A Comparative Analysis of Switchings in Static and Dynamic Power Grids

Master’s Thesis Presentation
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Overview

- Shift towards renewables ⇒ Decentrality & volatility
- **Switching**: Remove transmission lines to improve the power flow
- Can a static algorithm guarantee safe dynamic operation?
- Static and dynamic model for high voltage grid
Static Model \cite{GRW18}

Edge flow:
\[ f(e) = b(e) \cdot (\Theta(j) - \Theta(i)) \]

Capacity:
\[ \text{cap} : E \to \mathbb{R} \geq 0 \]

Potential angle:
\[ \Theta : V \to \mathbb{R} \geq 0 \]

Susceptance:
\[ b : E \to \mathbb{R} \geq 0 \]
Static Model \cite{GRW+18}

\begin{equation*}
\text{Flow} = 3
\end{equation*}

Edge flow: \[ f(e) \]

Capacity: \[ \text{cap} : E \rightarrow \mathbb{R}_{\geq 0} \]
Static Model [GRW⁺18]

Edge flow: \( f(e) = b(e) \cdot (\Theta(j) - \Theta(i)) \)

Capacity: \( \text{cap}: E \rightarrow \mathbb{R}_{\geq 0} \)

Potential angle: \( \Theta: V \rightarrow \mathbb{R}_{\geq 0} \)

Susceptance: \( b: E \rightarrow \mathbb{R}_{\geq 0} \)

Flow = \( \frac{3}{2} \)
Static Model \cite{GRW18}

\begin{align*}
\text{Edge flow:} & \quad f(e) = b(e) \cdot (\Theta(j) - \Theta(i)) \\
\text{Capacity:} & \quad \text{cap} : E \rightarrow \mathbb{R}_{\geq 0} \\
\text{Potential angle:} & \quad \Theta : V \rightarrow \mathbb{R}_{\geq 0} \\
\text{Susceptance:} & \quad b : E \rightarrow \mathbb{R}_{\geq 0}
\end{align*}
Switching Heuristic \cite{GRW+18}

Let \( \pi(s, t) \) a path from \( s \) to \( t \):

\[
\| \pi \| = \sum_{e \in \pi} b(e)^{-1} (= \text{length of } \pi)
\]
Switching Heuristic [GRW+18]

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\[
\text{cap}(\pi) = \min\{\text{cap}(e) \mid e \in \pi\}
\]

\[
\|\pi_1\| = 3 \\
\text{cap}(\pi_1) = 2
\]
Switching Heuristic \cite{GRW+18}

Let $\pi(s, t)$ a path from $s$ to $t$:

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$$\Delta \theta(\pi) = \|\pi\| \cdot \text{cap}(\pi)$$

$$\|\pi_1\| = 3$$

$$\text{cap}(\pi_1) = 2$$

$$\Delta \theta(\pi_1) = 3 \cdot 2 = 6$$
Switching Heuristic [GRW+18]

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\[
\Delta \theta(\pi) = \|\pi\| \cdot \text{cap}(\pi)
\]

\[
\pi_{\text{lim}}(s, t) = \min\{\Delta \theta(\pi) \mid \pi \text{ is s-t-path}\}
\]

\[
\|\pi_1\| = 3
\]
\[
\text{cap}(\pi_1) = 2
\]
\[
\Delta \theta(\pi_1) = 3 \cdot 2 = 6
\]
Switching Centrality: Which edge limits the power flow in most cases?
Switching Heuristic [GRW+18]

Switching Centrality: Which edge limits the power flow in most cases?

\[ c_{sw}(e) := \frac{\#\pi_{\text{lim}}(s, t, e)}{\#\pi_{\text{lim}}(s, t)} \]
Switching Heuristic \cite{GRW18}

Switching Centrality: Which edge limits the power flow in most cases?

\[
c_{sw}(e) := \frac{1}{m} \sum_{s \in V} \sum_{t \in V \setminus \{s\}} \frac{\# \pi_{lim}(s, t, e)}{\# \pi_{lim}(s, t)}
\]
Dynamic Model

What are the effects of switching over time?
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Switching of $e_1$ at $t_0 = 100$
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⇒ Phase angles $\phi_i$ adopt

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Dynamic Model

What are the effects of switching over time?
⇒ Phase angles $\phi_i$ adopt
⇒ Frequencies $\dot{\phi}_i$ are perturbed

Switching of $e_1$ at $t_0 = 100$
Coupled Oscillators [FNP08]

- How do phase and frequency develop over time?
- Characterize generators and loads by their rotations
Coupled Oscillators \[\text{[FNP08]}\]

- How do phase and frequency develop over time?
- Characterize generators and loads by their rotations

\[
\ddot{\phi}_i = \underbrace{P_i}_{\text{generated power}} - \underbrace{\alpha \dot{\phi}_i}_{\text{damping}} + \sum_{j=1}^{n} \underbrace{K_{ij}\sin(\phi_j - \phi_i)}_{\text{transmitted power}}
\]

![Diagram of coupled oscillators](image)
Coupled Oscillators [FNP08]

