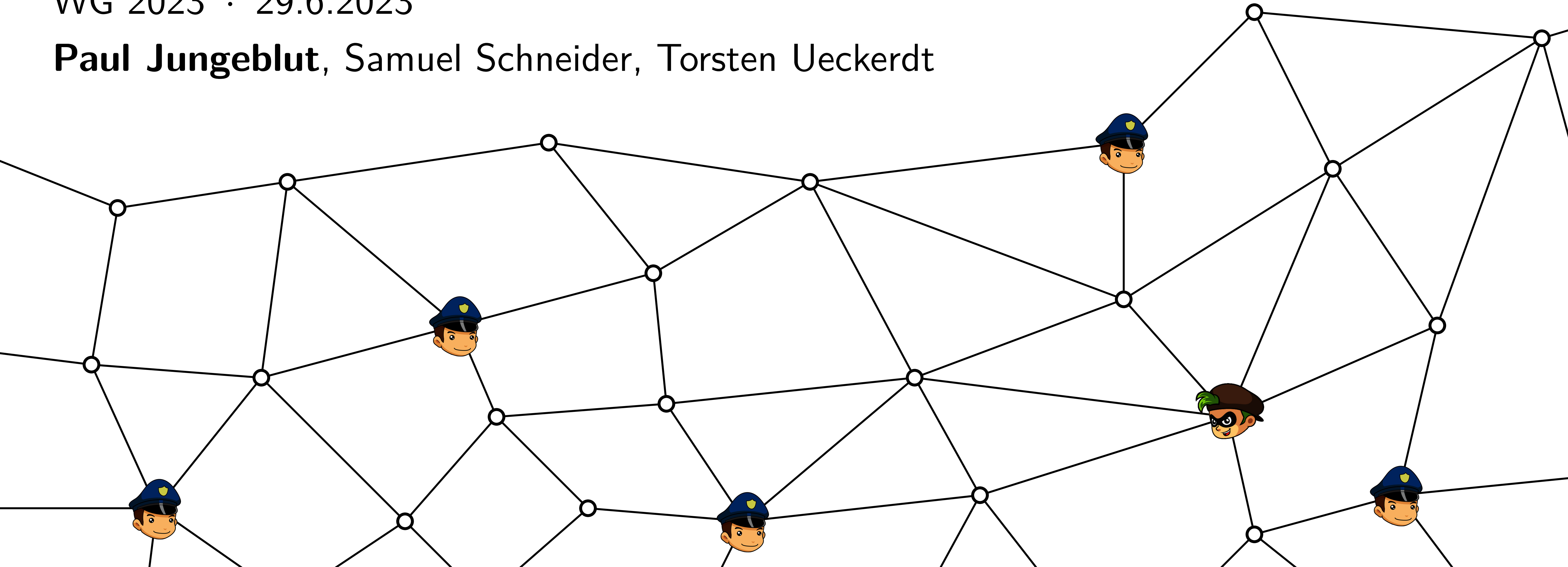
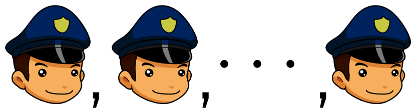
Cops and Robber: When Capturing is not Surrounding

WG 2023 · 29.6.2023

Paul Jungeblut, Samuel Schneider, Torsten Ueckerdt

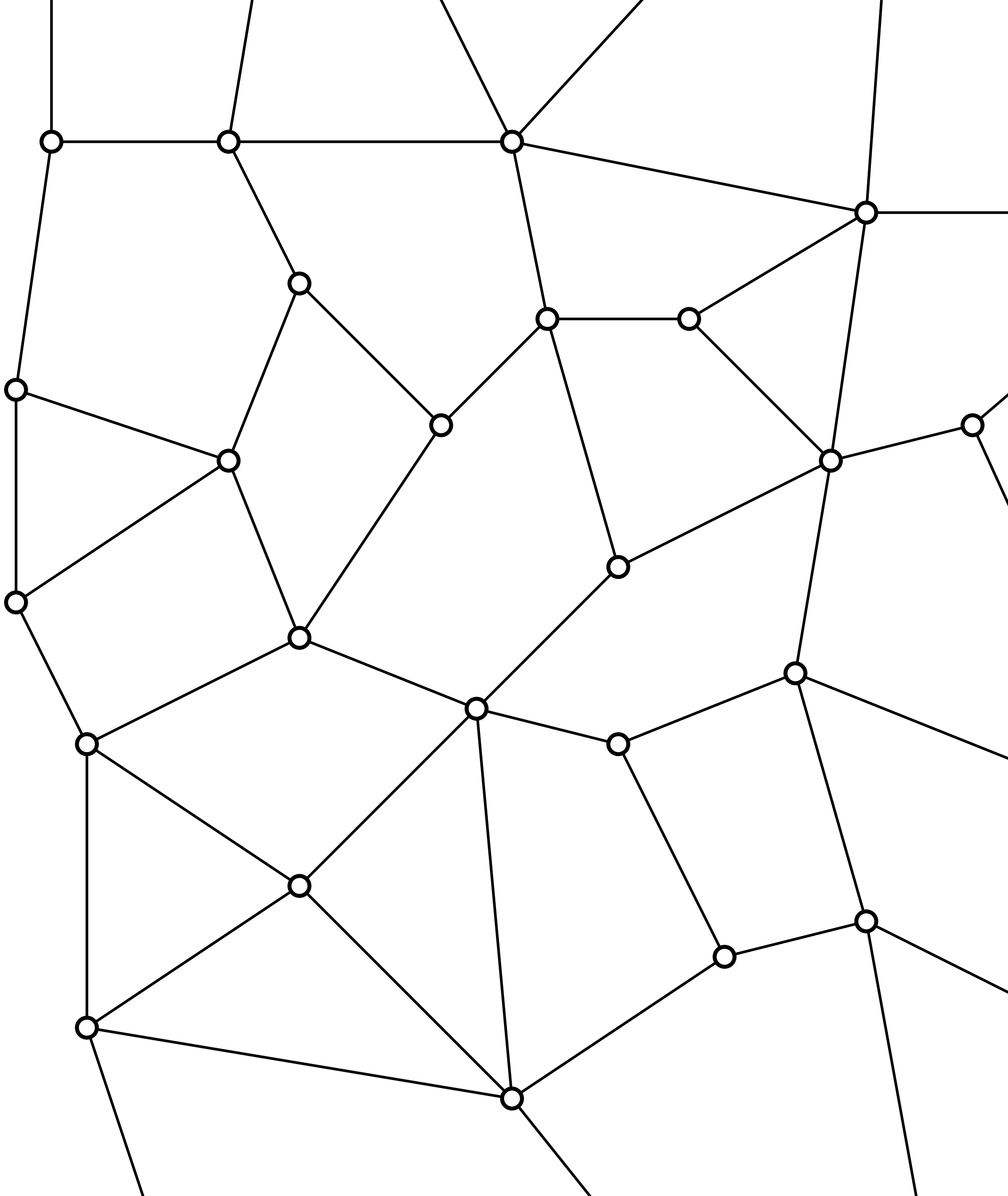


Cops and Robber

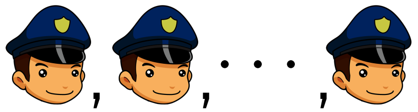
2-Players: k Cops 

1 Robber 

Rules:



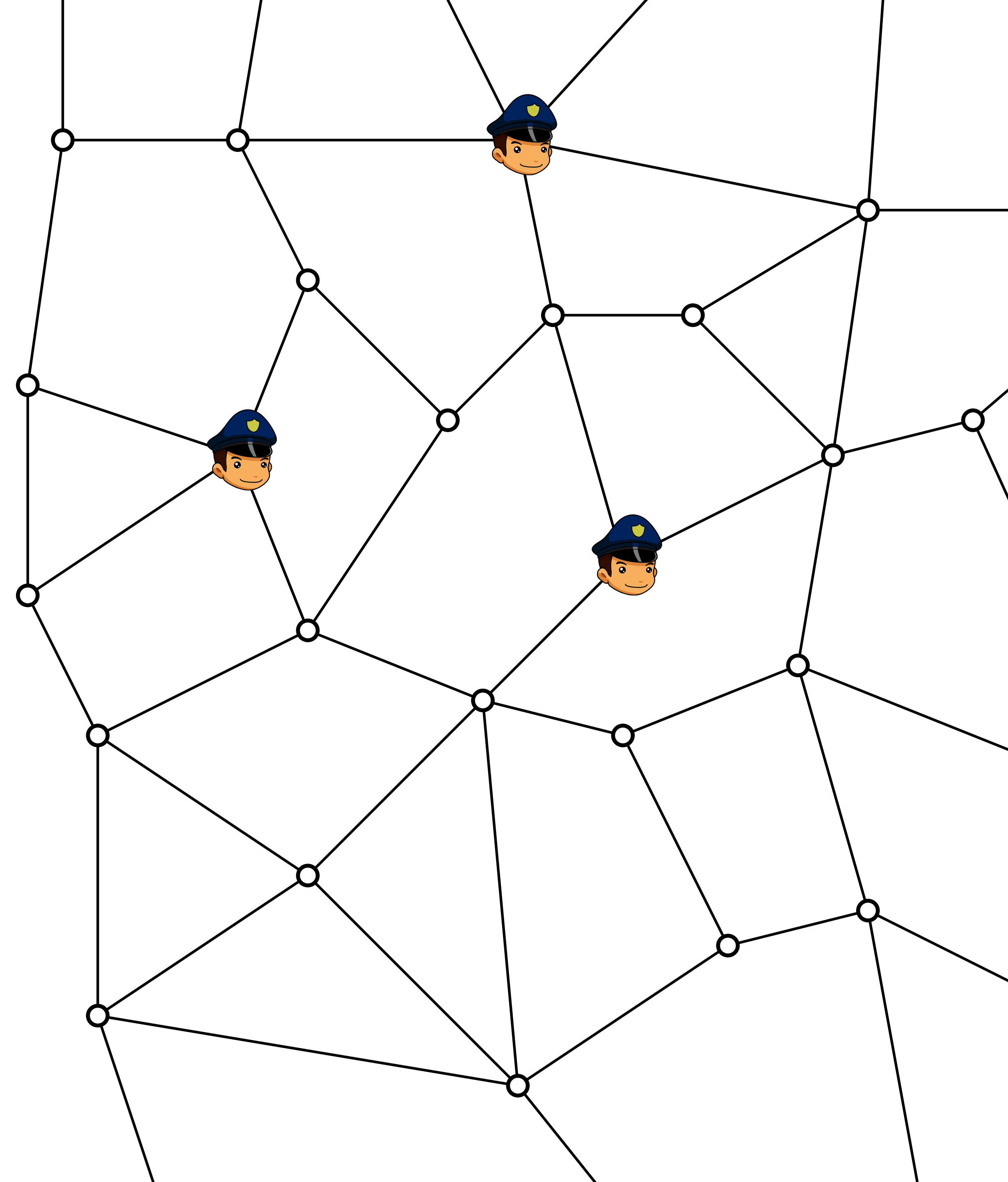
Cops and Robber

2-Players: k Cops 

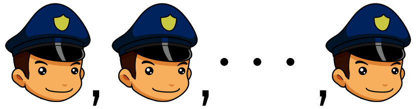
1 Robber 

Rules:

- Cops go first.



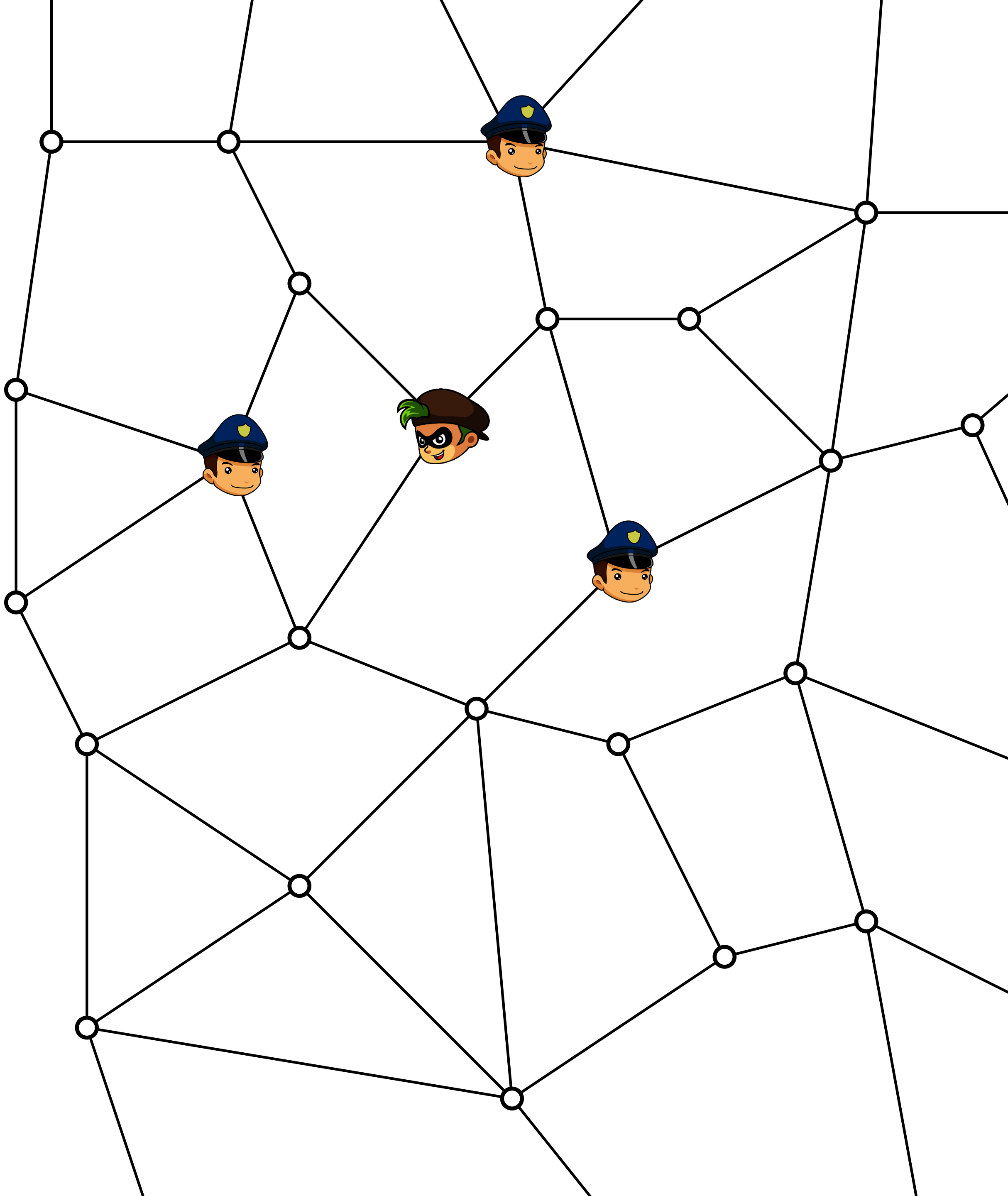
Cops and Robber

2-Players: k Cops 

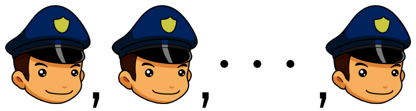
1 Robber 

Rules:

- Cops go first.
- Robber is second.



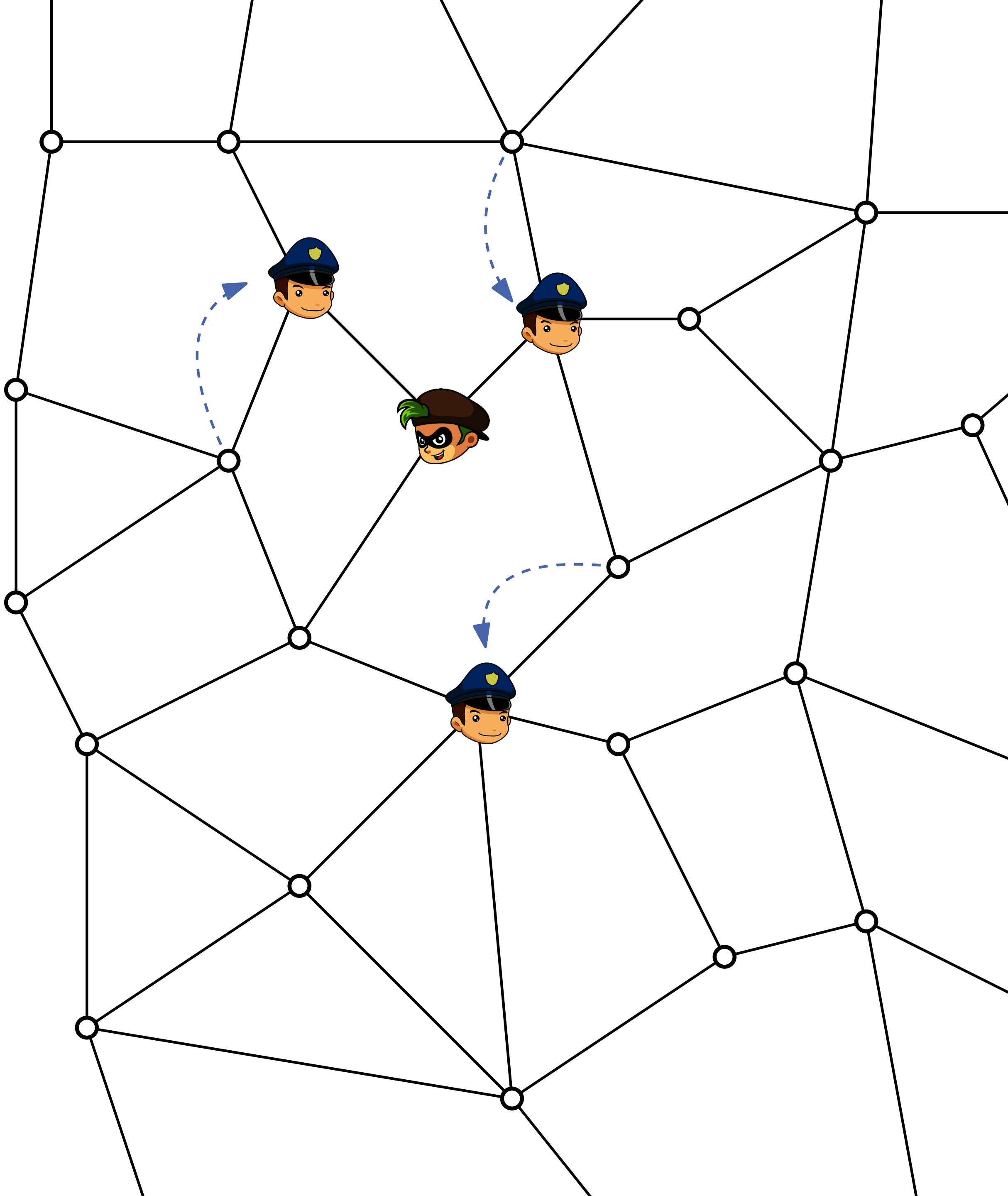
Cops and Robber

2-Players: k Cops 

1 Robber 

Rules:

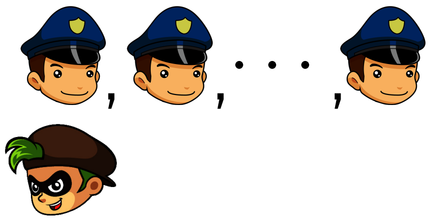
- Cops go first.
- Robber is second.
- Moves are between adjacent vertices.



