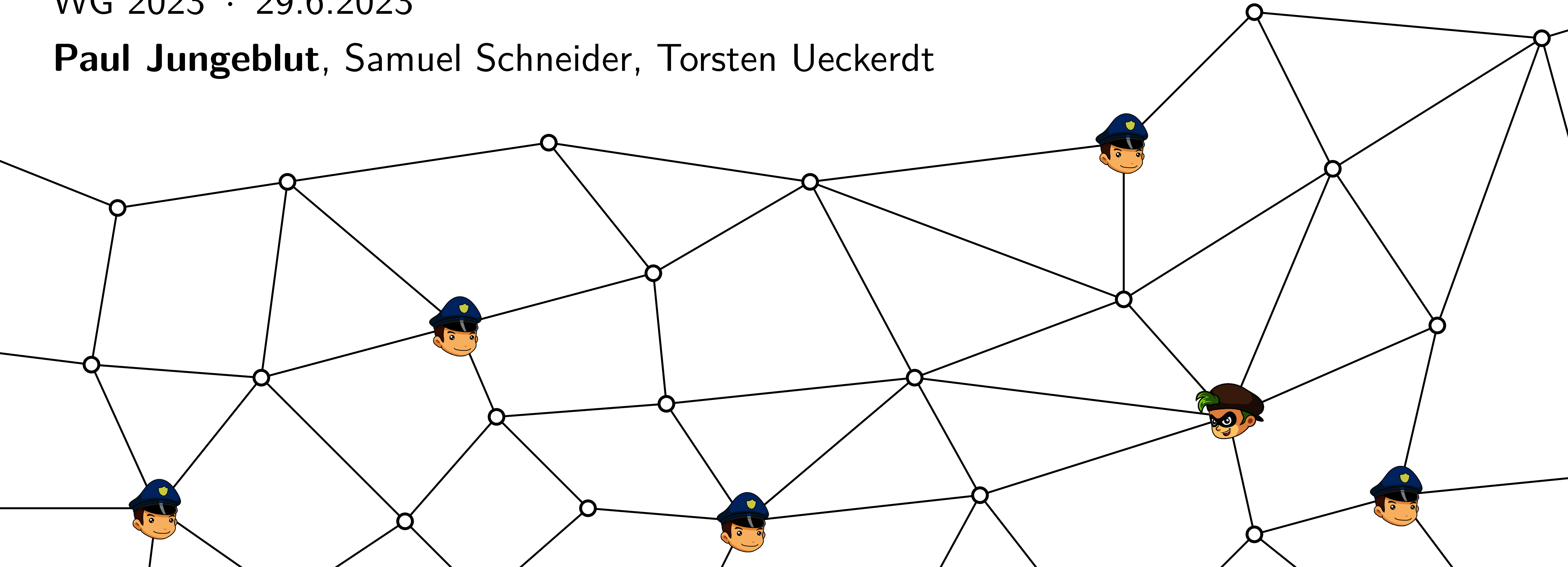
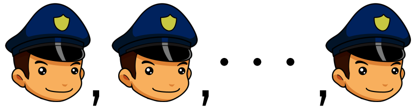
Cops and Robber: When Capturing is not Surrounding

WG 2023 · 29.6.2023

Paul Jungeblut, Samuel Schneider, Torsten Ueckerdt

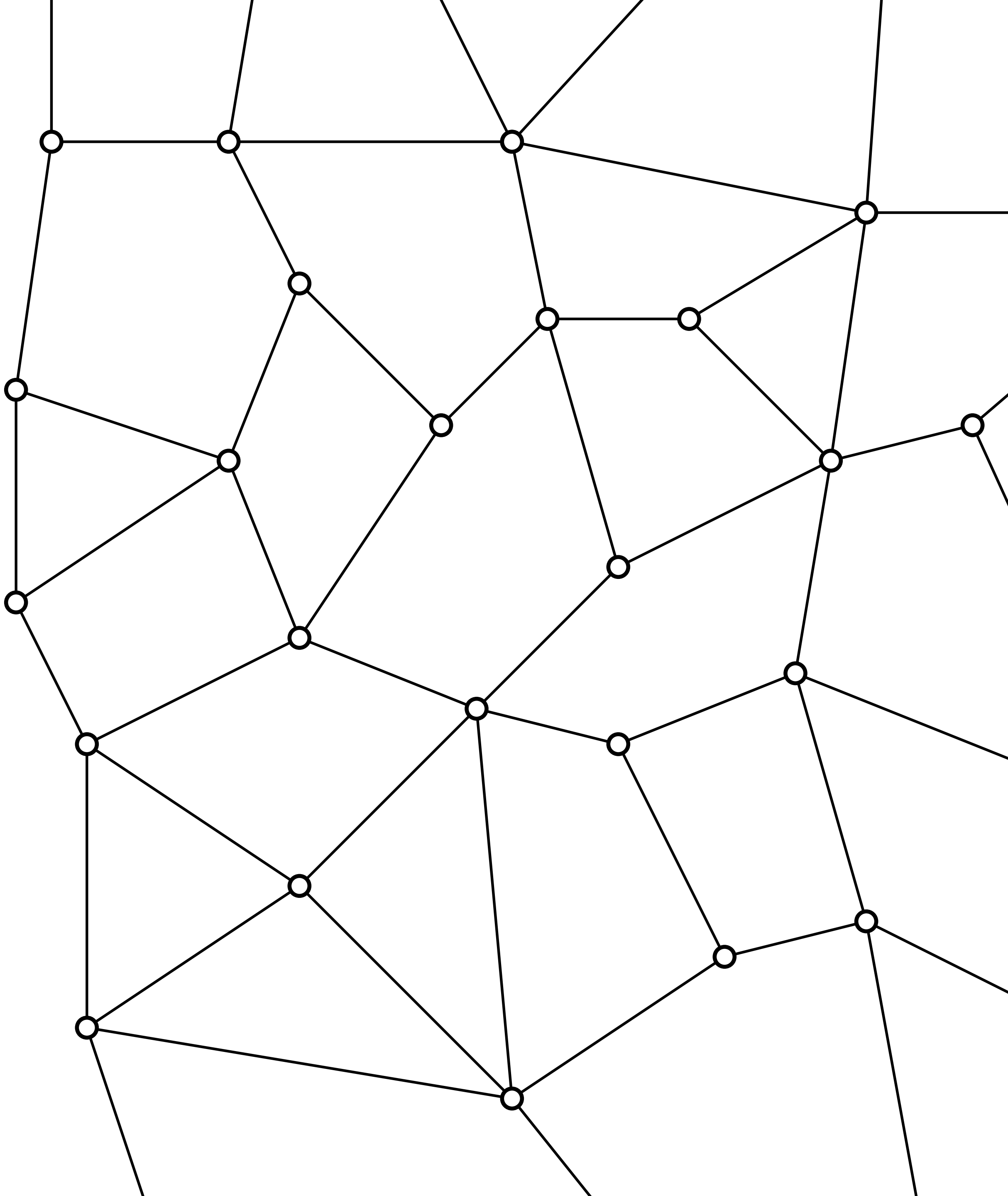


Cops and Robber

2-Players: k Cops 

1 Robber 

Rules:



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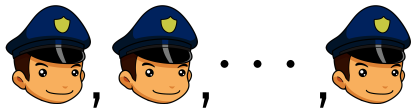
2-Players: k Cops 

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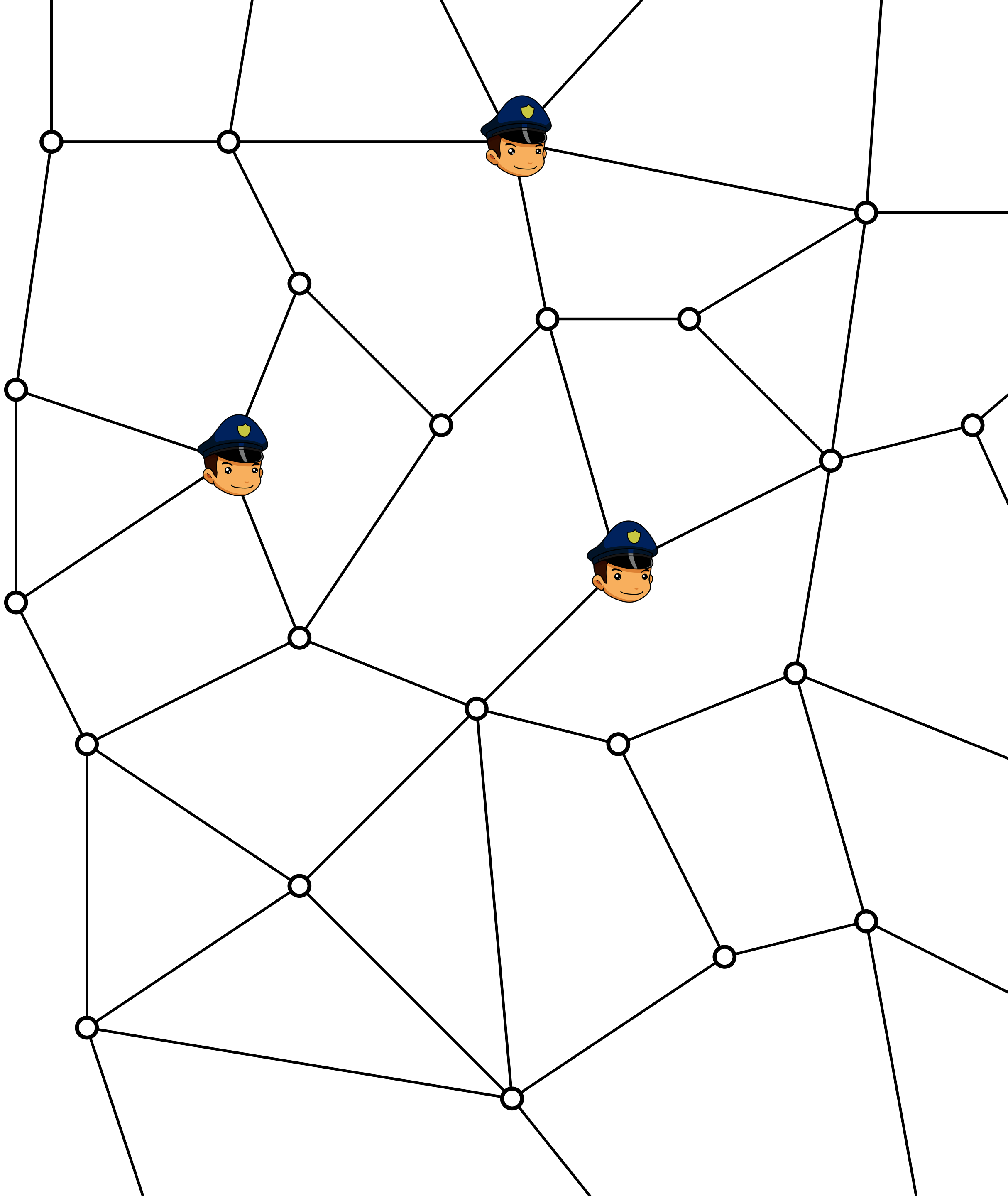

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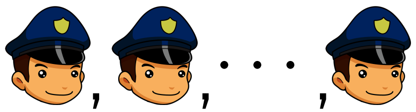
2-Players: k Cops 

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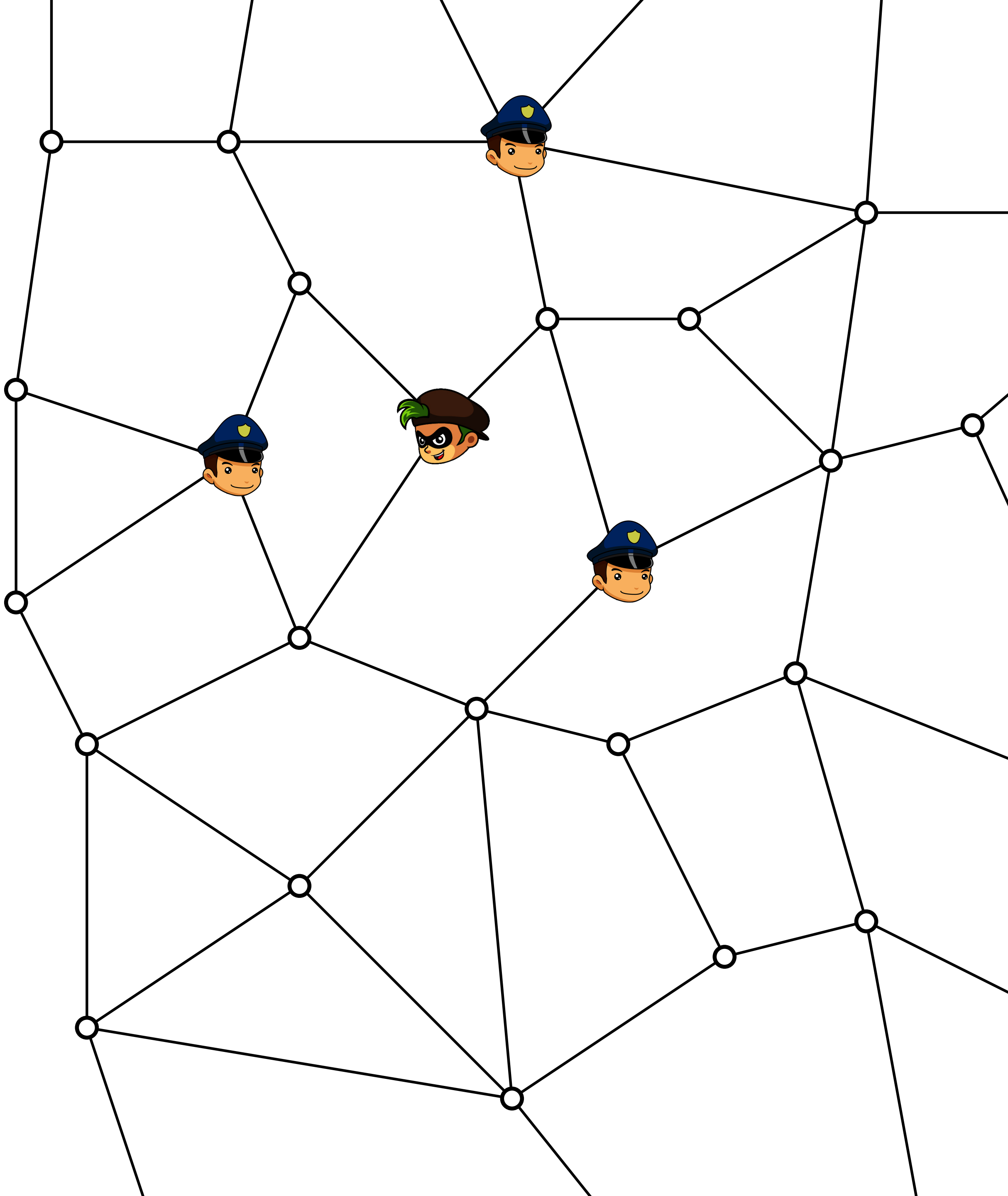

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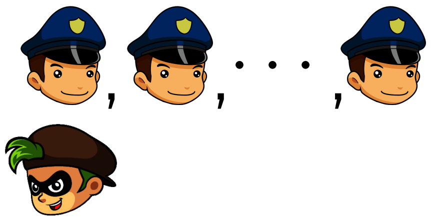

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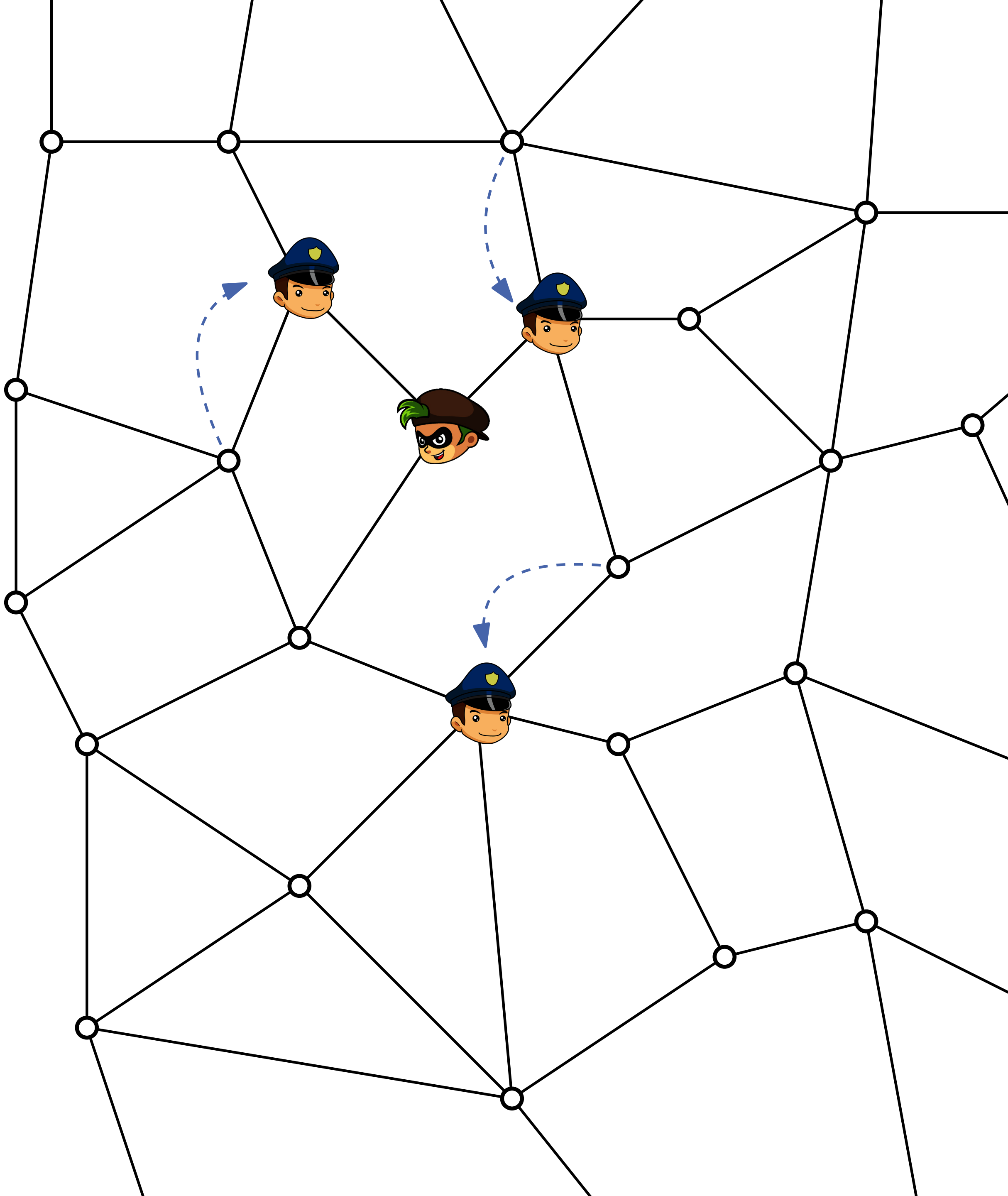
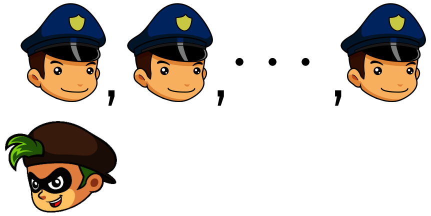


