The Maximum Transmission Switching Flow Problem

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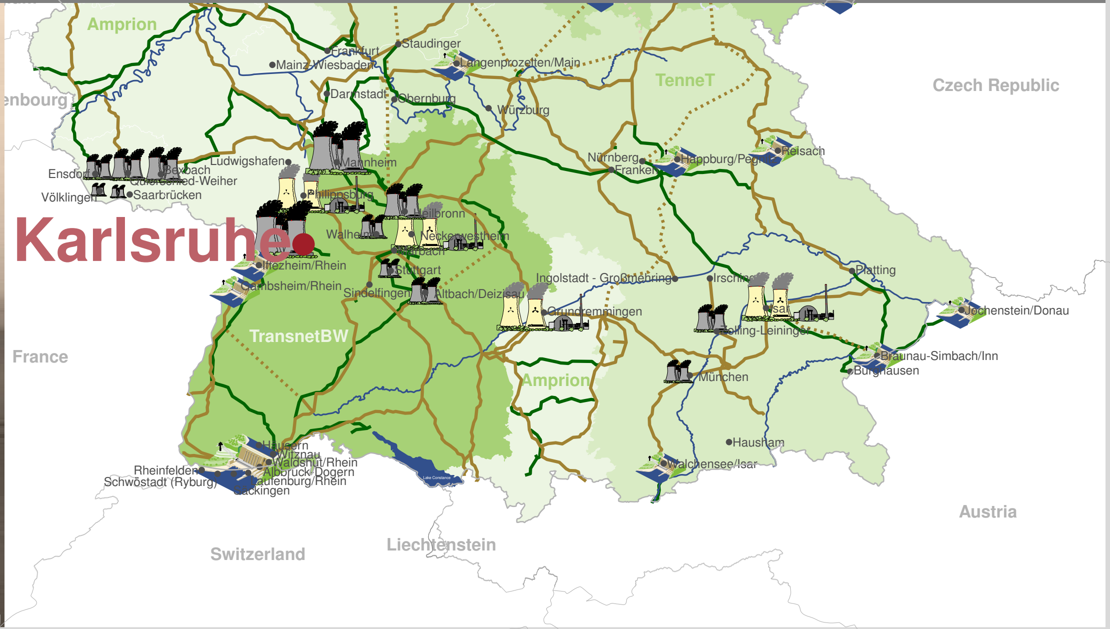
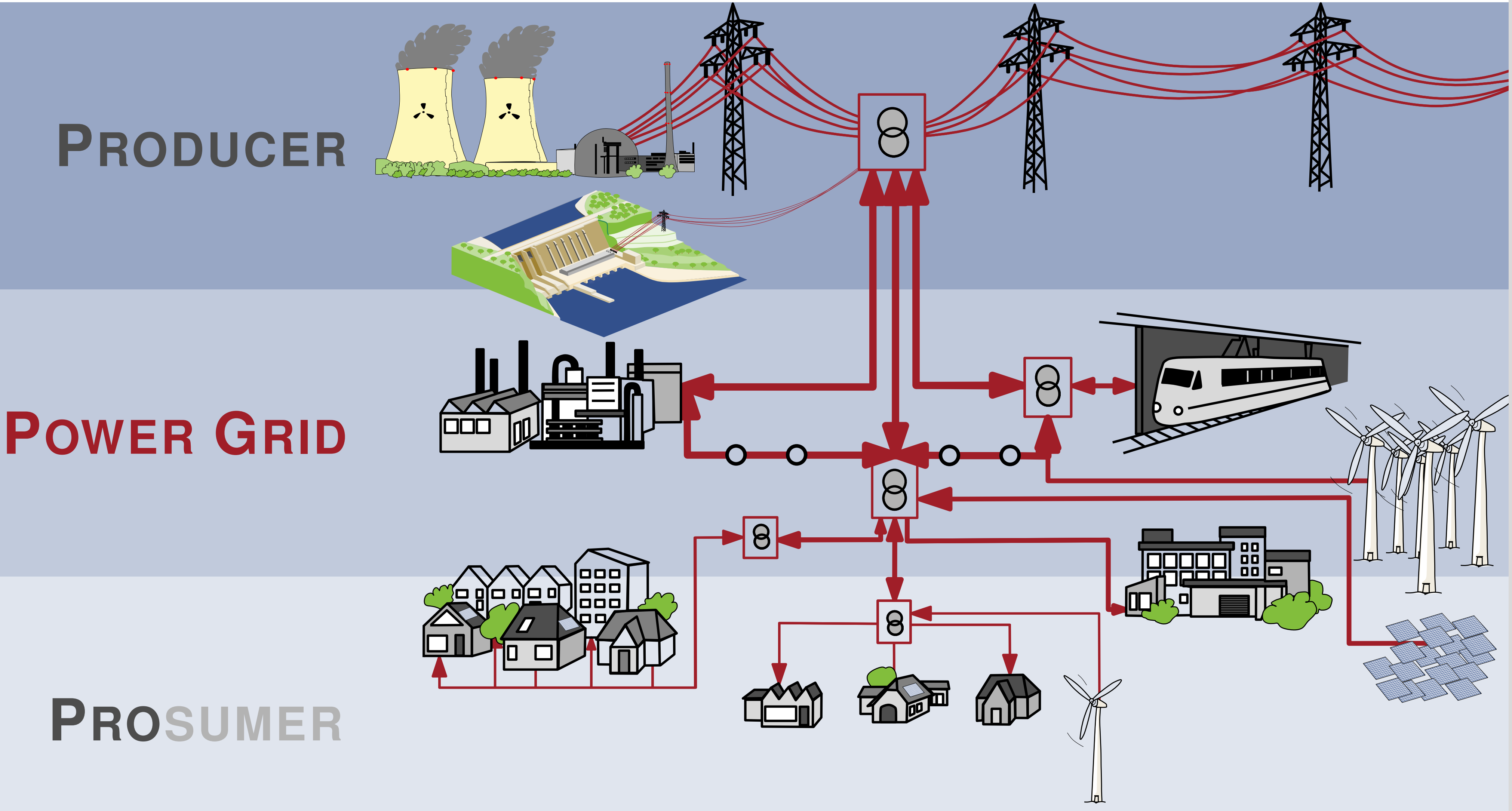
Alban Grastien, Ignaz Rutter, Dorothea Wagner, Franziska Wegner and Matthias Wolf

INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP

[1] http://img.welt.de/img/deutschland/crop124593297/5546936773-ci3x2l-w900/Strommasten.jpg



Recent Development in Power Grids



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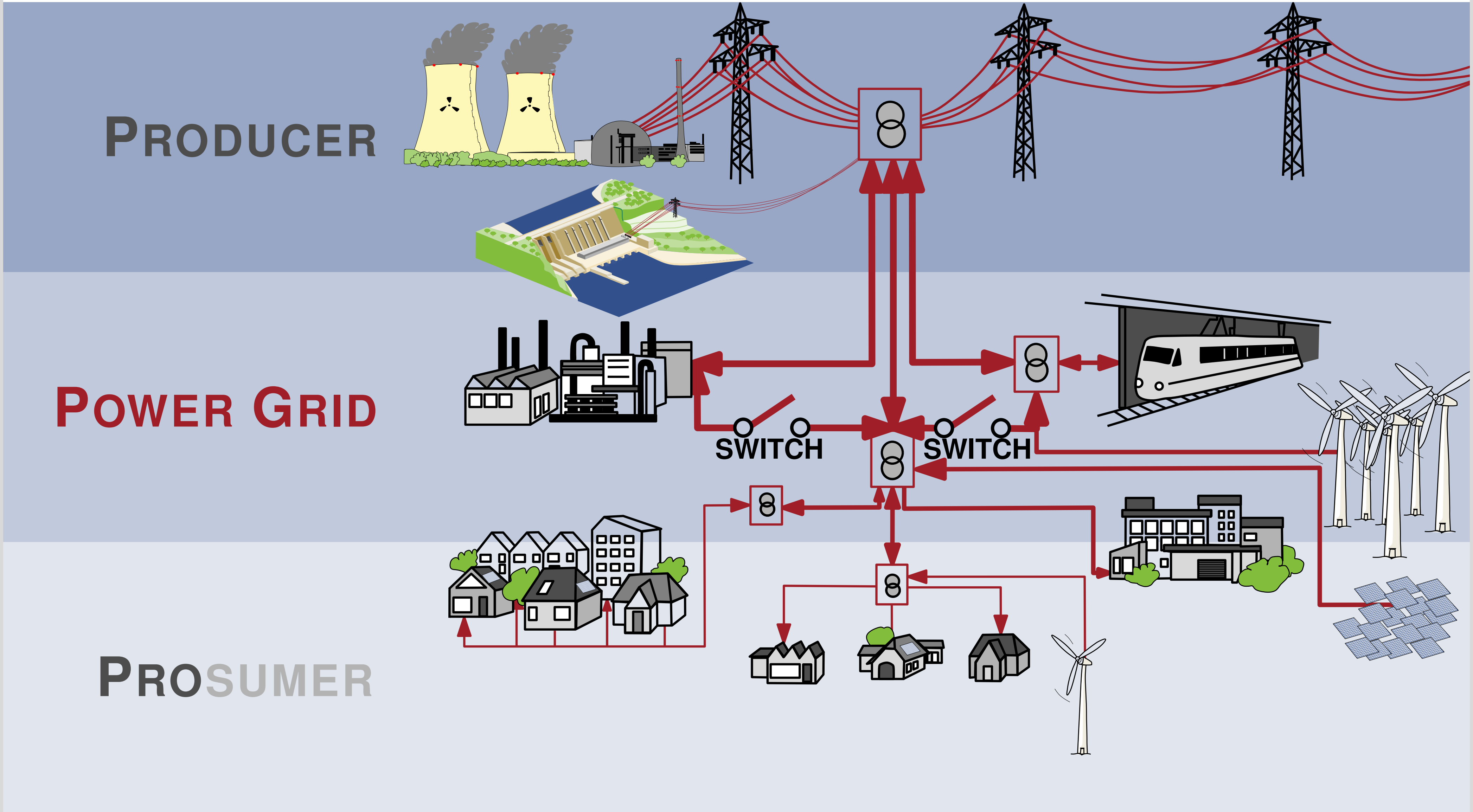
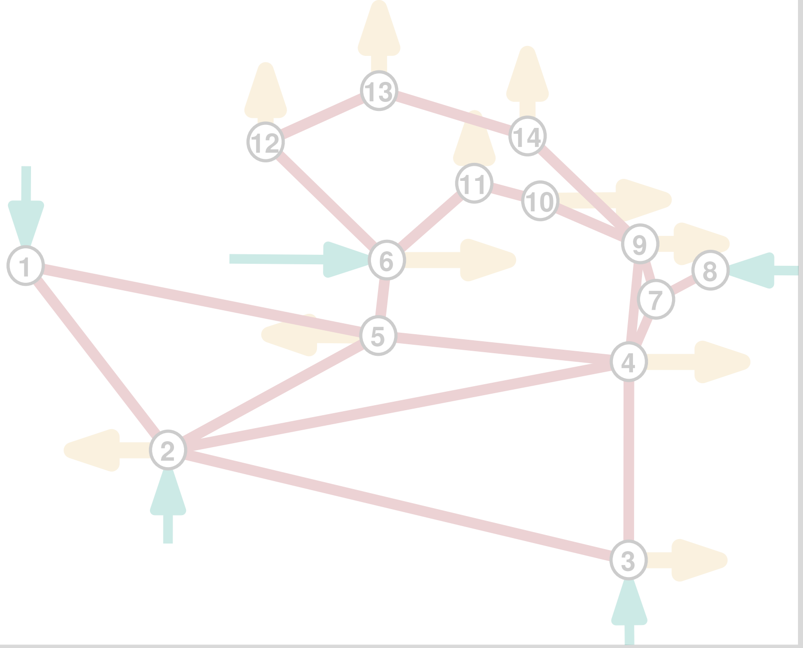