Cops and Robber

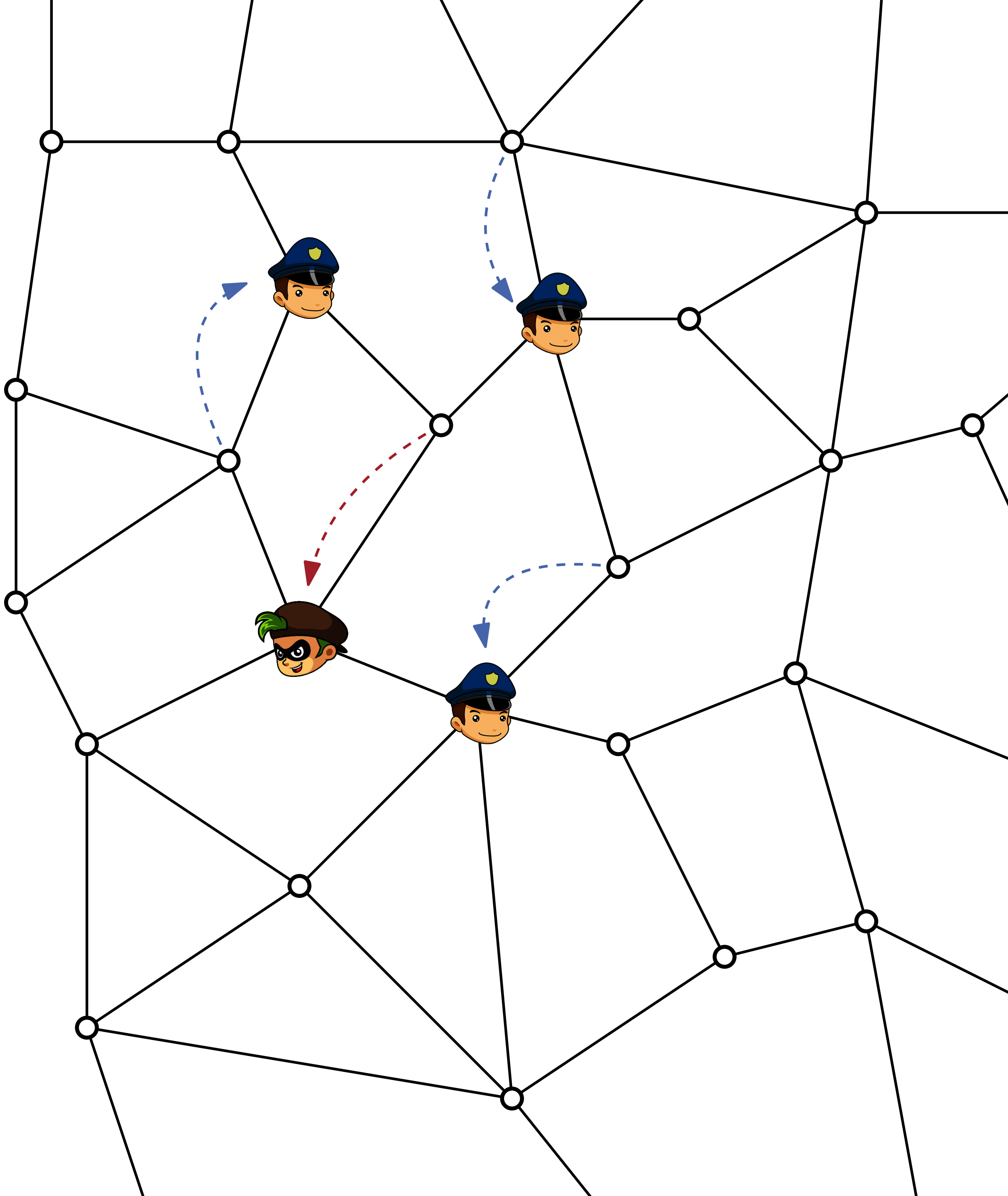
2-Players: k Cops

1 Robber

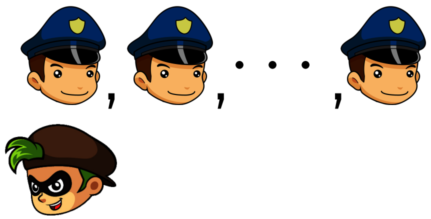


Rules:

- Cops go first.
- Robber is second.
- Moves are between adjacent vertices.
- Robber tries to flee indefinitely.



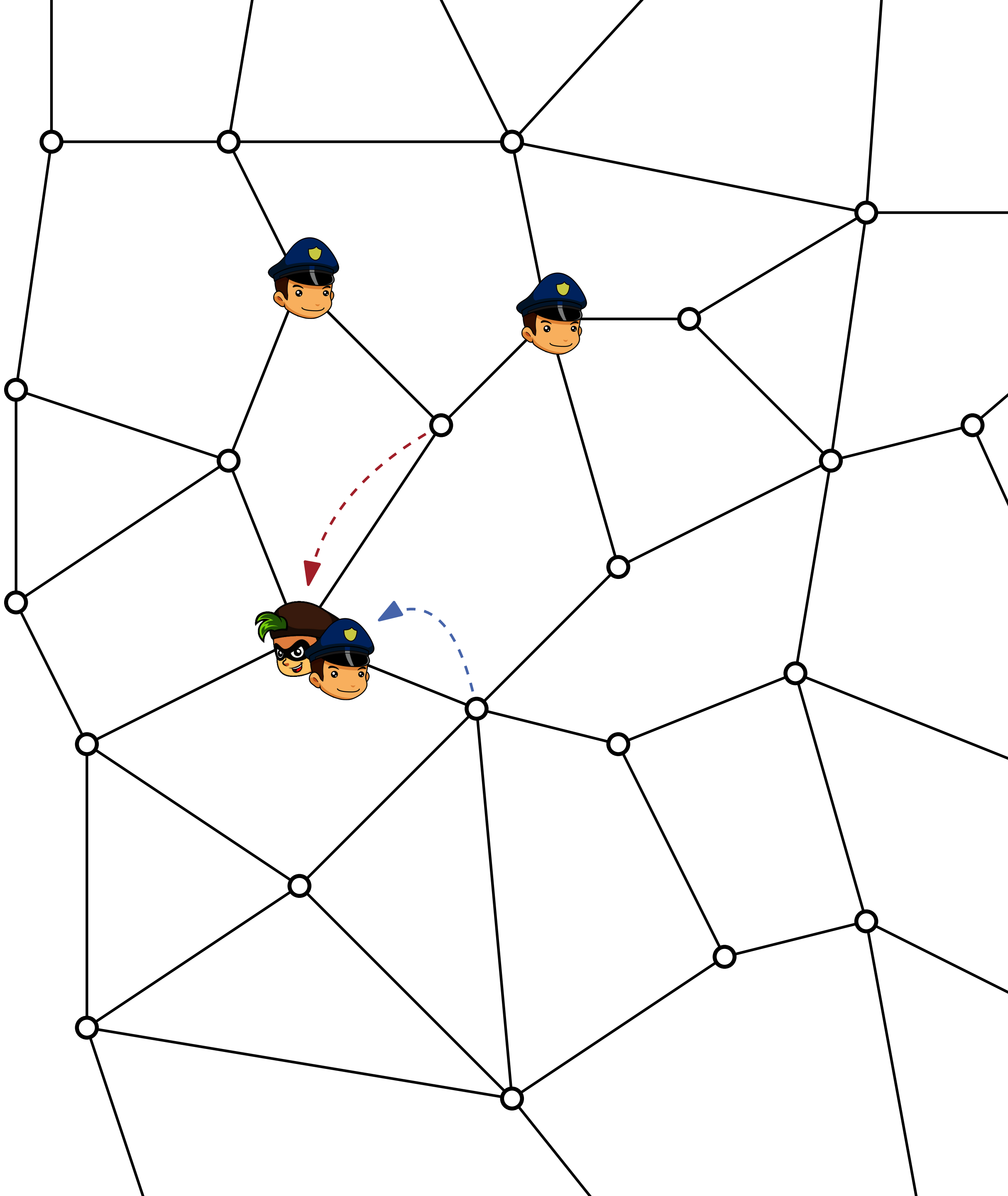
Cops and Robber

2-Players: k Cops 

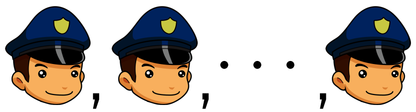
1 Robber

Rules:

- Cops go first.
- Robber is second.
- Moves are between adjacent vertices.
- Robber tries to flee indefinitely.
- Cops win by capturing the robber.



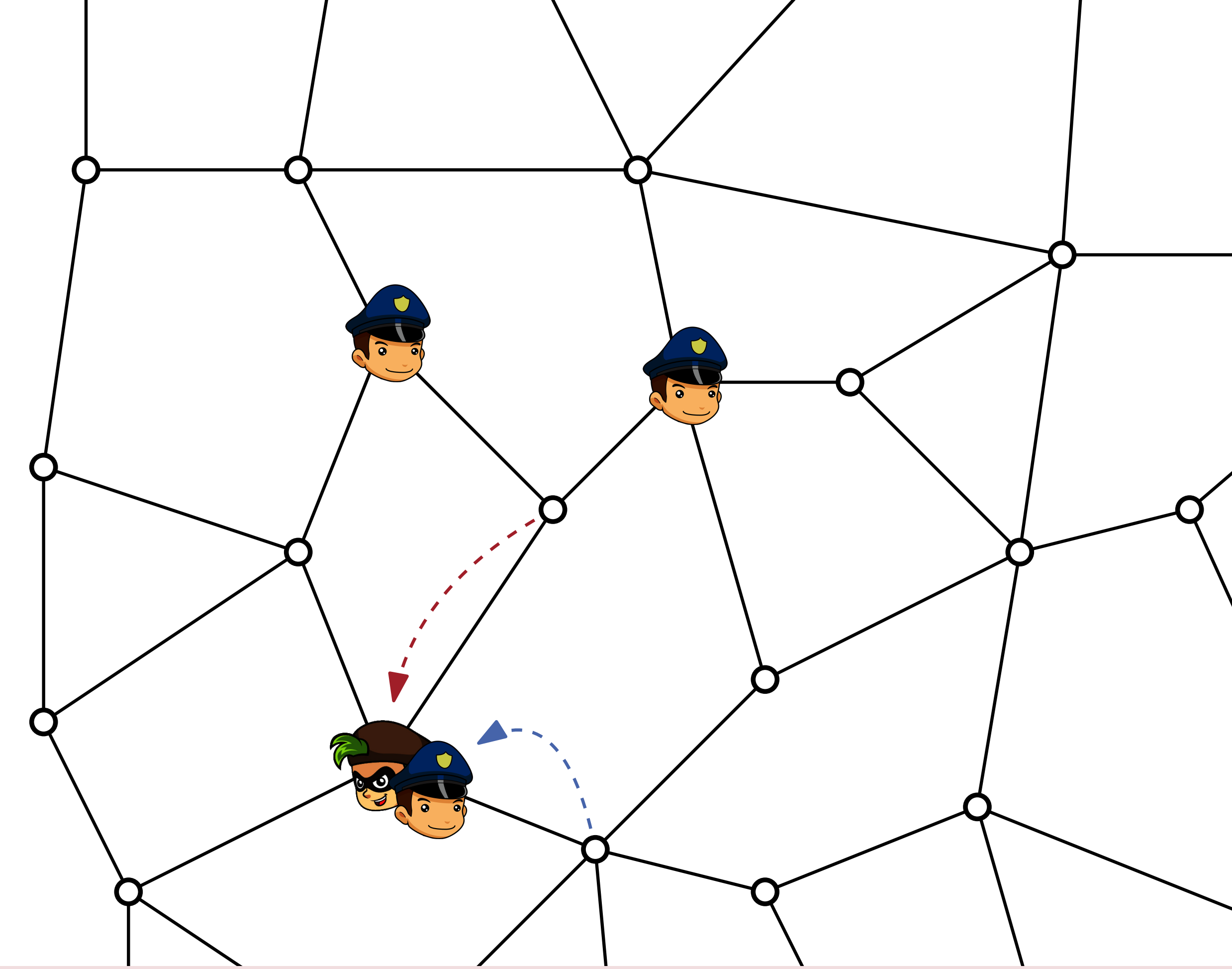
Cops and Robber

2-Players: k Cops 

1 Robber 

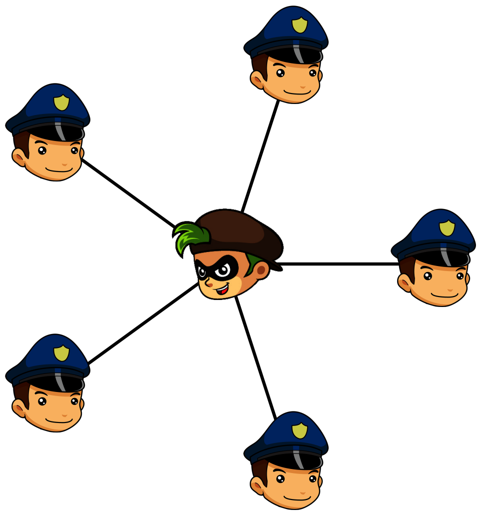
Rules:

- Cops go first.
- Robber is second.
- Moves are between adjacent vertices.
- Robber tries to flee indefinitely.
- Cops win by capturing the robber.



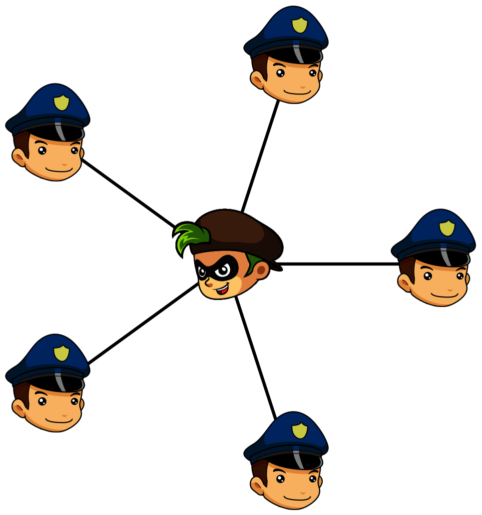
Cop number $c(G)$:
How many cops are necessary to capture the robber?