Cops and Robber

2-Players: k Cops, 1 Robber



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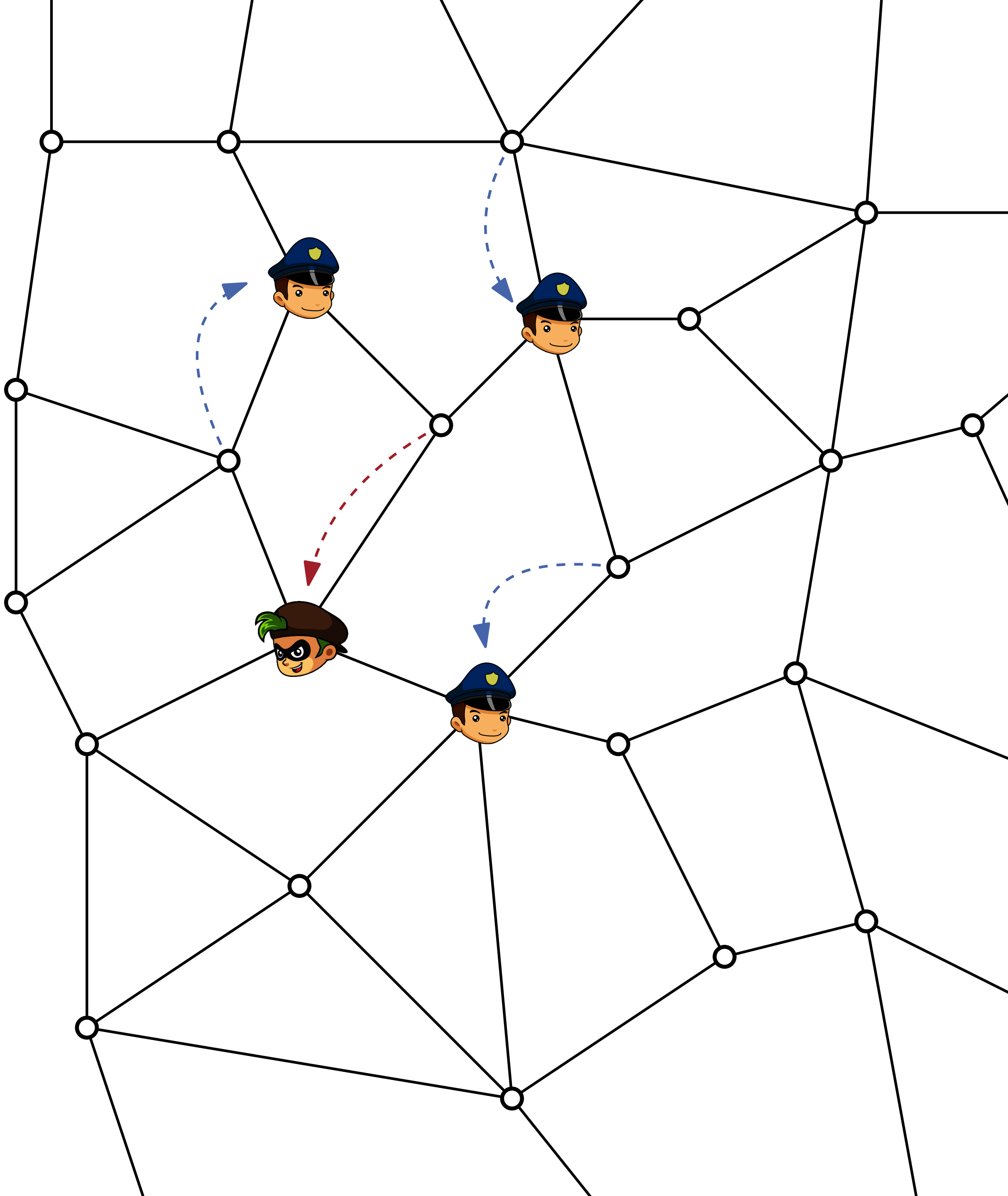

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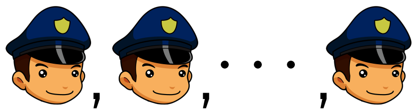
2-Players: k Cops 

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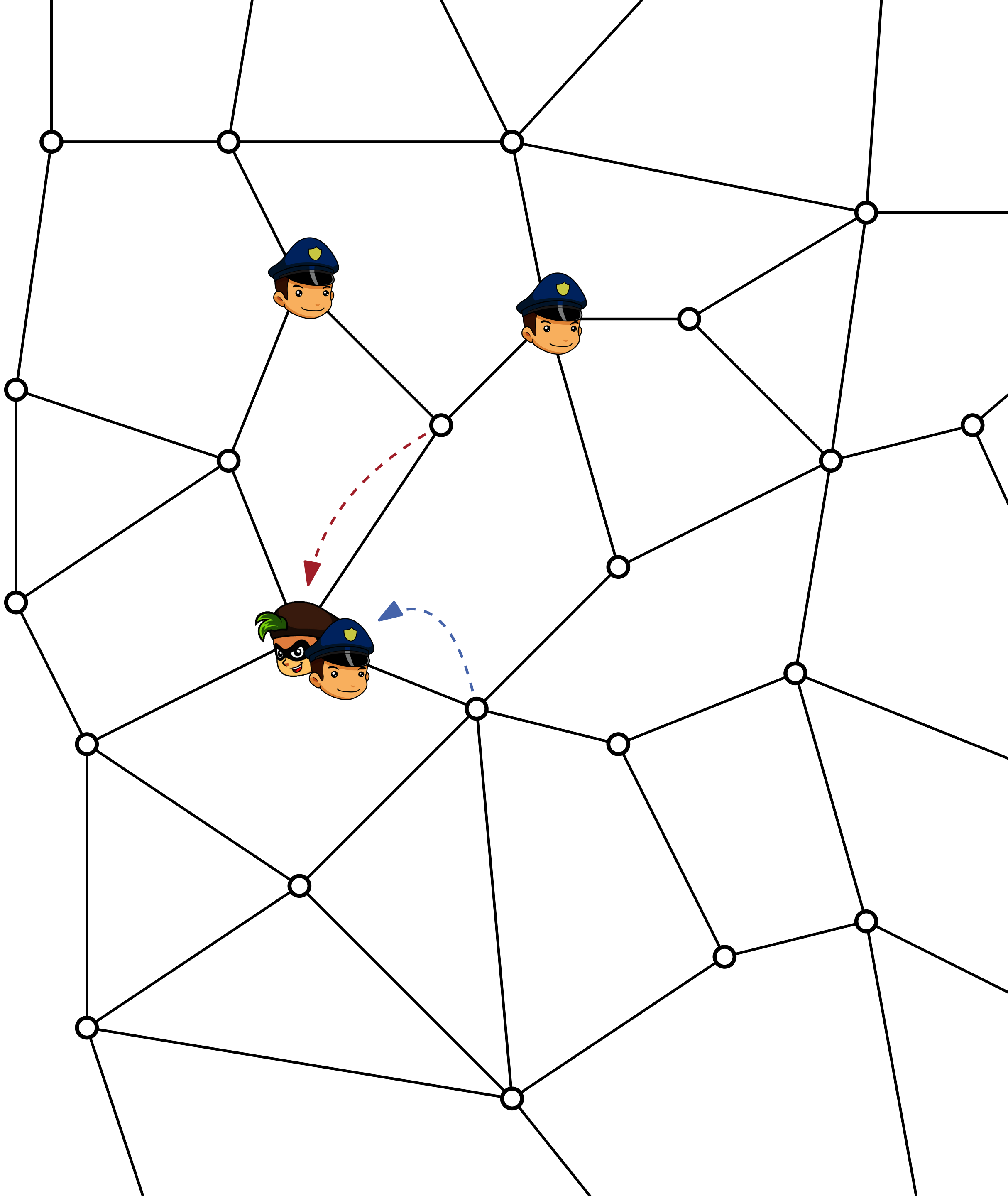
- Cops win by capturing the robber.



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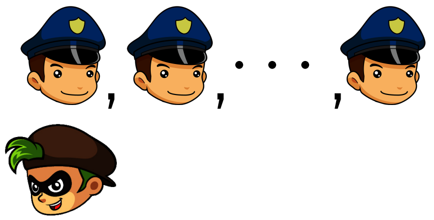

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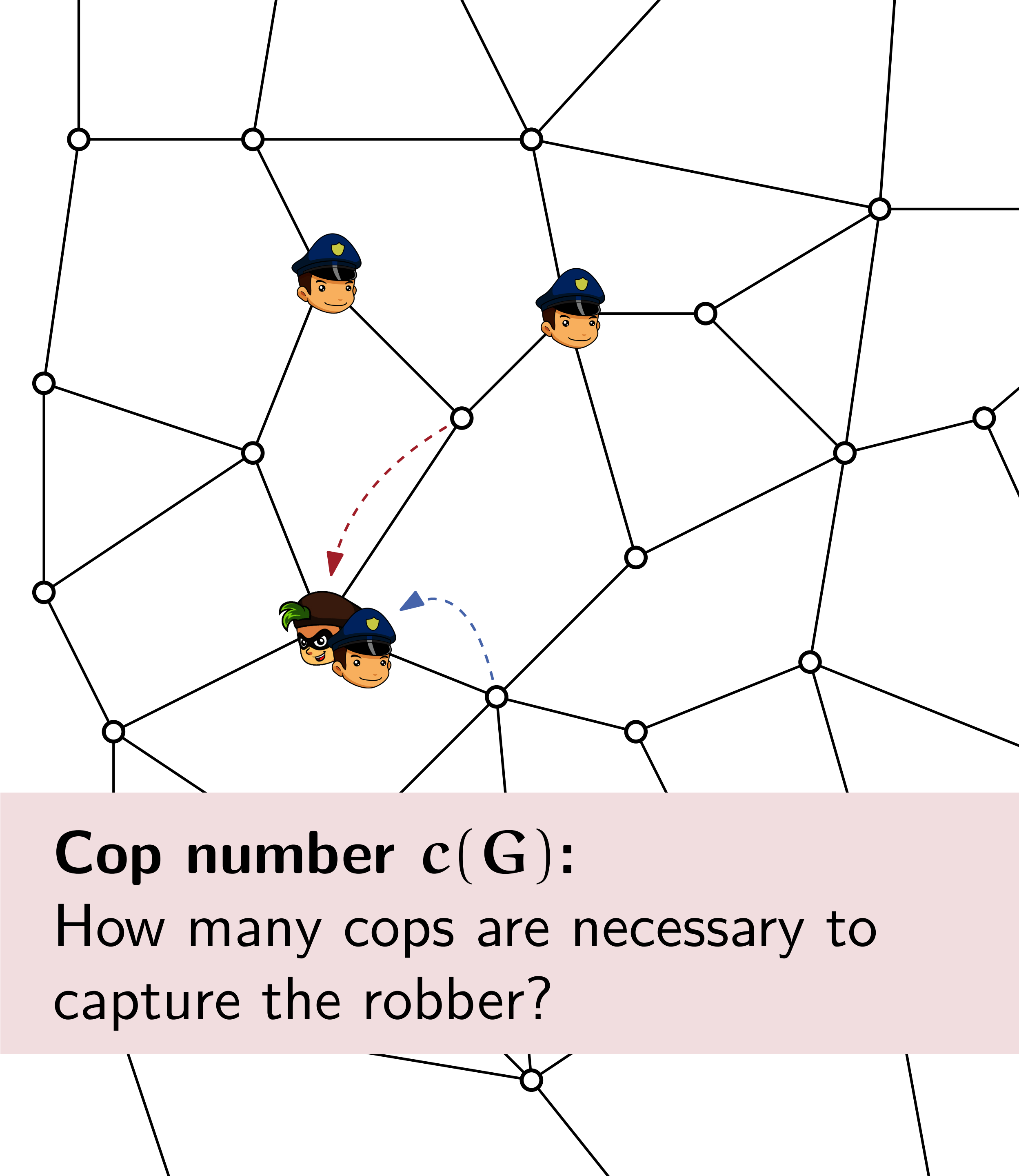
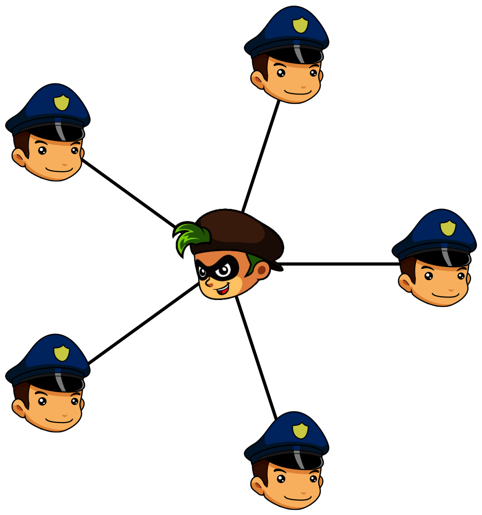


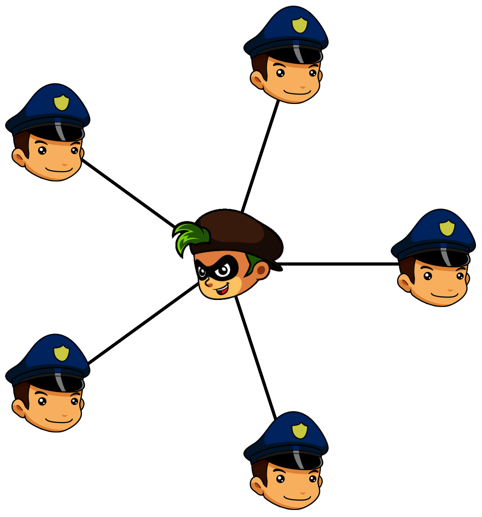
Cop number $\mathbf{c(G)}$:
How many cops are necessary to capture the robber?

Surrounding Variants



vertex surround

Surrounding Variants



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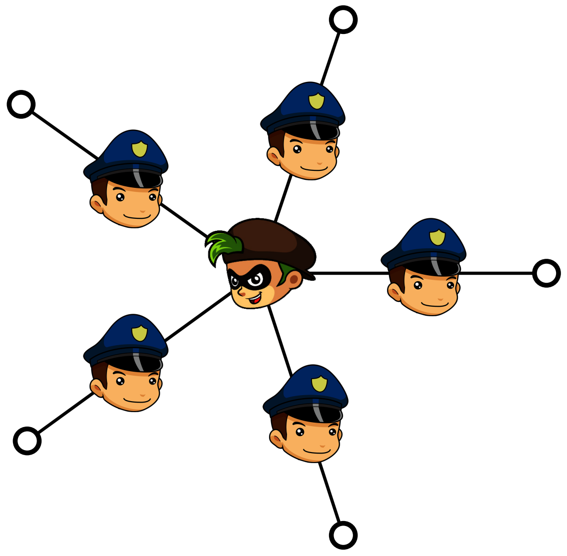

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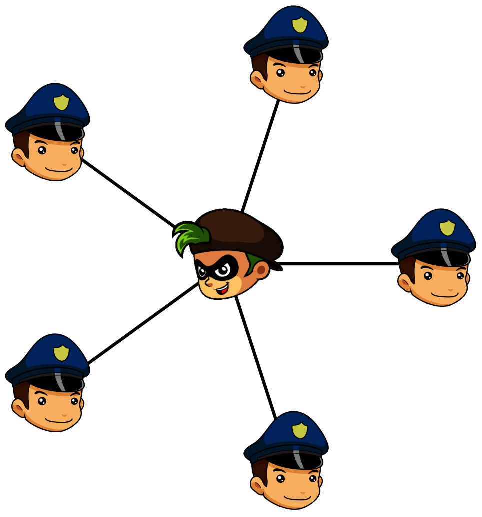

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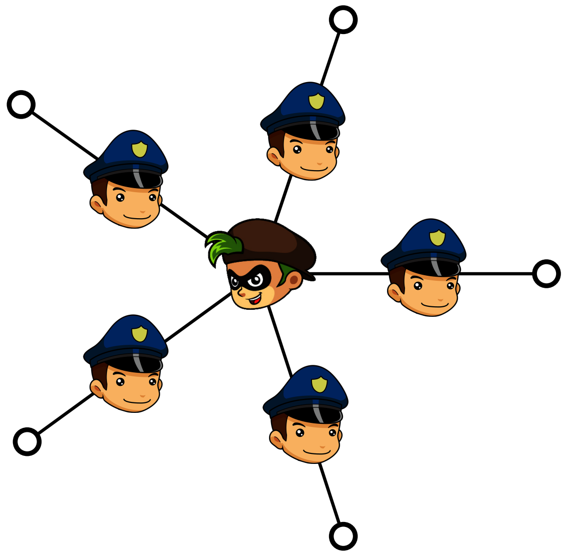

edge surround

$c_V(G)$ $c_E(G)$

How many cops are necessary to vertex/edge surround the robber?

