Karlsruhe Institute of Technology

The Local Queue Number of Graphs with Bounded Treewidth

Laura Merker and Torsten Ueckerdt | GD 2020

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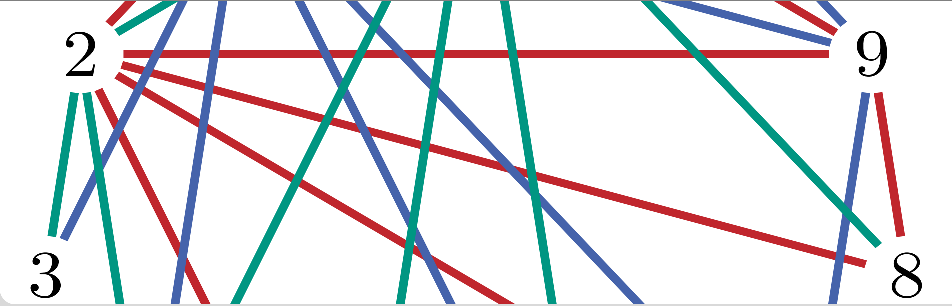
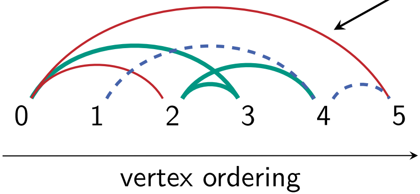
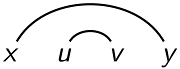




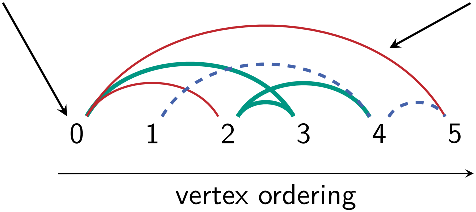
No  i.e. $xy \in Q, uv \in Q', x \prec u \prec v \prec y \implies Q = Q'$

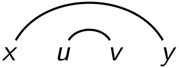
Queue Number

$\text{qn}(G) = \min k$ such that there is a k-queue layout for G

k-local, i.e. at most *k* queues at each vertex

partition of the edges into queues

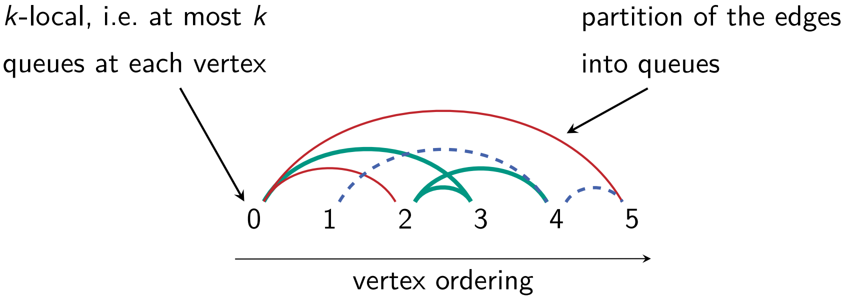


No 

i.e. $xy \in Q, uv \in Q', x \prec u \prec v \prec y \implies Q = Q'$

Local Queue Number

$\text{qn}_\ell(G) = \min k$ such that there is a *k*-local queue layout for *G*



Theorem 1

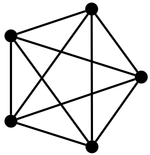
For any $d \geqslant 3$ and infinitely many n, there exist n-vertex graphs with local queue number at most $d + 2$ but queue number $\Omega(\sqrt{d} n^{1/2-1/d})$.

Maximum (Local) Queue Number

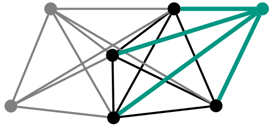
	max qn	max qn_ℓ
planar graphs	$\geqslant 4$ (Alam et al., 2020)	$\geqslant 3$
	$\leqslant 49$ (Dujmović et al., 2019)	$\leqslant 4$
treewidth 1	1 (Heath and Rosenberg, 1992)	1
treewidth 2	3 (Rengarajan and Veni Madhavan, 1995; Wiechert, 2017)	3
treewidth $k \geqslant 3$	$\geqslant k+1$ (Wiechert, 2017)	$\geqslant \lceil k/2 \rceil + 1$
	$\leqslant 2^k - 1$ (Wiechert, 2017)	$\leqslant k+1$

k-Trees

maximal graphs with treewidth k



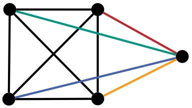
K_{k+1}



attach vertex to K_k

Theorem 3

Every k-tree admits a $(k+1)$-local queue layout.



k queues to parent clique

$+ 1$ queue to children

Theorem 2

The local queue number is tied to the maximum average degree.

series of two-player games

Theorem 4

There is a graph with treewidth 2 and local queue number 3.