THE MAXIMUM TRANSMISSION SWITCHING FLOW PROBLEM

Given V set of buses, $V_L \subseteq V$ set of loads (with capacities),
$V_G \subseteq V$ set of generators (with capacities)
E set of lines (each with **impedance**, **susceptance**, **capacity**)

inputs



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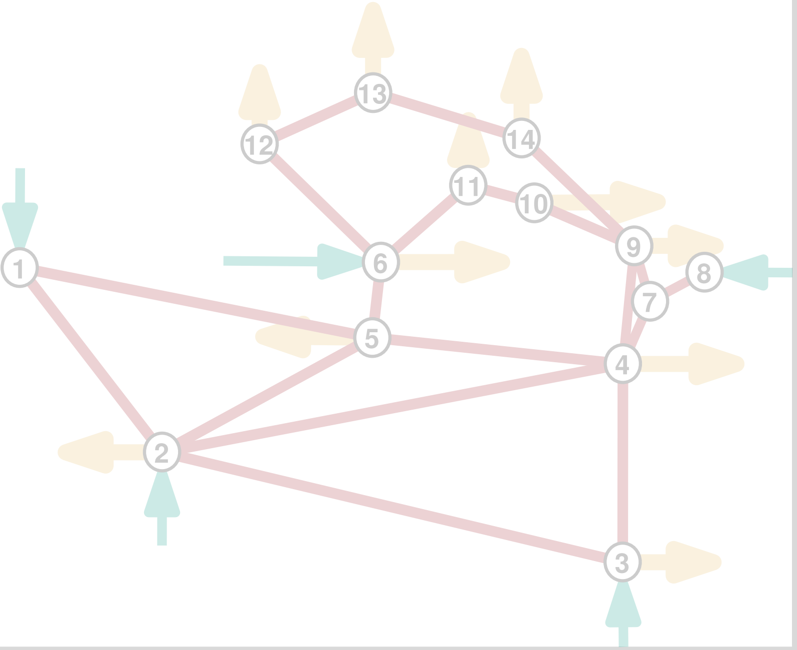
find for each line: **if the line is switched**

variables



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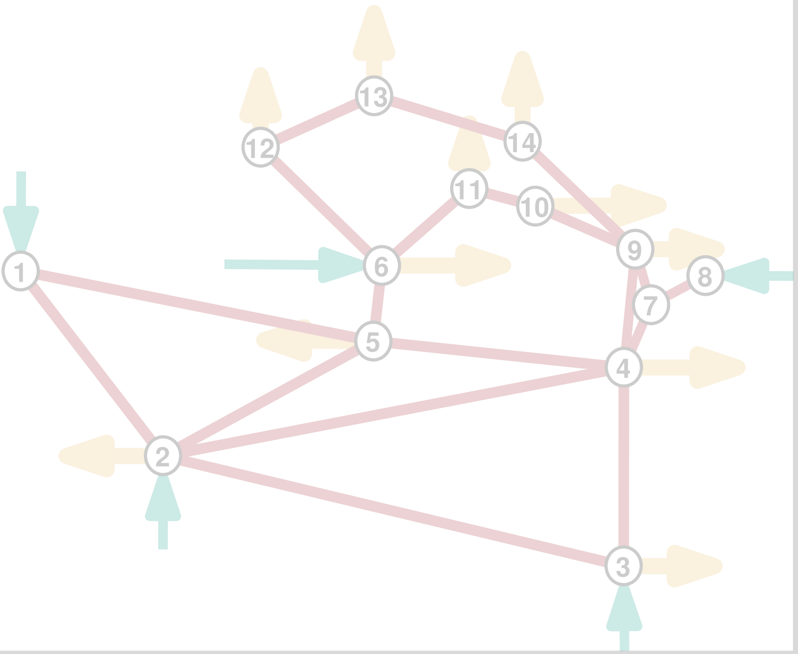
maximize **power production** **objective**



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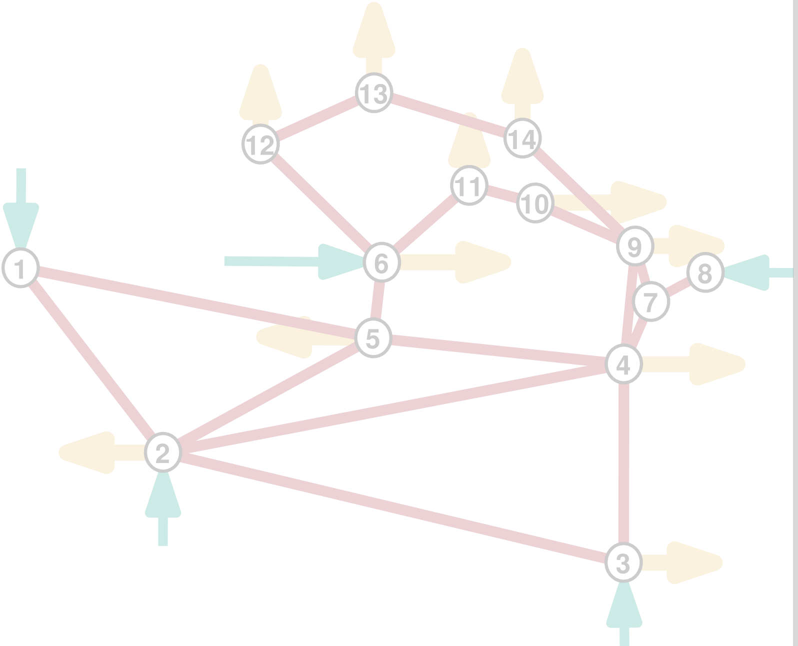
maximize **power production**

subject to line capacity constraints
load capacity constraints
power flow constraints



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The **AC** conservation of flow is a **subproblem** of the MTSF problem.

AC conservation of flow is already **NP-hard** on **trees**.
[Lehmann et al., 2015]

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→ Power grids are not easy.

→ **Linearized AC** conservation of flow is **easy** to solve.

