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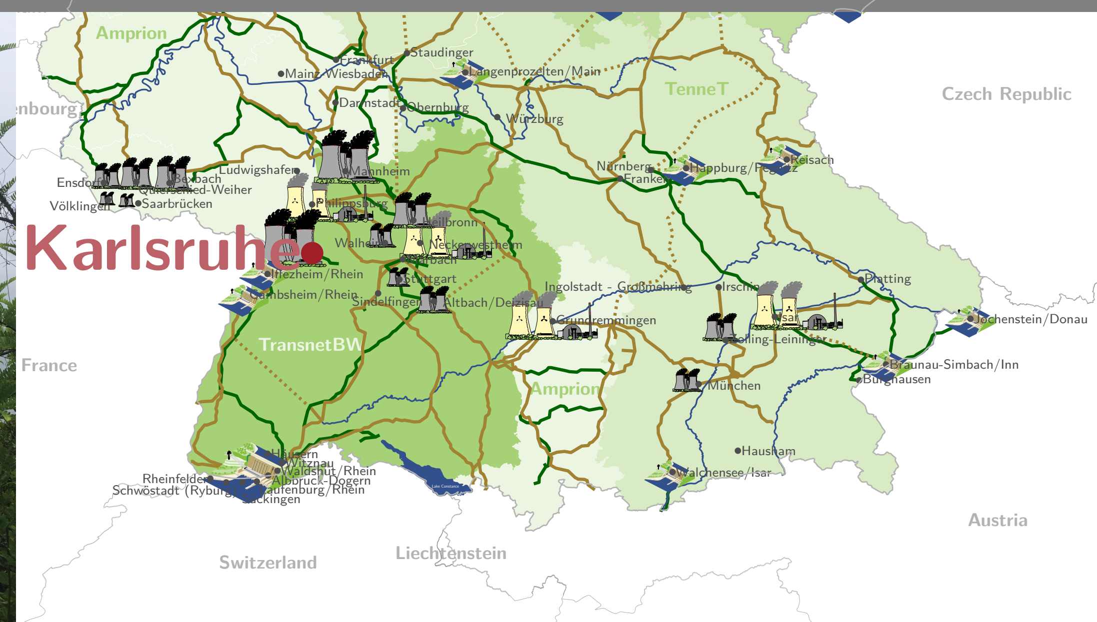
Analogies to the Power Flow

Towards Algorithms for the Power Flow and Maximum Power Flow Problem

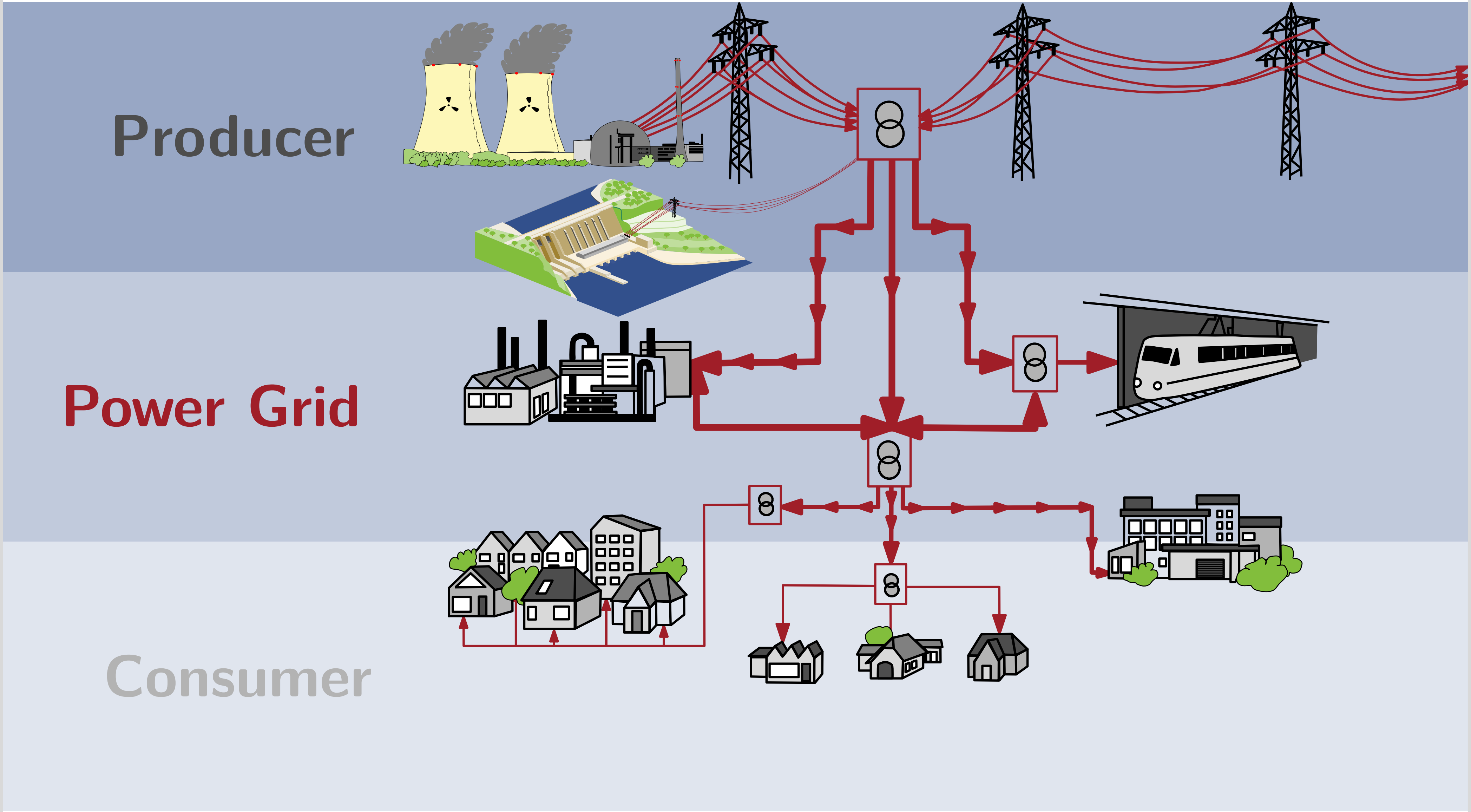
CONDYNET2 · Focus Workshop · July 22th, 2019

Franziska Wegner

INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP



Recent Development in Power Grids



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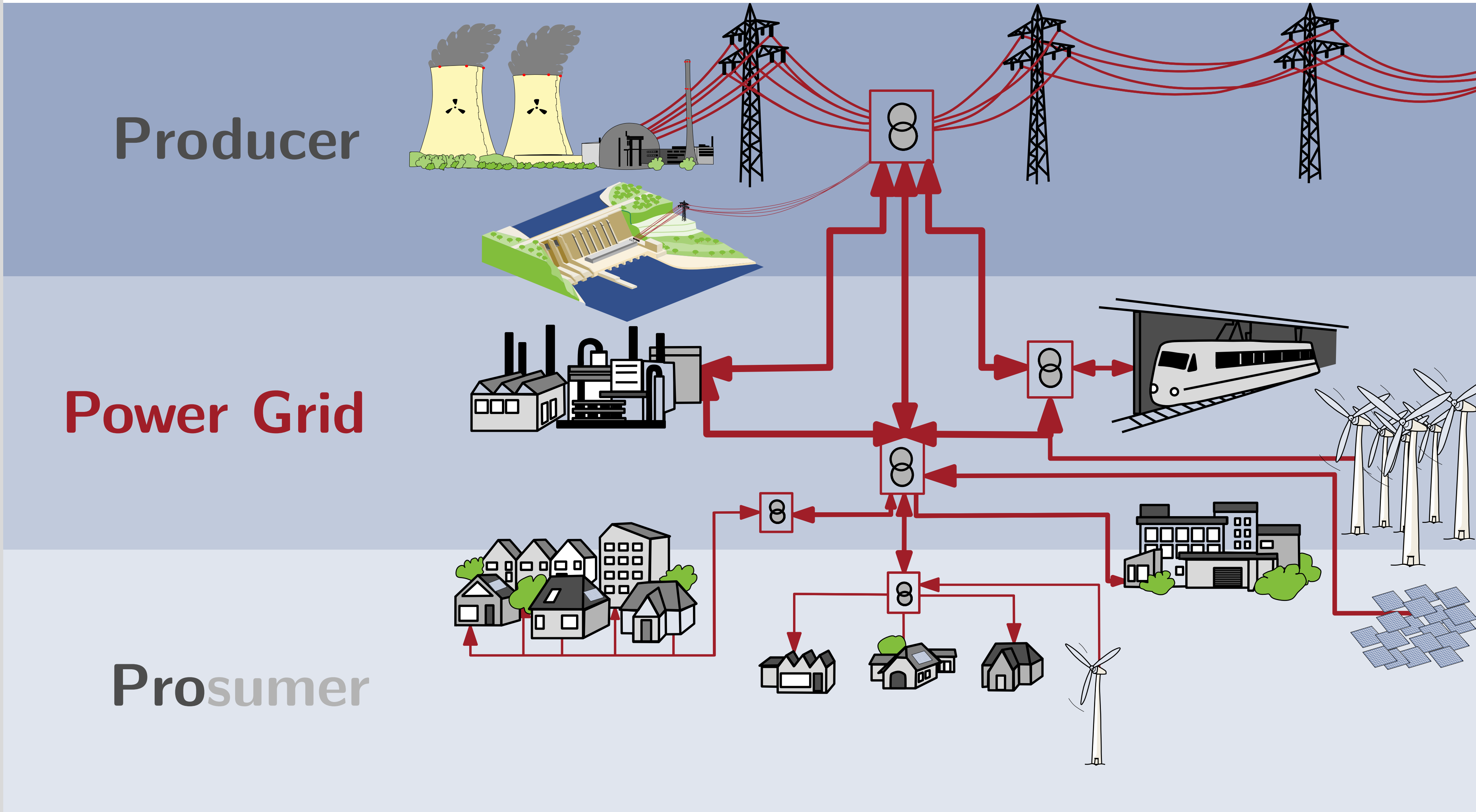

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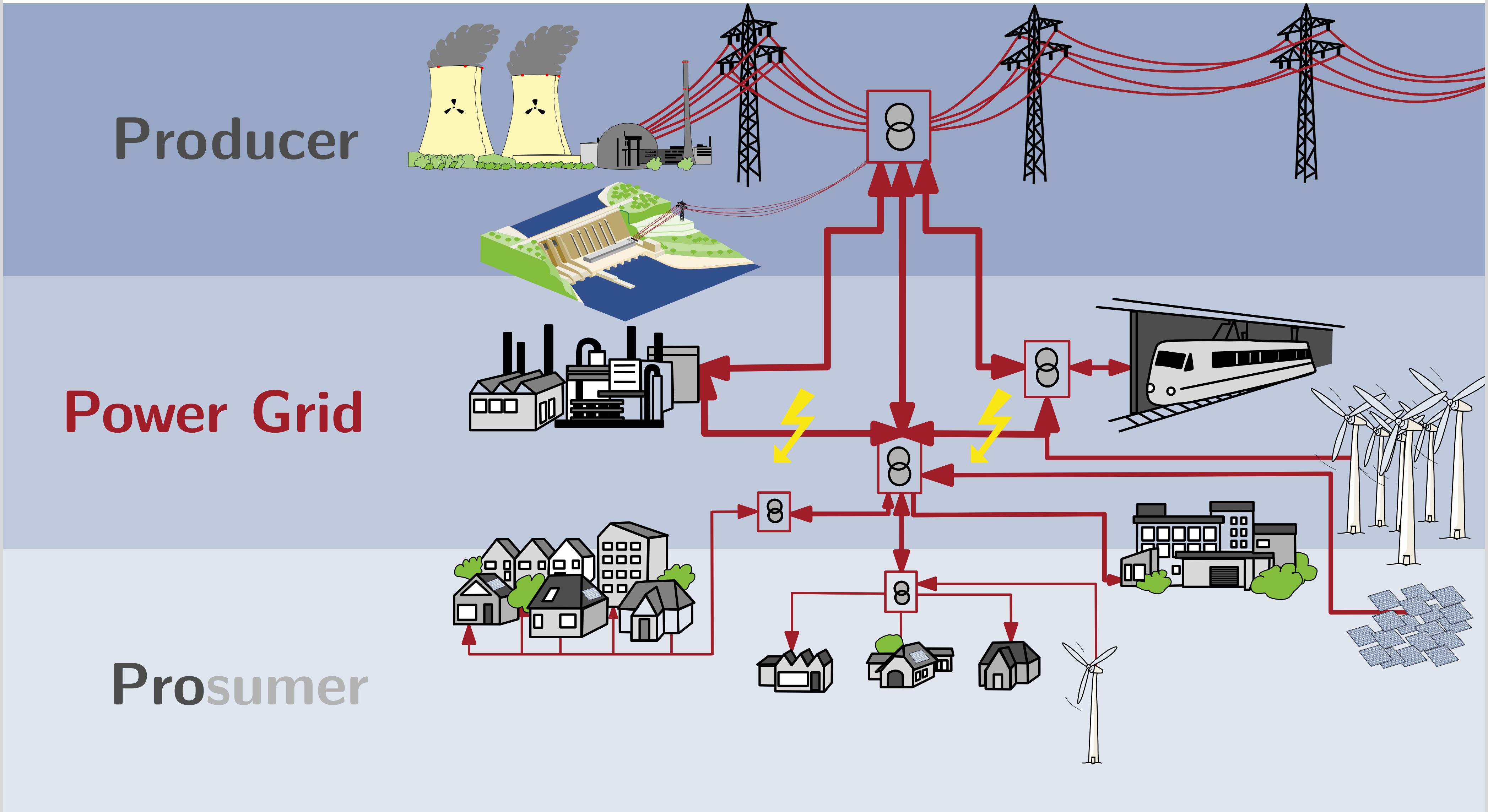

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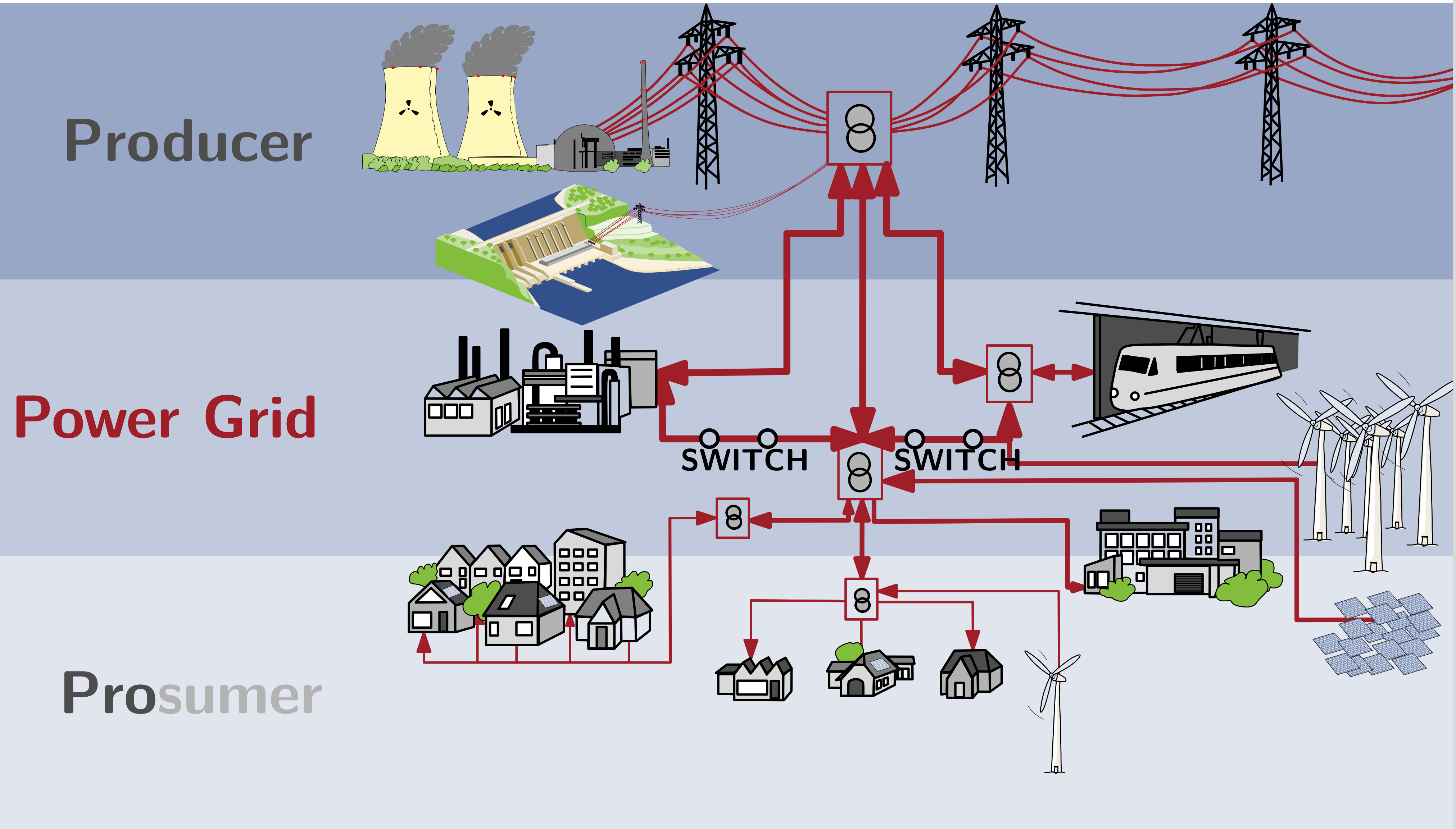

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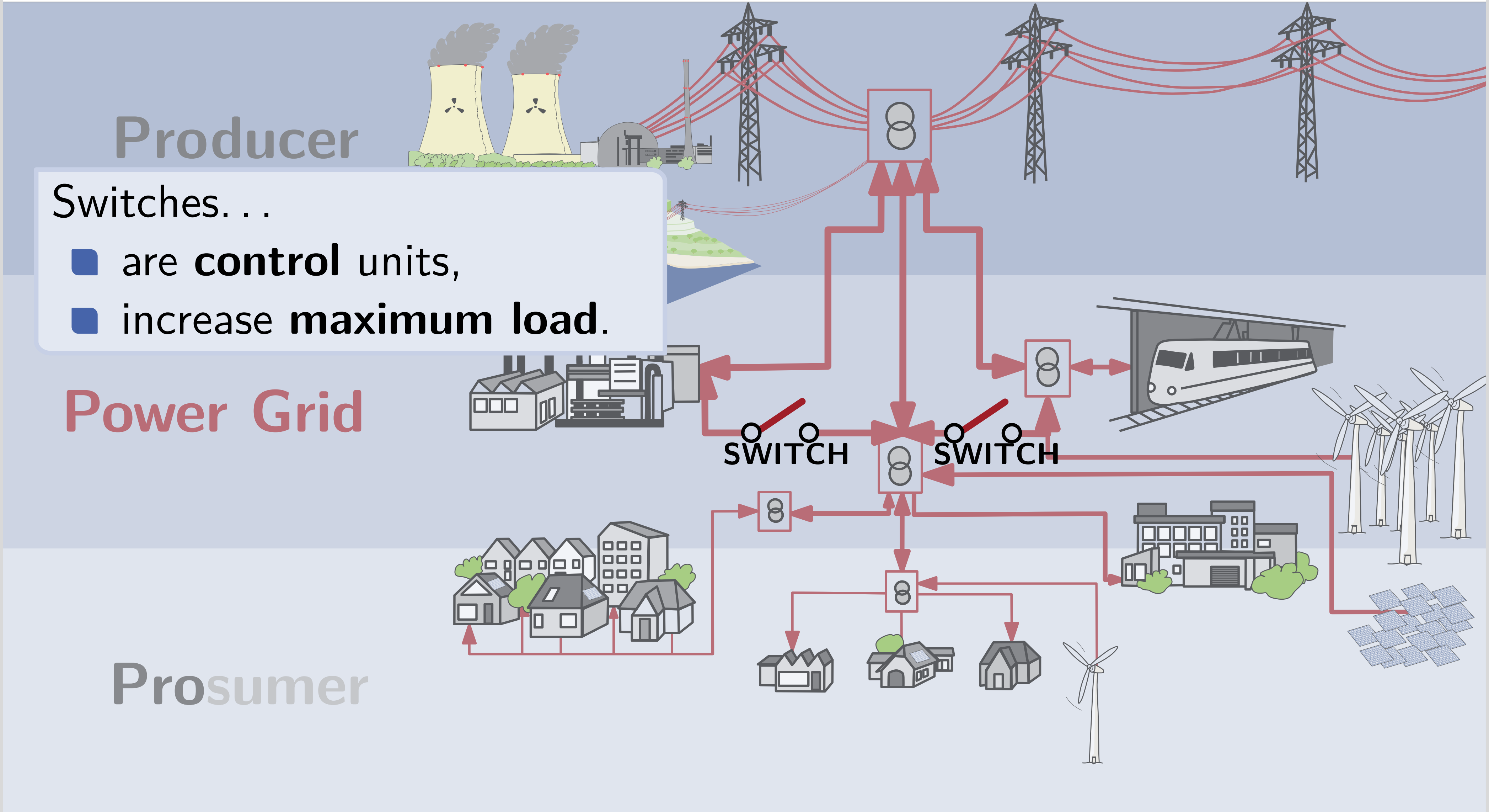
Switches...

- are **control** units,
- increase **maximum load**.



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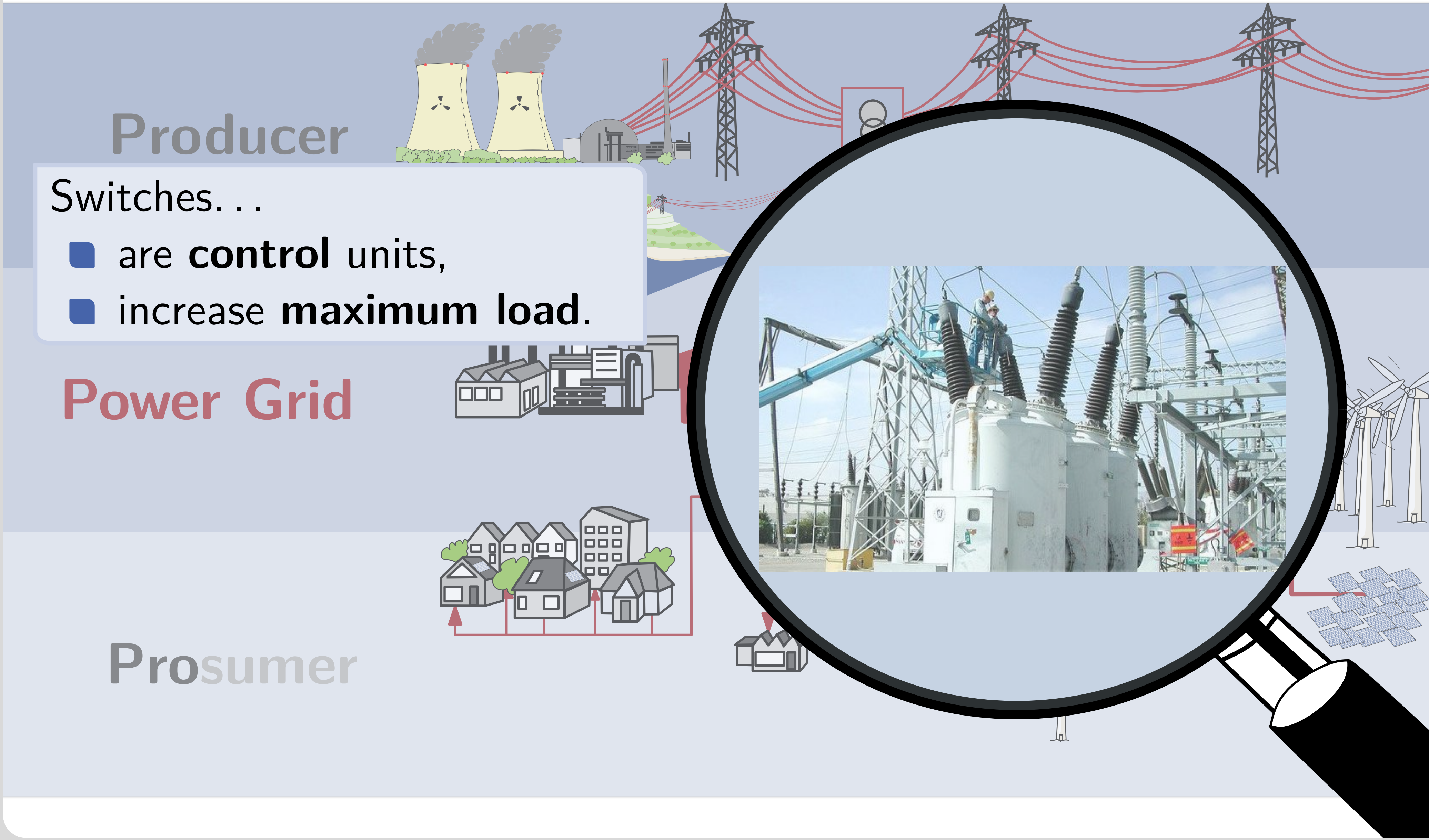

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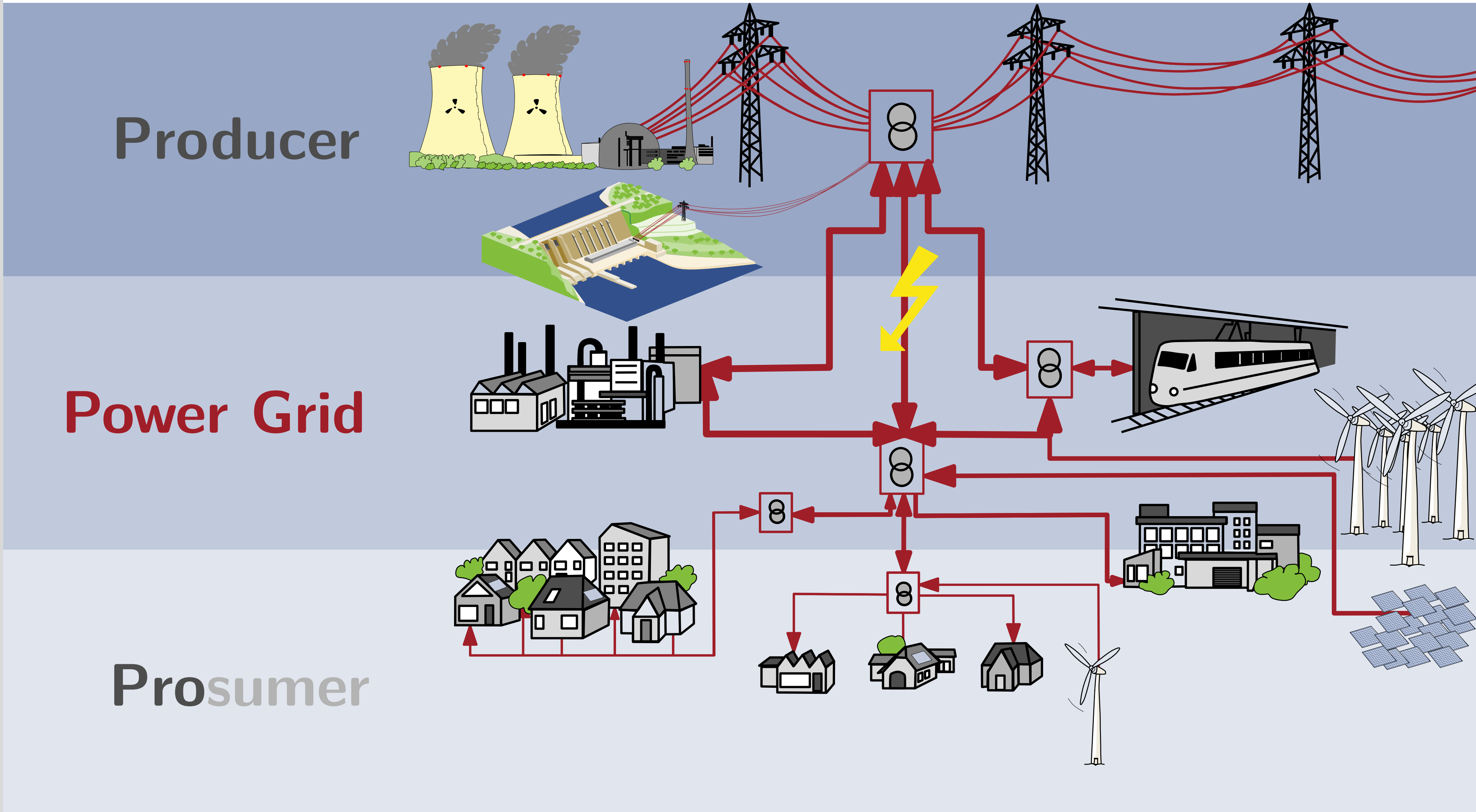

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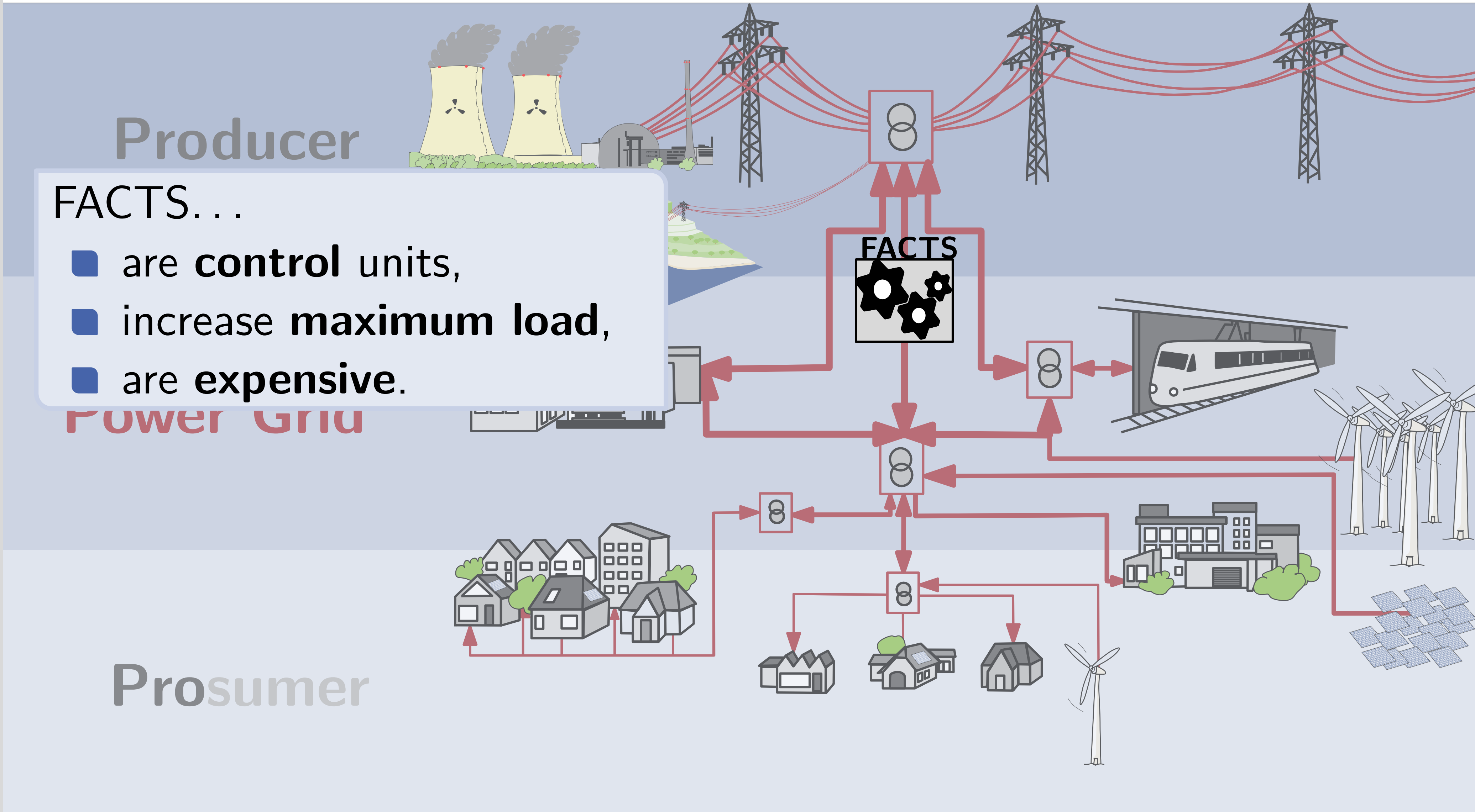

FACTS. . .

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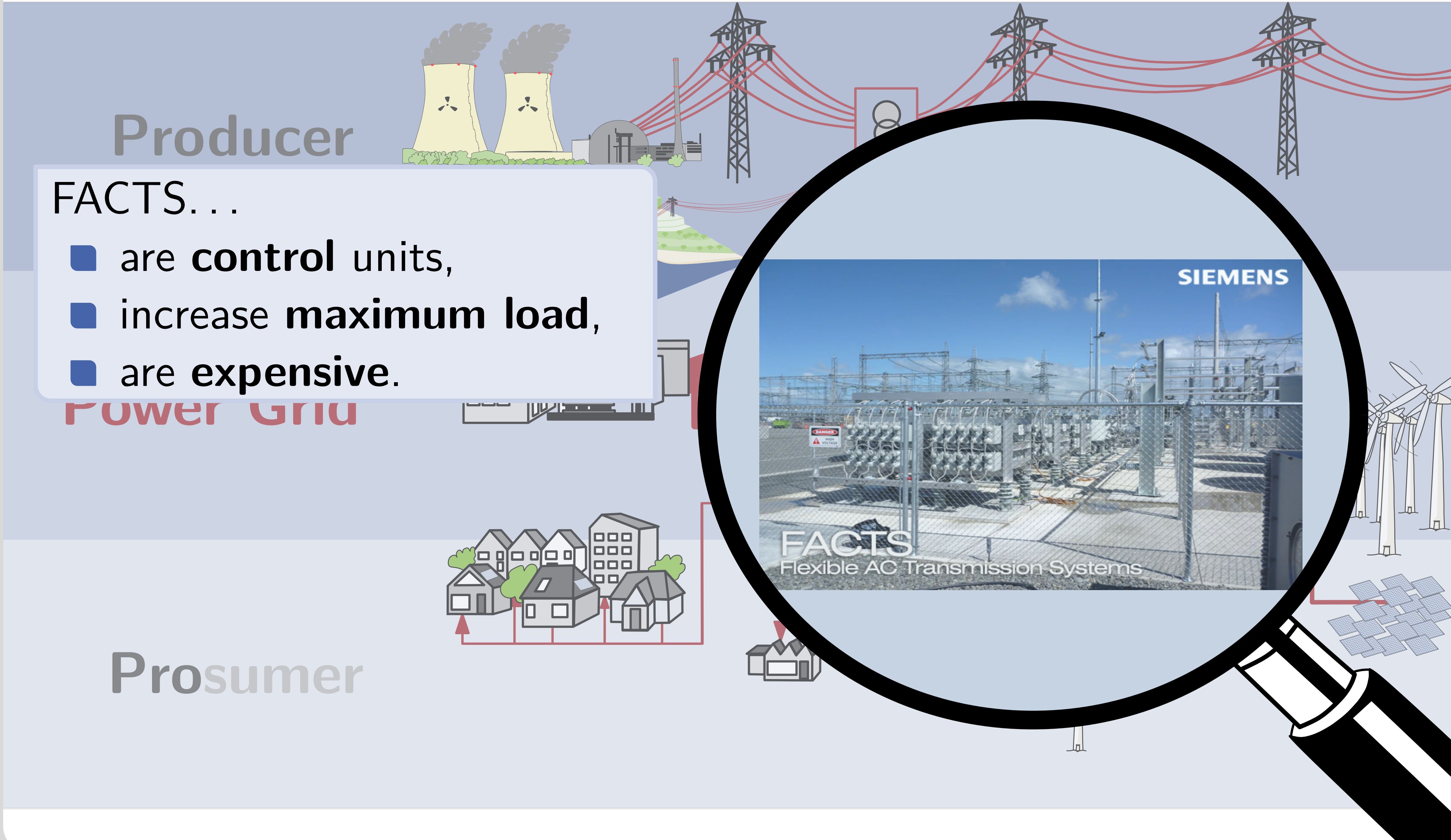
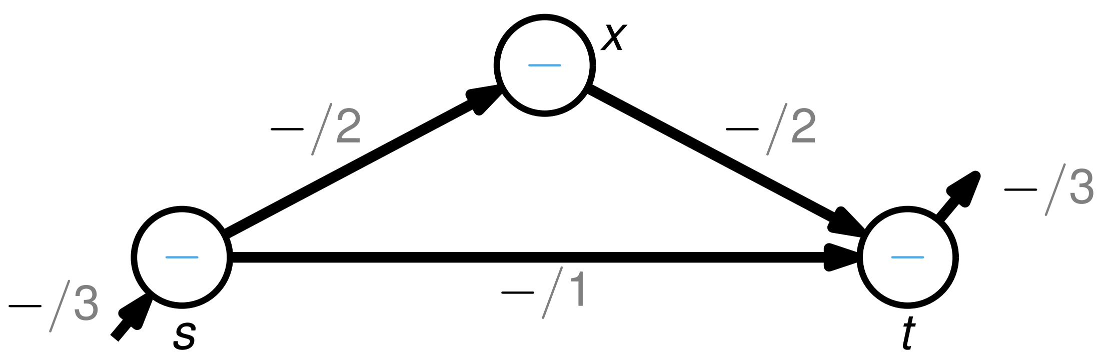


The Maximum Flow Problem (MFP)



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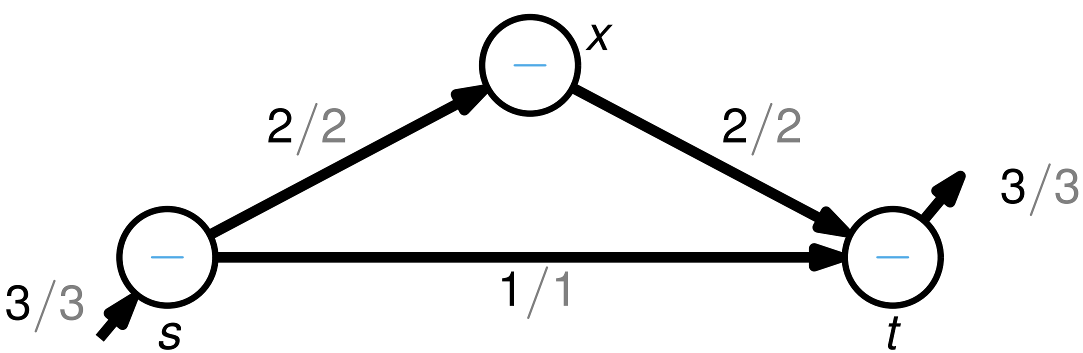

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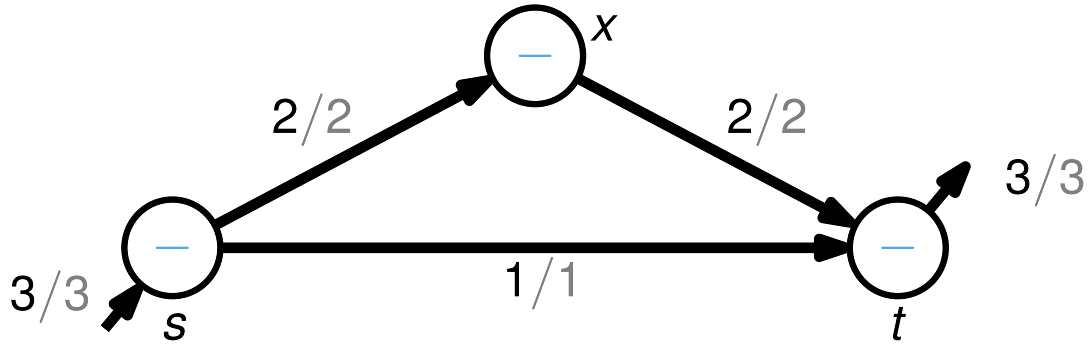
The Maximum Flow Problem (MFP)

- Flow $f: E \to \mathbb{R}$ with $f_{\text{net}}: V \to \mathbb{R}$ defined as $f_{\text{net}}(u) := \sum_{\{u,v\} \in E} f(u,v)$ and flow value $F(\mathcal{N}, f) := \sum_{u \in V_G} f_{\text{net}}(u)$



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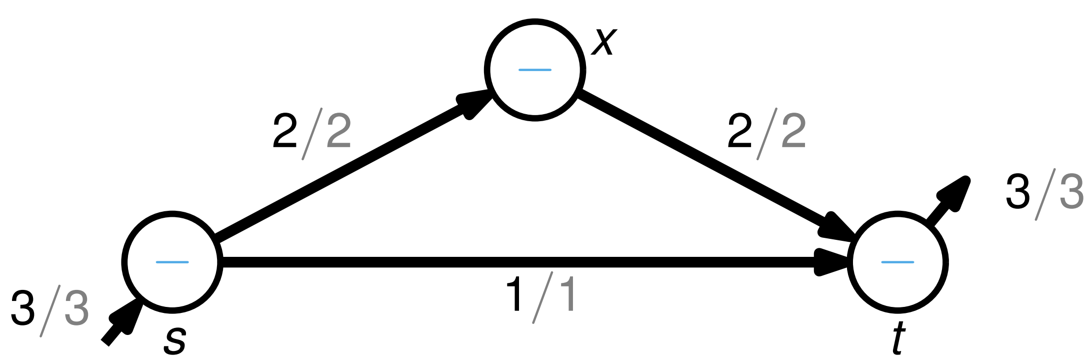

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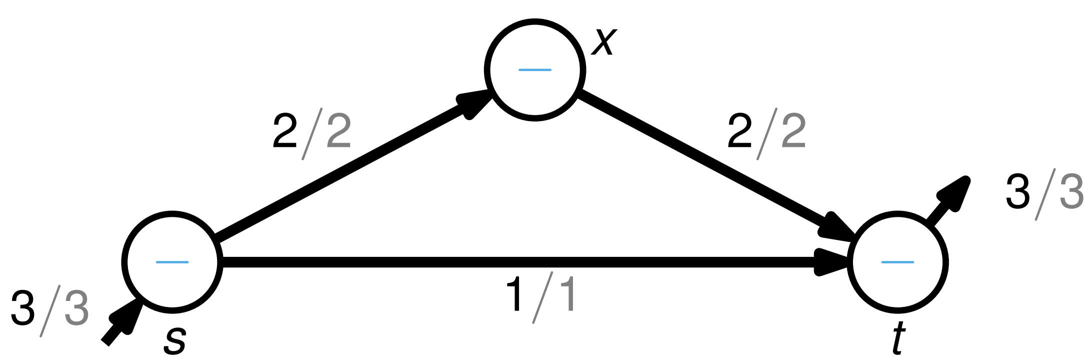

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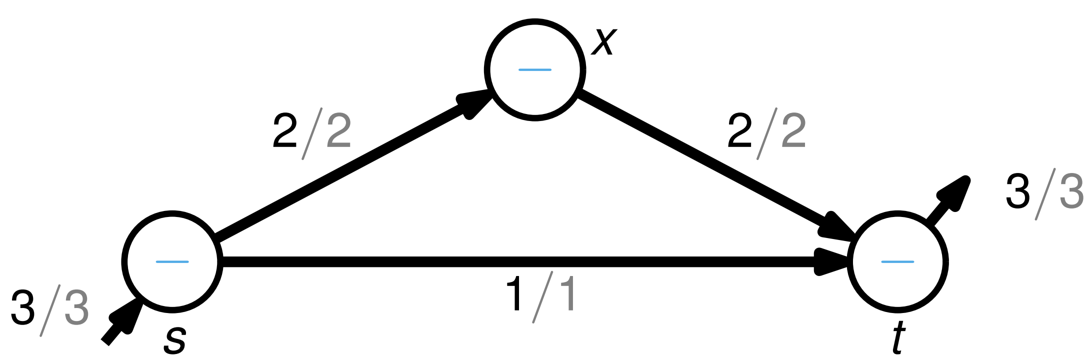

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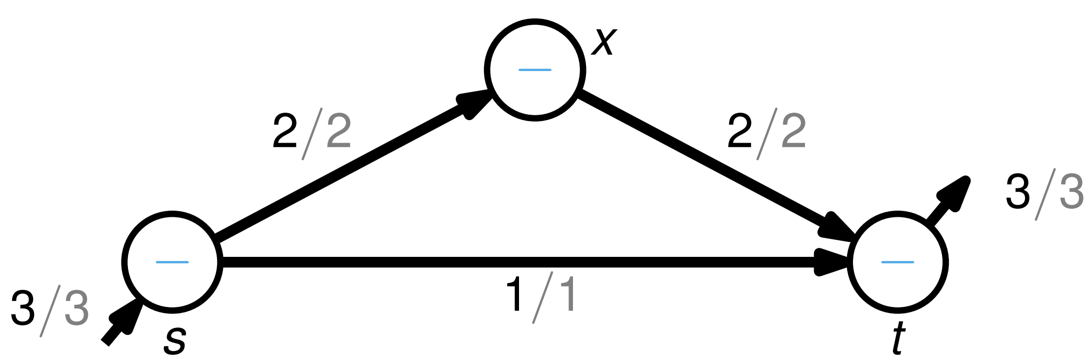

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-\infty \leq f_{\text{net}}(u) \leq -d & \qquad \forall u \in V_D \\
0 \leq f_{\text{net}}(u) \leq \infty & \qquad \forall u \in V_G \\
|f(u,v)| \leq \mathrm{cap}(u,v) & \qquad \forall (u,v) \in E
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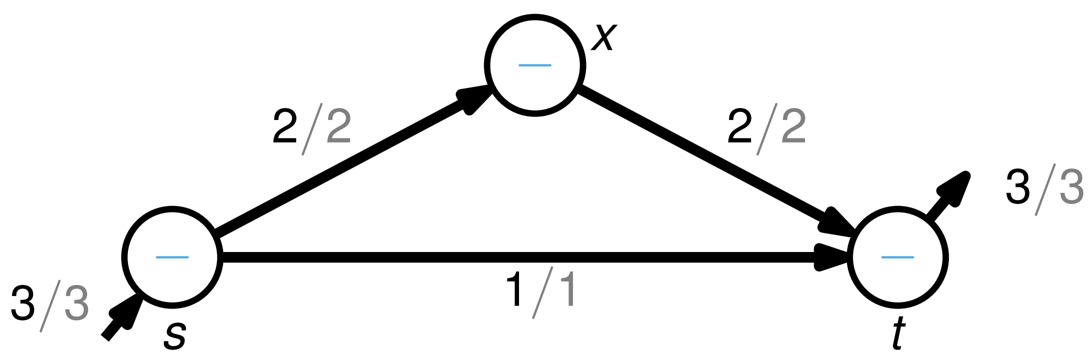

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Conservation of Flow

$$f_{\text{net}}(u) = \sum_{\{u,v\}\in E} f(u,v) = 0 \qquad \forall u \in V \setminus (V_G \cup V_D)$$

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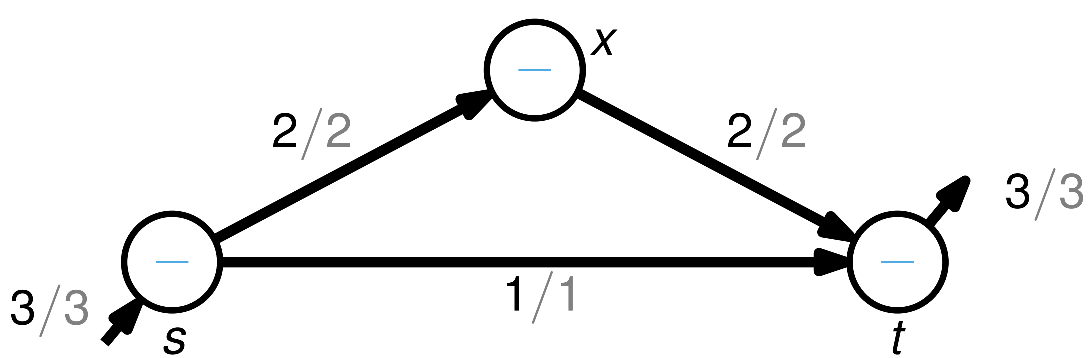

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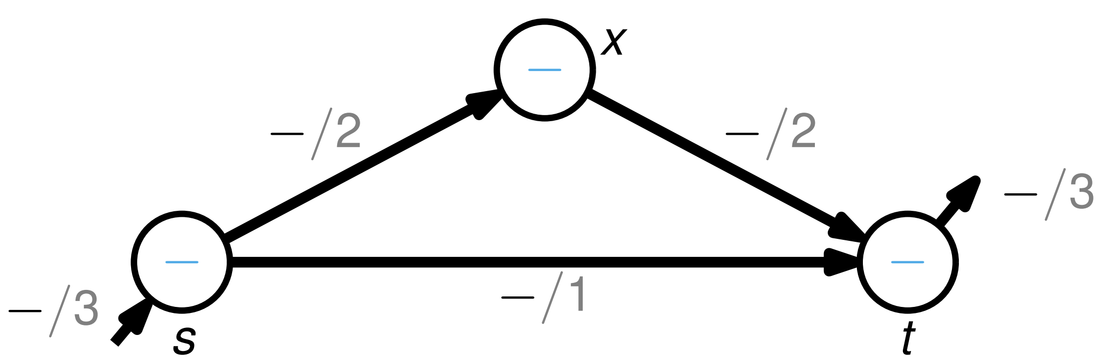
Maximum Power Flow (MPF)

[Zimmerman et al., 2011]

A feasible power flow has to satisfy (additional) physical constraints:

- The Kirchhoff's Current Law (KCL) which relates to flow conservation, i.e., $f_{\text{net}}(u) = 0$ for all $V \setminus (V_G \cup V_D)$



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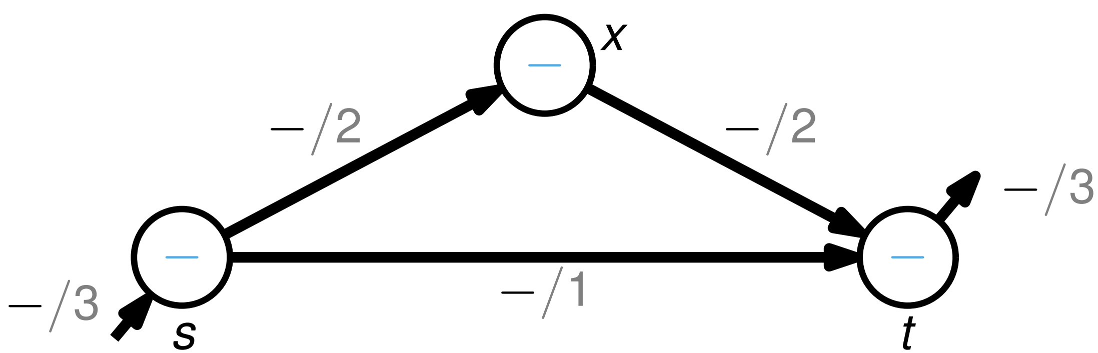

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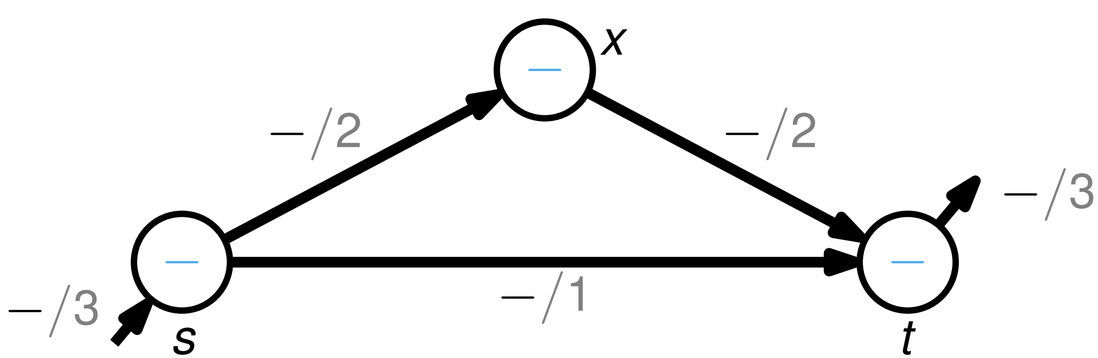

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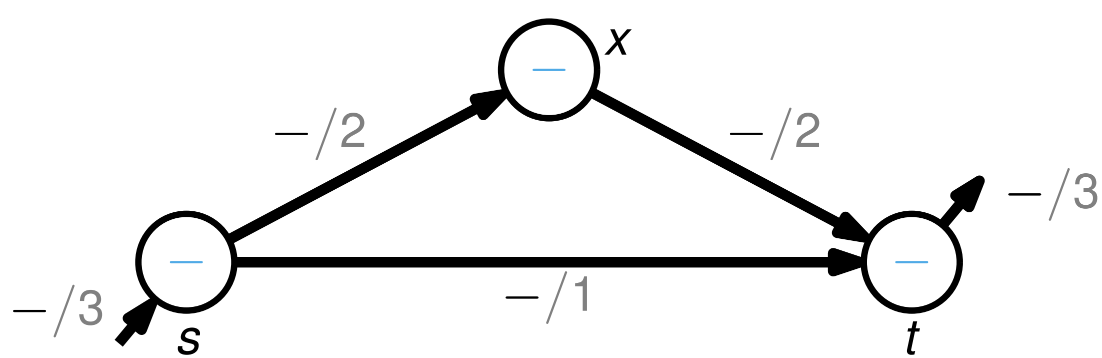

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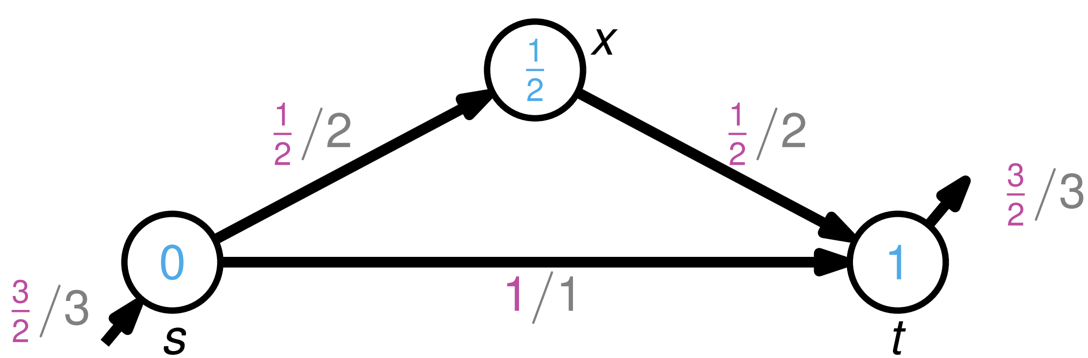

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A feasible power flow has to satisfy (additional) physical constraints:

$$\left.\begin{aligned}(\theta(x) - \theta(s)) &= f(s,x)\\ (\theta(t) - \theta(x)) &= f(x,t)\end{aligned}\right\} \quad (\theta(x) - \theta(s) + \theta(t) - \theta(x)) = f(s,x) + f(x,t)$$

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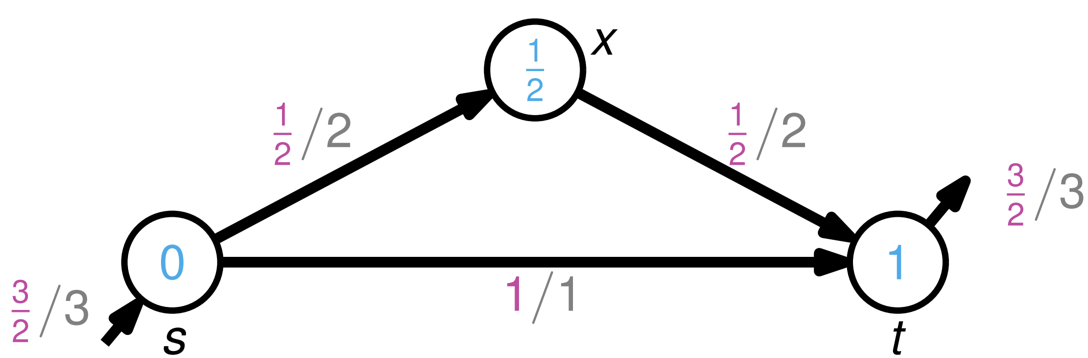

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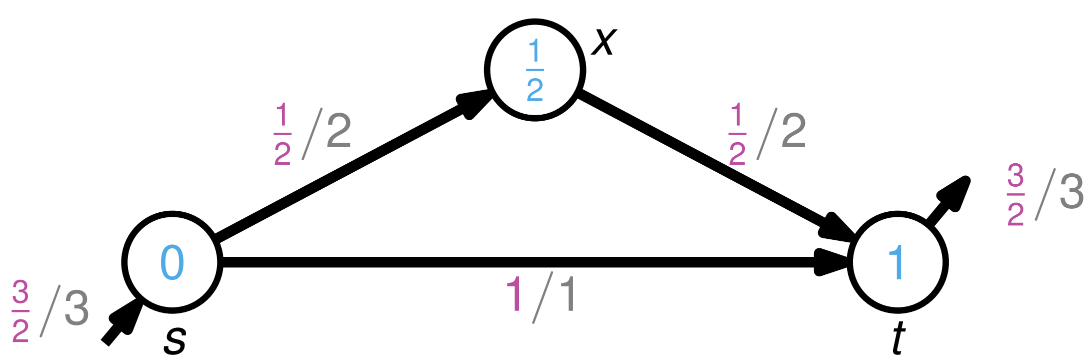

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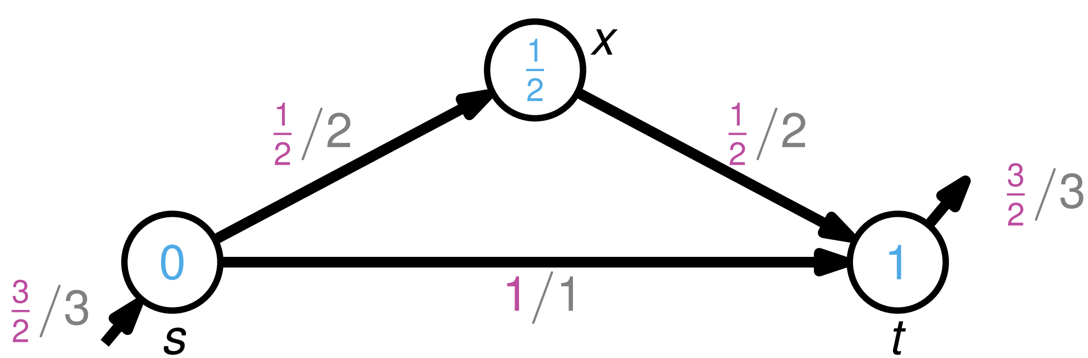

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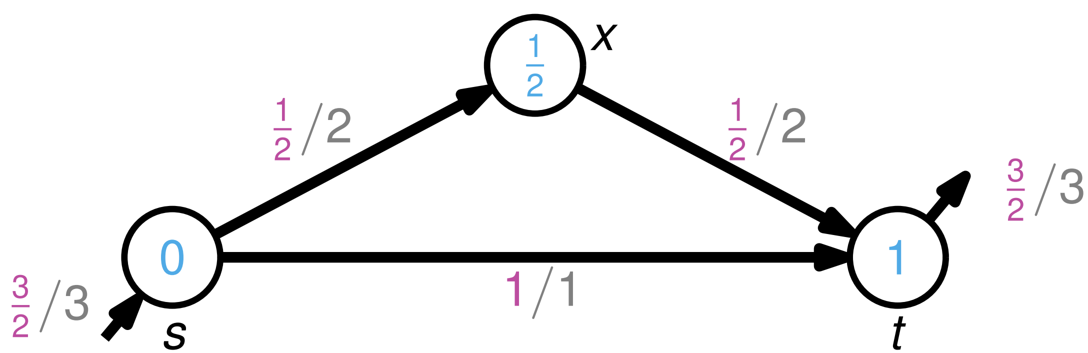

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Formulations

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Vertex-based Formulation		Vertex-/Cycle-based (Kirchhoff's) Formulation
$f_{\text{net}}(u) = \sum_{\{u,v\}\in E} f(u,v) = 0$ $-\infty \leq f_{\text{net}}(u) \leq -d$ $0 \leq f_{\text{net}}(u) \leq \infty$	**KCL**	$\mathbf{I}\vec{f} = \vec{0}$
$\theta^{v}(v) - \theta^{v}(u) = f(u,v)$	**KVL**	$\mathbf{B}\overrightarrow{\Delta\theta^{v}} = \vec{0}$
$\lvert f(u,v)\rvert \leq \text{cap}(u,v)$		$\vec{f} \leq \overrightarrow{\text{cap}}$

I – incidence matrix

B – circuite matrix

Structure of the Incidence and Circuit Matrix

7

5 | 2

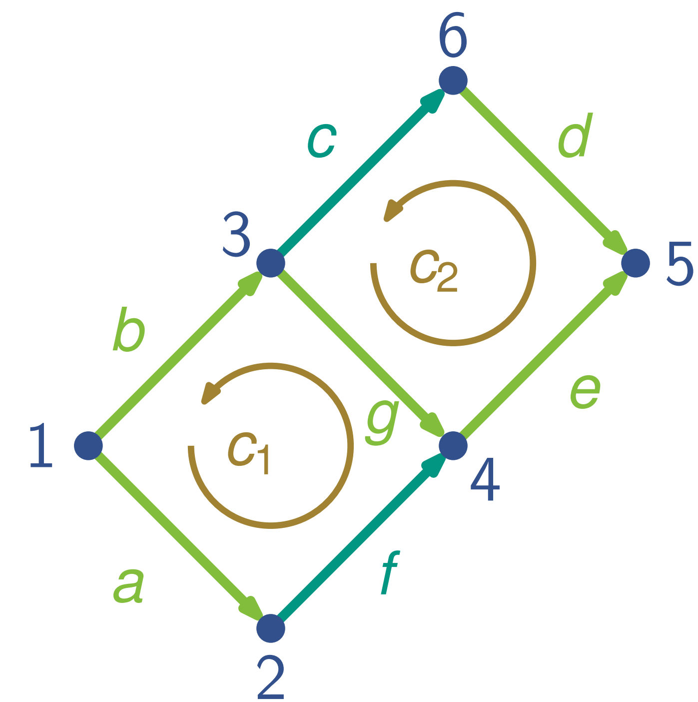
	Spanning Tree					Chords	
	g	e	d	b	a	f	c
3	-1	0	0	1	0	0	-1
4	1	-1	0	0	0	1	0
5	0	1	1	0	0	0	0
6	0	0	-1	0	0	0	1
1	0	0	0	-1	-1	0	0
2	0	0	0	0	1	-1	0
c_1	-1	0	0	-1	1	1	0
c_2	1	1	-1	0	0	0	1

5

2

7

unimodular submatrices



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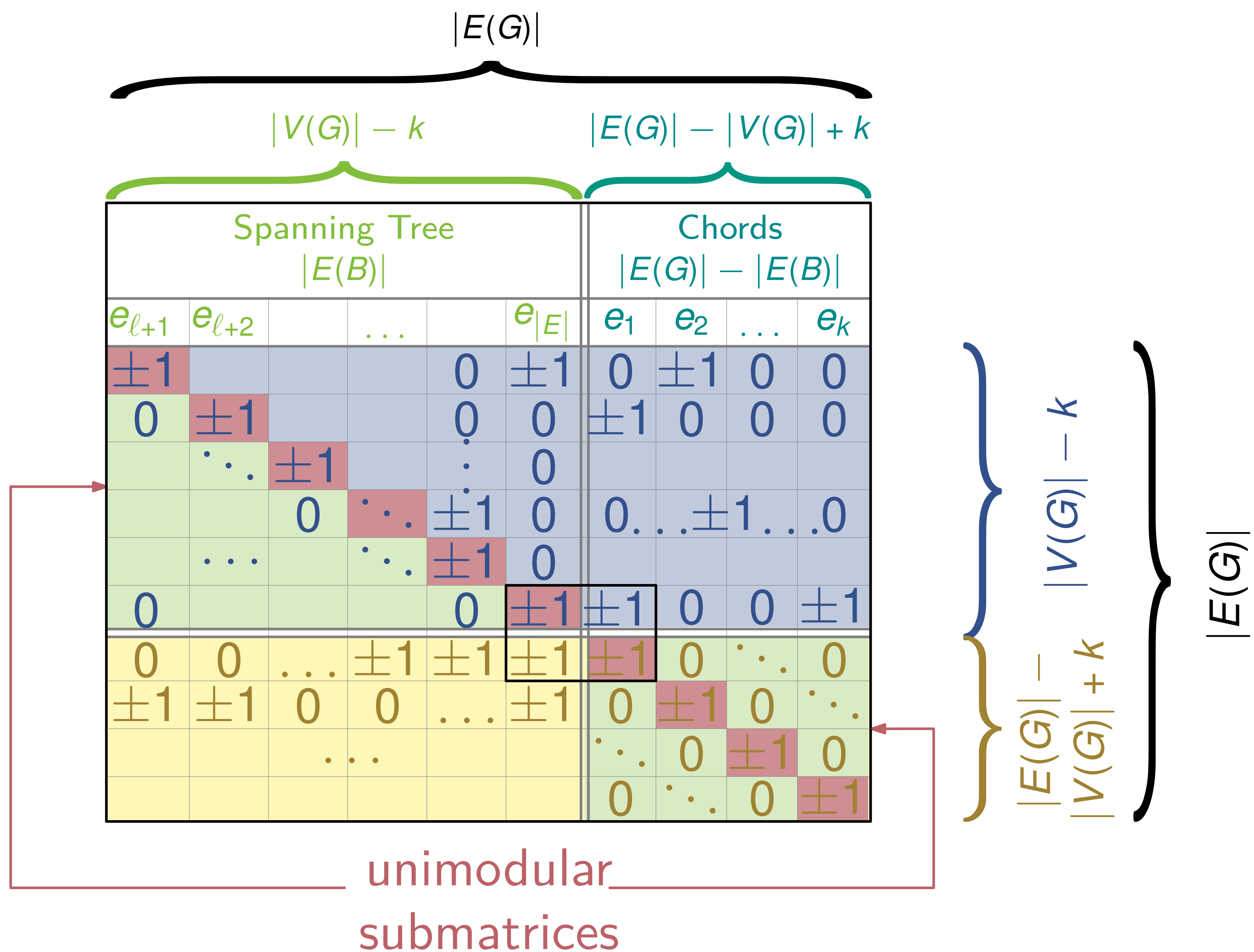
General Structure



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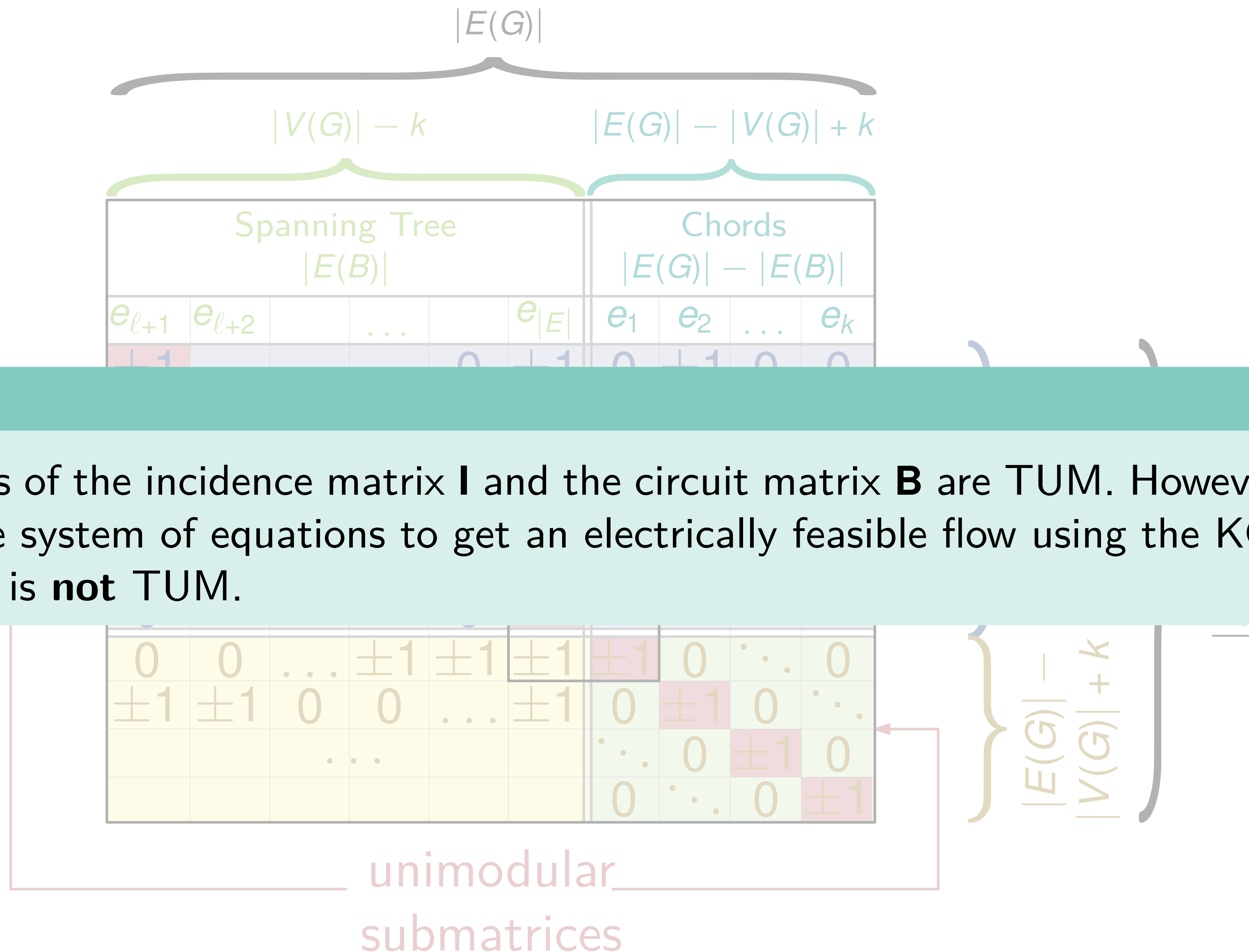

Lemma 1

The bases of the incidence matrix **I** and the circuit matrix **B** are TUM. However, the whole system of equations to get an electrically feasible flow using the KCL and KVL is **not** TUM.