Surrounding Variants

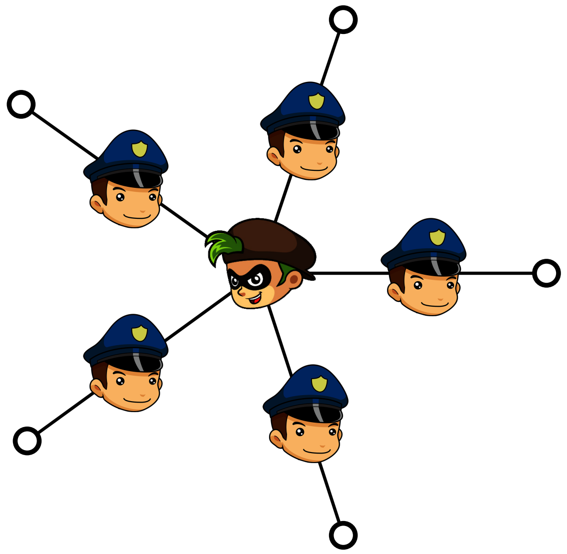


vertex surround

Surrounding Variants

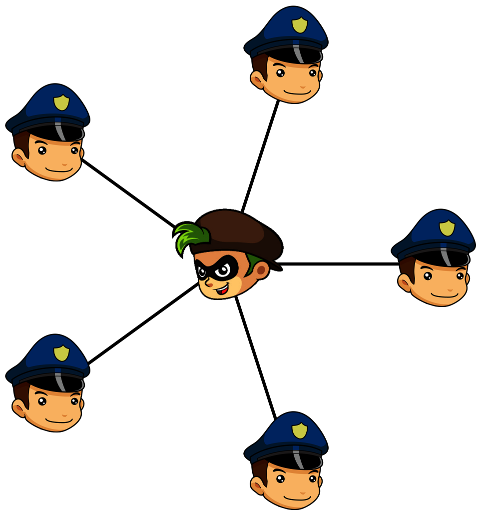


vertex surround

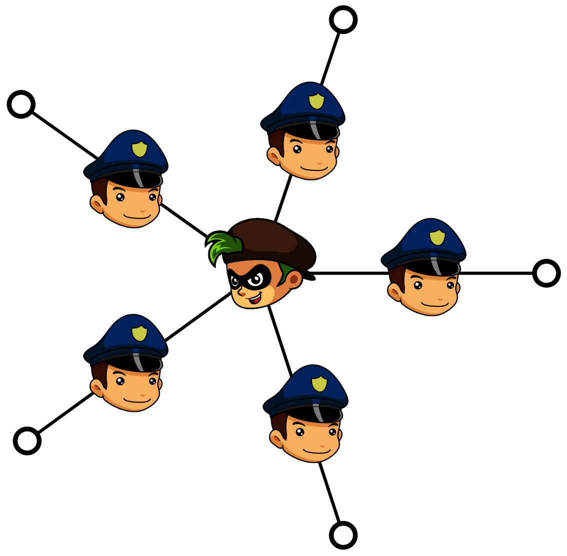


edge surround

Surrounding Variants



vertex surround

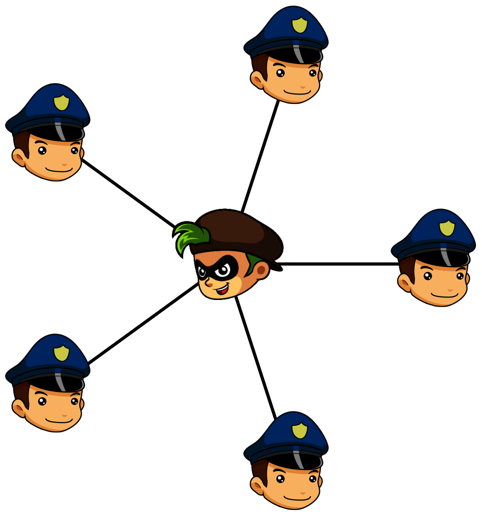


edge surround

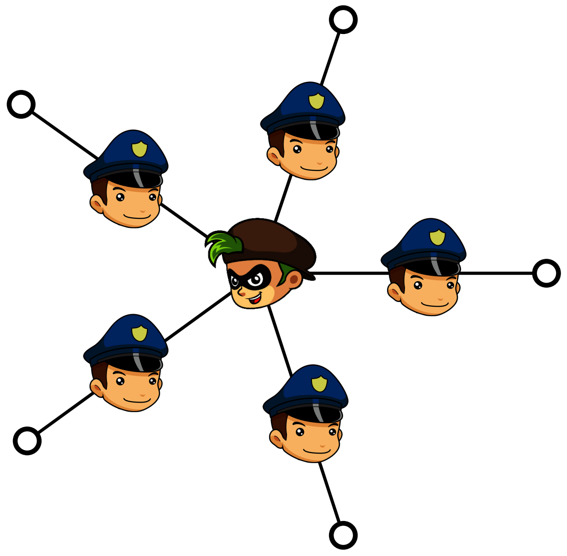
$\mathbf{c_V(G)}$ $\mathbf{c_E(G)}$

How many cops are necessary to vertex/edge surround the robber?

Surrounding Variants



vertex surround



edge surround

$c_V(G)$ $c_E(G)$

How many cops are necessary to vertex/edge surround the robber?

Restrictive variants:

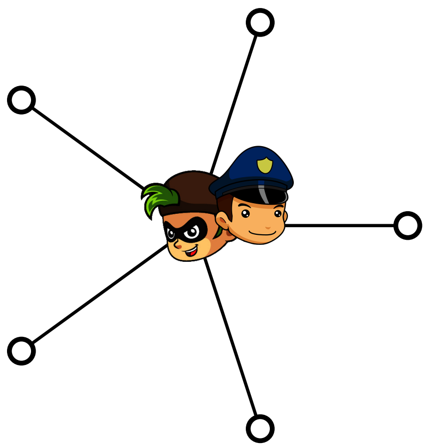
$c_{V,r}(G)$:

Robber must not end his move on a cop.

$c_{E,r}(G)$:

Robber must not move through a cop.

A Little Bit of History



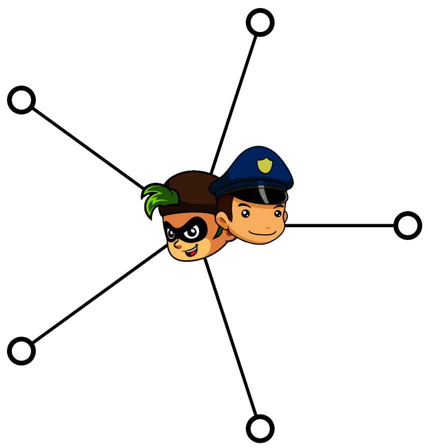
$\mathbf{c(G)}$:

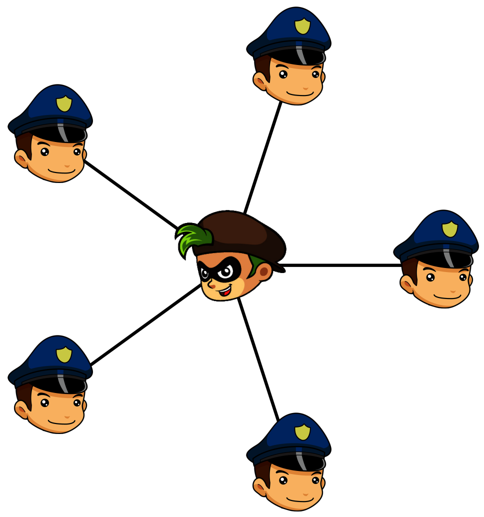
Quilliot ’78

Nowakowski, Winkler ’83

Aigner, Fromme ’84

A Little Bit of History





$c(G)$:

Quilliot '78

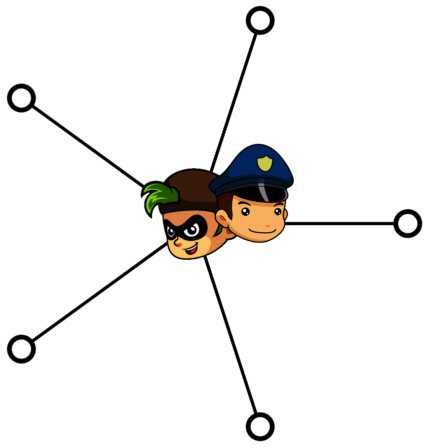
Nowakowski, Winkler '83

Aigner, Fromme '84

$c_{V,r}(G)$:

Burgess et al. '20

A Little Bit of History

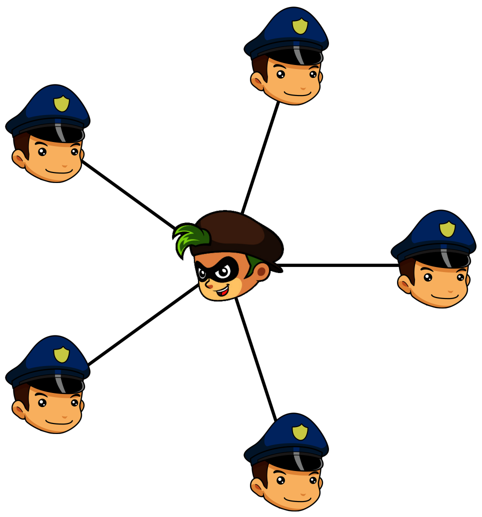


$\mathbf{c(G)}$:

Quilliot '78

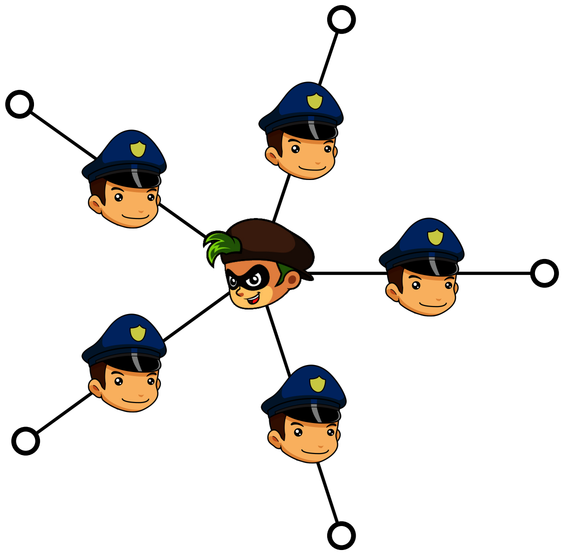
Nowakowski, Winkler '83

Aigner, Fromme '84



$\mathbf{c_{V,r}(G)}$:

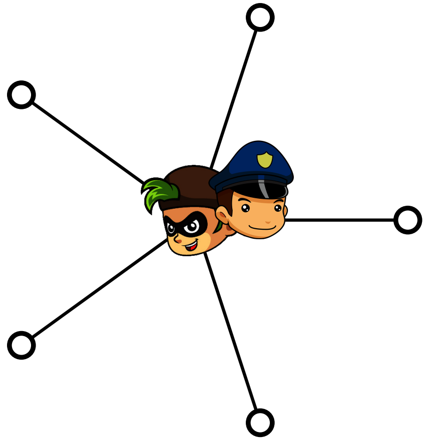
Burgess et al. '20



$\mathbf{c_{E,r}(G)}$:

Crytser et al. '20

A Little Bit of History

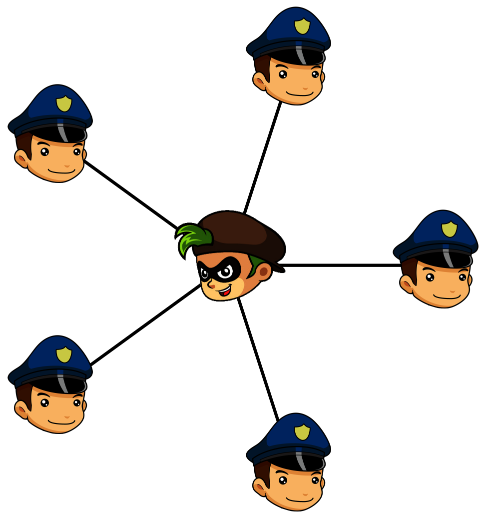


$c(G)$:

Quilliot '78

Nowakowski, Winkler '83

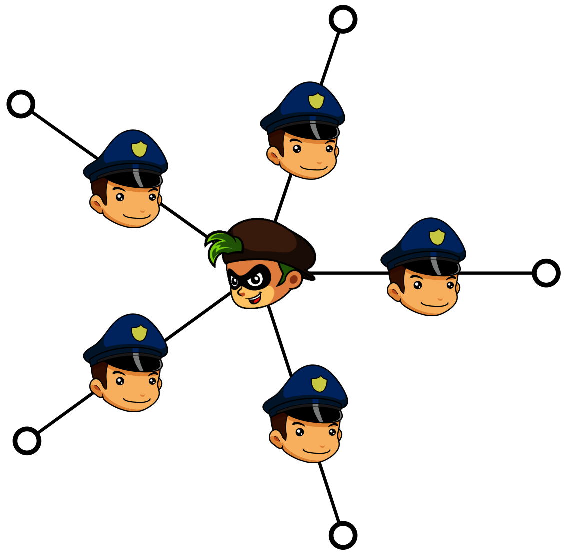
Aigner, Fromme '84



$c_{V,r}(G)$:

Burgess et al. '20

$c_V(G)$: new



$c_{E,r}(G)$:

Crytser et al. '20

$c_E(G)$: new

Our Results

Unification

of the four surrounding variants

Theorem: (informal)
Strategy for k cops in some variant gives a strategy for $\leqslant k \cdot 2\Delta$ cops in any other variant.

max. degree

Our Results

Unification

of the four surrounding variants

Theorem: (informal)
Strategy for k cops in some variant gives a strategy for $\leqslant k \cdot 2\Delta$ cops in any other variant.

max. degree

Separation

from classical variant

Conjecture: (Crytser et al.)
$c_{E,r}(G) \leqslant c(G) \cdot \Delta(G)$

Our Results

Unification

of the four surrounding variants

Theorem: (informal)
Strategy for k cops in some variant gives a strategy for $\leqslant k \cdot 2\Delta$ cops in any other variant.

max. degree

Separation

from classical variant

Conjecture: (Crytser et al.)
$c_{E,r}(G) \leqslant c(G) \cdot \Delta(G)$

Wrong!

Theorem:
There are infinitely many G with:

- $c(G) = 2$ and $\Delta(G) = 3$
- $c_V(G)$, $c_{V,r}(G)$, $c_E(G)$ and $c_{E,r}(G)$ are unbounded

Unifying the Surrounding Variants

Upper bounds: Simulation

	not restricted	restricted
vertices	$c_V(G)$	$c_{V,r}(G)$
edges	$c_E(G)$	$c_{E,r}(G)$

Unifying the Surrounding Variants

Upper bounds: Simulation

	 not restricted	 restricted
 vertices	$c_V(G)$	$c_{V,r}(G)$
 edges	$c_E(G)$	$c_{E,r}(G)$

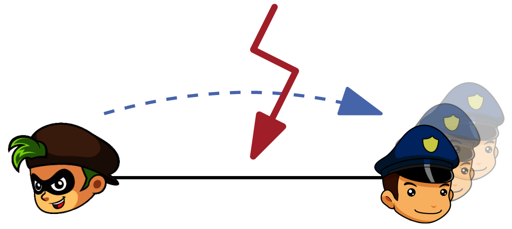
A restricted robber is a weaker robber.

Unifying the Surrounding Variants

Upper bounds: Simulation

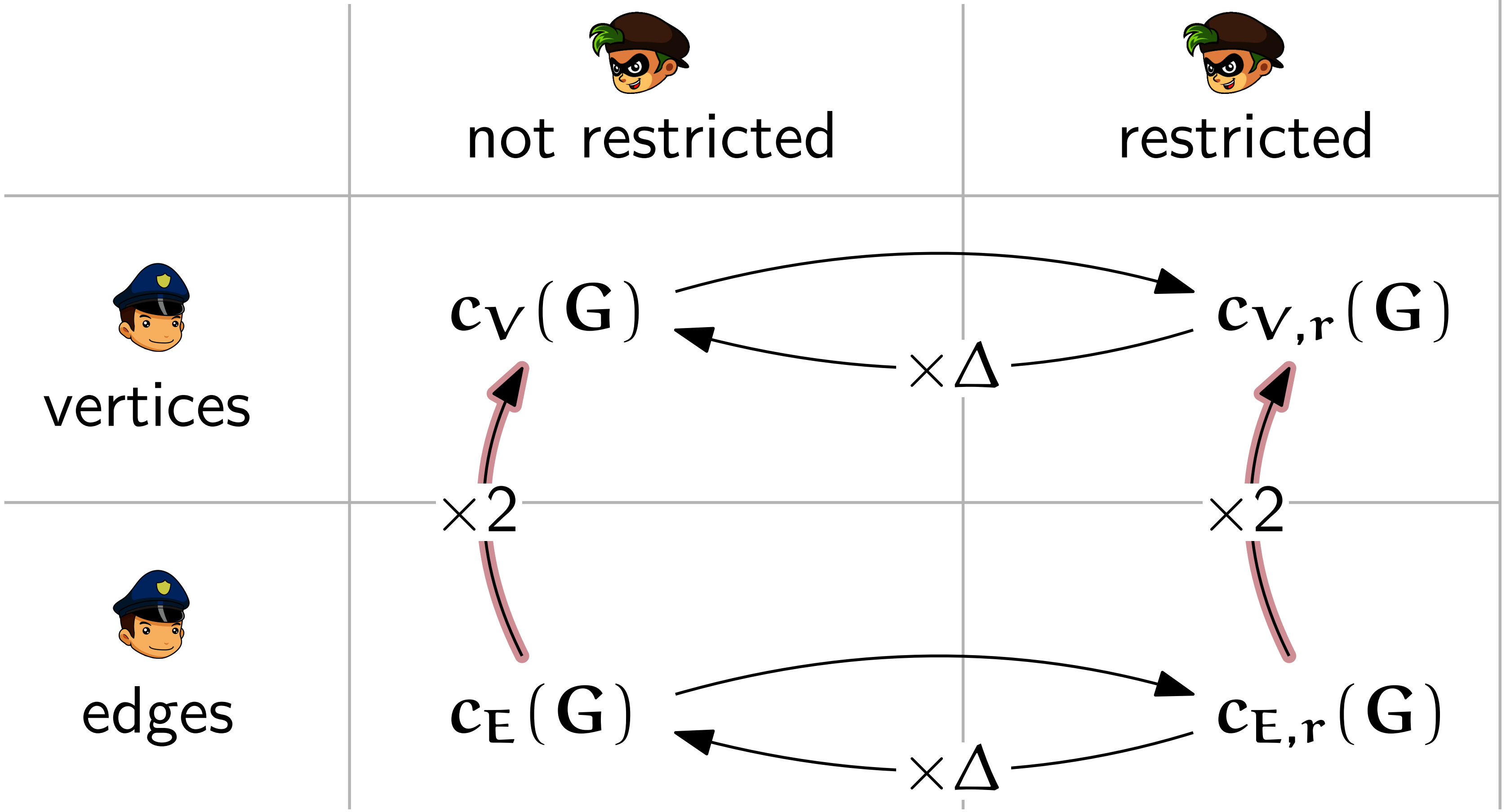
	not restricted		restricted
vertices	$c_V(G)$	$\rightarrow$ / $\leftarrow \times\Delta$	$c_{V,r}(G)$
edges	$c_E(G)$	$\rightarrow$ / $\leftarrow \times\Delta$	$c_{E,r}(G)$

Replace each cop by a group of $\Delta(G)$ cops.