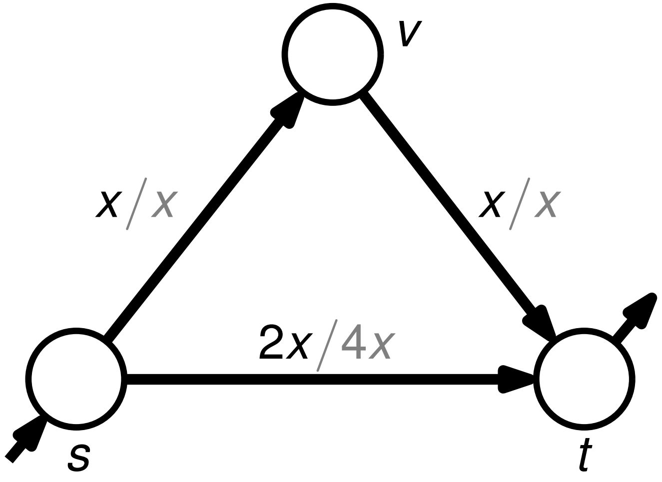
m

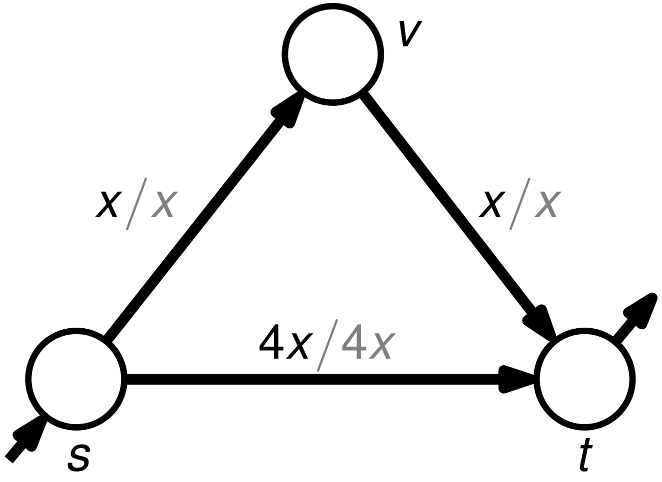
su

load capacity constraints

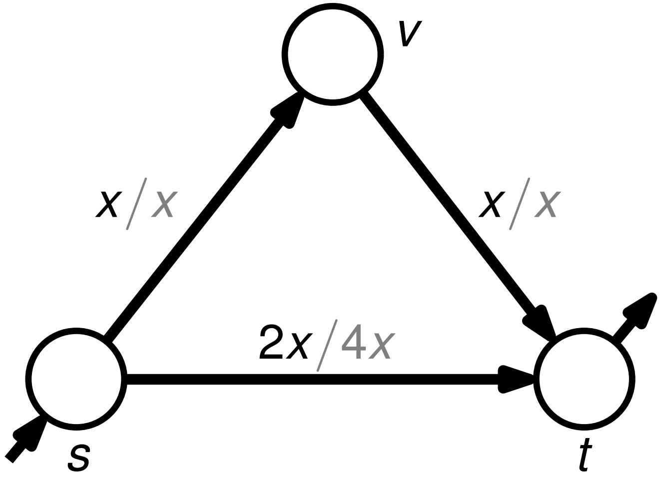
power flow constraints

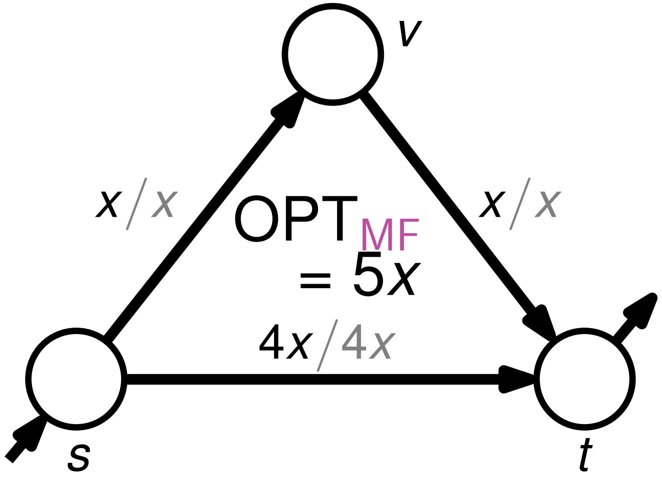
Power Flow Constraints





Power Flow Constraints

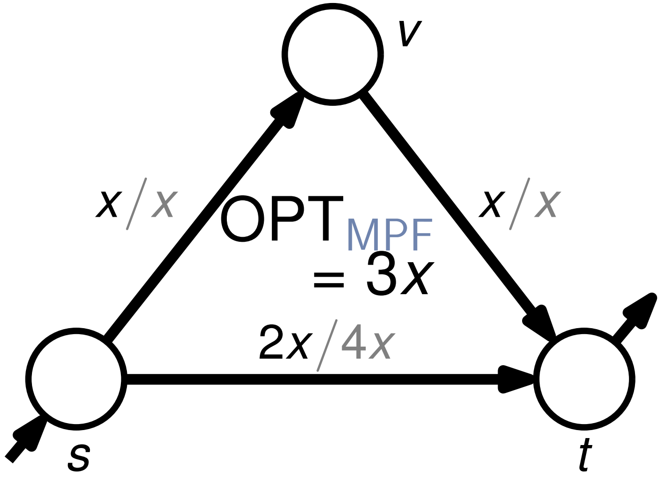


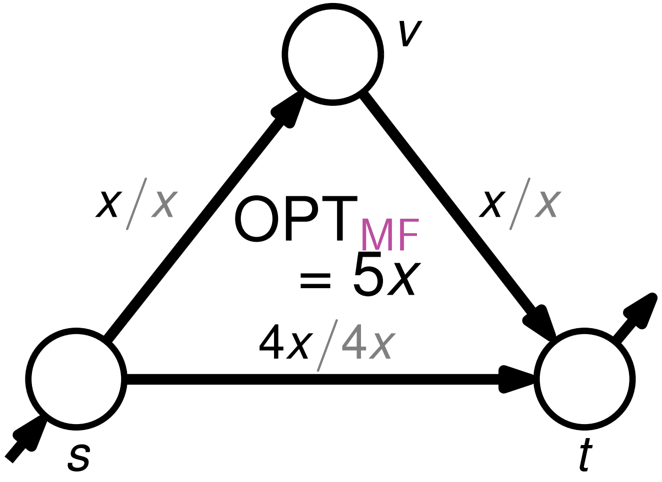


flow model

upper bound

Power Flow Constraints





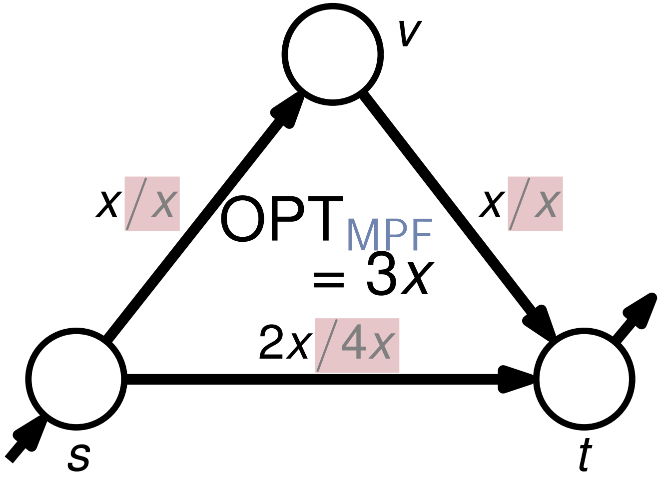
physical model
(AC linearization)

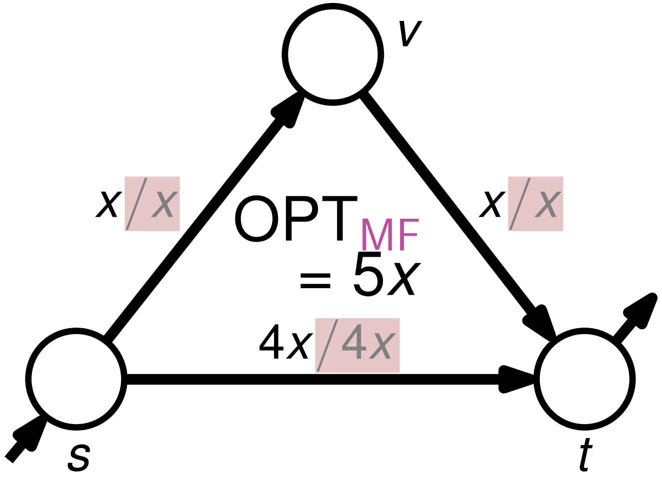
