Karlsruhe Institute of Technology

HELMHOLTZ
RESEARCH FOR GRAND CHALLENGES

SCI / RU1 Networks and System Integration

The Maximum Transmission Switching Flow Problem

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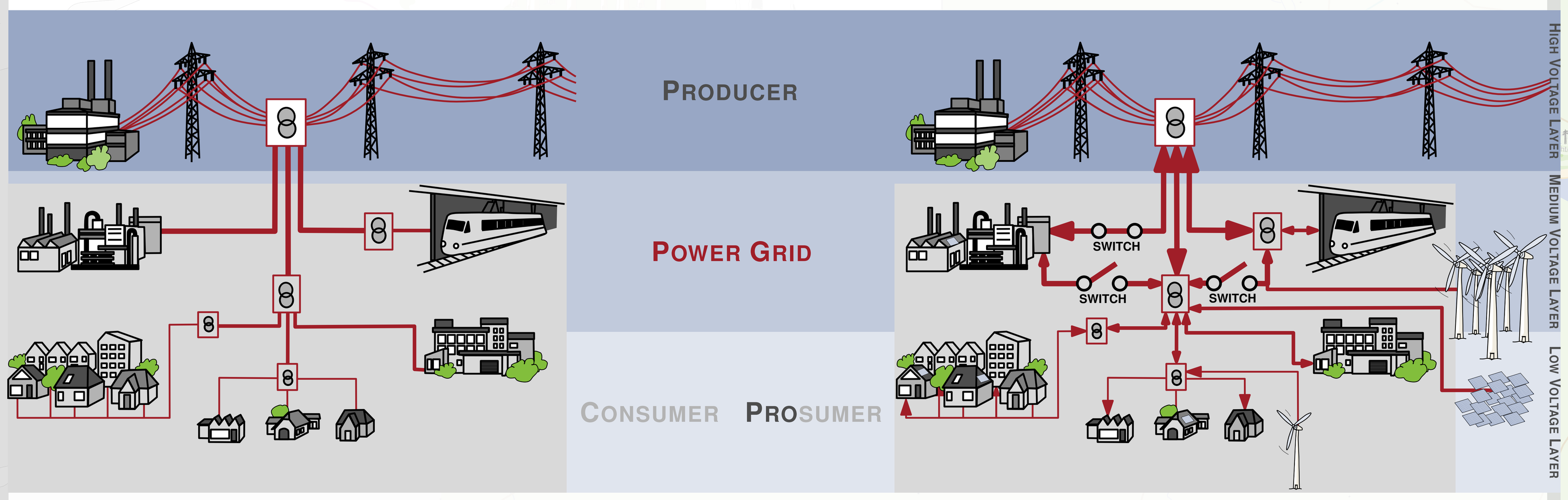
Operation of Power Grids [2]



Challenges

- Shift towards renewable distributed energy production,
  - → Changing energy flow patterns,
- Independent power producers,
- Volatile power flows and flow directions,
- ⇒ Operation of the power grid becomes more demanding.

Strategies to cope with the challenges

1. Extend the grid with additional transmission lines,
2. Install control units like circuit breakers (switches) to enhance grid utilization (our approach).