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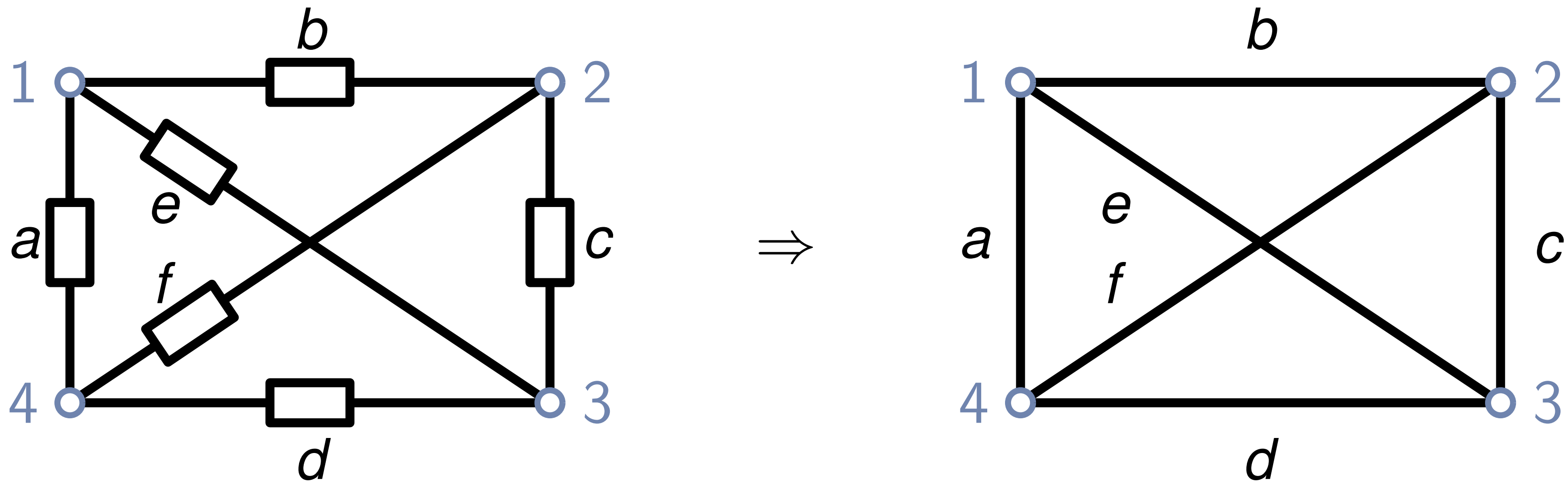
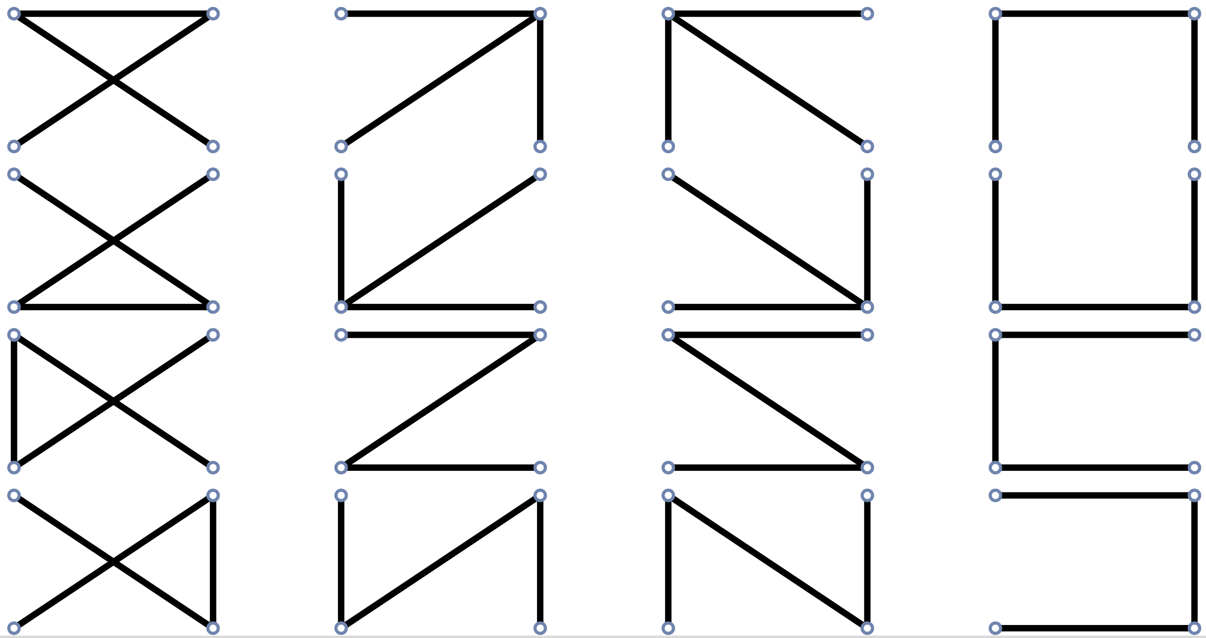
Circuits and Spanning Trees



All spanning trees T



An Ancient Algorithm for the Power Flow

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- A first algorithm that represents a structural result

Lemma 2 [p.36, Lemma 1; Shapiro, 1987]

Let every edge of G have a resistor of 1 ohm. Let N denote the number of spanning trees and let $N(s, a \to b, t)$ be the spanning trees that contain the edge (a, b) in that particular direction. Put a 1-ampere current between s and t and let $i(a, b) = {}^{(N(s,a \to b,t) - N(s,b \to a,t))}/_{N}$. Then $i(a, b)$ is the current in the edge ab oriented from a to b.

- Apply Binet-Cauchy theorem on matrix $\mathbf{Y_n} = \mathbf{I}\ \mathbf{Y_e}\ \mathbf{I^T}$
- $\triangle_n = \det(\mathbf{Y_n}) = \det(\mathbf{I}\ \mathbf{Y_n}\ \mathbf{I^T}) = \sum_{T \in \mathcal{T}}$ (Tree-Admittance Product of T)
- Tree-Admittance Product $\sum_{(u,w) \in E(T)} \mathbf{Y_{u,w}}$

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Primal and Dual Graphs

Theorem 3 [p.522, Theorem 23; Whitney, 1935]

Let $\mathcal{E}$ be a planar embedding of a primal graph G with $G(\mathcal{E})$ being isomorph to G. The graphs G and $G^{\star}$ are duals if and only if there is a bijection $\mu_{\text{dual}} \colon E(G) \to E(G^{\star})$ between their edges such that bases in one correspond to base complements in the other.

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Primal and Dual Graphs

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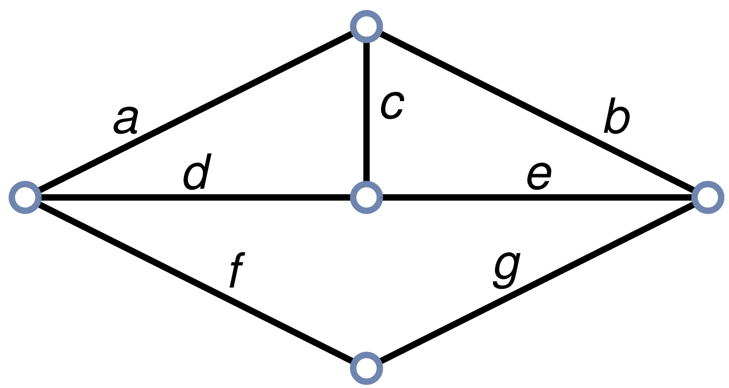
Theorem 3 [p.522, Theorem 23; Whitney, 1935]

Let $\mathcal{E}$ be a planar embedding of a primal graph G with $G(\mathcal{E})$ being isomorph to G. The graphs G and $G^\star$ are duals if and only if there is a bijection $\mu_{\text{dual}} : E(G) \to E(G^\star)$ between their edges such that bases in one correspond to base complements in the other.

primal graph G



Primal and Dual Graphs

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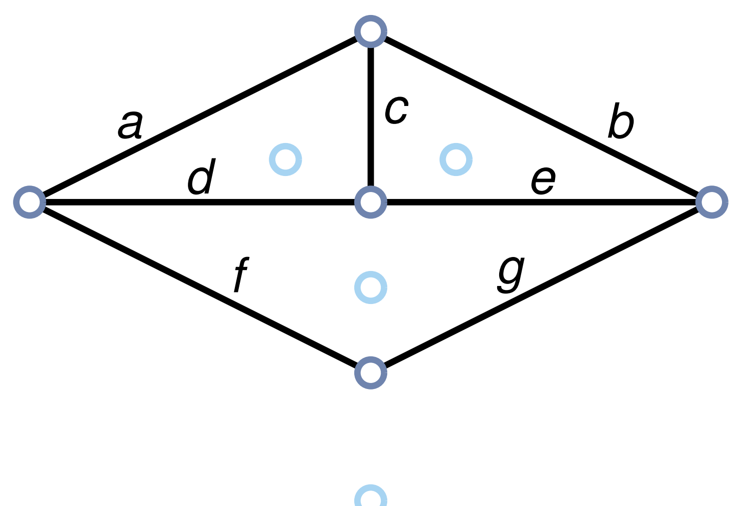

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Primal and Dual Graphs

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primal graph G

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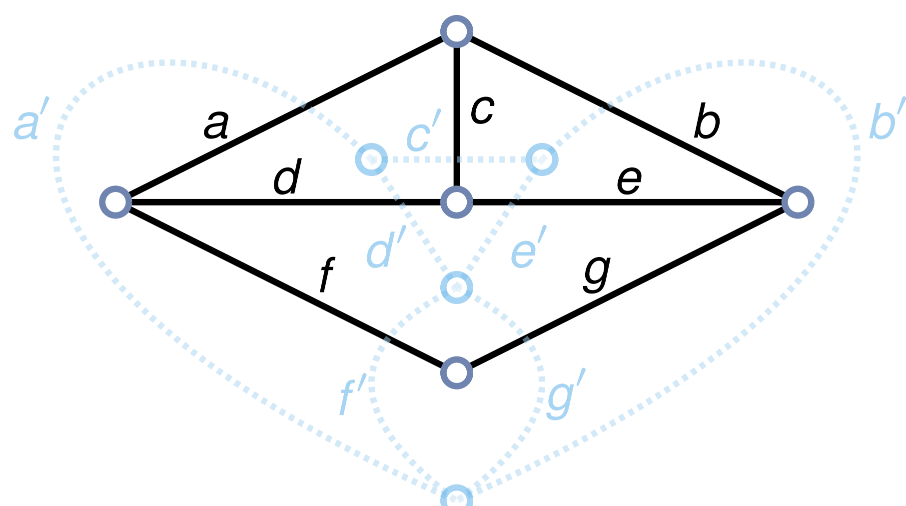

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Primal and Dual Graphs

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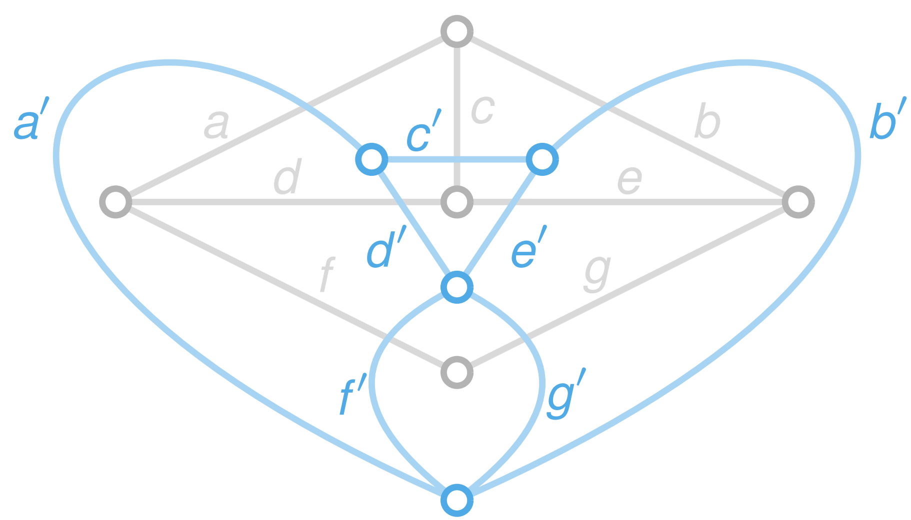

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Corollary 4 [p.85, Corollary 4-24; Seshu and Reed, 1961]

If G and $G^\star$ are dual graphs, the incidence matrix of either graphs is a circuit matrix of the other (with the proper rank, and each row representing a cycle); that is

$$\mathbf{I}_1 = \mathbf{B}_2 \text{ and } \mathbf{I}_2 = \mathbf{B}_1 .$$

Planar s-t PF and MPF

Instance: A plane s-t-graph G, its dual graph $G^\star$ and a bijection $\mu_{\text{dual}}: E(G) \to E(G^\star)$.

Objective: Find feasible flows $f_G, f_{G^\star}: E \to \mathbb{R}_{\geq 0}$ in G and $G^\star$ such that for every edge $e \in E(G)$ we have

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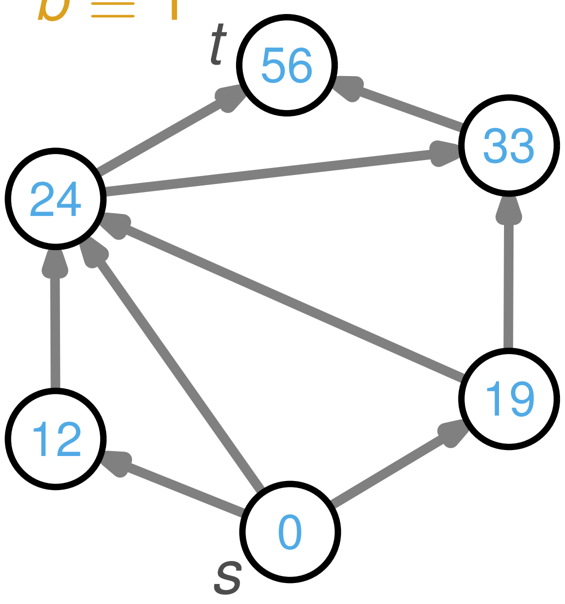
$b \equiv 1$



Power Flow Problem Reformulation

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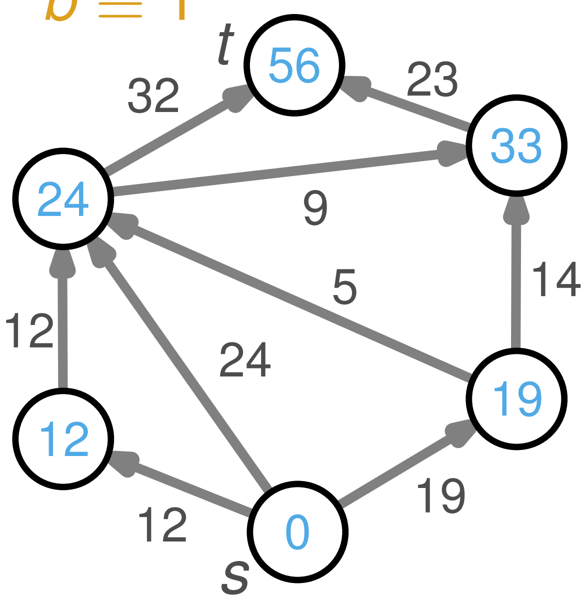

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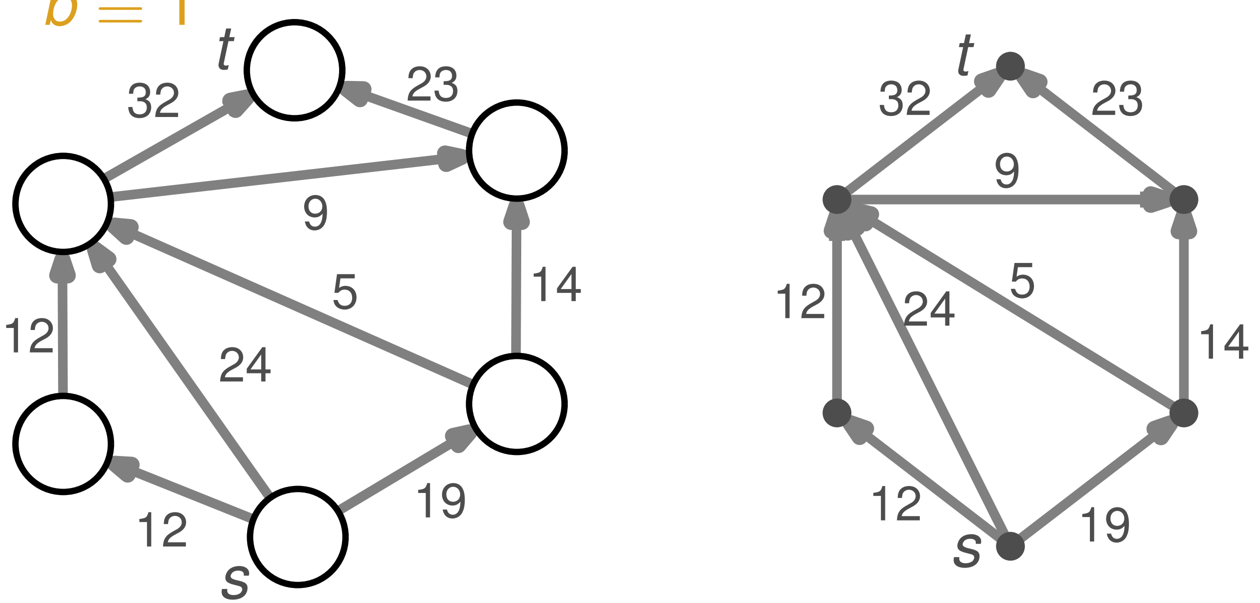

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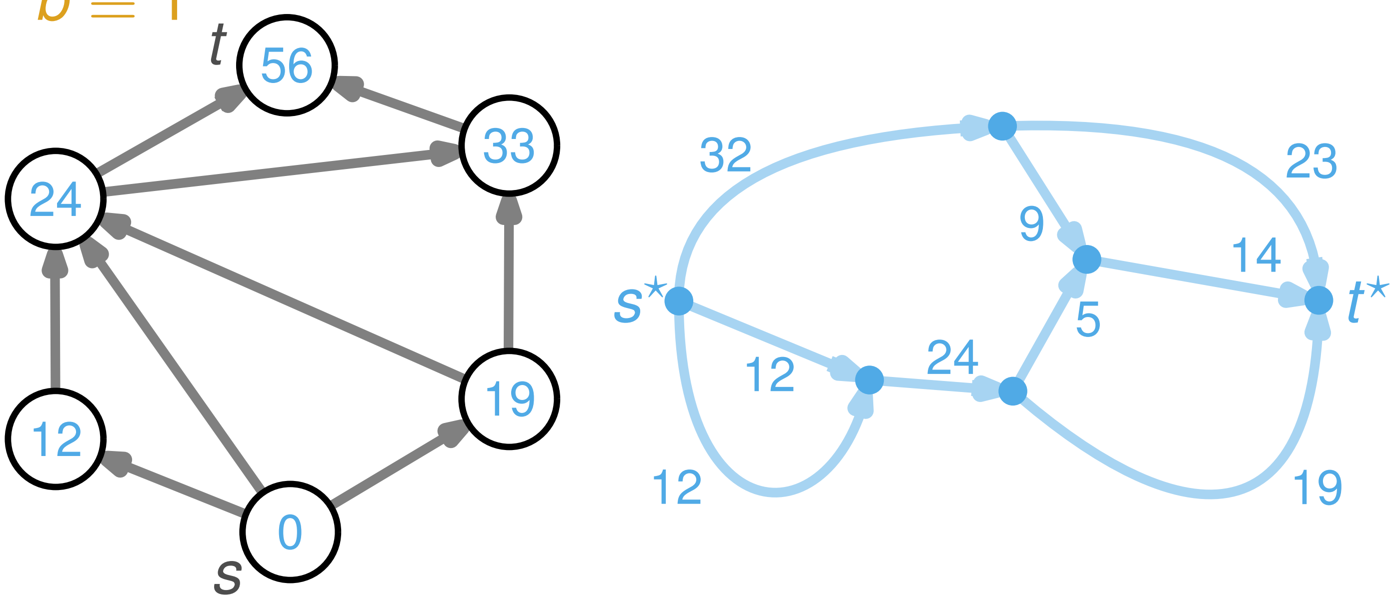

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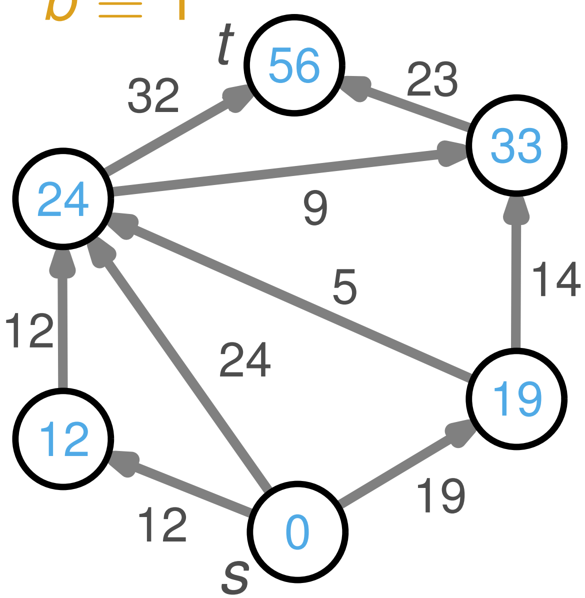

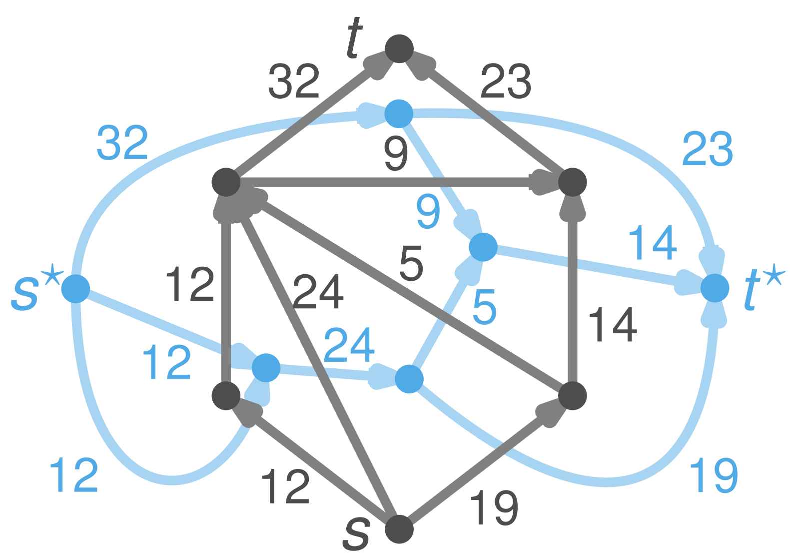

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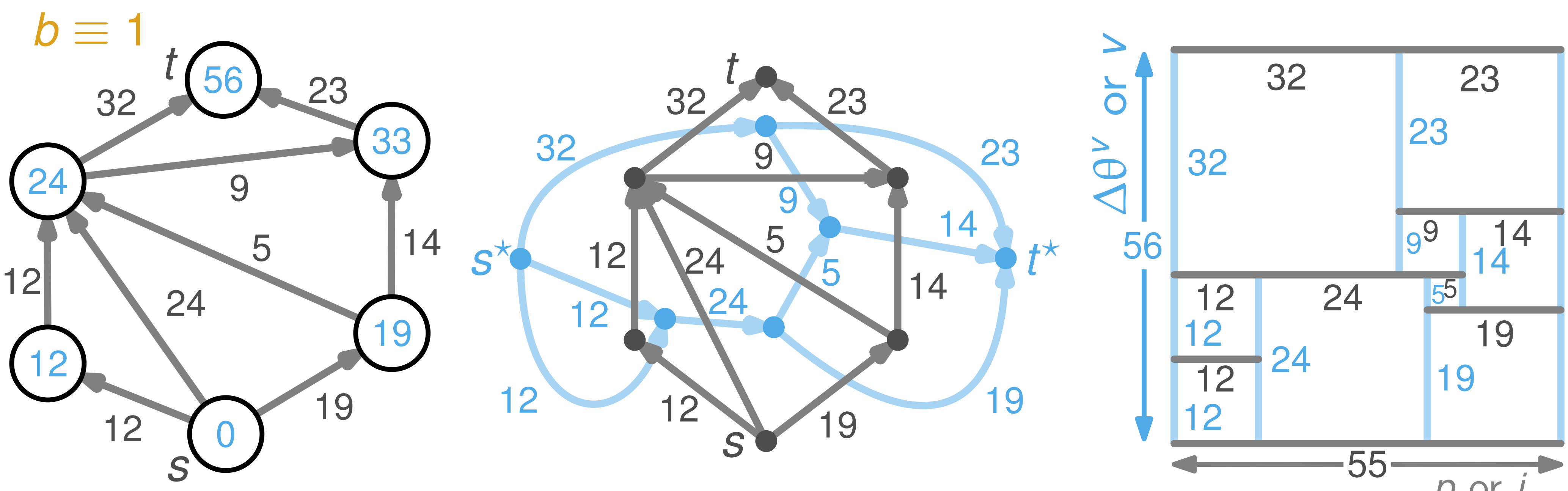

[Felsner, 2013]

Power Flow Problem Reformulation

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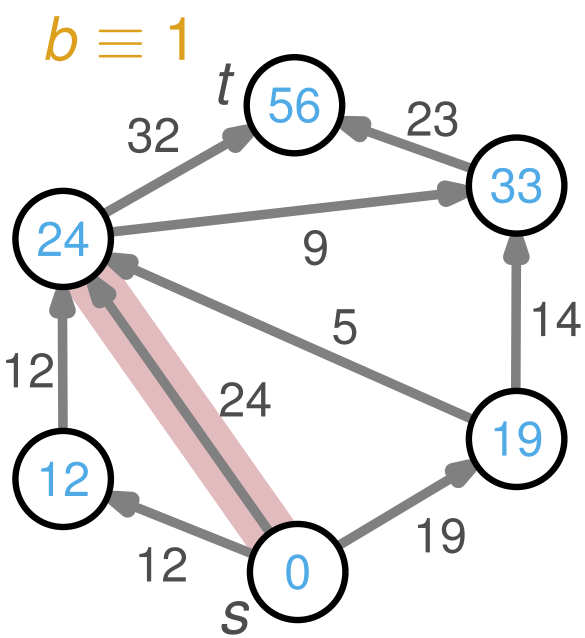

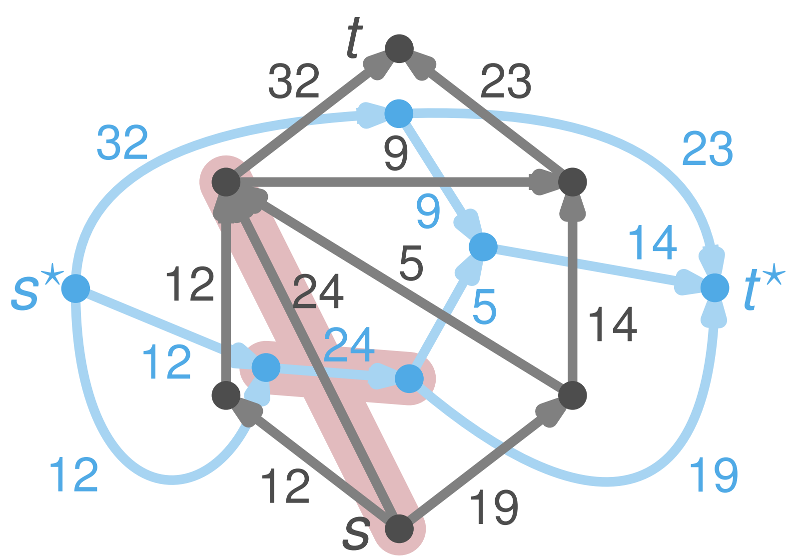

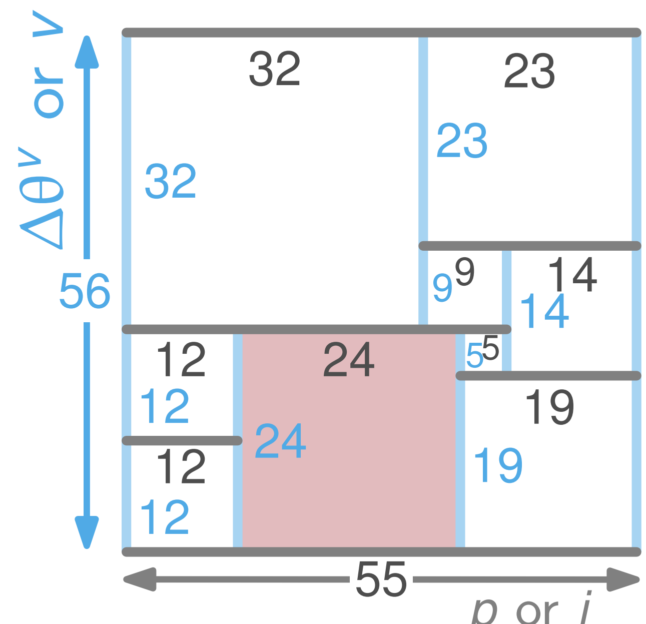

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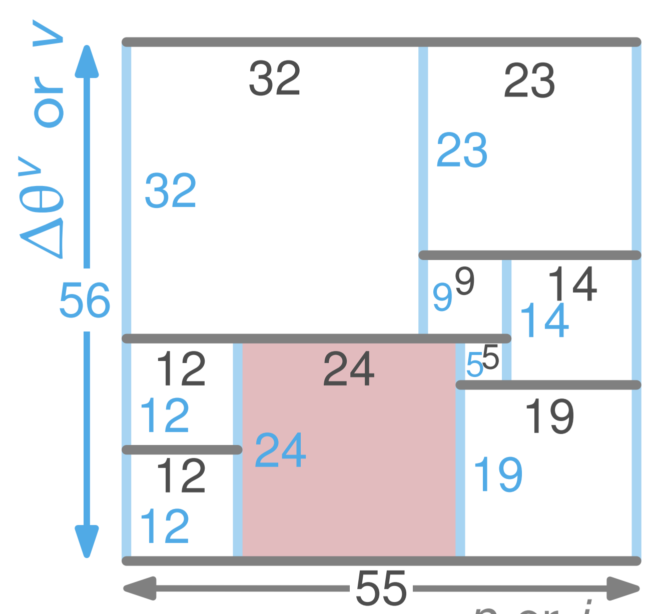
- the two graphs model a quadratic relationship
- $p = v \cdot i$



[Felsner, 2013]

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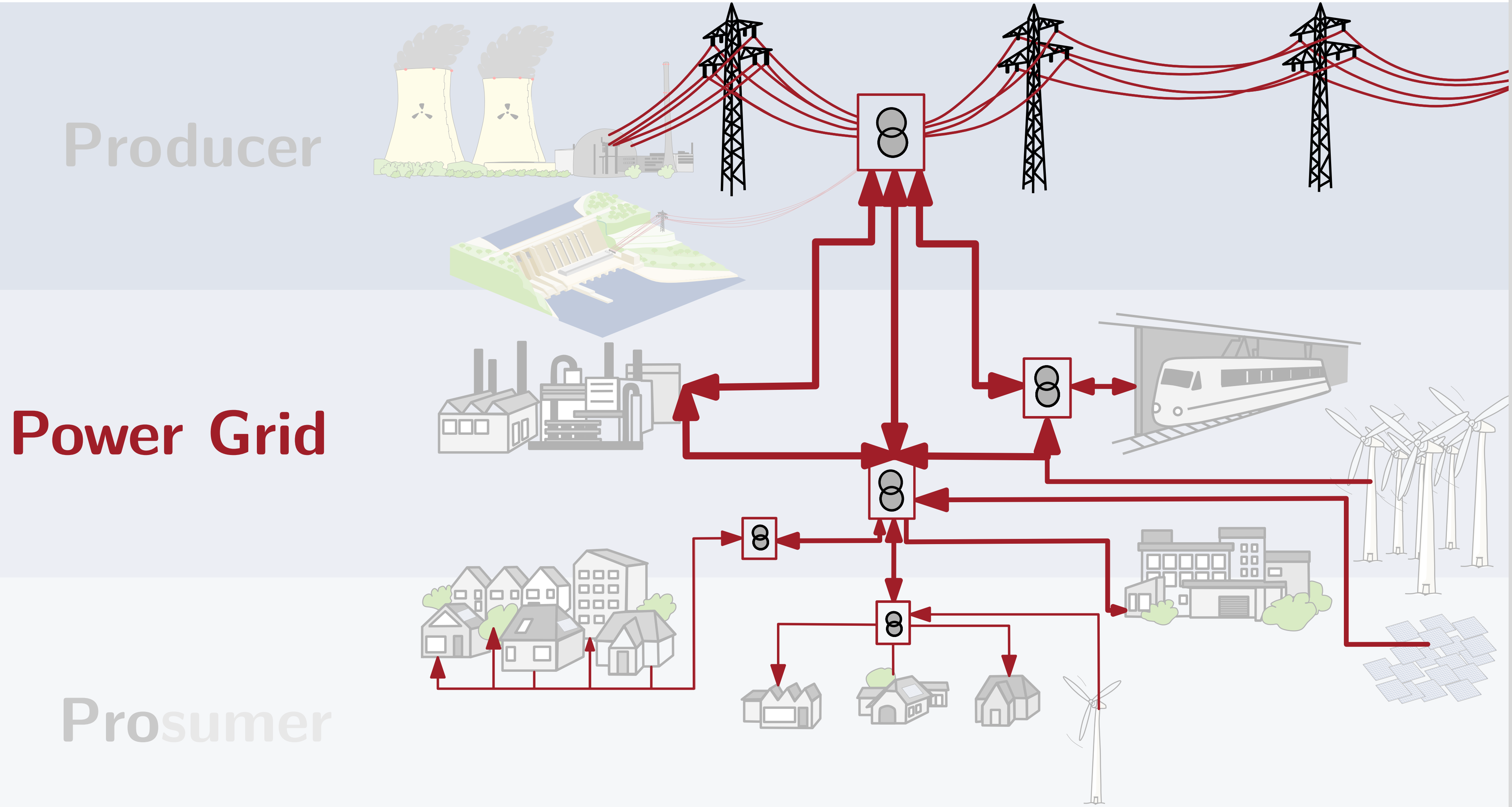
Towards an Algorithm for the Power Flow



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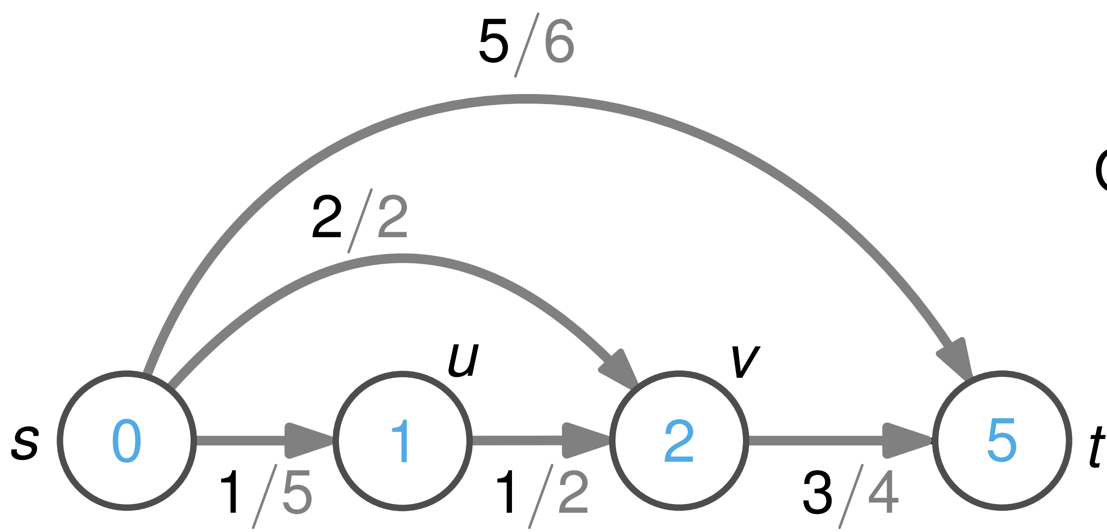
Prisoner's Dilemma

Karlsruhe Institute of Technology



$\mathrm{OPT}_{\mathrm{MPF}}$
$= 8$

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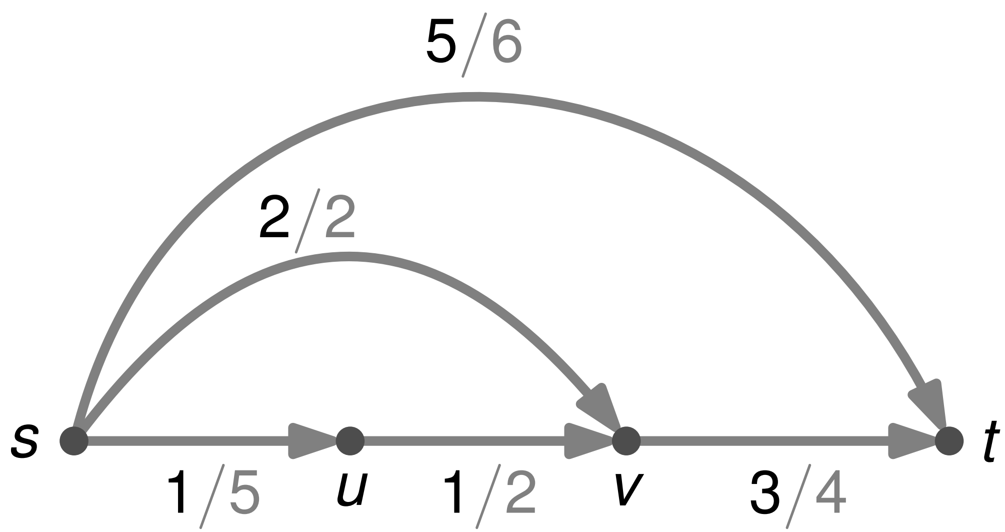
Prisoner's Dilemma



OPT_{MPF}
$= 8$

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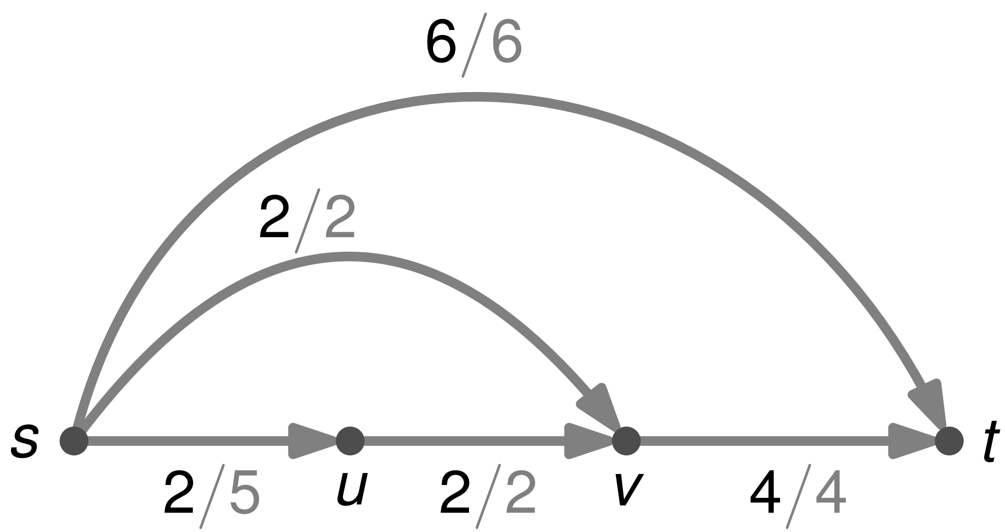
Prisoner's Dilemma

Prisoner 1



$OPT_{MF} = 10$

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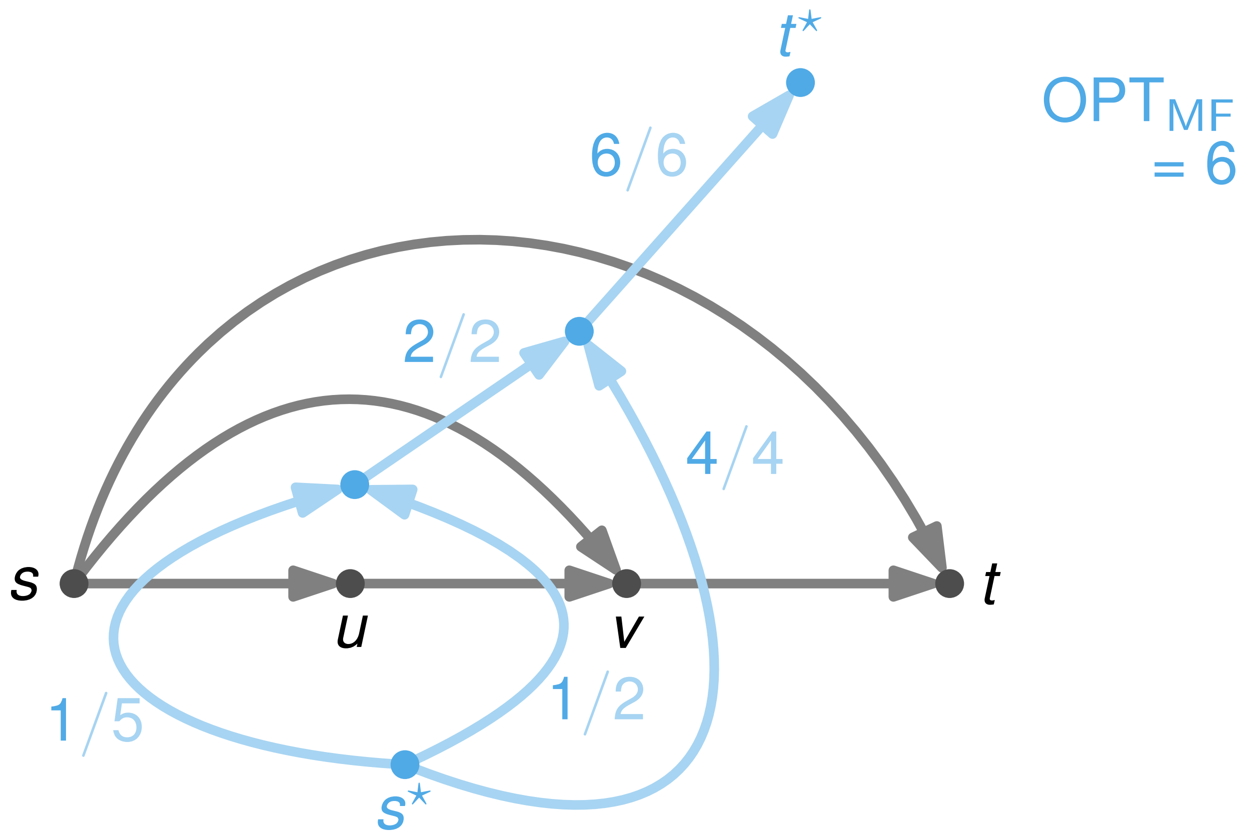
Prisoner's Dilemma

Prisoner 1

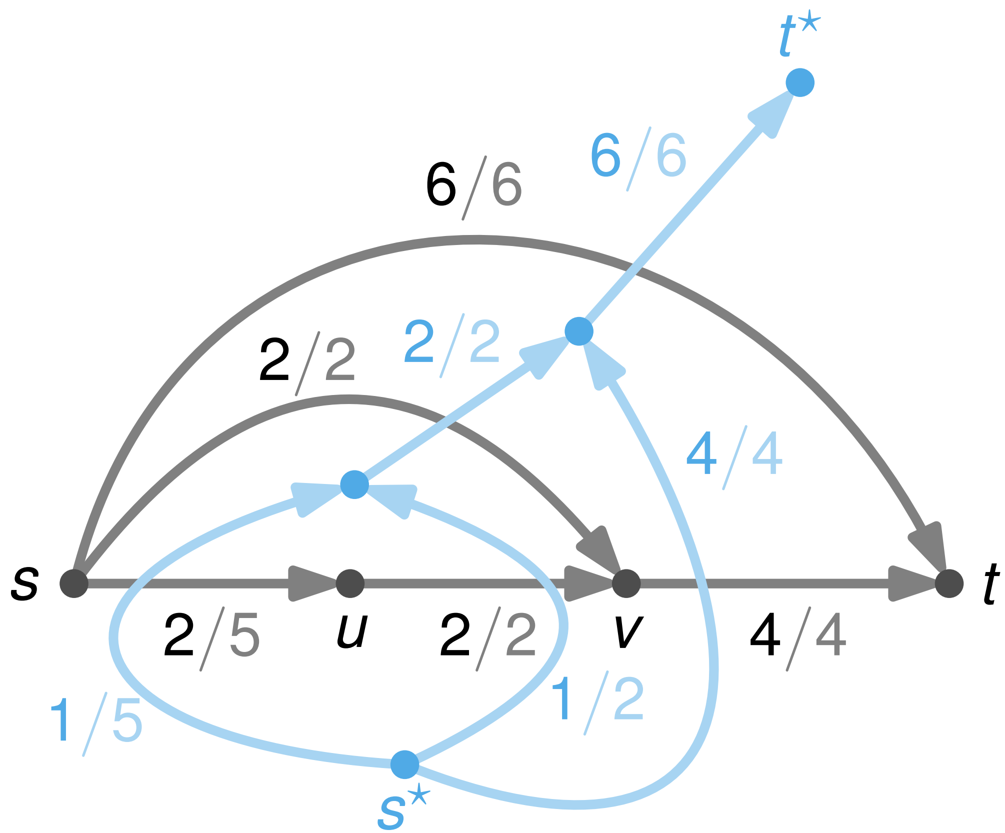
Prisoner 2



Prisoner's Dilemma

Prisoner 1

Prisoner 2



$\mathrm{OPT}_{\mathrm{MF}} = 6$

$\mathrm{OPT}_{\mathrm{MF}} = 10$

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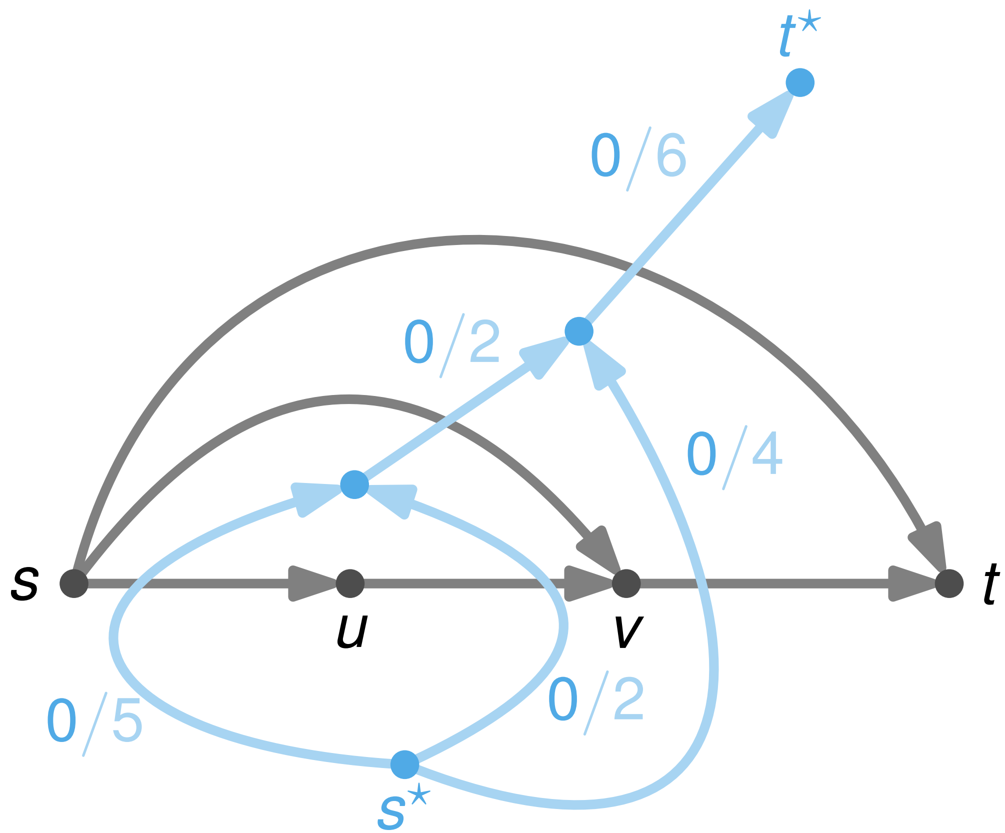
Prisoner's Dilemma

Prisoner 1

Prisoner 2



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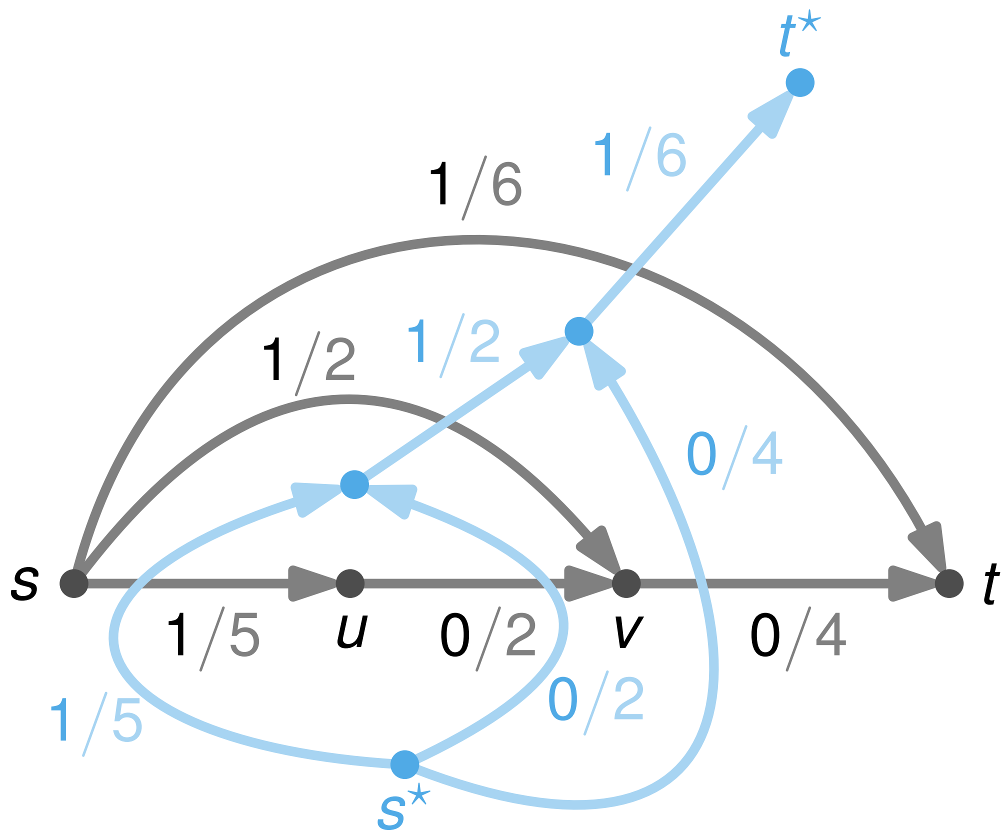
Prisoner's Dilemma

Prisoner 1
Prisoner 2



Prisoner's Dilemma

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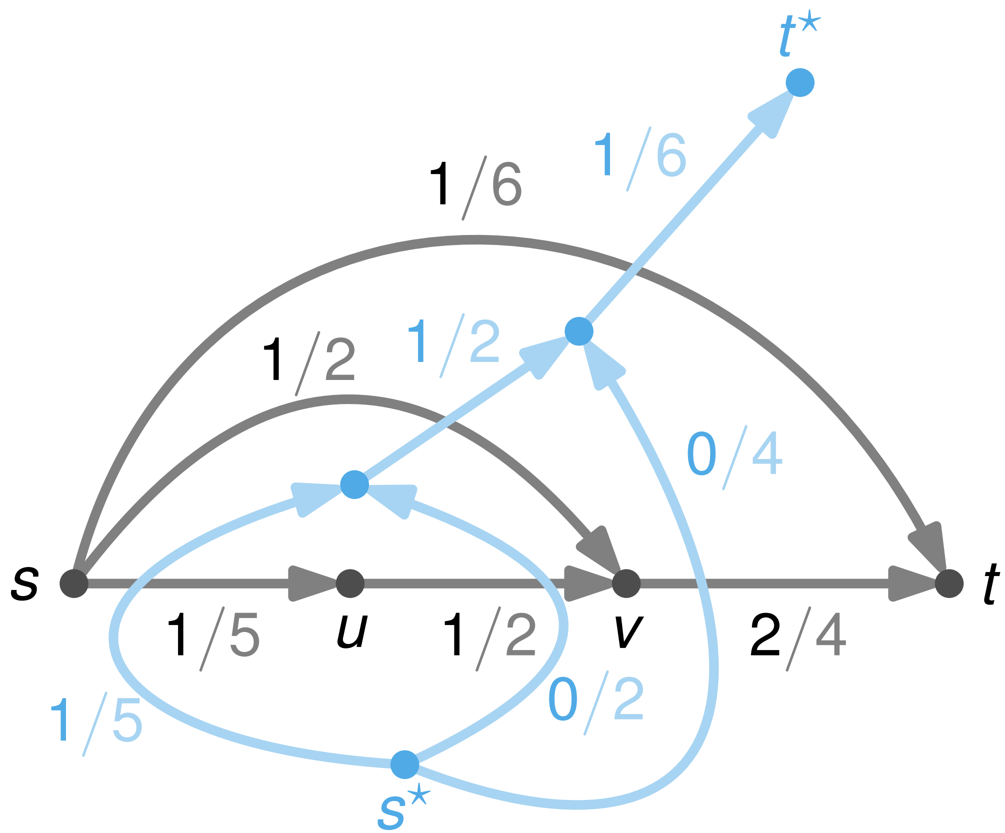

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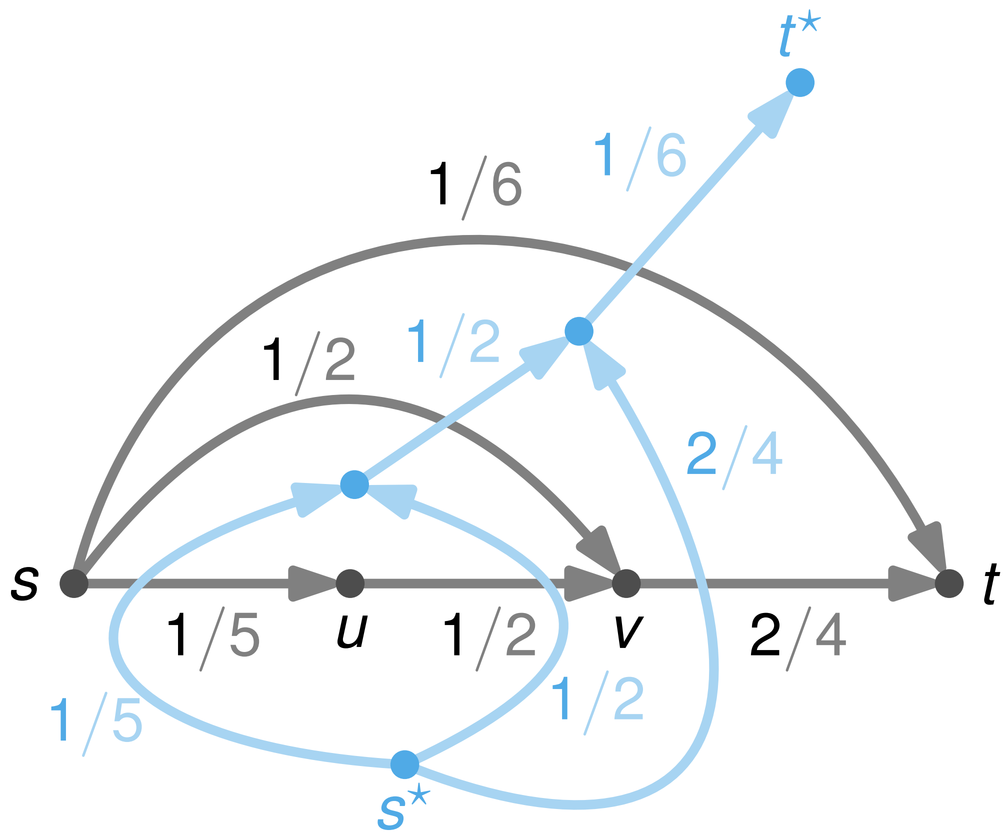

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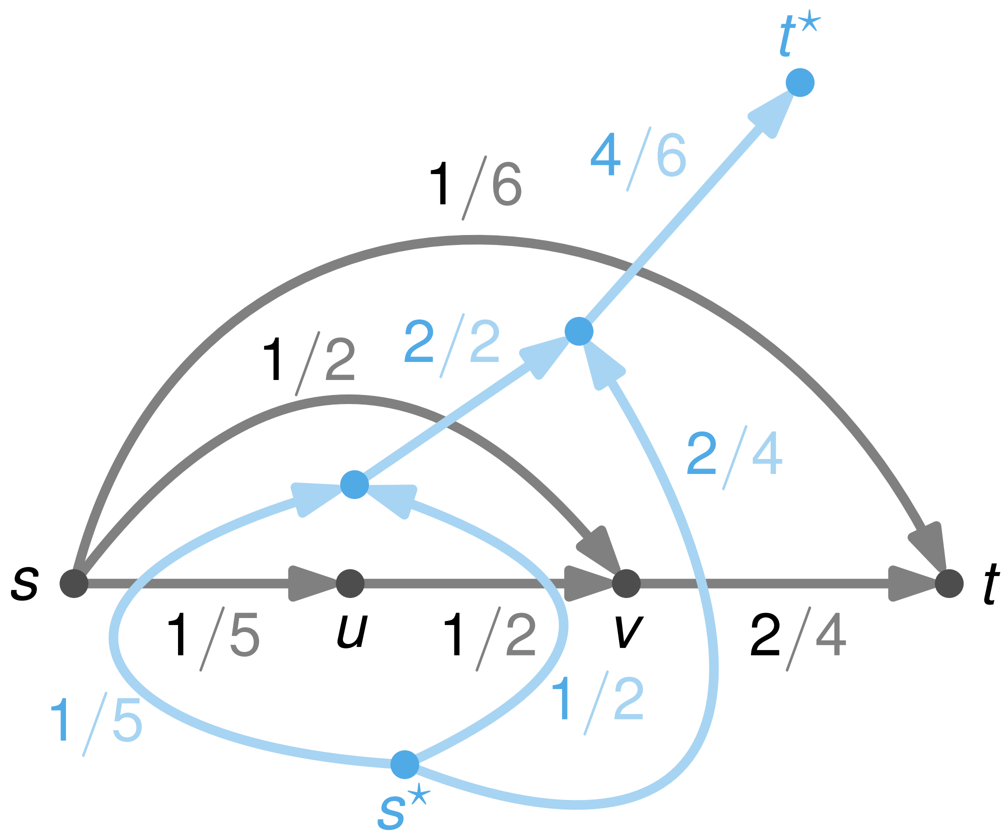