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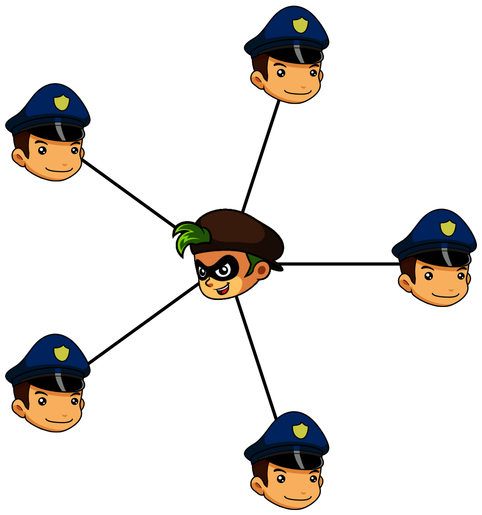

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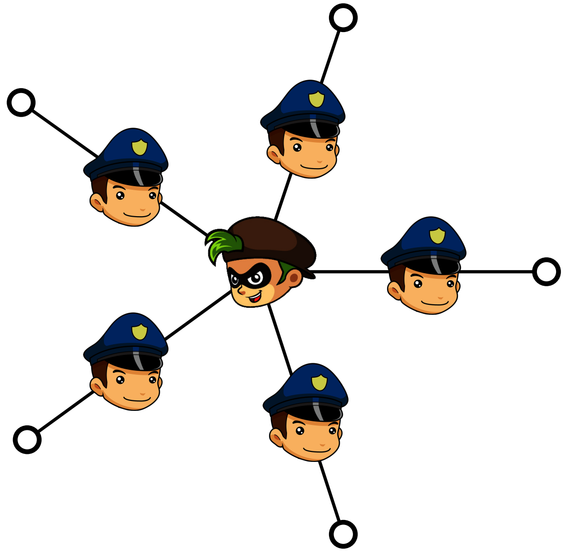


edge surround

$\mathbf{c_V(G)}$ $\mathbf{c_E(G)}$

How many cops are necessary to vertex/edge surround the robber?

Restrictive variants:

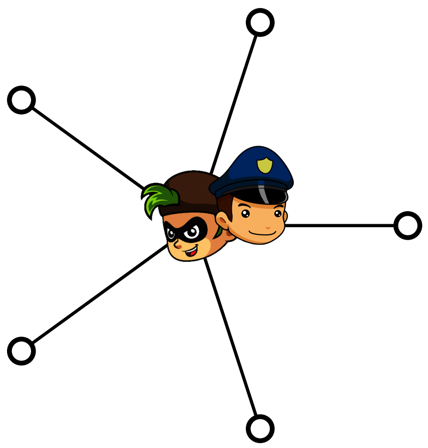
$\mathbf{c_{V,r}(G)}$:

Robber must not end his move on a cop.

$\mathbf{c_{E,r}(G)}$:

Robber must not move through a cop.

A Little Bit of History



$\mathbf{c(G)}$:

Quilliot '78

Nowakowski, Winkler '83

Aigner, Fromme '84

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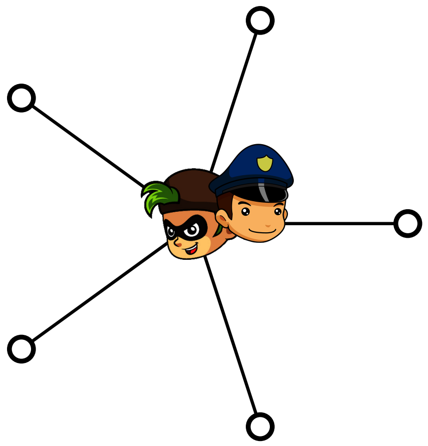

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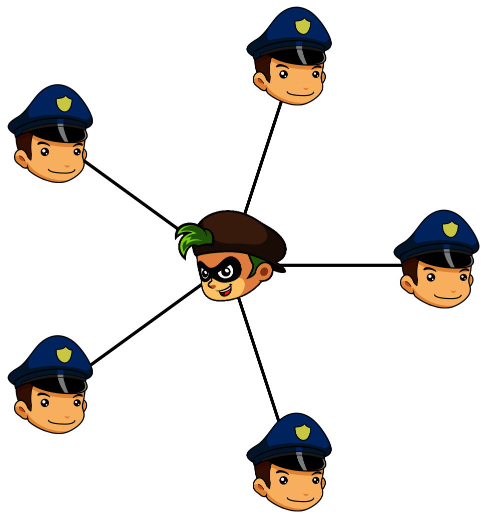

$\mathbf{c_{V,r}(G)}$:

Burgess et al. '20

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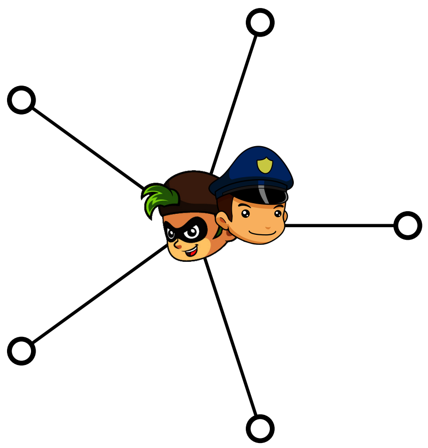

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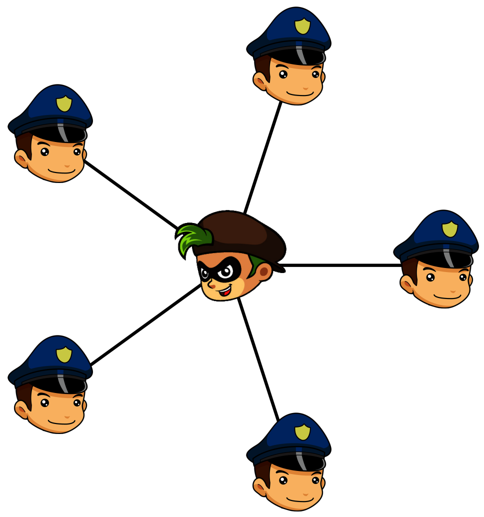

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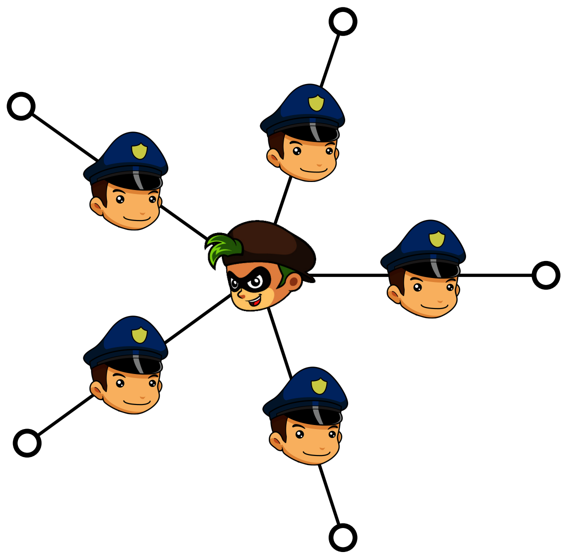

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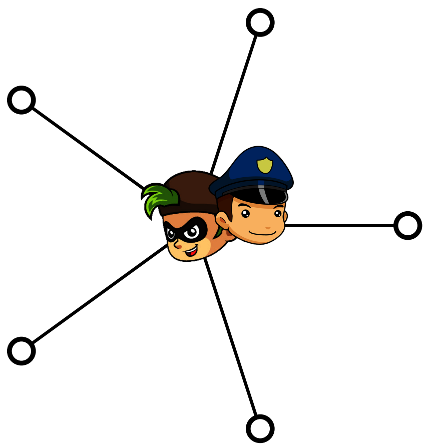

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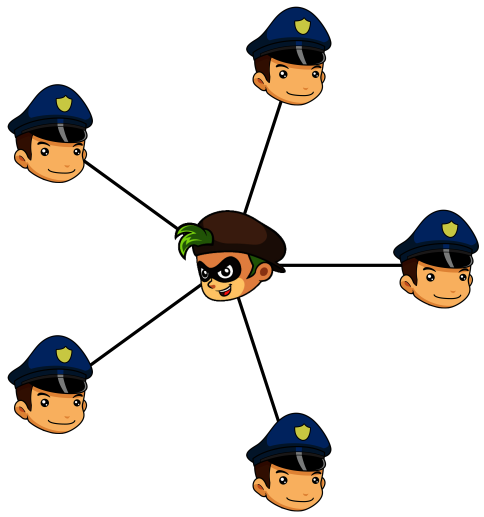
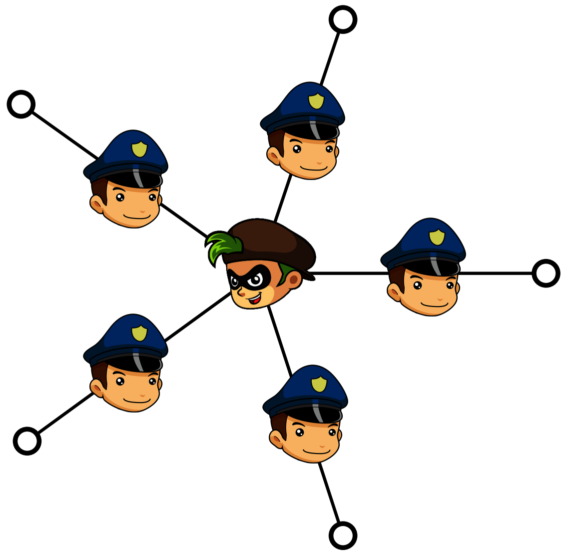


$c_{V,r}(G)$:

Burgess et al. '20

$c_V(G)$: new



$c_{E,r}(G)$:

Crytser et al. '20

$c_E(G)$: new

Our Results

Unification

of the four surrounding variants

Theorem: (informal)
Strategy for k cops in some variant gives a strategy for $\leqslant k \cdot 2\Delta$ cops in any other variant.

max. degree

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from classical variant

Conjecture: (Crytser et al.)
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Wrong!

Theorem:
There are infinitely many G with:

- $c(G) = 2$ and $\Delta(G) = 3$
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Unifying the Surrounding Variants

Upper bounds: Simulation

	not restricted	restricted
vertices	$c_V(G)$	$c_{V,r}(G)$
edges	$c_E(G)$	$c_{E,r}(G)$

Unifying the Surrounding Variants

Upper bounds: Simulation

	 not restricted	 restricted
 vertices	$c_V(G)$ →	$c_{V,r}(G)$
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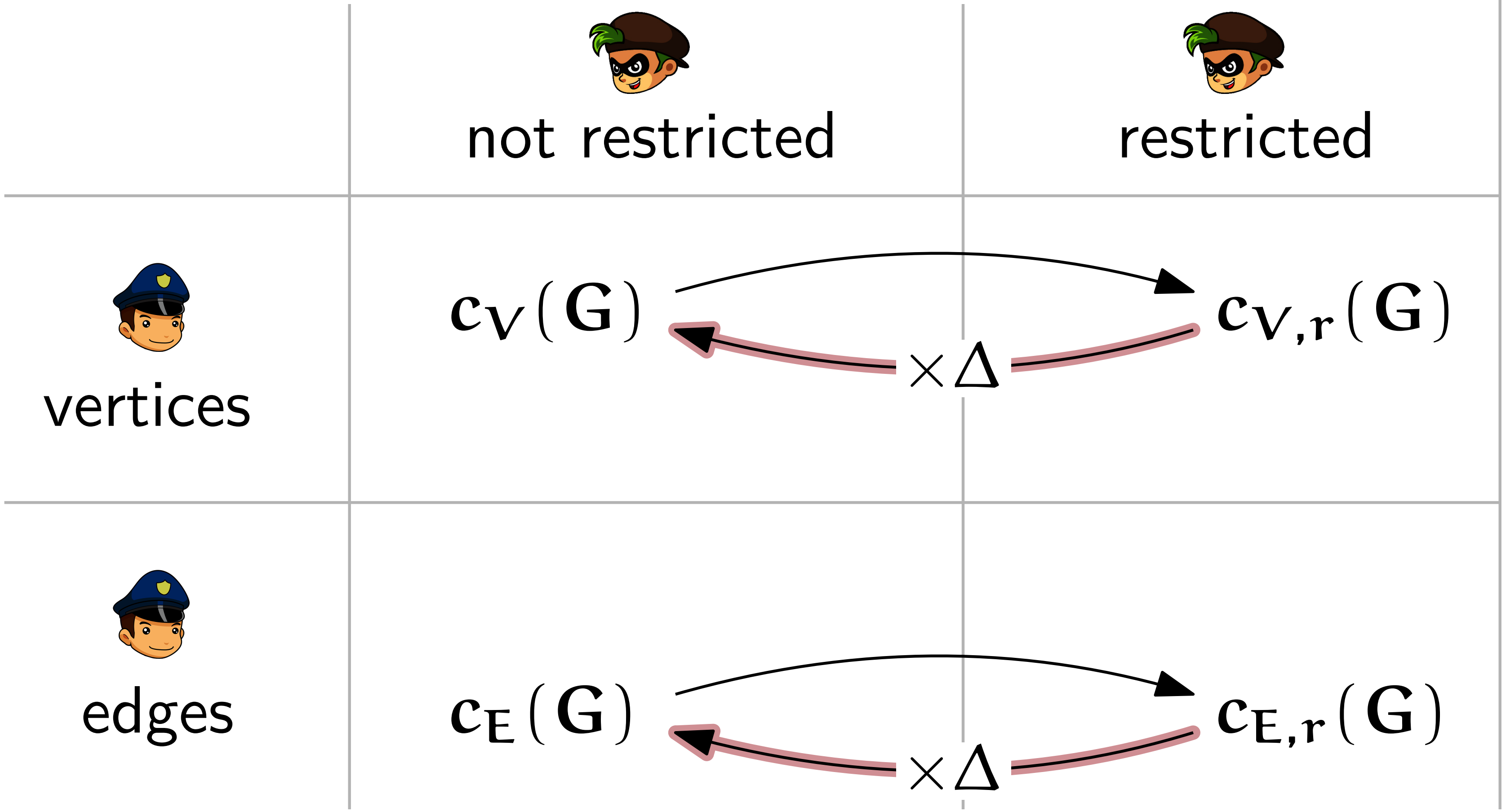
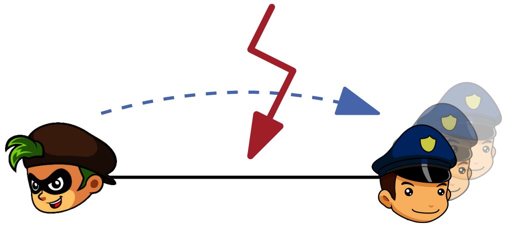
A restricted robber is a weaker robber.

Unifying the Surrounding Variants

Upper bounds: Simulation



Replace each cop by a group of $\Delta(G)$ cops.



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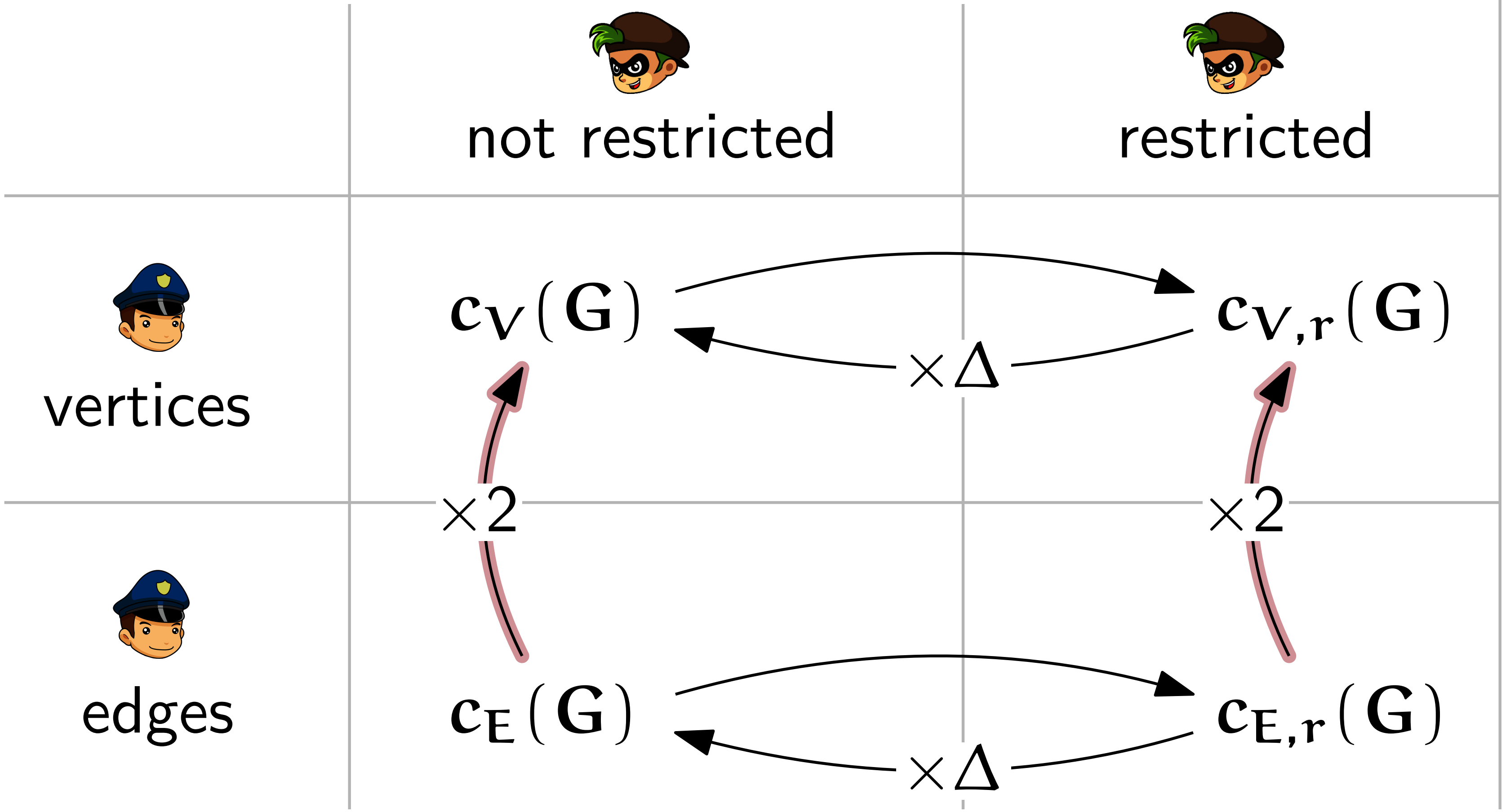
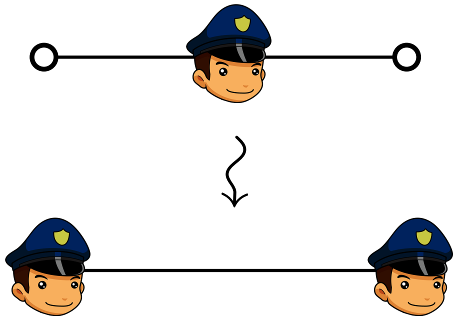


Replace edge cop by two vertex cops.