lower bound

flow model

upper bound

Power Flow Constraints





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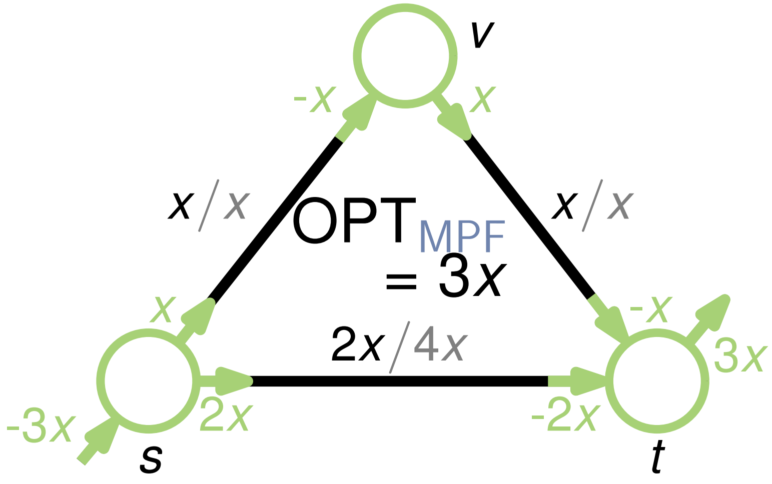
lower bound

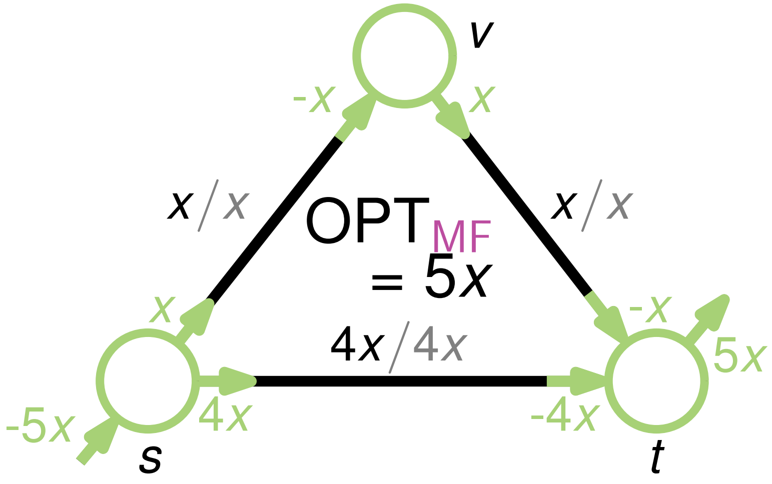
capacity constraints

flow model

upper bound

Power Flow Constraints





physical model

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flow model

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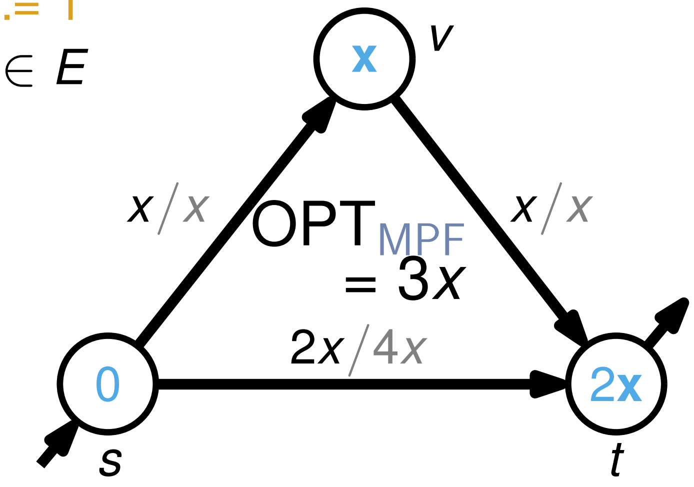
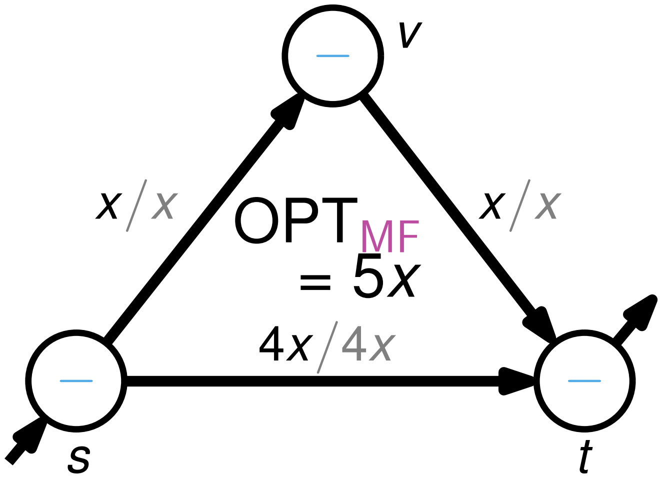
capacity constraints