Source-Sink-Network

Idea

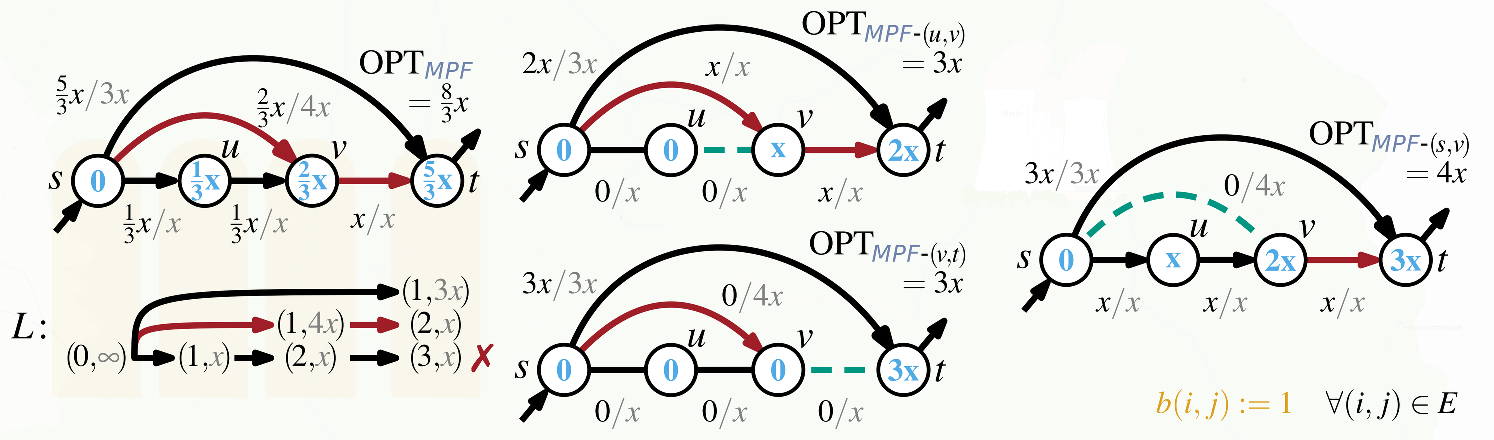
- *MTSF* on networks with one generator $s \in V_G$ and one load $t \in V_C$,
- Function that calculates the voltage angle difference on any u-v-path $\pi(u,v)$

$$\Delta\Theta(\pi(u,v)) := \sum_{e\in\pi(u,v)} b(e)^{-1} \cdot \min_{(i,j)\in\pi(u,v)} \mathrm{cap}(i,j),$$

- s-t-networks are often dominated by a path having the smallest $\Delta\Theta(\pi(s,t))$

$$DTP(u,v) := \Delta\Theta_{\min}(u,v) := \min \Delta\Theta(\pi(u,v)),$$

- Calculated using modified *multi-criteria shortest-path algorithm.*



On **general networks** the *switching centrality* $c_S\colon E \to \mathbb{R}_{\geq 0}$ is defined by

$$c_S(e) := \frac{1}{m_B} \sum_{s\in V} \sum_{t\in V\setminus\{s\}} \frac{\sigma_{DTP}(s,t,e)}{\sigma_{DTP}(s,t)},$$

where $\sigma_{DTP}(s,t,e)$ is the number of *DTP*-paths between s and t that use e, $\sigma_{DTP}(s,t)$ is the total number of *DTP*-paths from s to t and $m_B = |V|(|V|-1)$.

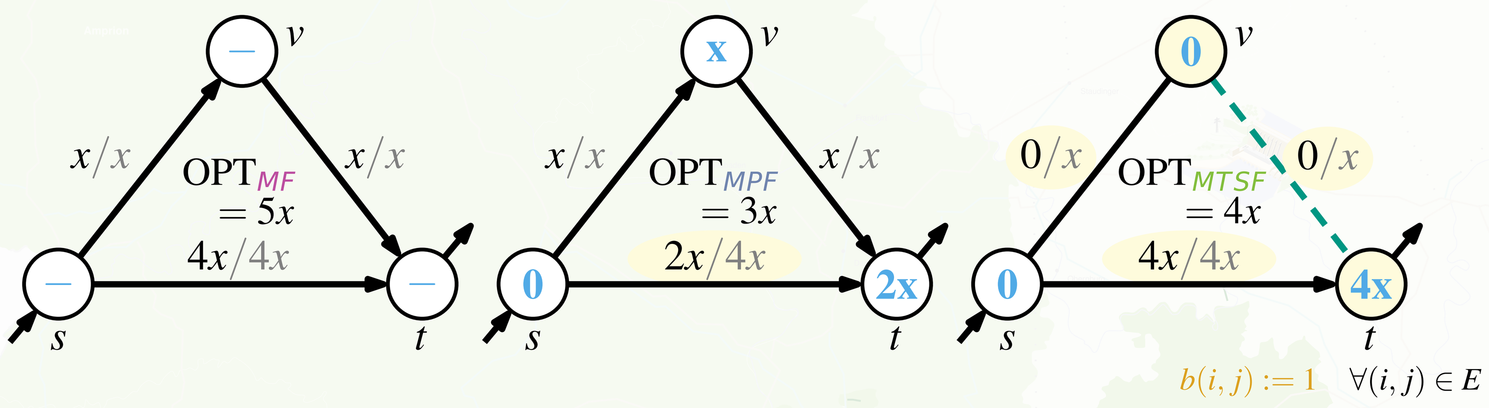
Problem Statement [2, 3]