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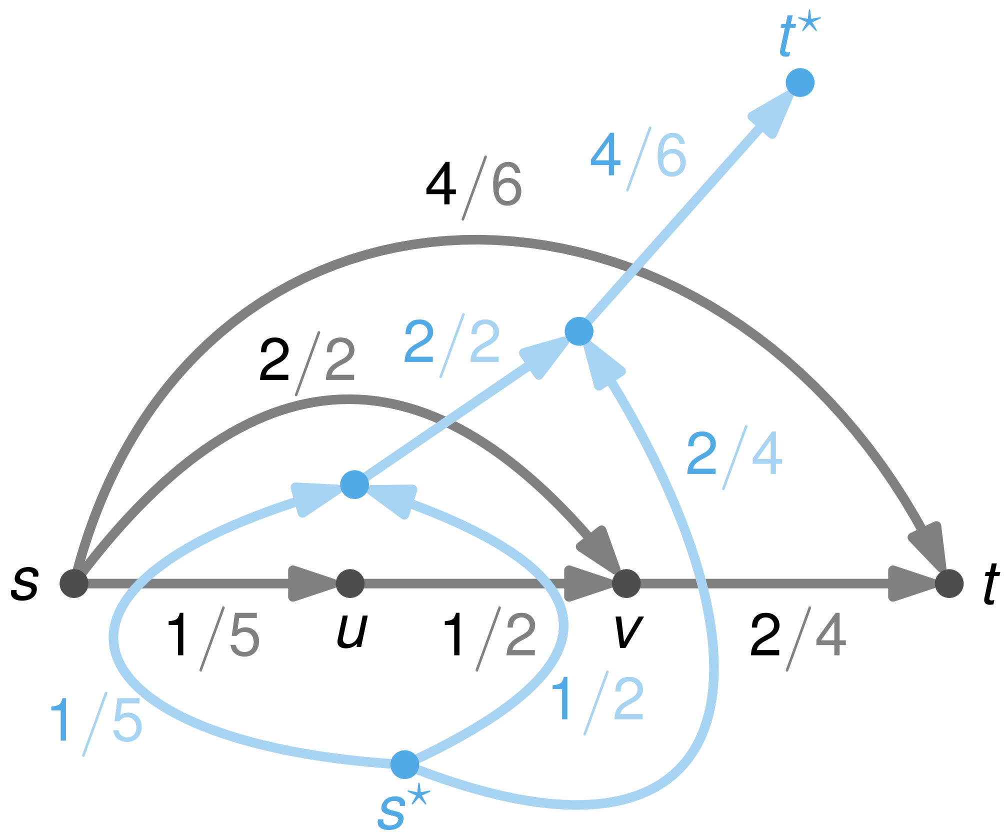

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Prisoner's Dilemma

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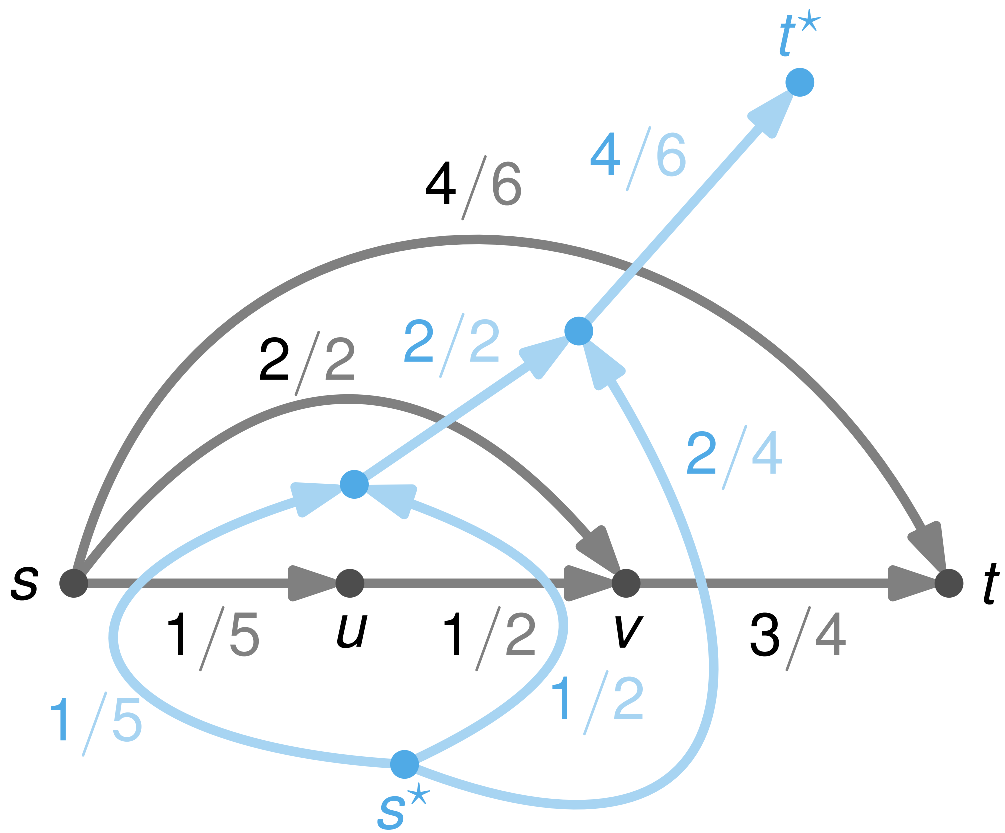

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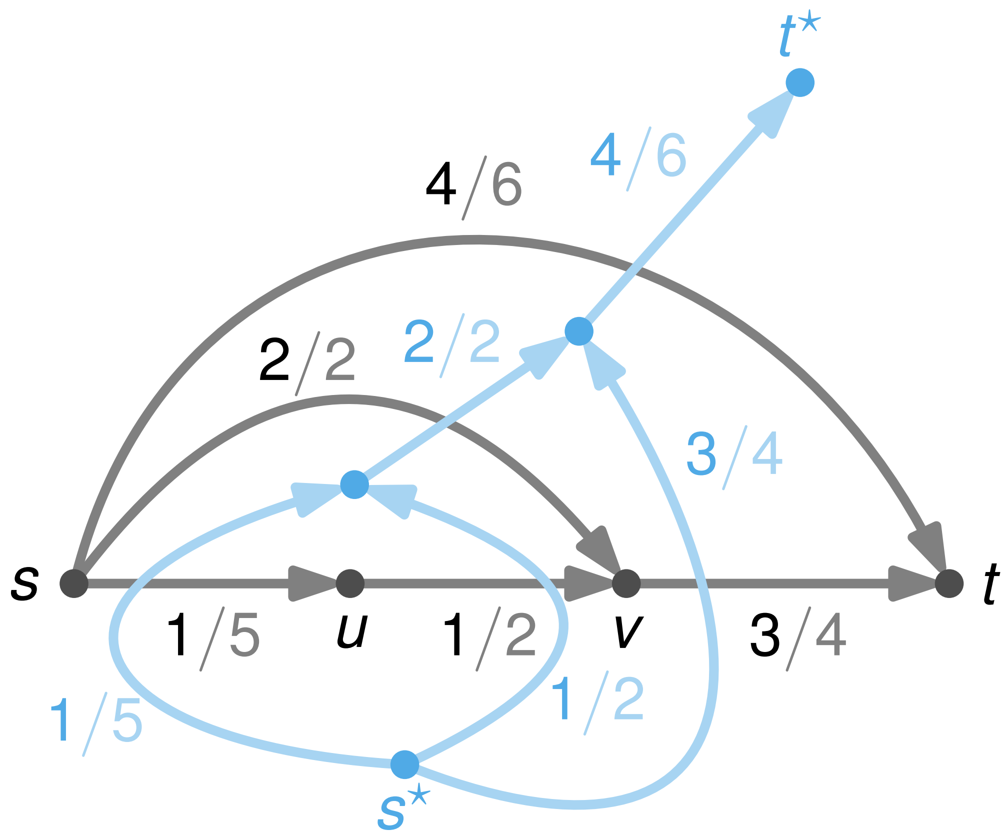

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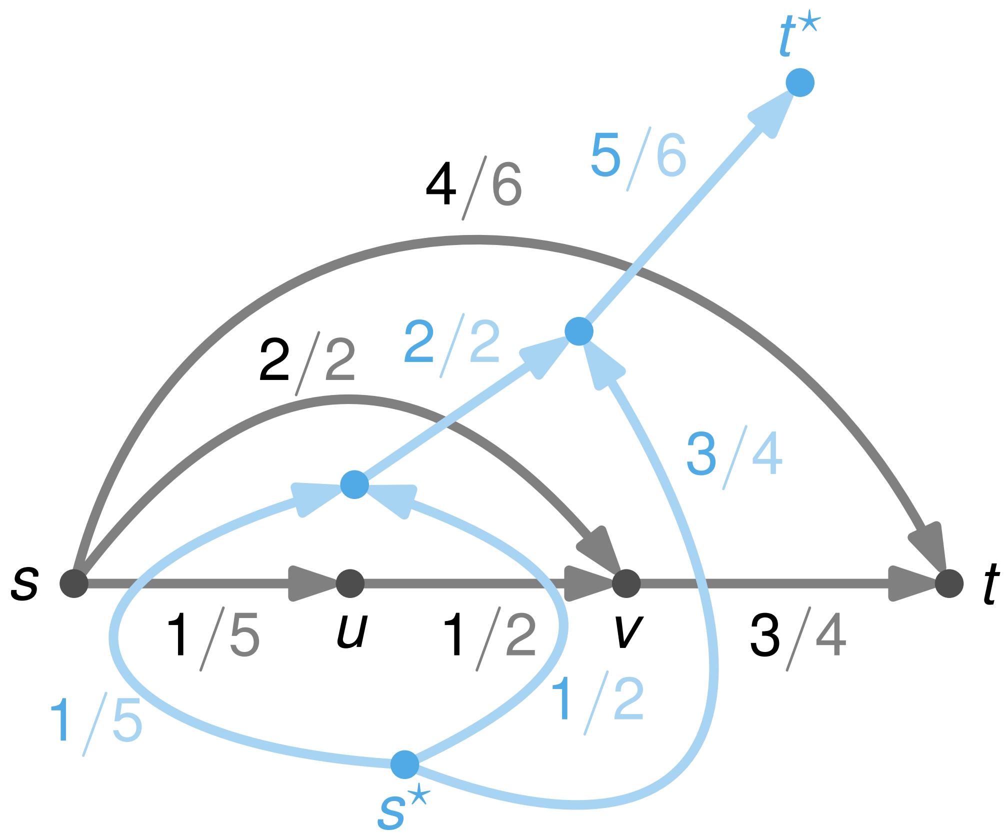

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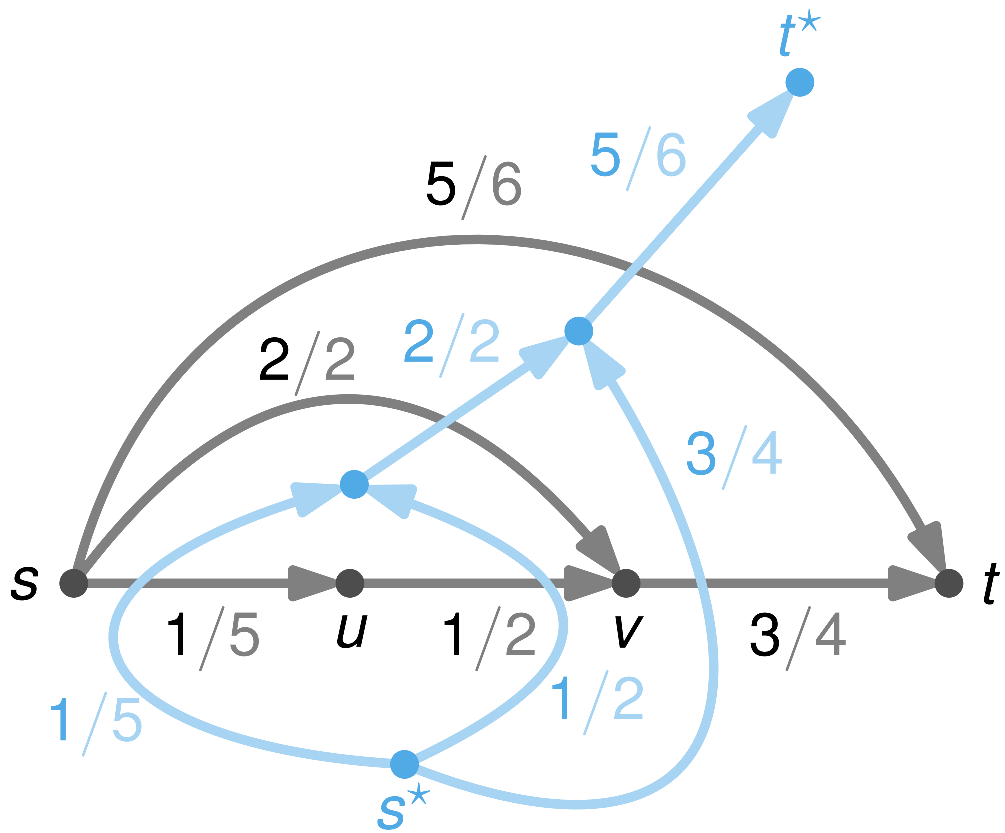

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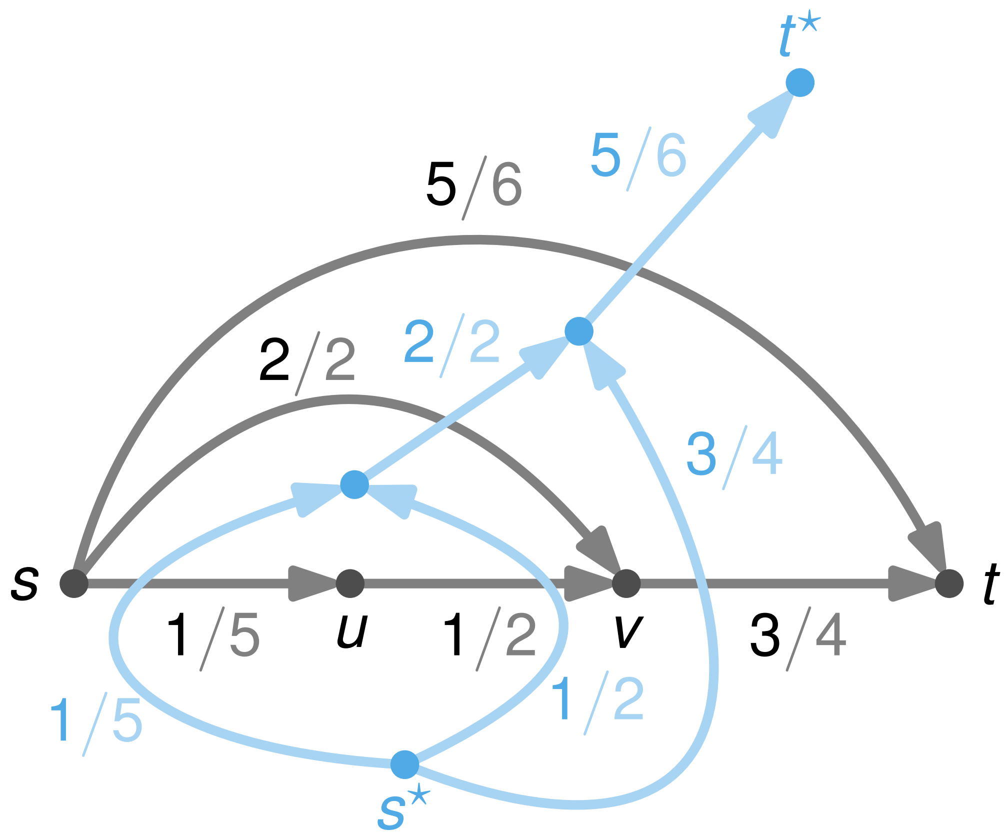
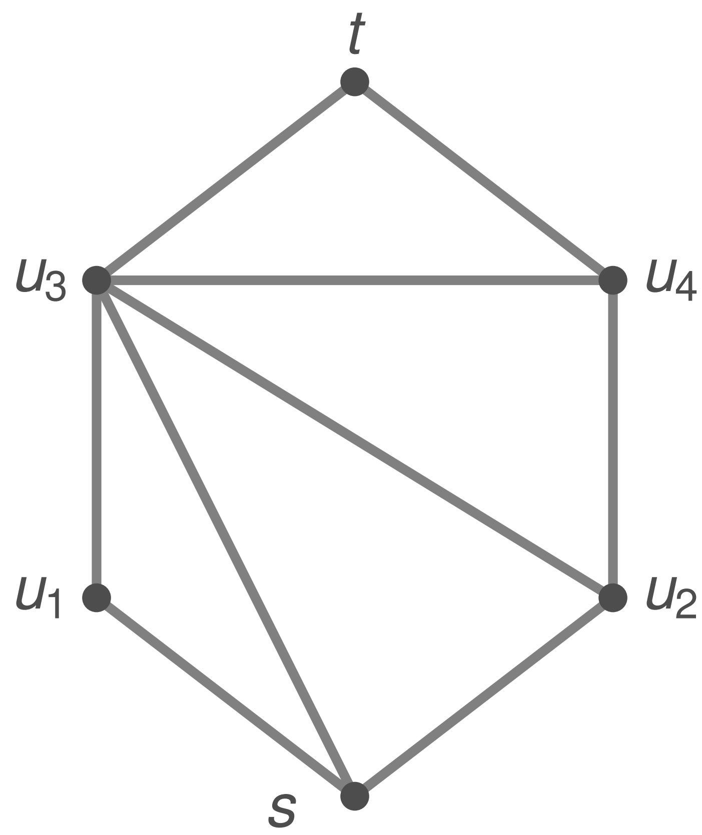


$\mathrm{OPT}_{\mathrm{MPF}} = 5$

$\mathrm{OPT}_{\mathrm{MPF}} = 8$

Algorithmic Sketch



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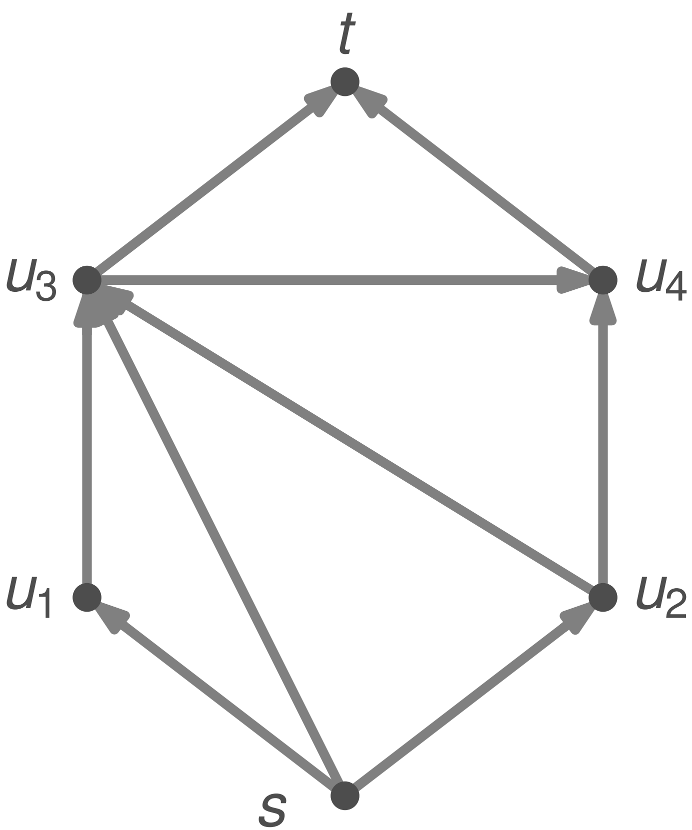
Algorithmic Sketch

$G = \texttt{bipolarSubgraphOf}(G, s, t)$



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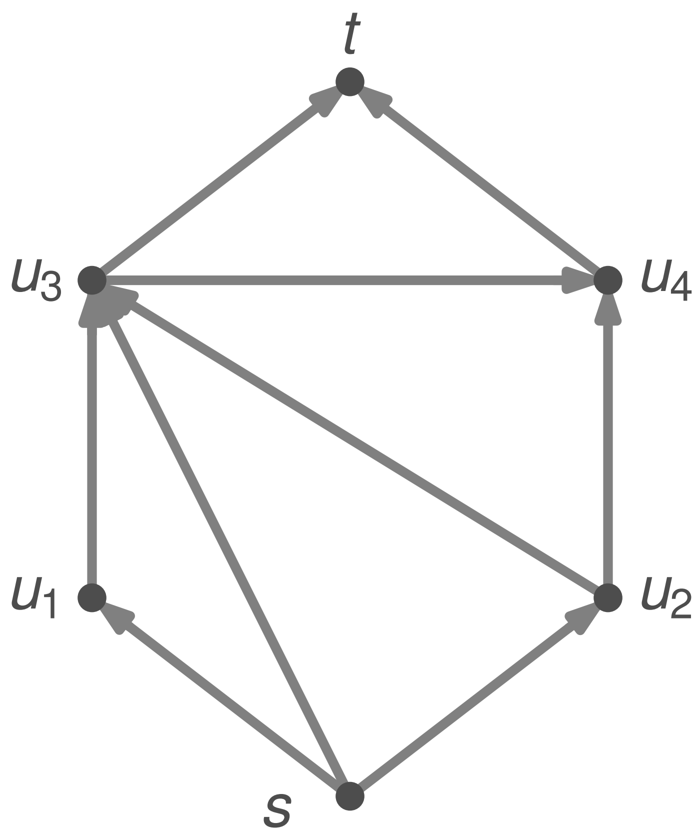

▷ PQ-Tree; Invariant $G(\mathcal{E}) \cong G$

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Algorithmic Sketch

$G = \texttt{bipolarSubgraphOf}(G, s, t)$

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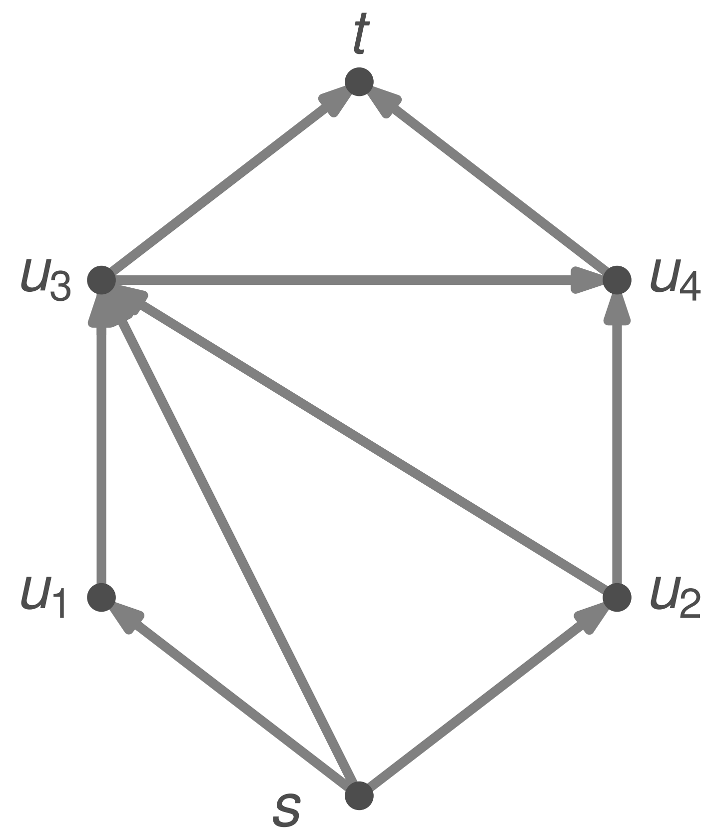
$G^\star = \texttt{constructDualGraphOf}(G(\mathcal{E}))$



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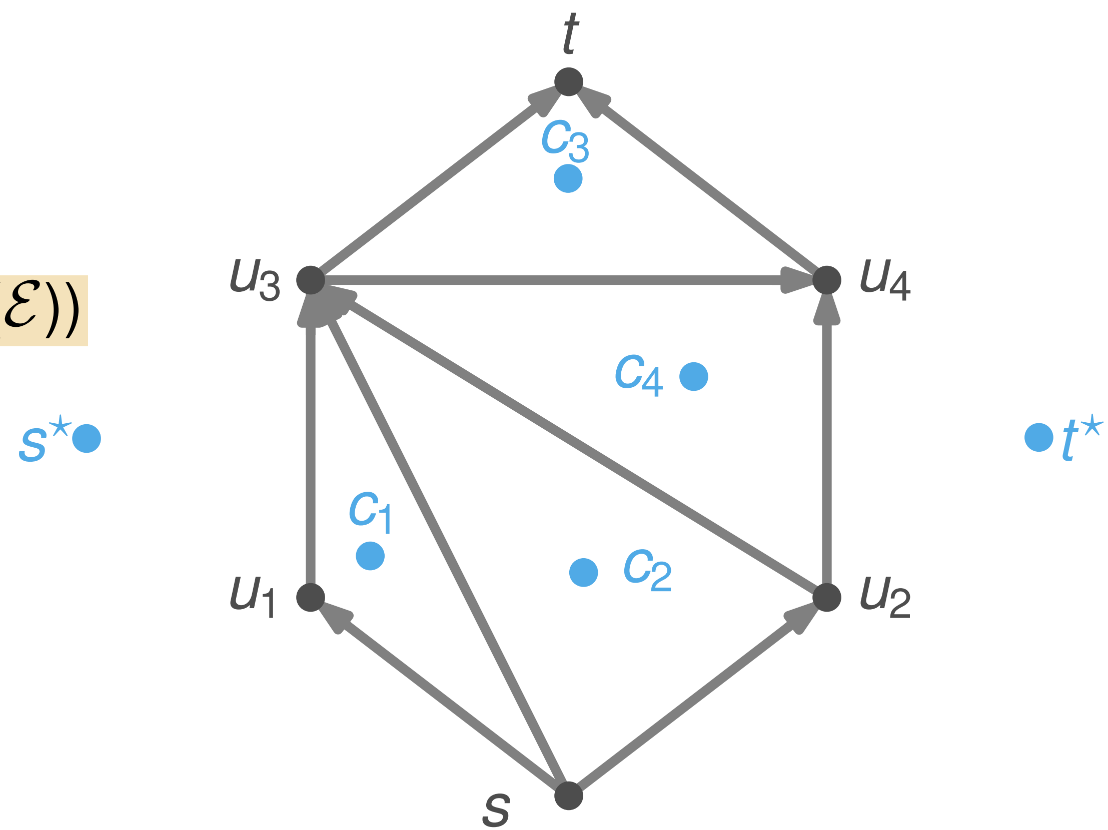

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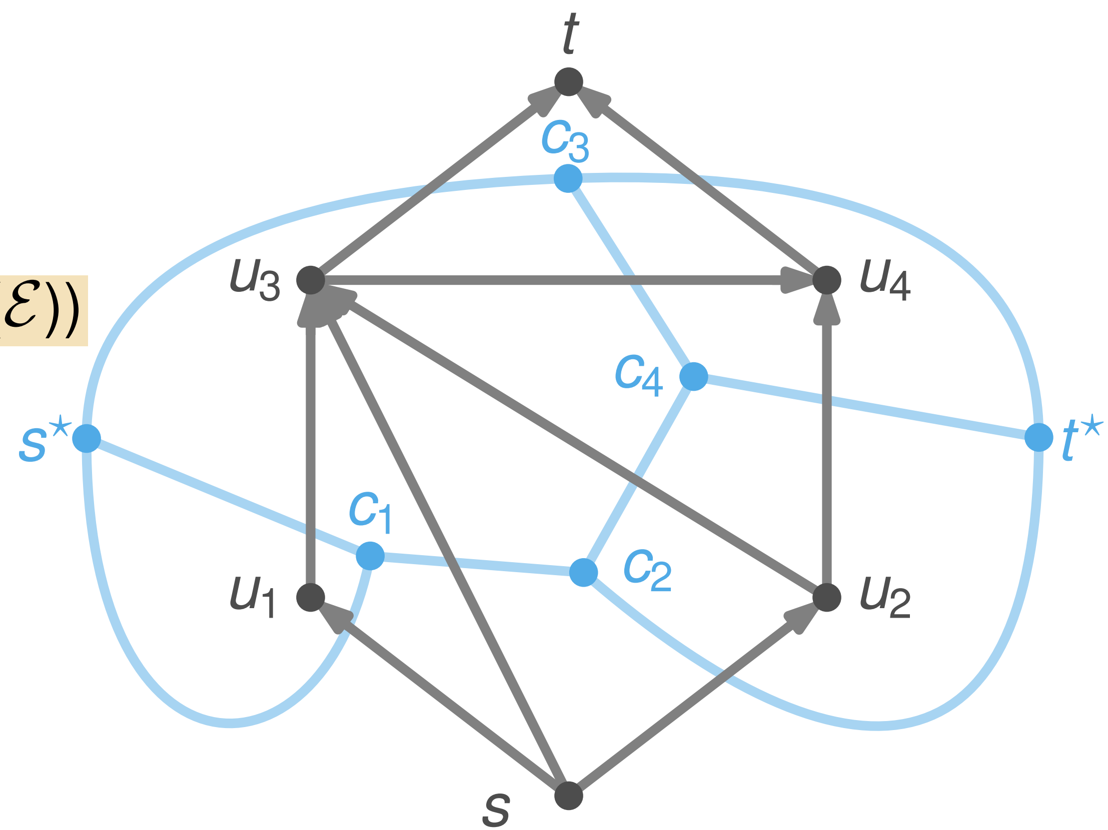

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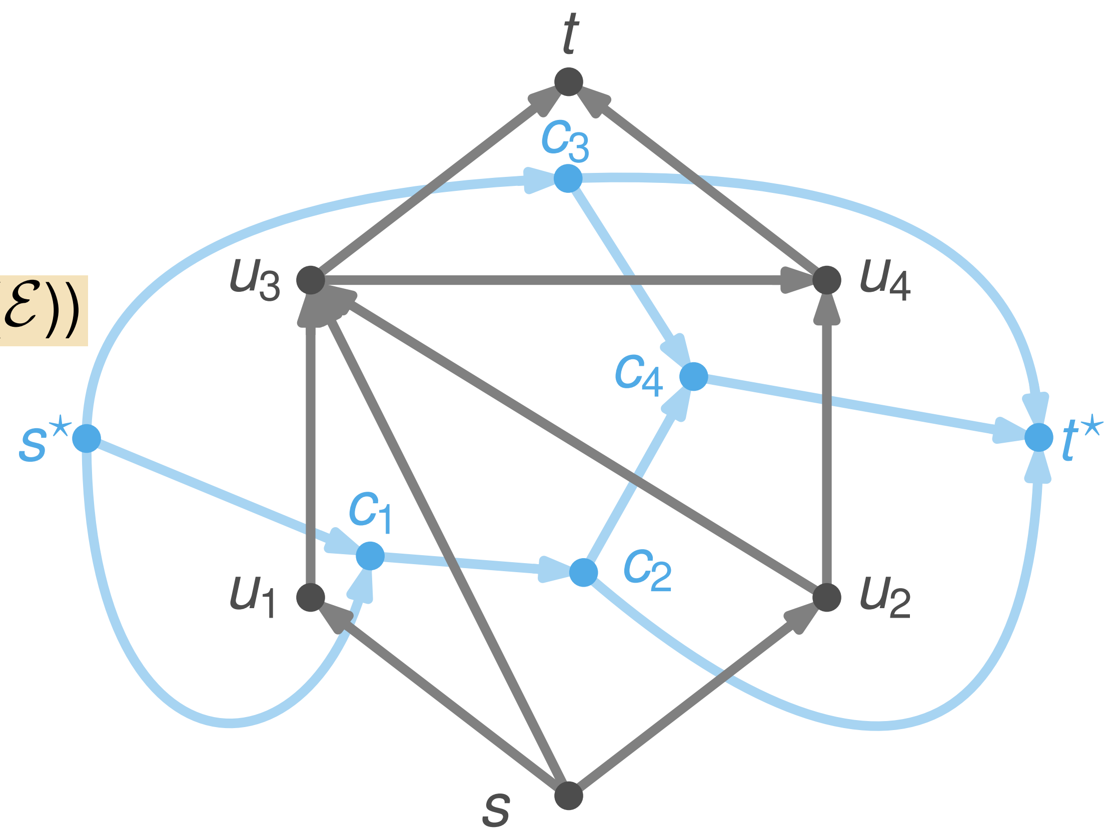

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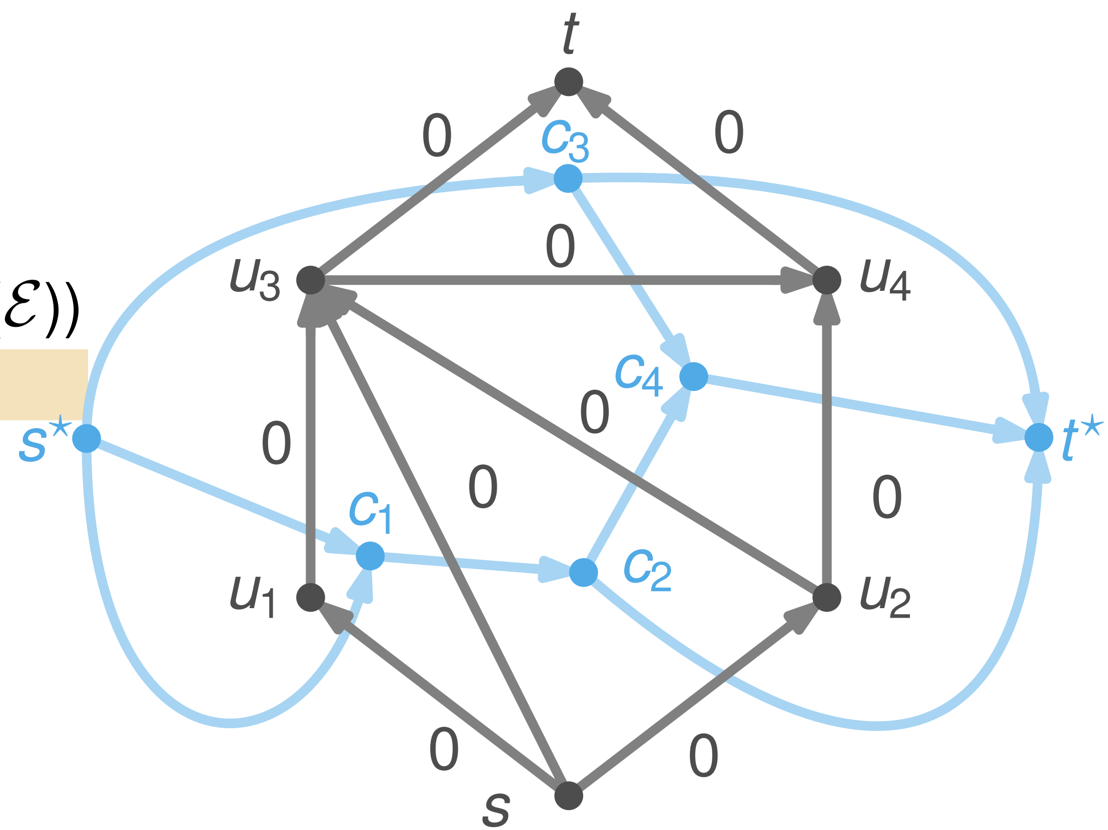

▷ Augment flow along an incident edge at source s

Algorithmic Sketch

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$f \equiv 0$

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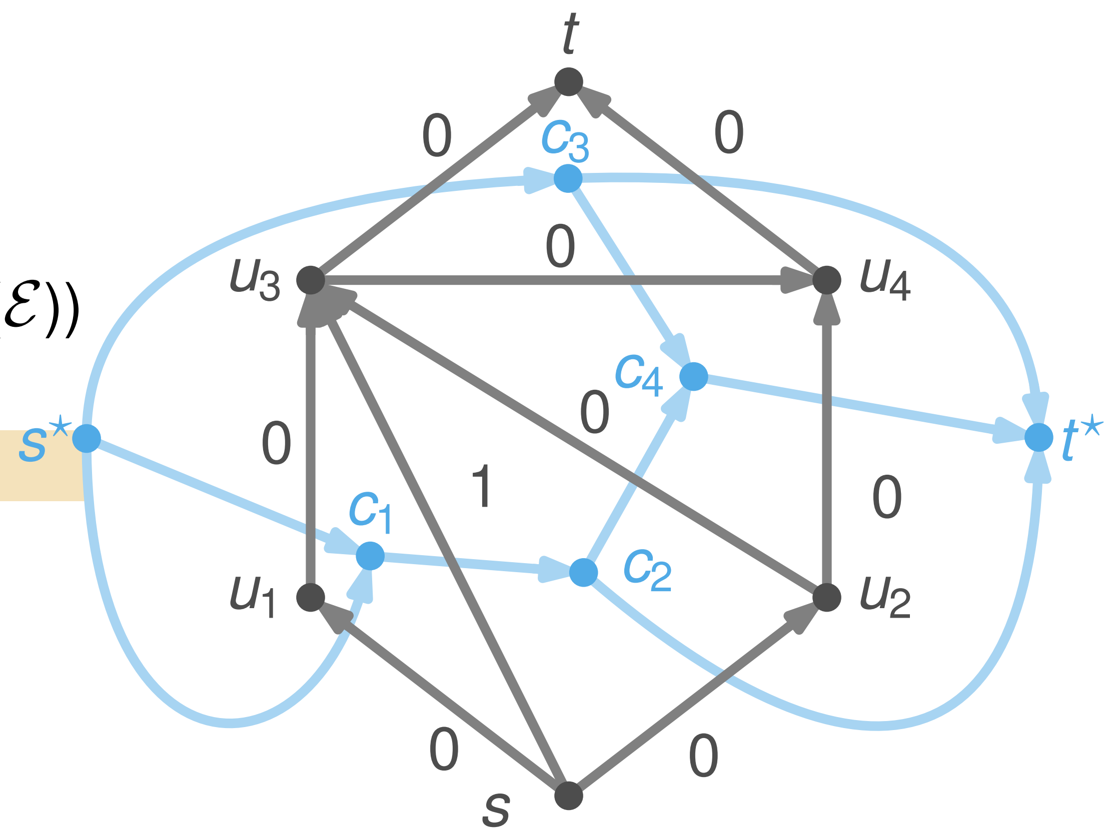

Algorithmic Sketch

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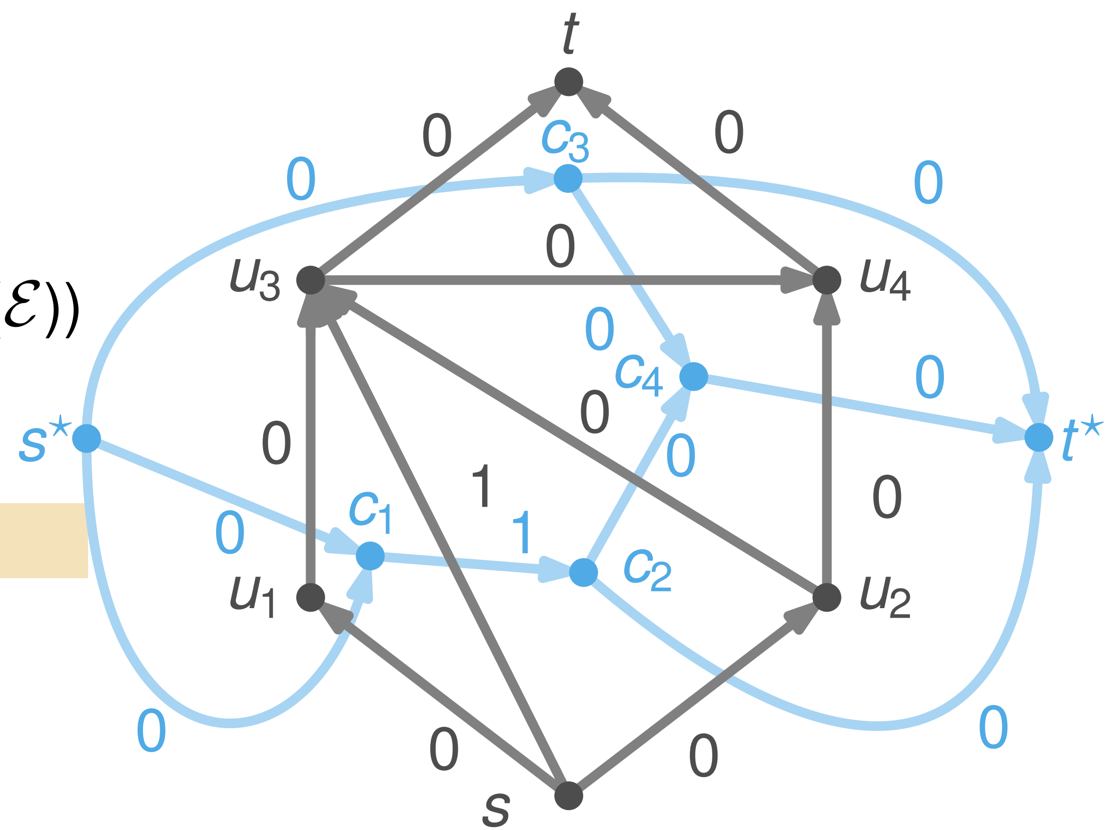

▷ Bijective function

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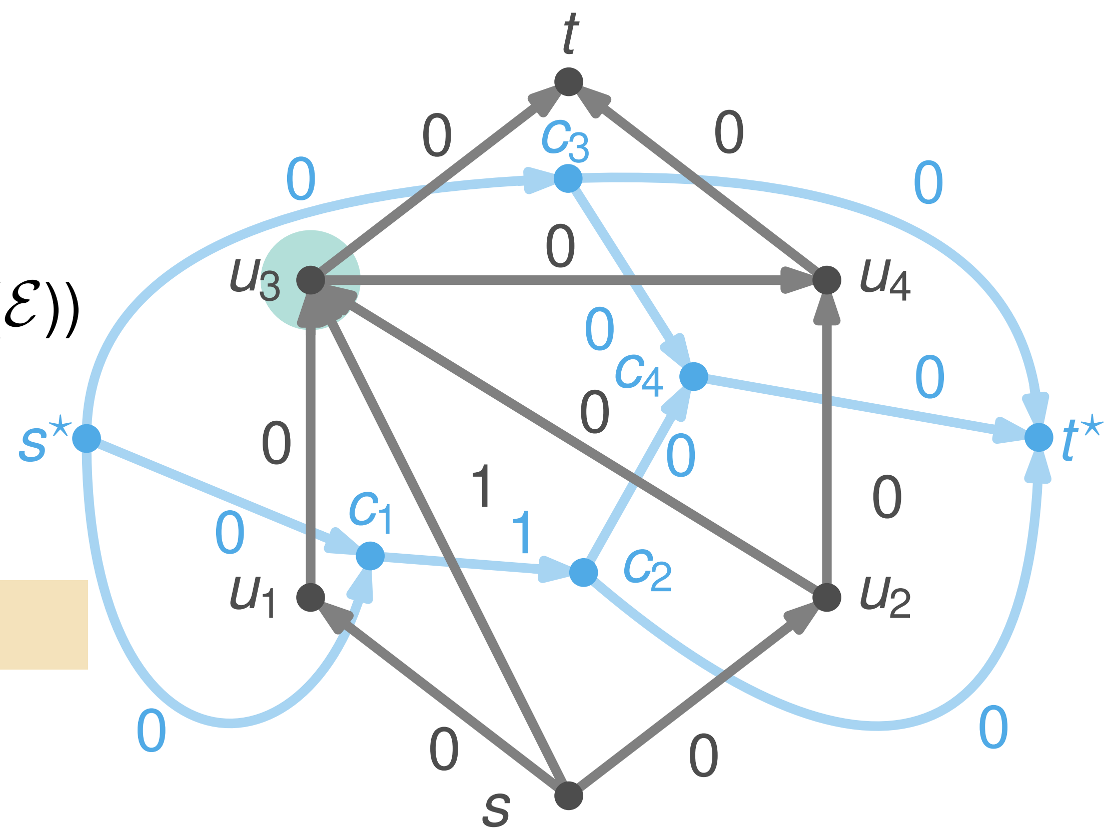
$\mu_{\text{dual}} : E(G) \to E(G^\star)$
$X = \bigcup_{u \in V(G):\ f_{\text{net}}(u) \neq 0} \{u\}$



▷ Check KCL property in graph G

Algorithmic Sketch

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$f \equiv 0$

$f(s, u) = 1$ for some $u \in V(G)$

$\mu_{\text{dual}} : E(G) \rightarrow E(G^\star)$

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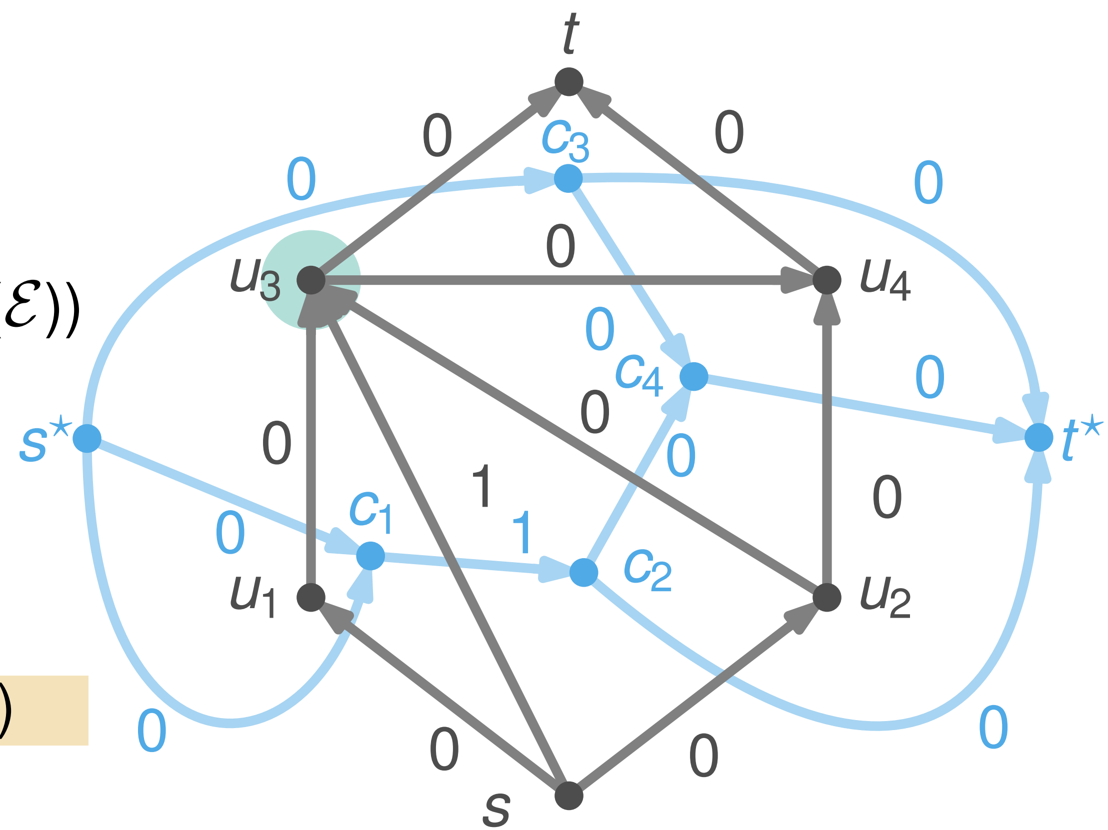
$f = \texttt{resolveConflict}(G, s, t, f, X)$



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Algorithmics Group

Algorithmic Sketch

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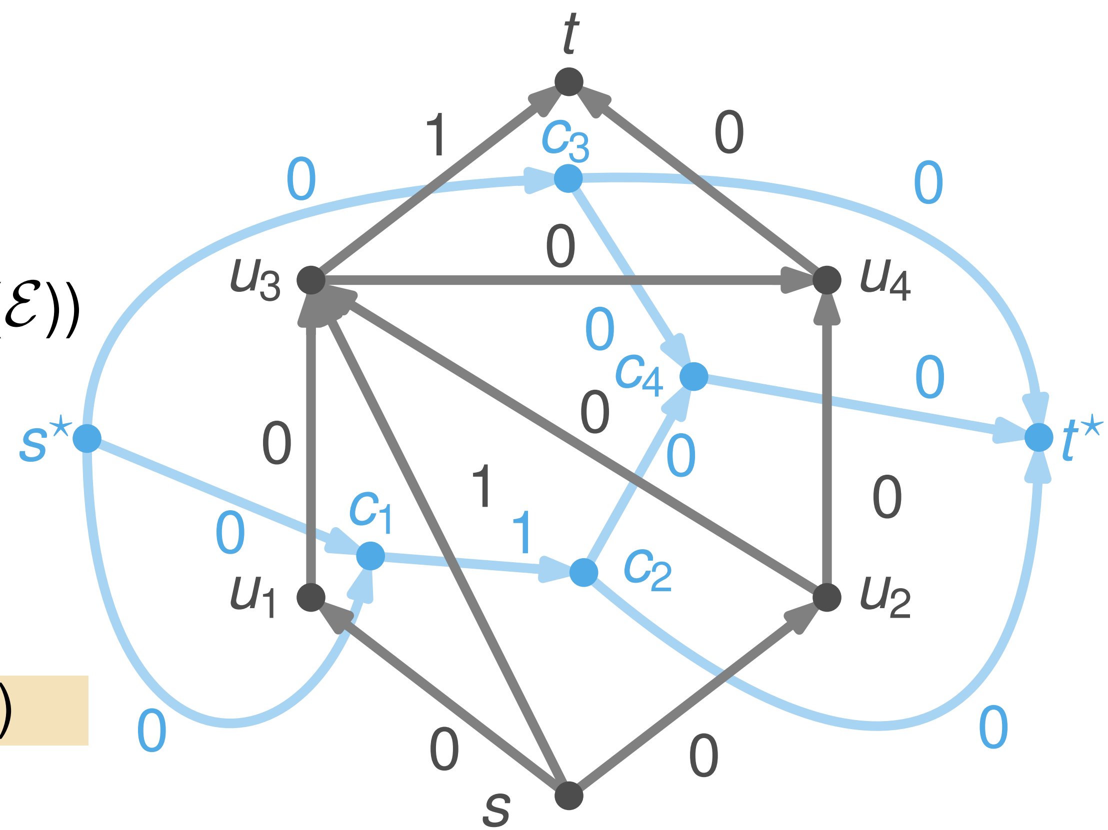

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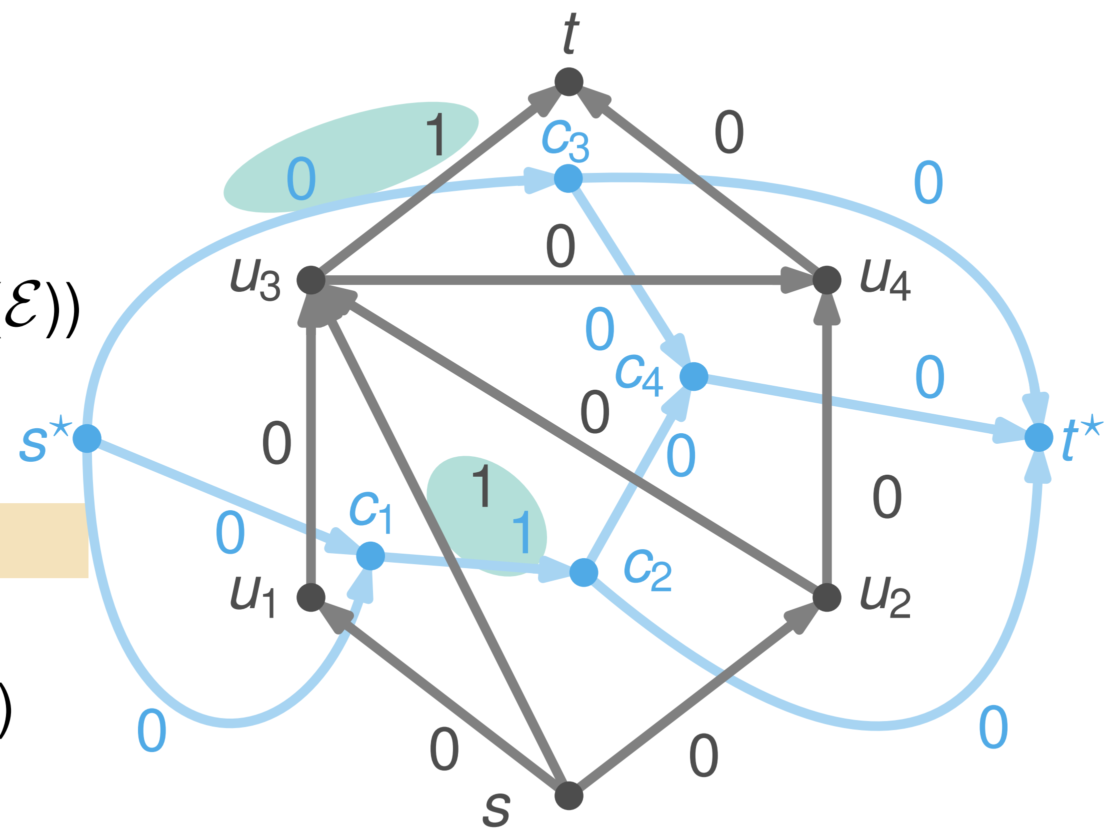

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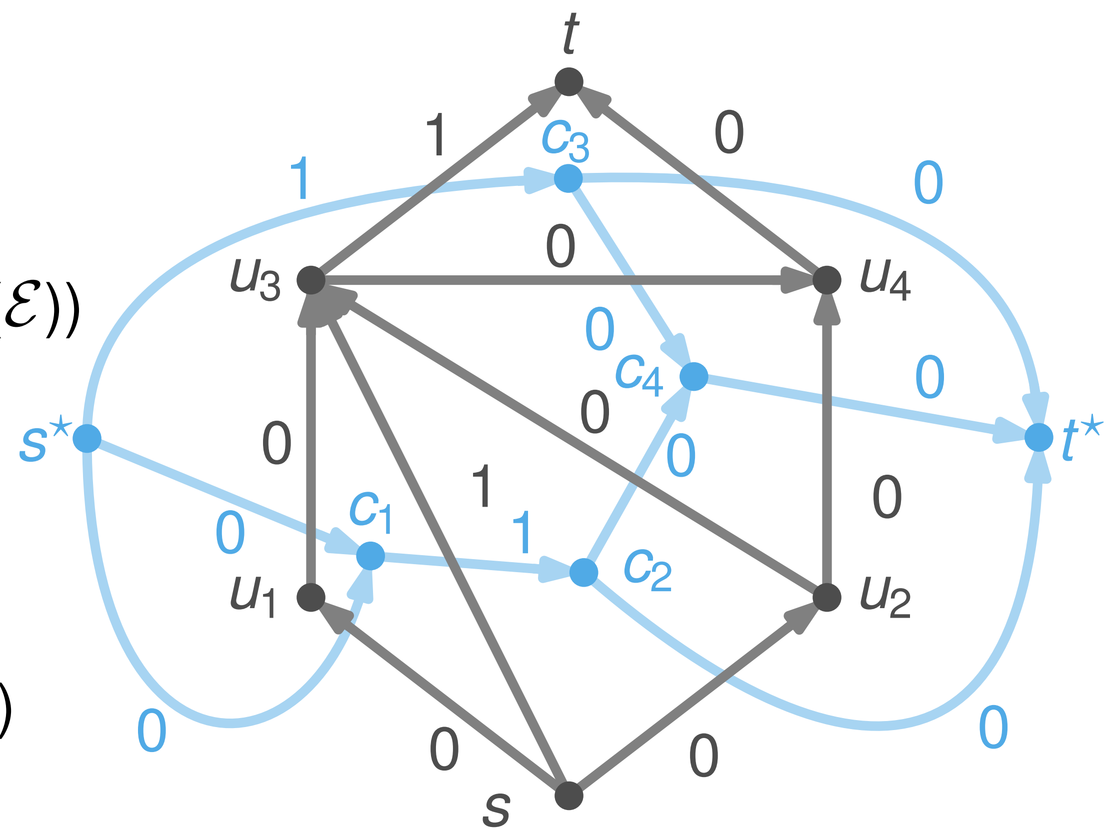

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Algorithmic Sketch

Karlsruhe Institute of Technology

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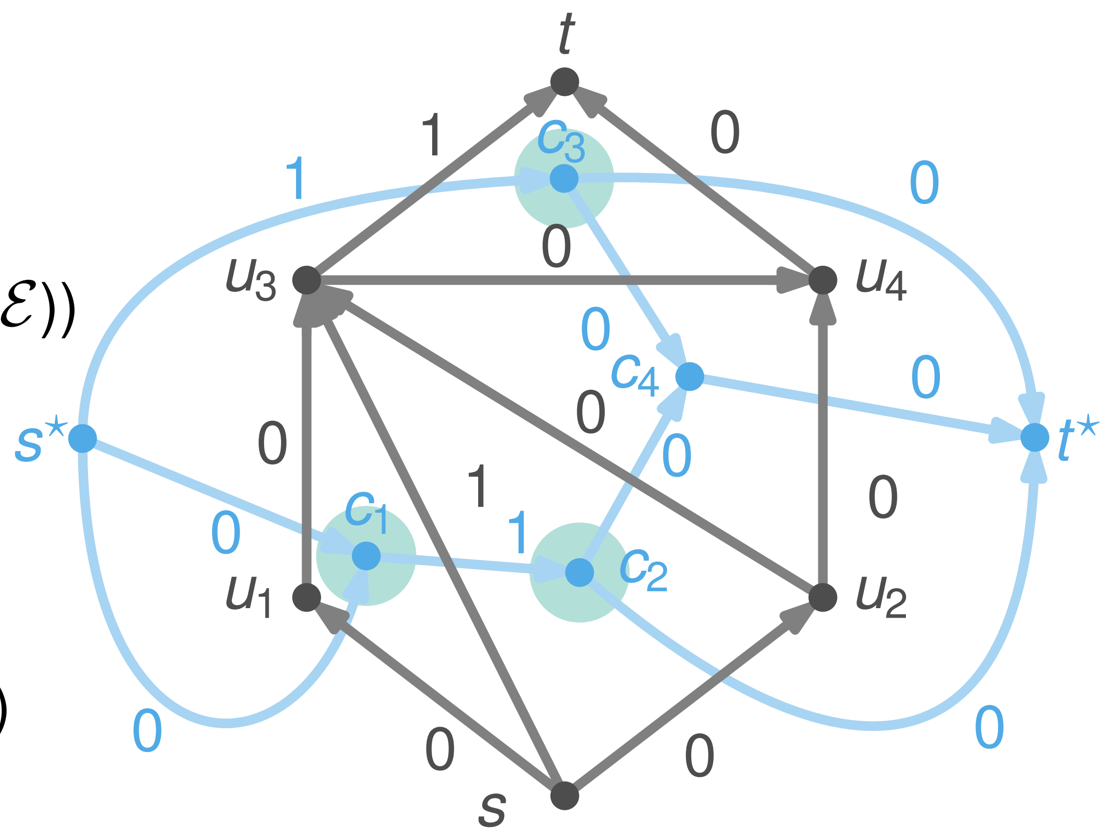

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Algorithmic Sketch

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$G^\star$ = constructDualGraphOf($G(\mathcal{E})$)

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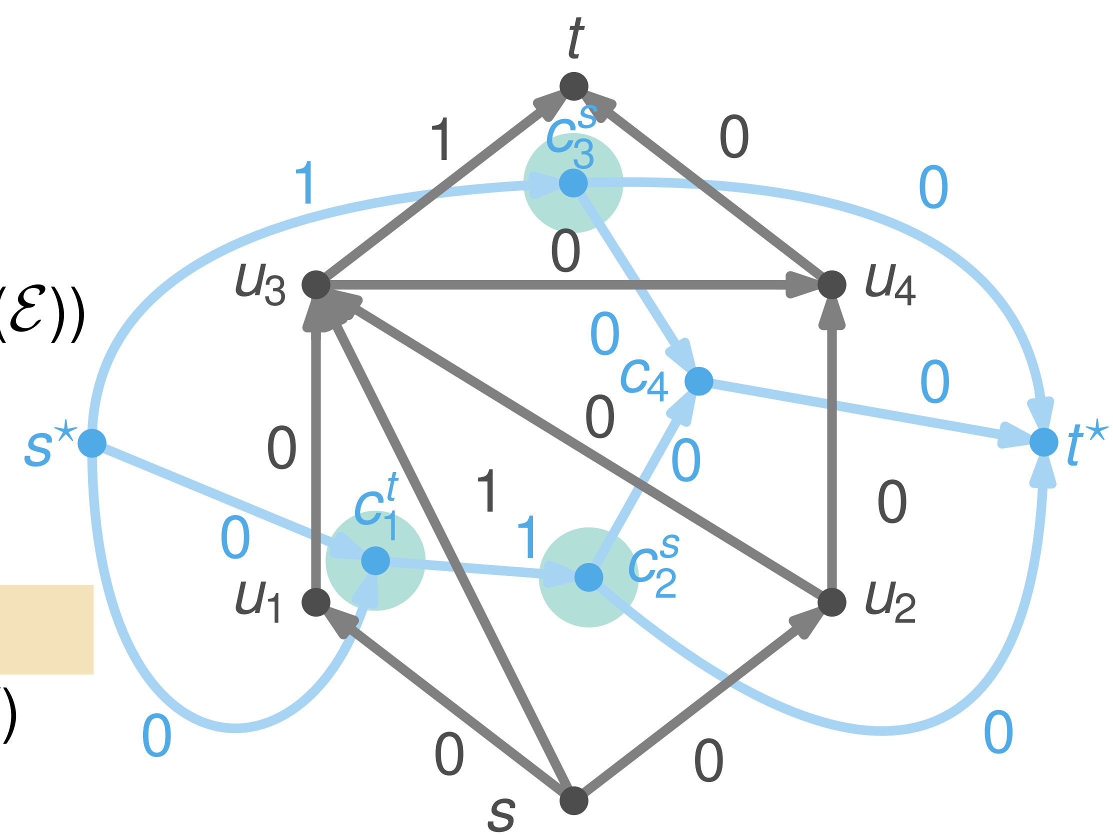

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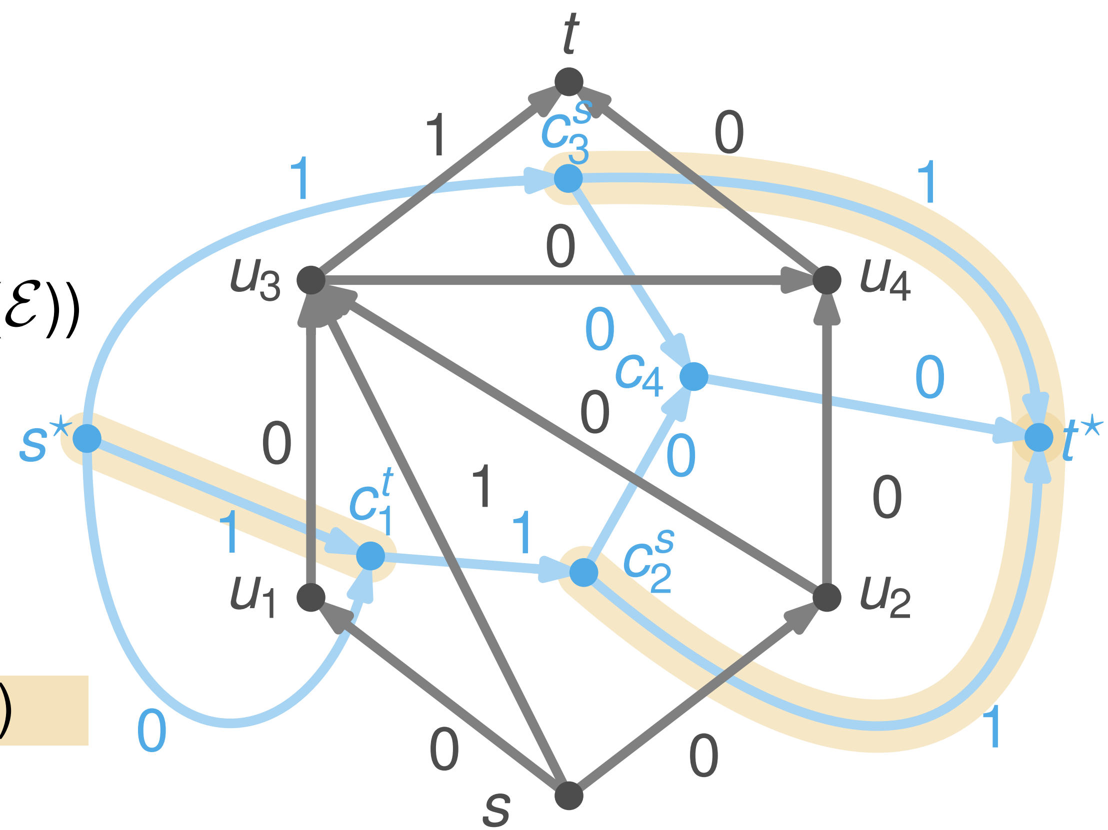