Kirchhoff's Current Law (KCL)

Power Flow Constraints

$b(i, j) := 1$
$\forall (i, j) \in E$





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DC power flow constraints

$$\forall (u, v) \in E\colon\ f(u, v) = b(u, v)\big(\theta(v) - \theta(u)\big)$$

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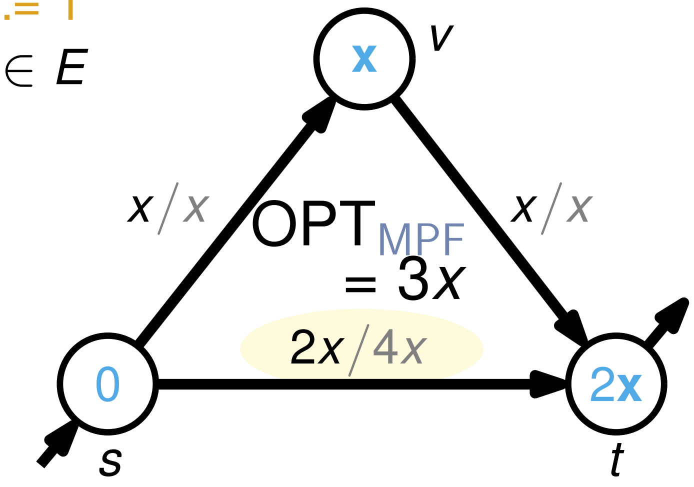

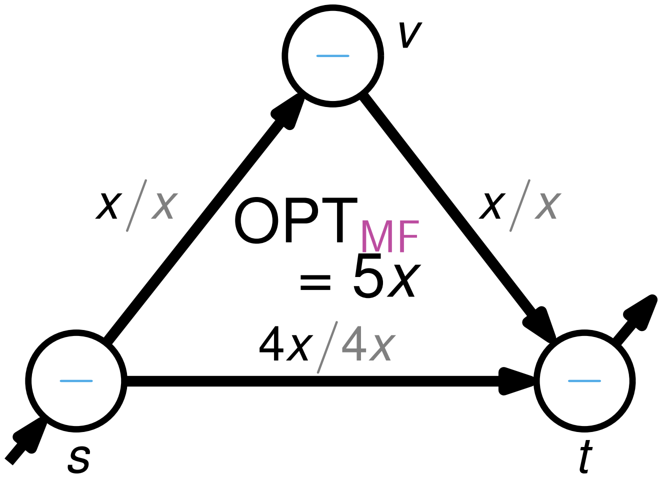

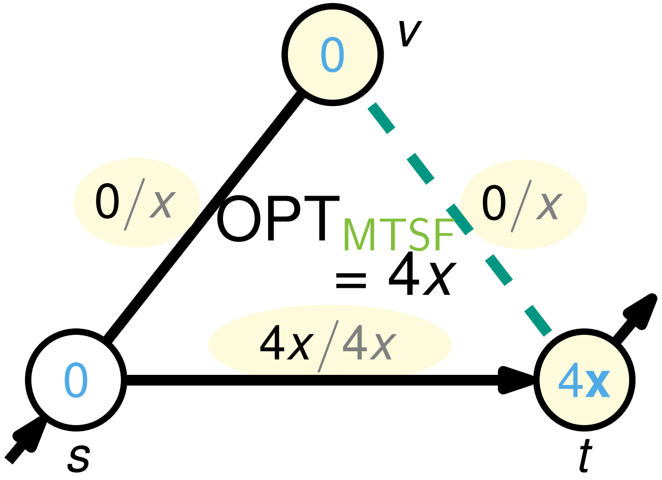

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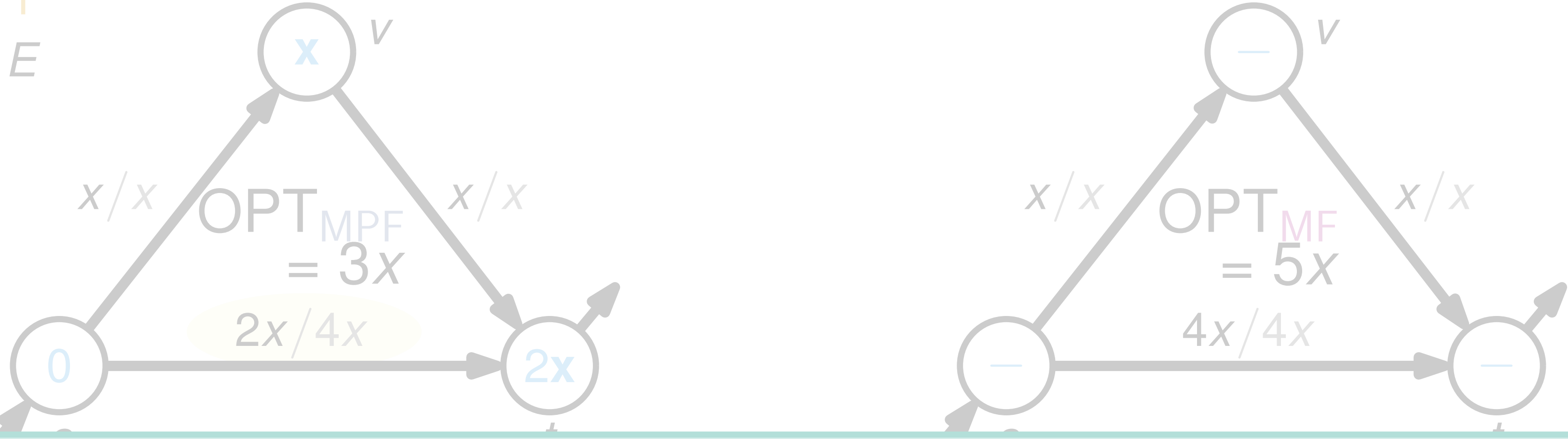
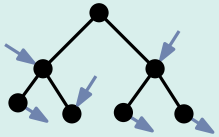
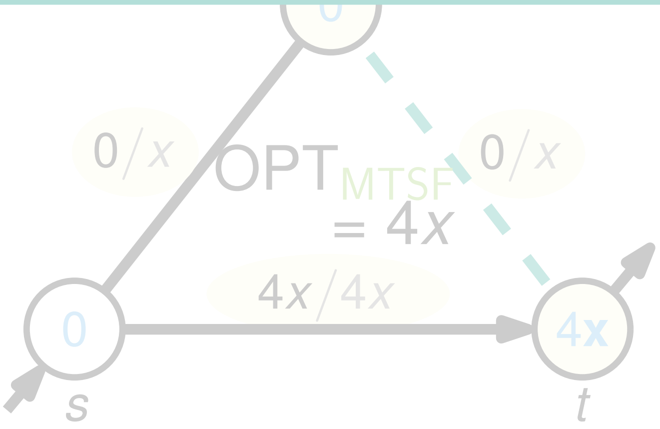