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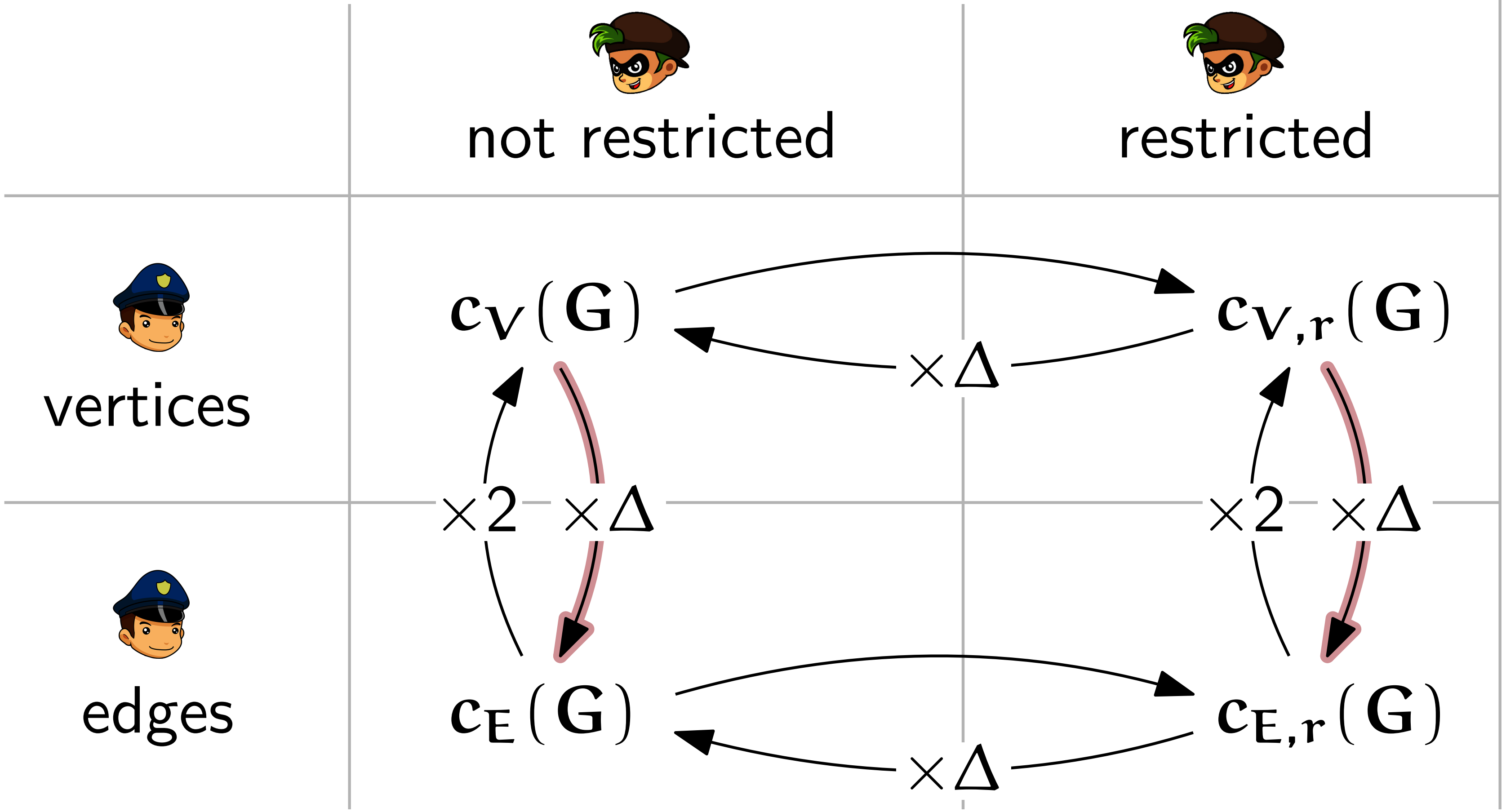
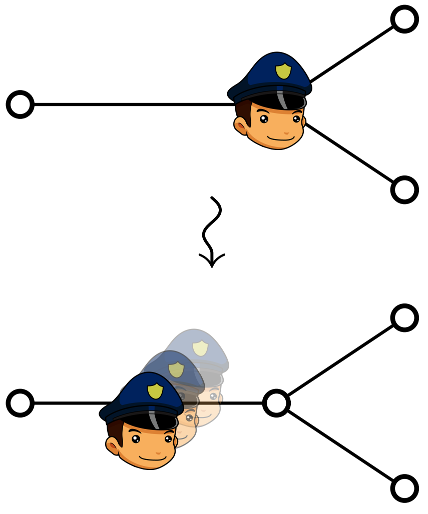


Replace vertex cop by a group of $\Delta(G)$ edge cops.



follow original cop

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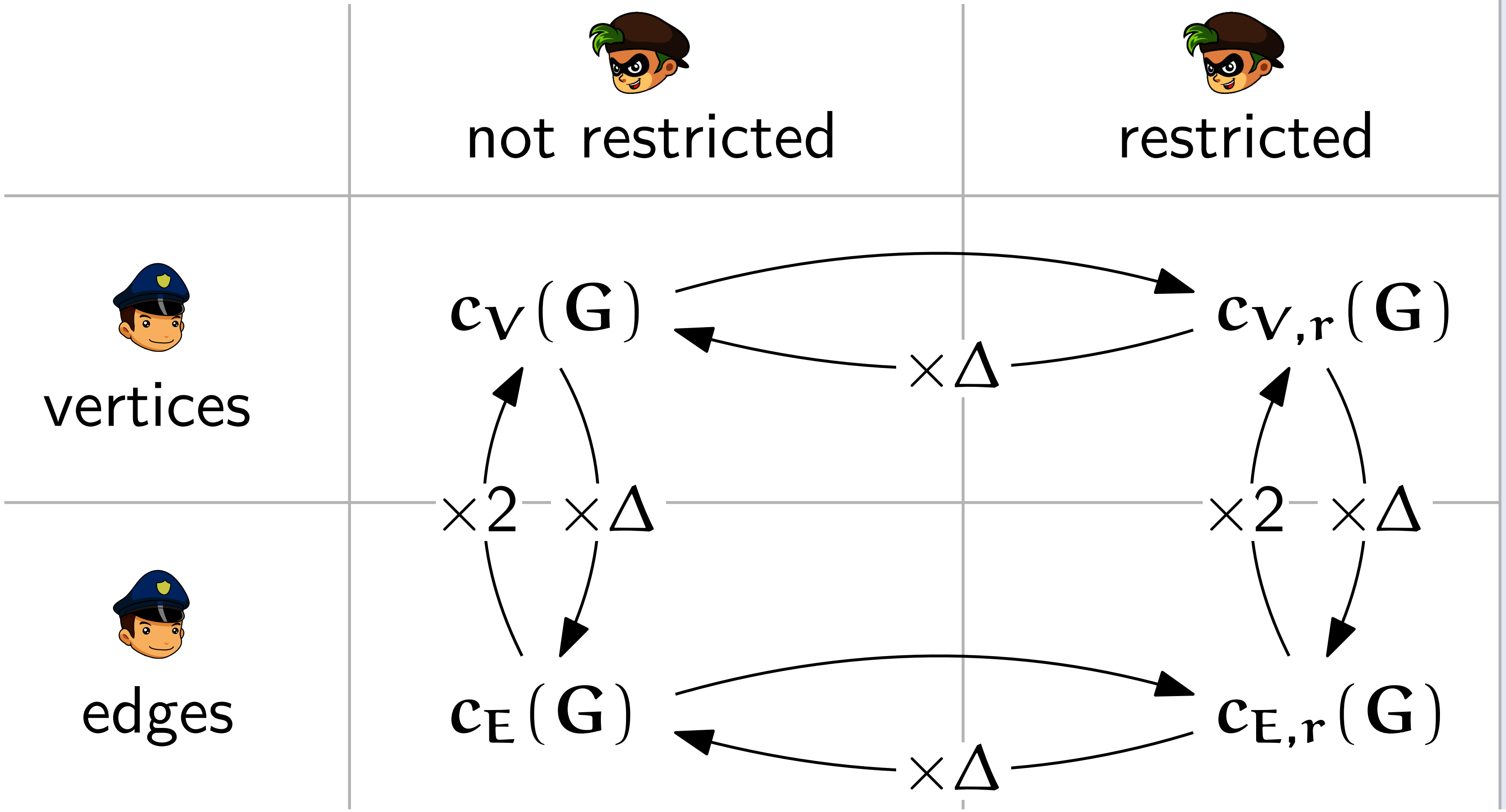


Lower bounds: Constructions

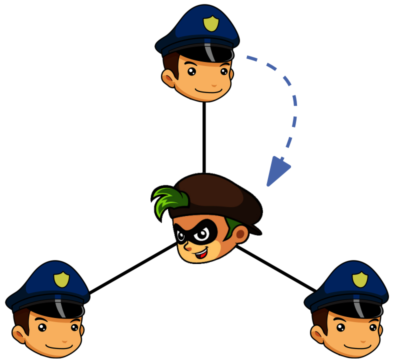
Tight examples for all claimed inequalities:

- complete (bipartite) graphs
- regular graphs (with "leaves")
- based on "MOLS" (mutually orthogonal Latin squares)
- line graphs of complete graphs

Paul Jungeblut, Samuel Schneider, Torsten Ueckerdt

Separating "Capturing" and "Surrounding"

Observation:
surrounding $\Longrightarrow$ capturing
$\rightsquigarrow c_{X(,r)}(G) \geqslant c(G)$



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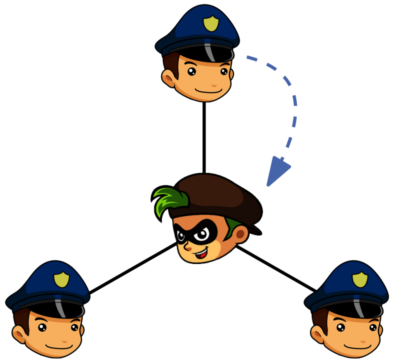

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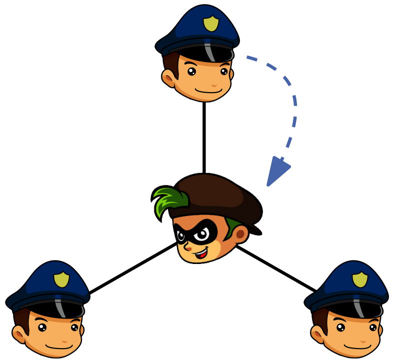

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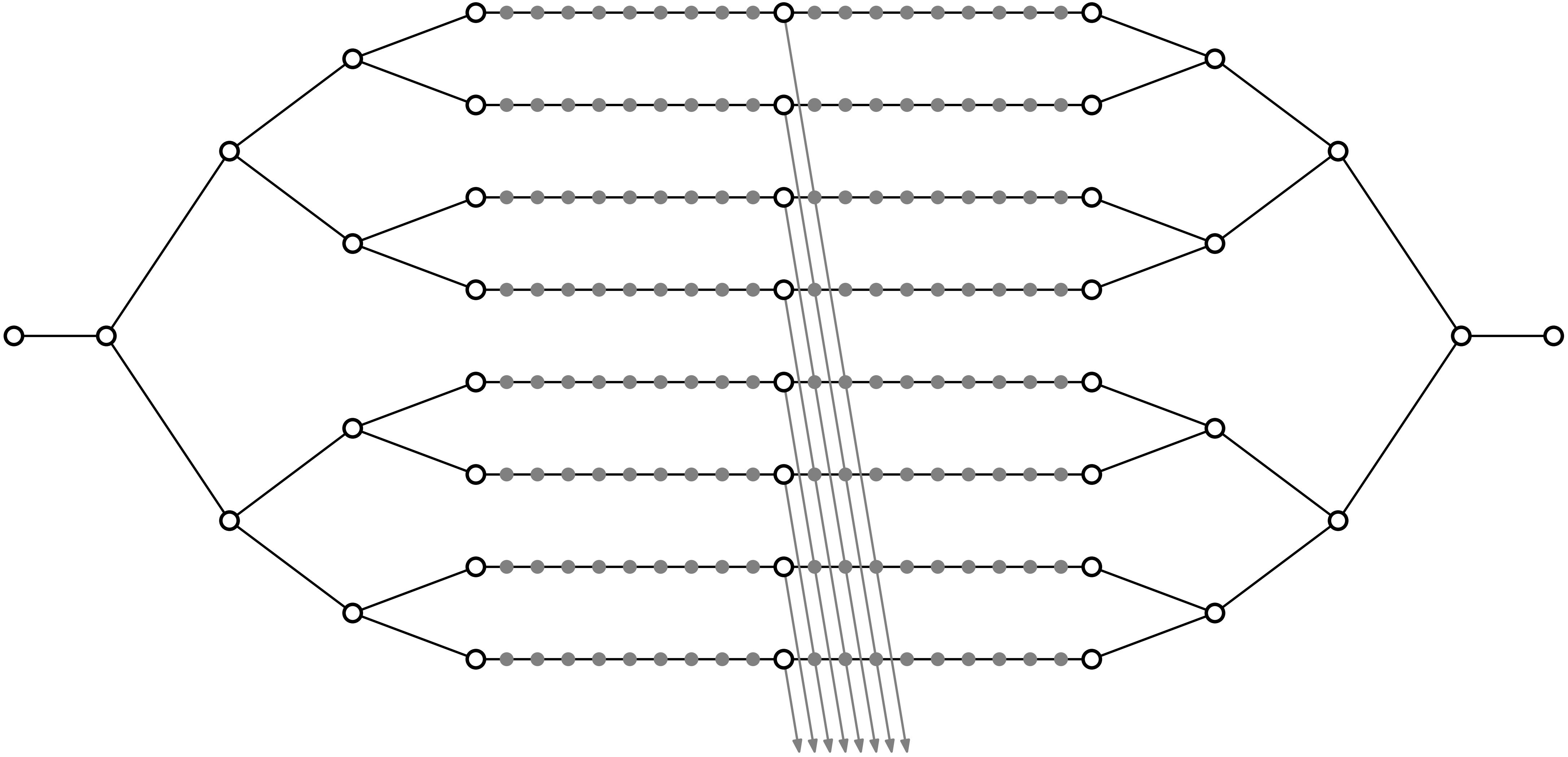