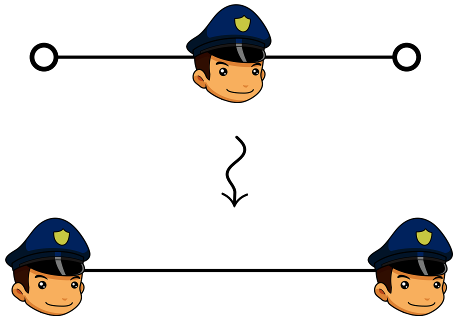


Unifying the Surrounding Variants

Upper bounds: Simulation

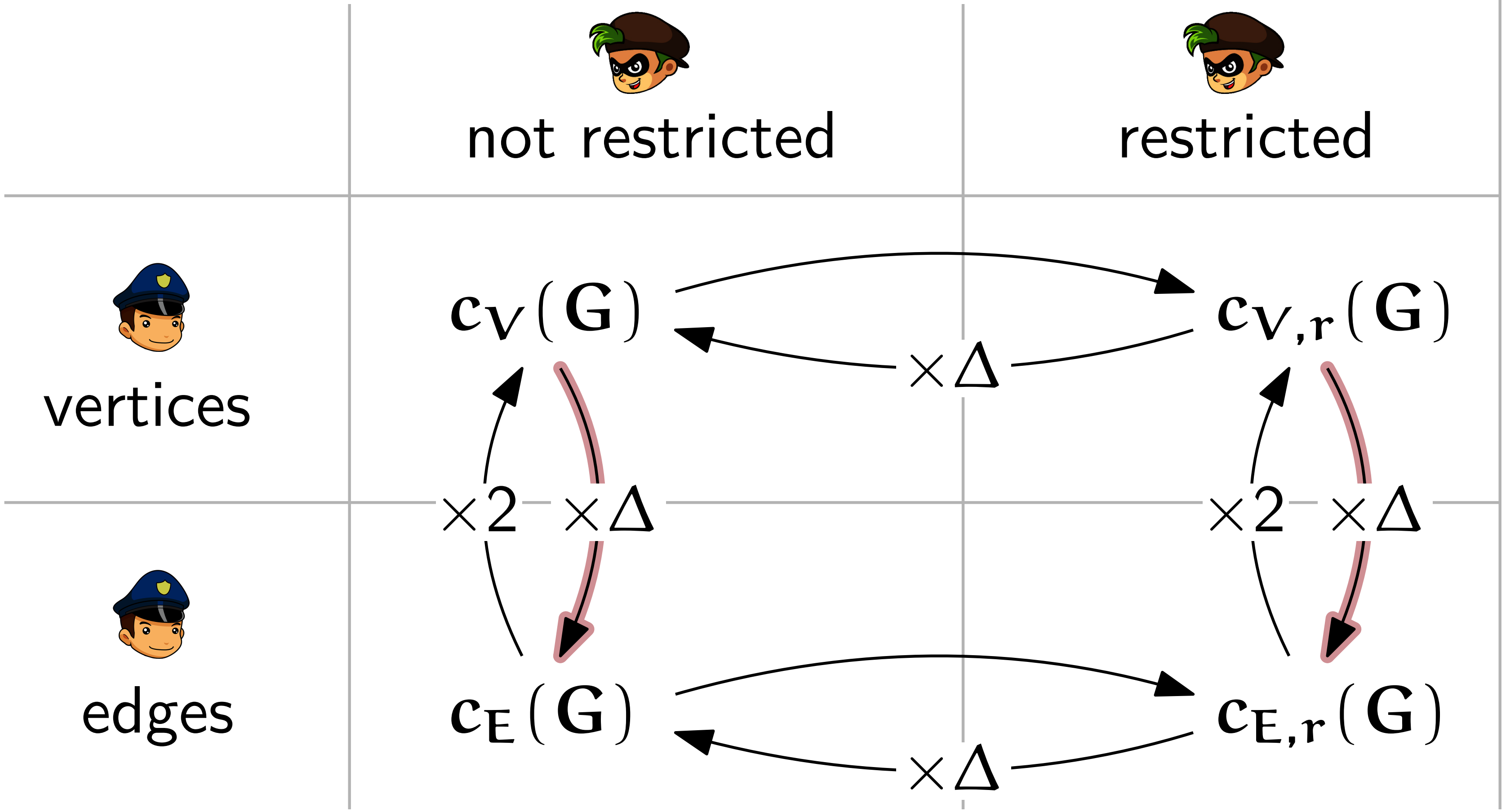


Replace edge cop by two vertex cops.

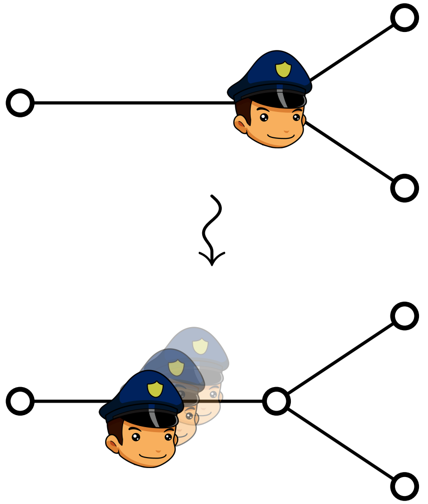


Unifying the Surrounding Variants

Upper bounds: Simulation



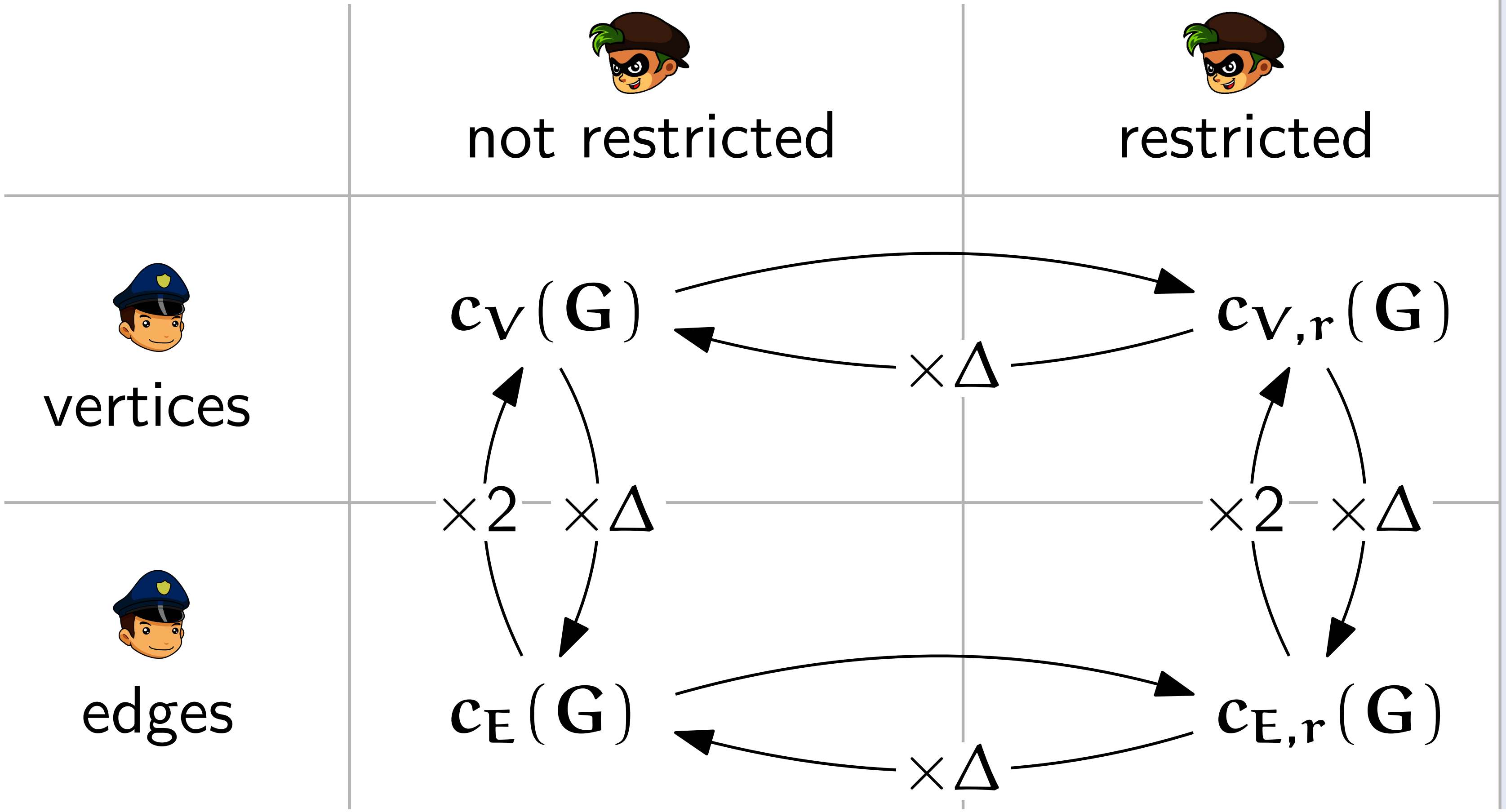
Replace vertex cop by a group of $\Delta(G)$ edge cops.



follow original cop

Unifying the Surrounding Variants

Upper bounds: Simulation



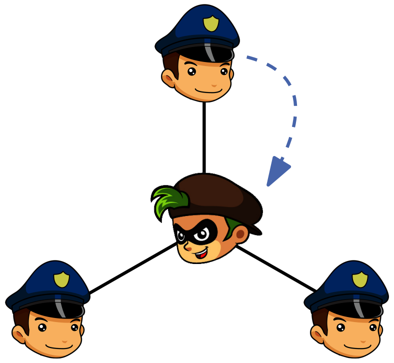
Lower bounds: Constructions

Tight examples for all claimed inequalities:

- complete (bipartite) graphs
- regular graphs (with “leaves”)
- based on “MOLS” (mutually orthogonal Latin squares)
- line graphs of complete graphs

Separating “Capturing” and “Surrounding”

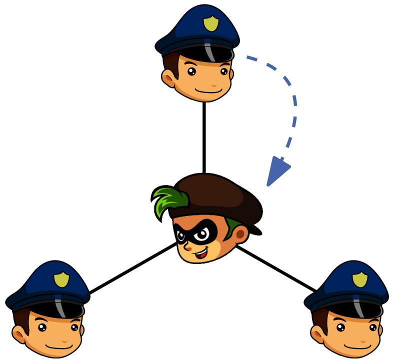
Observation:
surrounding $\Longrightarrow$ capturing
$\rightsquigarrow c_{X(,r)}(G) \geqslant c(G)$



Question: other direction?

Separating “Capturing” and “Surrounding”

Observation:
surrounding $\implies$ capturing
$\rightsquigarrow c_{X(,r)}(G) \geqslant c(G)$

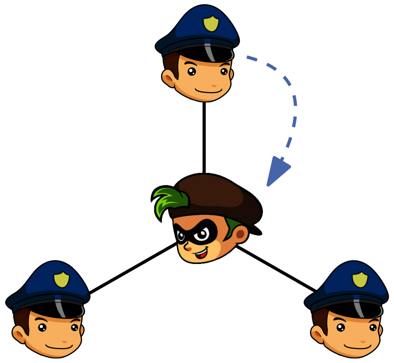


Question: other direction?

Conjecture: (Crytser et al.)
$c_{E,r}(G) \leqslant c(G) \cdot \Delta(G)$

Separating "Capturing" and "Surrounding"

Observation:
surrounding $\implies$ capturing
$\rightsquigarrow c_{X(,r)}(G) \geqslant c(G)$



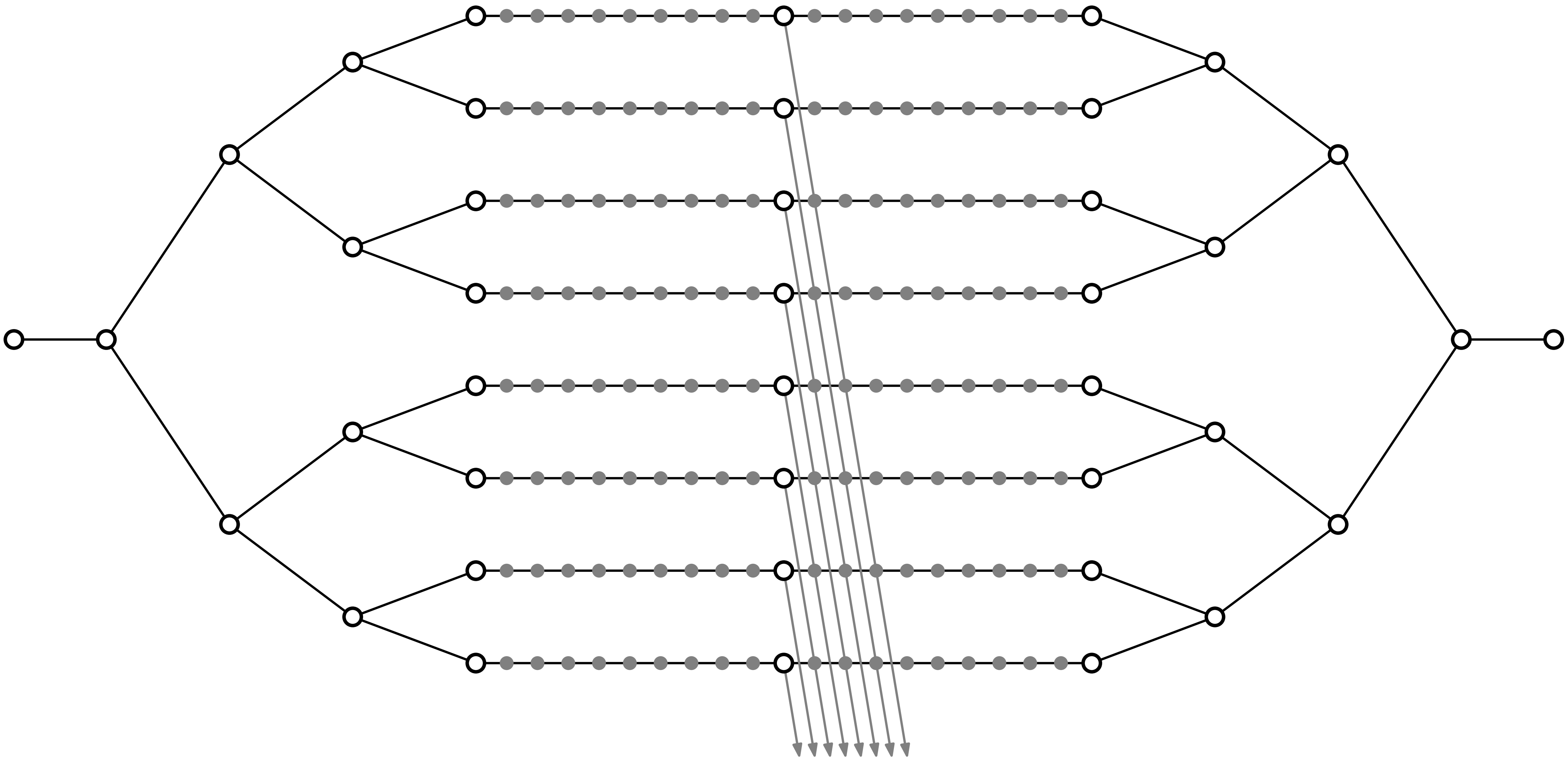
Question: other direction?