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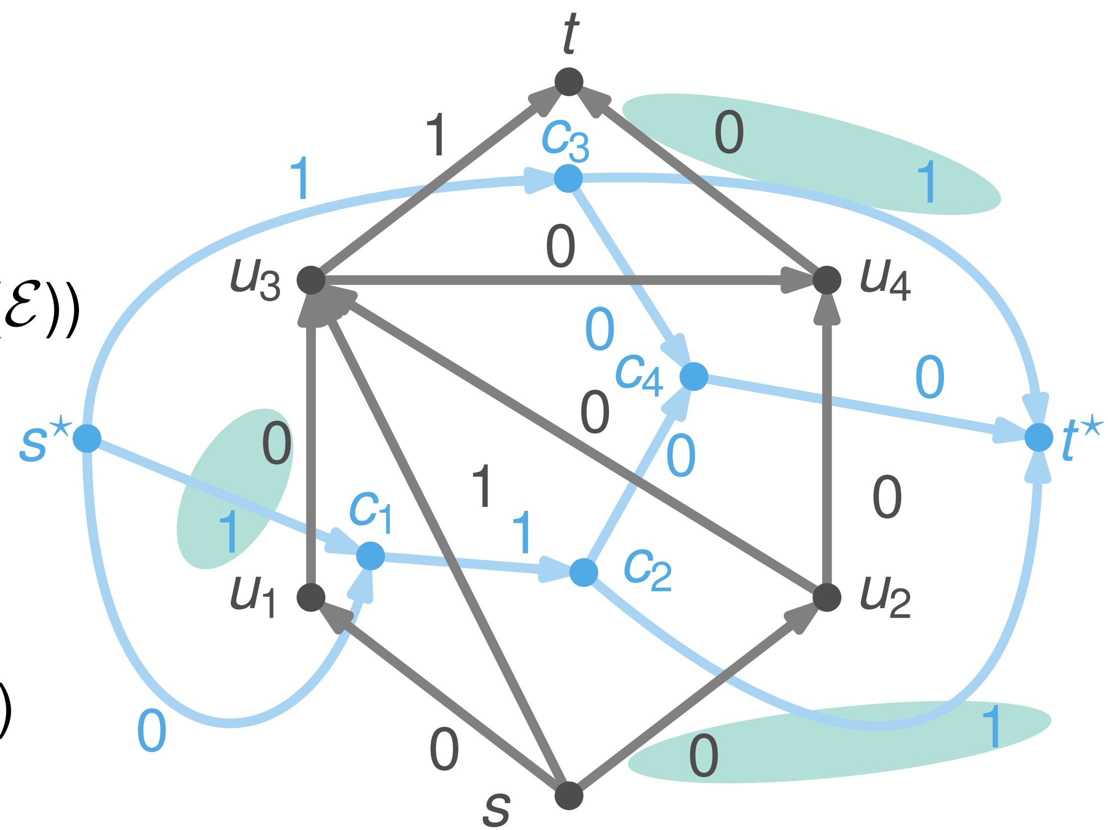

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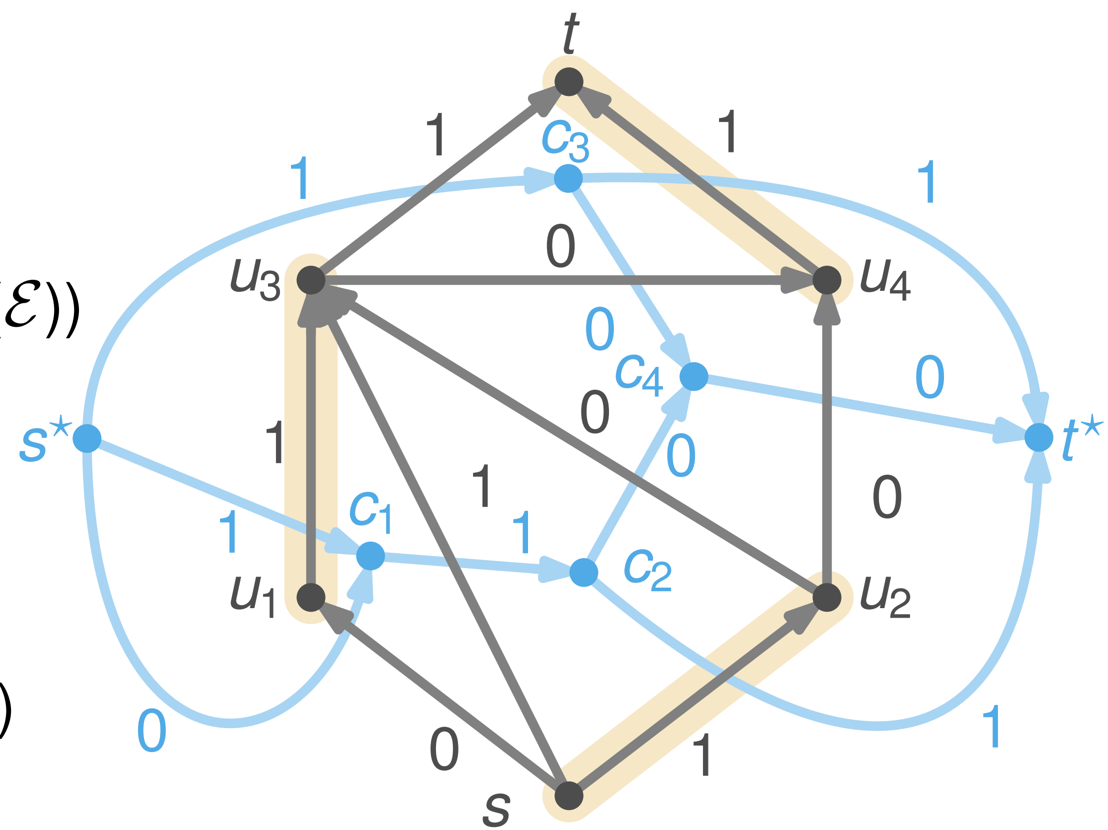

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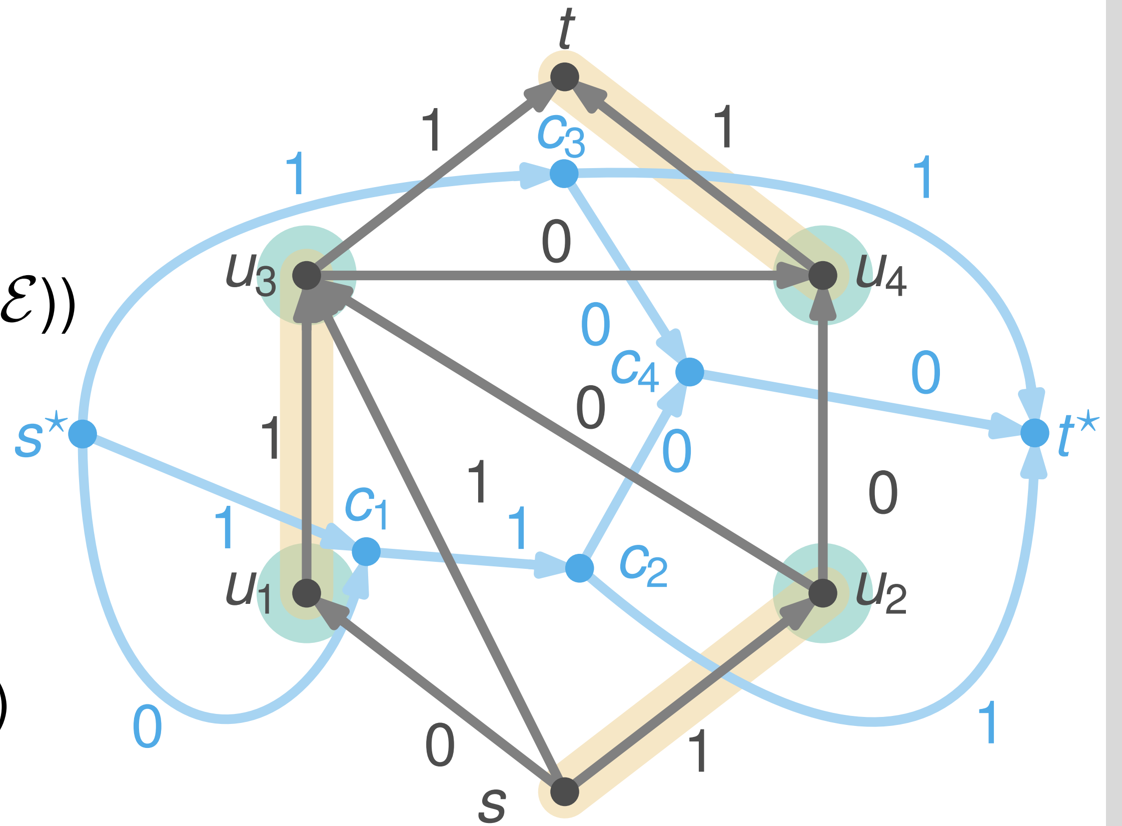

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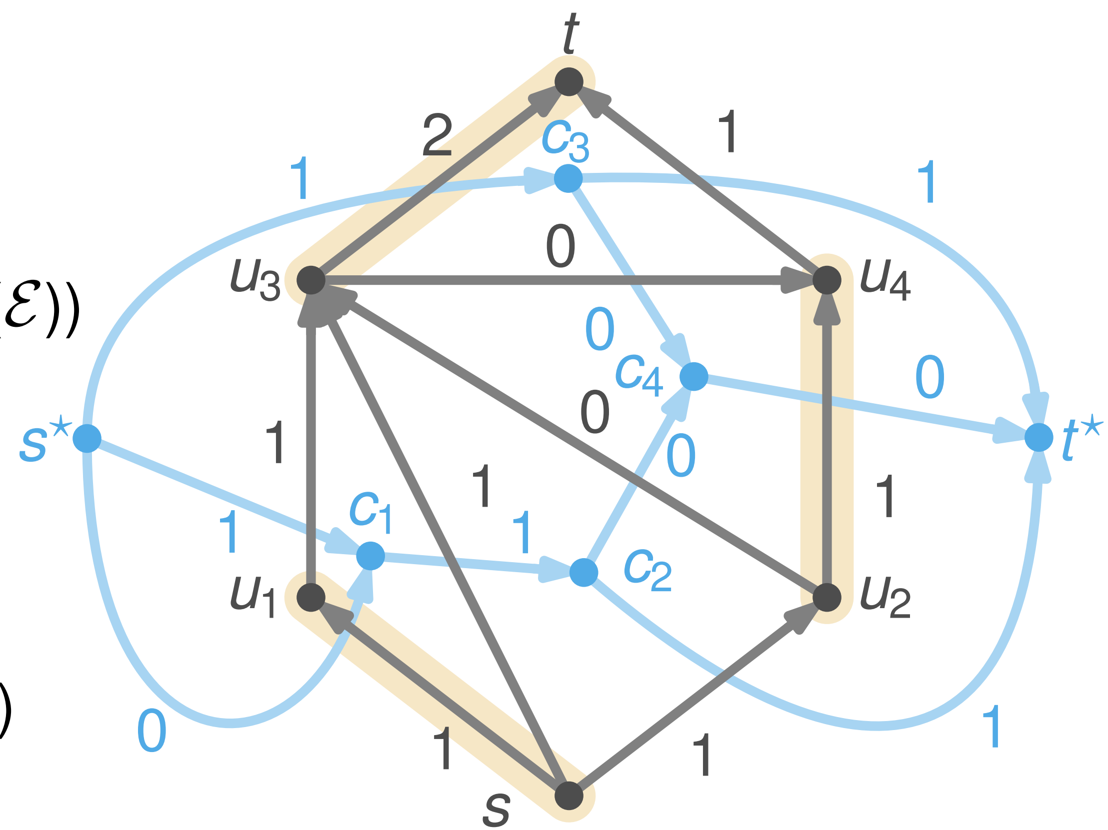

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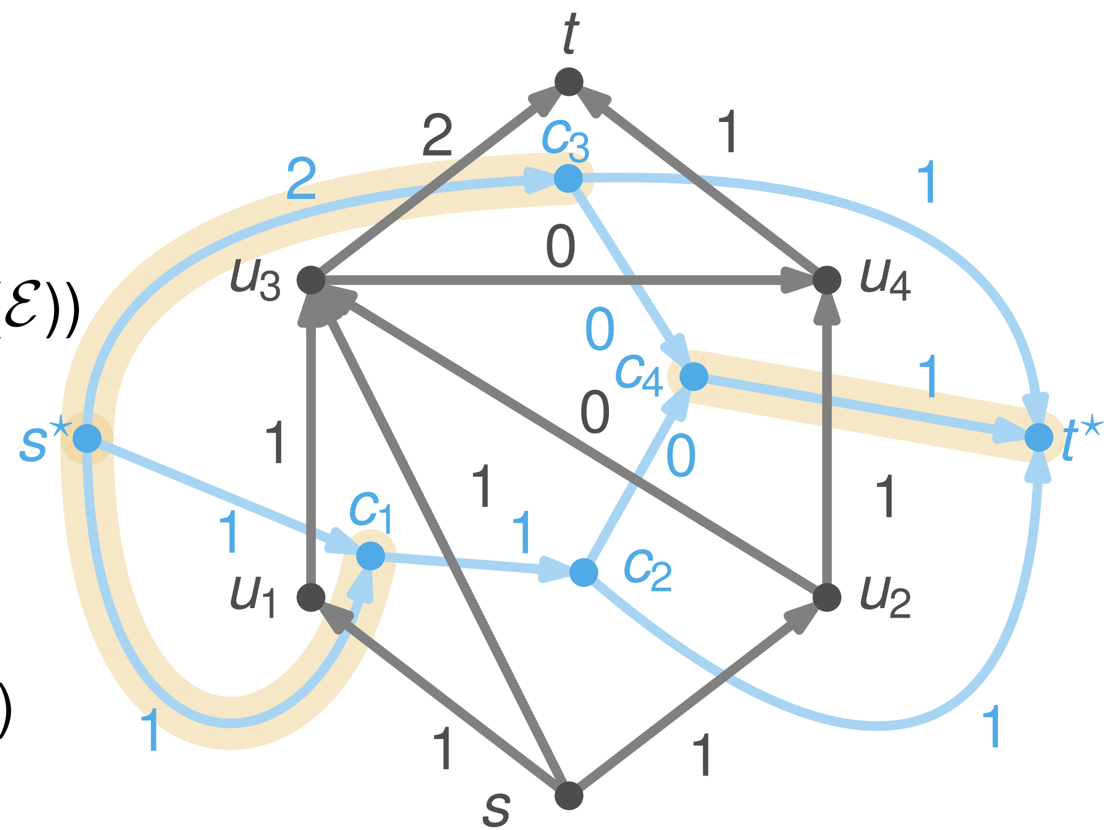

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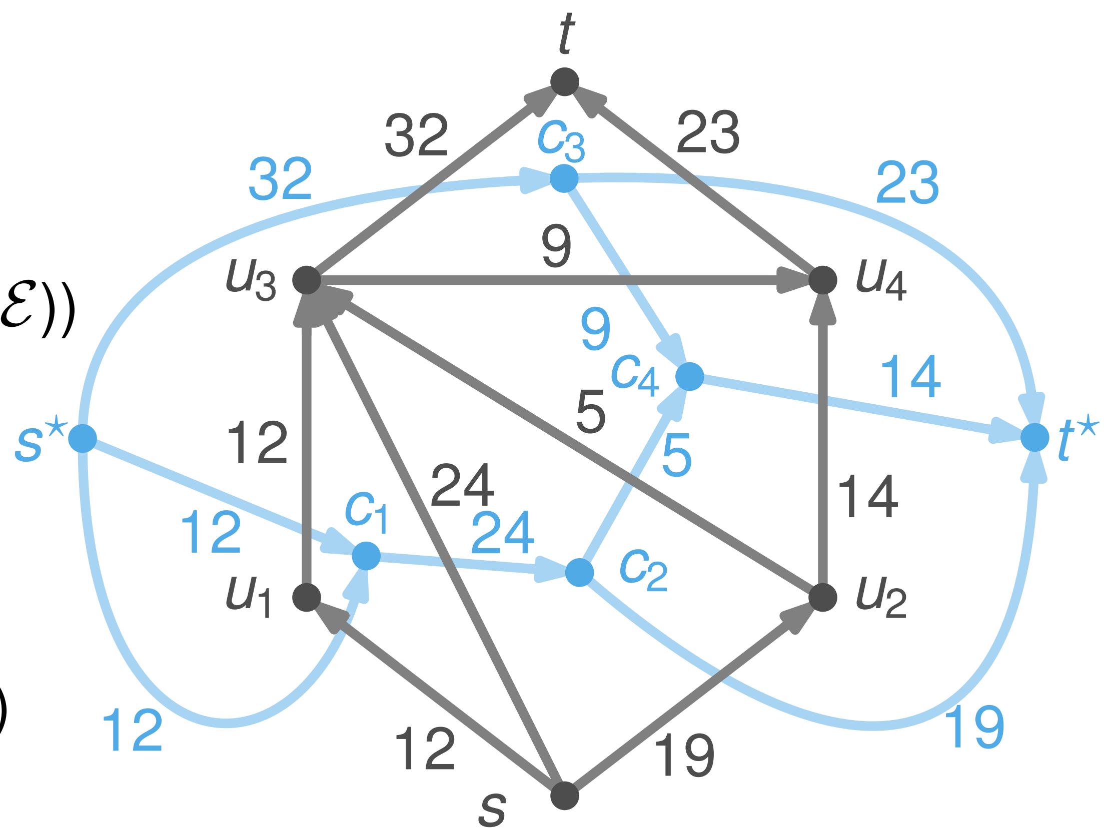
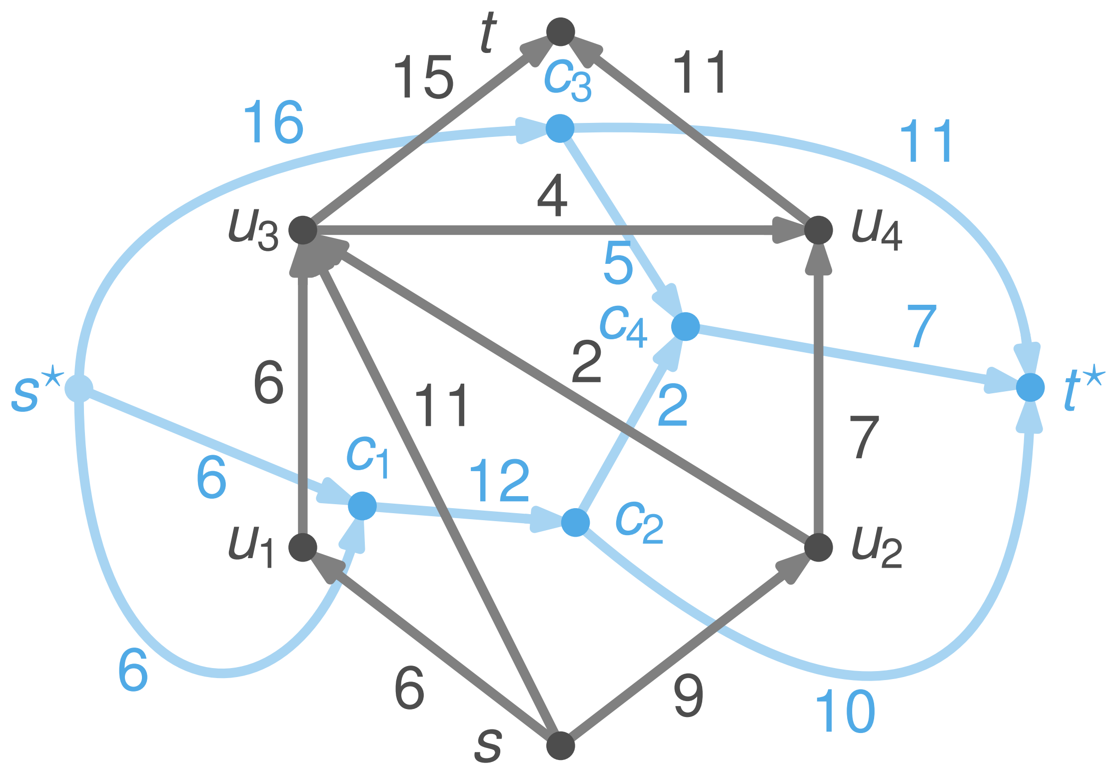
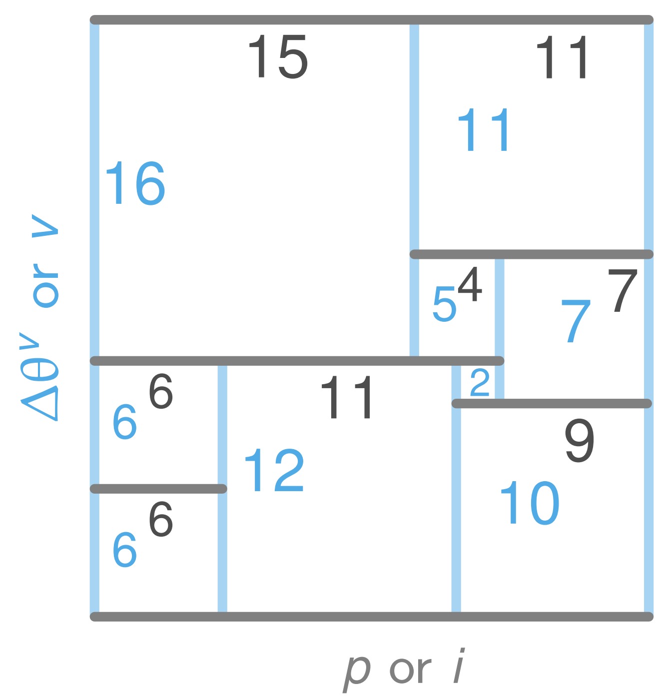


Geometric Interpretation of a KCL Conflict





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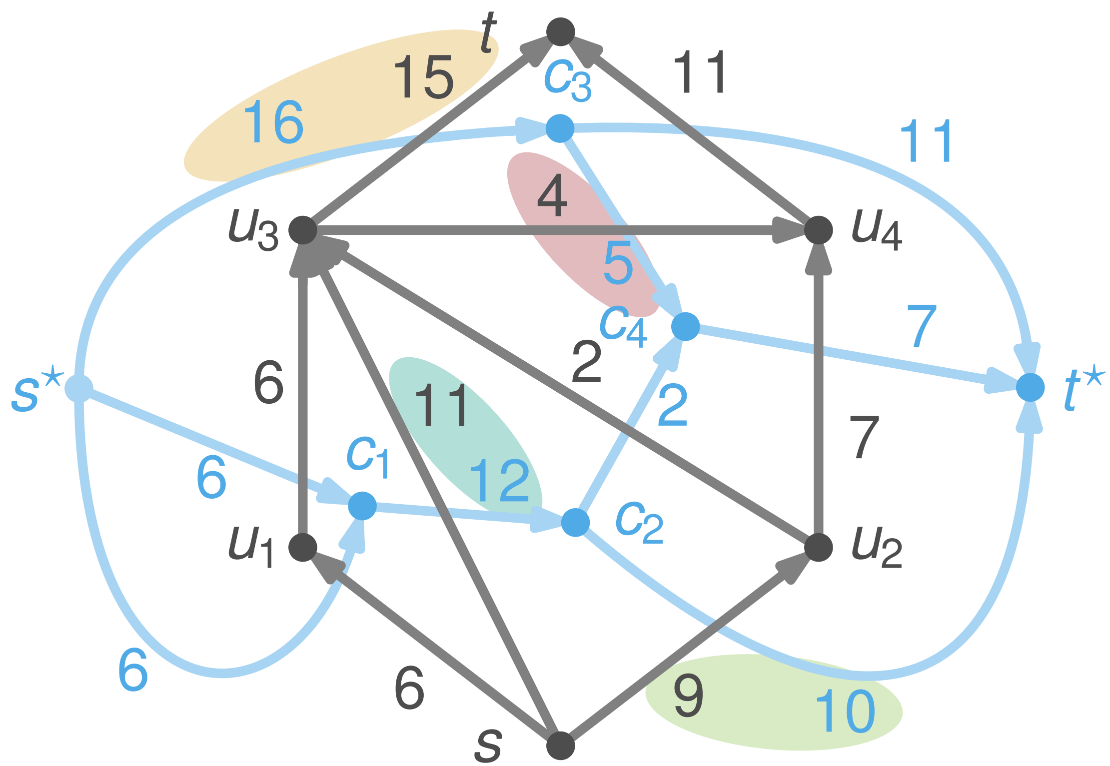

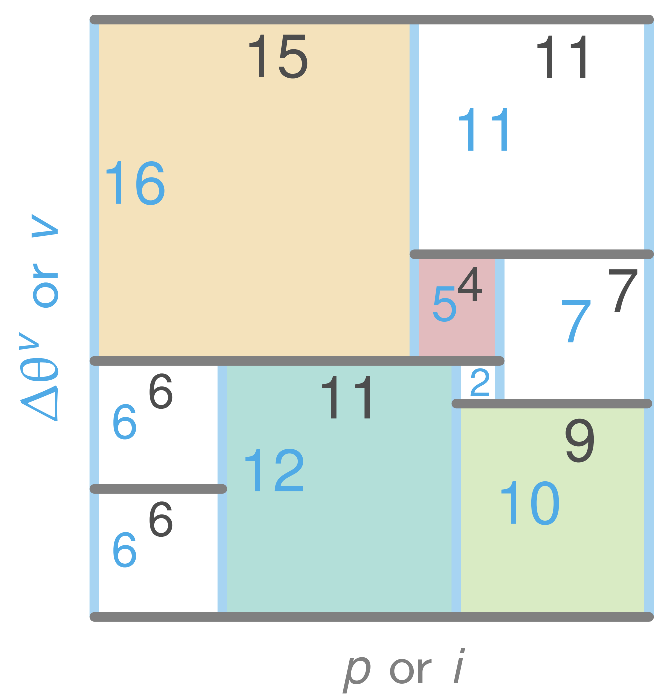

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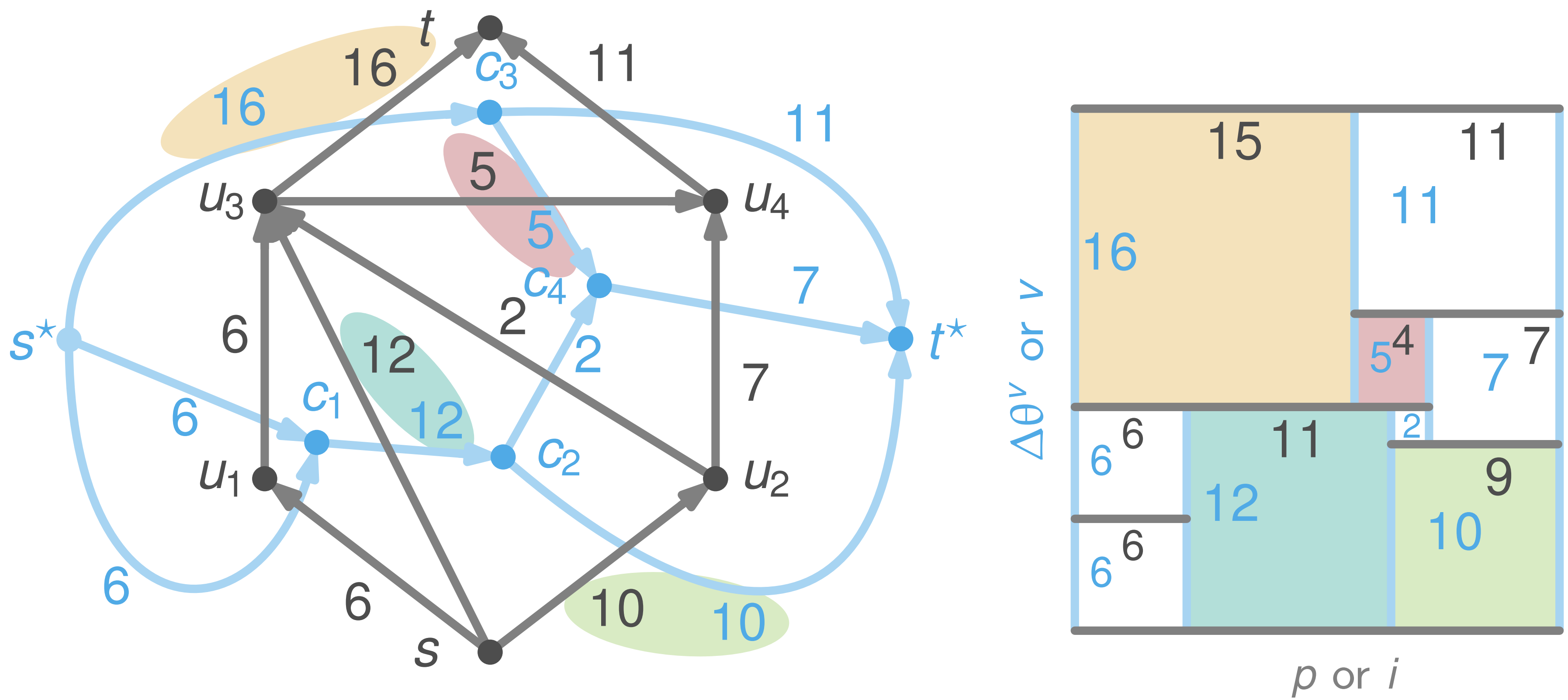
Geometric Interpretation of a KCL Conflict



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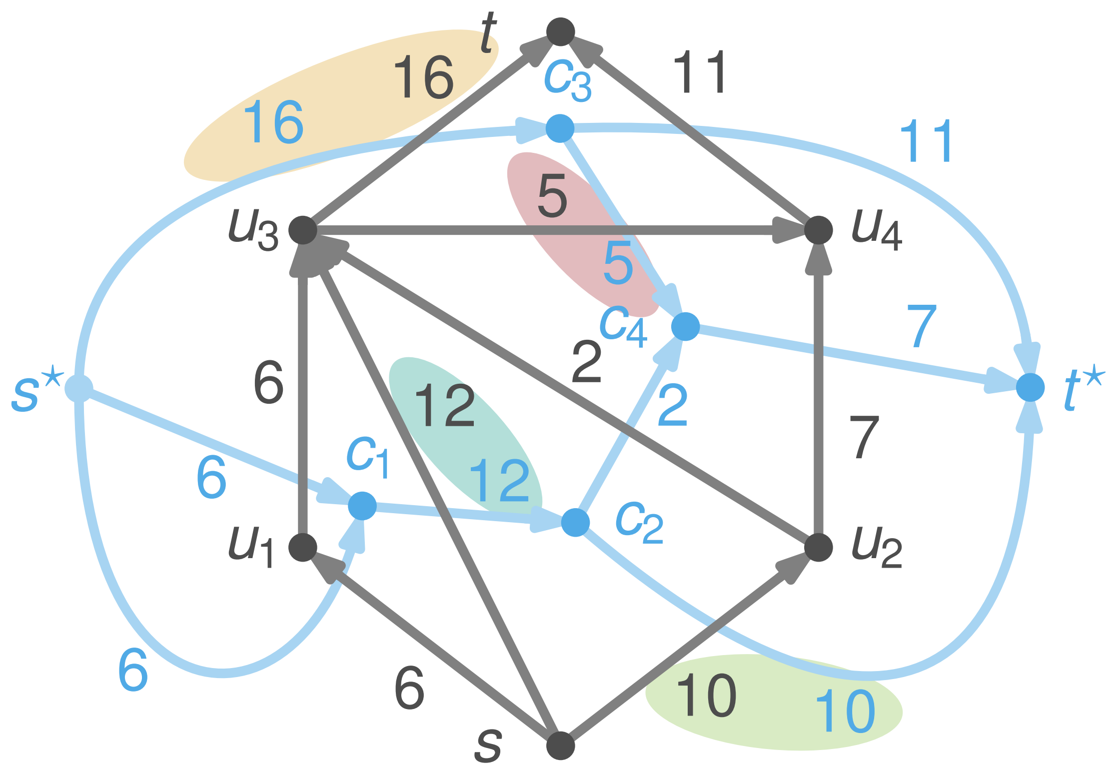

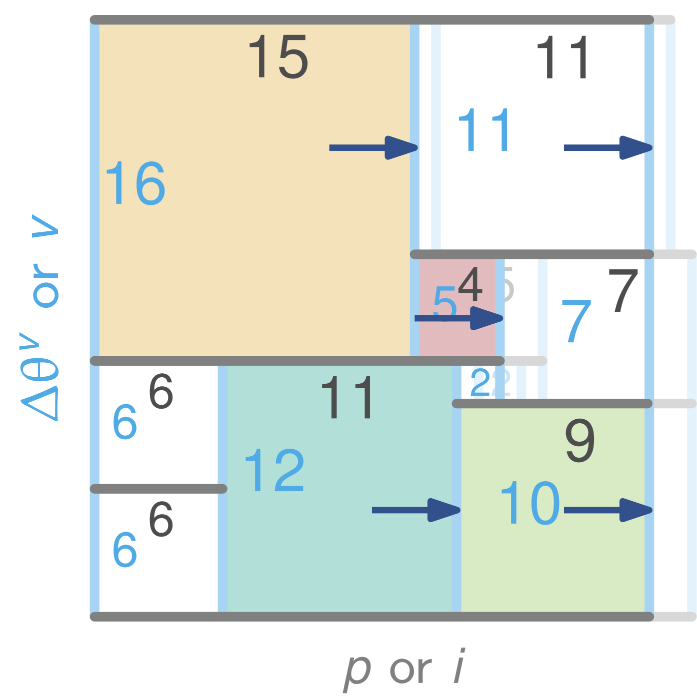

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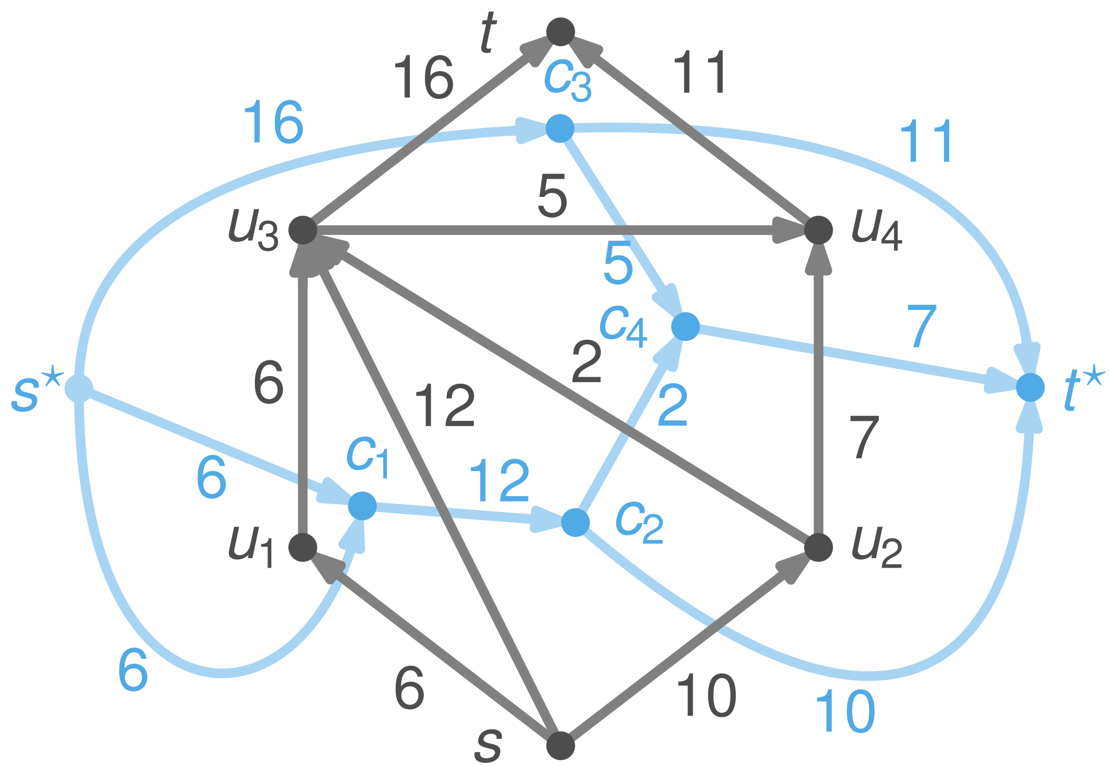

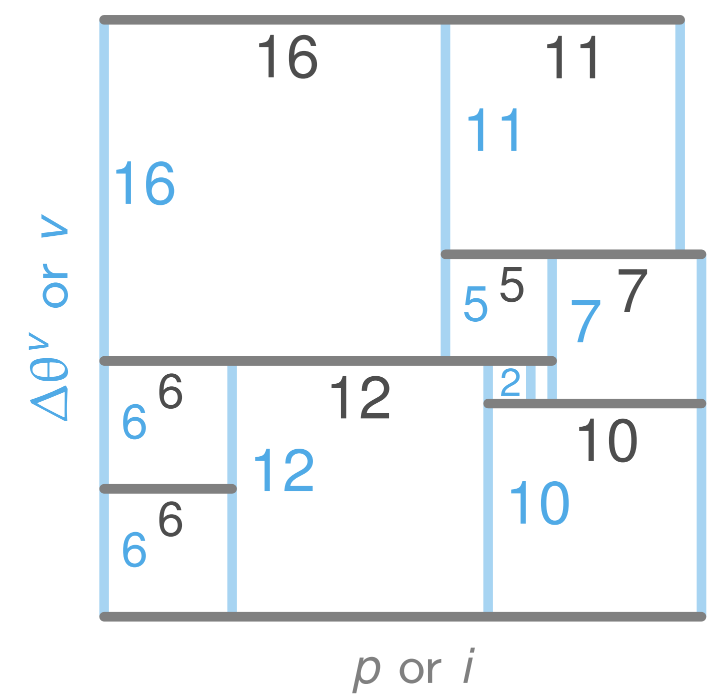

Geometric Interpretation of a KCL Conflict

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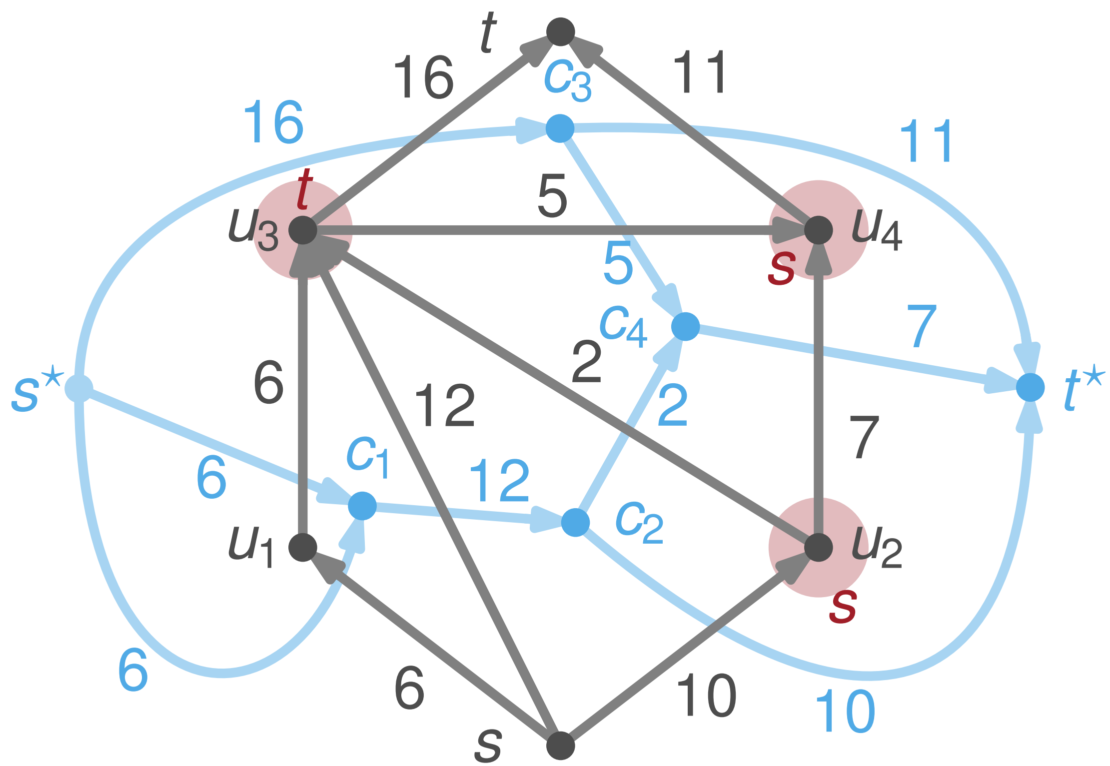

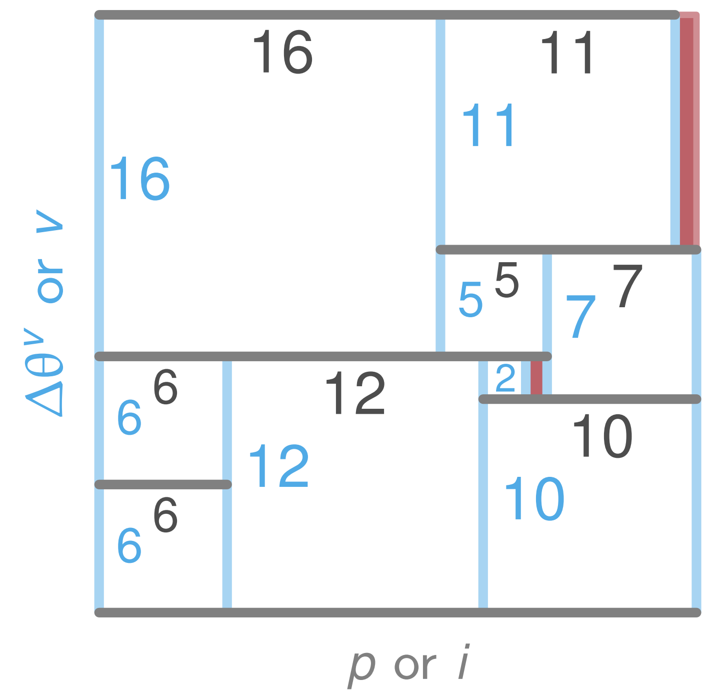

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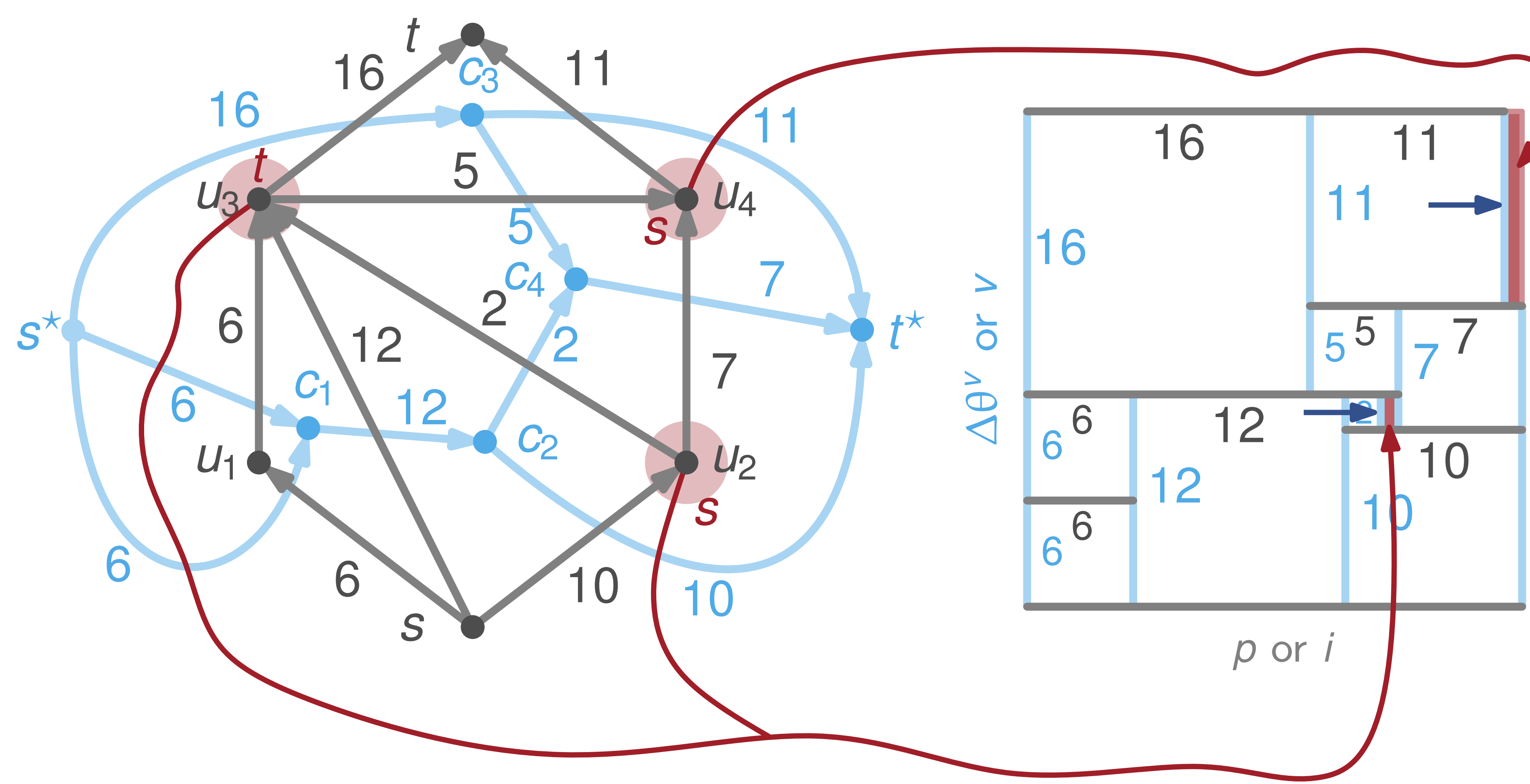
Geometric Interpretation of a KCL Conflict



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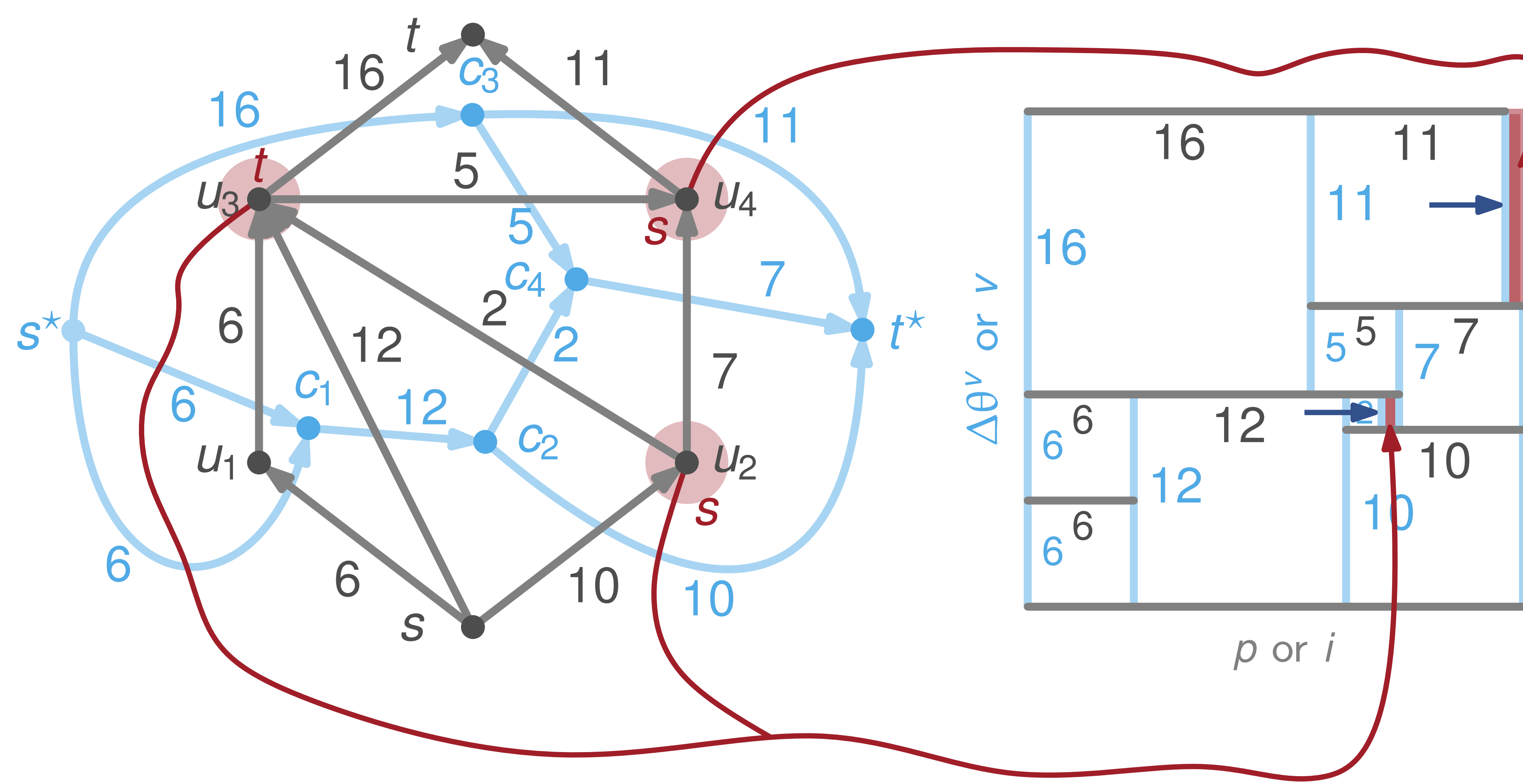

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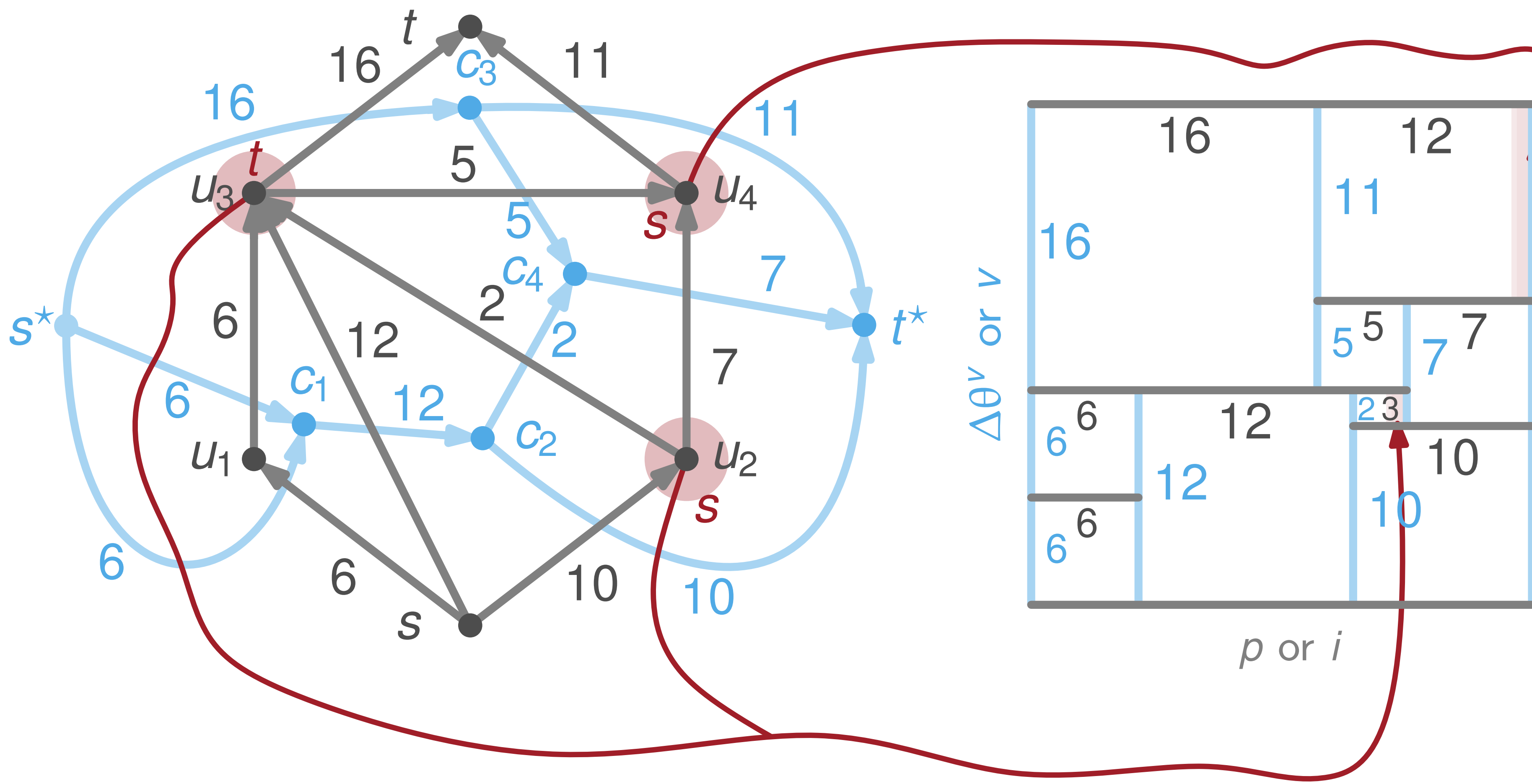

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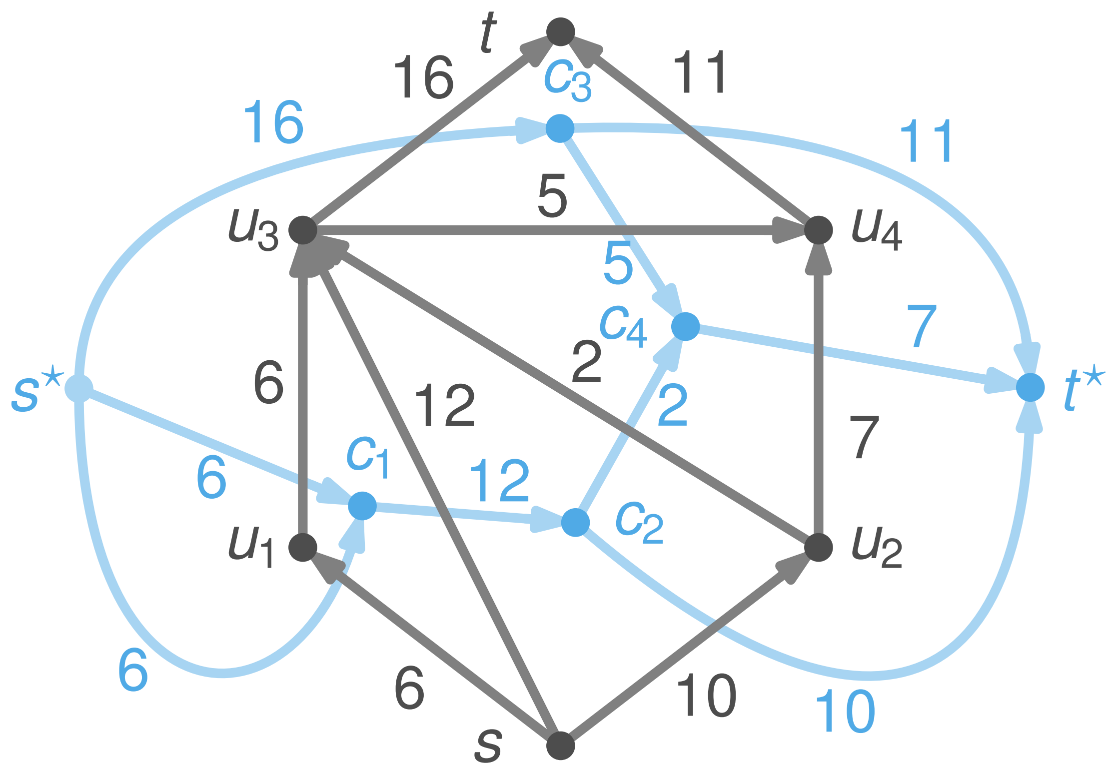

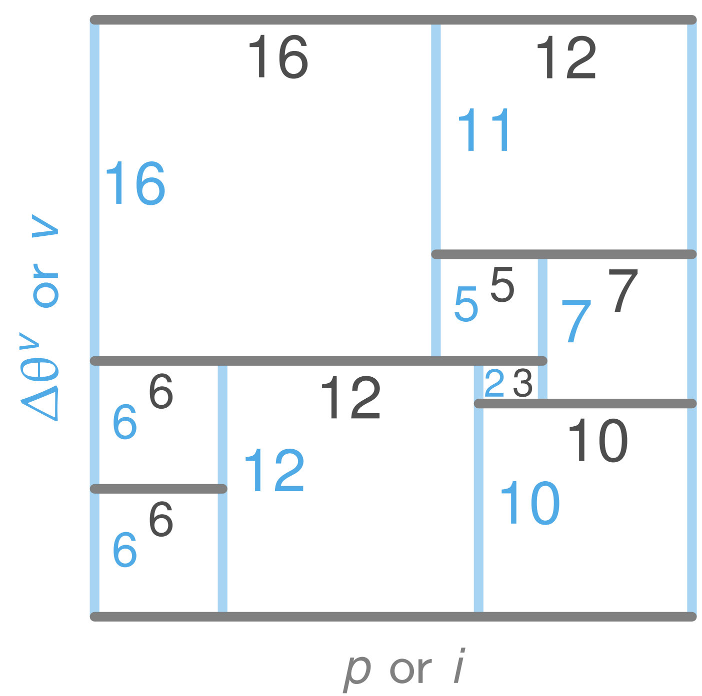

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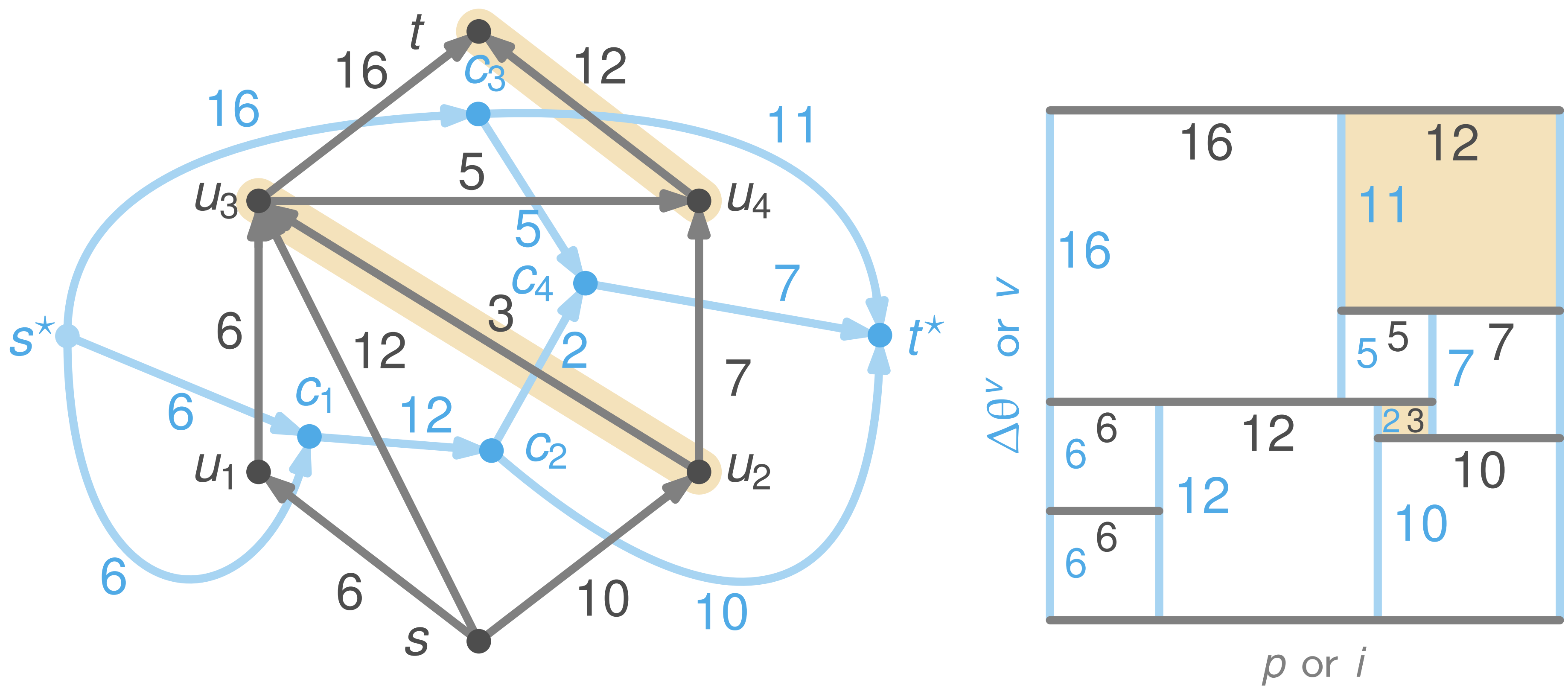

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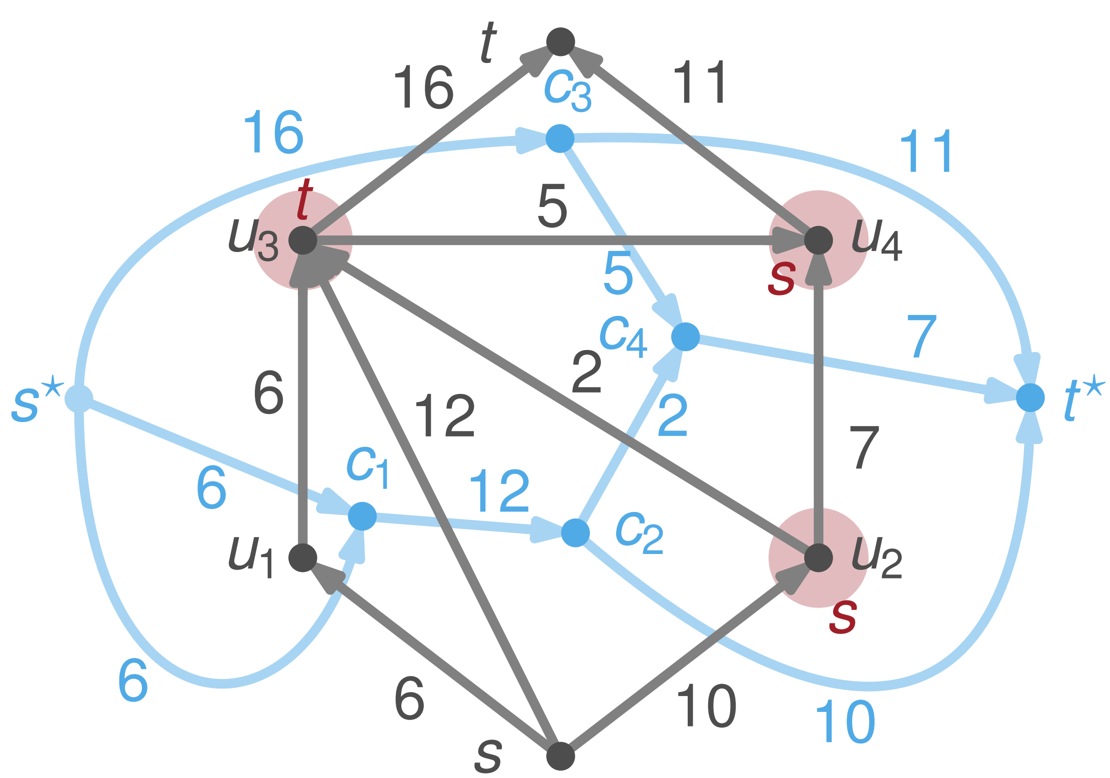

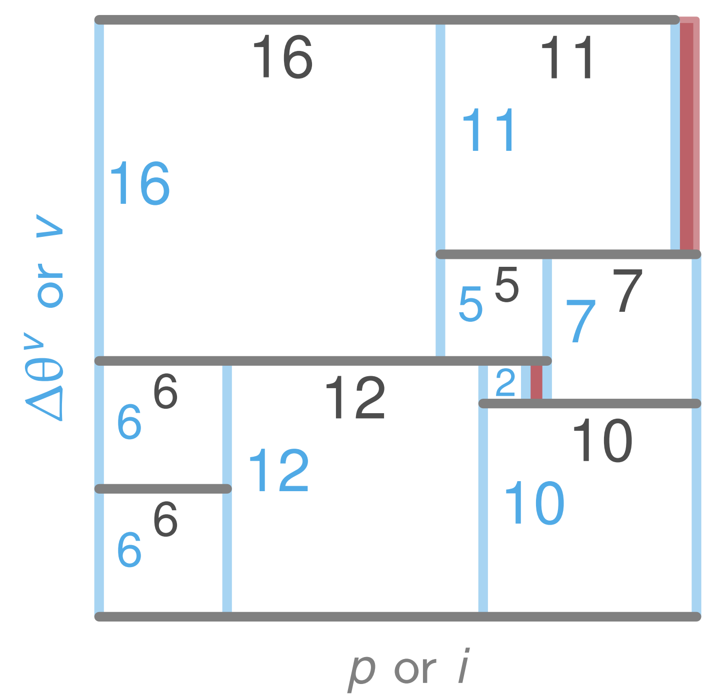

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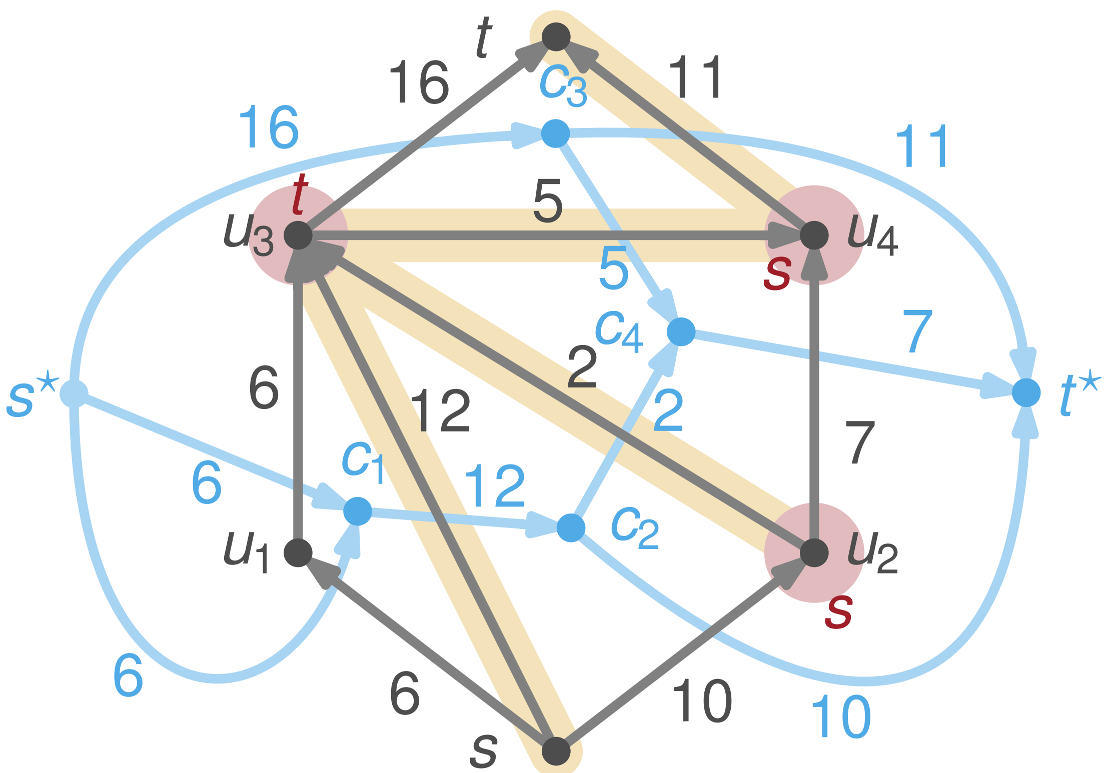

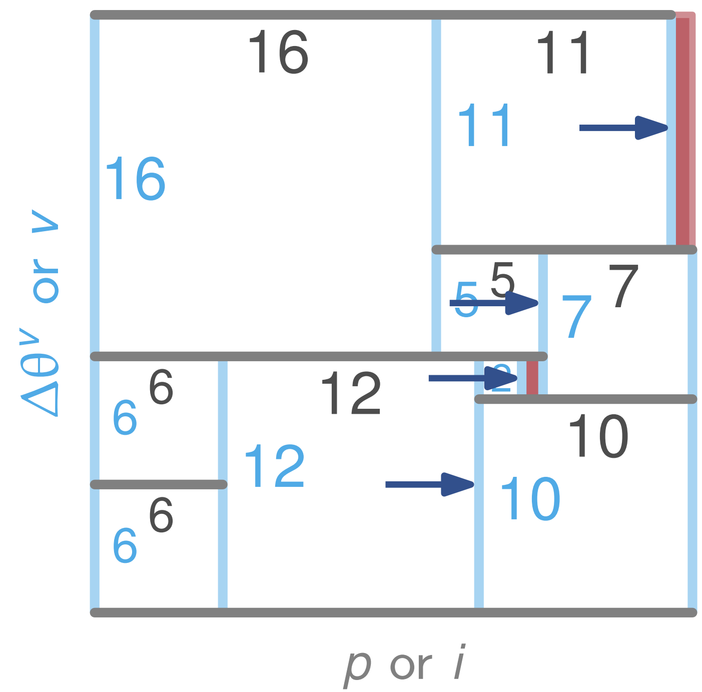

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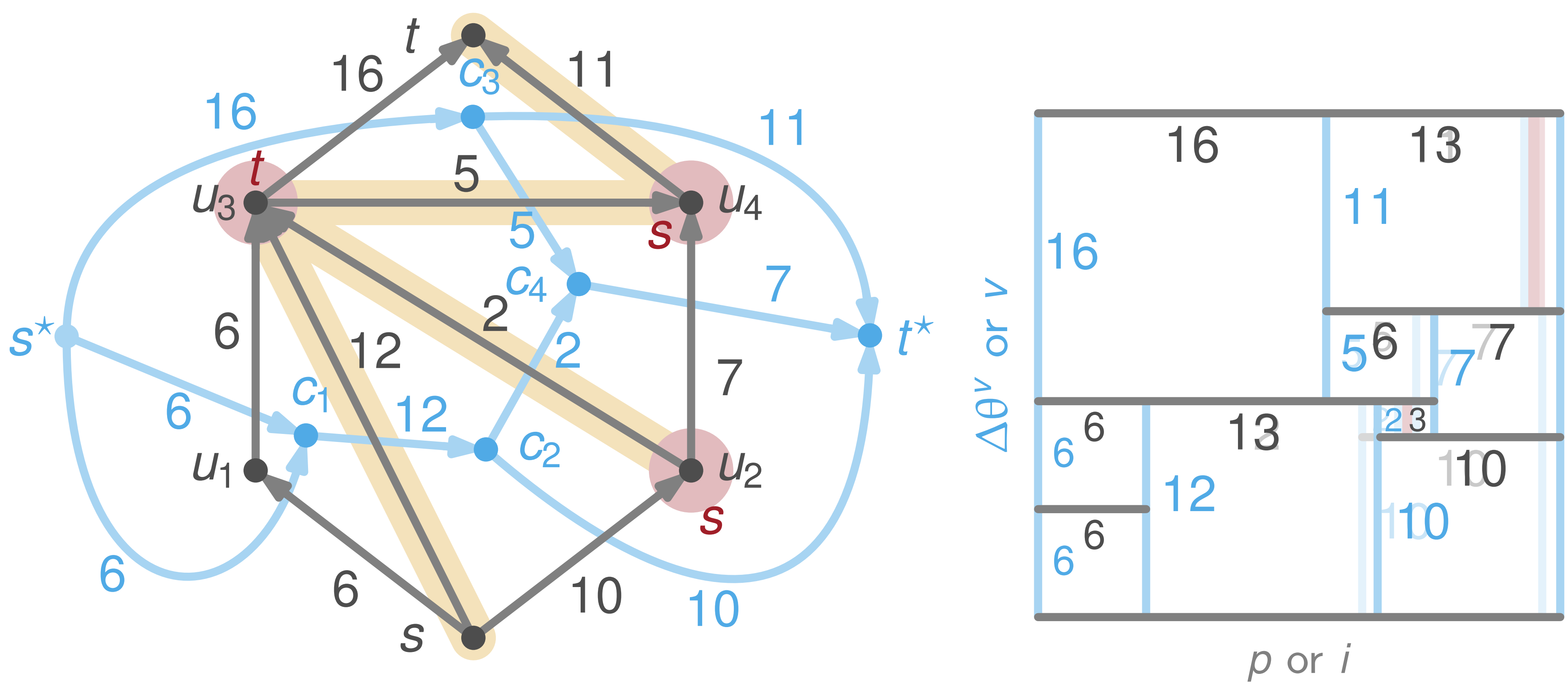

Observation 5 [Resolve KCL Conflicts]

Minimize in each `resolveConflict` step the total resizing of the outer rectangle, since a too large increase might skip a valid solution.