- How do phase and frequency develop over time?
- Characterize generators and loads by their rotations

\[
\ddot{\phi}_i = P_i - \alpha \dot{\phi}_i + \sum_{j=1}^{n} K_{ij} \sin(\phi_j - \phi_i)
\]

- generated power
- damping
- transmitted power

\[
\phi_0 \quad \phi_1 \quad \phi_2
\]
Coupled Oscillators [FNP08]

- How do phase and frequency develop over time?
- Characterize generators and loads by their rotations

\[
\ddot{\phi}_i = P_i - \alpha \dot{\phi}_i + \sum_{j=1}^{n} K_{ij} \sin(\phi_j - \phi_i)
\]

- \(\phi\) represents the phase
- \(P\) represents generated power
- \(\alpha\) represents damping
- \(K\) represents transmitted power

Diagram:

- \(\phi_0\), \(\phi_1\), \(\phi_2\)
- \(K_{12}\)
Evaluation

- NESTA Dataset [CGS14]
- Compute switching centrality
- Solve dynamic equations
- Compare properties
Does the switching centrality correlate to system stability?

- Small-signal analysis
- Mean-field theory
Stability

Does the switching centrality correlate to system stability?

Switchings with highest centrality yield a stable network.

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Stability

Does the switching centrality correlate to system stability?

⇒ Switchings with highest centrality yield a stable network.
Load Redistribution

Is more load redistributed after switching with lower centrality?

- Edge load: Capacity saturation with load flow
Load Redistribution

Is more load redistributed after switching with lower centrality?

- Edge load: Capacity saturation with load flow

Highest centrality

Lowest centrality
Load Redistribution

- Switching of edge (22, 23)
Accumulated Load Redistribution

Load redistributed over all edges: $\Delta L_{acc}$

Lower centrality leads to more redistribution in many cases.
Accumulated Load Redistribution

- Load redistributed over all edges: $\Delta L_{acc}$

$\Rightarrow$ Lower centrality leads to more redistribution in many cases.
Conclusion

- Connect static and dynamic model
- Centrality measure provides useful switchings
- Stable net for high centrality
- More load redistribution for low centrality
- Exceptions due to topological properties
Future Work

- Limit heuristic to certain topologies (2-connected, ...)
- Refine dynamic model
- Time-expanded networks
  - Invalid states
  - Continuous Time-expanded
Sources


Small-Signal Analysis

1) Reduce to first order

\[ \dot{\phi}_i = \omega_i \]

\[ \dot{\omega}_i = P_i - \alpha \omega_i + \sum_{j=1, j \neq i}^{n} K_{ij} \sin(\phi_j - \phi_i) = f_i(\phi, \omega) \]

2) Find equilibrium point \( a = (\phi'_1, \ldots, \phi'_N, \omega'_1, \ldots, \omega'_N) \)

\[ 0 = \omega'_i \]

\[ 0 = P_i + \sum_{j=1, j \neq i}^{n} K_{ij} \sin(\phi'_j - \phi'_i) \]
3) Linearization:

\[
\begin{pmatrix}
\dot{\phi} \\
\dot{\omega}
\end{pmatrix} = \begin{pmatrix}
\omega \\
f(a)
\end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix}
\dot{\phi} \\
\dot{\omega}
\end{pmatrix} = \begin{pmatrix}
\omega' \\
f'(a)
\end{pmatrix} + J(a) \cdot (\begin{pmatrix}
\phi \\
\omega
\end{pmatrix} - a)
\]

4) Calculate eigenvalues $\lambda_i$ for $J(a)$:

- Stable: $\Re(\lambda_i) < 0 \ \forall i$
- Unstable: $\exists i : \Re(\lambda_i) > 0$
- Unknown (“critical”): $\Re(\lambda_i) \leq 0 \ \forall i \land \exists i : \Re(\lambda_i) = 0$
Edge Load

\[ \ddot{\phi}_i = P_i - \alpha \dot{\phi}_i + \sum_{j=1}^{n} K_{ij} \sin(\phi_j - \phi_i) = F_{ji} \]

Edge load: \[ L_{ij} = \frac{F_{ij}}{K_{ij}} = \sin(\phi_j - \phi_i) \]
Accumulated Load Redistribution

\[ L_{ij}^1 = L_{ij}(t_0 - \delta), \quad L_{ij}^2 = L_{ij}(T) \]

\[ \Delta L_{ij_{acc}} := \sum_{\{i,j\} \in E} |L_{ij}^1 - L_{ij}^2| \]
Mean-Fields

- Centroid on complex plane
- $r$: phase coherence
- $\psi$: average phase angle

$$re^{i\psi} = \frac{1}{N} \sum_{j=1}^{n} e^{i\phi_j}$$
Mean-Fields

\[ r_{\text{mean}} := \text{Re} \left( \frac{1}{N} \sum_{i=1}^{n} e^{i \phi_j(t)} \right) \]
Mean-Fields

\[ r_{\text{mean}} := \text{Re} \left( \frac{1}{N} \langle \sum_{i=1}^{n} e^{i\phi_j(t)} \rangle_t \right) \]