Theorem:
There are infinitely many G with:

- $c(G) = 2$ and $\Delta(G) = 3$
- $c_V(G)$, $c_{V,r}(G)$, $c_E(G)$ and $c_{E,r}(G)$ are unbounded

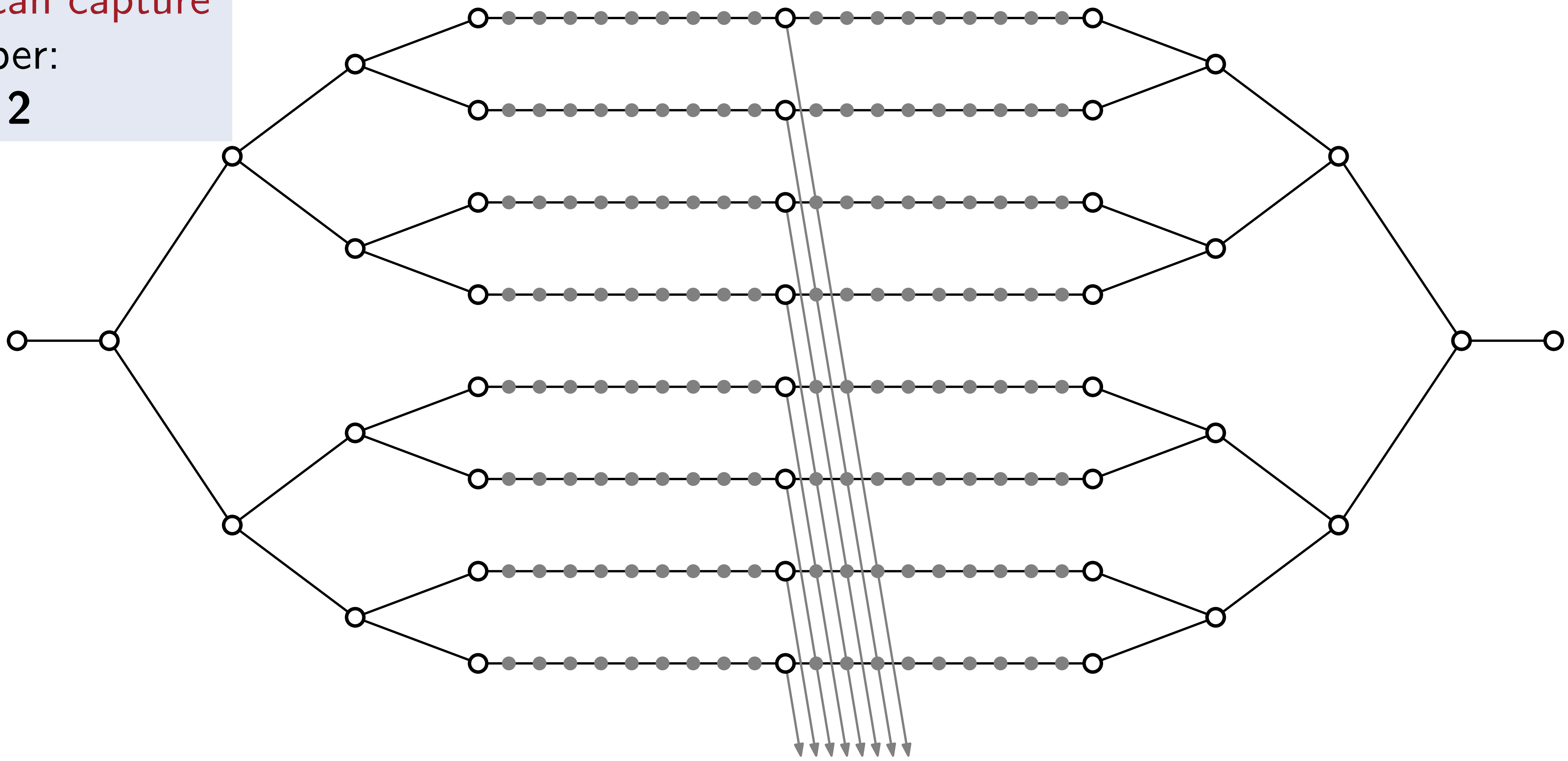
Conjecture: (Crytser et al.)
$c_{E,r}(G) \leqslant c(G) \cdot \Delta(G)$

Separating “Capturing” and “Surrounding”



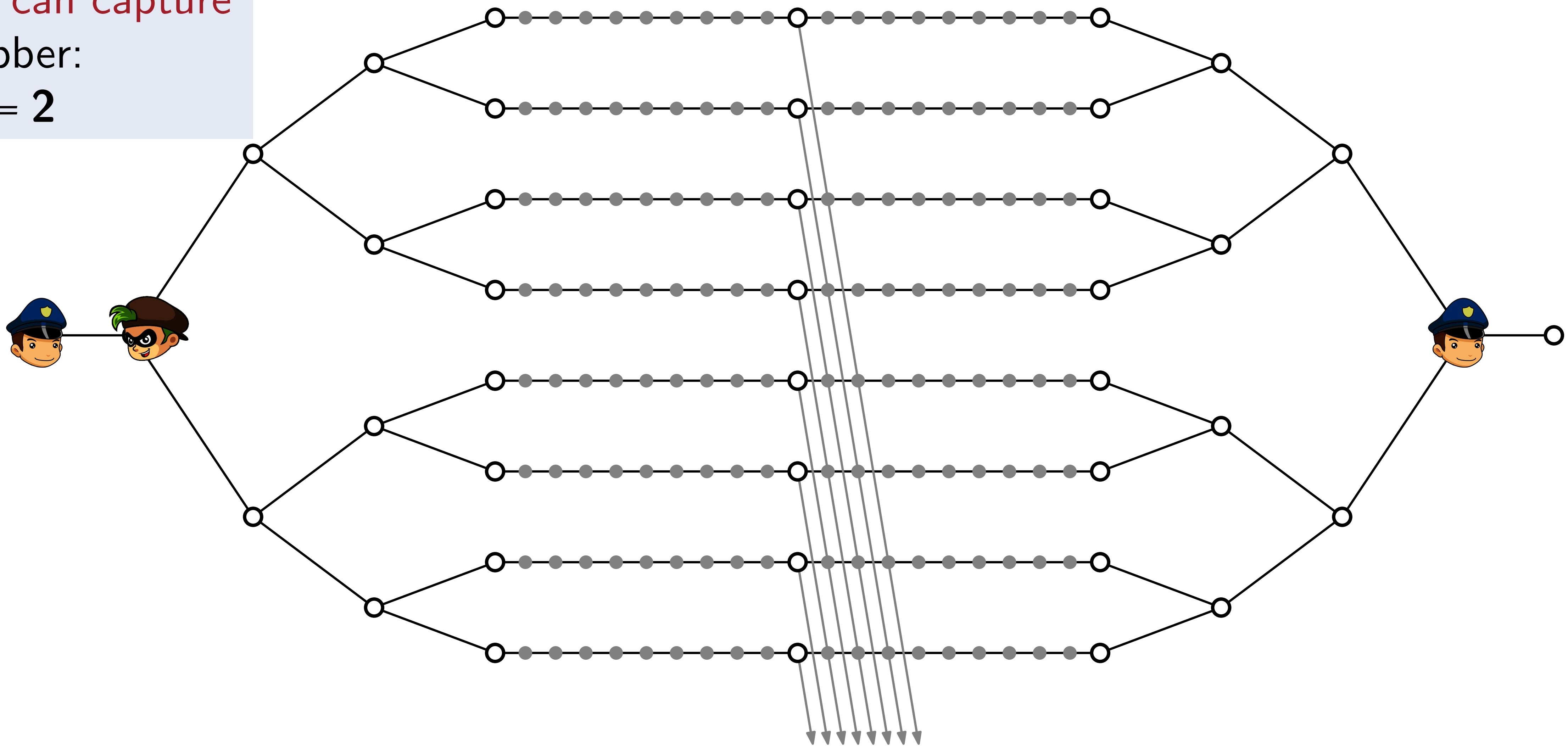
Separating “Capturing” and “Surrounding”

2 cops can capture the robber:
$\mathbf{c(G) = 2}$



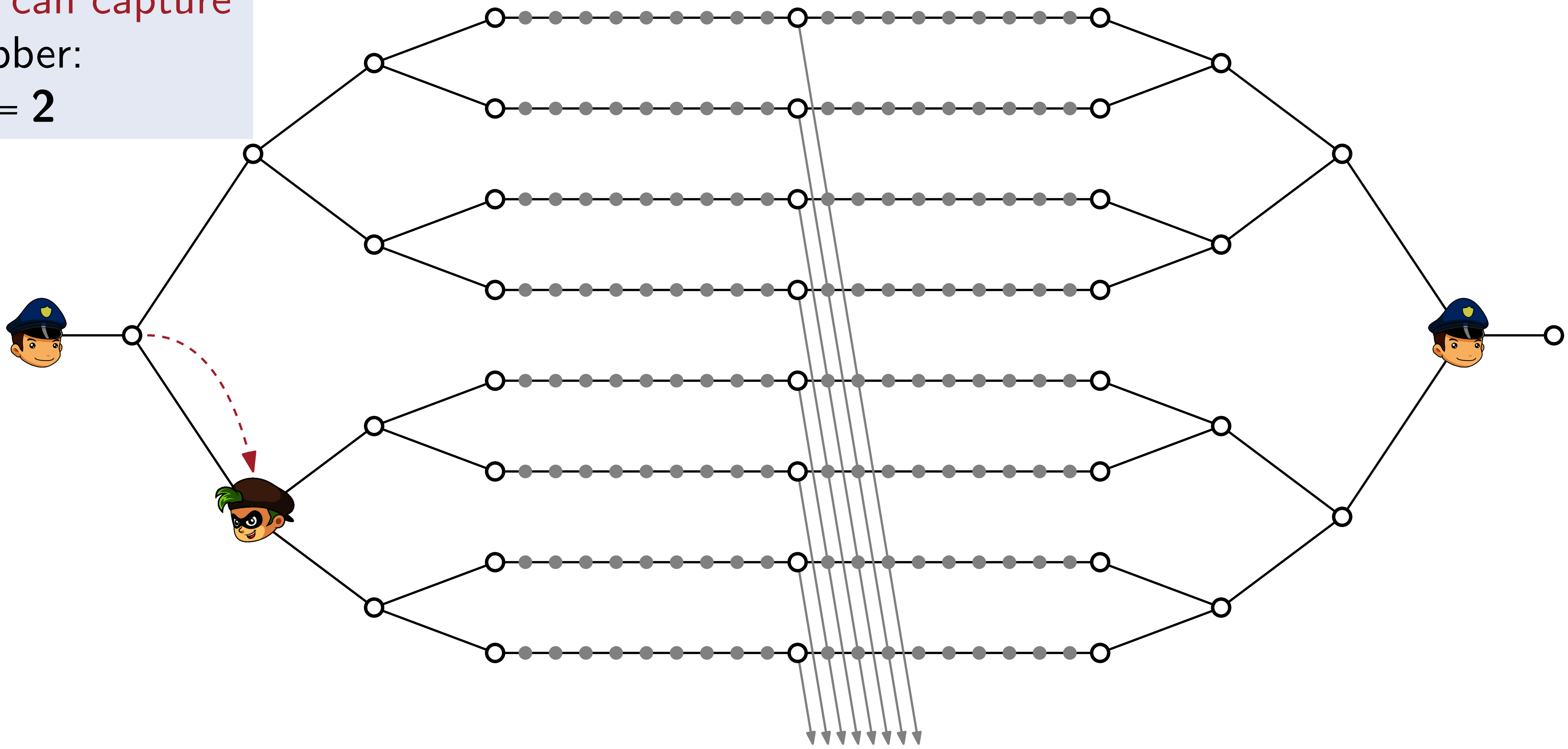
Separating "Capturing" and "Surrounding"

2 cops can capture the robber:
$\mathbf{c(G) = 2}$



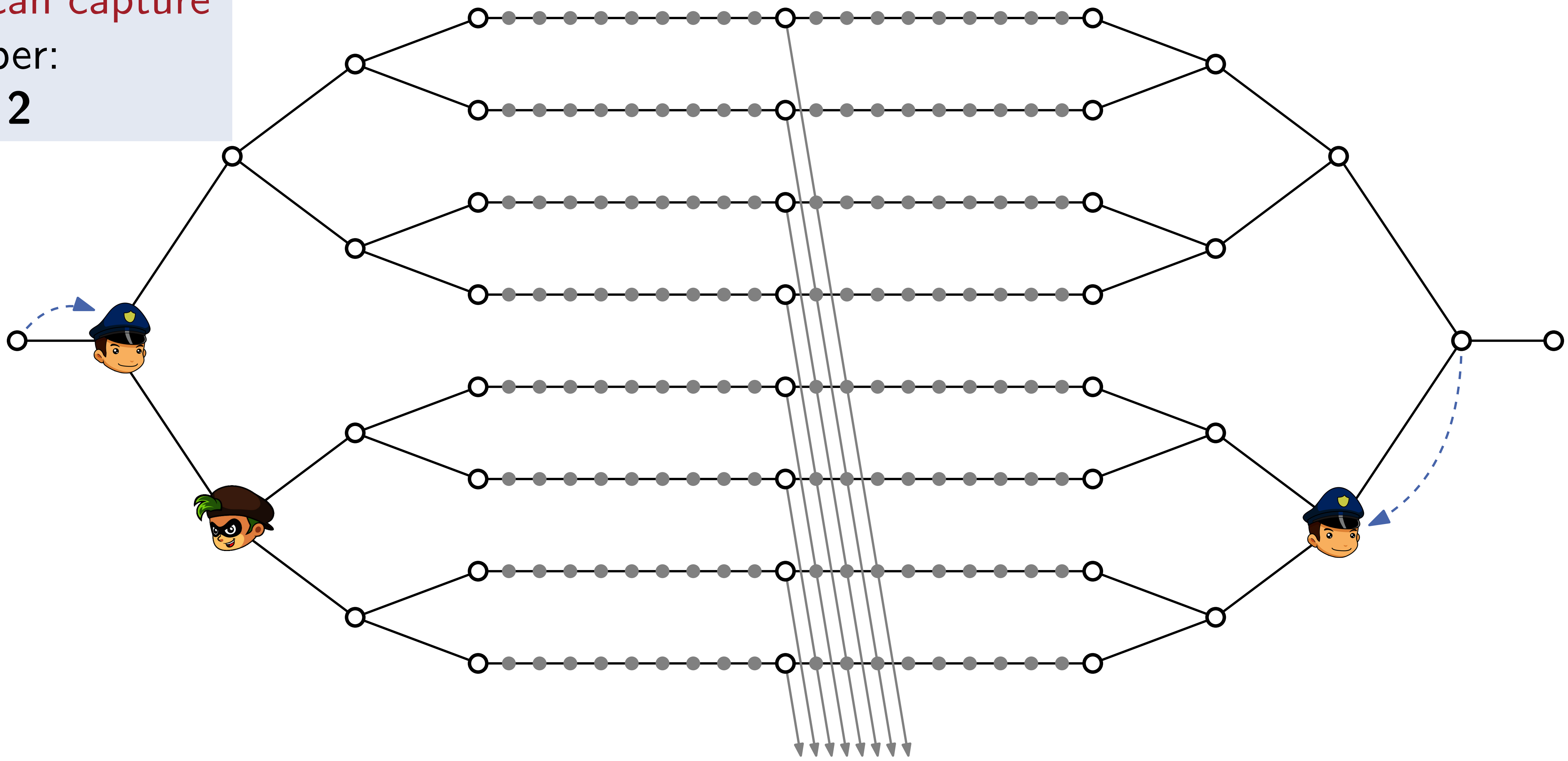
Separating "Capturing" and "Surrounding"

2 cops can capture the robber:
$\mathbf{c(G) = 2}$



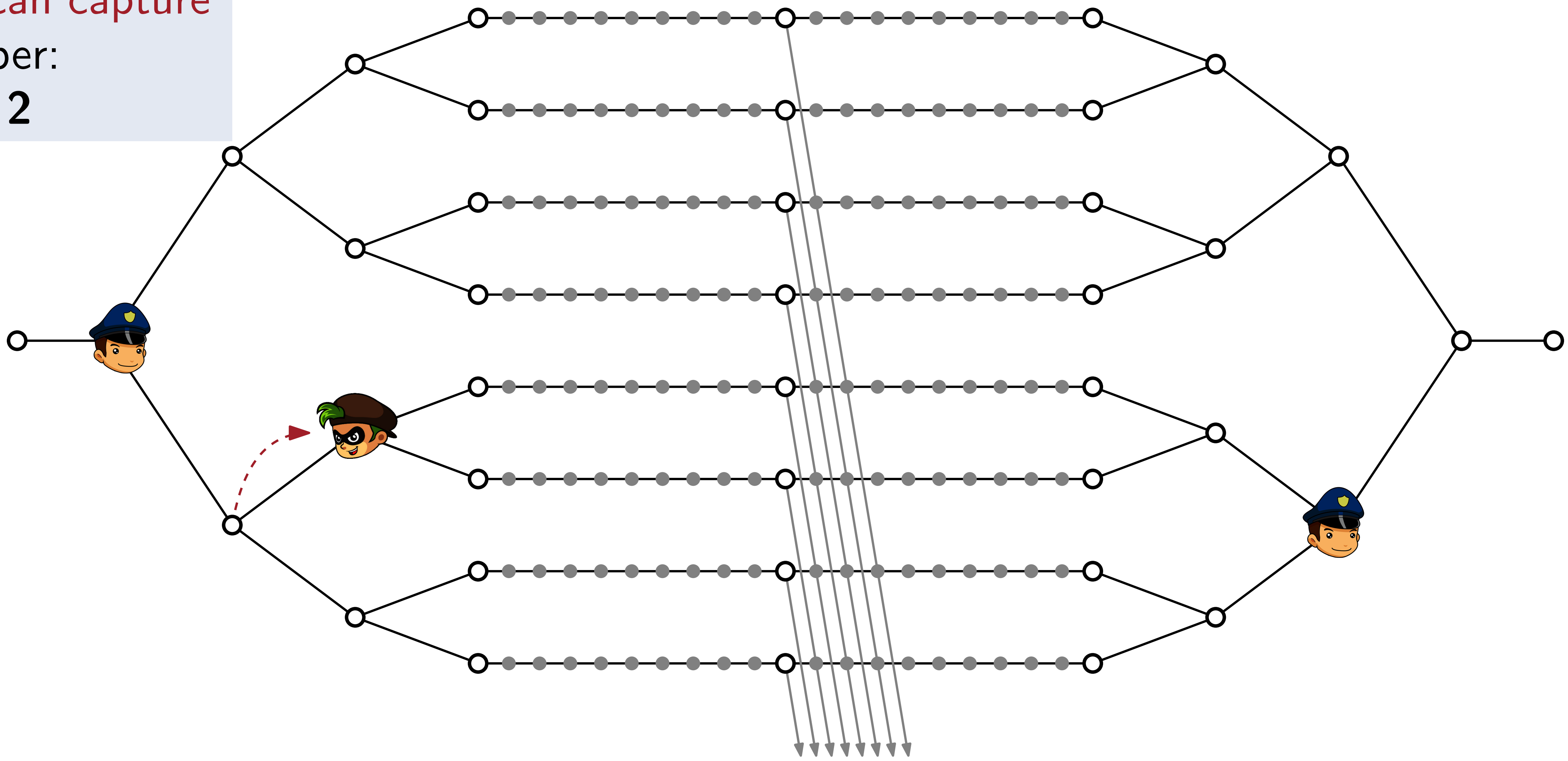
Separating “Capturing” and “Surrounding”