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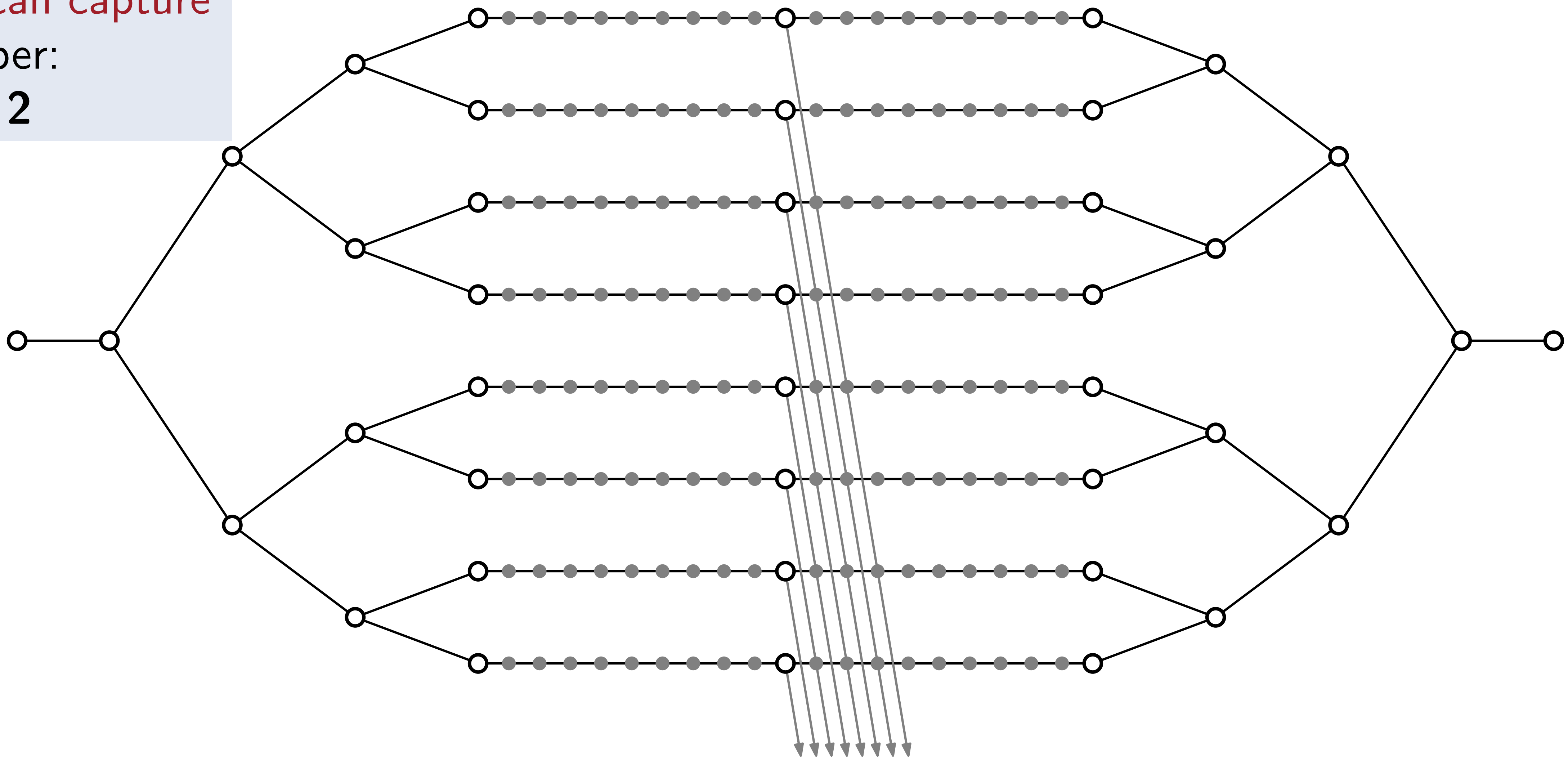
2 cops can capture the robber:
$\mathbf{c(G) = 2}$



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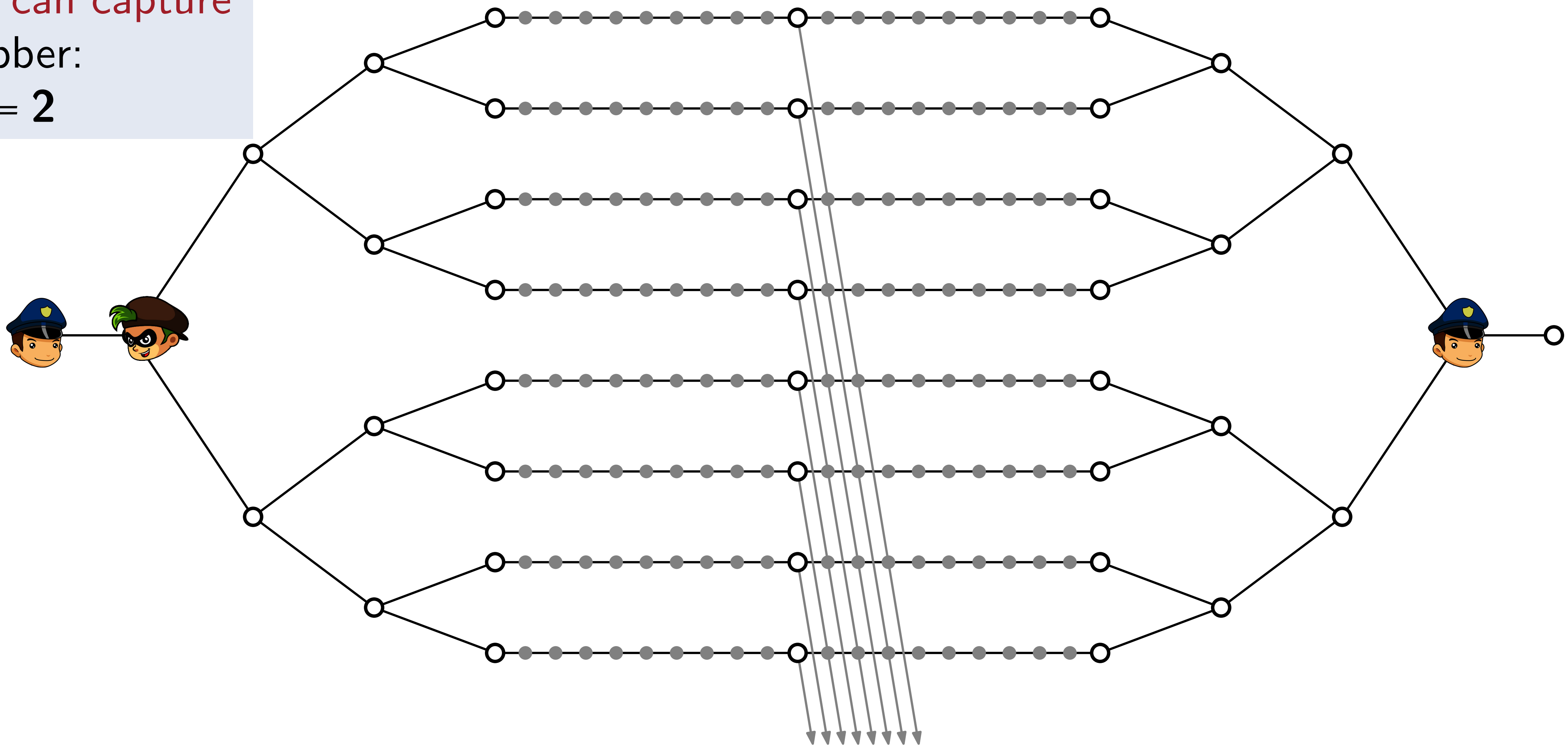

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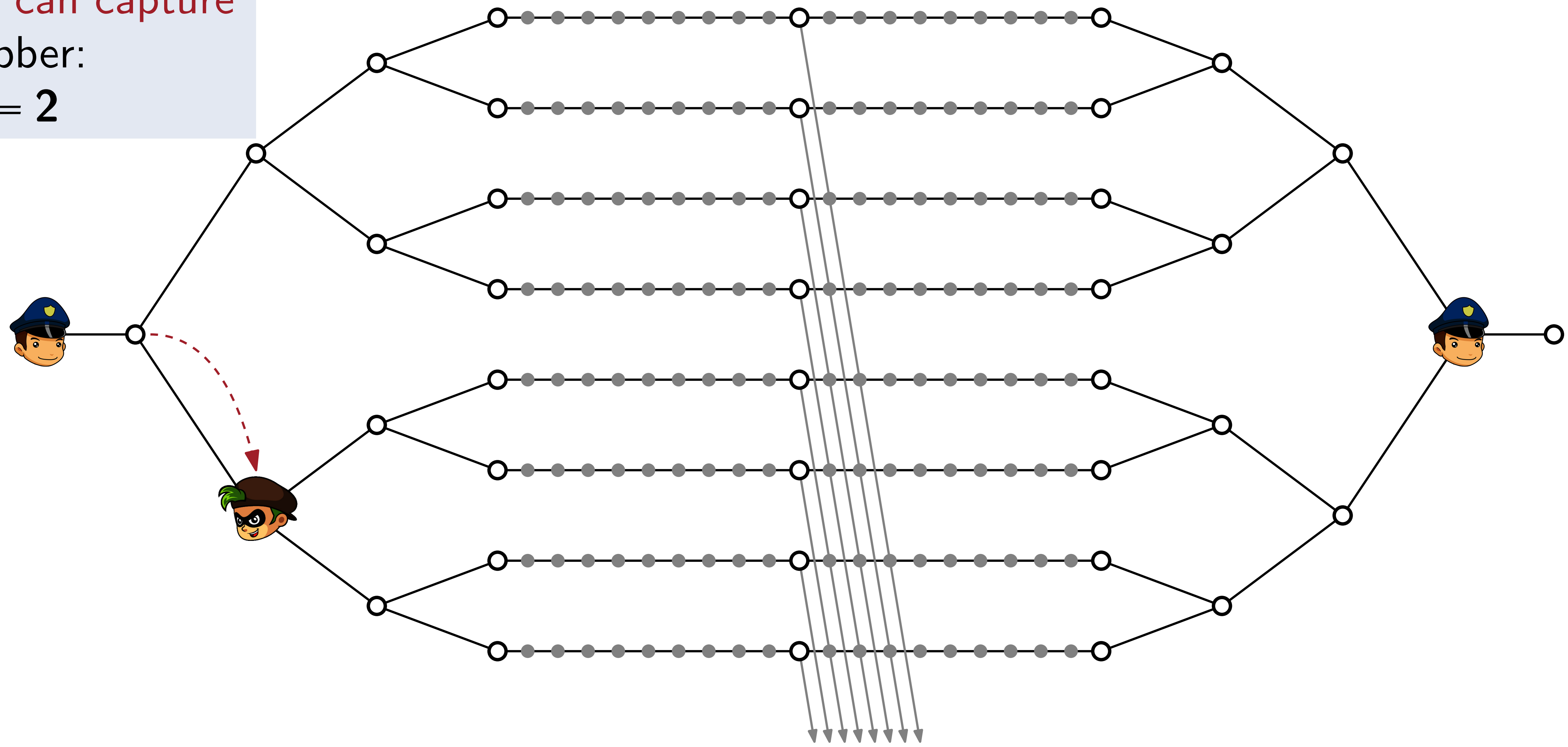

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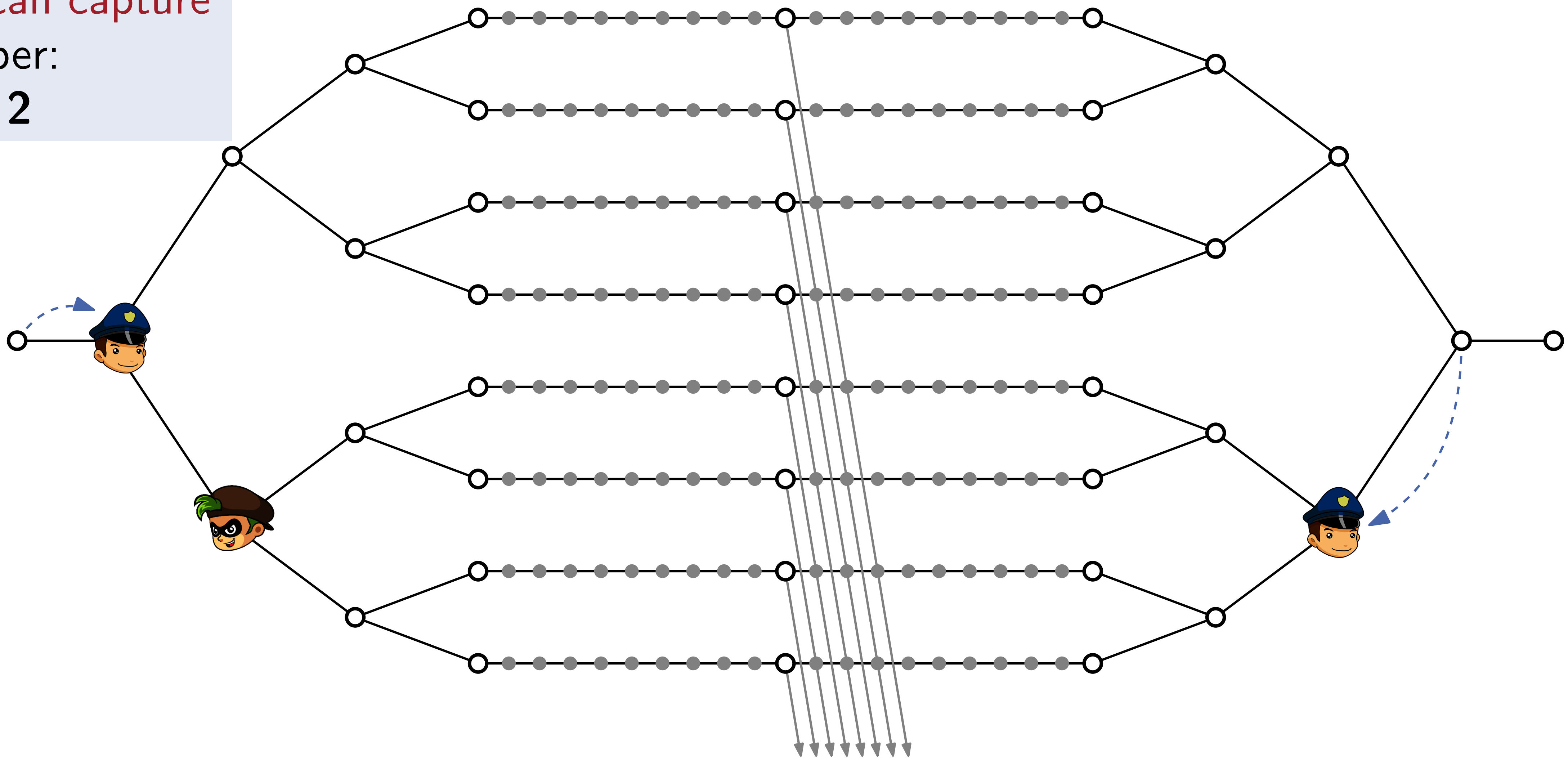

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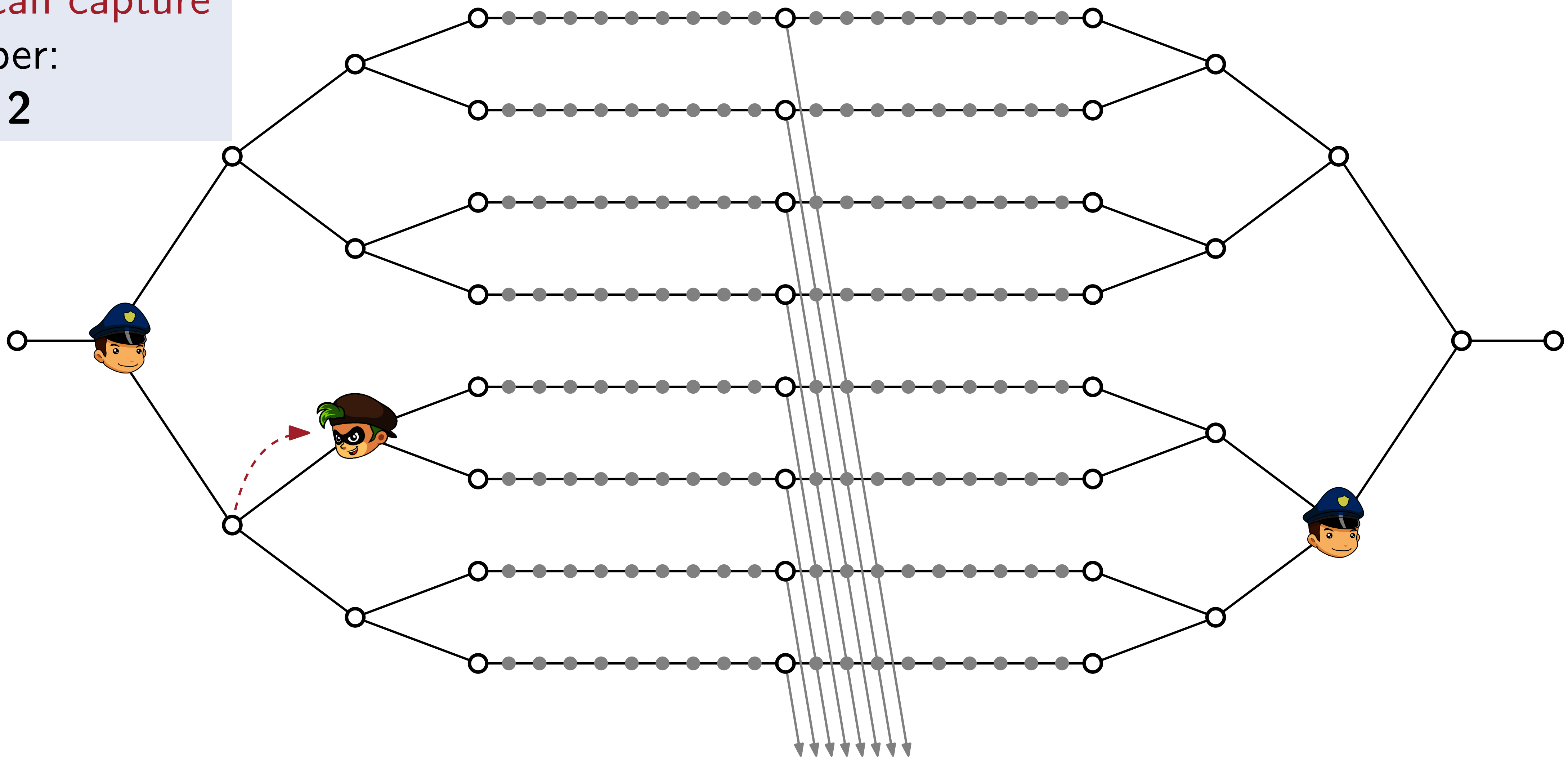