Physical Model (MPF) = Maximum Switching Flow (MTSF) = Flow Model (MF)



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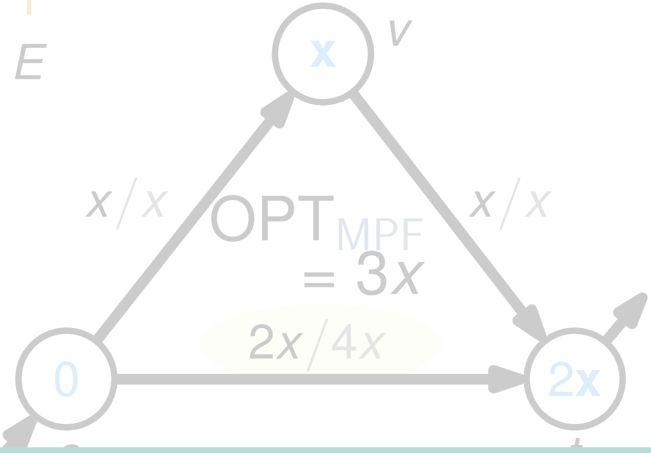
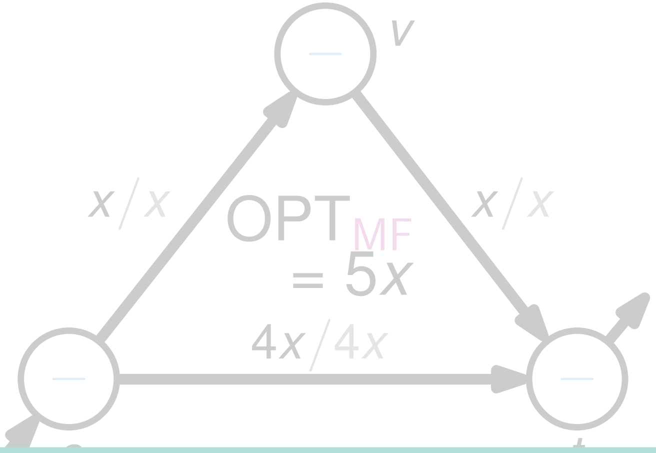
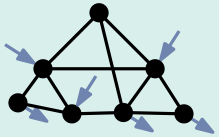
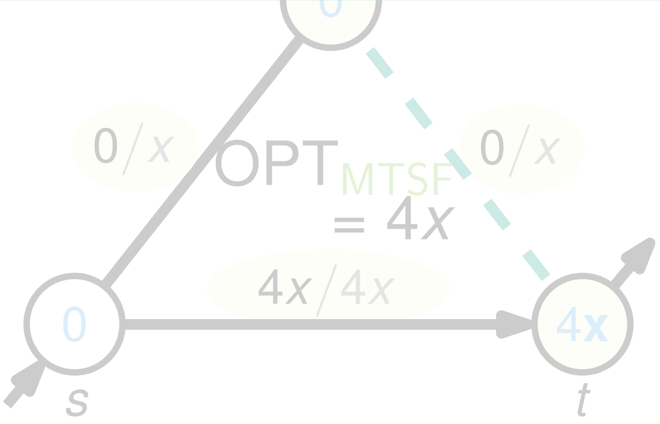






Physical Model (MPF) $\leq$ Maximum Switching Flow (MTSF) $\leq$ Flow Model (MF)



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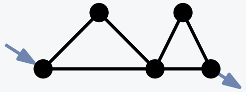
Overview of the MTSF Results

Graph Structure	Complexity	Algorithm

Overview of the MTSF Results

		Graph Structure		Complexity	Algorithm
easy	one generator, one load	cacti		✗	✗

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complexity

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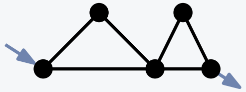
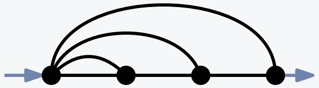
one generator, one load	cacti		✗	✗
	series-parallel graphs		✗	✗

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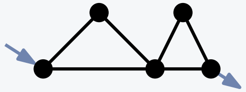
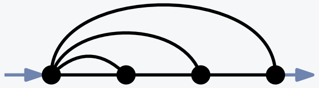
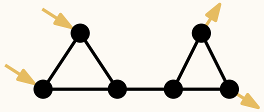
one generator, one load	cacti 	✗	✗
	series-parallel graphs 	✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3 	NP-hard [Lehmann et al., 2014]	✗