Theorem 5

For every $k > 1$, there is a graph G with treewidth k and $\mathrm{qn}_\ell(G) \geqslant \lceil k/2 \rceil + 1$.

Corollary 6

The maximum local queue number of planar graphs is either 3 or 4.

Lower bounds: Two-player games

Start with a k-clique (here: $k = \ell = 2$)

In each round:

Alice chooses a k-clique and a number of vertices to add

Bob inserts the new vertices into the vertex ordering and assigns the new edges to queues such that ...

Game (i): the layout is ℓ-local

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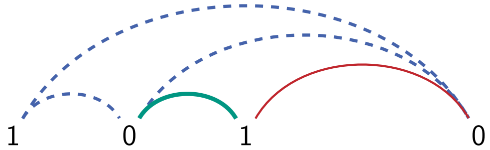

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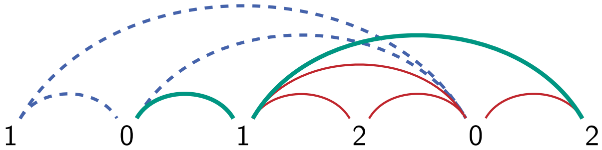

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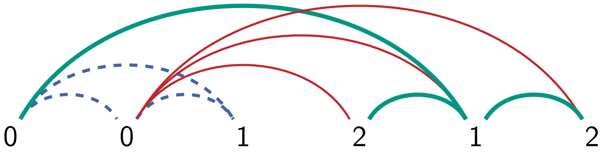
Game (ii): children to the right in the first round

plus all previous rules



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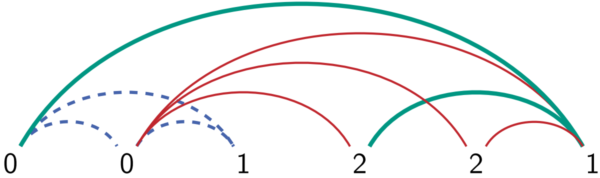
Game (iii): new vertices consecutively

plus all previous rules



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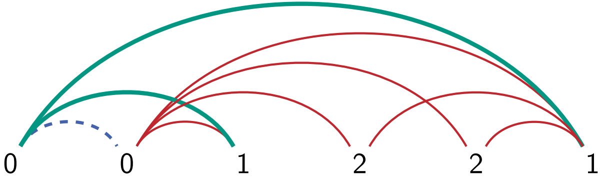
Game (iv): twin edges in the same queue

plus all previous rules



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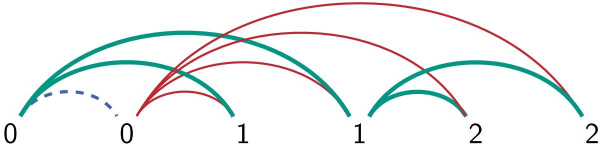
Game (v): all children to the right and edges to parents pw different

plus all previous rules



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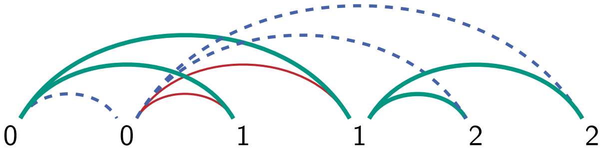
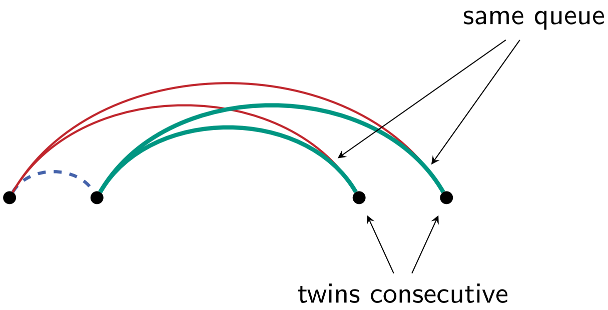
Game (v'): new edges not in the same queue as parent edge

plus all previous rules



Next rule: all children to the right



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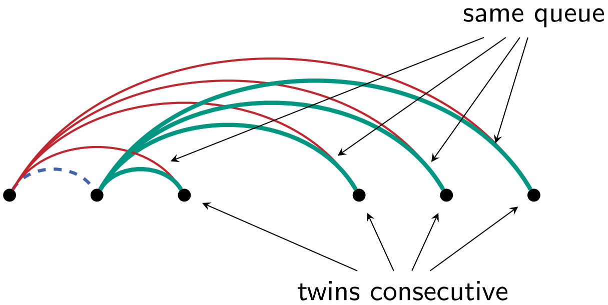