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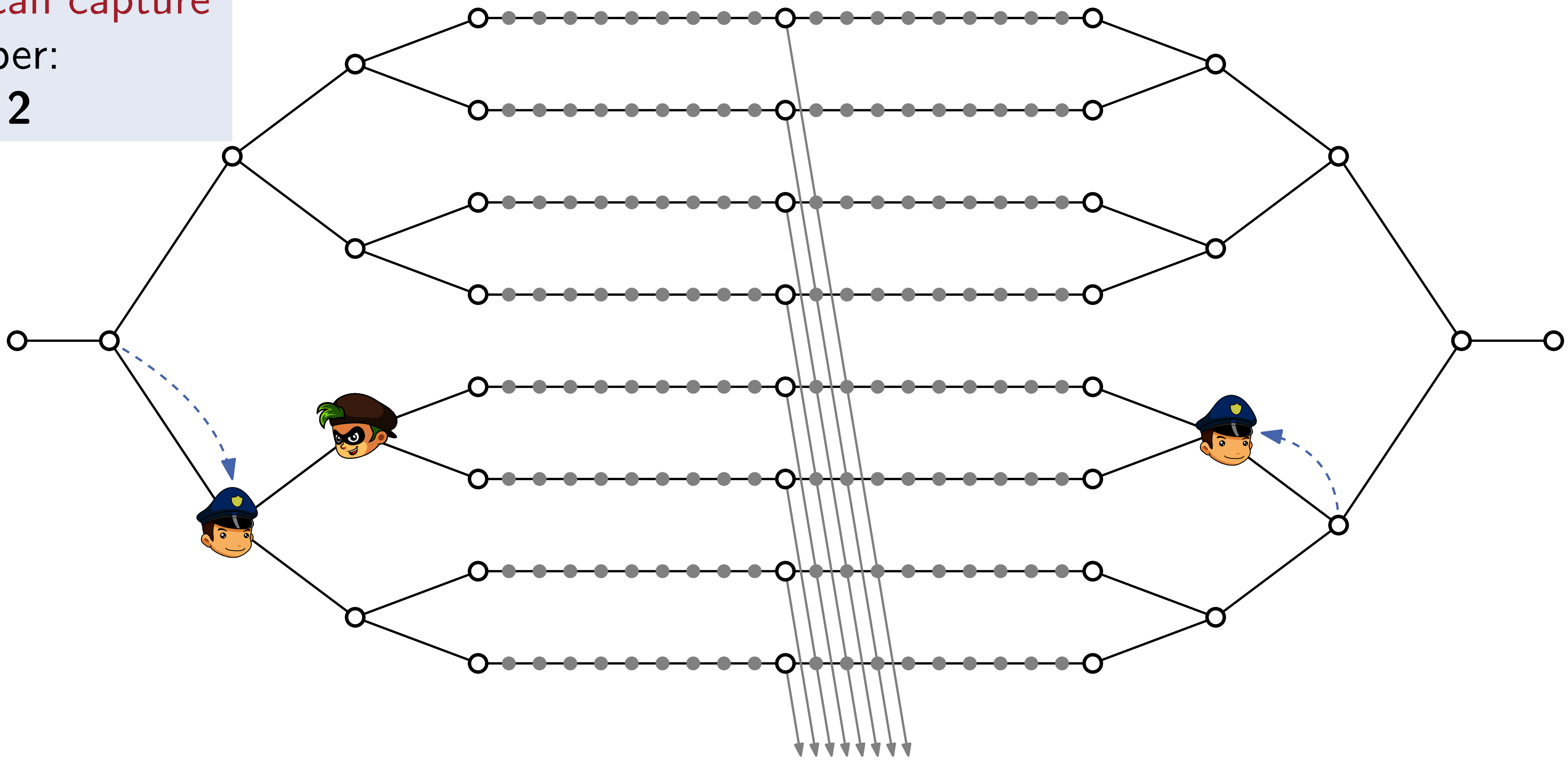

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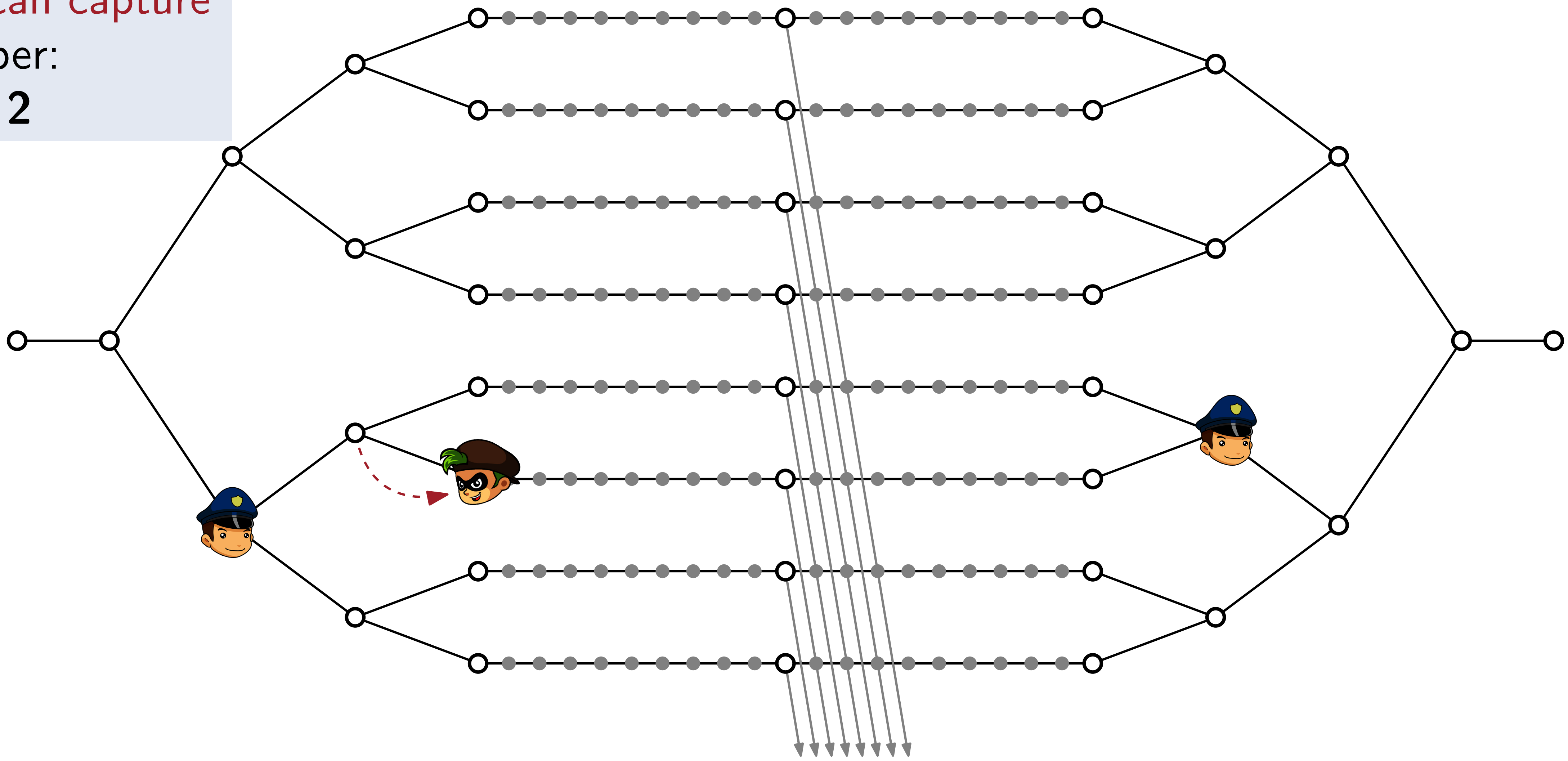

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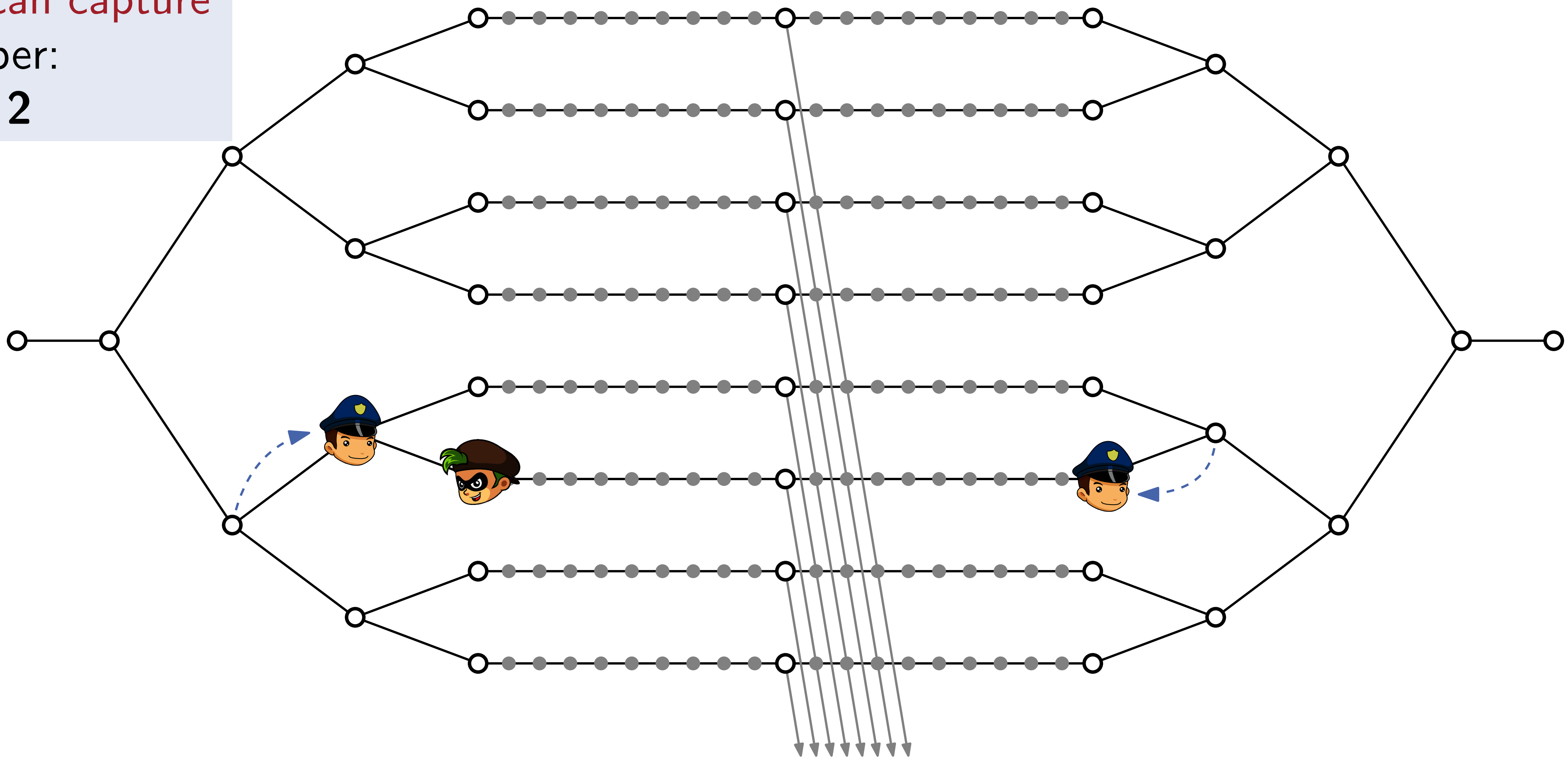

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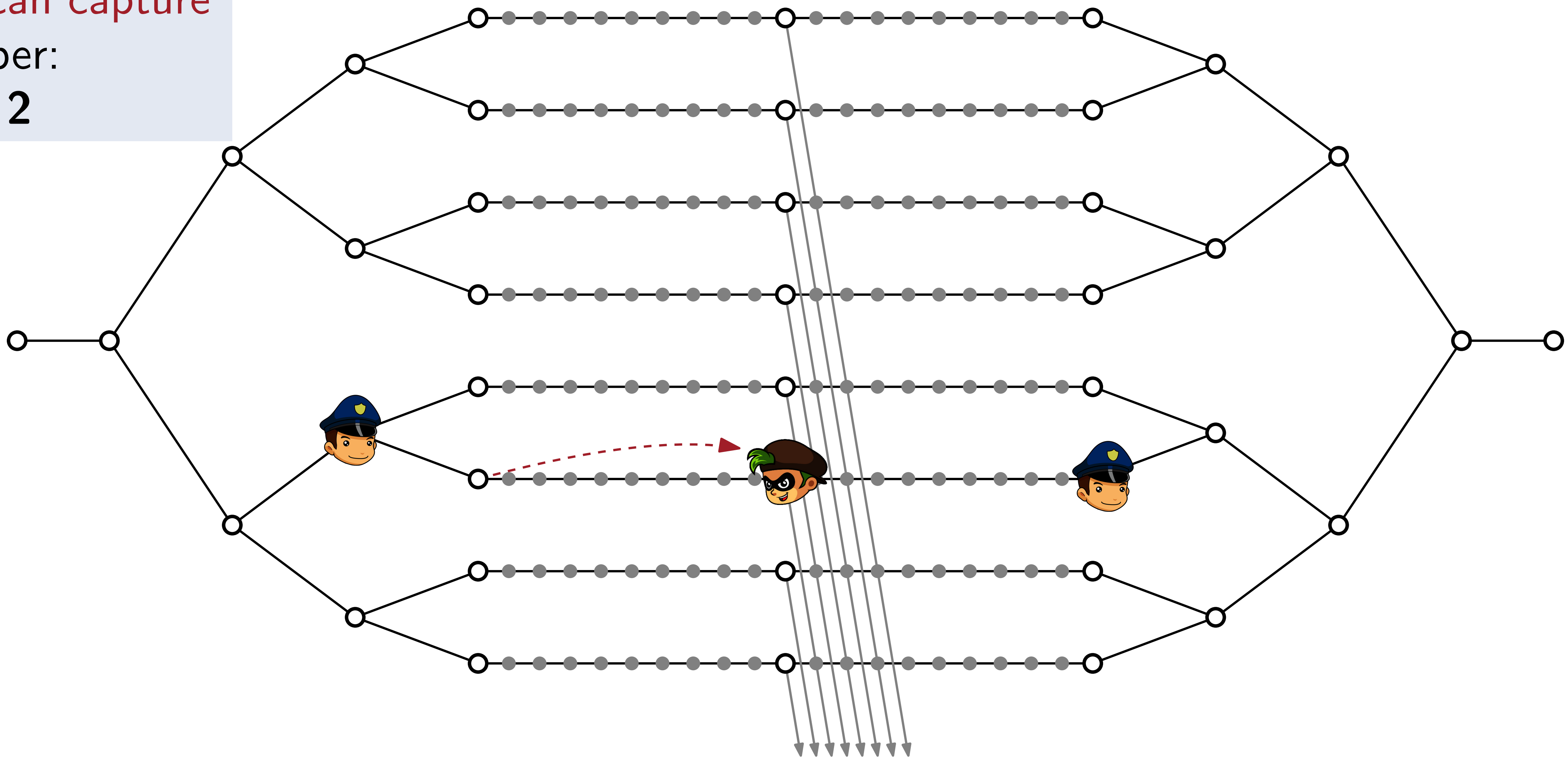

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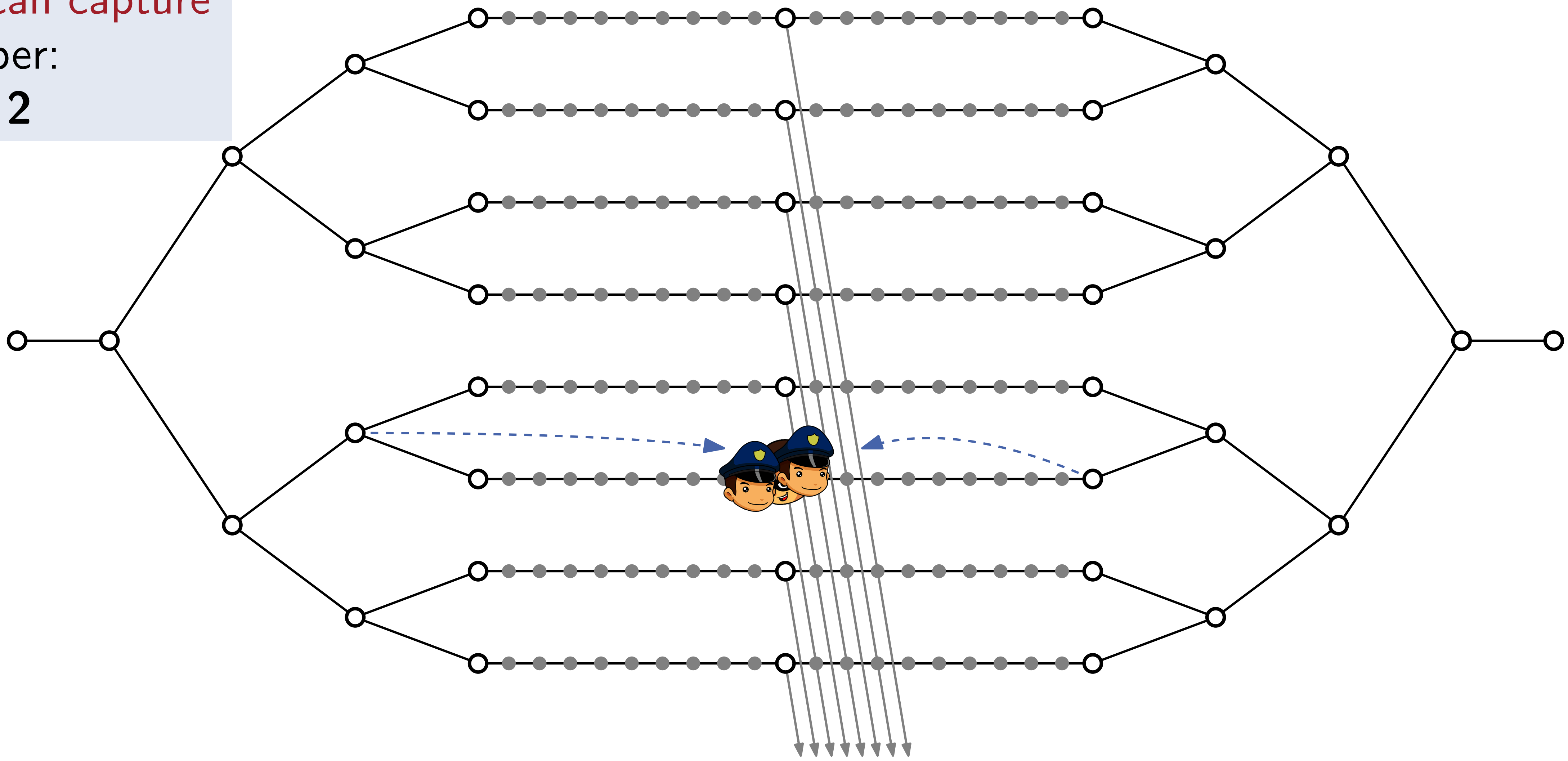