Geometric Interpretation of a KCL Conflict

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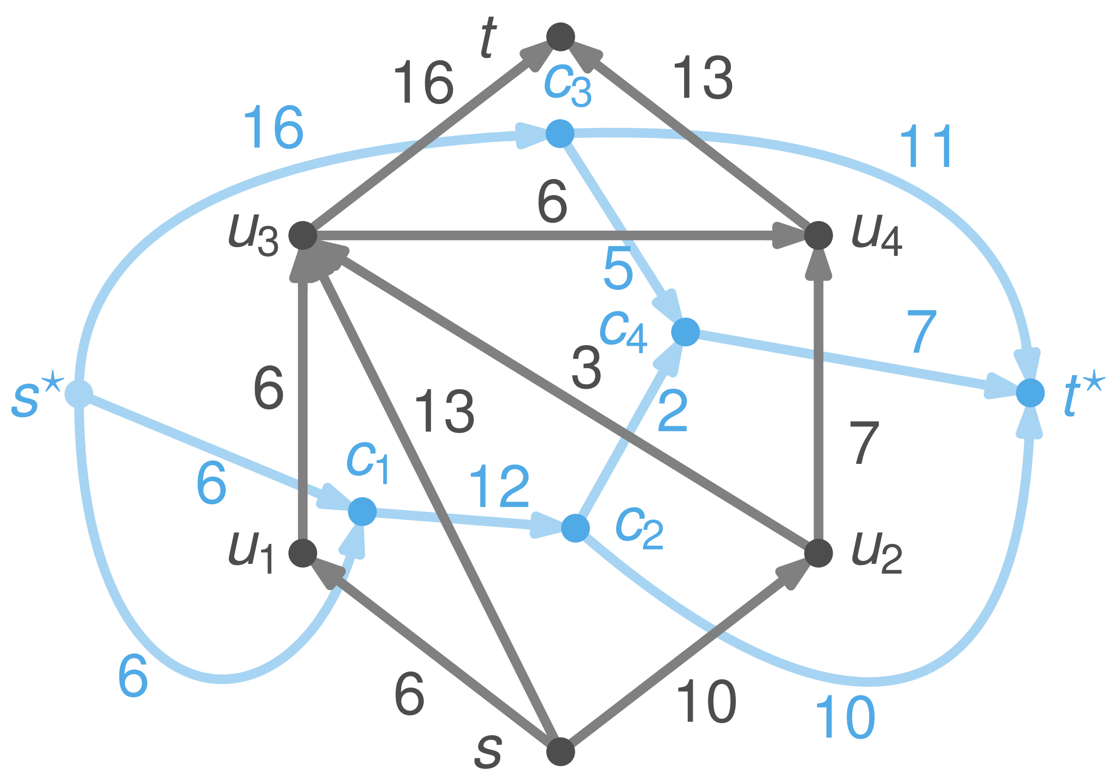

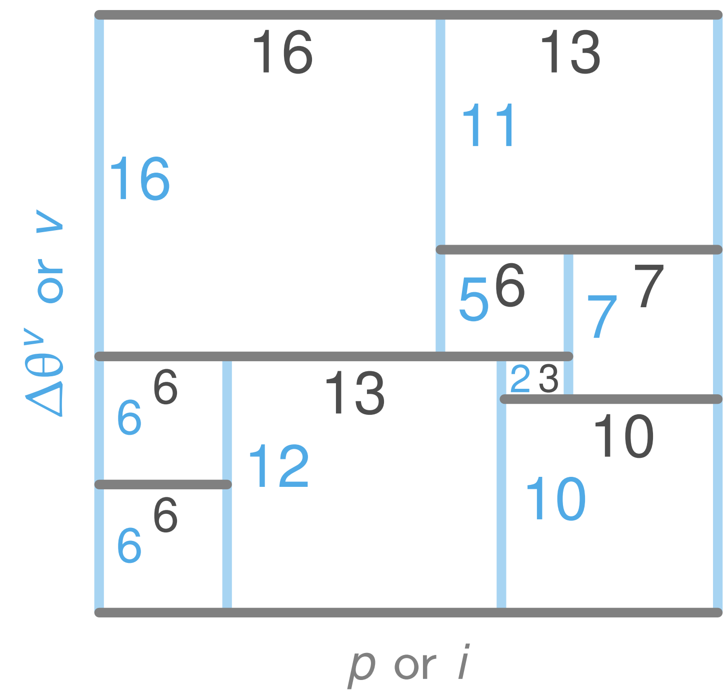

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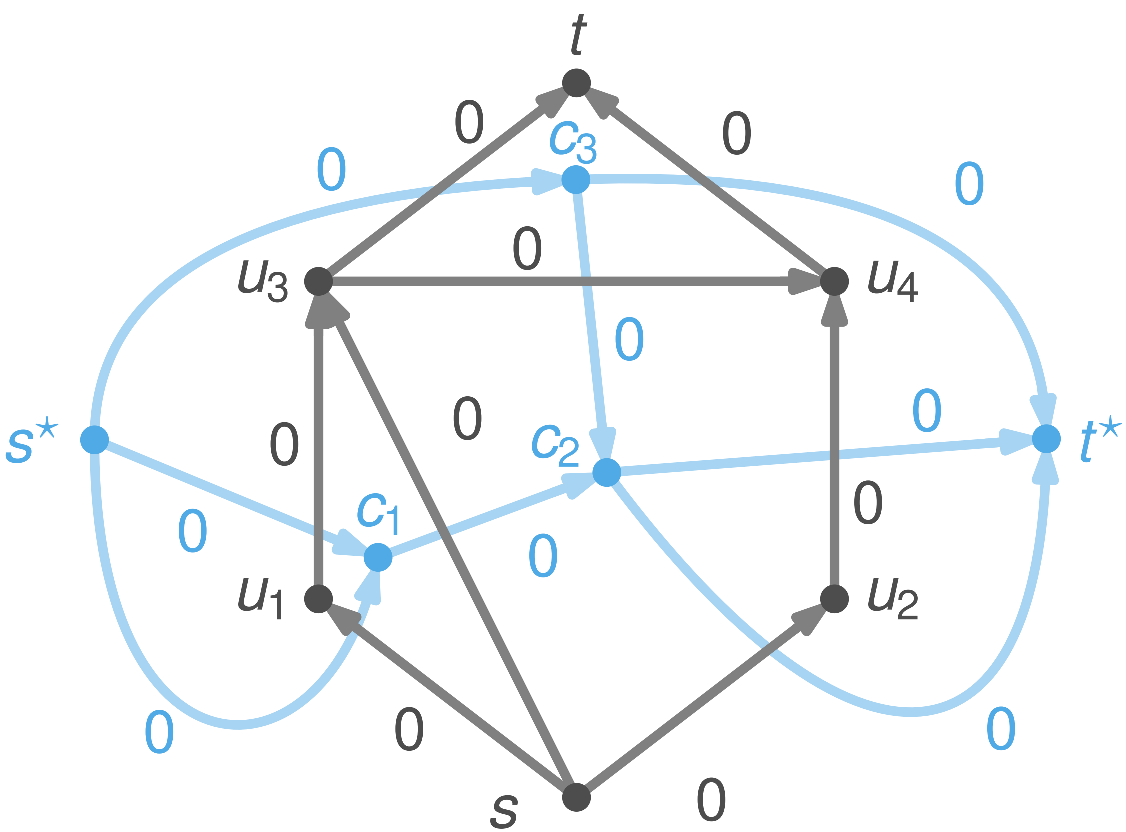
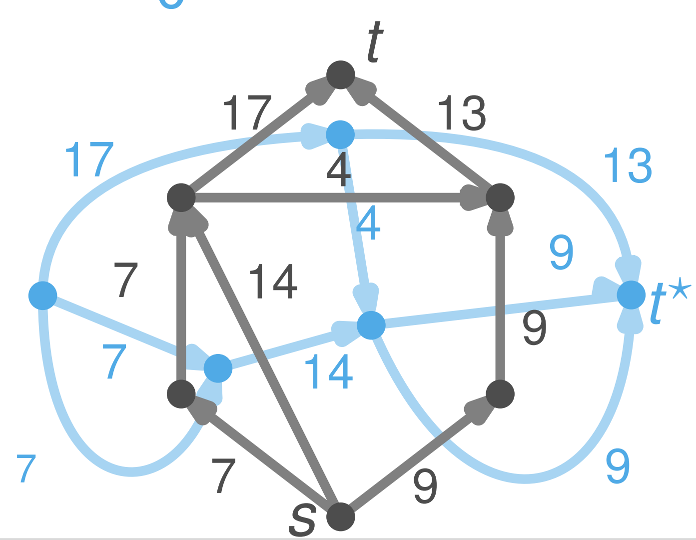
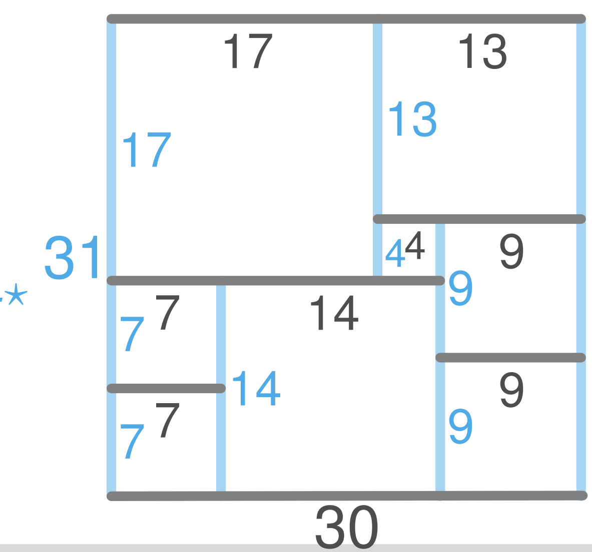
Wrong Conflict Resolution







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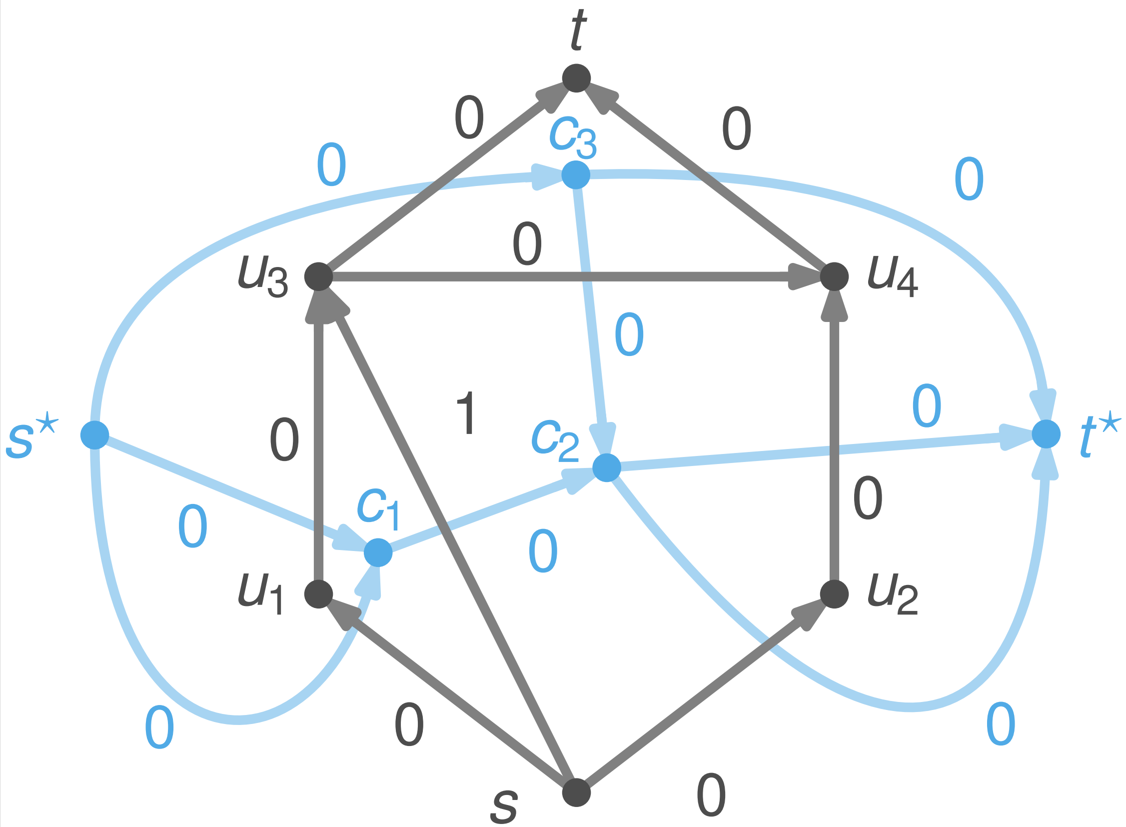

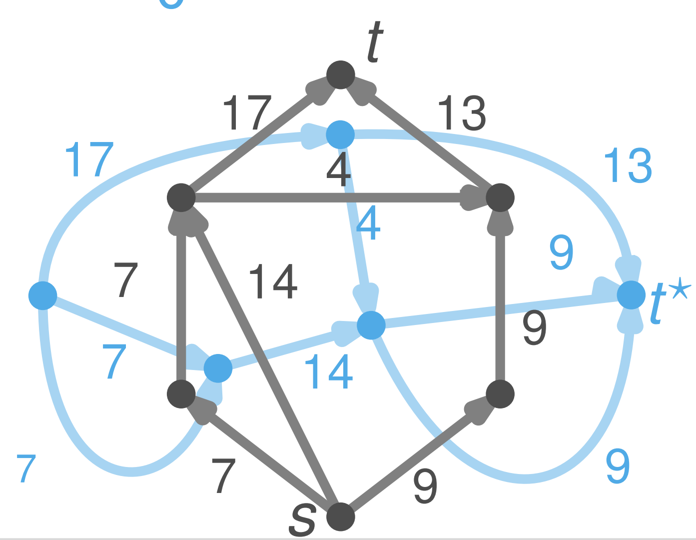

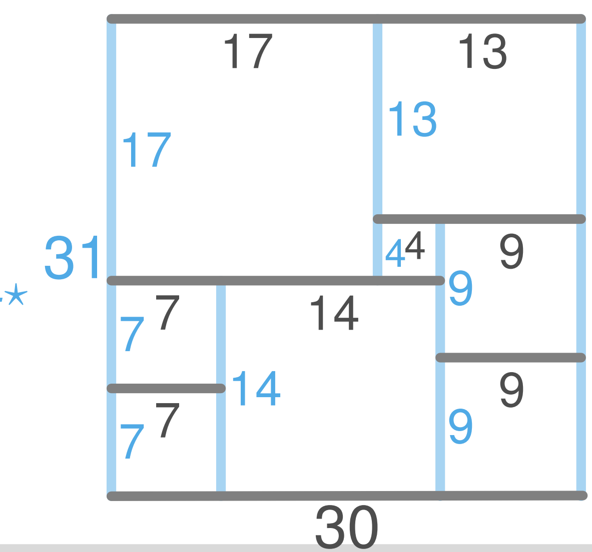

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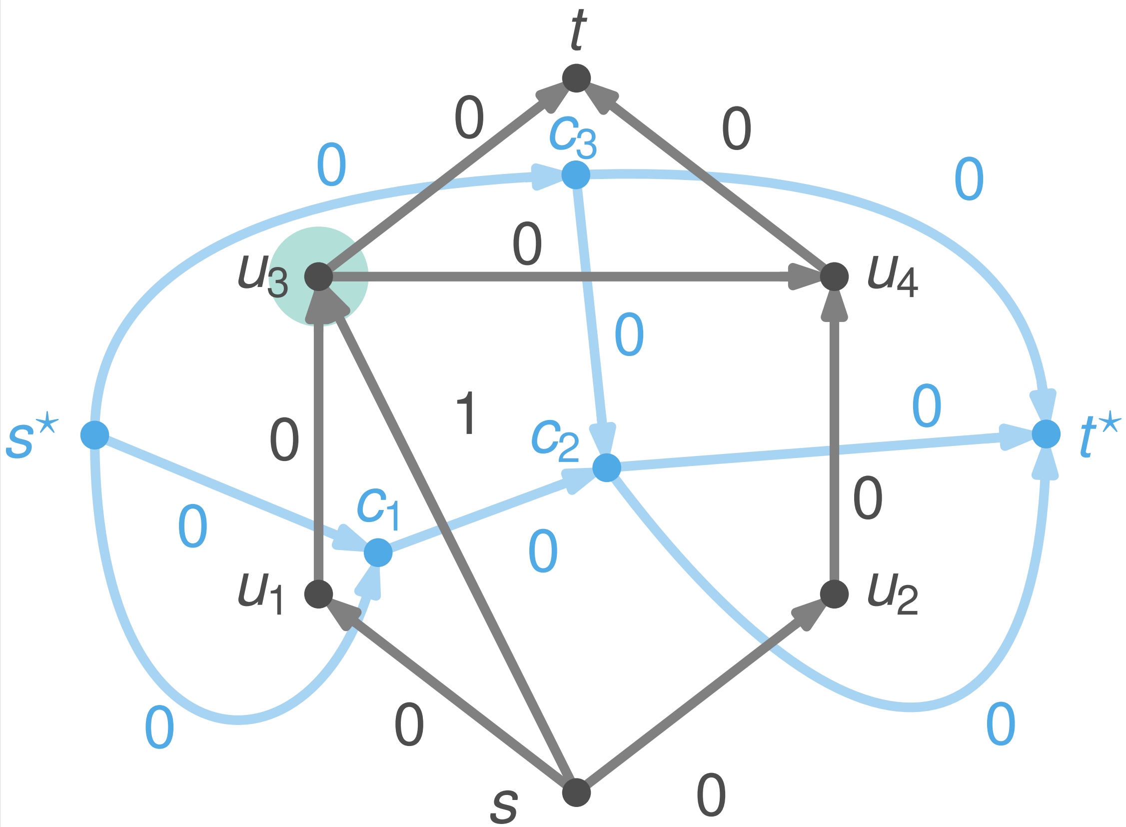

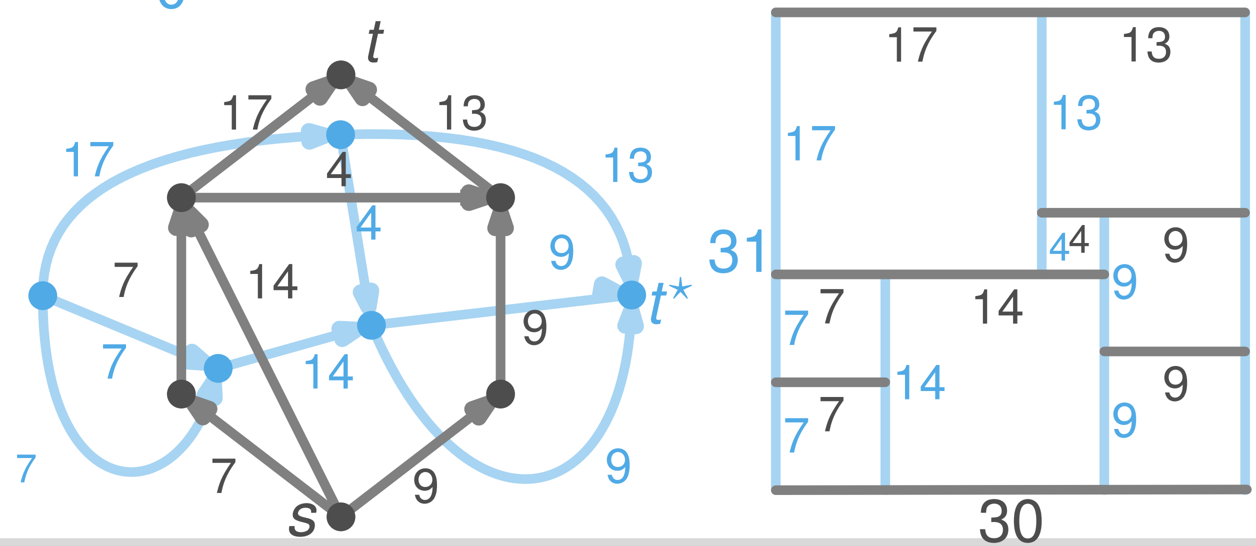

Wrong Conflict Resolution

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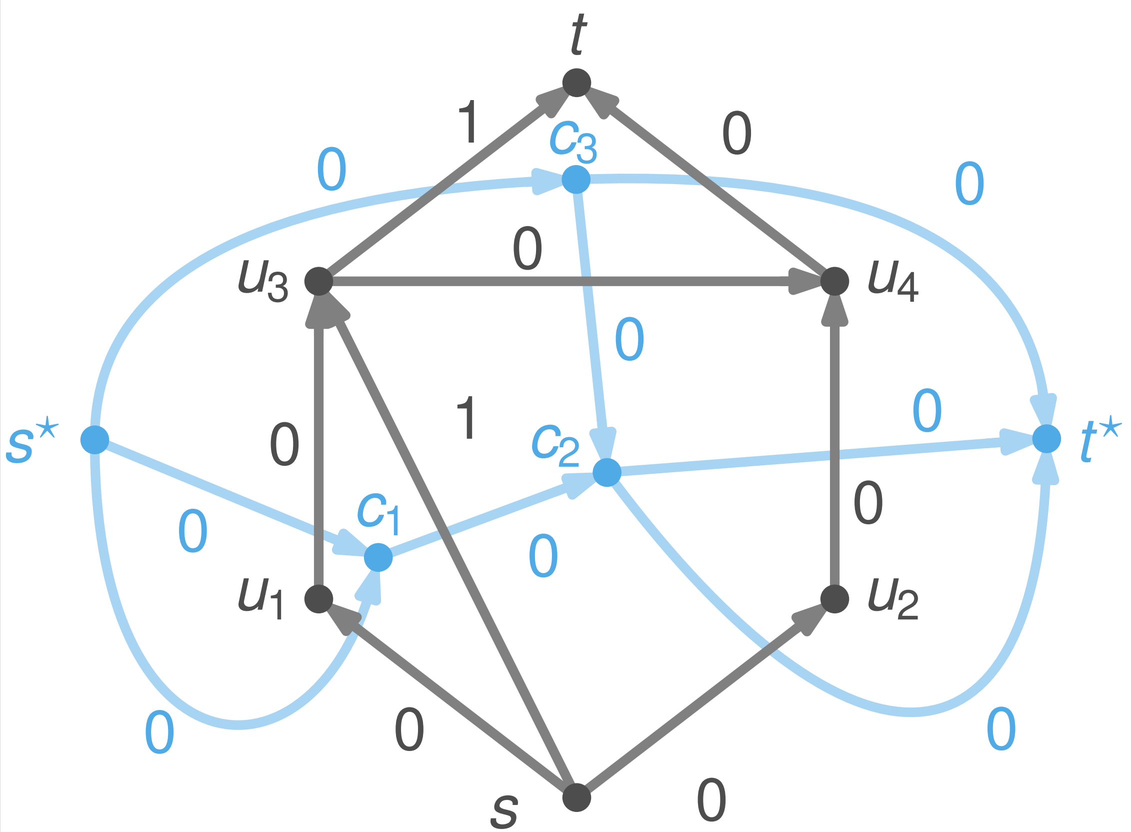

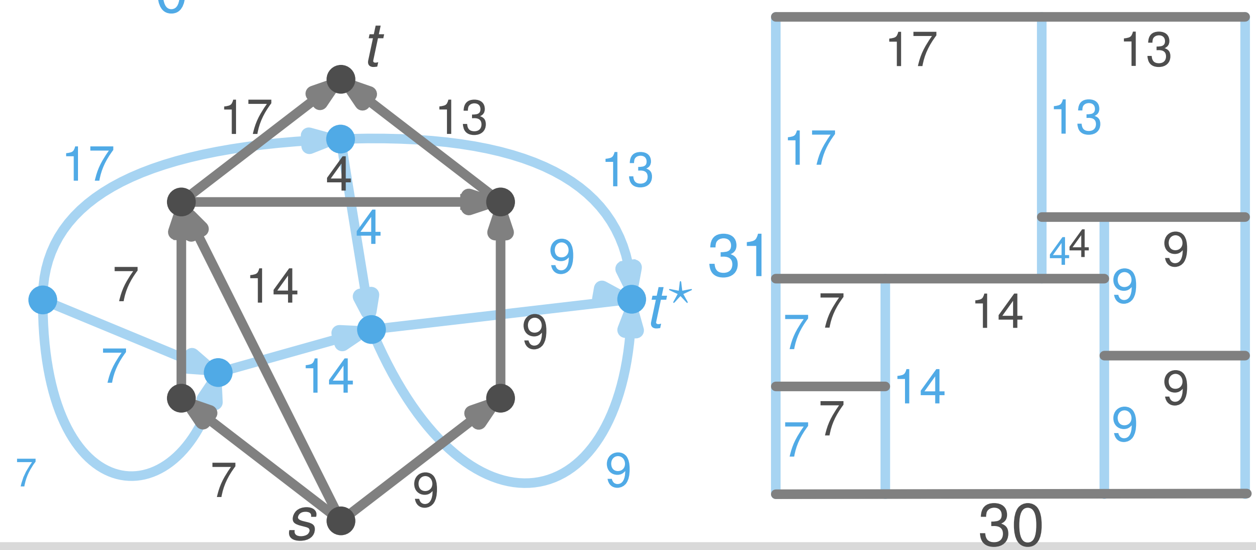

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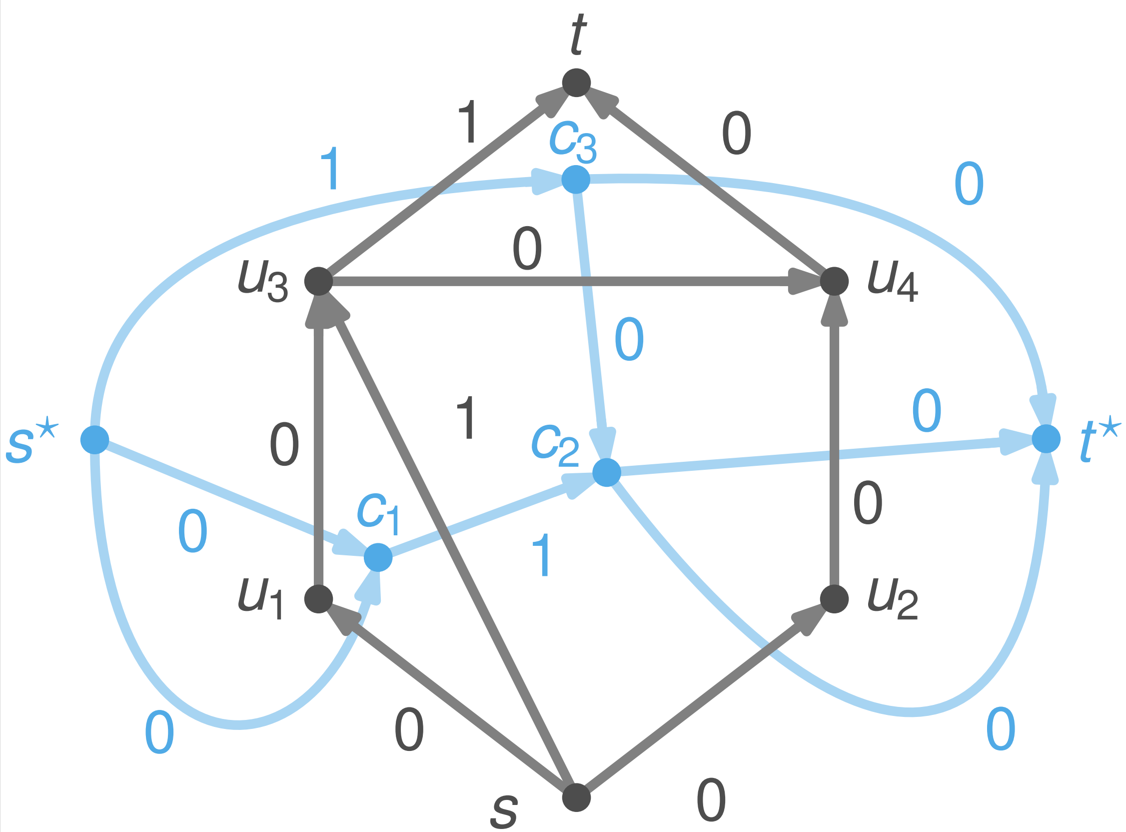

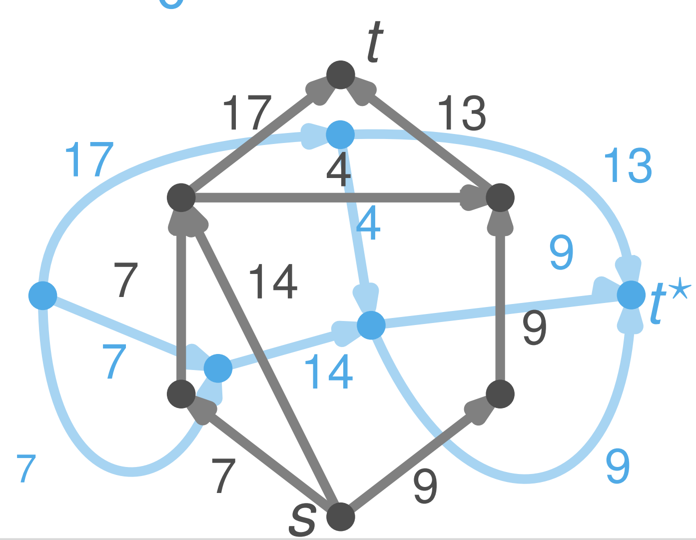

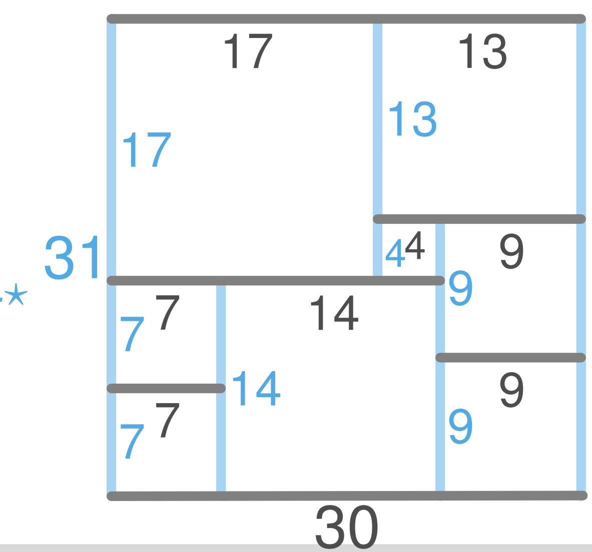

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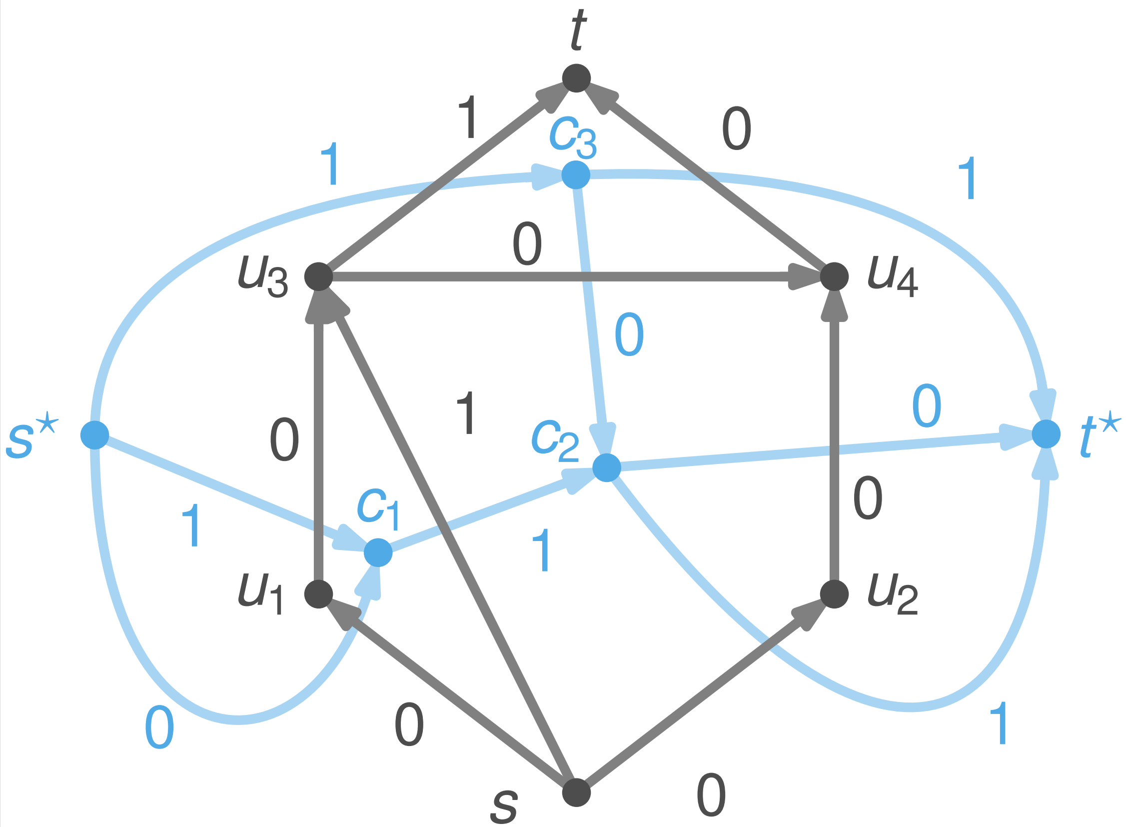

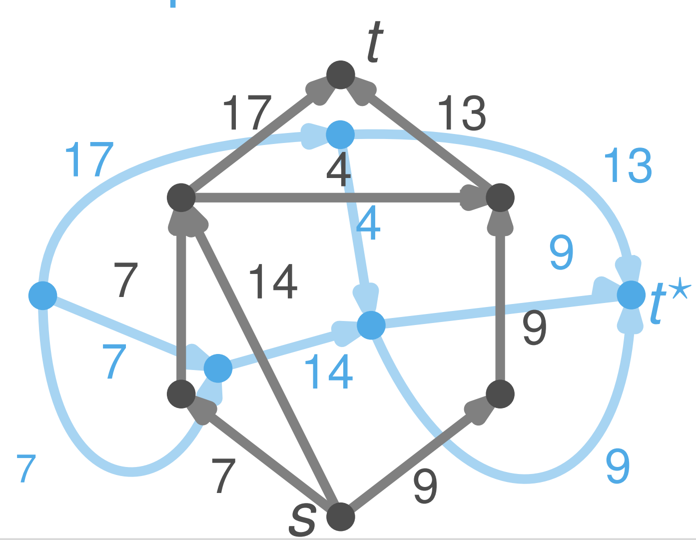

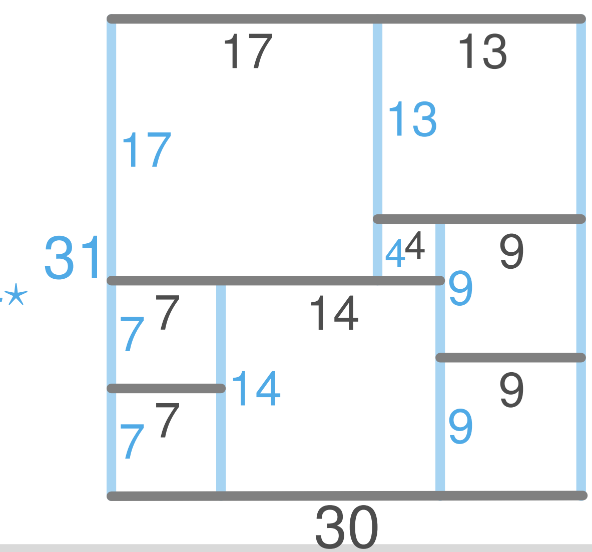

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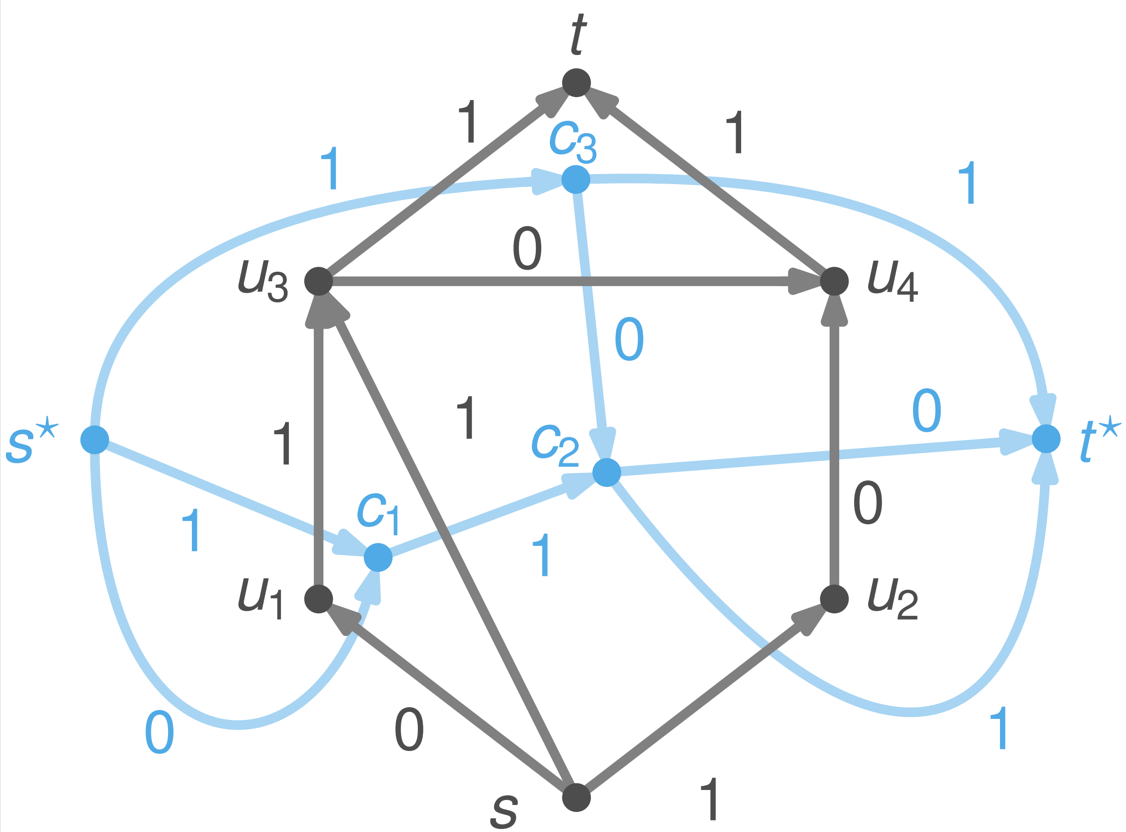

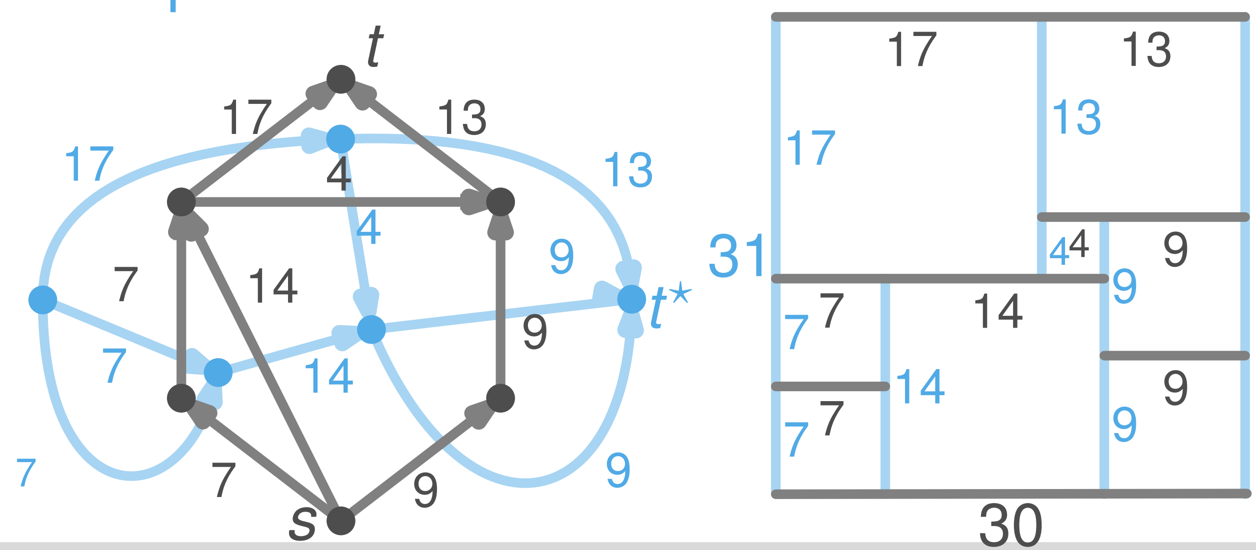

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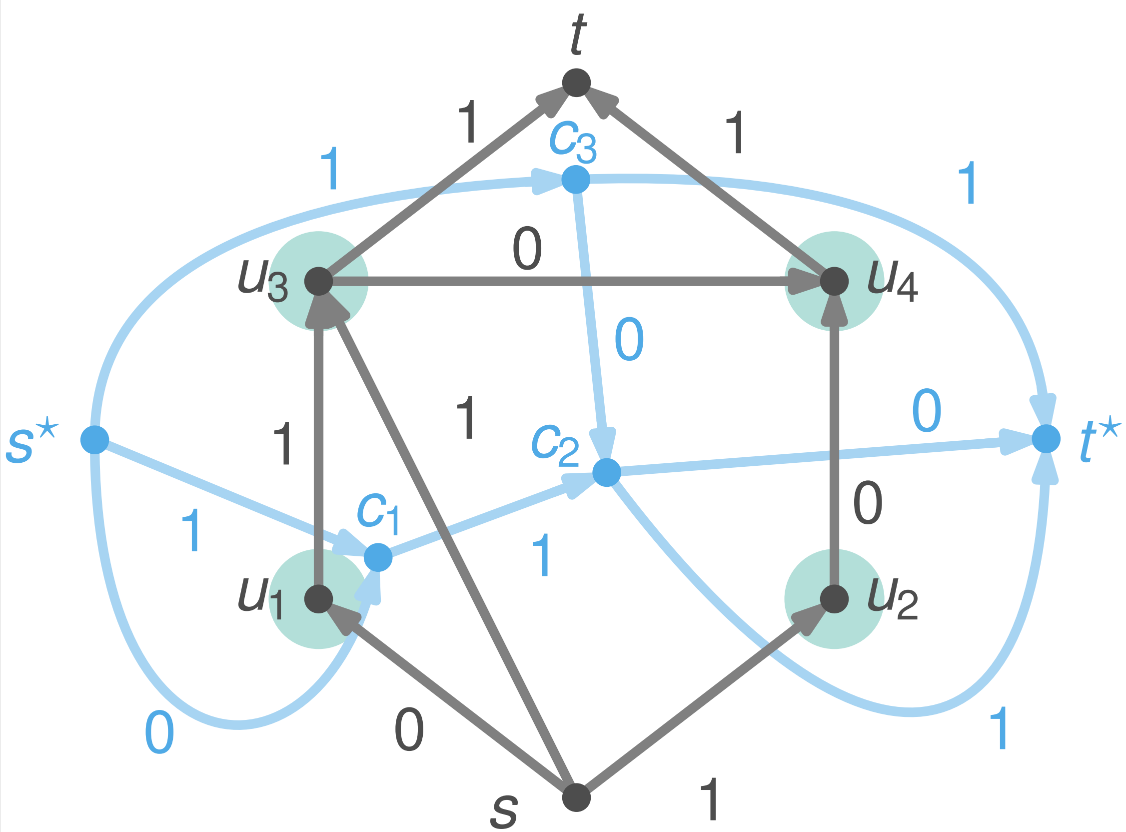

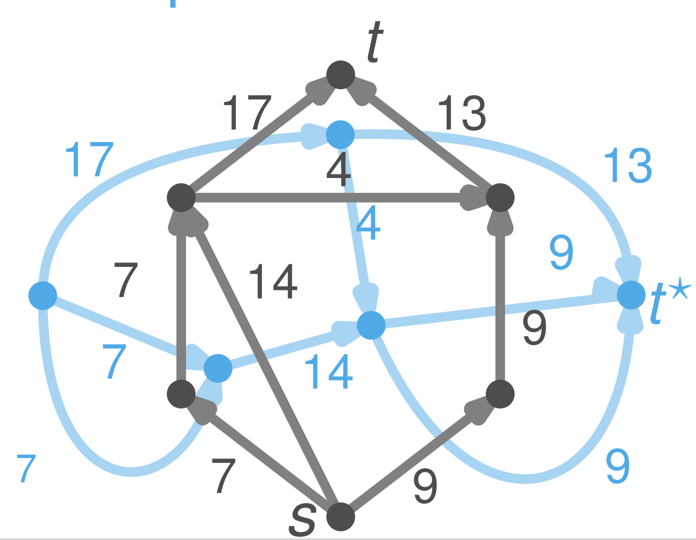

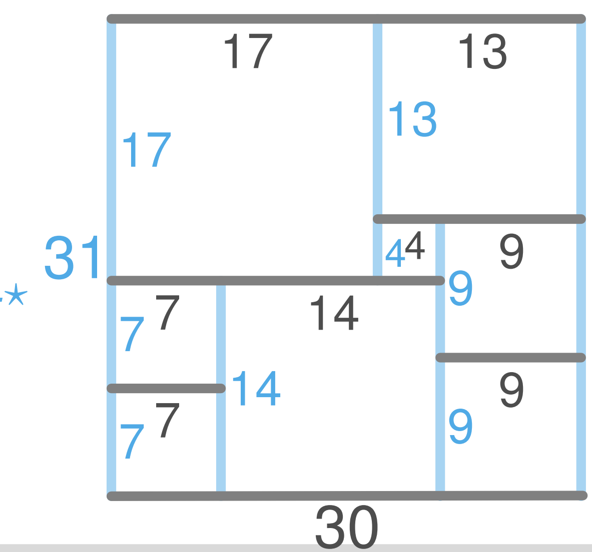

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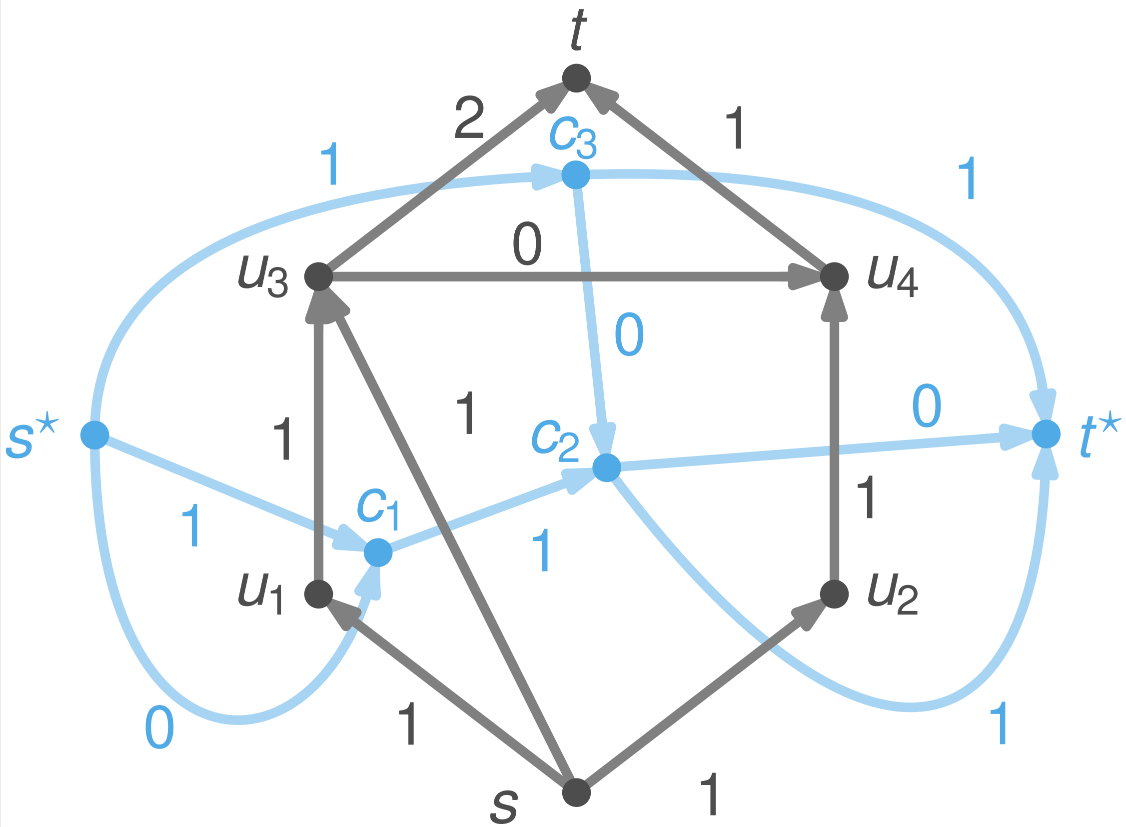

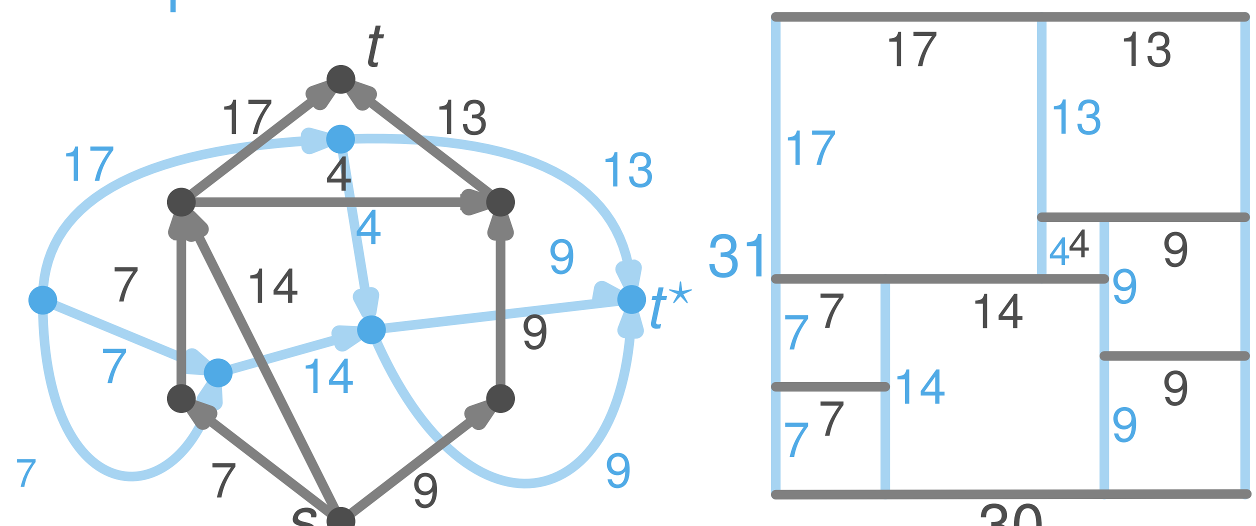

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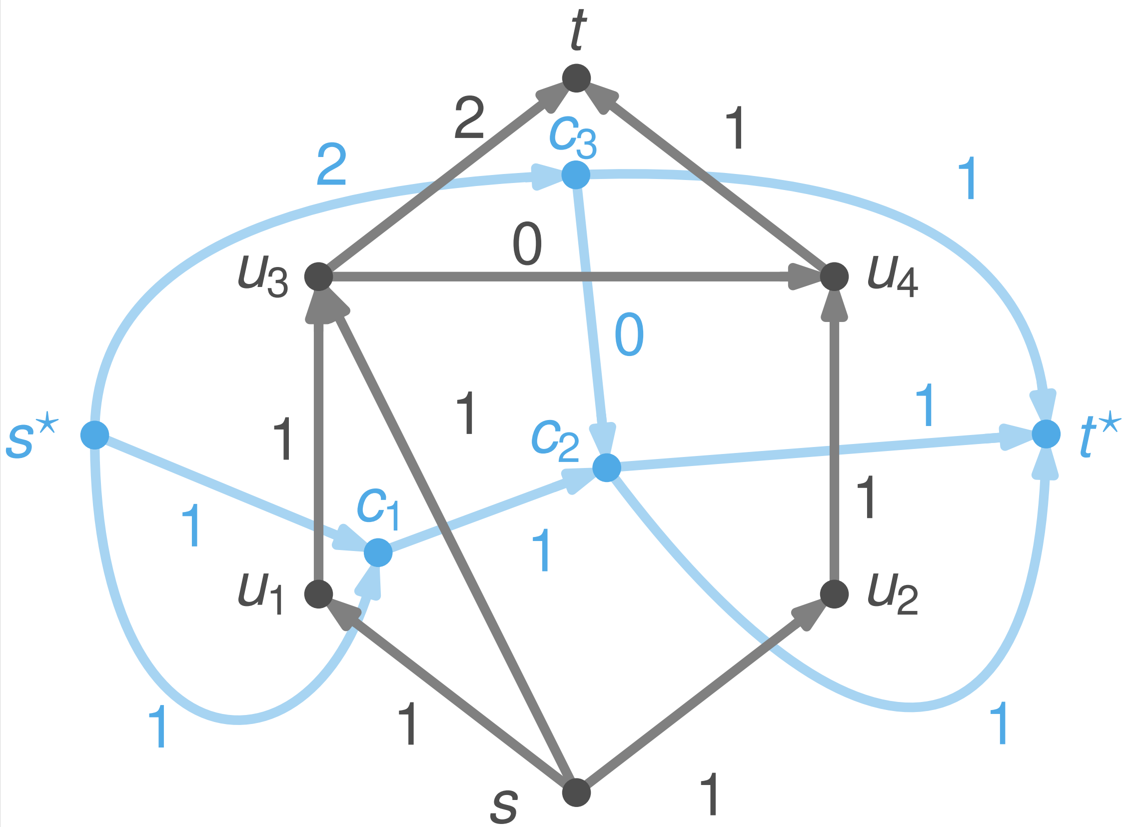