2 cops can capture the robber:
$\mathbf{c}(\mathbf{G}) = \mathbf{2}$



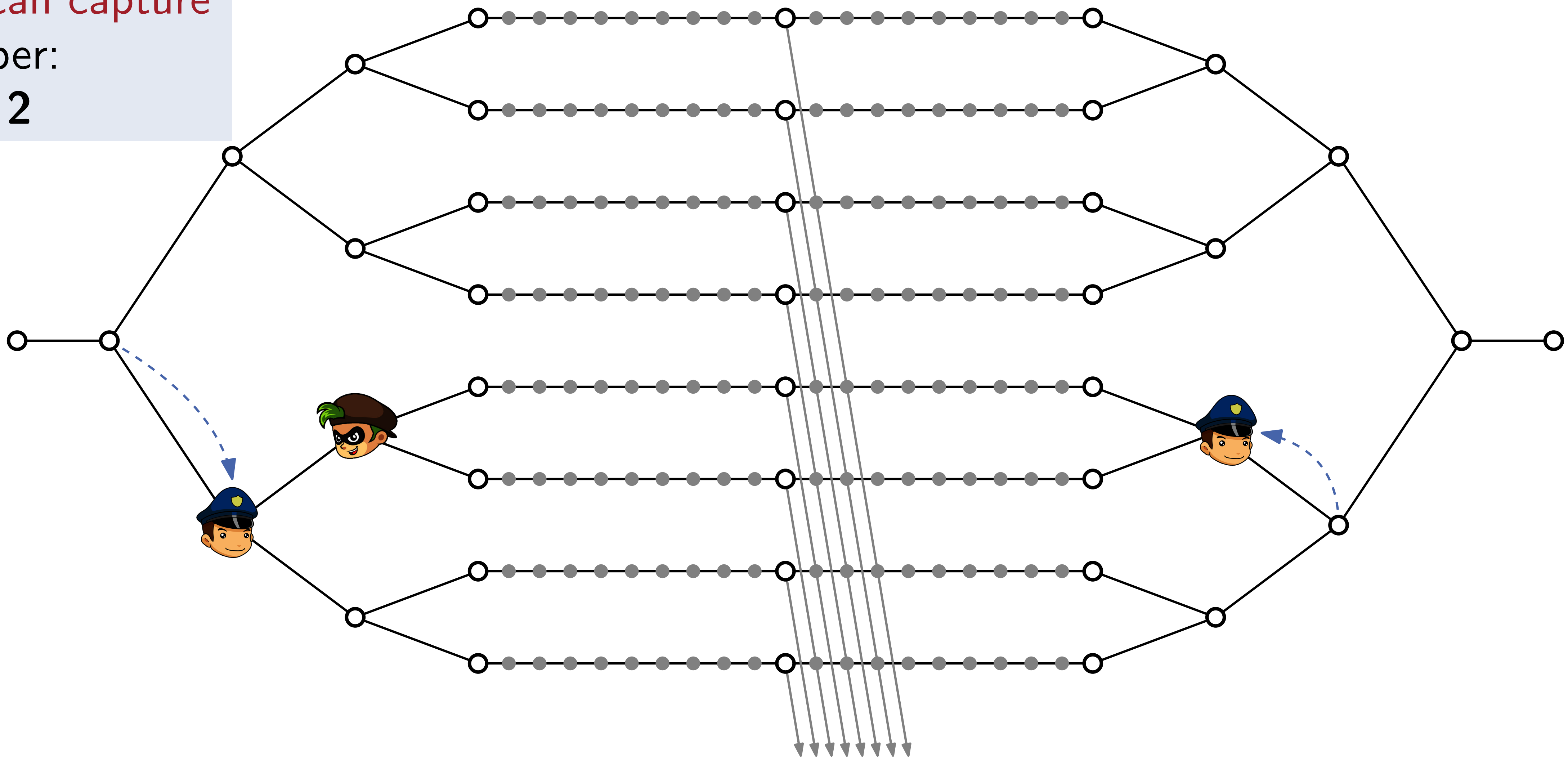
Separating “Capturing” and “Surrounding”

2 cops can capture the robber:
$\mathbf{c}(\mathbf{G}) = \mathbf{2}$



Separating “Capturing” and “Surrounding”

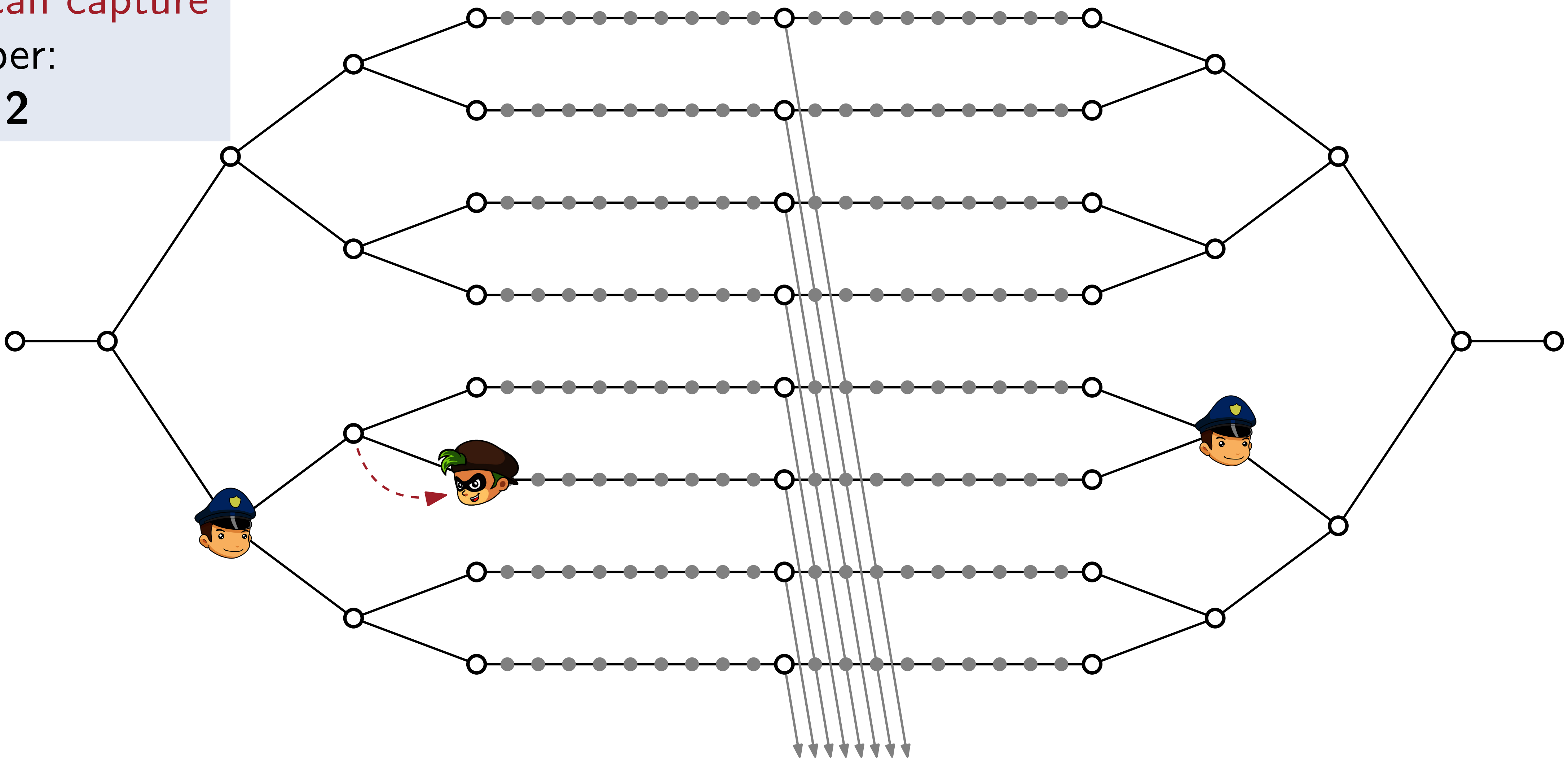
2 cops can capture
the robber:
$\mathbf{c}(\mathbf{G}) = \mathbf{2}$



Separating "Capturing" and "Surrounding"

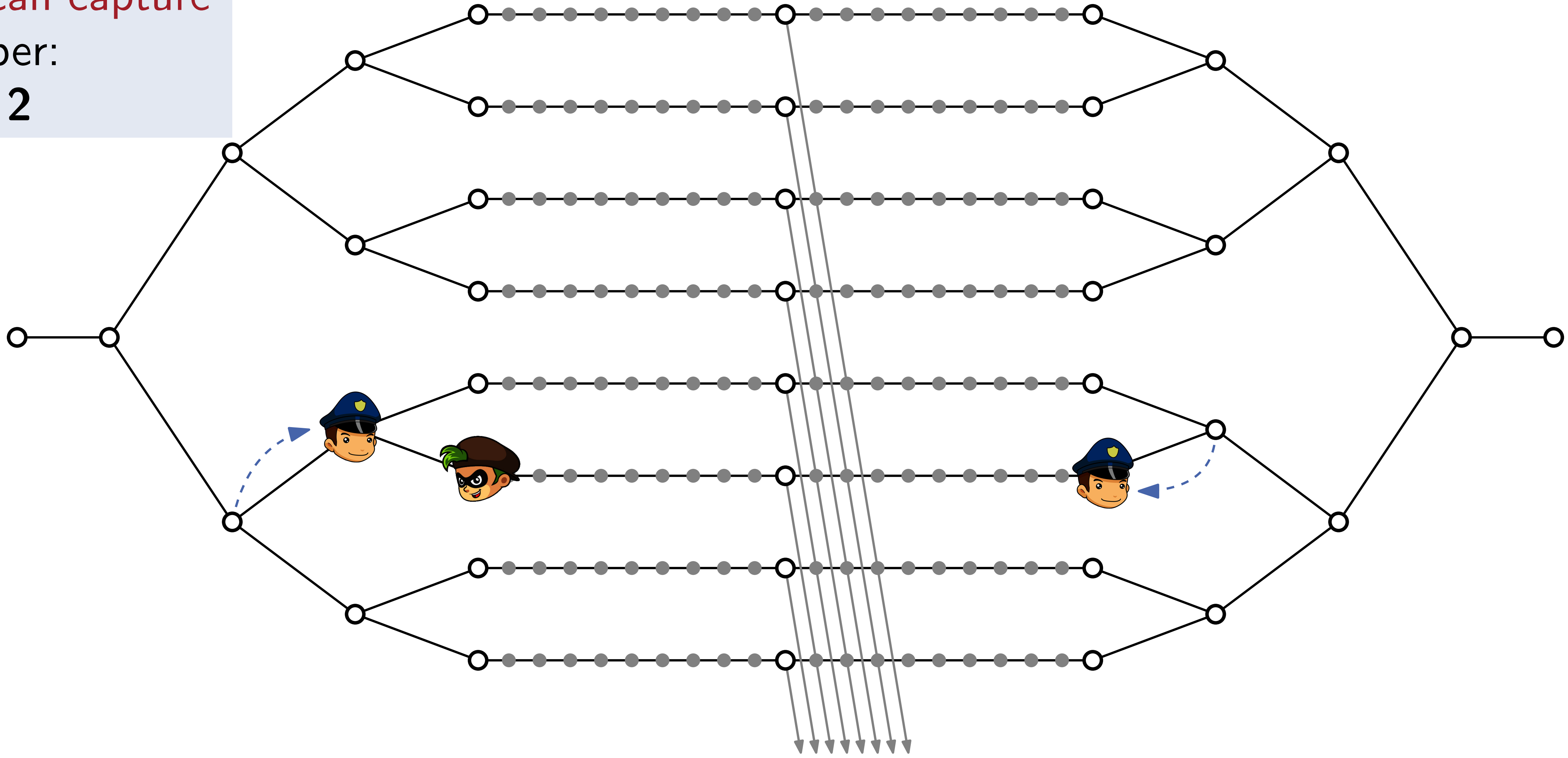
2 cops can capture the robber:

$\mathbf{c}(\mathbf{G}) = \mathbf{2}$



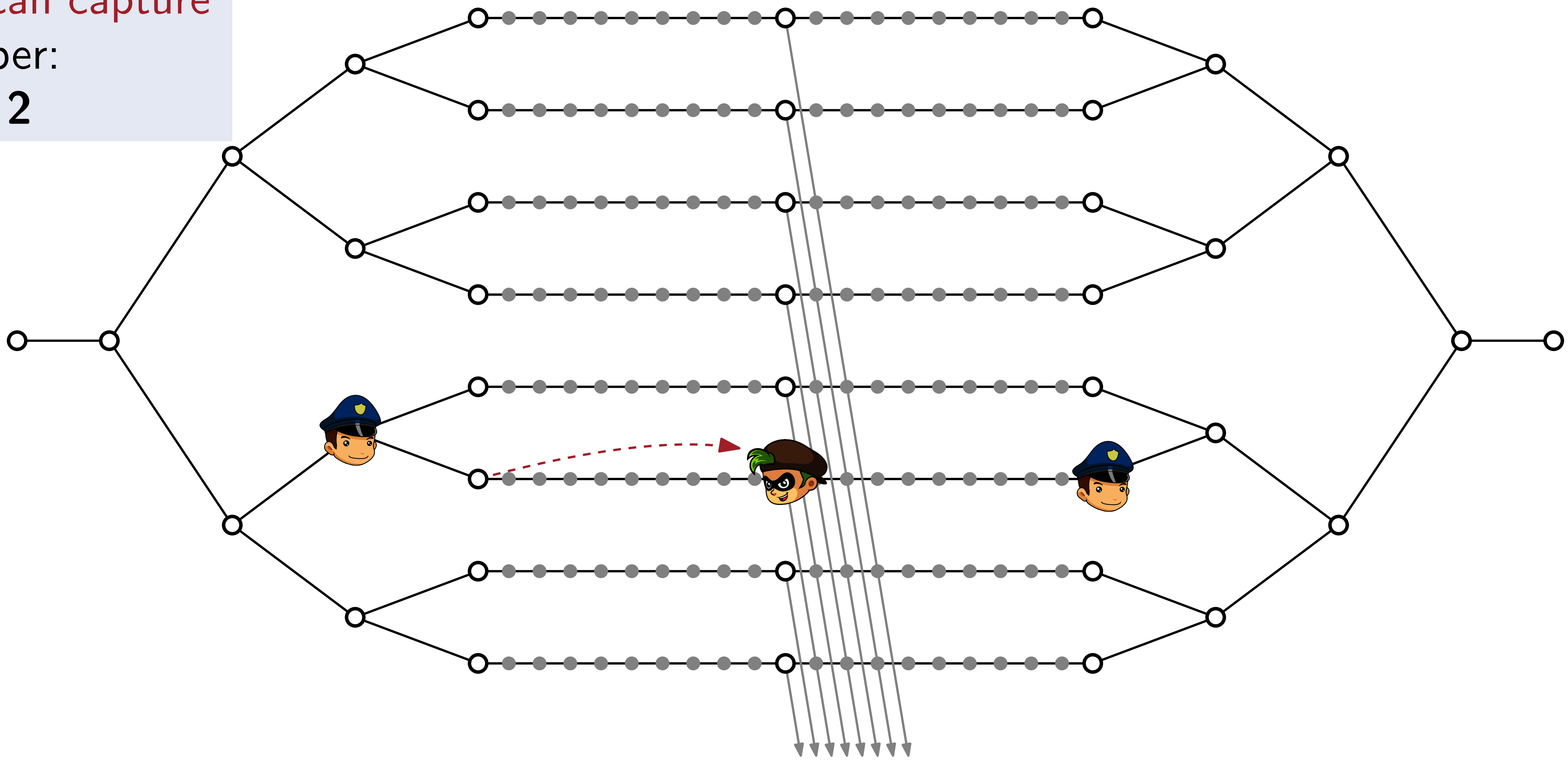
Separating "Capturing" and "Surrounding"