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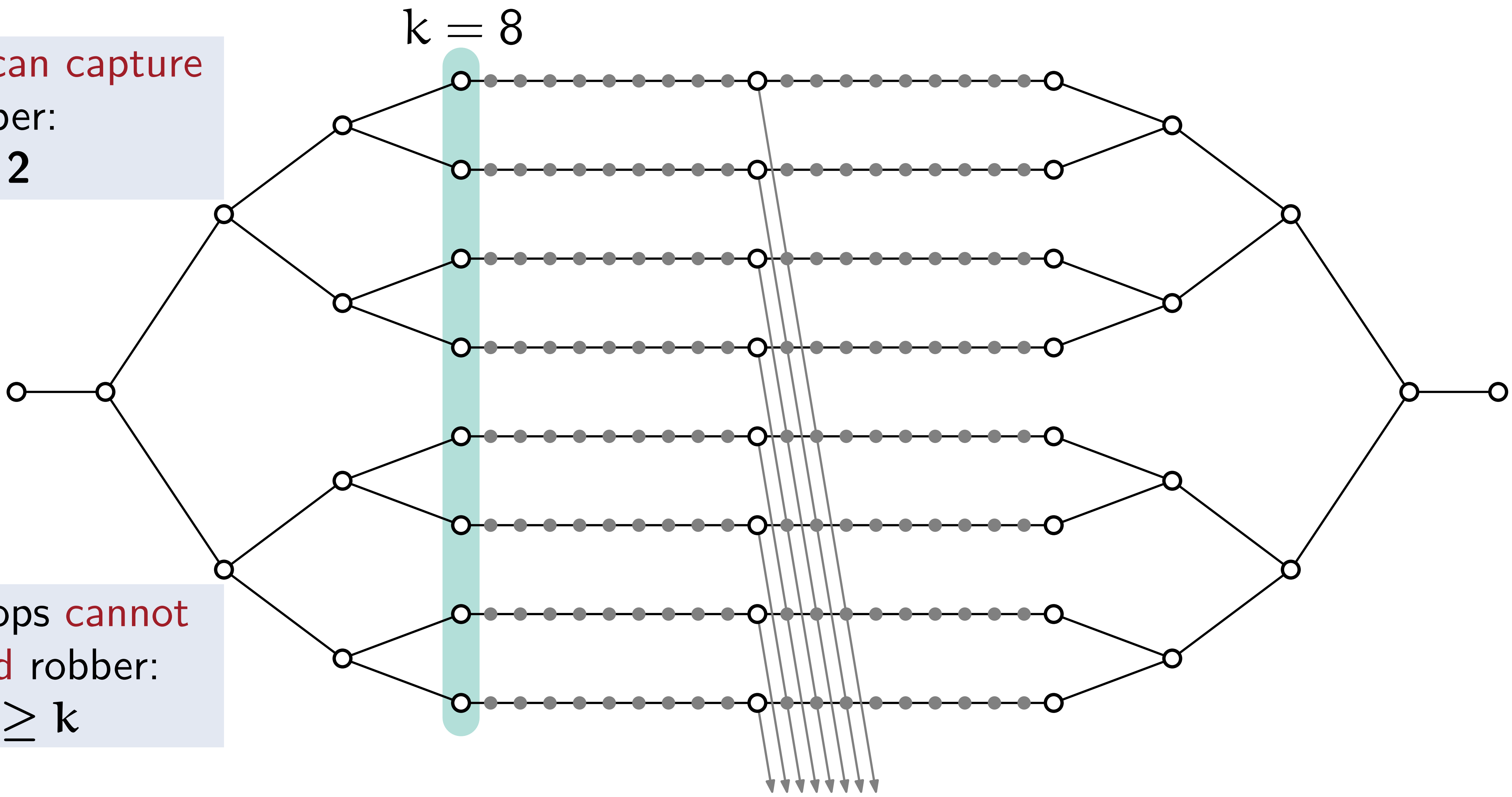

$k - 1$ cops cannot surround robber:

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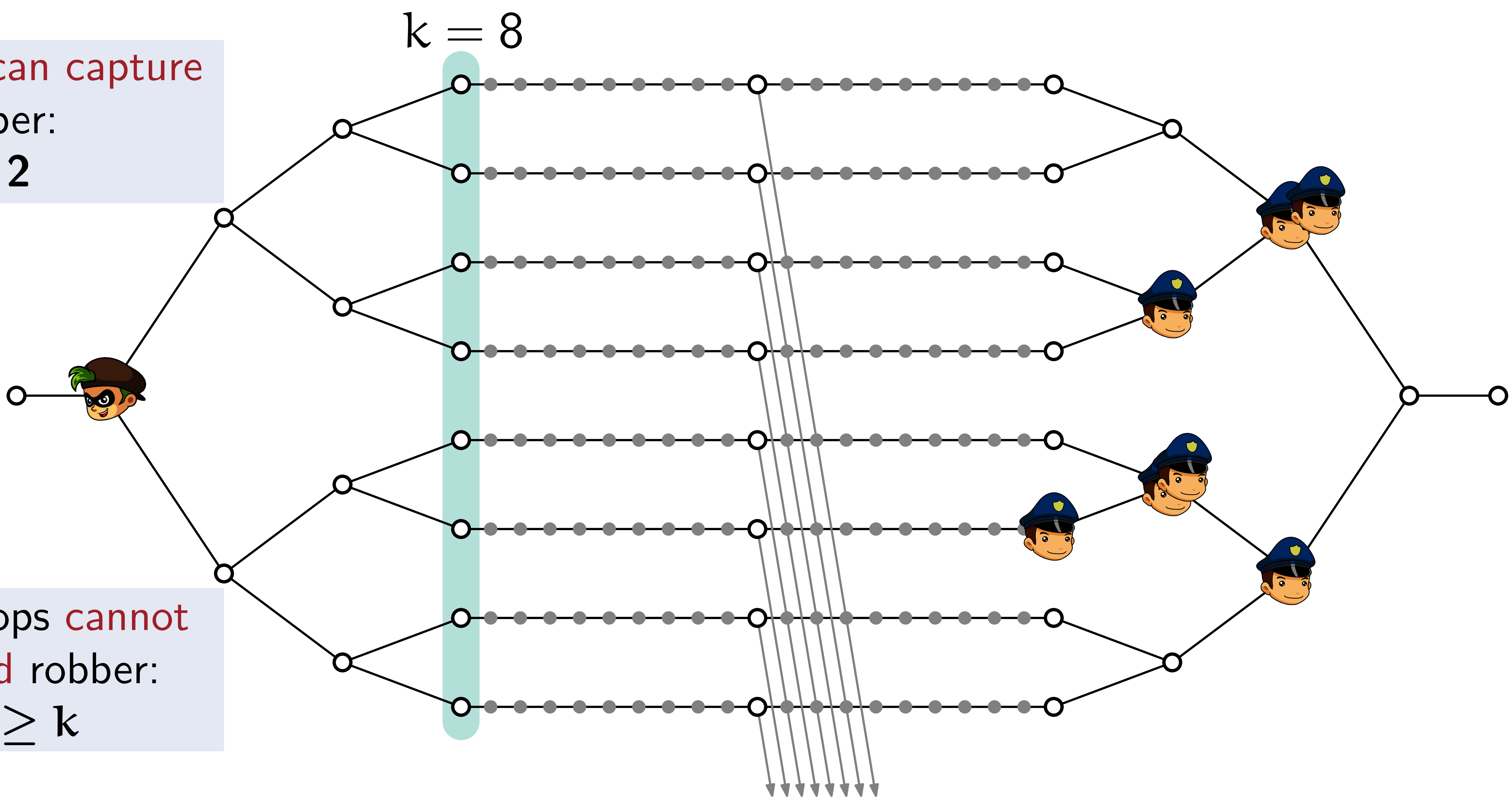

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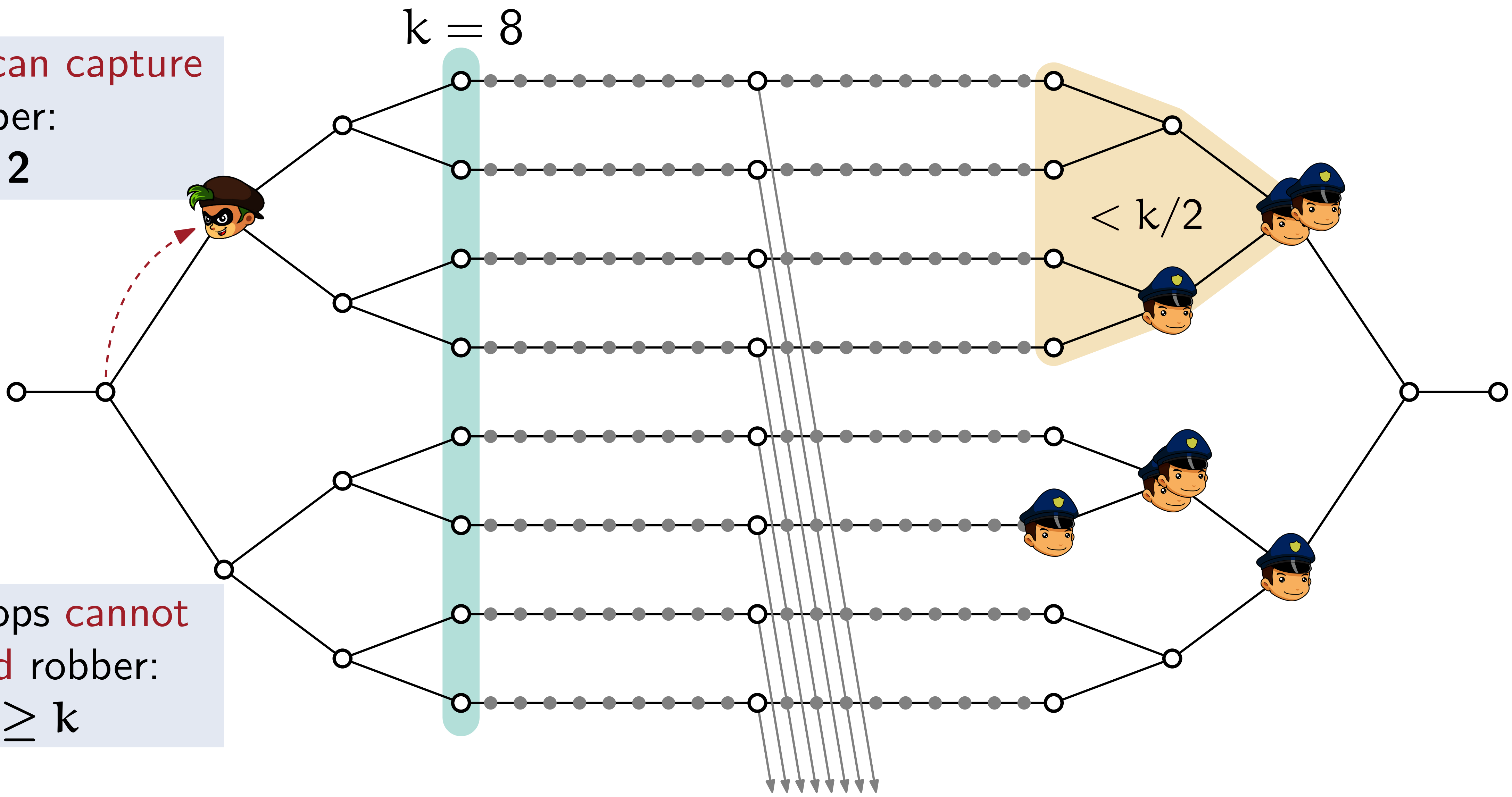

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2 cops can capture the robber:
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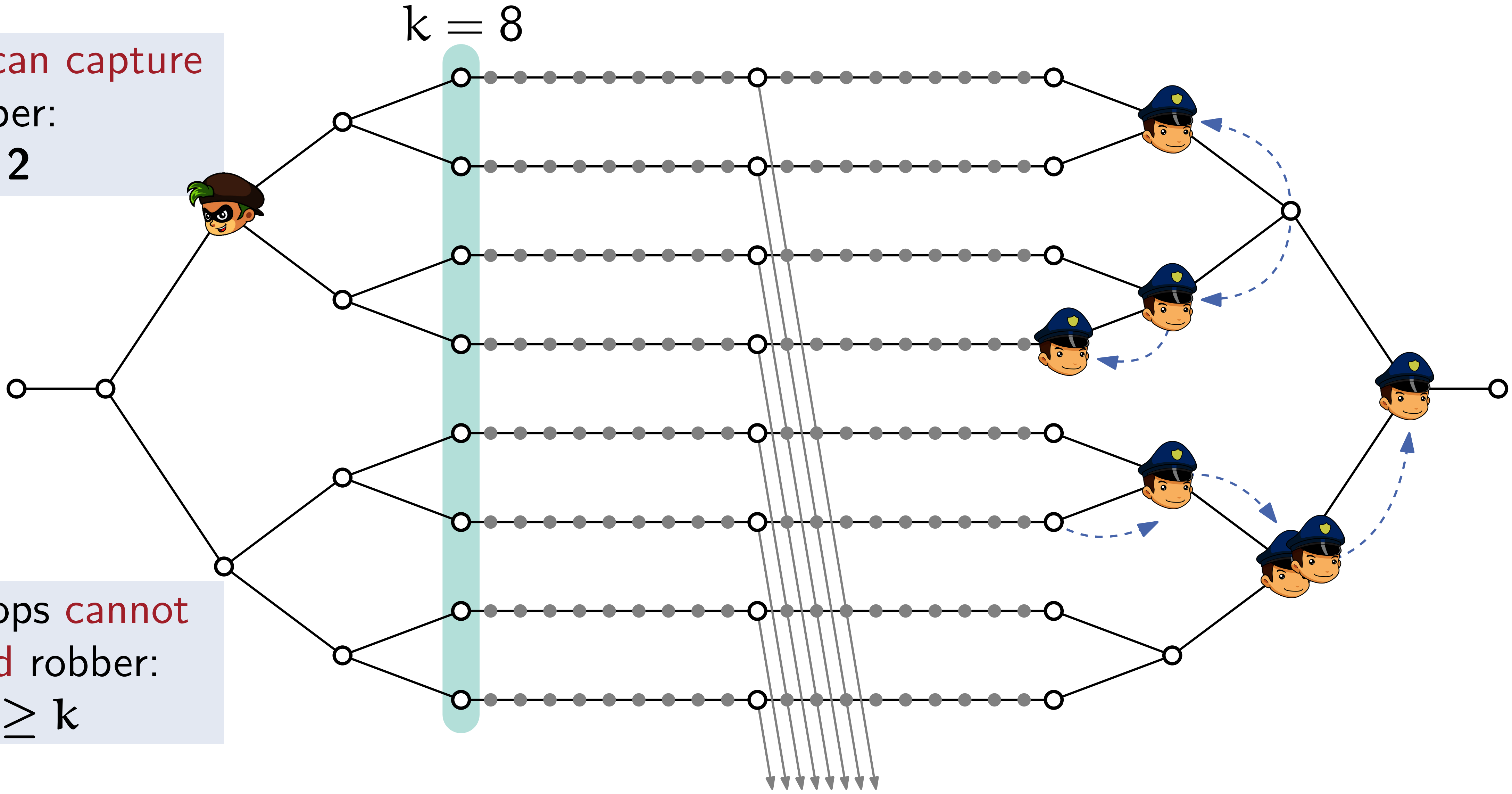