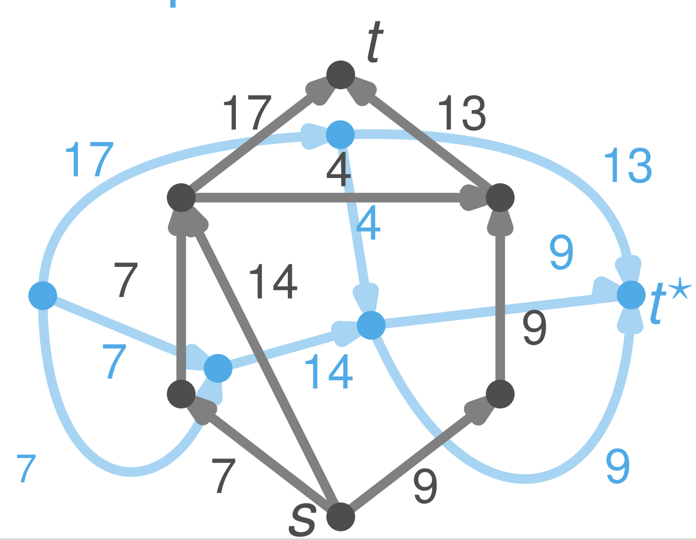

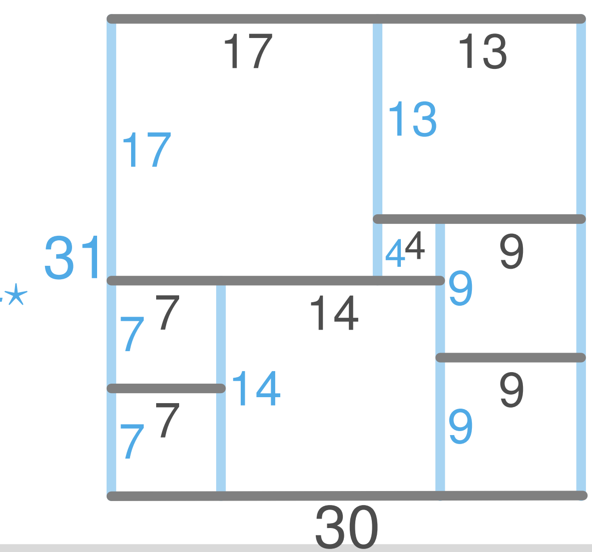

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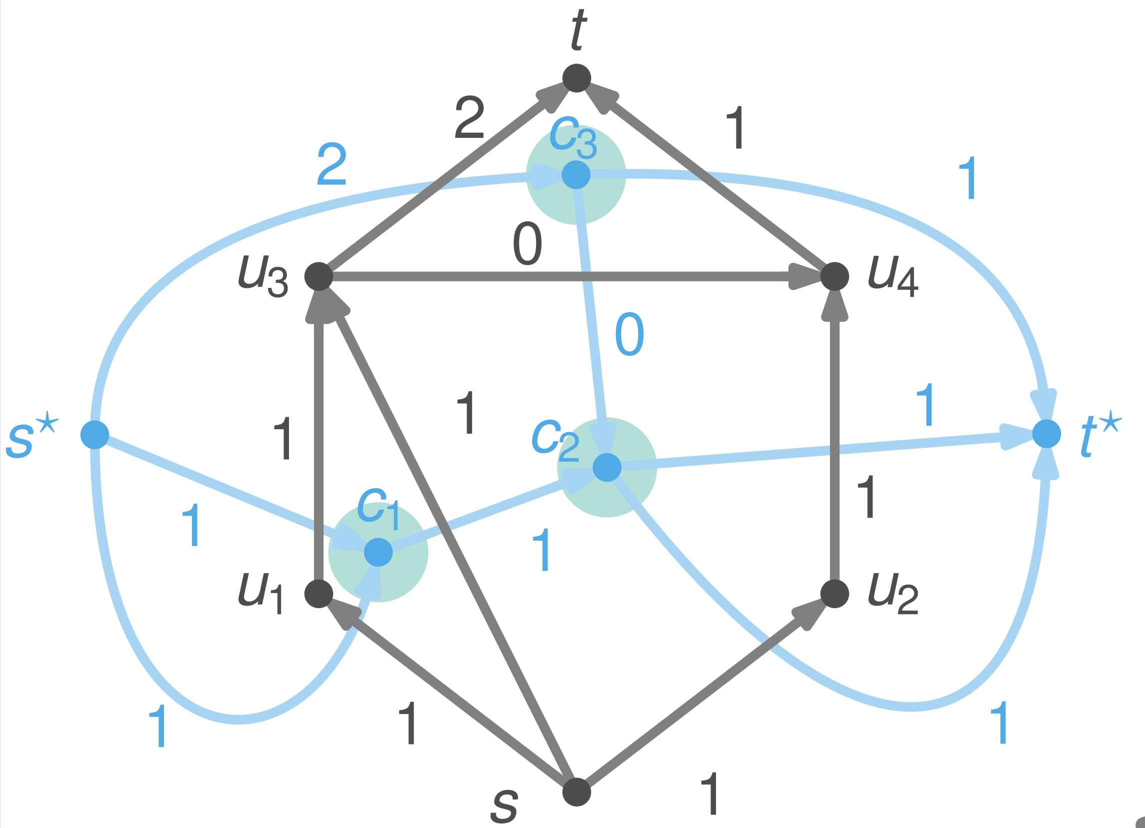

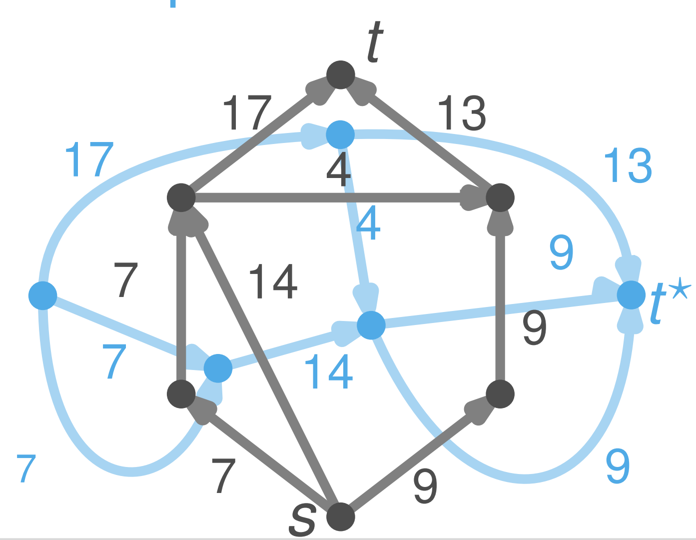

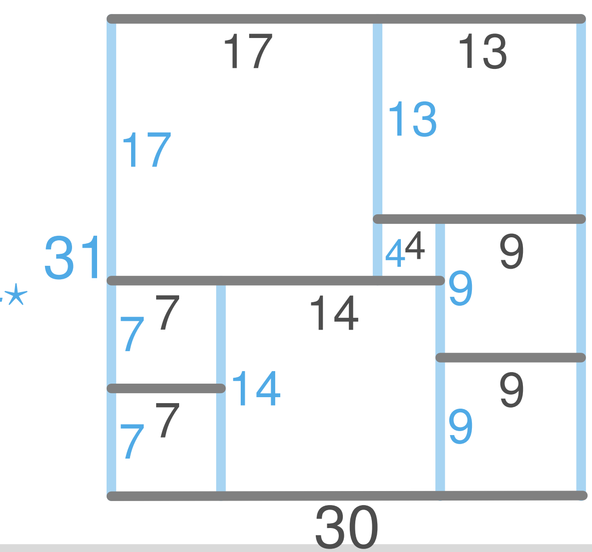

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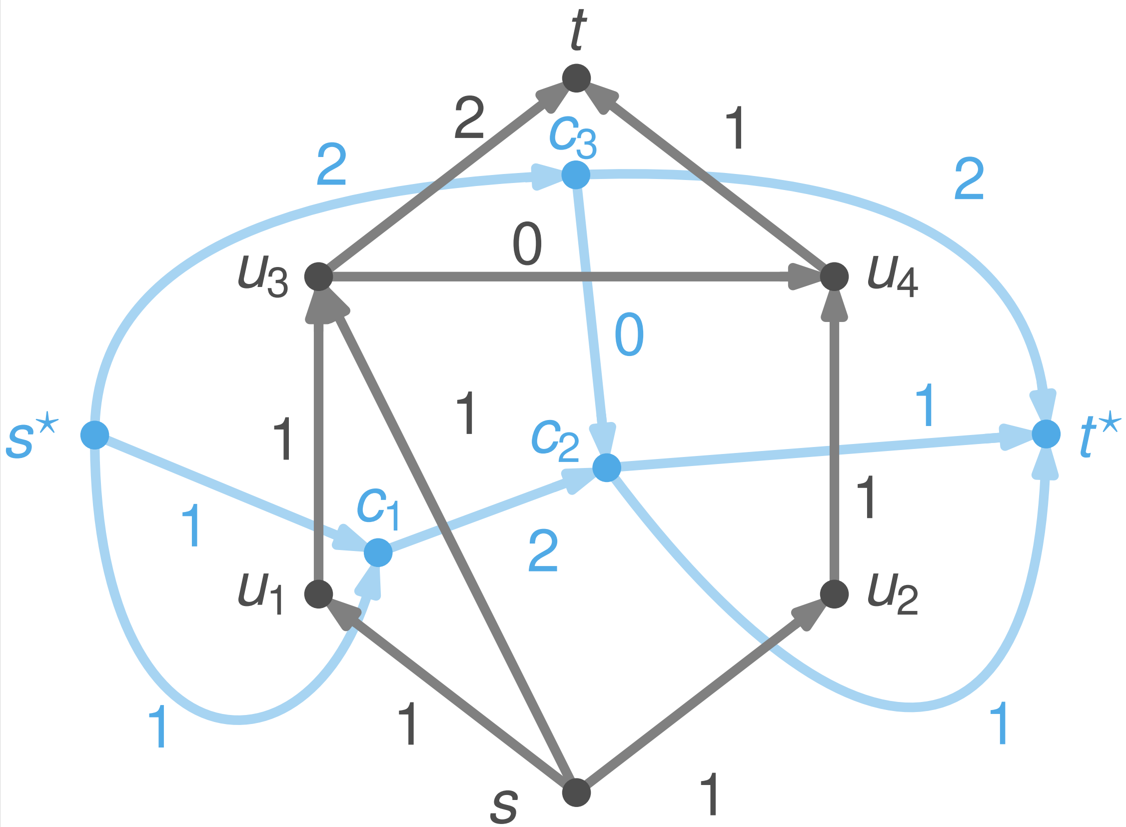

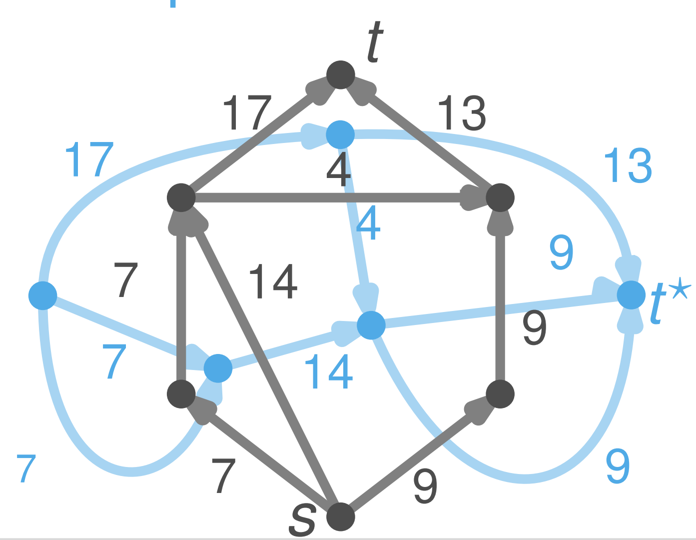

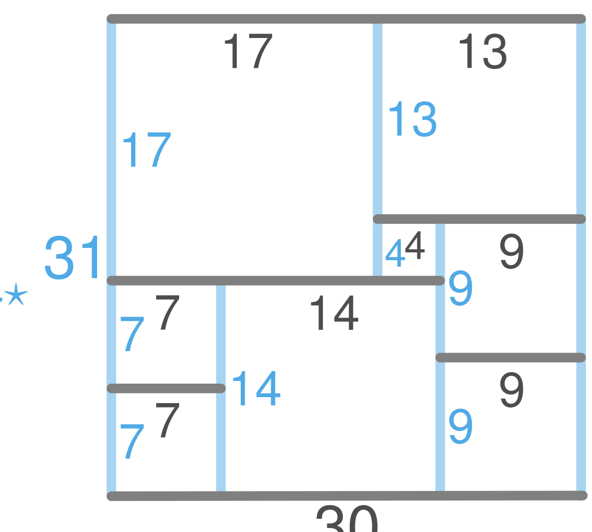

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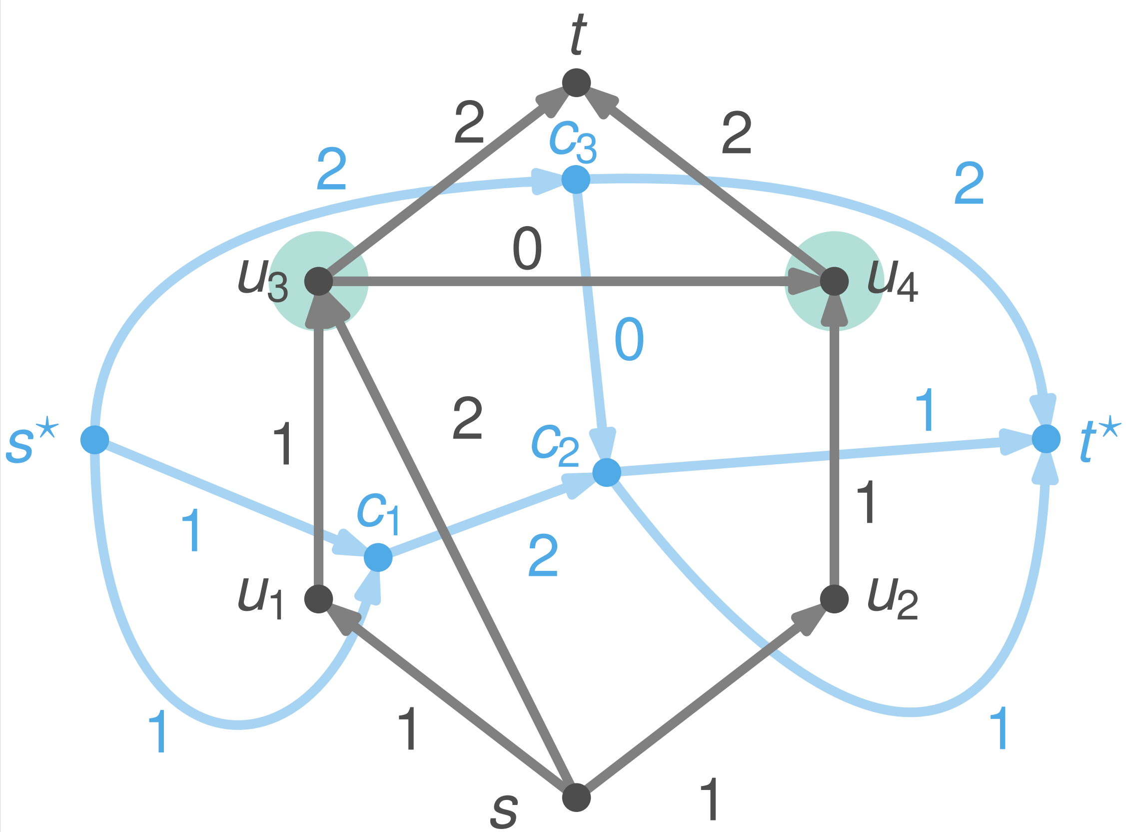

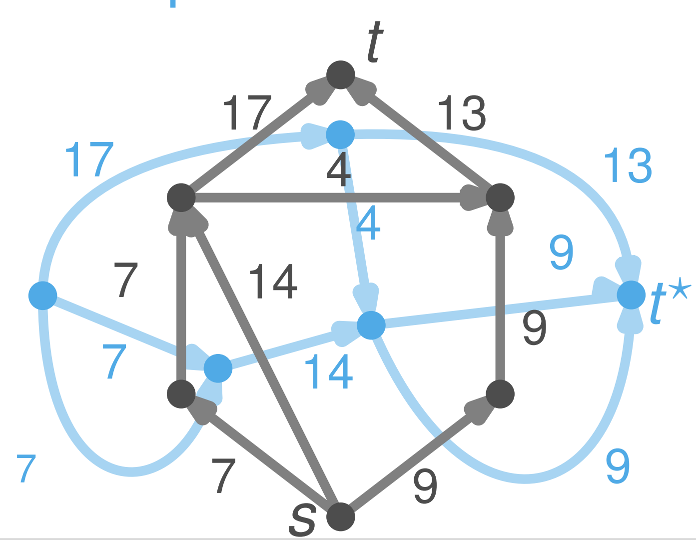

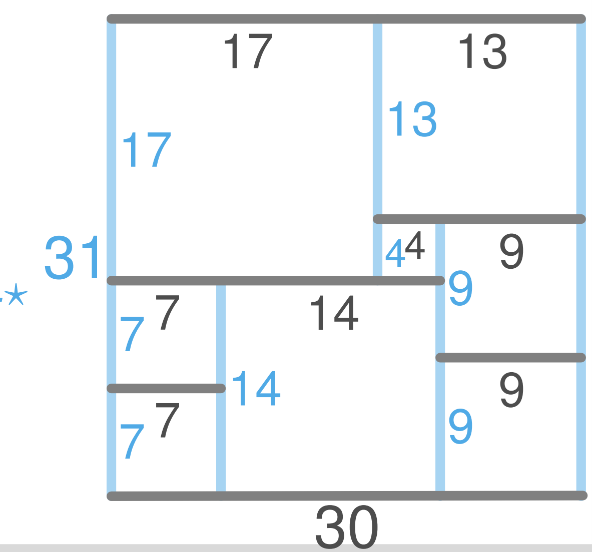

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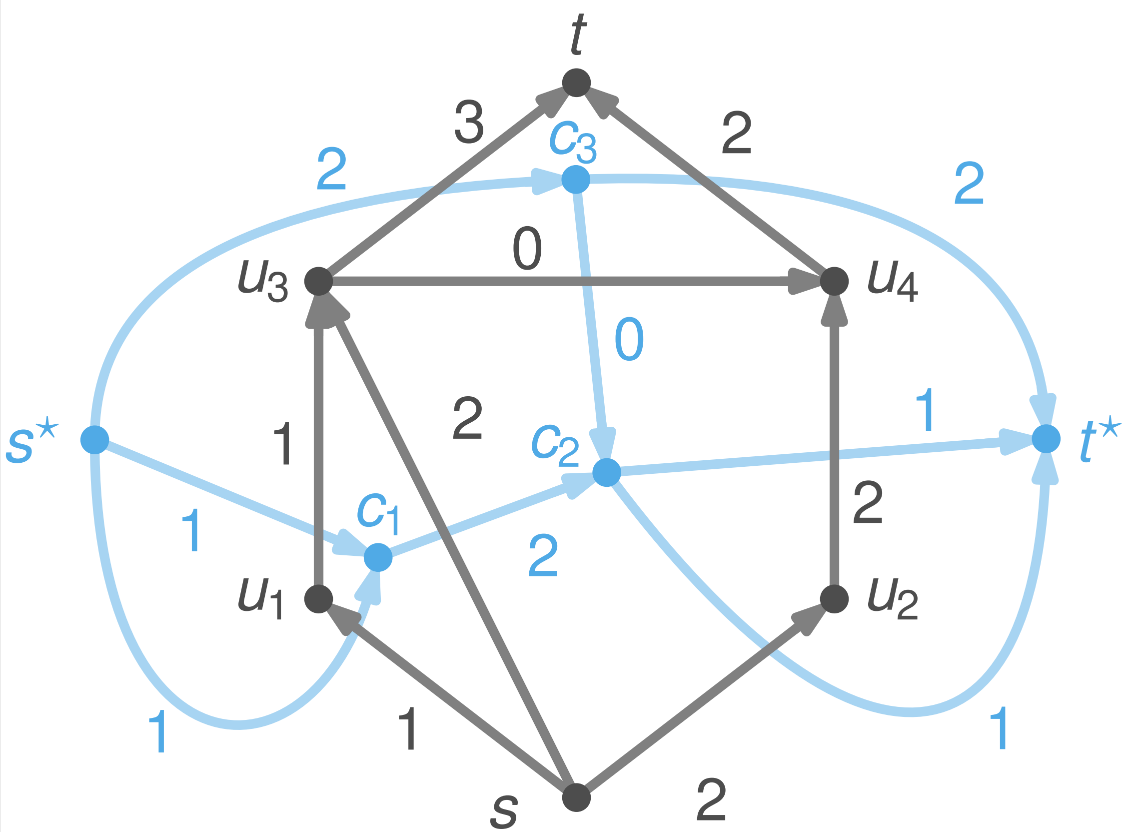

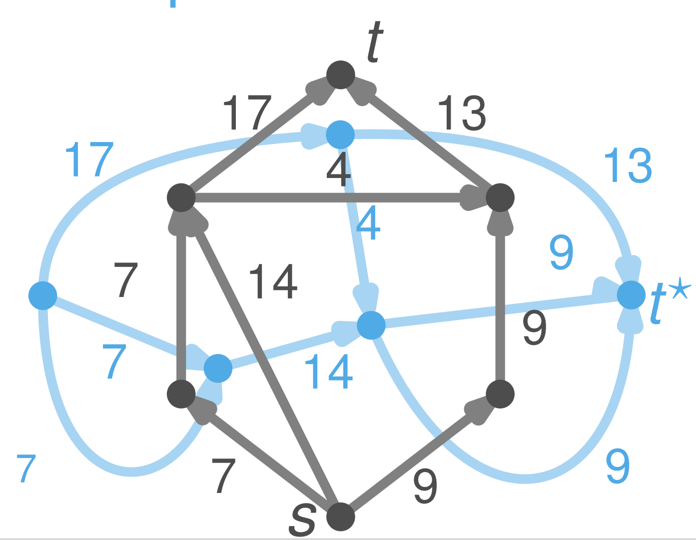

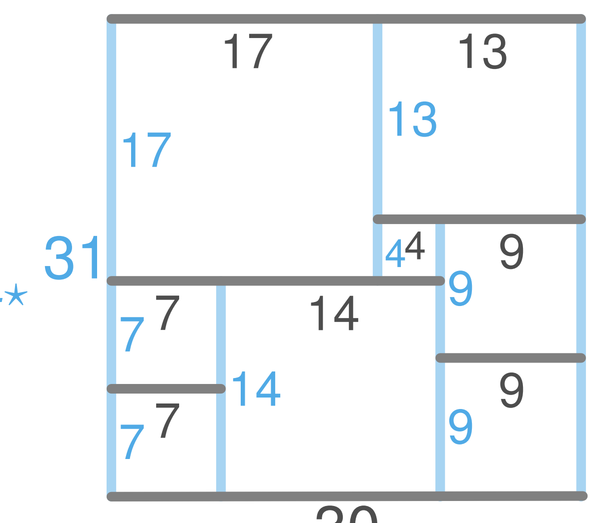

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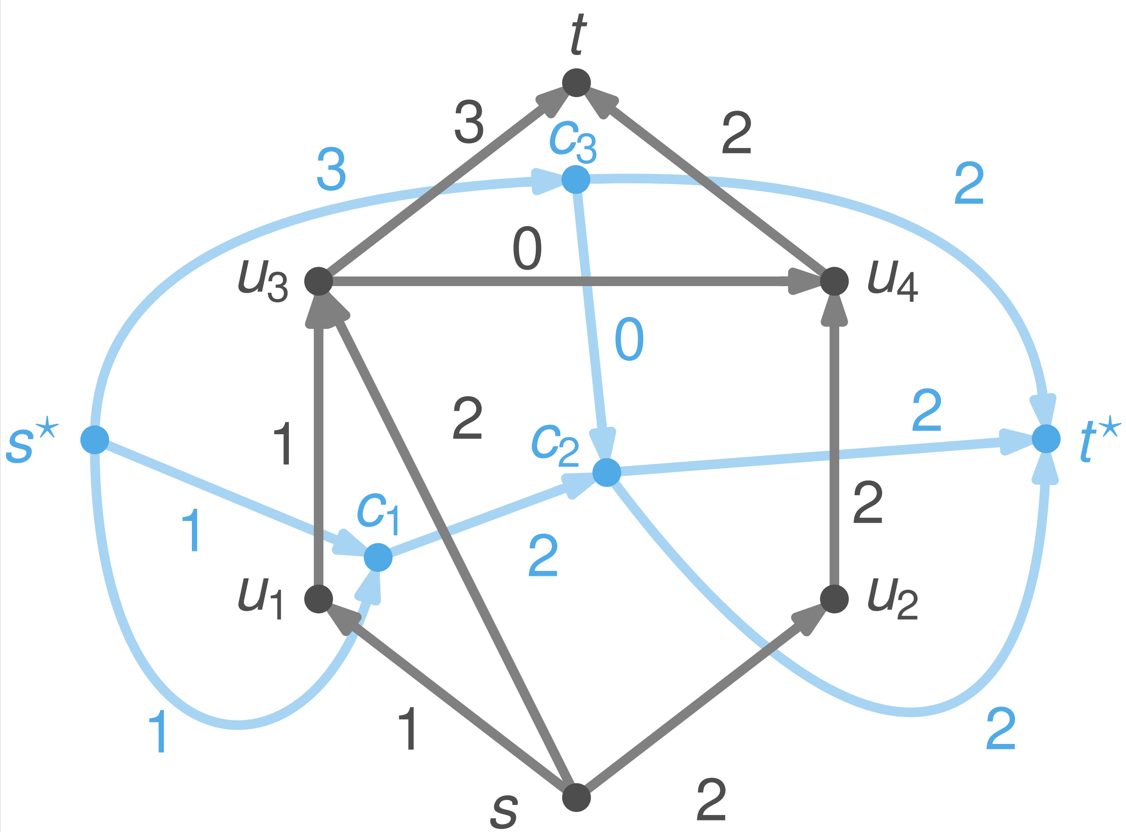

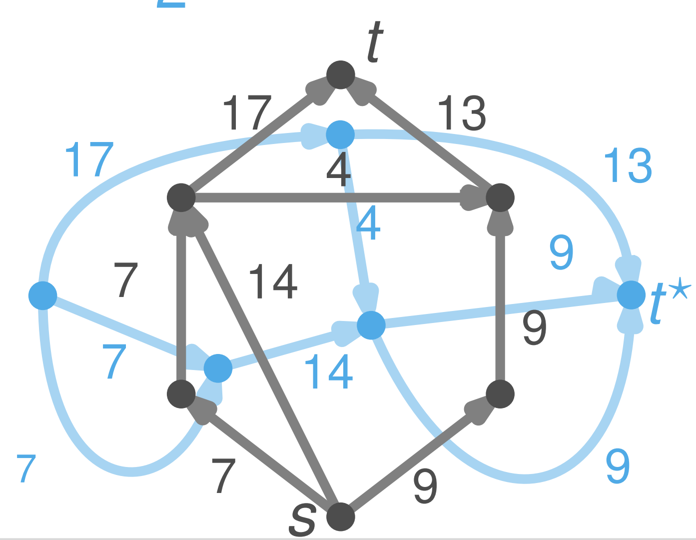

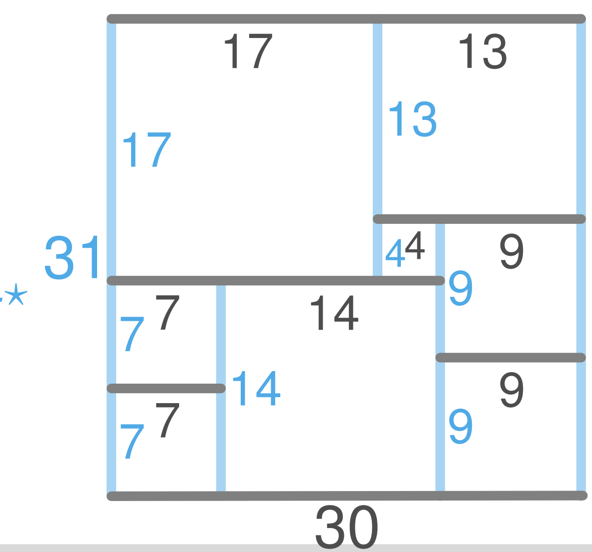

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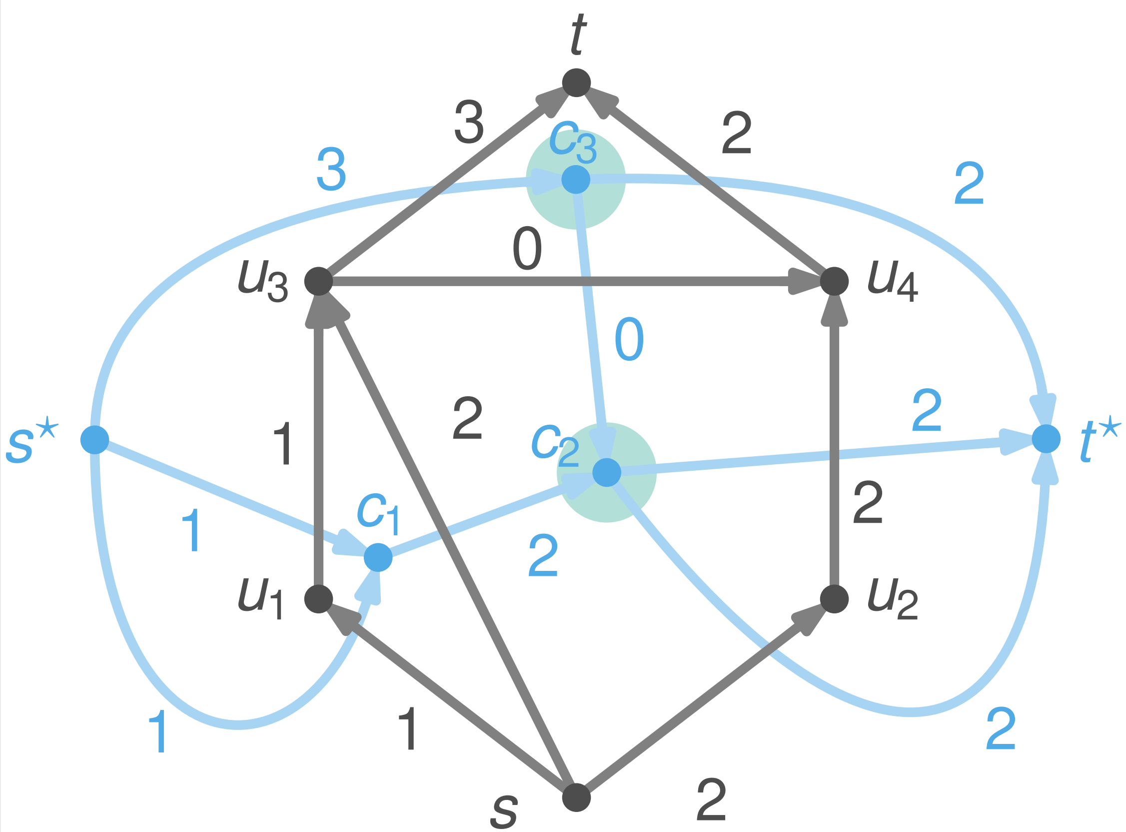

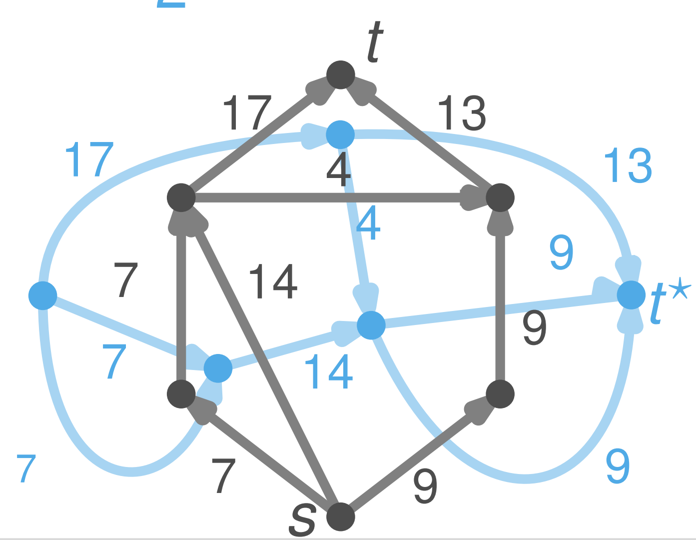

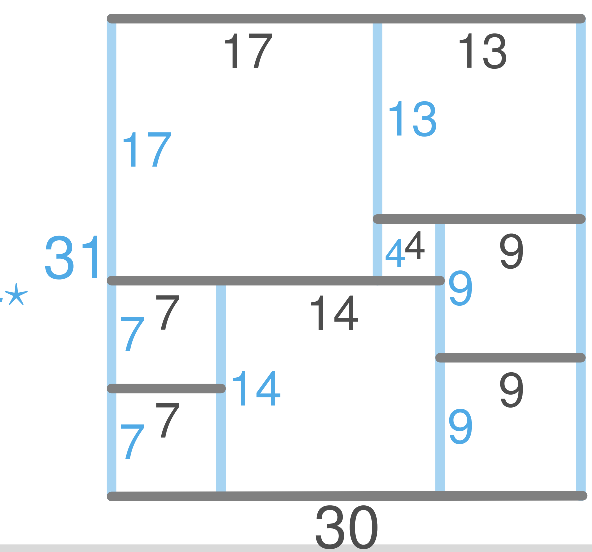

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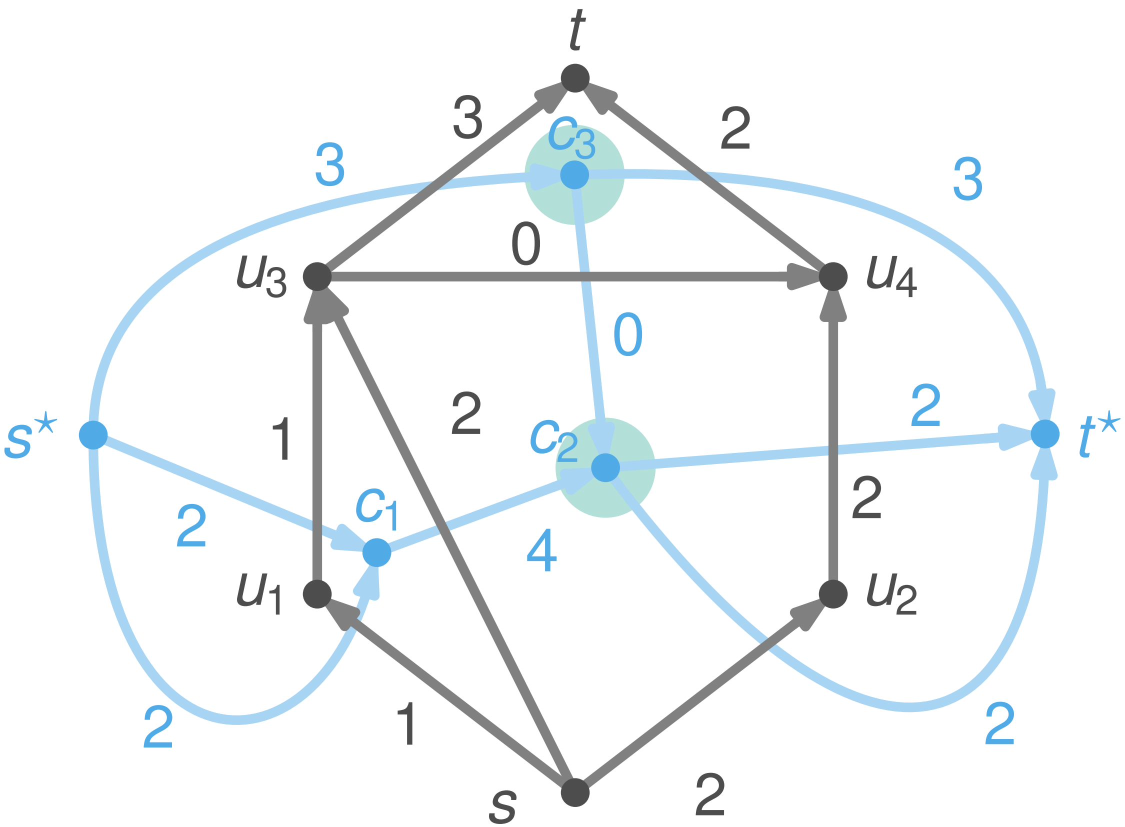

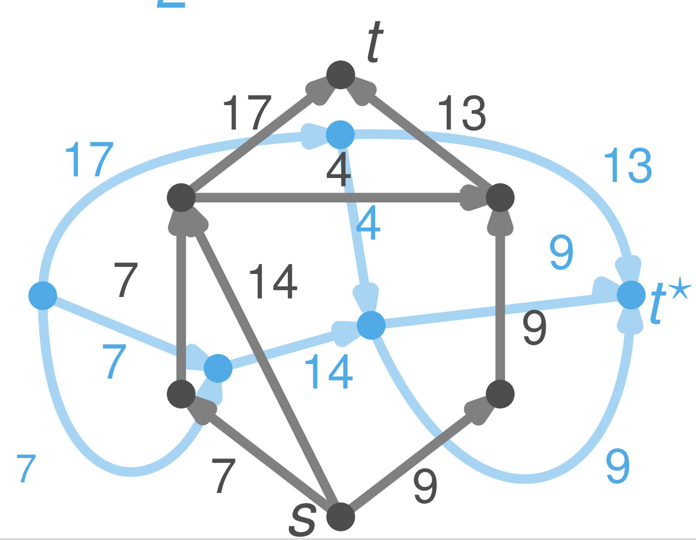

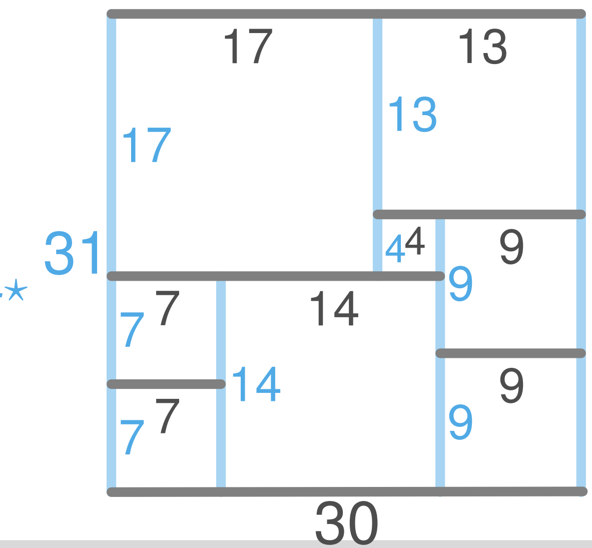

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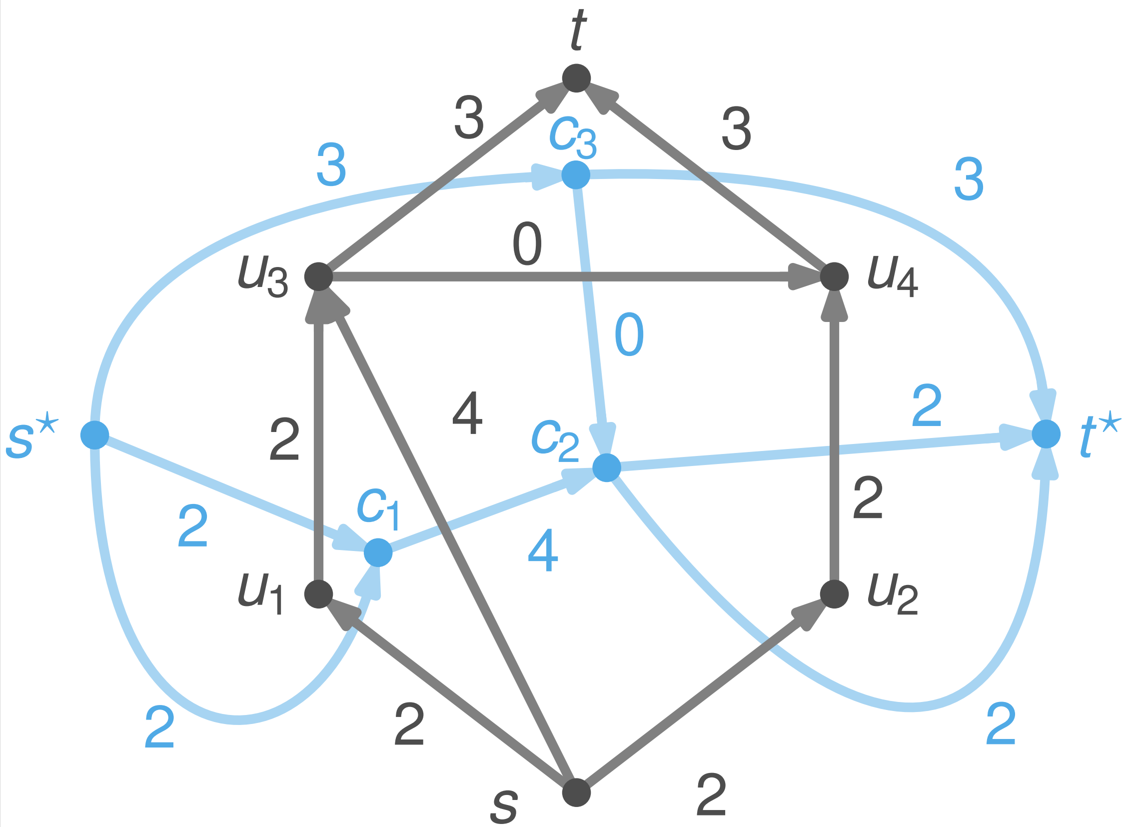

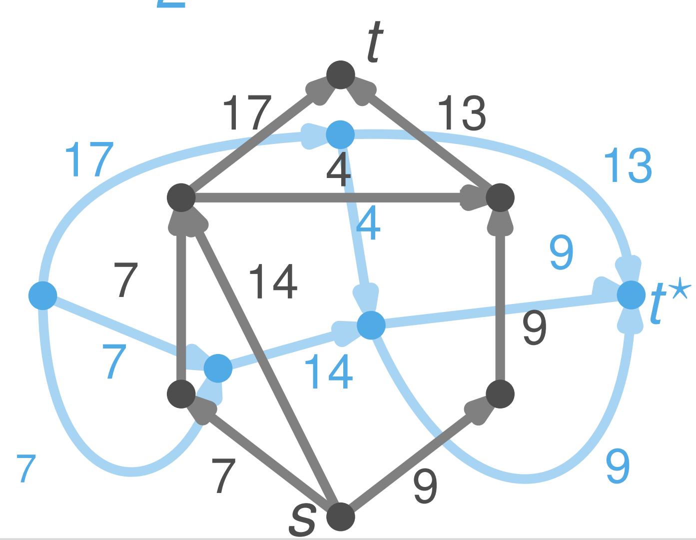

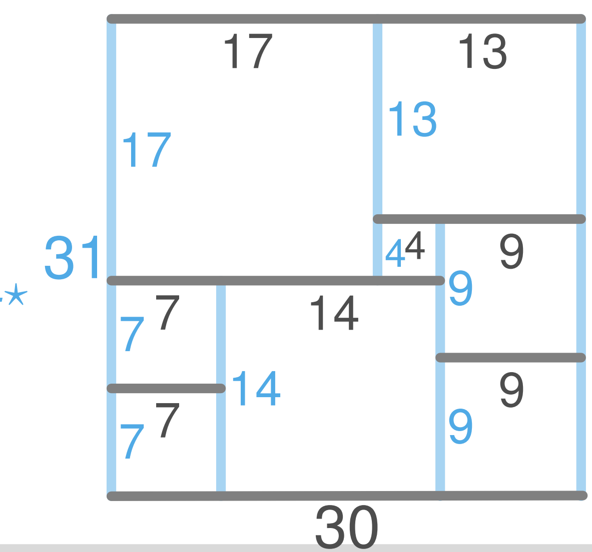

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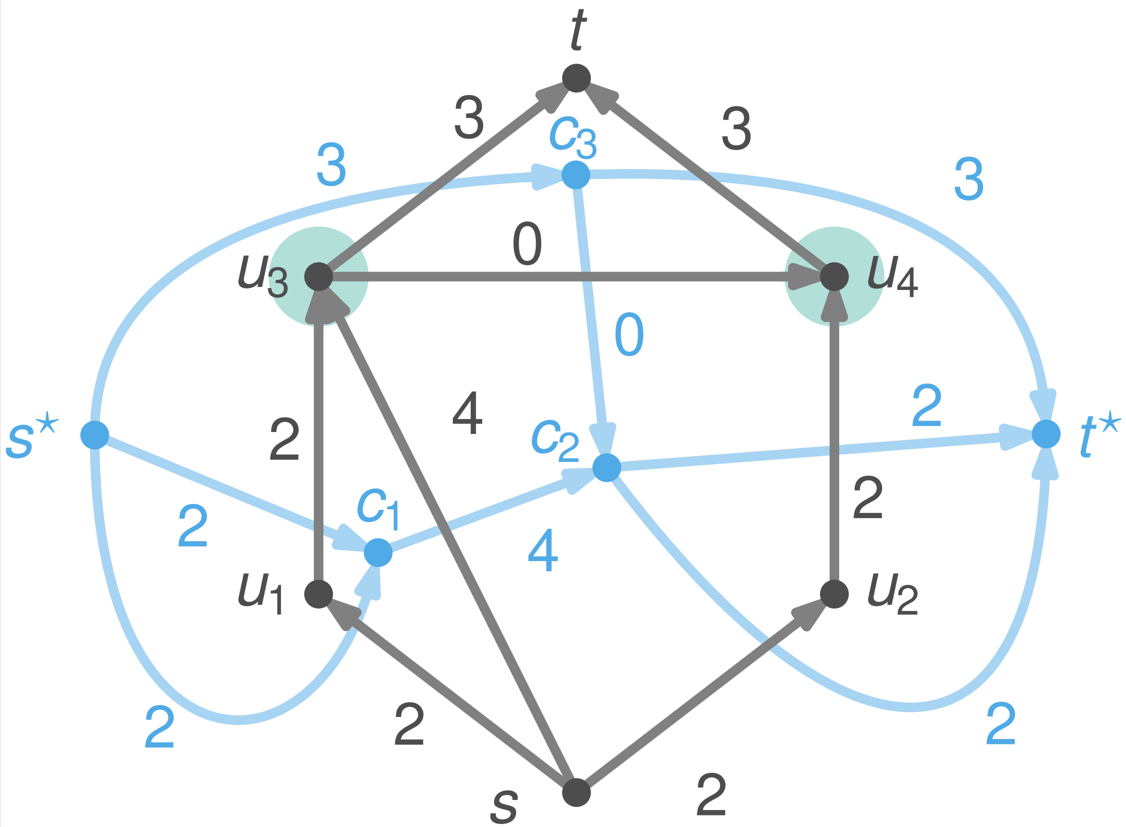

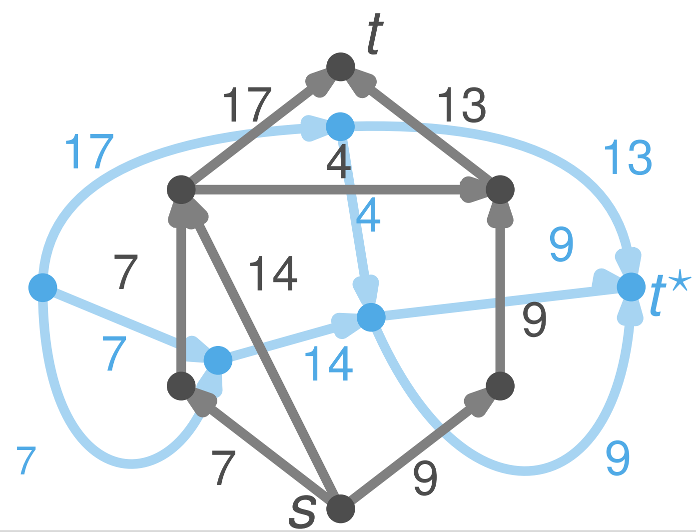

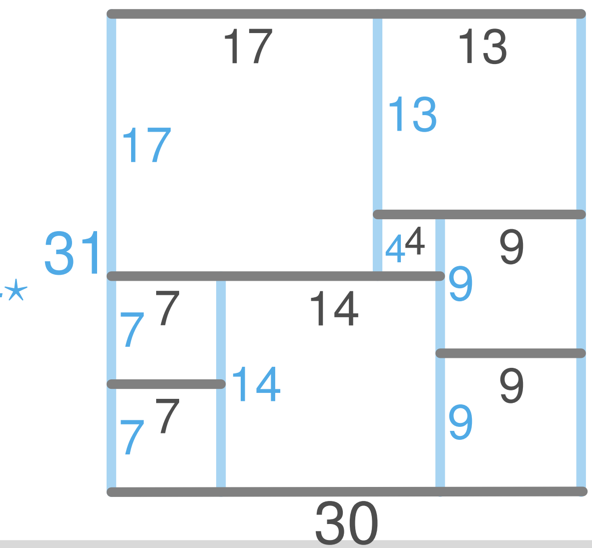

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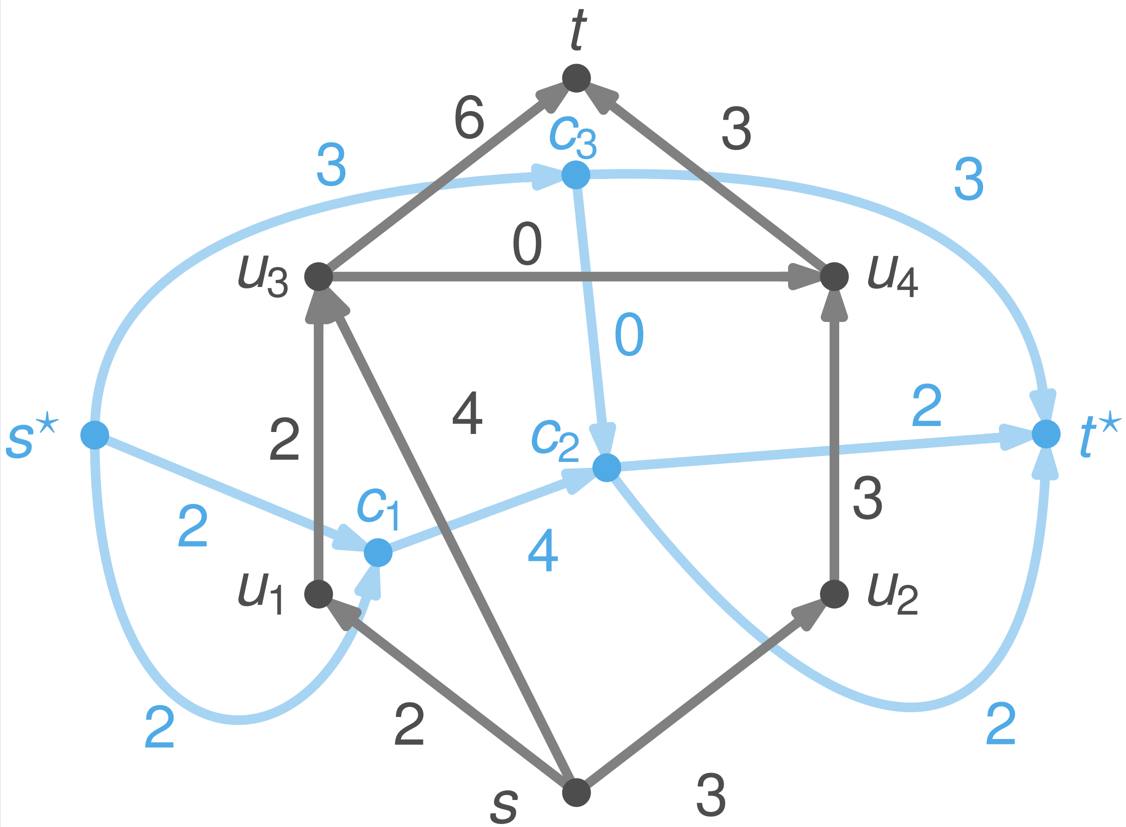

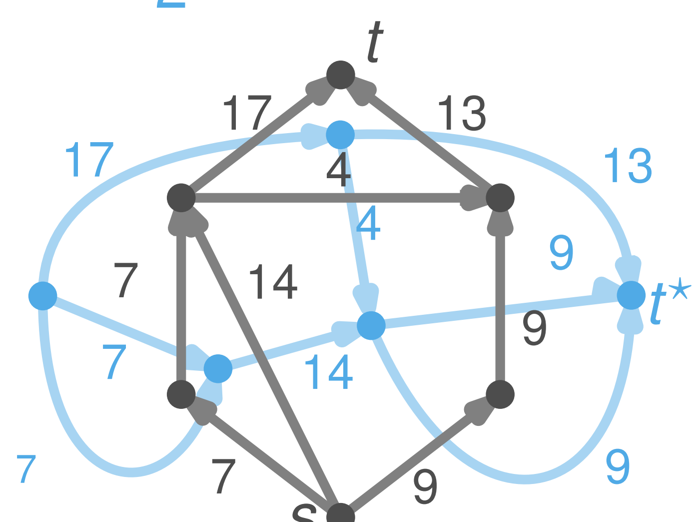

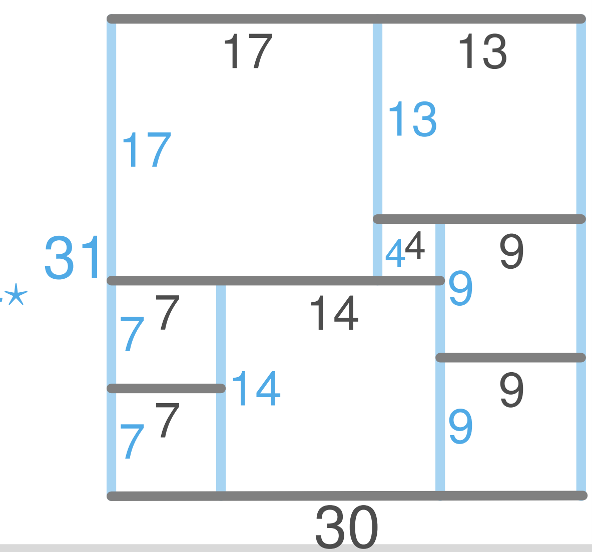

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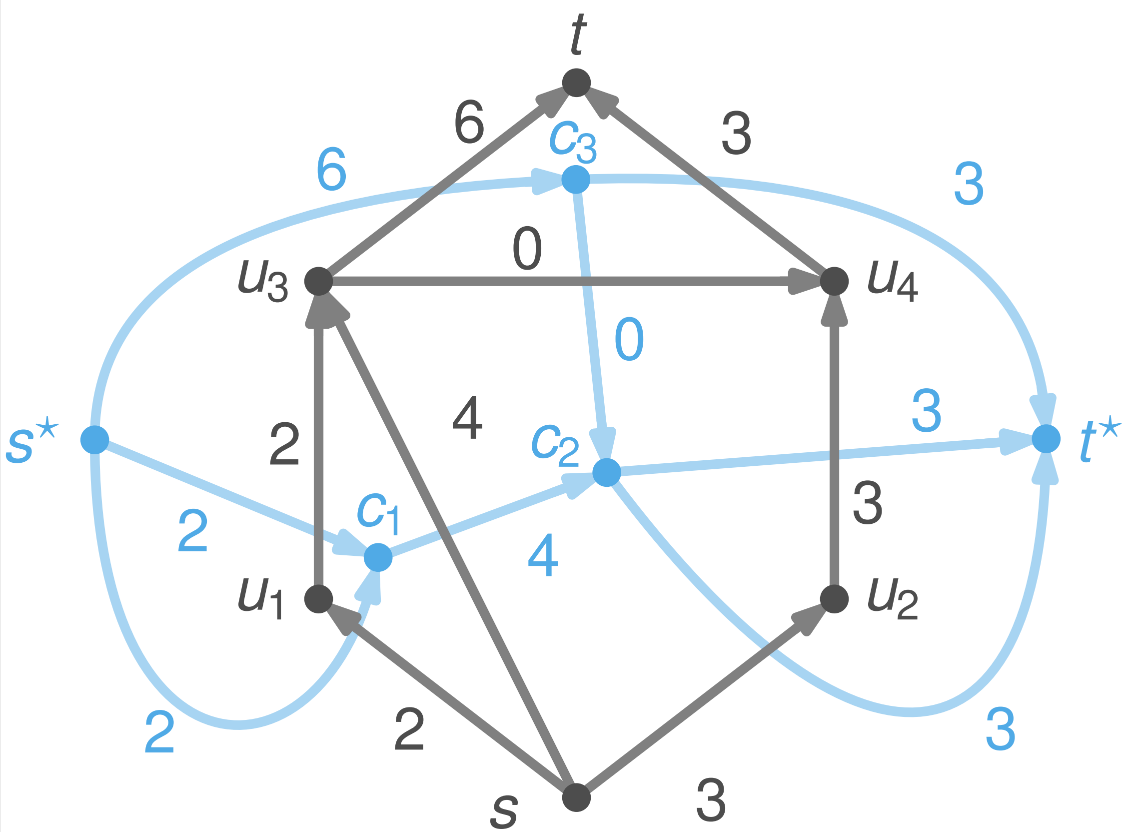

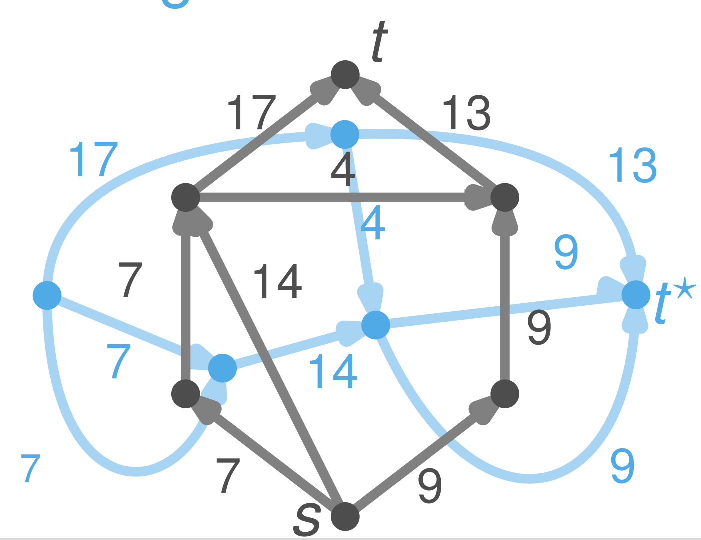

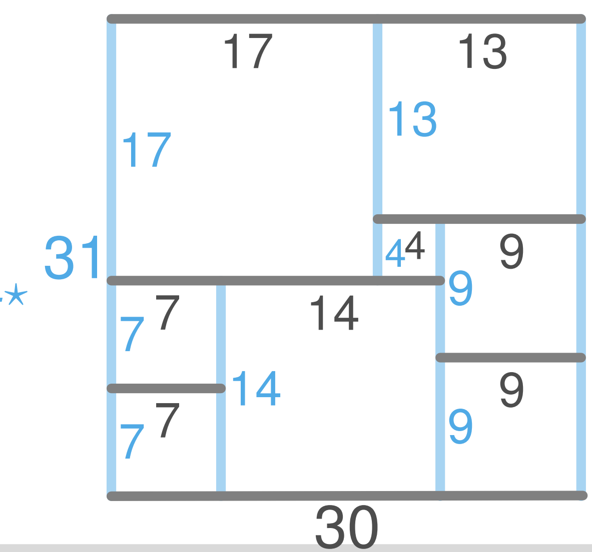

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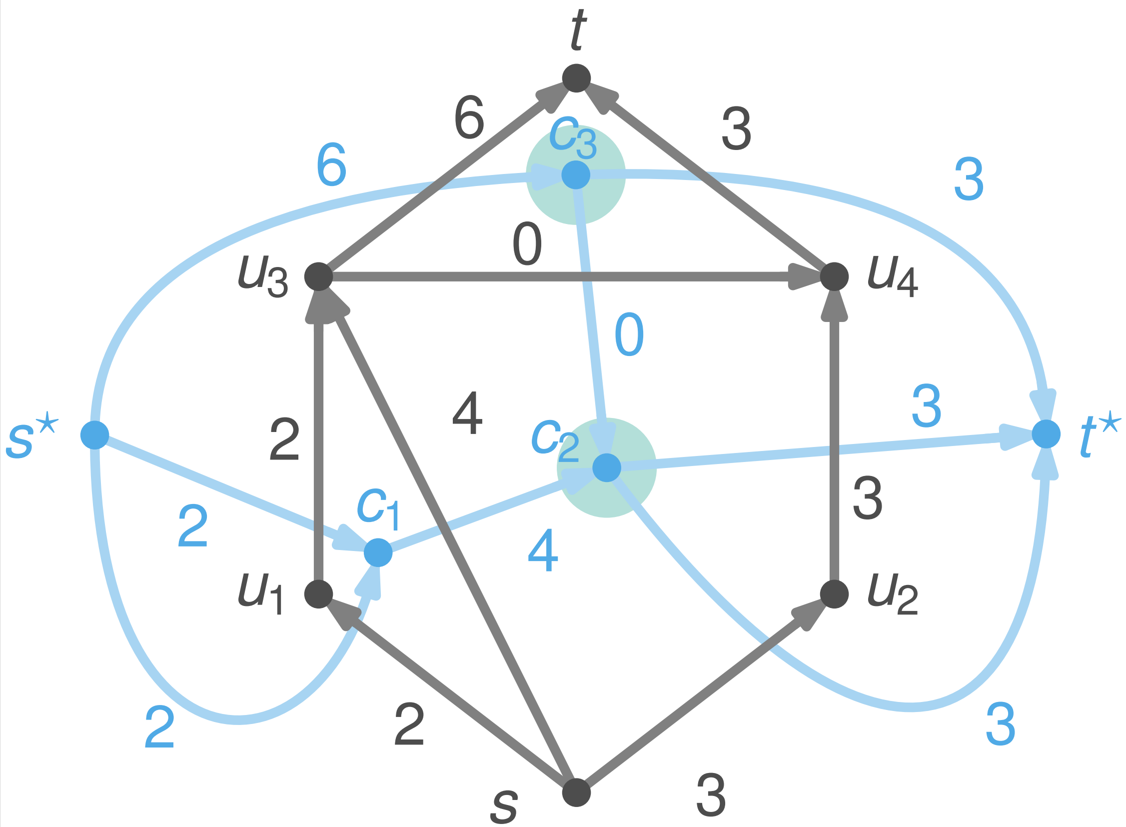

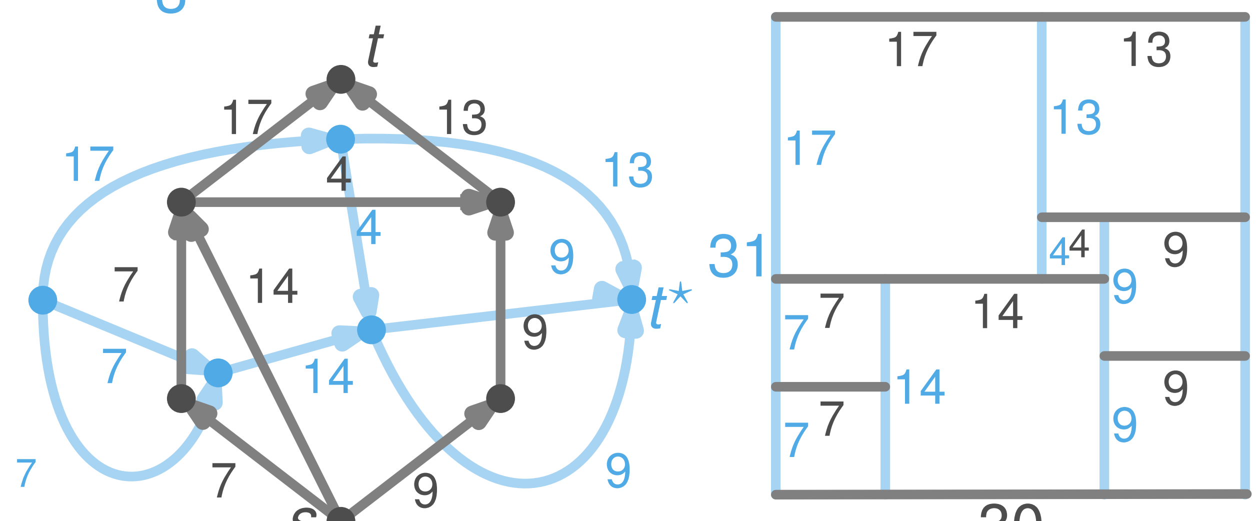

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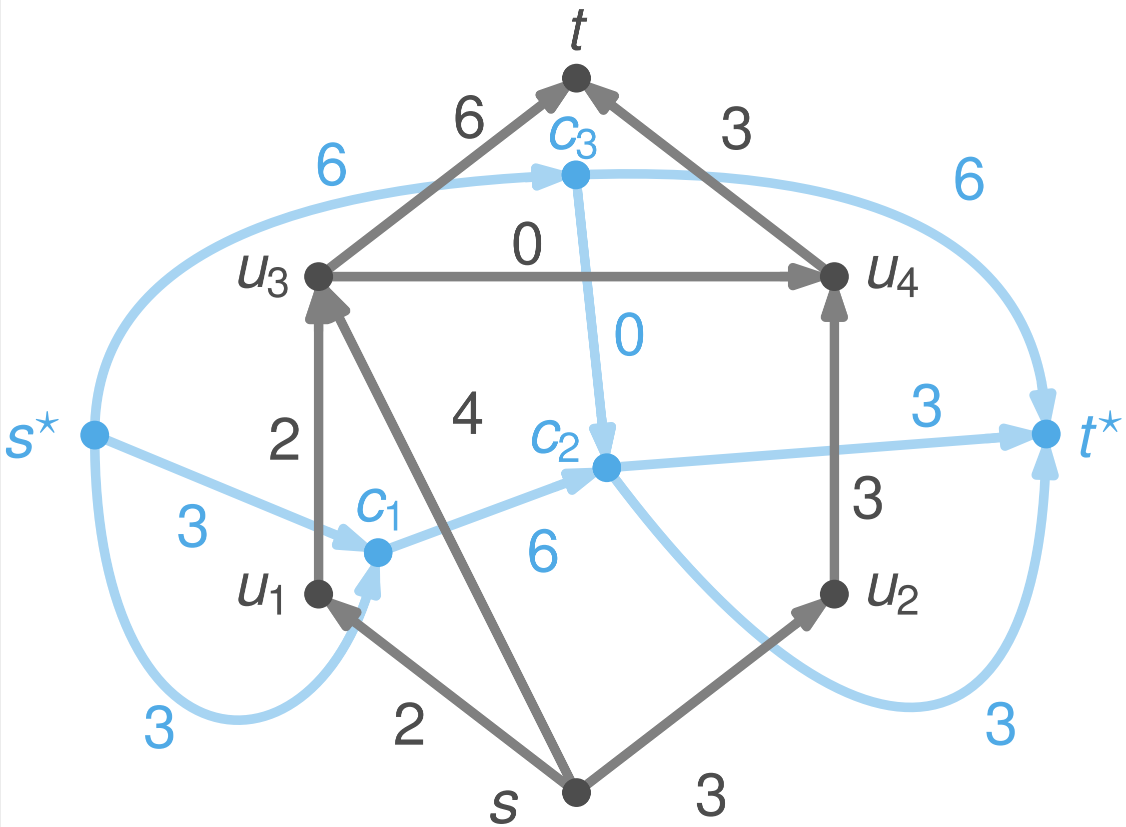