2 cops can capture the robber:
$\mathbf{c(G) = 2}$



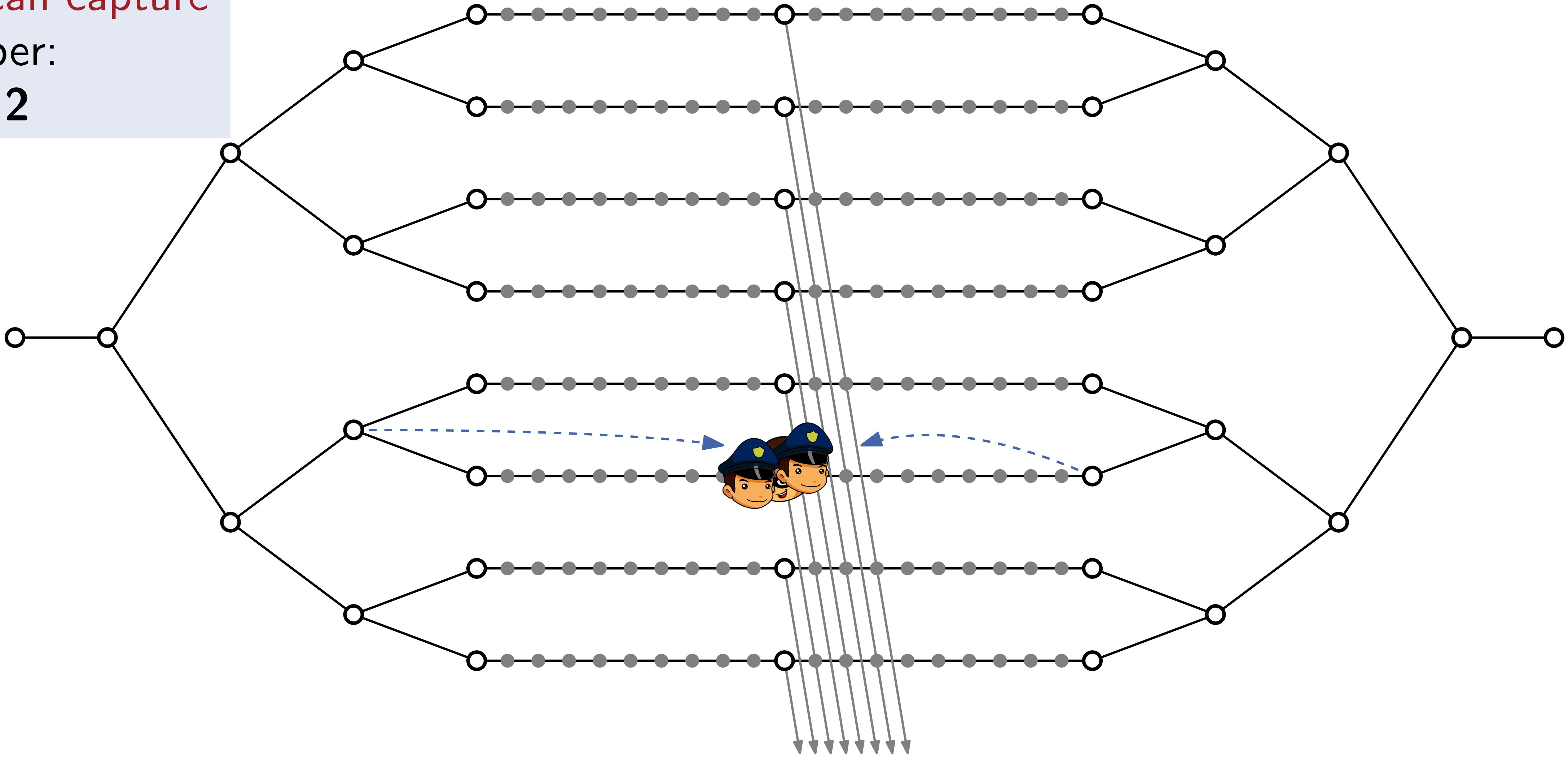
Separating “Capturing” and “Surrounding”

2 cops can capture the robber:
$\mathbf{c(G) = 2}$



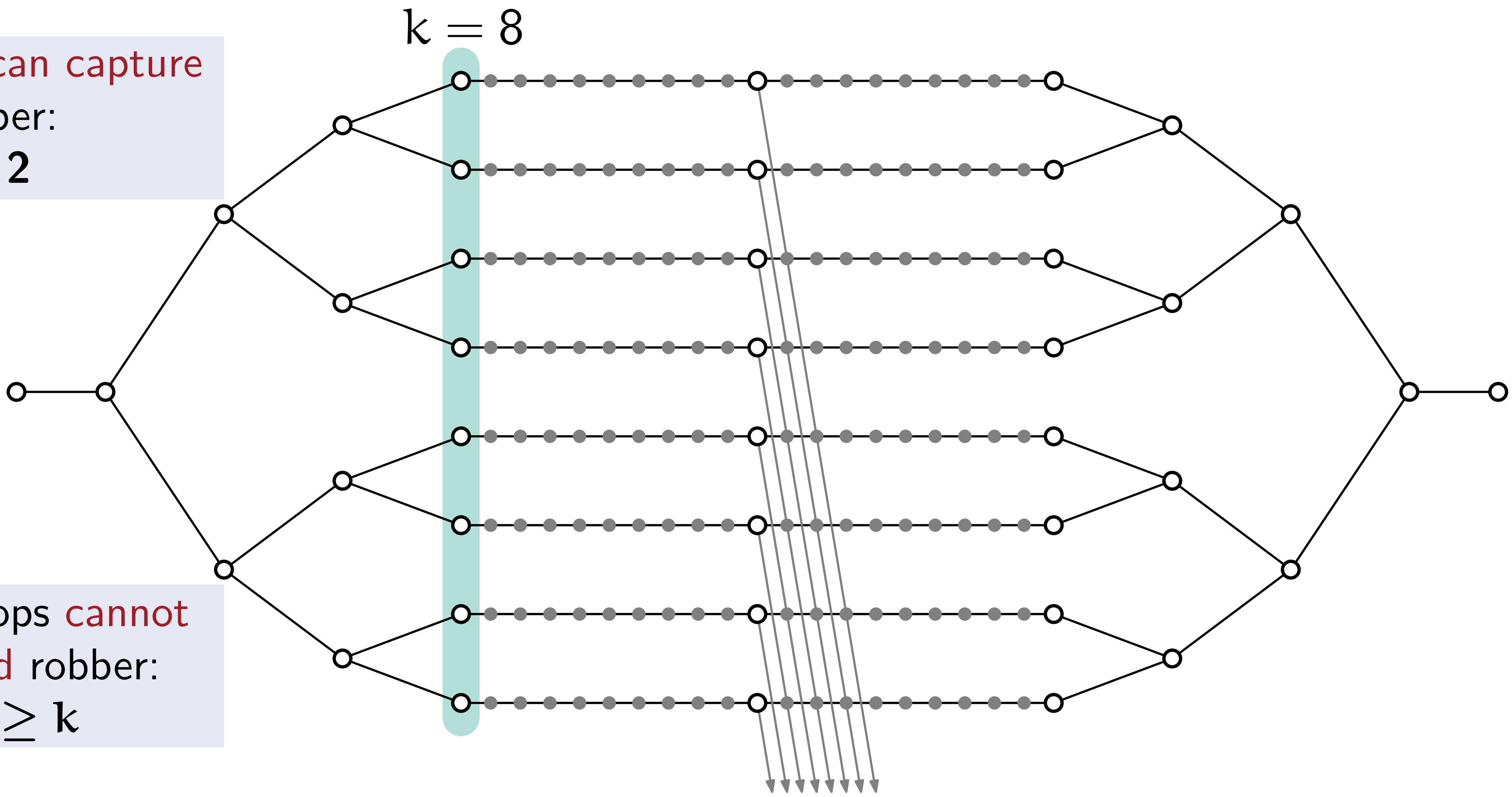
Separating “Capturing” and “Surrounding”

2 cops can capture the robber:
$\mathbf{c(G) = 2}$



Separating “Capturing” and “Surrounding”

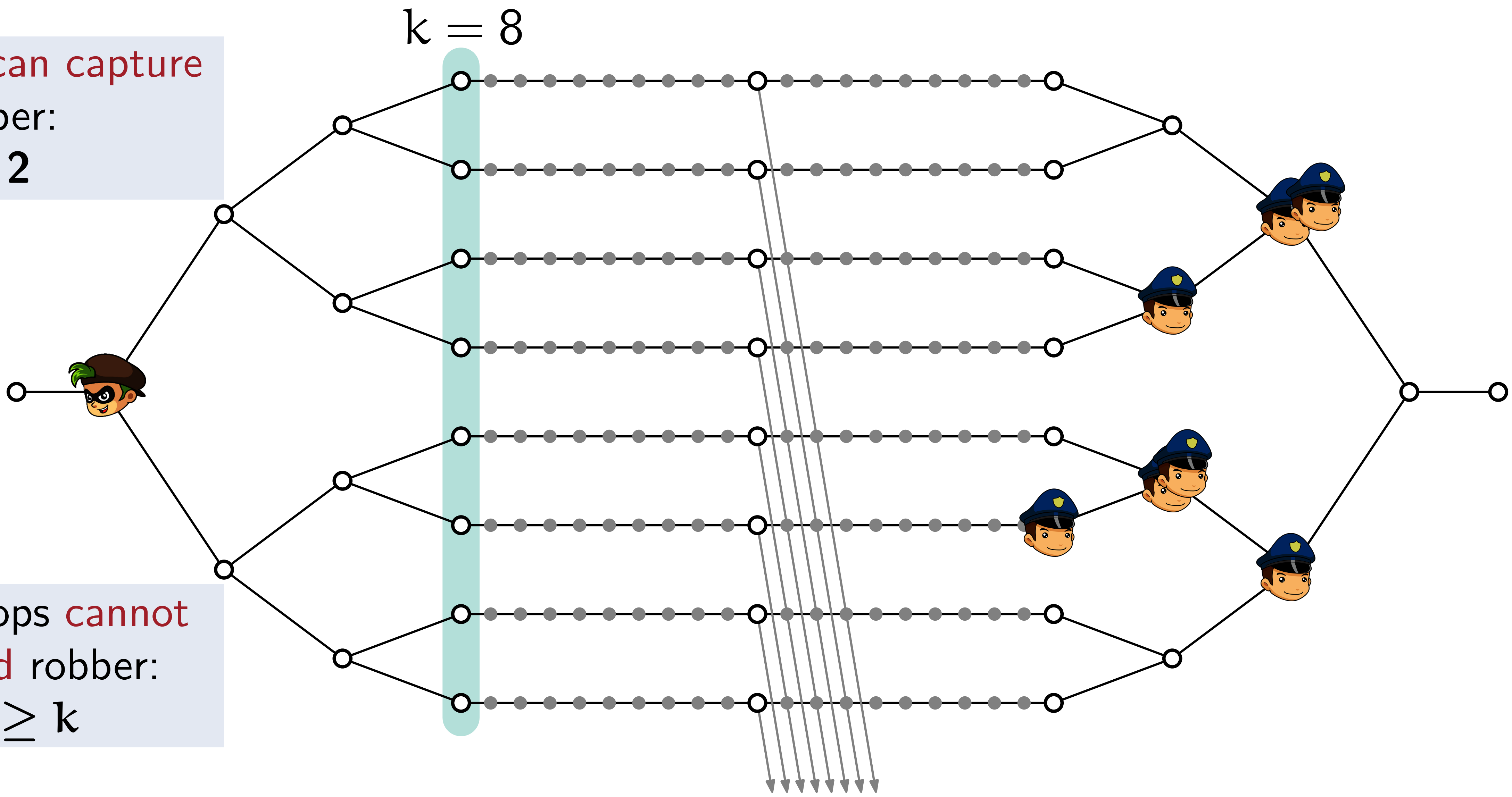
2 cops can capture the robber:
$\mathbf{c(G) = 2}$



$k - 1$ cops cannot surround robber:
$\mathbf{c_V(G) \geq k}$

Separating “Capturing” and “Surrounding”

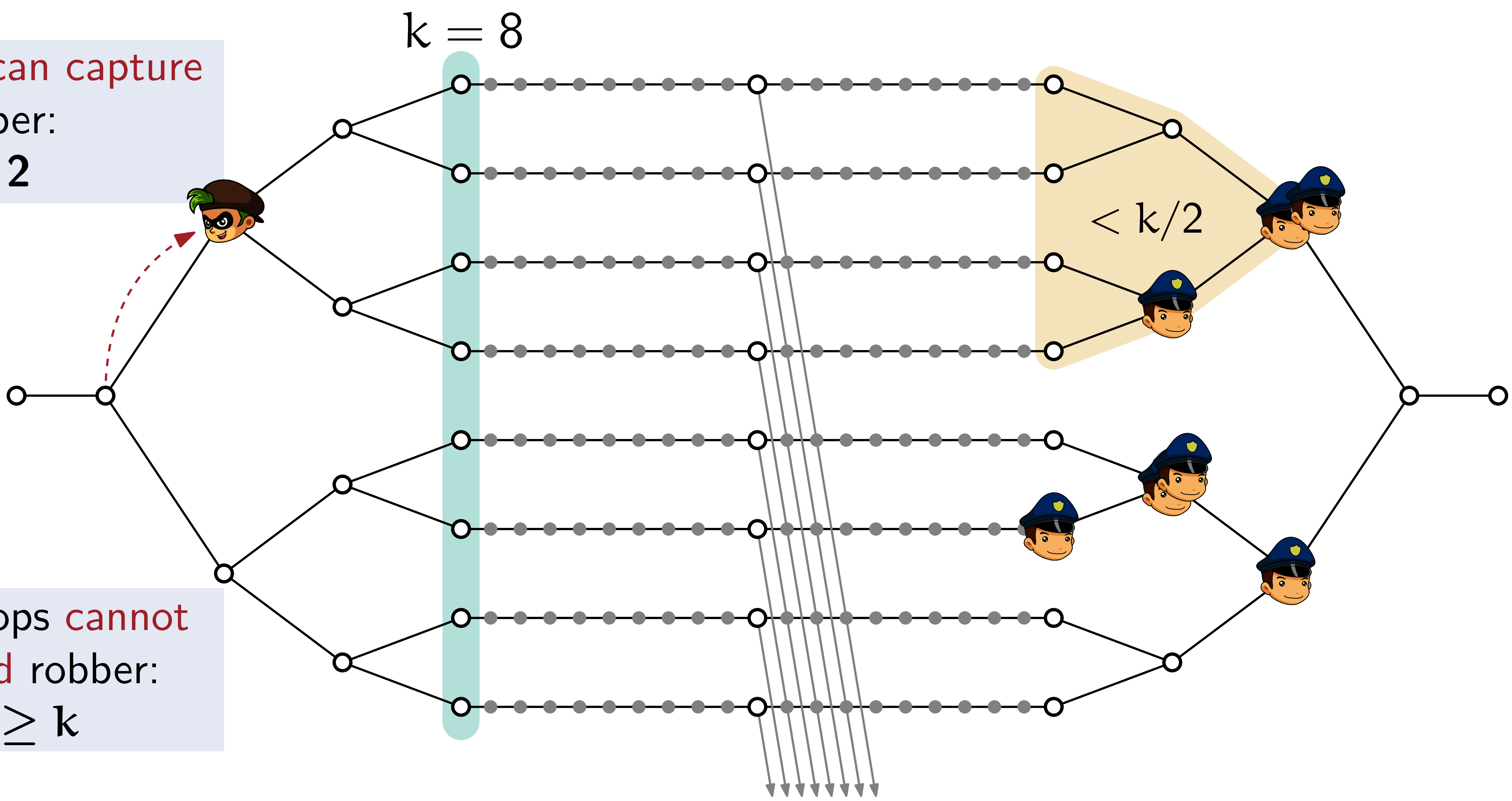
2 cops can capture the robber:
$\mathbf{c(G) = 2}$



$k - 1$ cops cannot surround robber:
$\mathbf{c_V(G) \geq k}$

Separating "Capturing" and "Surrounding"

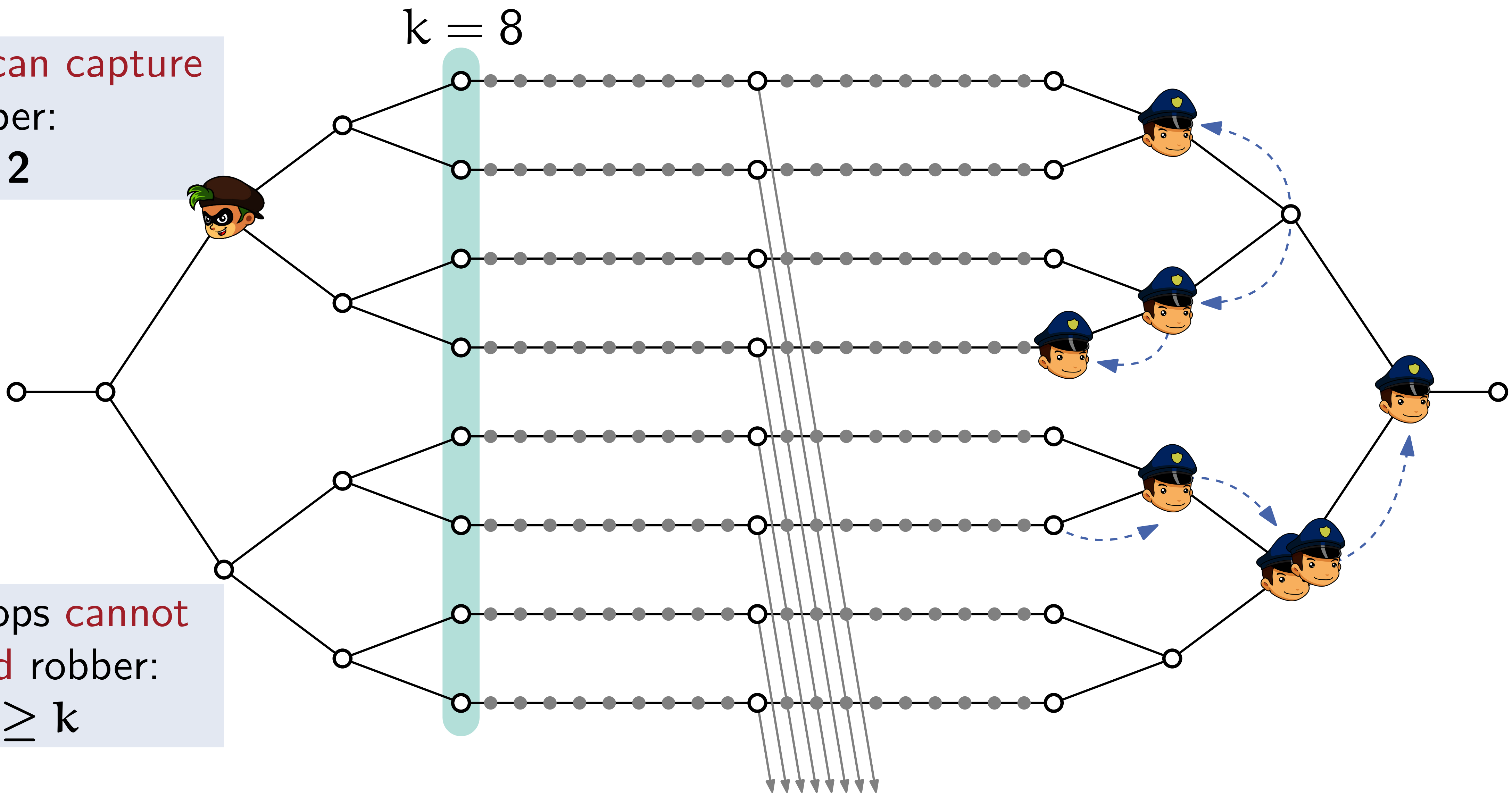
2 cops can capture the robber:
$\mathbf{c(G) = 2}$