Goal

- Maximize the throughput of the network.

Input

- Network $N := (G = (V,E), V_G, V_C, \mathrm{cap}, b)$ representing power grid with buses, lines, generators $s \in V_G$, loads $t \in V_C$, capacities and susceptances,
- Set $S \subseteq E$ of switched edges (circuit breakers),
- **Braess' Paradox** meaning adding a line might increase the cost.



Models

- Maximum Power Flow (*MPF*) assumes $S = \emptyset$ and requires an *electrically feasible flow* obeying the conservation of flow, feasible thermal line limits, and voltage laws [2]. Latter is defined by

$$b(u,v)\cdot(\Theta(u) - \Theta(v) - \Theta_{\mathrm{shift}}(u,v)) = f(u,v) \qquad \forall (u,v) \in E,$$
$$\Theta_{\min}(u) \leq \Theta(u) \leq \Theta_{\max}(u) \qquad \forall u \in V.$$

- Maximum Transmission Switching Flow (*MTSF*) assumes $S \subseteq E$ and requires an electrically feasible flow on $N - S$ with $MTSF(N) := \max_{S\subseteq E} MPF(N-S)$

$$b(u,v)\cdot z(u,v)\cdot(\Theta(u) - \Theta(v)) = f(u,v) \qquad \forall (u,v) \in E,$$
$$|f(u,v)| \leq z(u,v)\cdot \mathrm{cap}(u,v) \qquad \forall (u,v) \in E,$$

where $z\colon E \to \{0,1\}$.

Find

- The lines to be switched and the voltage angles.

Maximum Spanning Tree

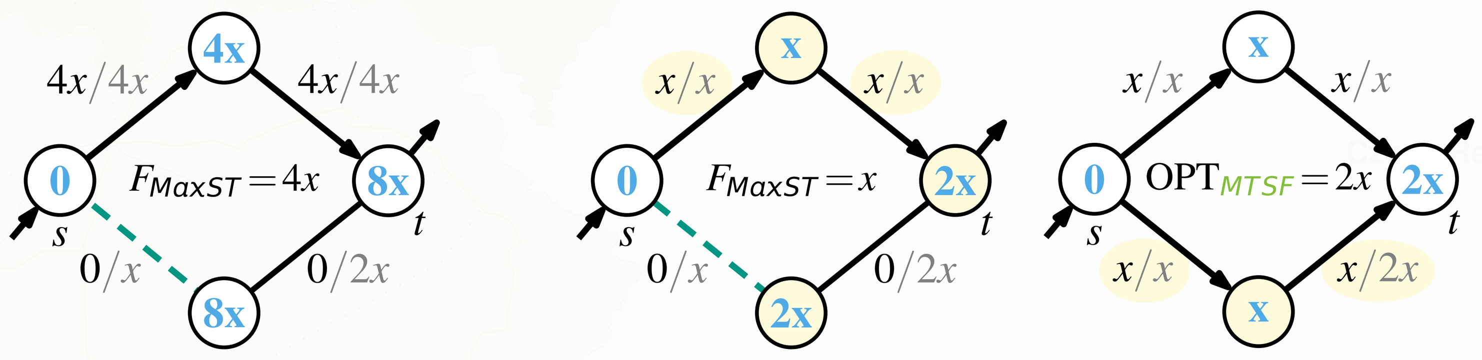
Description

The Maximum Spanning Tree (*MaxST*) removes from each cycle the line with the smallest capacity.