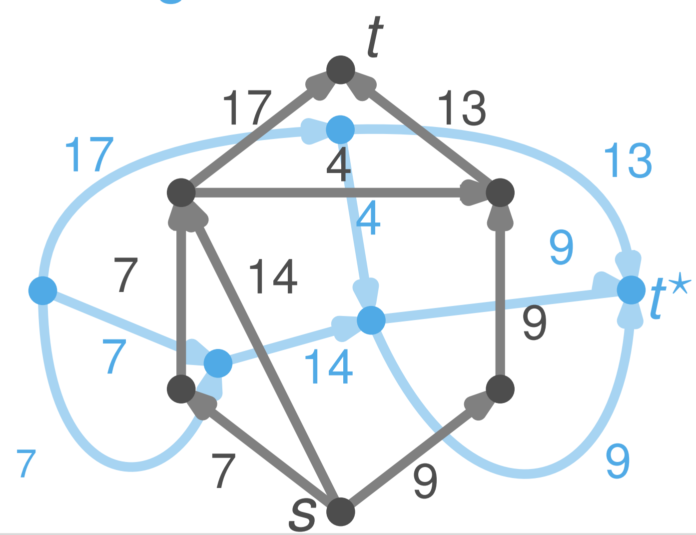

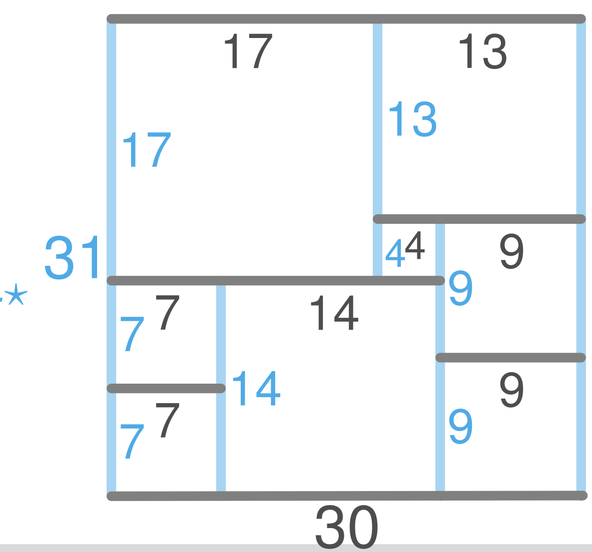

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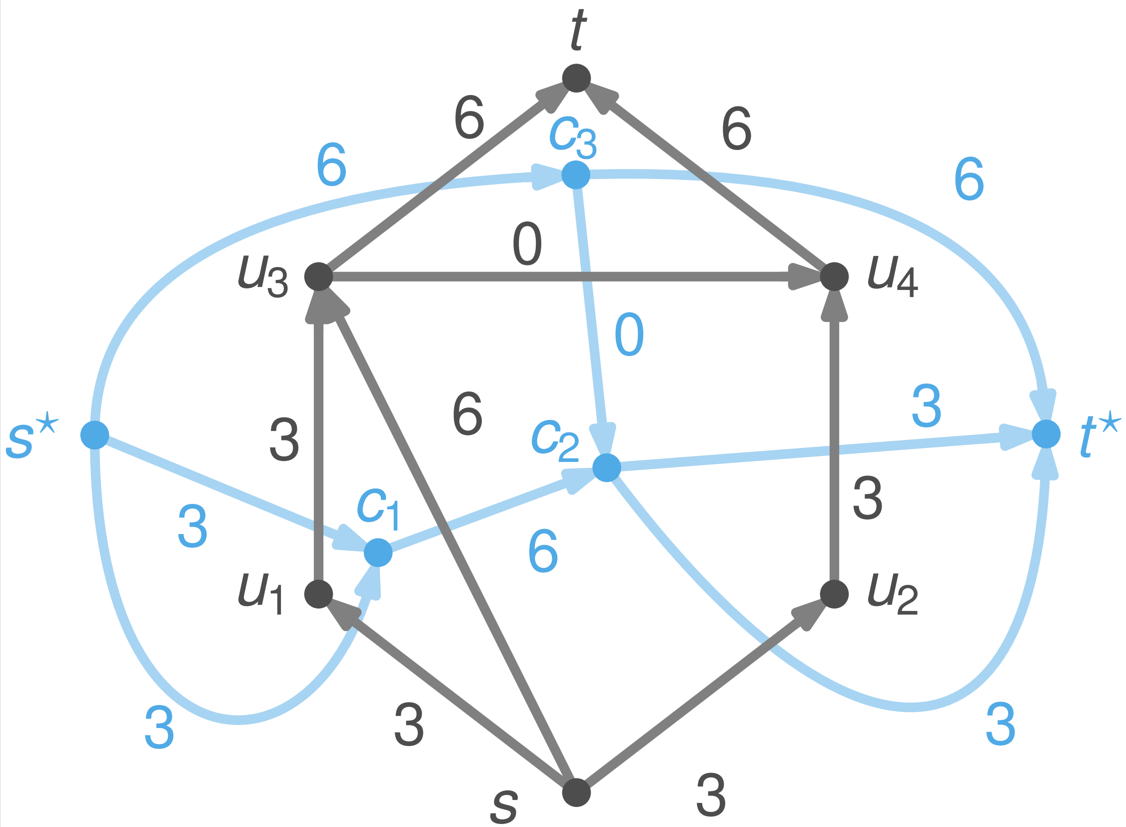

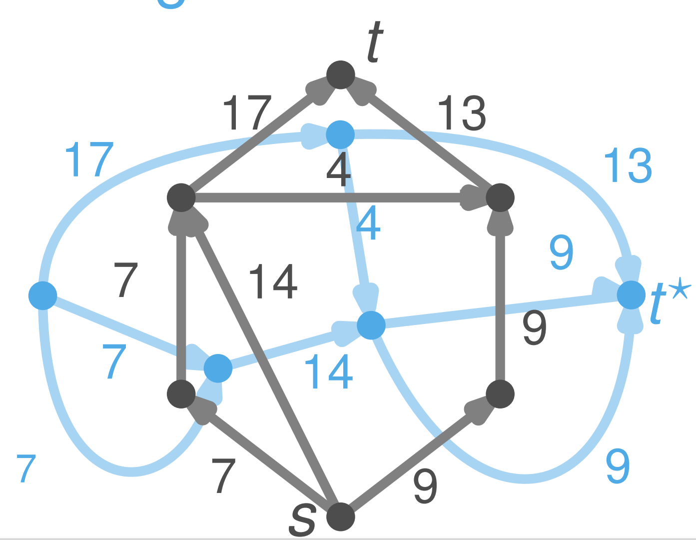

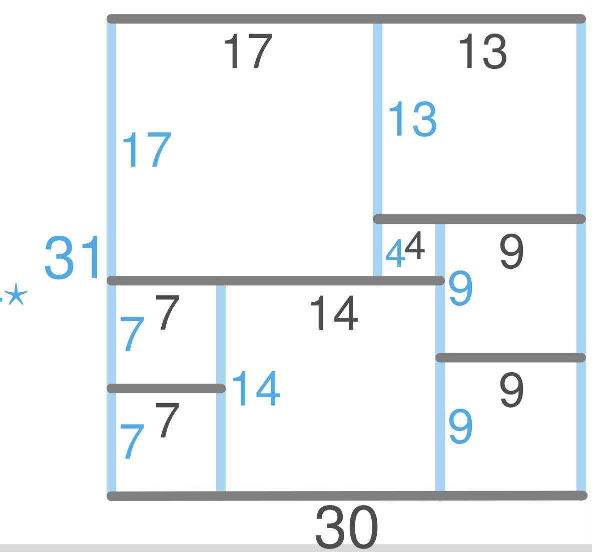

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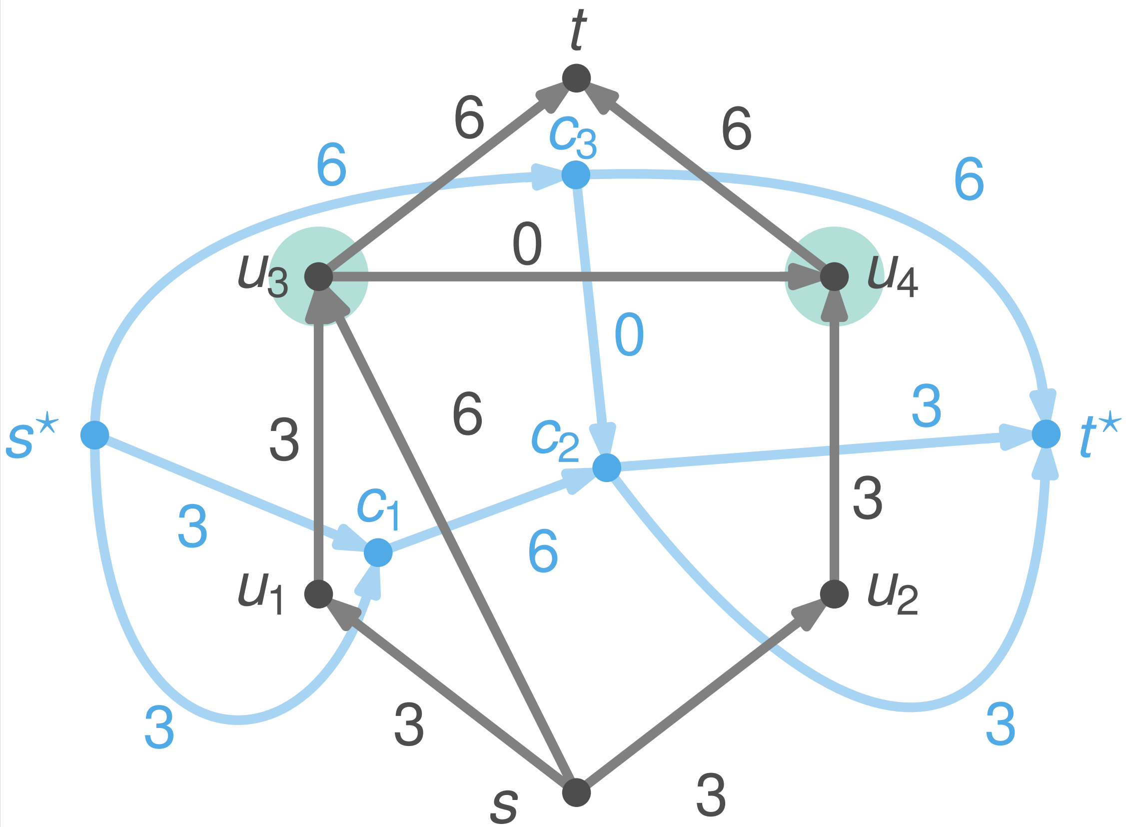

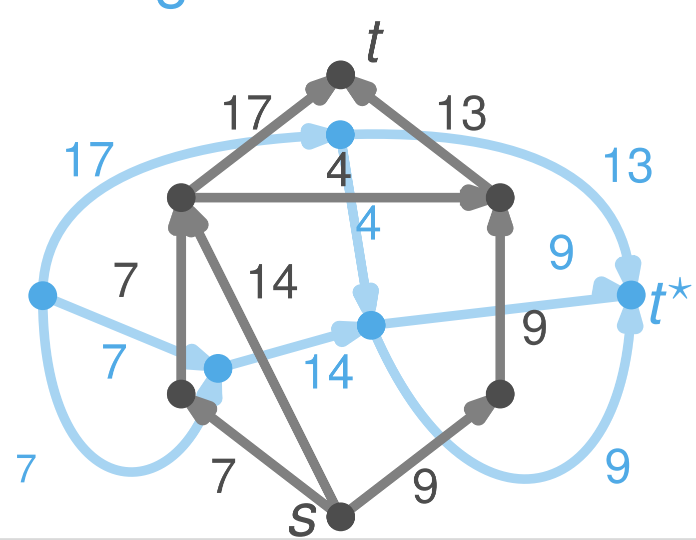

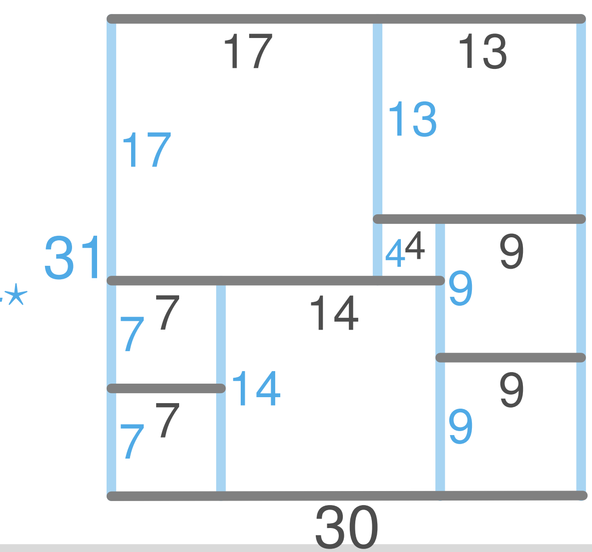

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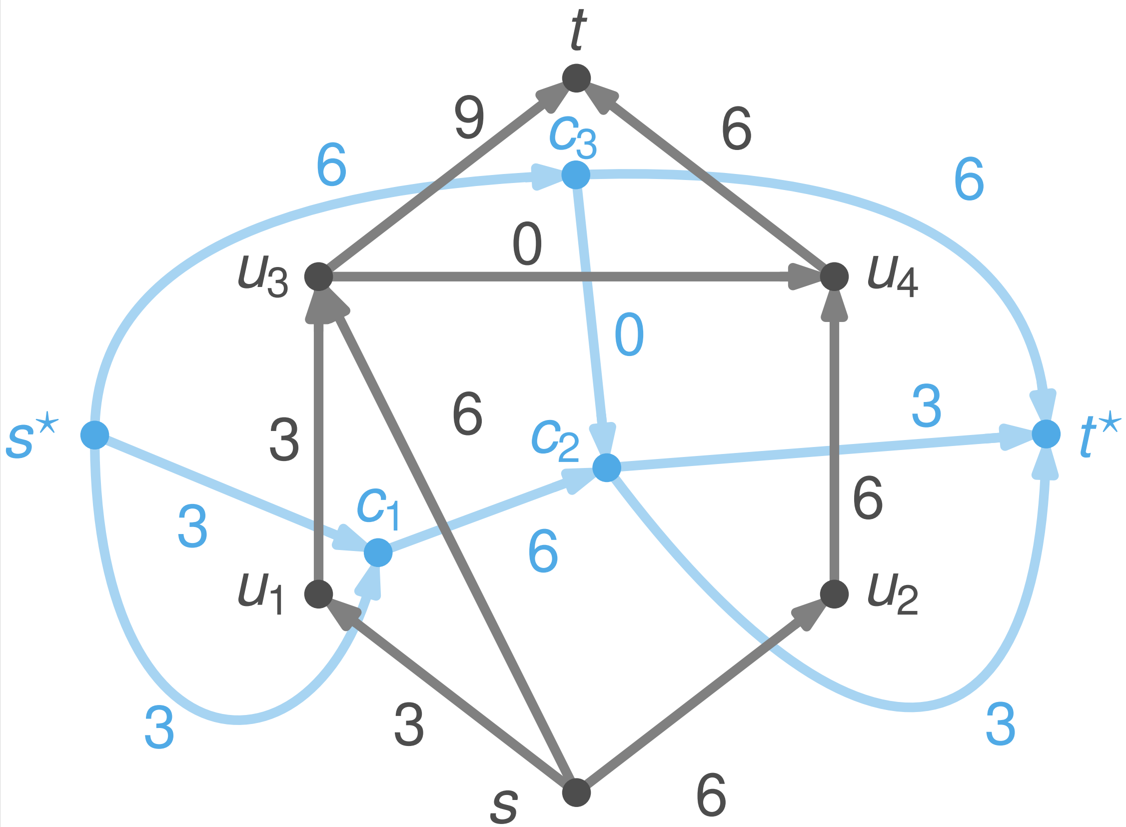

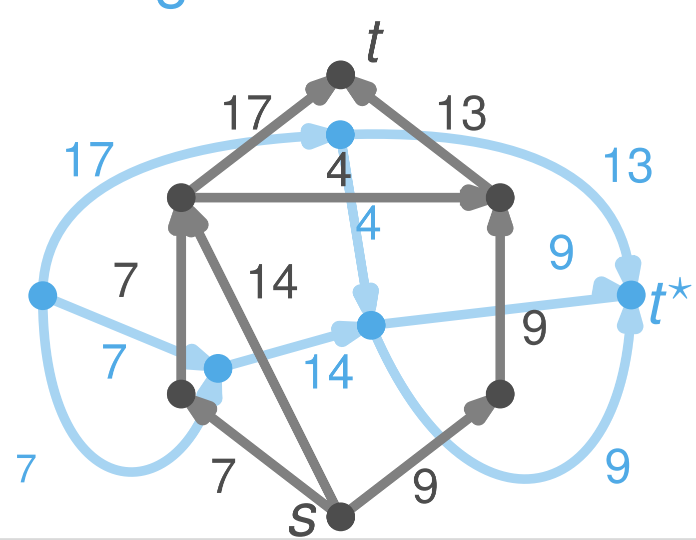

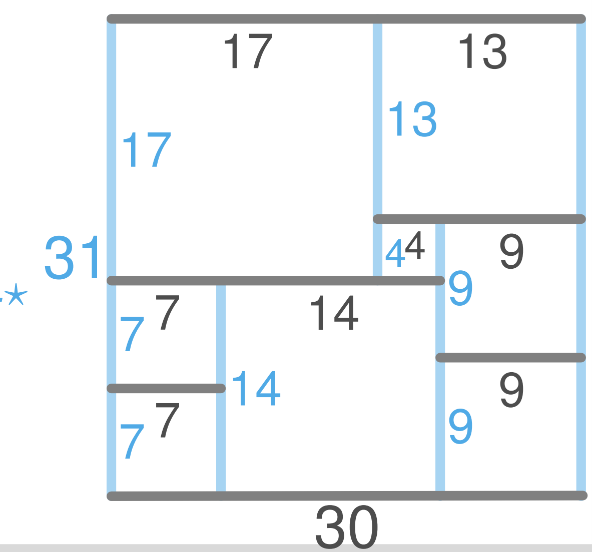

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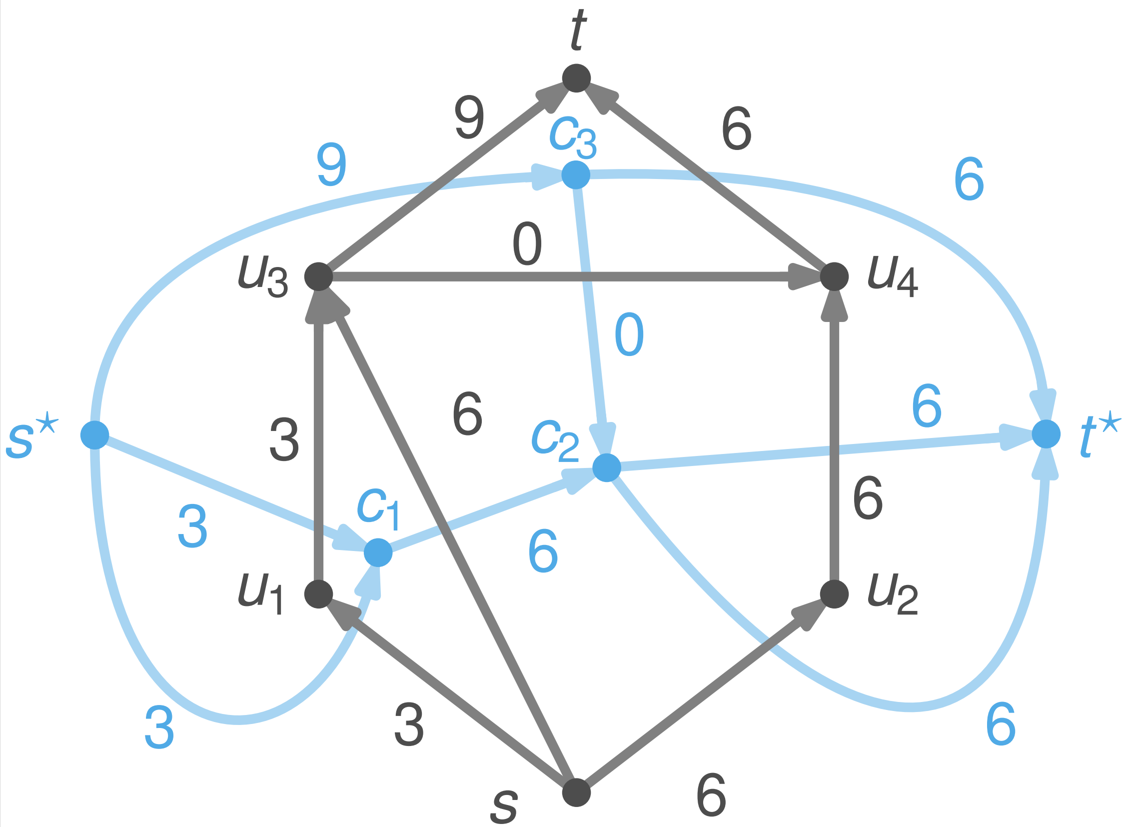

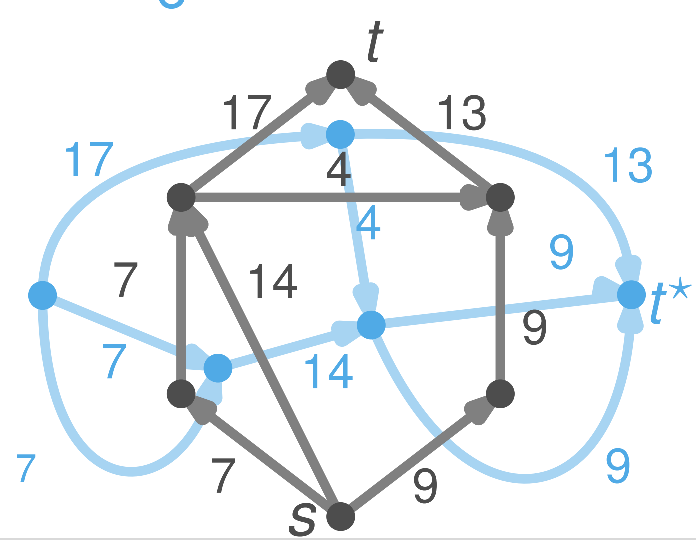

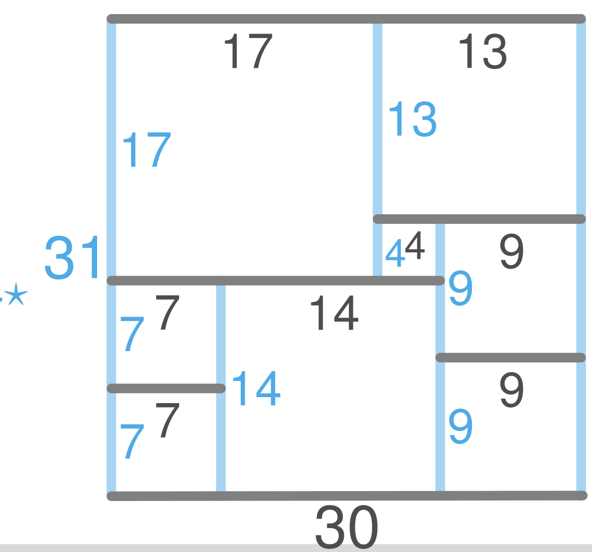

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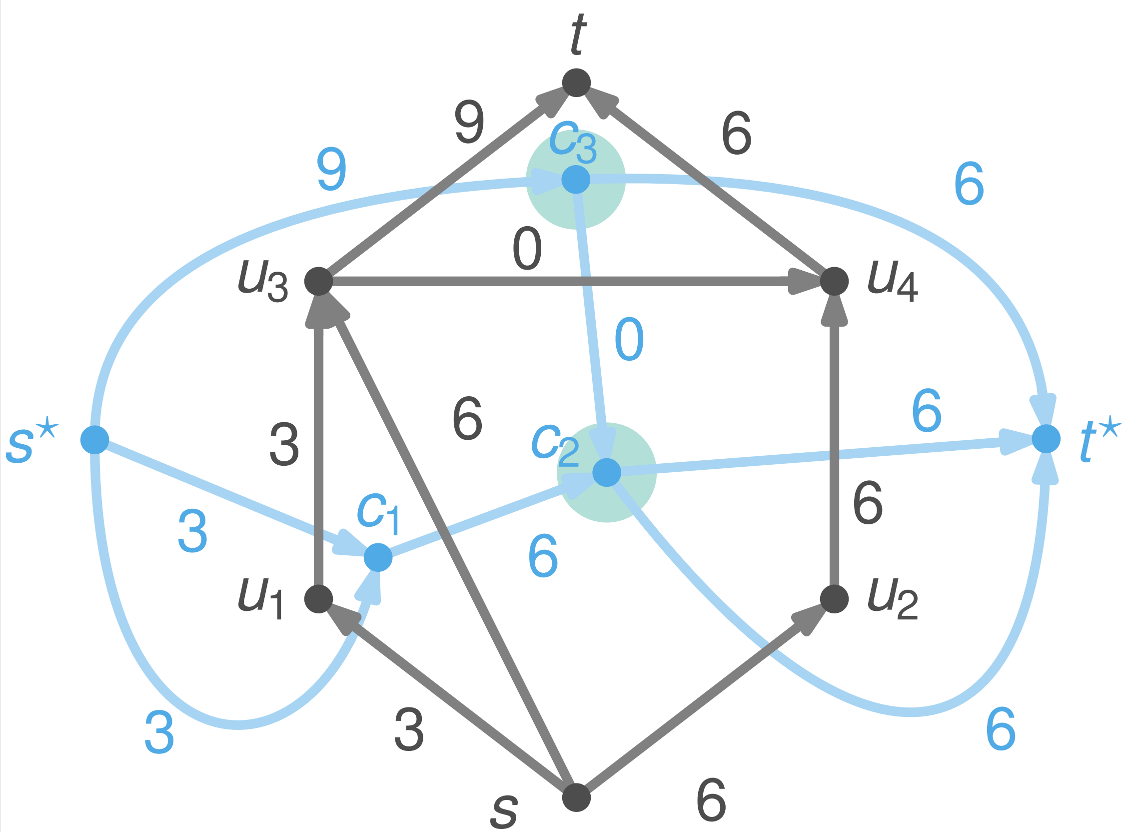

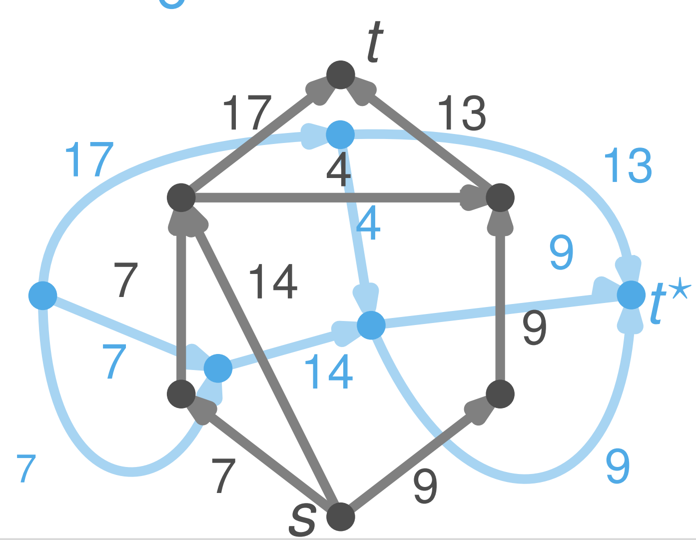

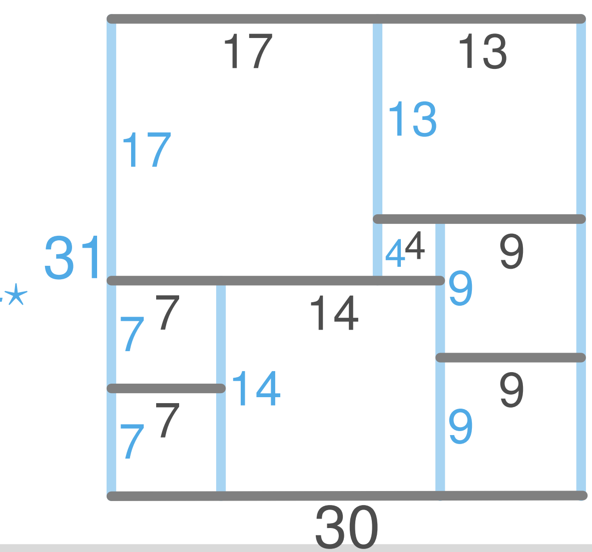

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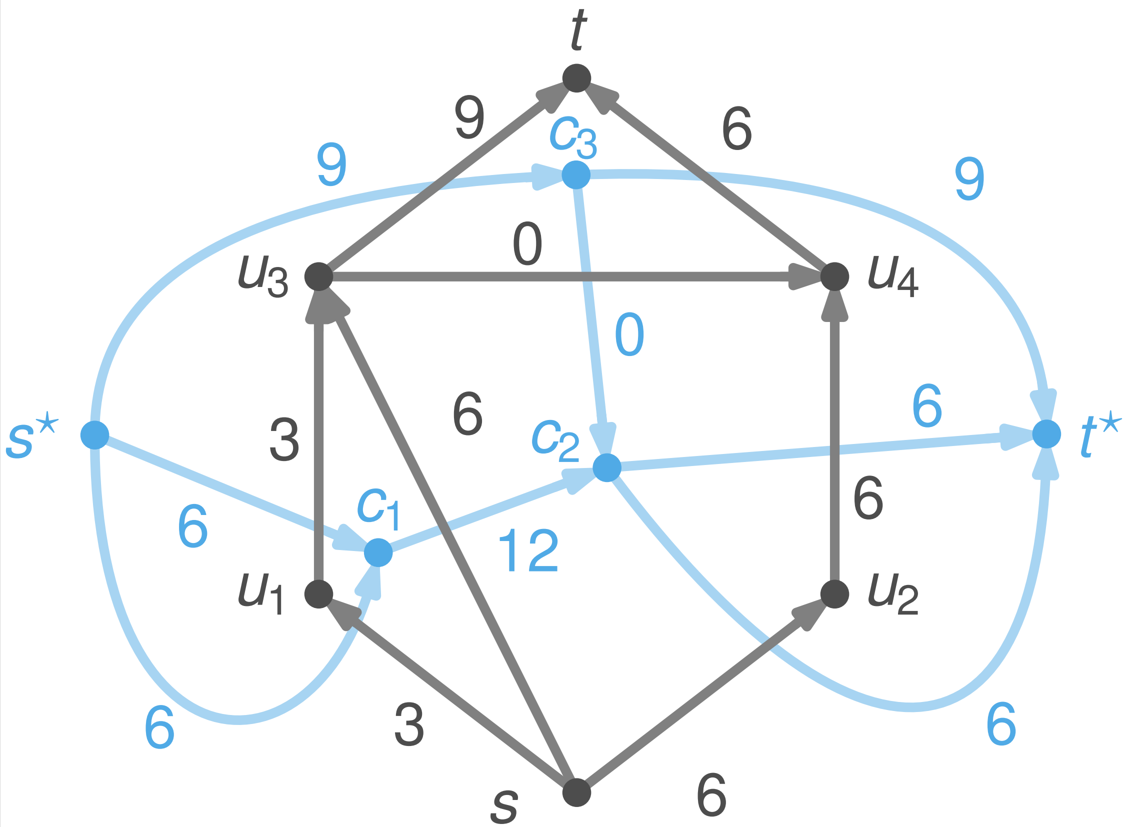

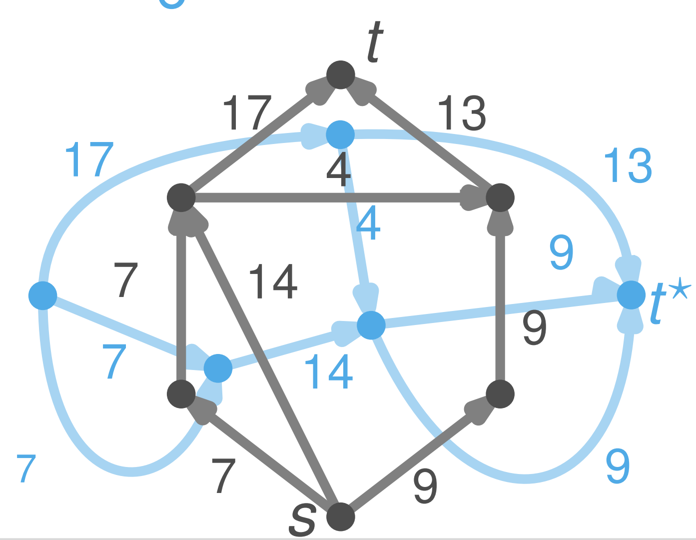

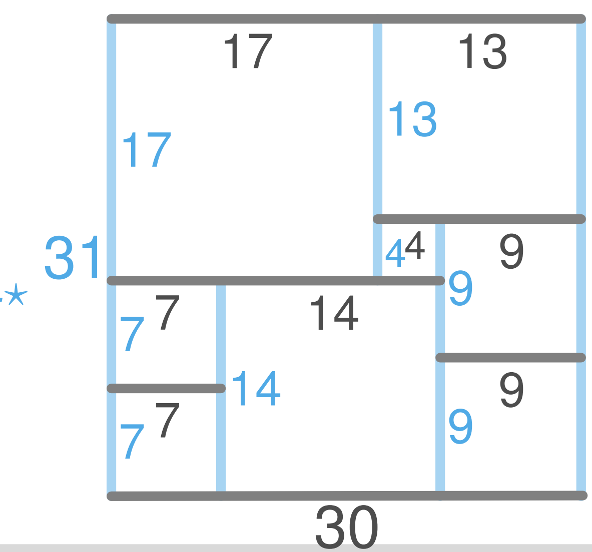

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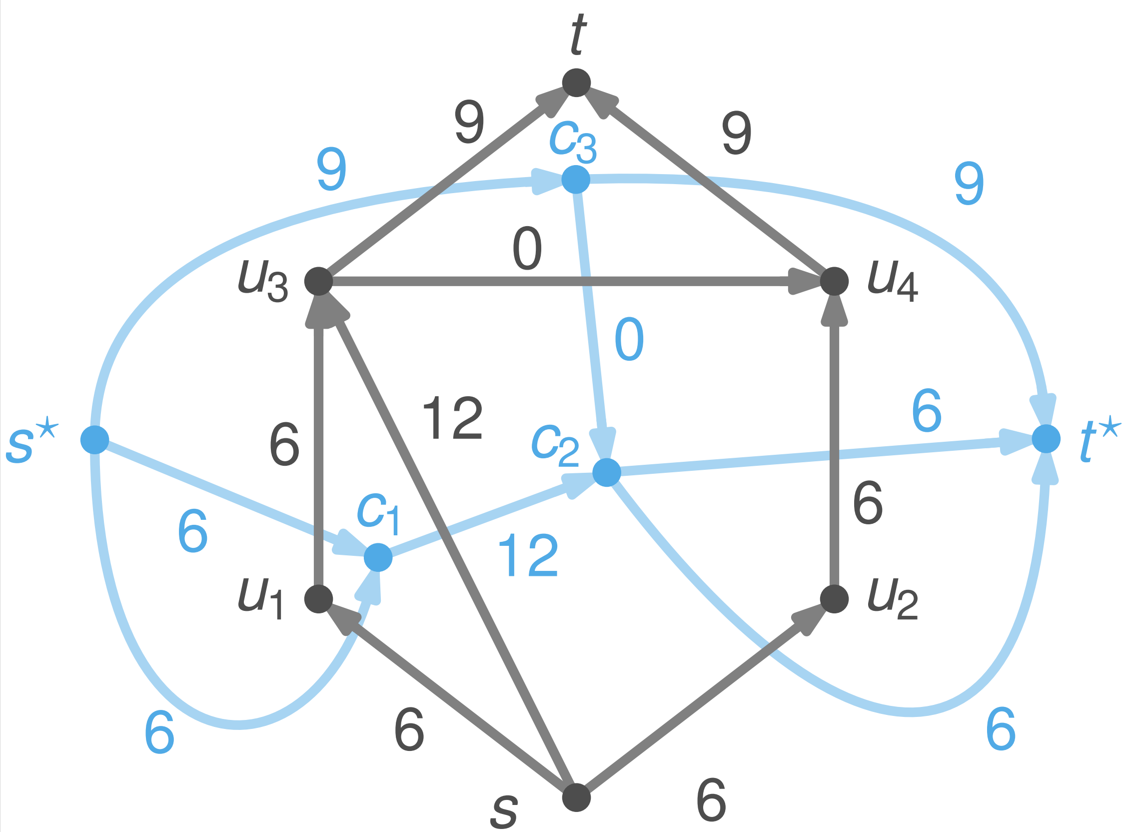

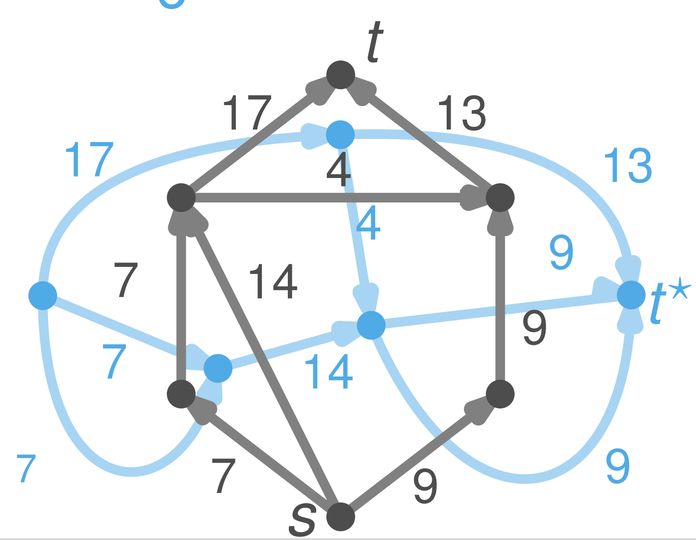

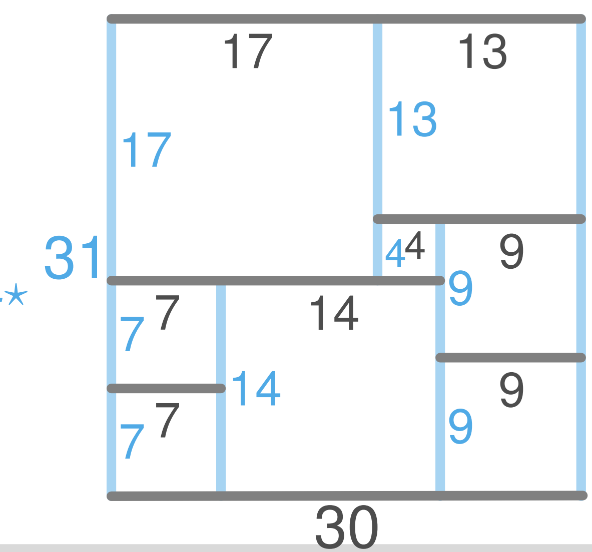

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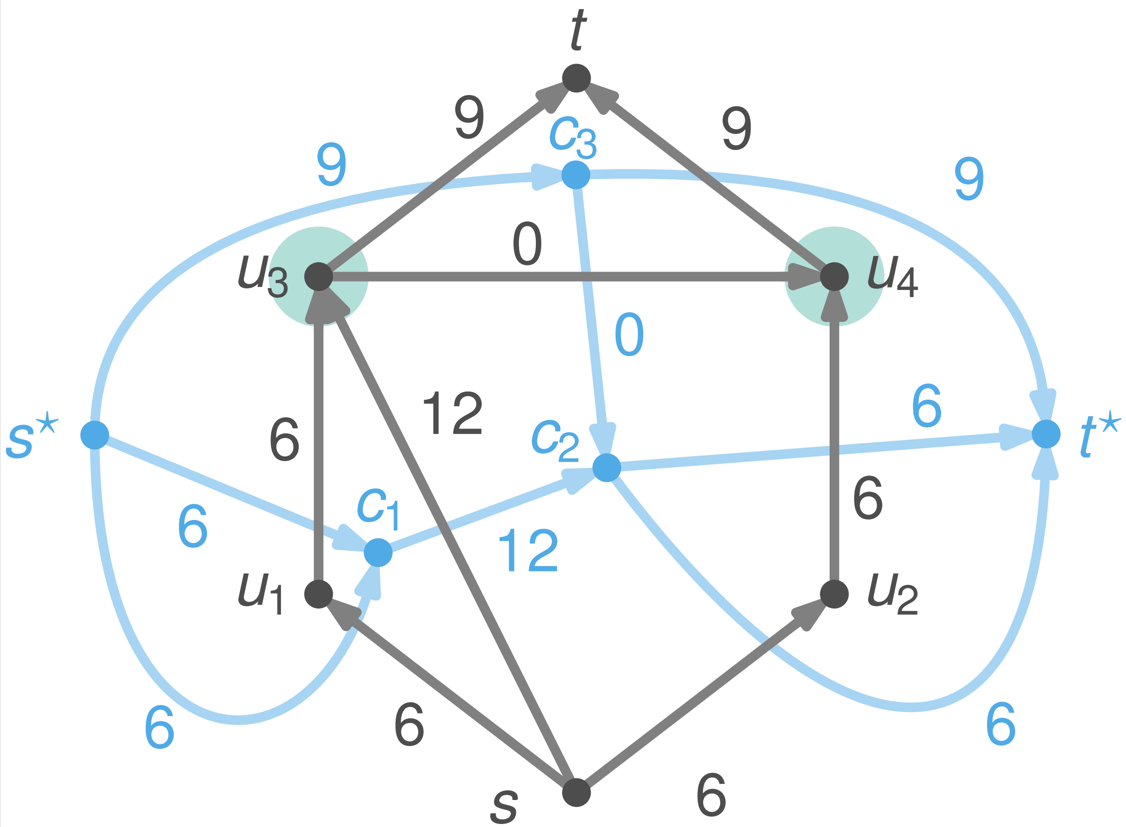

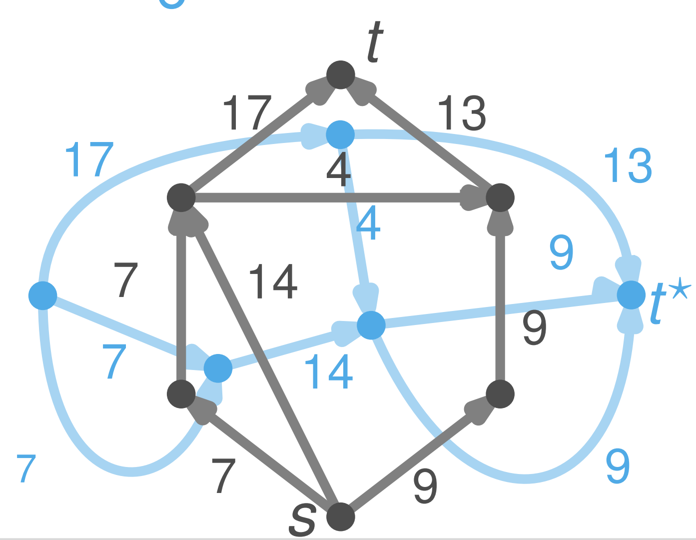

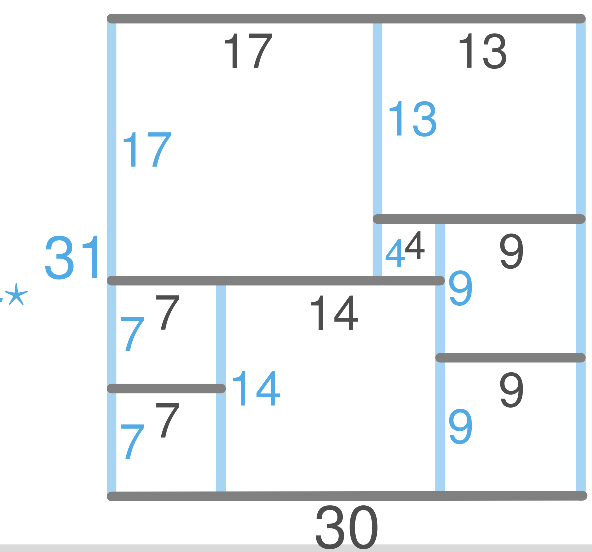

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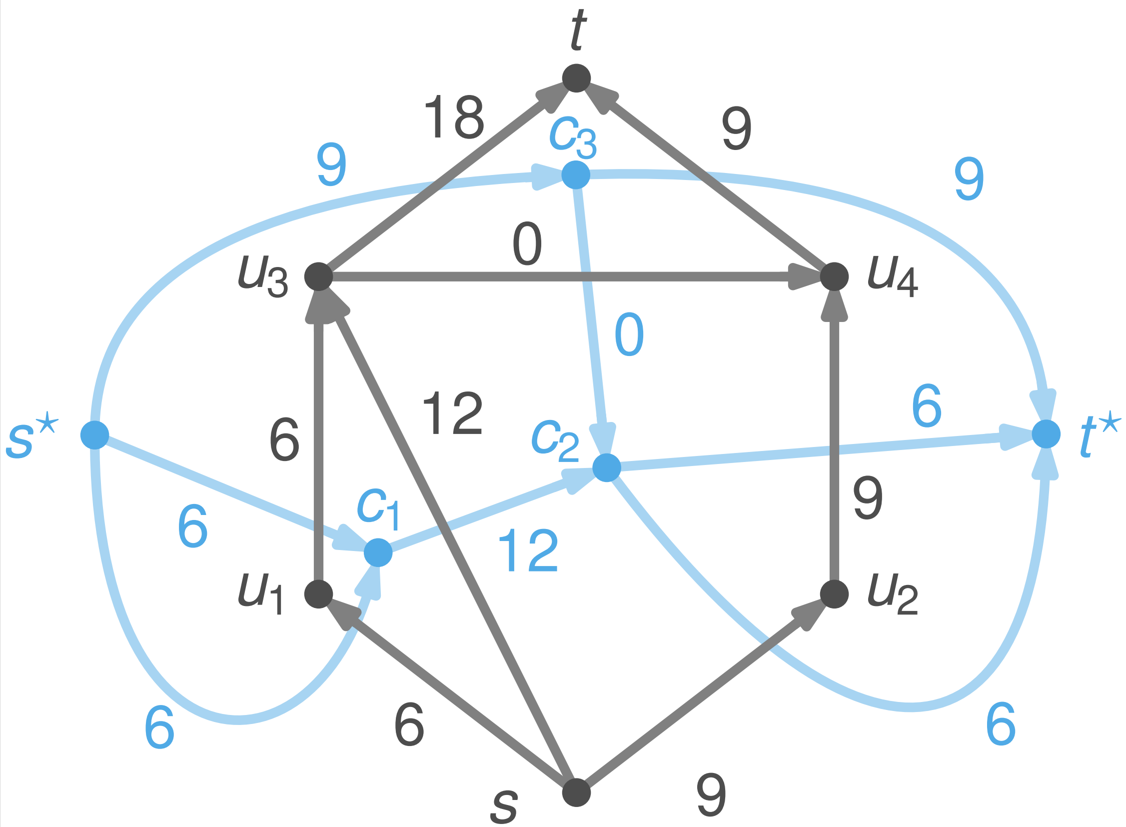

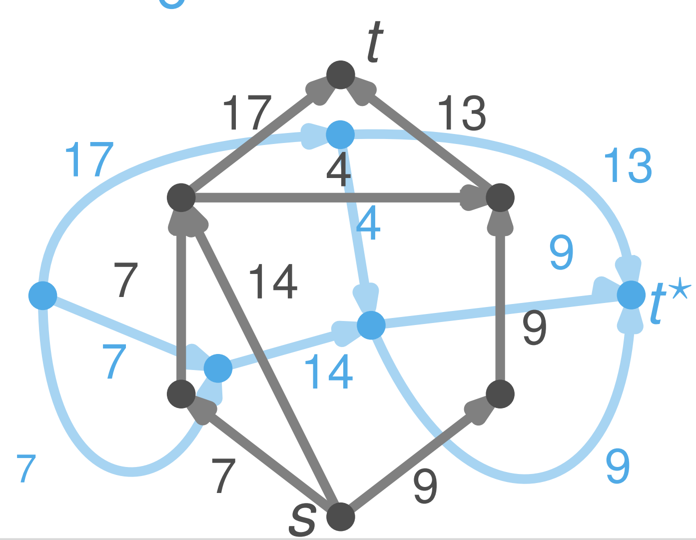

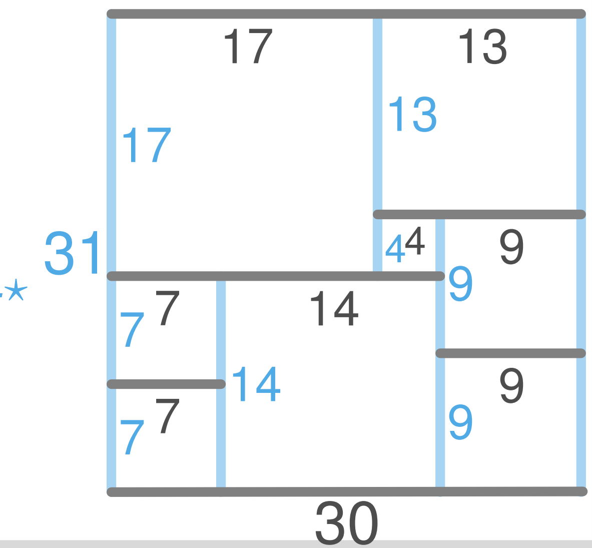

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Wrong Conflict Resolution

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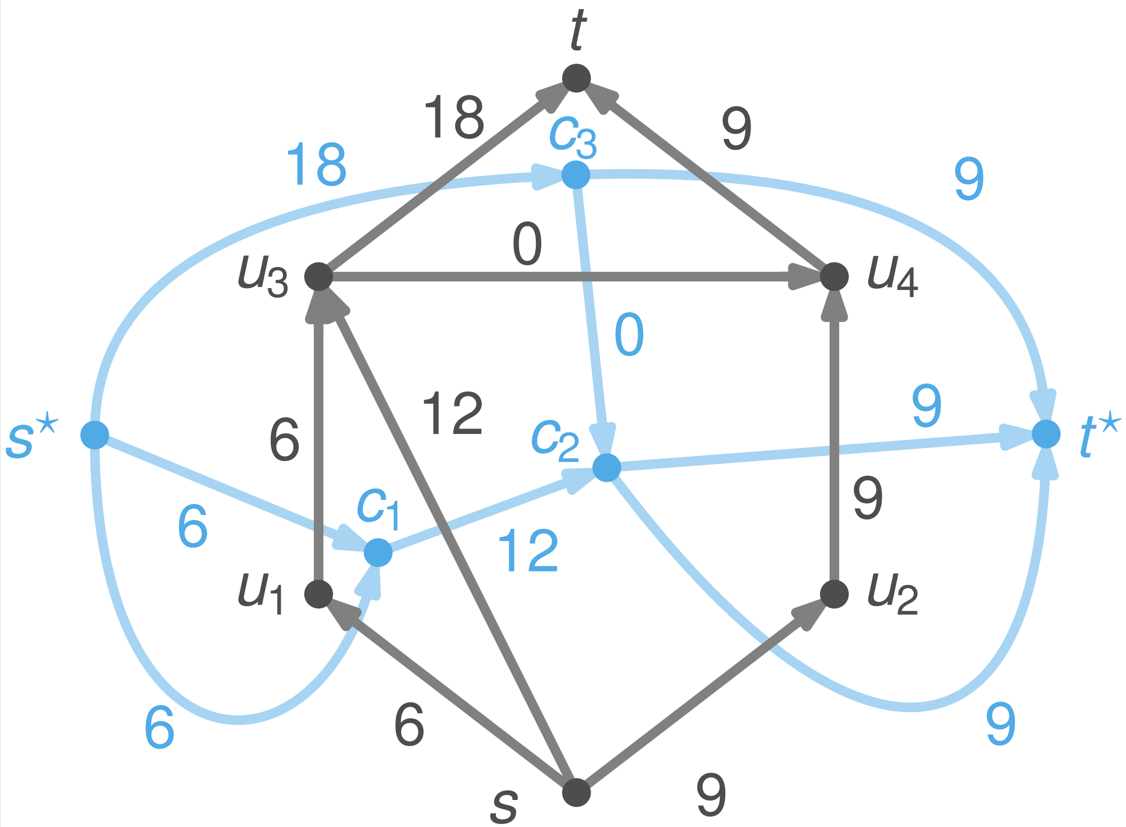
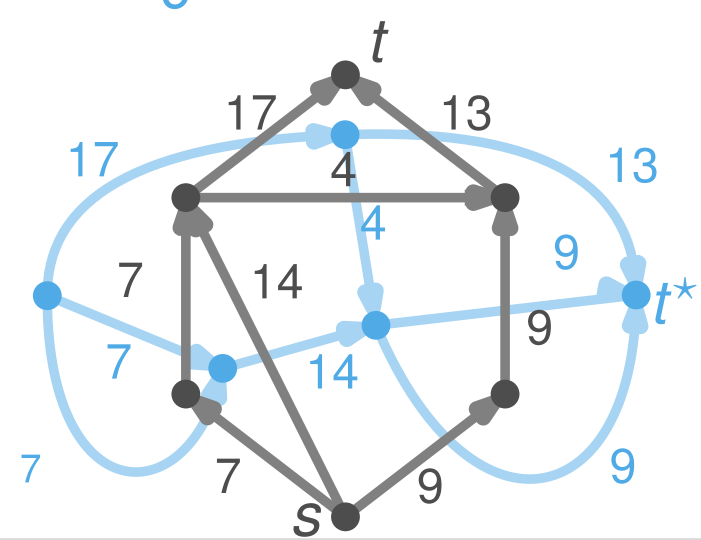
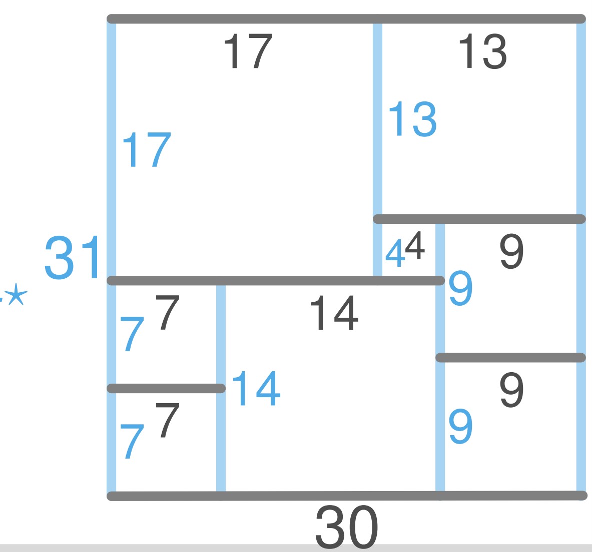
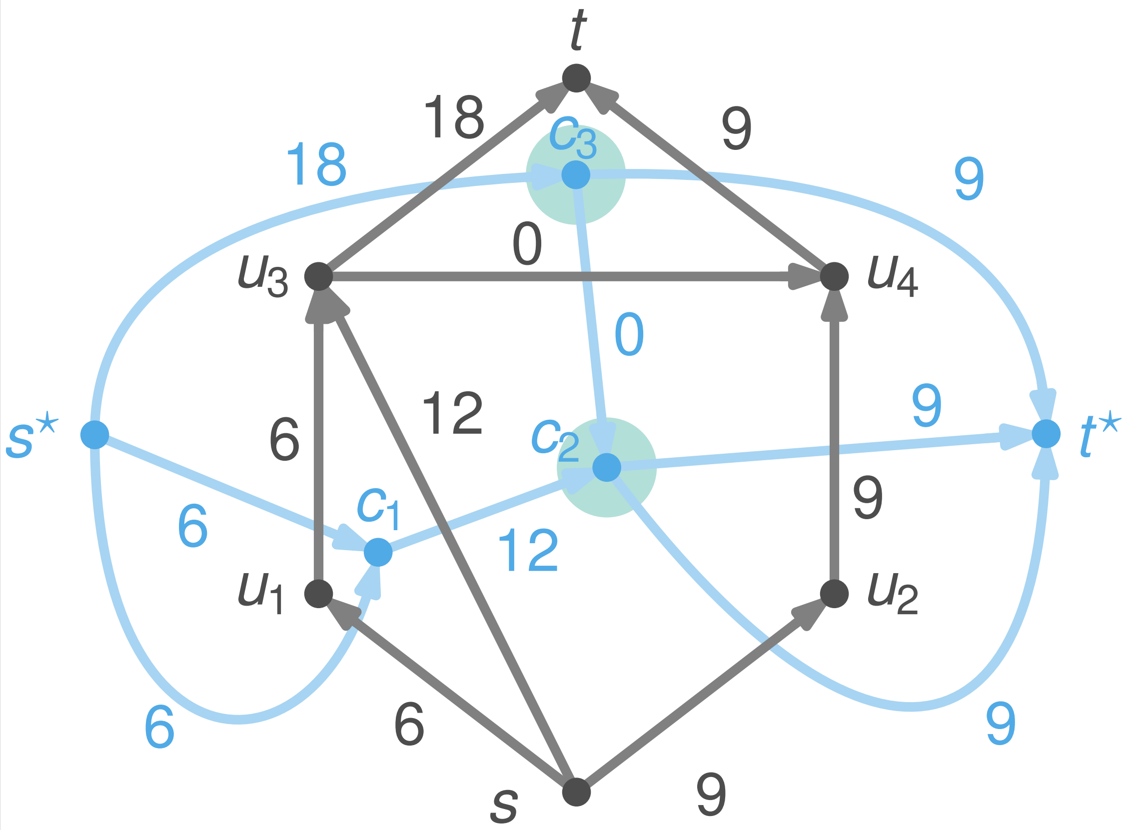
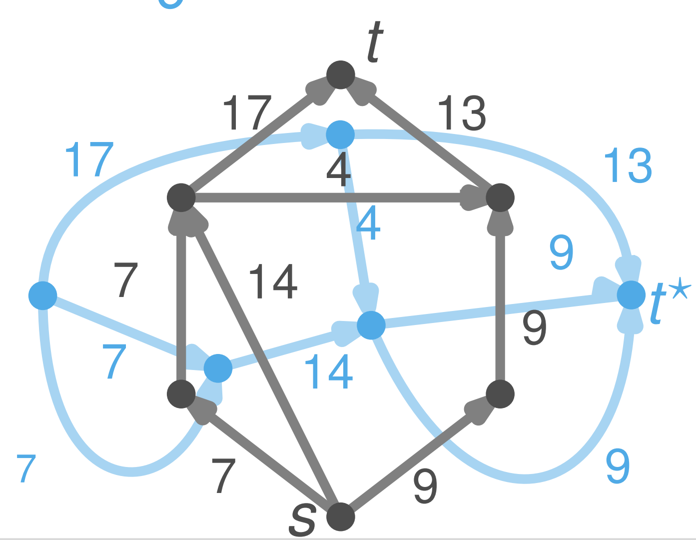


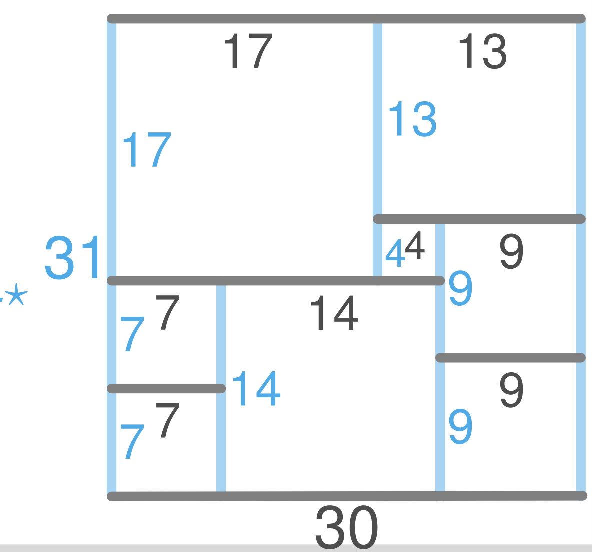




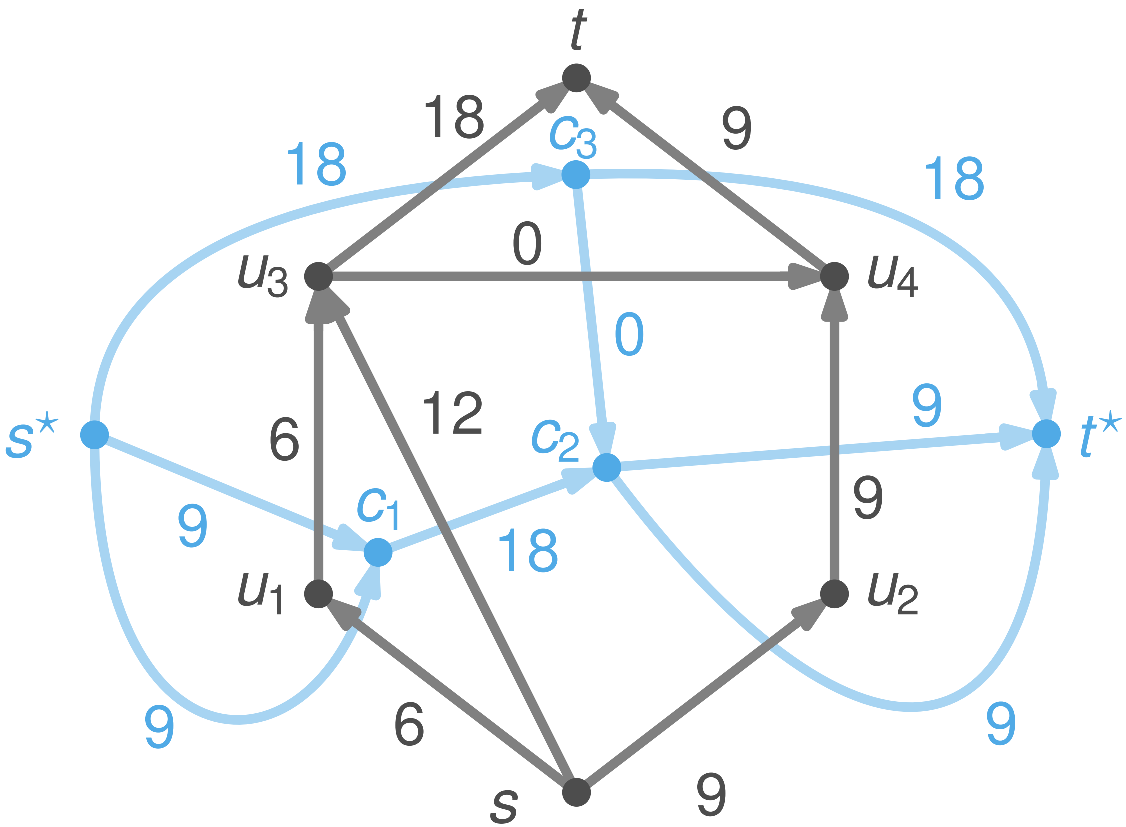
Wrong Conflict Resolution

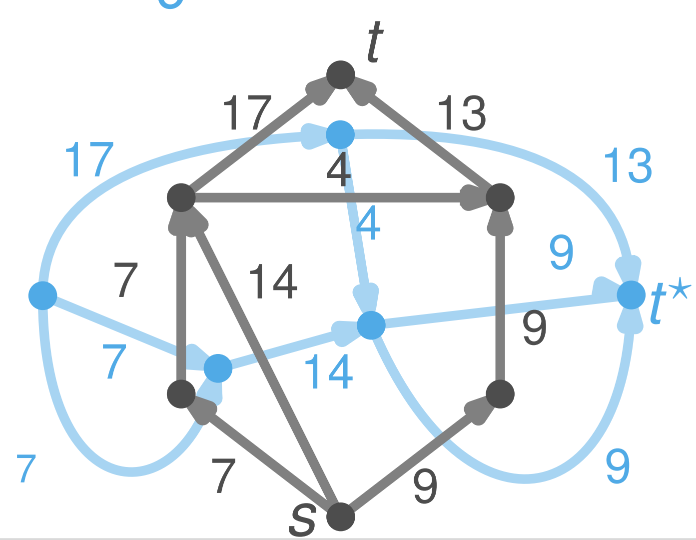


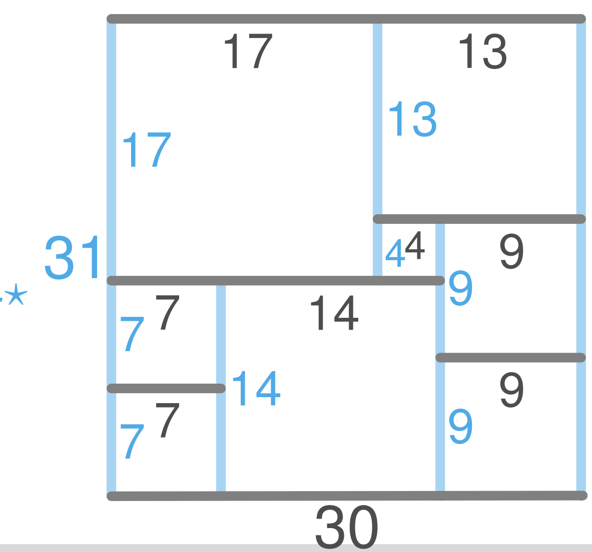




Wrong Conflict Resolution







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Wrong Conflict Resolution

Observation 6

A wrong conflict resolution might never lead to a feasible power flow.