$k - 1$ cops cannot surround robber:
$\mathbf{c_V(G) \geq k}$

Separating "Capturing" and "Surrounding"

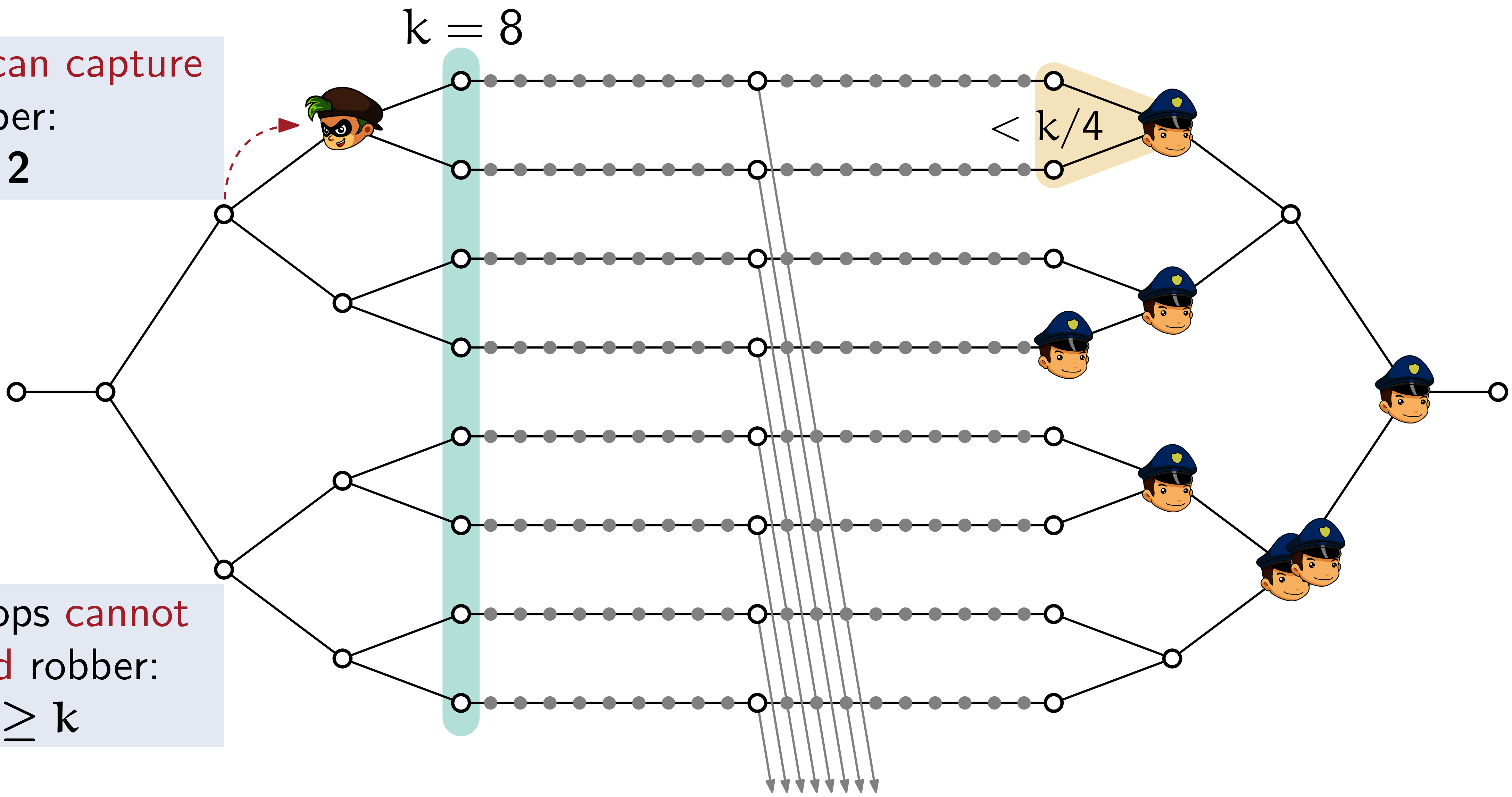
2 cops can capture the robber:
$\mathbf{c(G) = 2}$



$k - 1$ cops cannot surround robber:
$\mathbf{c_V(G) \geq k}$

Separating "Capturing" and "Surrounding"

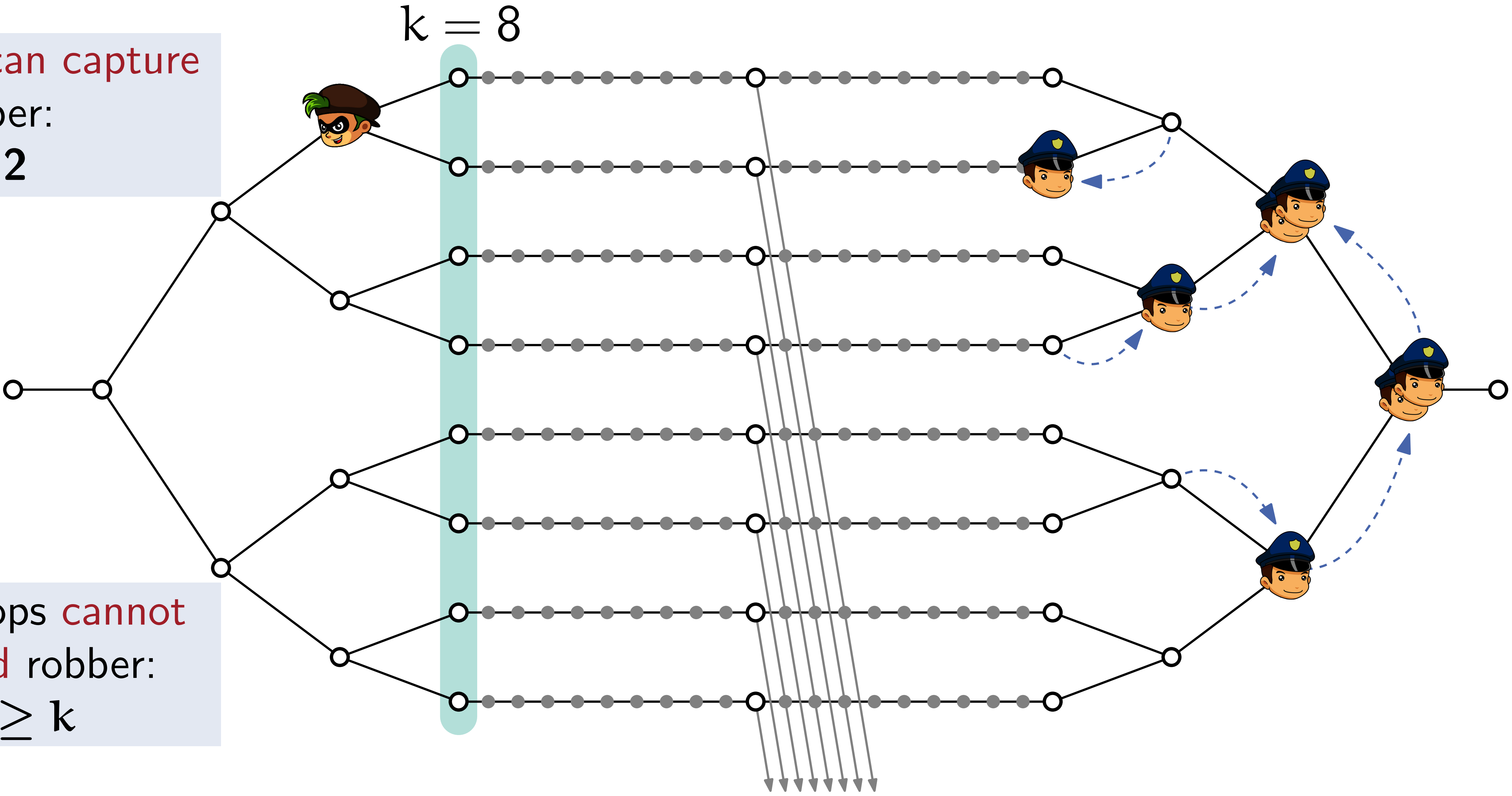
2 cops can capture the robber:
$\mathbf{c(G) = 2}$



$k - 1$ cops cannot surround robber:
$\mathbf{c_V(G) \geq k}$

Separating "Capturing" and "Surrounding"

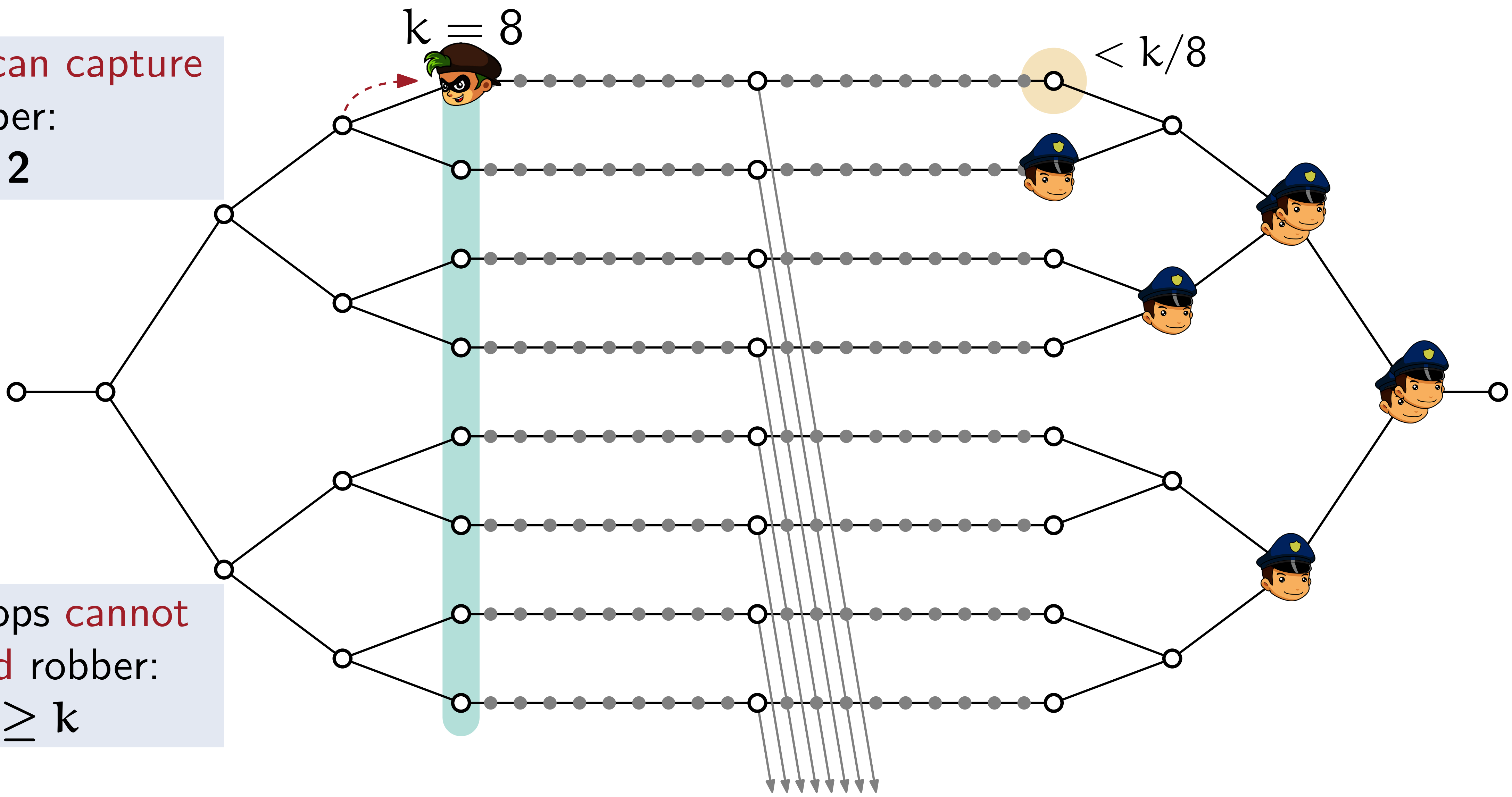
2 cops can capture the robber:
$\mathbf{c(G) = 2}$



$k - 1$ cops cannot surround robber:
$\mathbf{c_V(G) \geq k}$

Separating "Capturing" and "Surrounding"

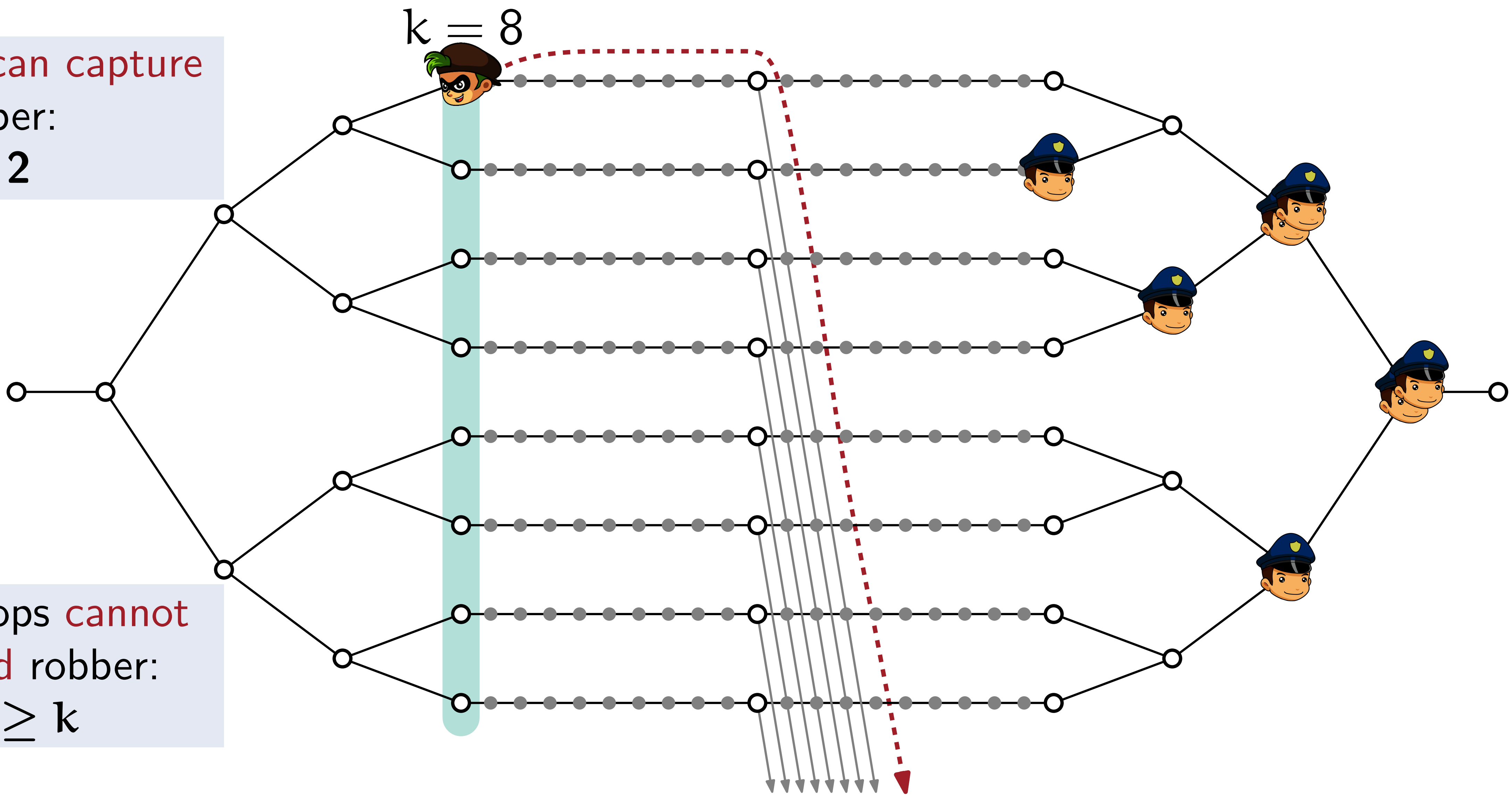
2 cops can capture the robber:
$\mathbf{c(G) = 2}$



$k - 1$ cops cannot surround robber:
$\mathbf{c_V(G) \geq k}$

Separating "Capturing" and "Surrounding"

2 cops can capture the robber:
$\mathbf{c(G) = 2}$



$k - 1$ cops cannot surround robber:
$\mathbf{c_V(G) \geq k}$