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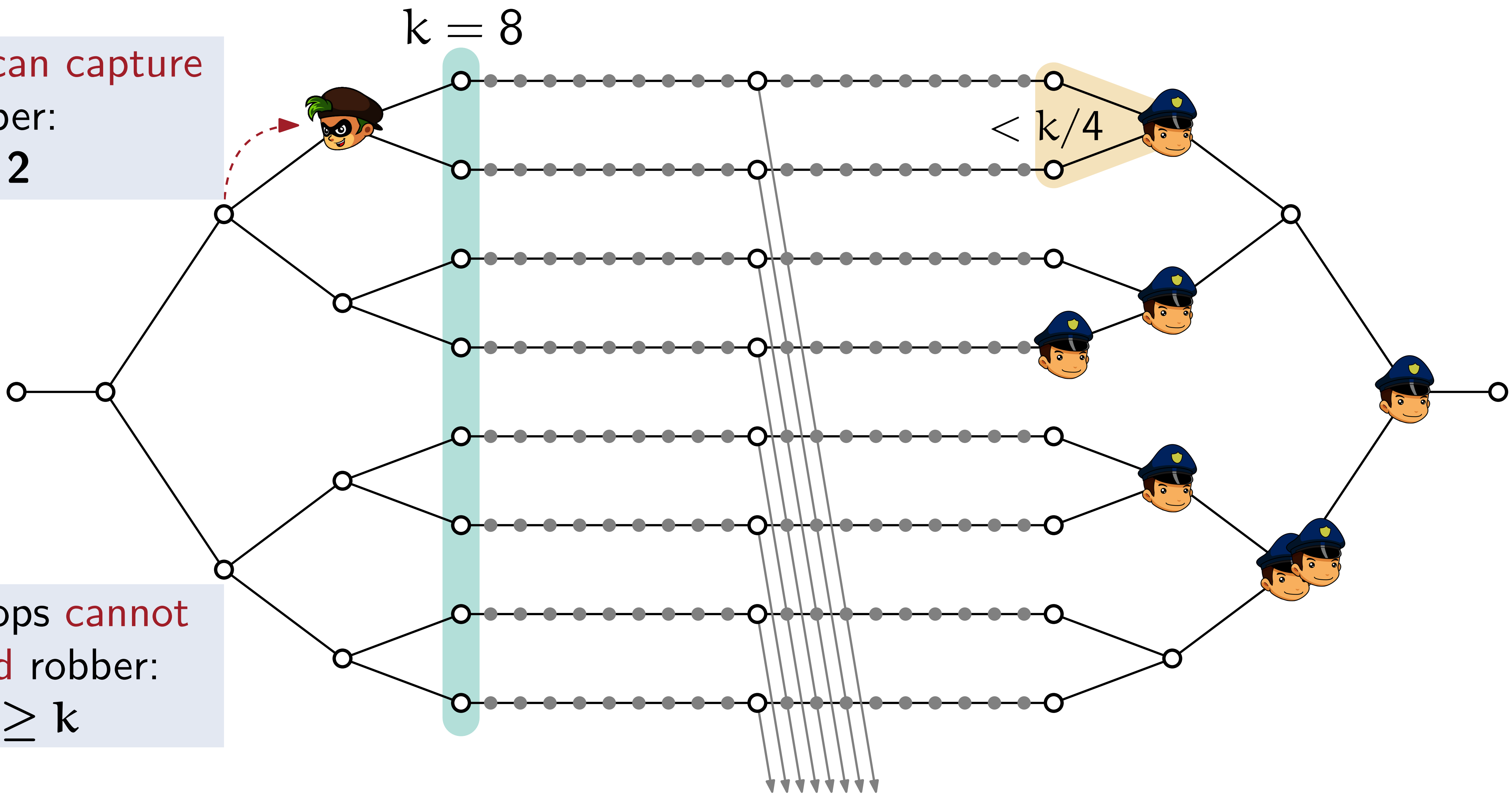

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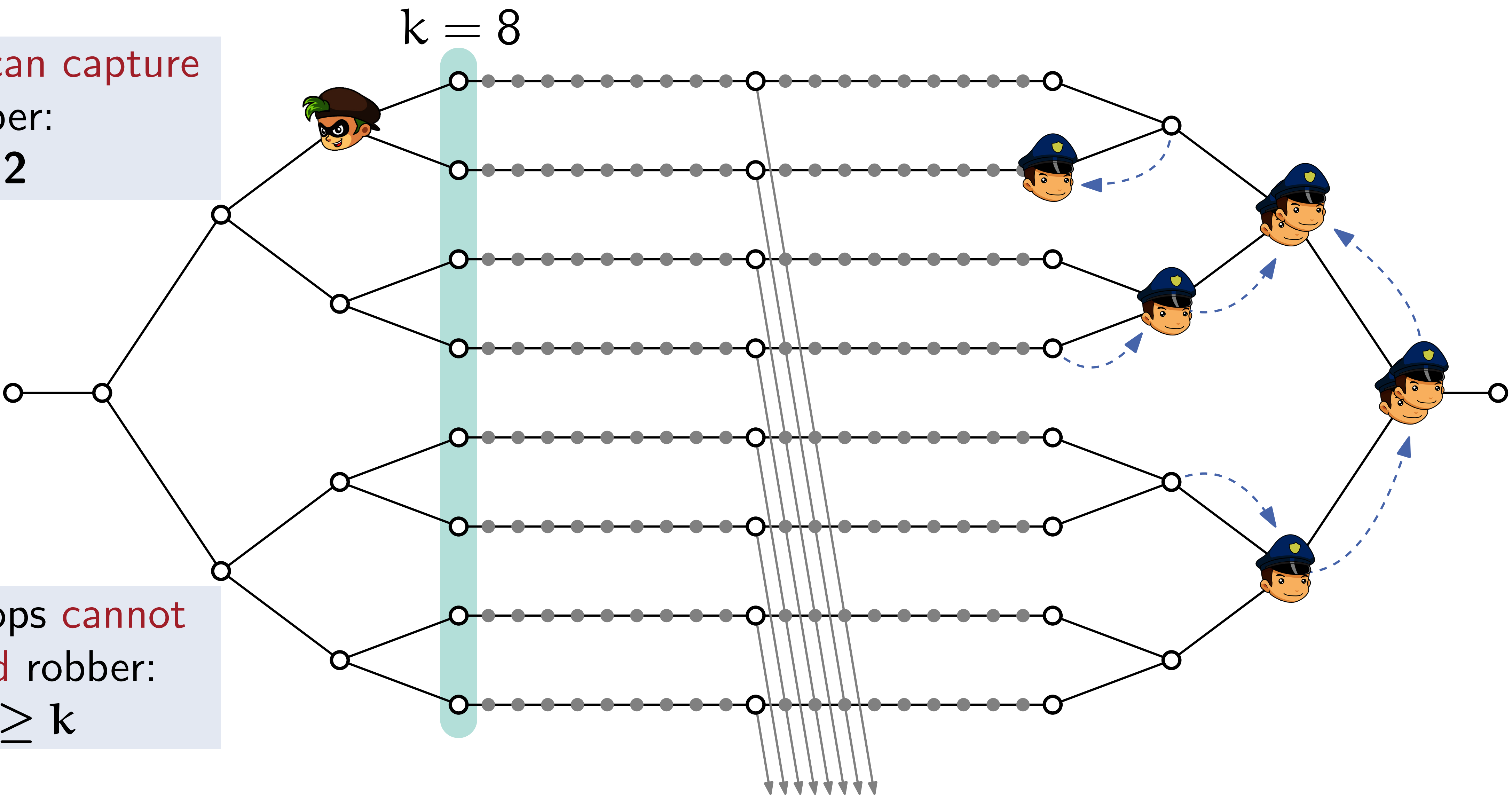

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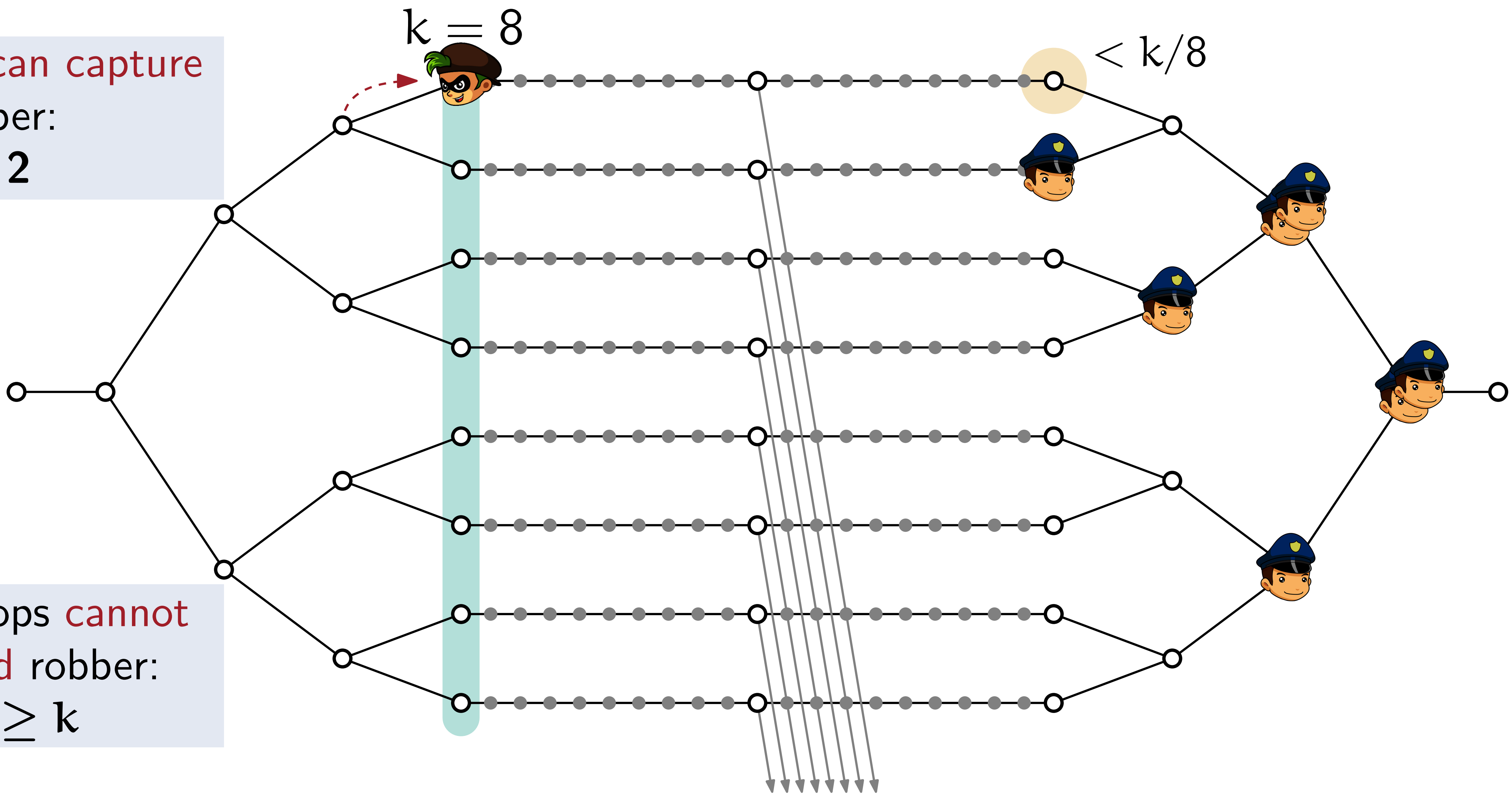

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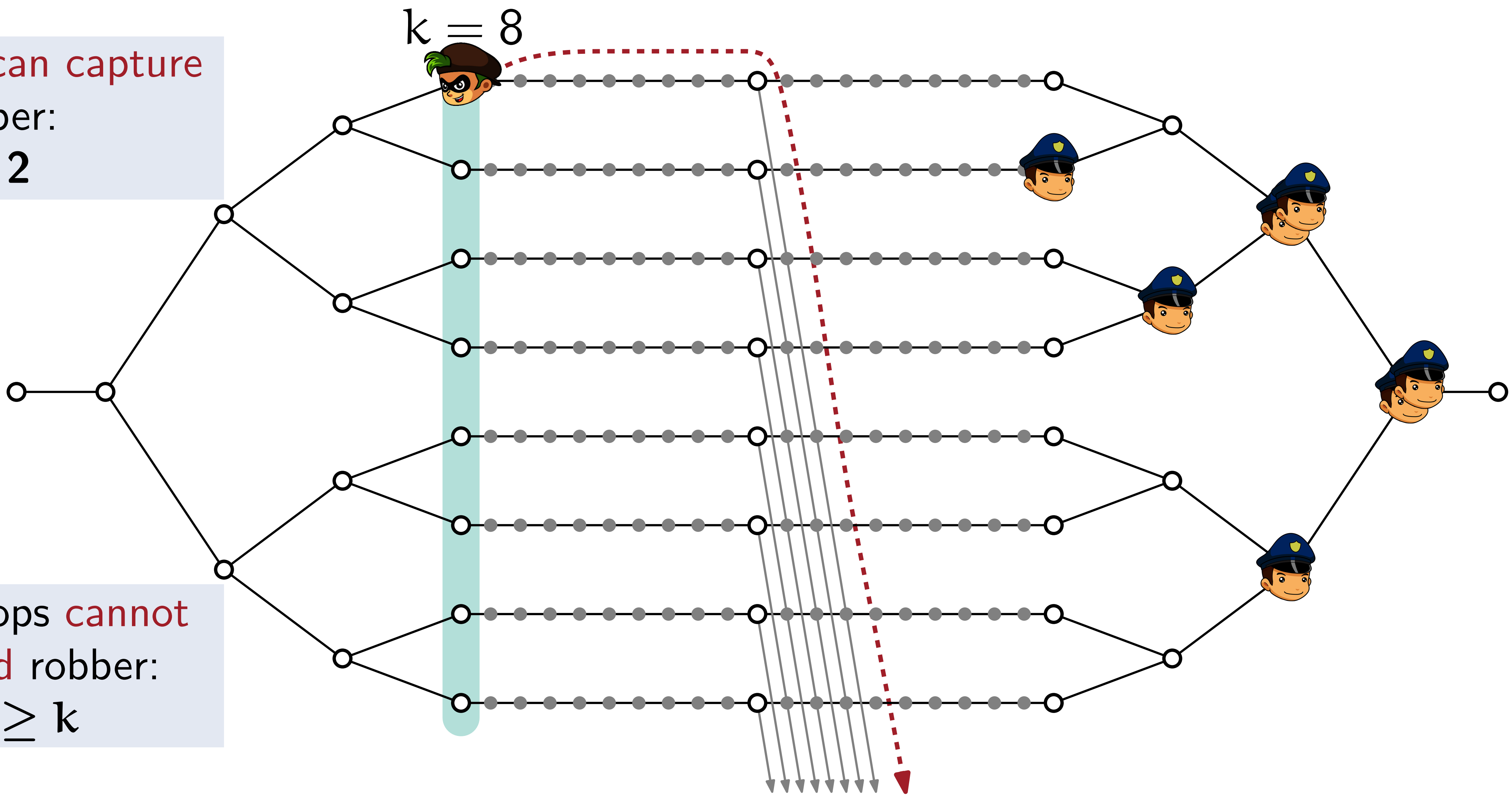

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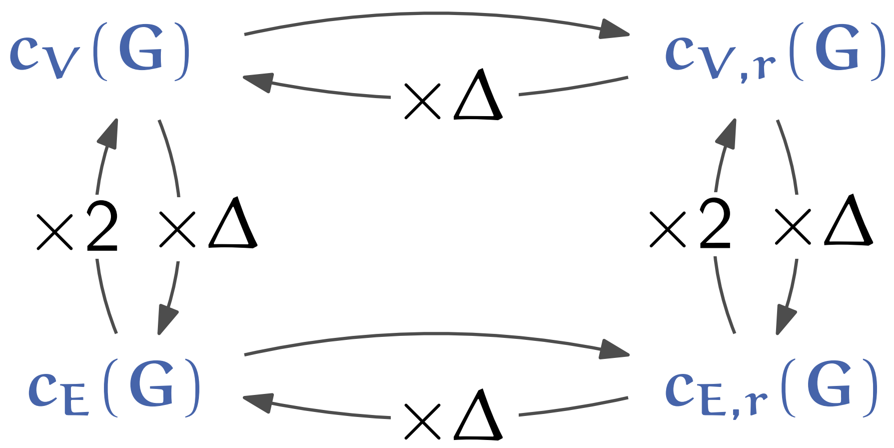
Conclusion

- All surrounding cop numbers are within 2Δ of each other.



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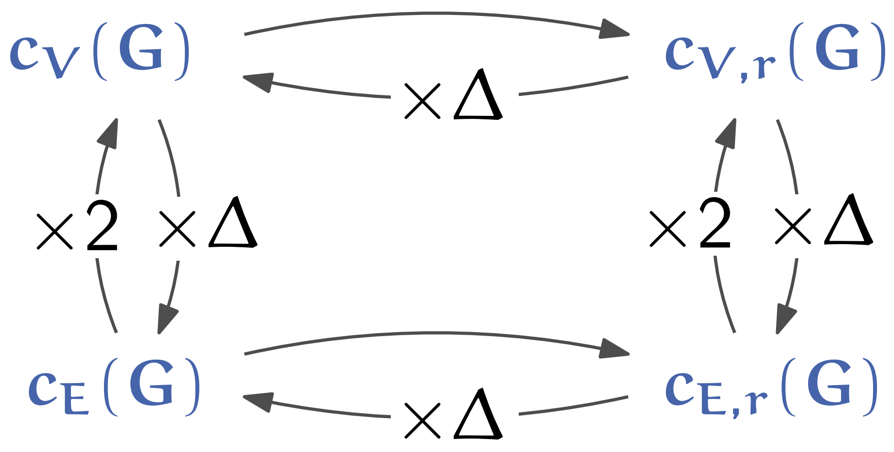

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Open Problems

- Improve our (small) additive and multiplicative constants.
- Other upper bounds of surrounding cop numbers in terms of $c(G)$?

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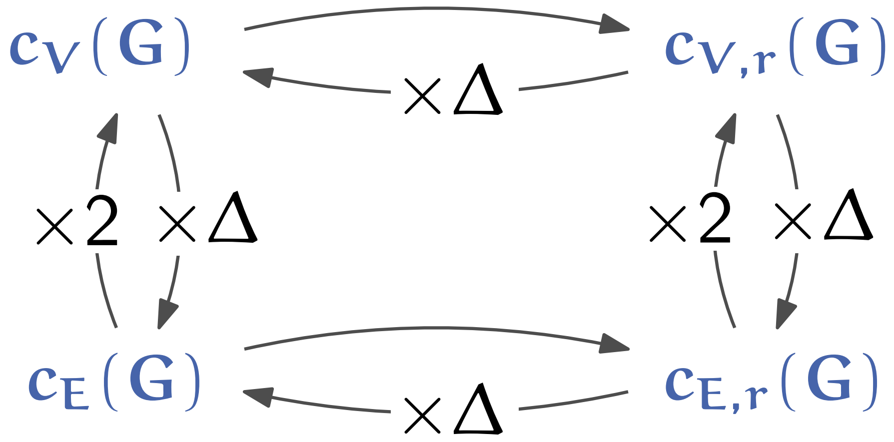

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Thank you!