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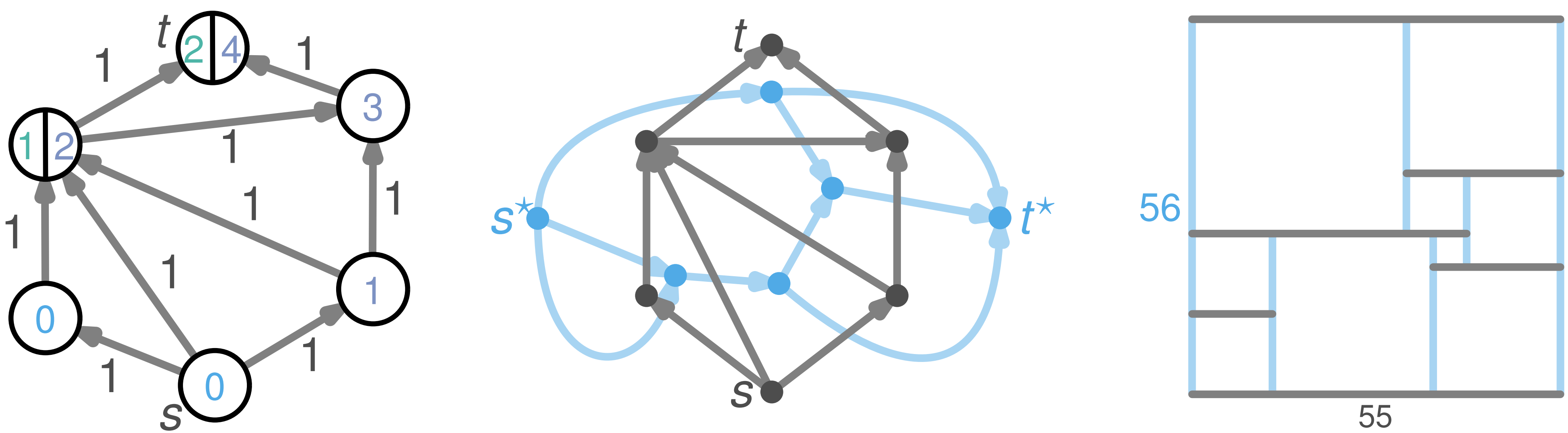
The Property of Balancing



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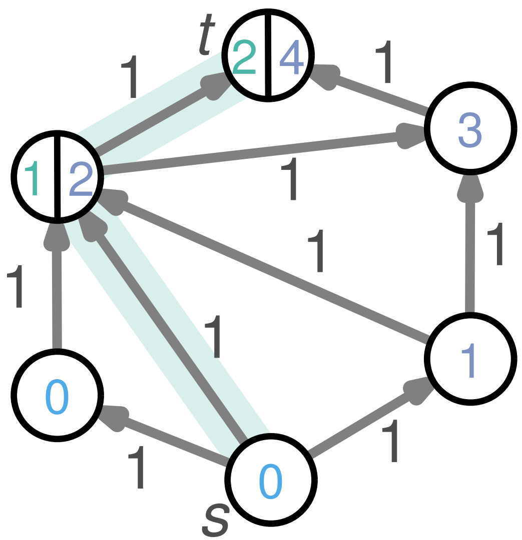
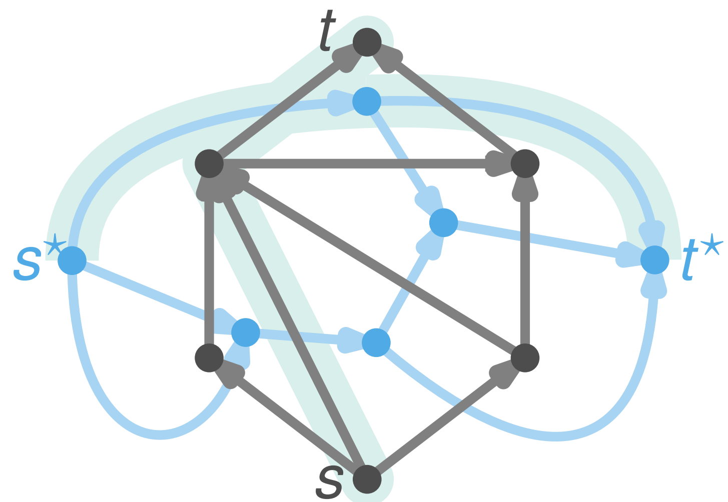
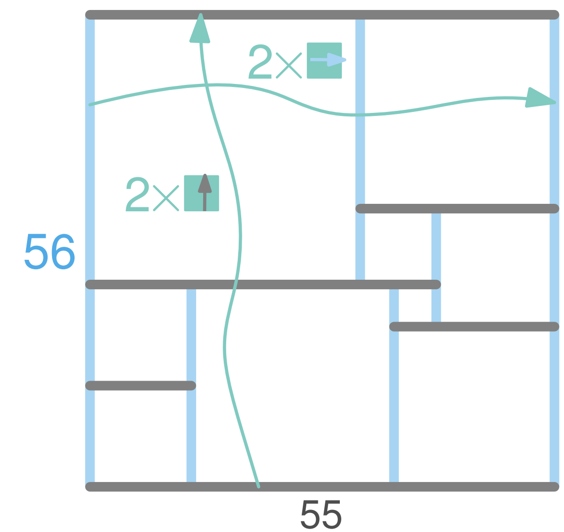
The Property of Balancing

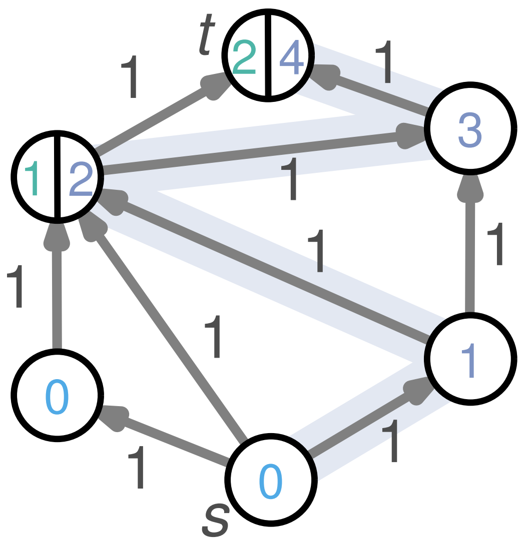


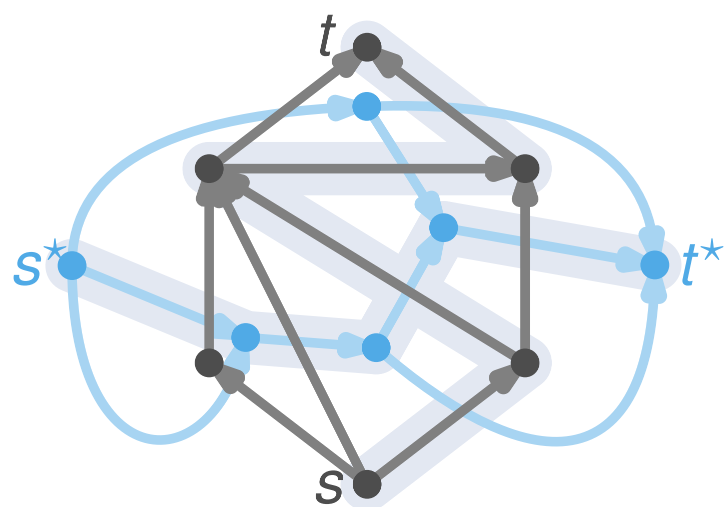


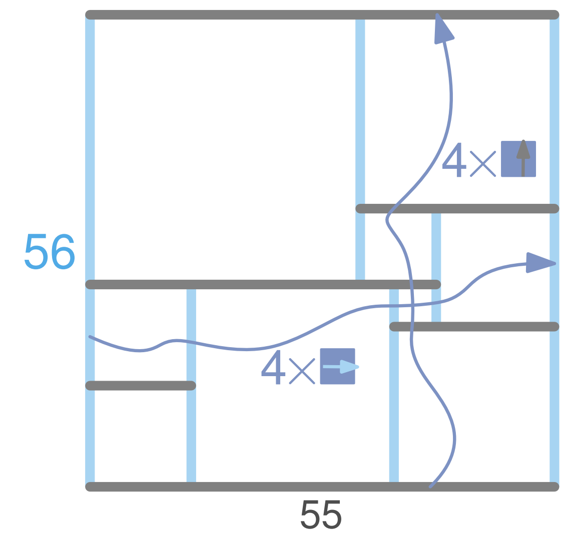


shortest path

The Property of Balancing







longest path

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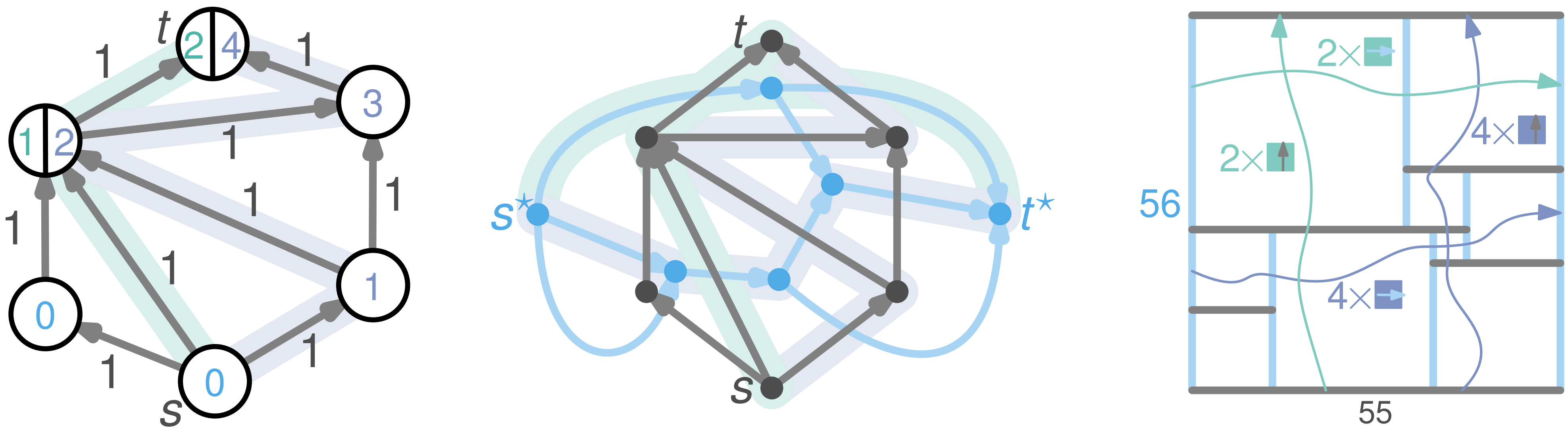
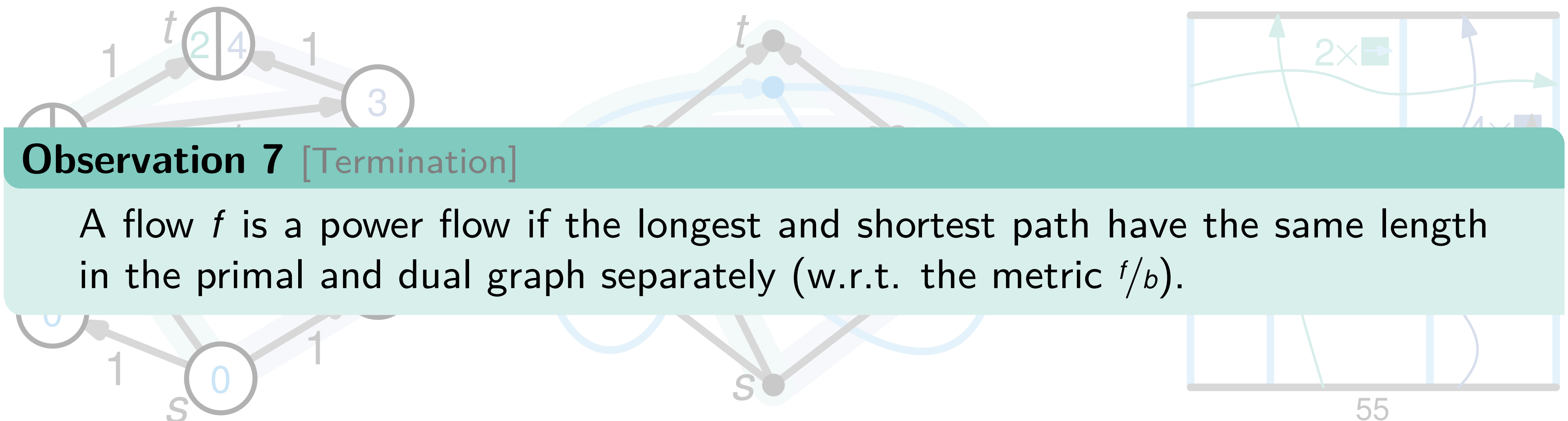
The Property of Balancing



The Property of Balancing



Observation 7 [Termination]

A flow f is a power flow if the longest and shortest path have the same length in the primal and dual graph separately (w.r.t. the metric ${}^{f}/_{b}$).

shortest path

longest path

Scalability of Power Flows

- Graph-theoretical flow algorithms use scaling techniques
    1. Capacity scaling [Edmonds and Karp, 1972]
    2. Excess scaling [Ahuja and Orlin, 1989]
- Power flows excluding a trivial power flow ($f \equiv 0$) can be scaled up and down by a factor χ

Lemma 8 [Scaling]

Every non-zero electrical flow $f' \colon E \to \mathbb{R}_{>0}$ can be rescaled to a new feasible electrical flow f by applying a scaling factor

$$0 \leq \chi \leq \min_{e' \in E} \frac{\mathrm{cap}(e')}{f'(e')} =: \overline{\chi} \tag{1}$$

to $f(e) = f'(e) \cdot \chi$ for all $e \in E$.

Continous Changes to the Power Grid

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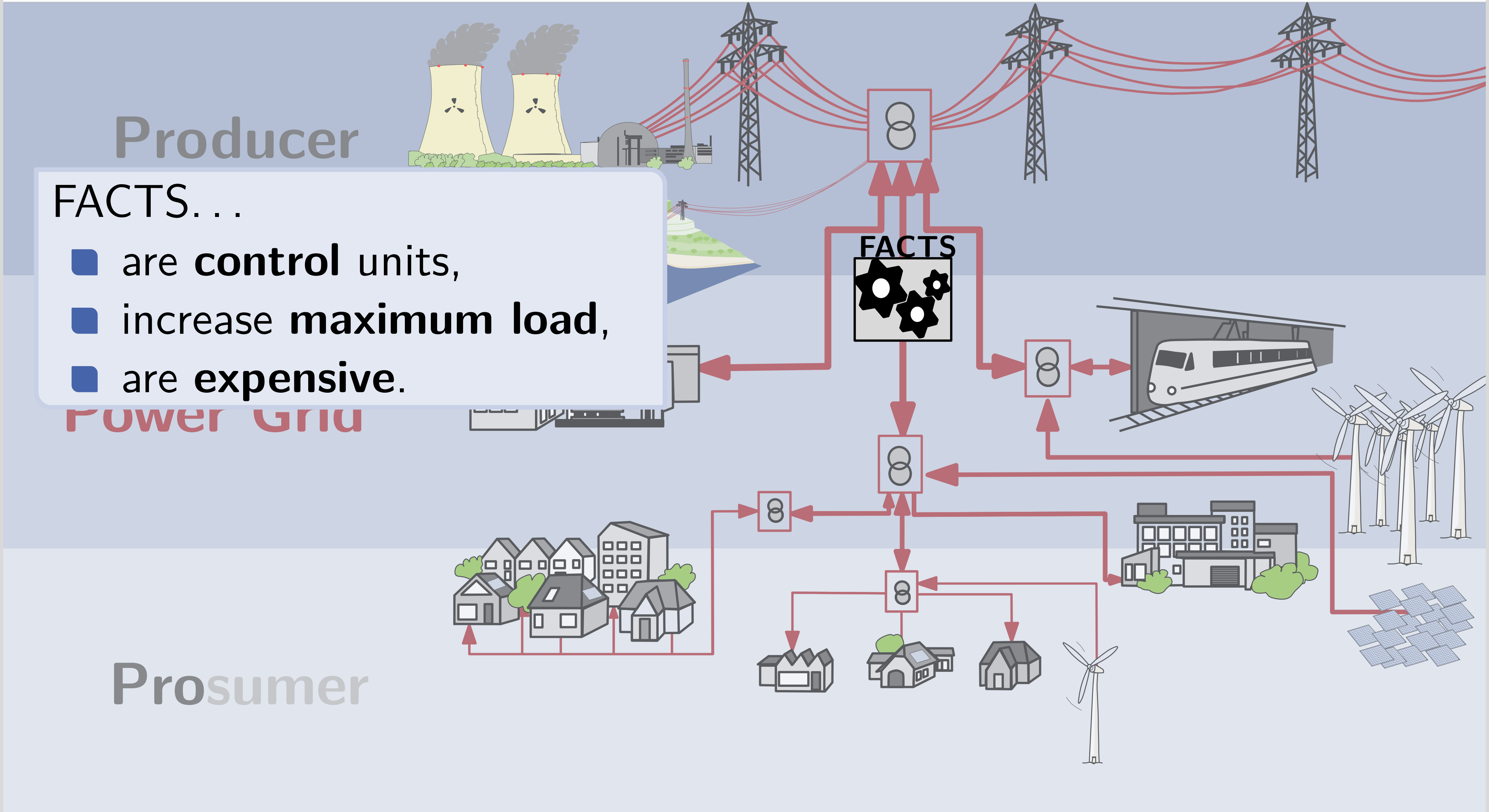
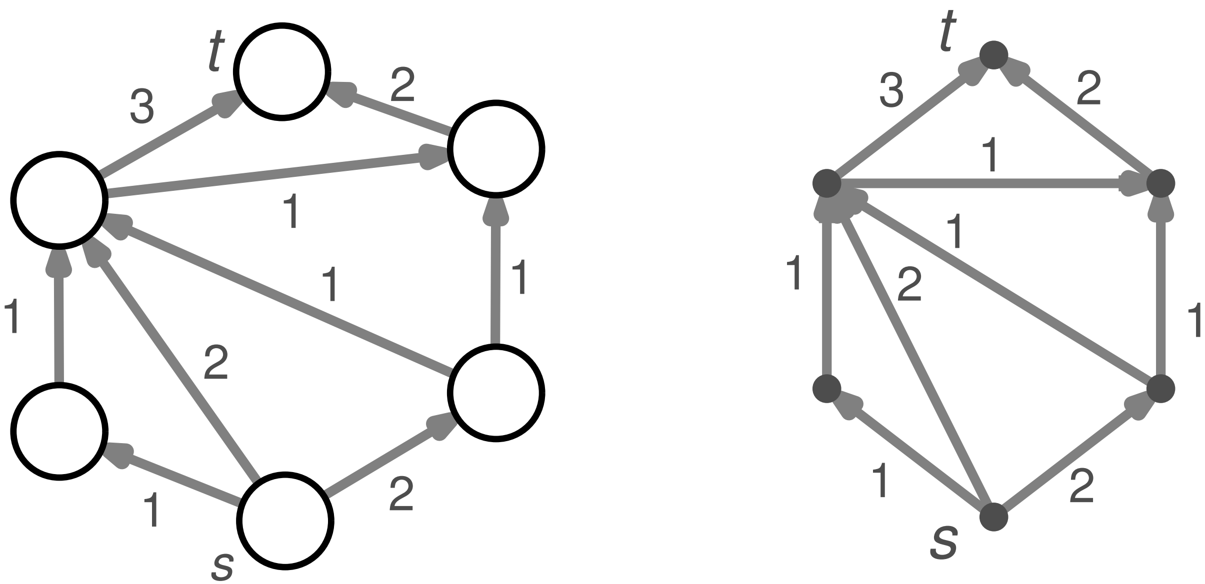
FACTS. . .

- are **control** units,
- increase **maximum load**,
- are **expensive**.



Susceptance Scaling

- Apply a feasible flow



Observation 9 [KCL and KVL Duality]

KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^{\star}$

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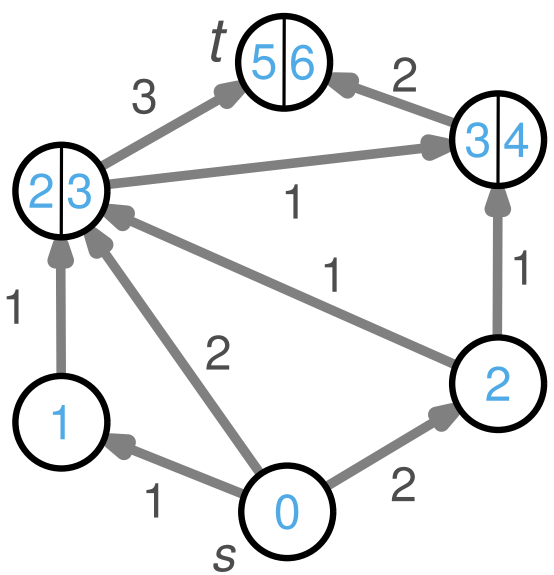
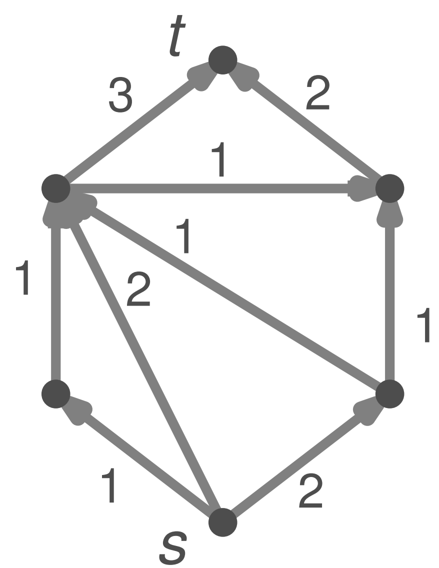
Susceptance Scaling

- Apply a feasible flow



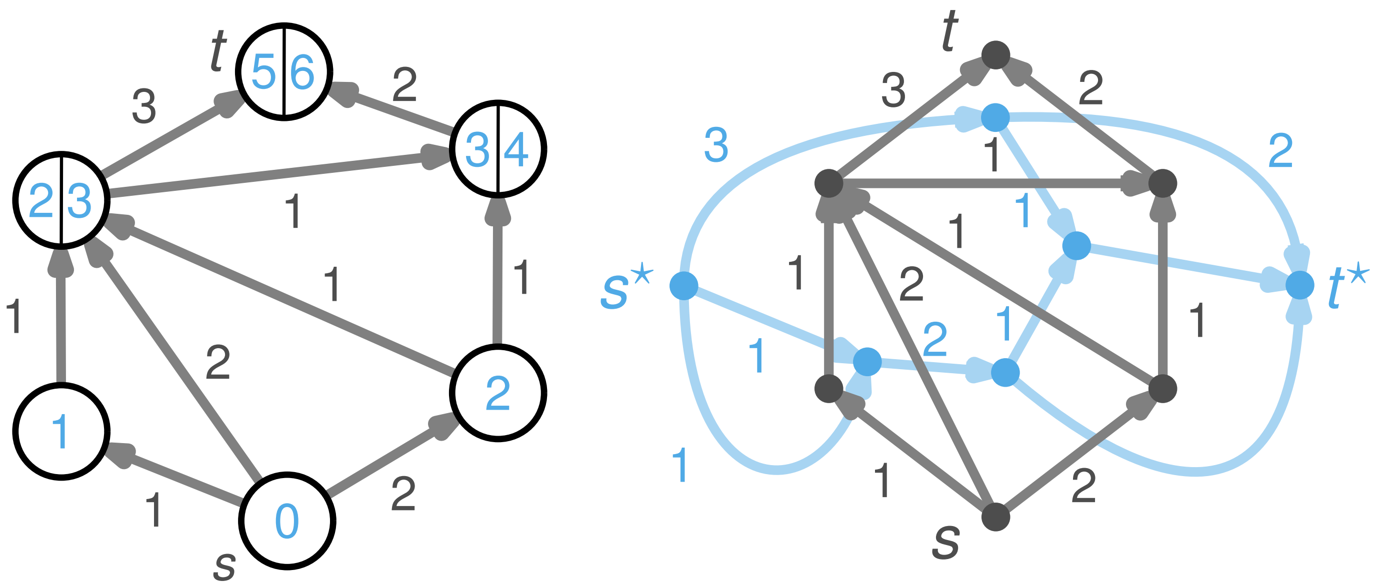


Observation 9 [KCL and KVL Duality]

KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^{\star}$

Susceptance Scaling

- Apply a feasible flow
- Can we make it electrical feasible?



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KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^\star$

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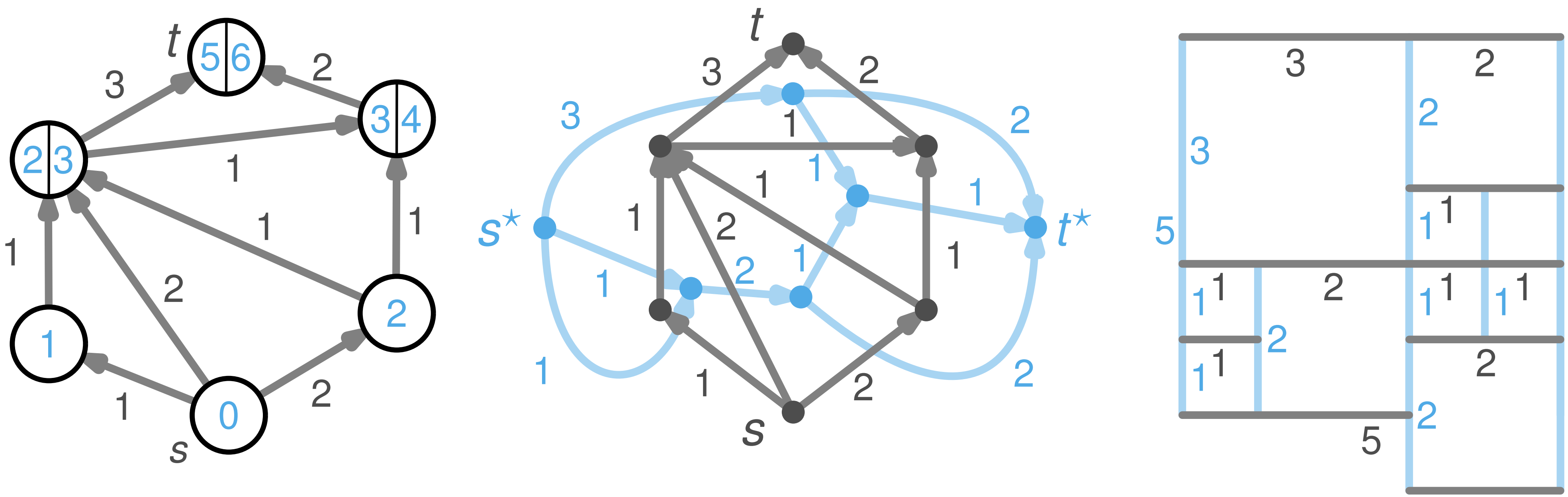

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Susceptance Scaling

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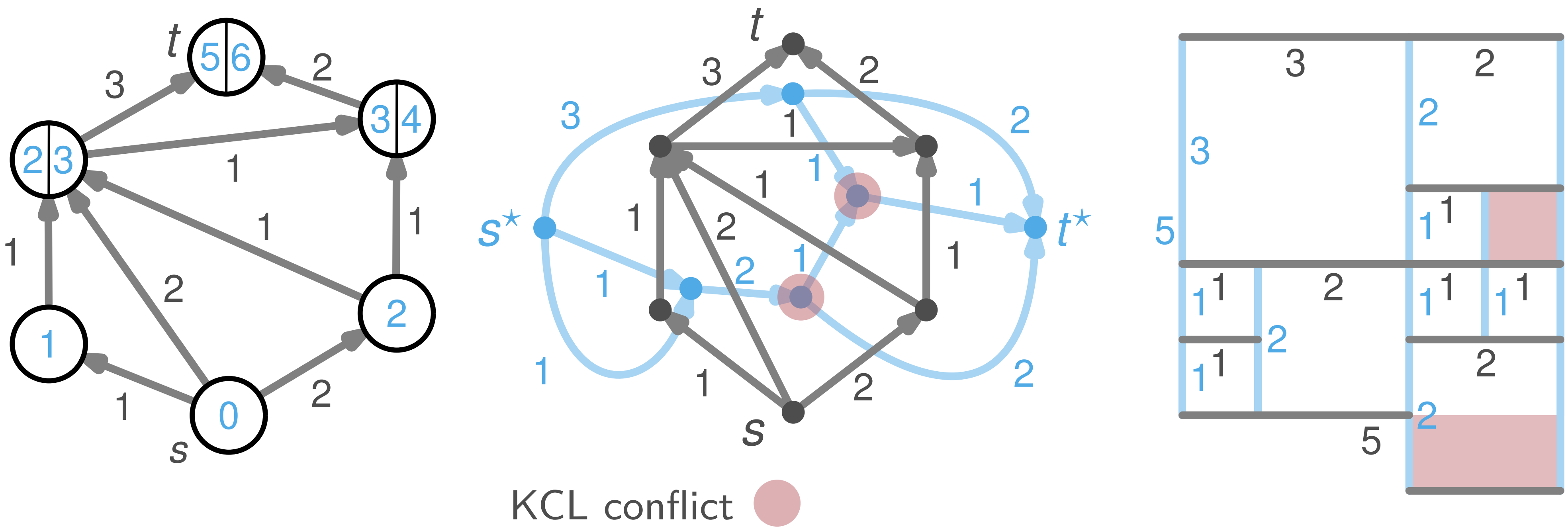

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Susceptance Scaling

- Apply a feasible flow
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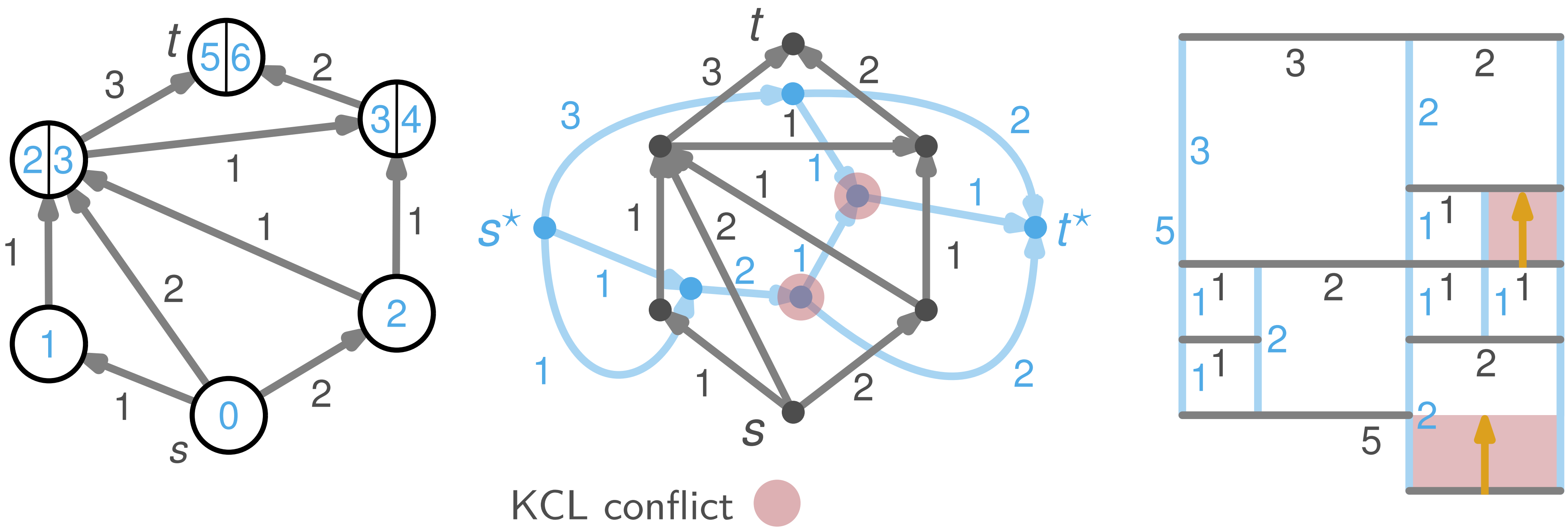

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Susceptance Scaling

- Apply a feasible flow
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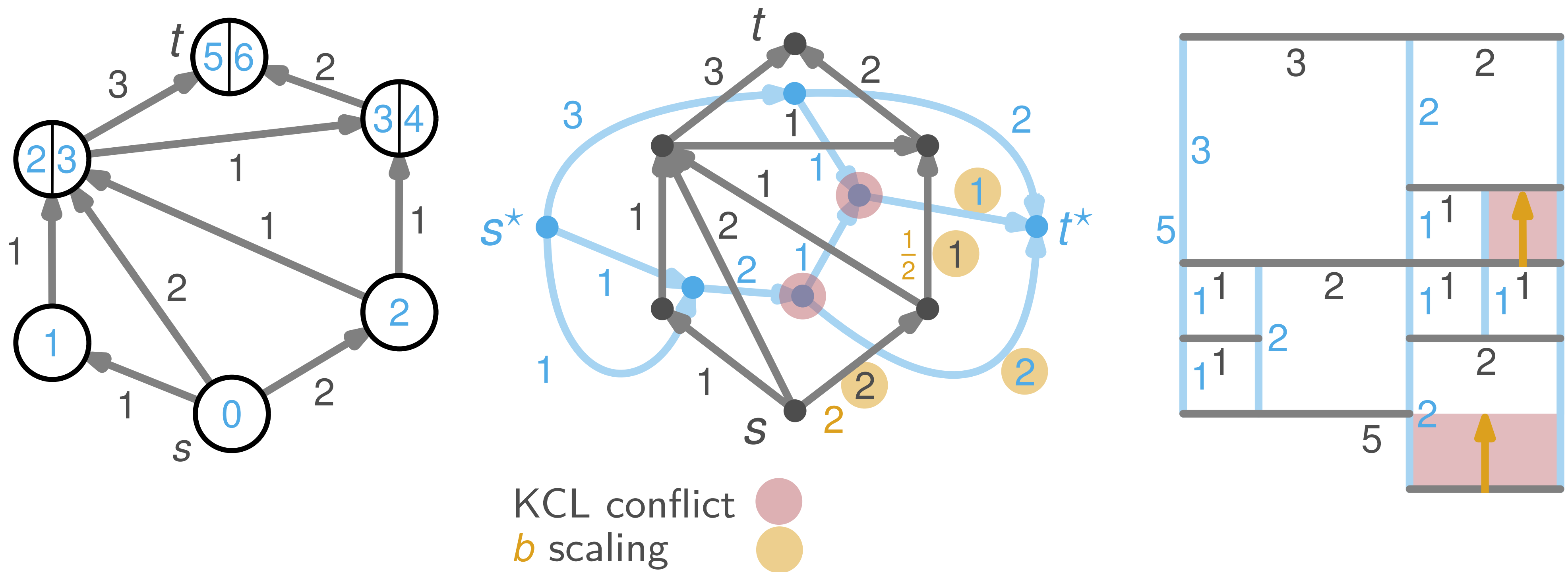
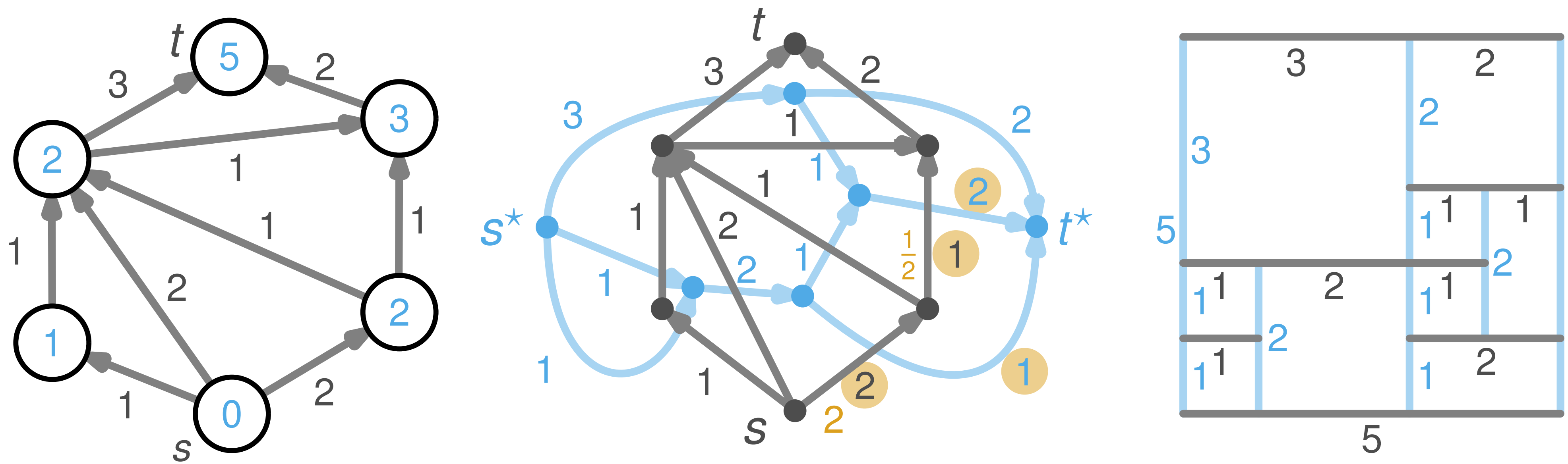


Observation 9 [KCL and KVL Duality]

KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^{\star}$

Susceptance Scaling

- Apply a feasible flow
- Can we make it electrical feasible? ✓



b scaling

Observation 9 [KCL and KVL Duality]

KVL conflict in the primal graph G is a KCL conflict in the dual graph $G^\star$

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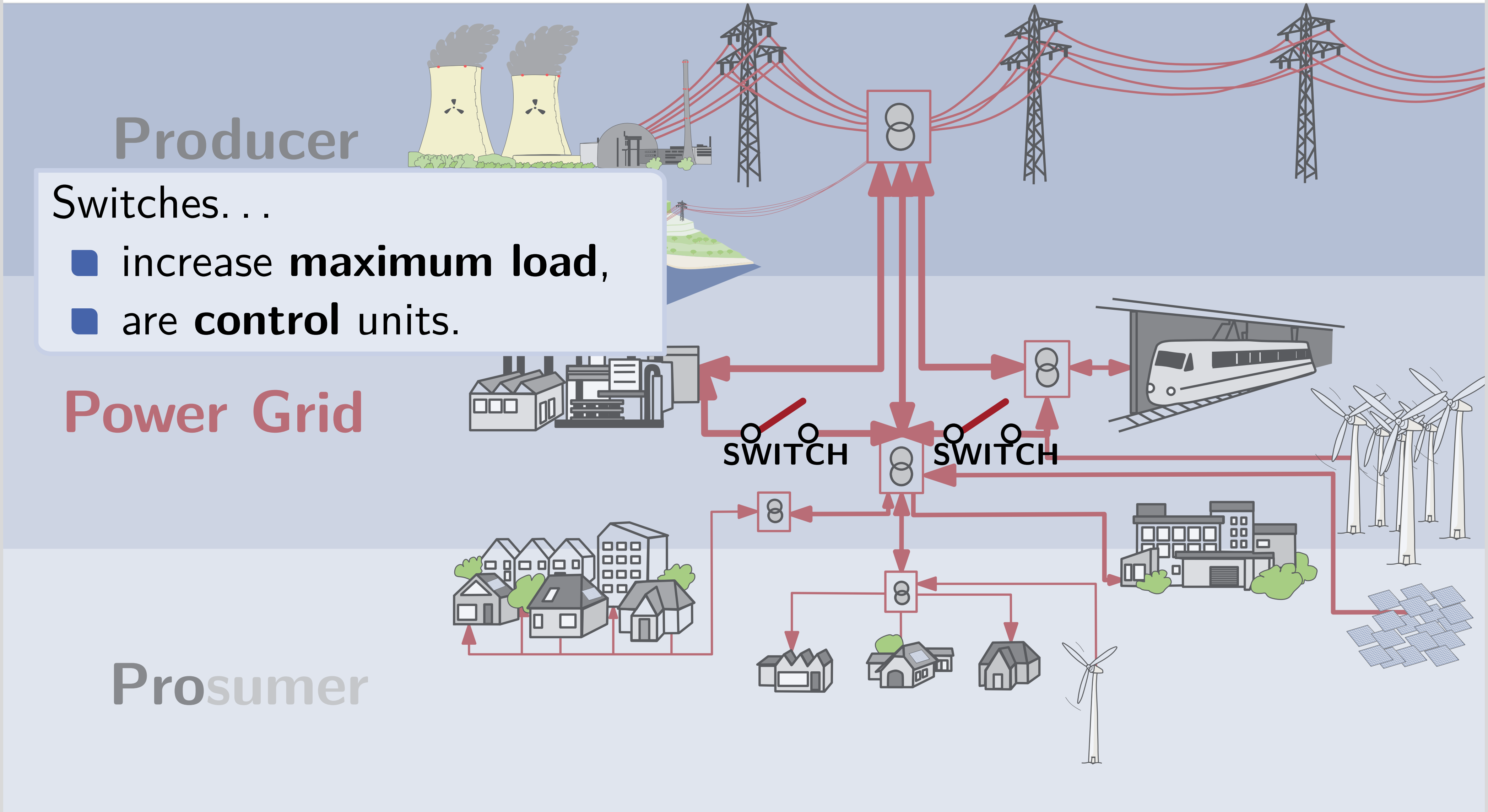
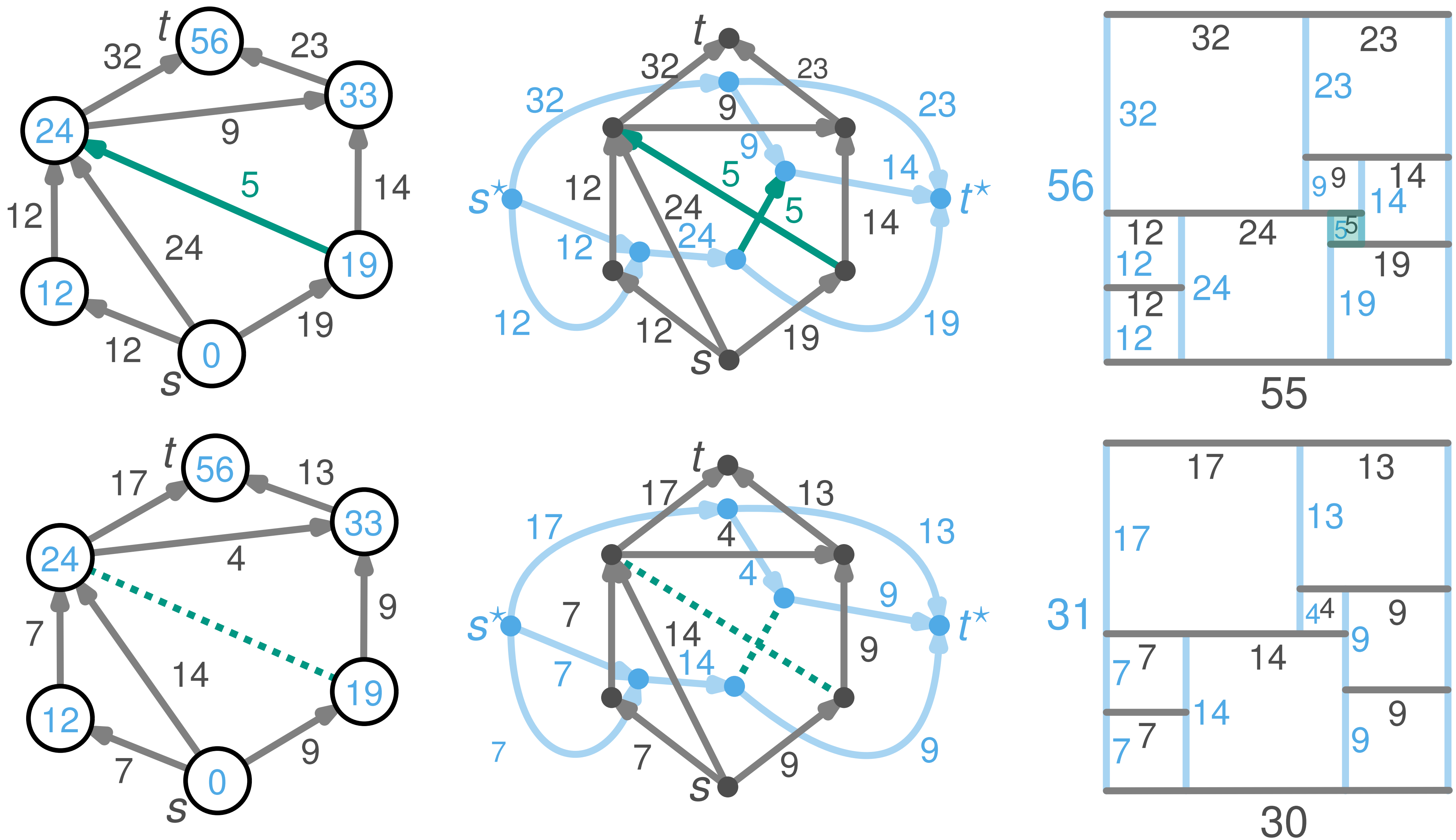
Discrete Changes to the Power Grid



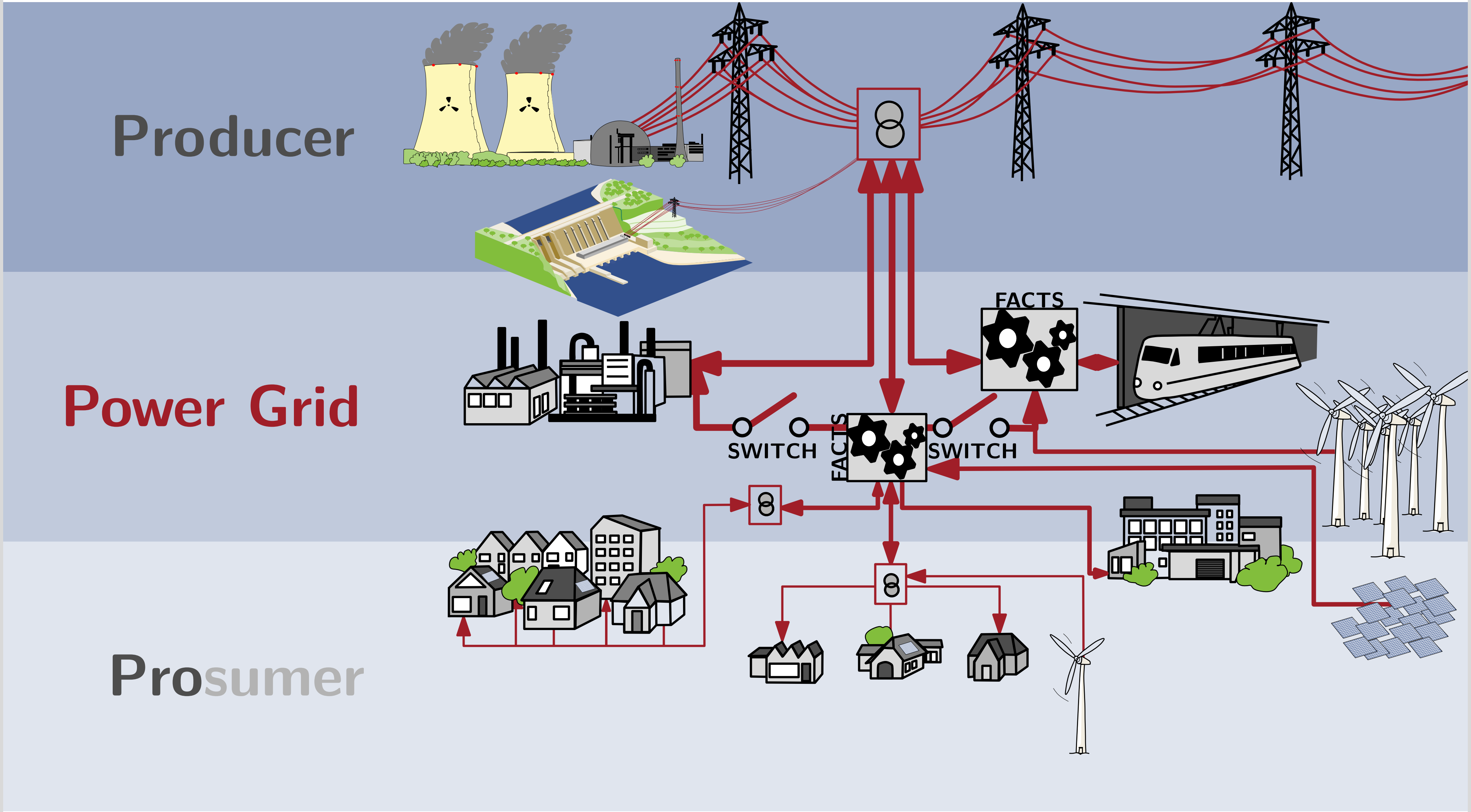
Switches. . .

- increase **maximum load**,
- are **control** units.

Switching



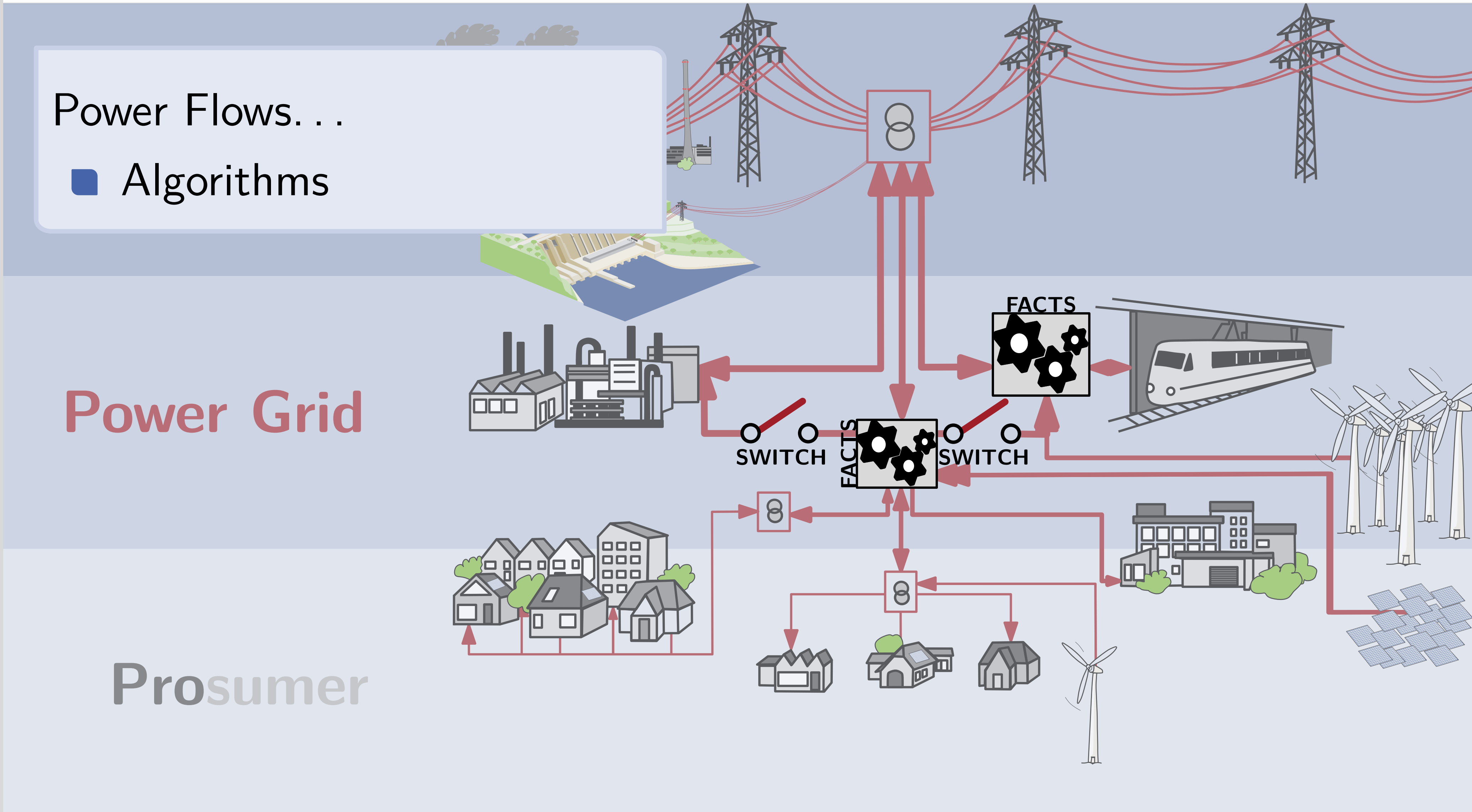
Summary



Summary

Power Flows...

- Algorithms



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Summary

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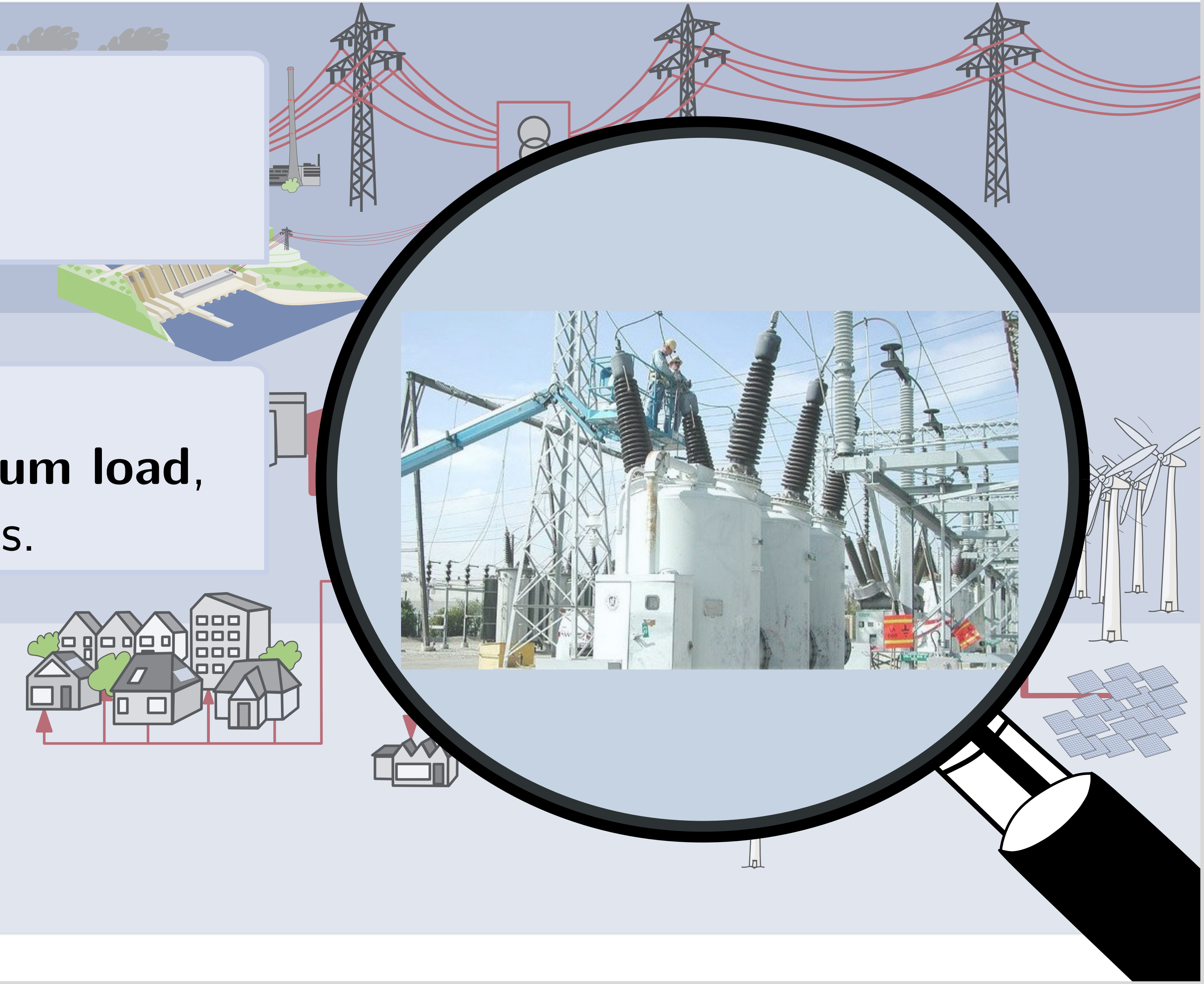
Power Flows. . .

- Algorithms

Switches. . .

- increase **maximum load**,
- are **control** units.



Prosumer

Summary

Power Flows...

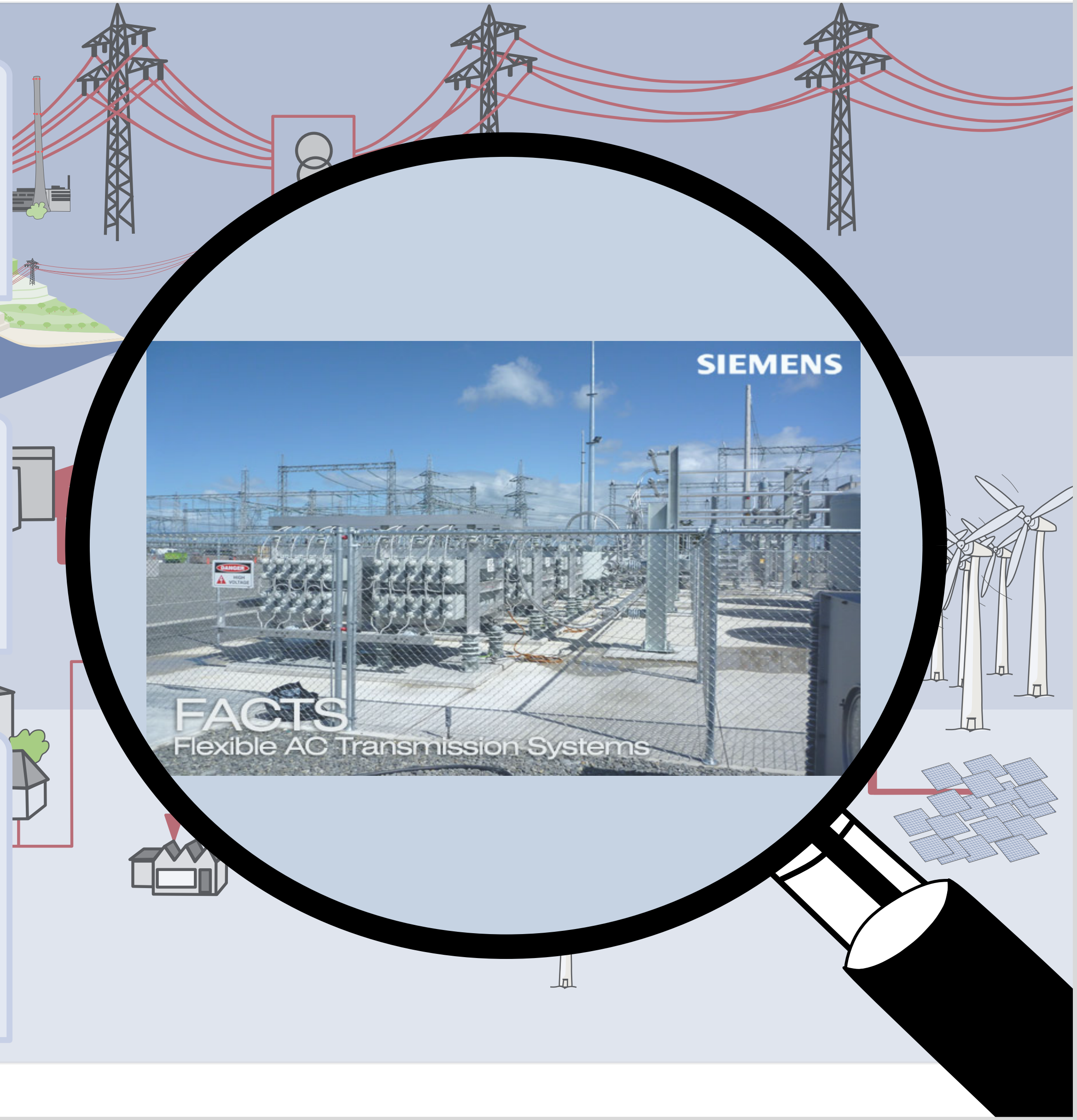
- Algorithms

Switches...

- increase **maximum load**,
- are **control** units.

FACTS...

- increase **maximum load**,
- are **control** units,
- are **expensive**.



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Guido Brückner

Dorothea Wagner

References

1. Wilhelm Cauer. *Théorie der Linearen Wechselstromschaltungen.* Berlin: Akademie Verlag, 1954. English translation “Theory of Linear AC Circuits”, New York: McGraw-Hill, 1958.
2. Ronald Martin Foster. *Topologie and Algebraic Considerations in Network Synthesis.* Proceedings in Polytechnic Institute of Brooklyn Symposium on Modern Network Synthesis. I: 8–18. 1952.
3. Stefan Felsner. *Rectangle and Square Representations of Planar Graphs.* Thirty Essays on Geometric Graph Theory. 213–248. 2013.
4. Ernst Adolph Guillemin. *Introductory Circuit Theory.* New York: Wiley, 1953.
5. Gustav Kirchhoff. *Über die Auflösung der Gleichungen, auf welche man bei der Untersuchungen der Linearen Verteilung Galvanisher Ströme geführt wird.* Poggendorf Annalen der Physik, 72: 497–508. DOI: 10.1002/andp.18471481202, 1847. English translation, Transactions of the Institute of Radio Engineers **CT–5**, 4–7.1958.
6. S. Seshu, and M. B. Reed. *On Cut Sets of Electrical Networks.* Proceedings 2nd Midwest Symposium on Circuit Theory, Michigan State. 1.1–1.13. 1956.
7. Hassler Whitney. *Non-seperable and Planar Graphs.* Transactions of the American Mathematical Society, 34: 339–362, 1932.
8. Hassler Whitney. *2-Isomorphic Graphs.* American Journal of Mathematics, 55: 245–254 (1933).
9. Hassler Whitney. *On the Abstract Properties of Linear Dependence.* American Journal of Mathematics, 57: 509–533 (1935).
10. C. Kuratowski. *Sur le Problème des Courbes Gauches en Topologie.* Fund. Math., 15: 271–283 (1930).