***MaxST* on Cacti**

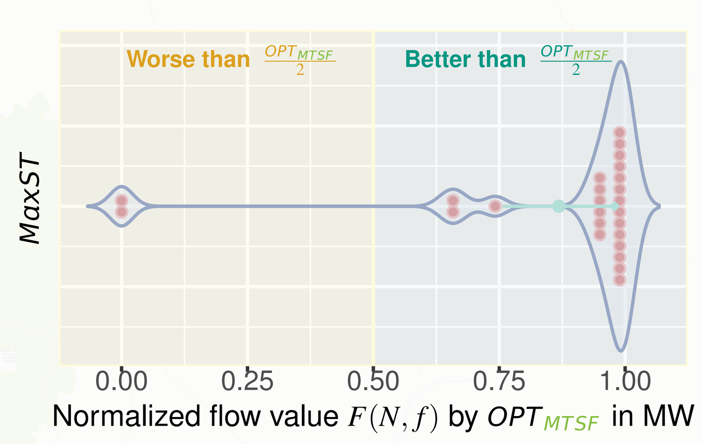
- Note that *MTSF* is *NP*-hard on cacti [4].

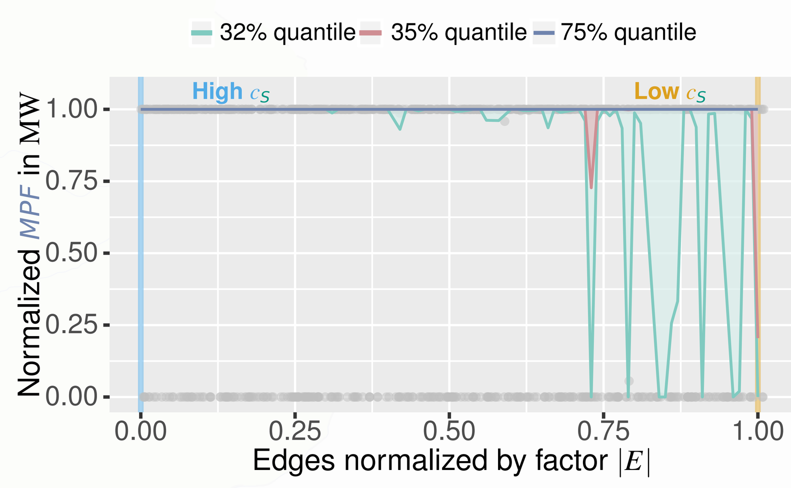
Theorem 1
MaxST is a factor 2 approximation algorithm for the *MTSF* problem on cacti.



Simulations on General Networks

- Simulations on NESTA benchmark sets that are more realistic than the IEEE benchmark sets, e.g., with regards to thermal line limits.





MaxST on **general graphs** is in most cases very close to the OPT_{MTSF}.

The *MPF* decreases mainly for lines having a small centrality c_S.

Future Work

- Can the *DTP* be computed in polynomial time?
- Is there a PTAS on cacti?
- What happens if we minimize the number of switches or fix a set of non-switchable edges?

Literature

[1] A. Grastien, I. Rutter, D. Wagner, F. Wegner, and M. Wolf. *The Maximum Transmission Switching Flow Problem.* ACM e-Energy. 340–360. 2018. Doi: 10.1145/3208903.3208910.

[2] S. Frank, I. Steponavice, and S. Rebennack. *Optimal power flow: a bibliographic survey I.* Energy Systems, 3(3):221–258, 2012.

[3] E. B. Fisher, R. P. O'Neill, and M. C. Ferris. *Optimal transmission switching.* IEEE Transactions on Power Systems, 23(3):1346–1355, 2008. ISSN 0885-8950. Doi: 10.1109/TPWRS.2008.922256.

[4] K. Lehmann, A. Grastien, and P. Van Hentenryck. *The complexity of DC-switching problems.* CoRR, abs/1411.4369, 2014.