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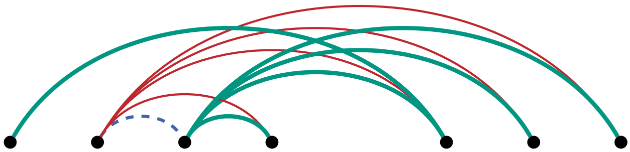

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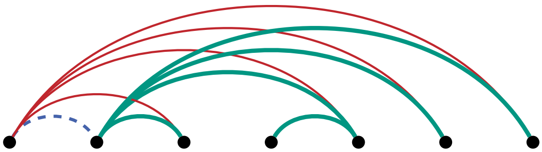

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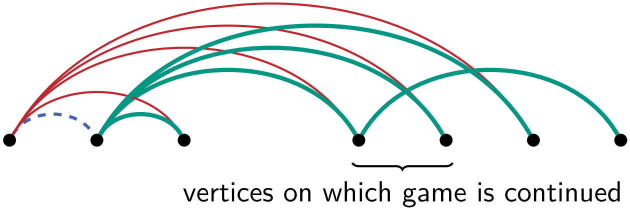


Game (v') with 2-trees and a 2-local queue layout

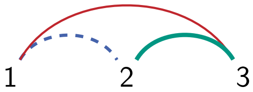
Theorem 4

There is a graph with treewidth 2 and local queue number 3.

Corollary 6

There is a planar graph with local queue number at least 3.

Game (v') with 2-trees and a 2-local queue layout



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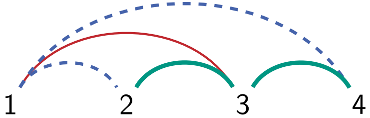

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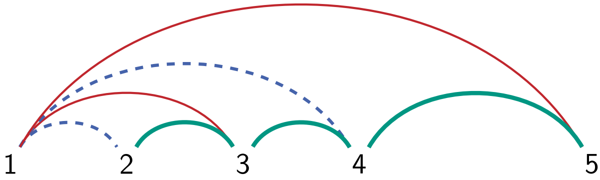

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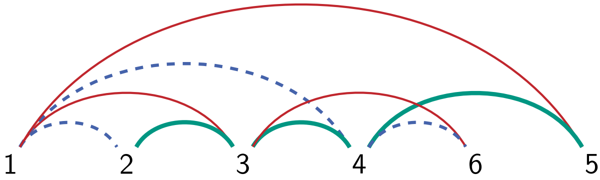

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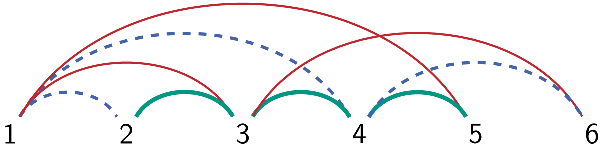

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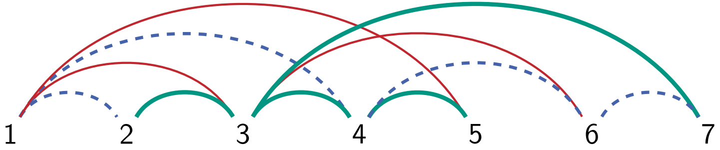

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... two more games

Lemma

If there is a k-tree such that every ℓ-local queue layout contains a k'-clique with non-nesting children,

$\Downarrow$

then there is a k-tree with local queue number at least $\ell + 1$,

where $1 < k' \leqslant k$ and $\ell \leqslant k'$.

true for $k = \ell = 2$

← true for $\ell \leqslant \lceil k/2 \rceil$

true for all $\ell \leqslant k$?

otherwise:

Theorem 5

For every $k > 1$, there is a graph G with treewidth k and $\mathrm{qn}_\ell(G) \geqslant \lceil k/2 \rceil + 1$.

Open Questions

- What is the maximum local queue number of k-trees?
  (lower bound: $\lceil k/2 \rceil + 1$, upper bound: $k + 1$)
  $\rightsquigarrow$ find a clique with non-nesting children
- Can our techniques be used to improve the lower bound on the queue number of k-trees?
  (Wiechert (2017): lower bound: $k + 1$, upper bound: $2^k - 1$)
- Is the maximum local queue number of planar graphs 3 or 4?

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Thank you!