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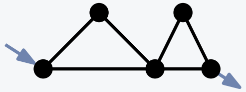
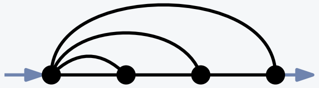
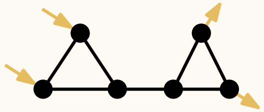
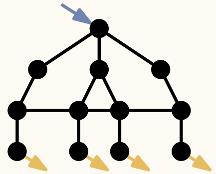
one generator, one load	cacti		✗	✗
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	✗
	2-level trees		NP-hard [Lehmann et al., 2014]	✗

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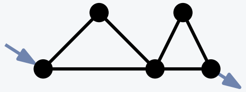
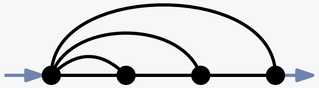
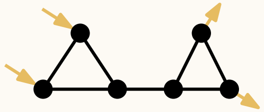
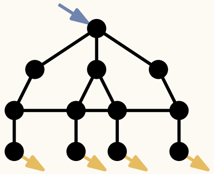
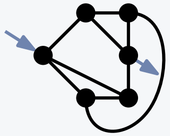
one generator, one load	cacti 	✗	✗
	series-parallel graphs 	✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3 	NP-hard [Lehmann et al., 2014]	✗
	2-level trees 	NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3 	strongly NP-hard [Lehmann et al., 2014]	✗

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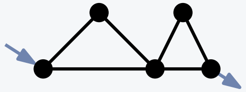
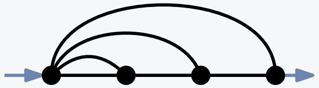
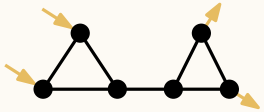
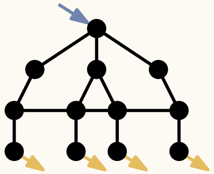
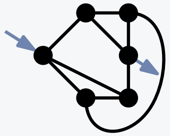
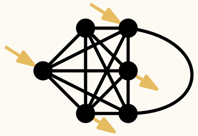
one generator, one load	cacti		✗	✗
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	✗
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\|V_G\|=2$, $\|V_C\|=2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Overview of the MTSF Results

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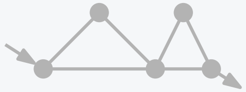
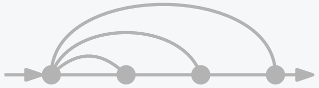
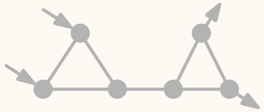
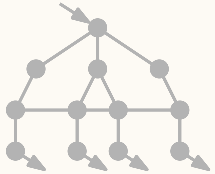
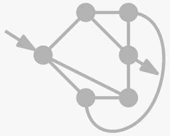
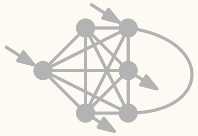
one generator, one load	cacti 	✗	✗
	series-parallel graphs 	✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3 	NP-hard [Lehmann et al., 2014]	✗
	2-level trees 	NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3 	strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G \rvert = 2$, $\lvert V_C \rvert = 2$	arbitrary graphs 	non-APX [Lehmann et al., 2014]	✗

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complexity

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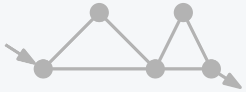
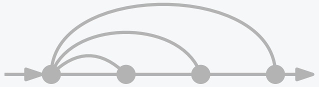
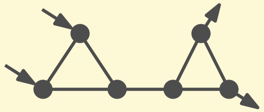
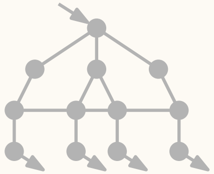
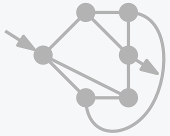
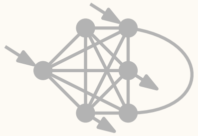
one generator, one load	cacti		✗	✗
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	**cacti with max degree of 3**		**NP-hard** [Lehmann et al., 2014]	**2-approx.**
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G \rvert = 2$, $\lvert V_C \rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

2-approximation on Cacti

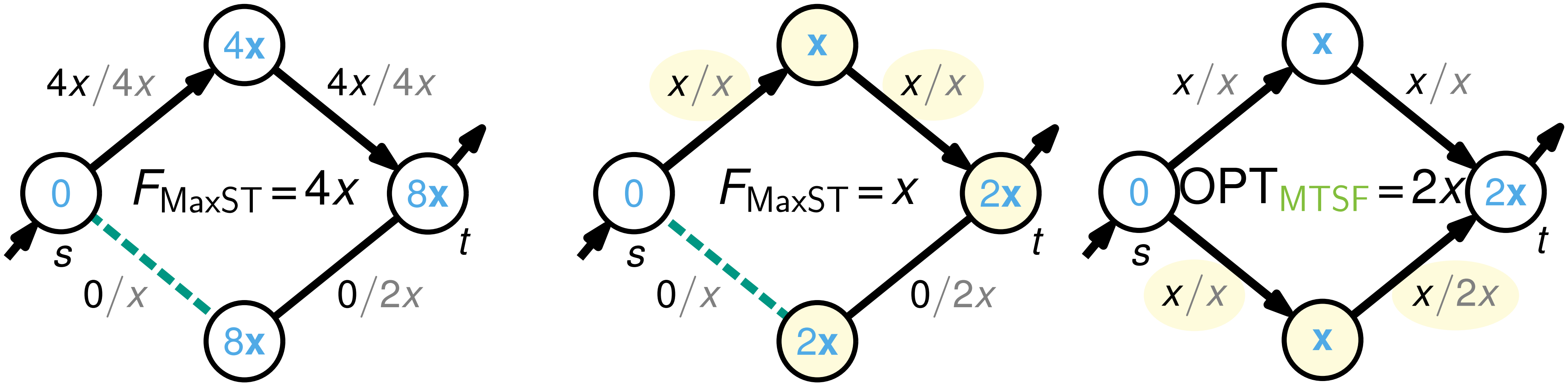
Description

- Remove from each cycle the edge with the smallest capacity

⇔ the MAXIMUM SPANNING TREE (MaxST)

MaxST on Cacti

- MTSF is NP-hard on cacti [Lehmann et al., 2014]

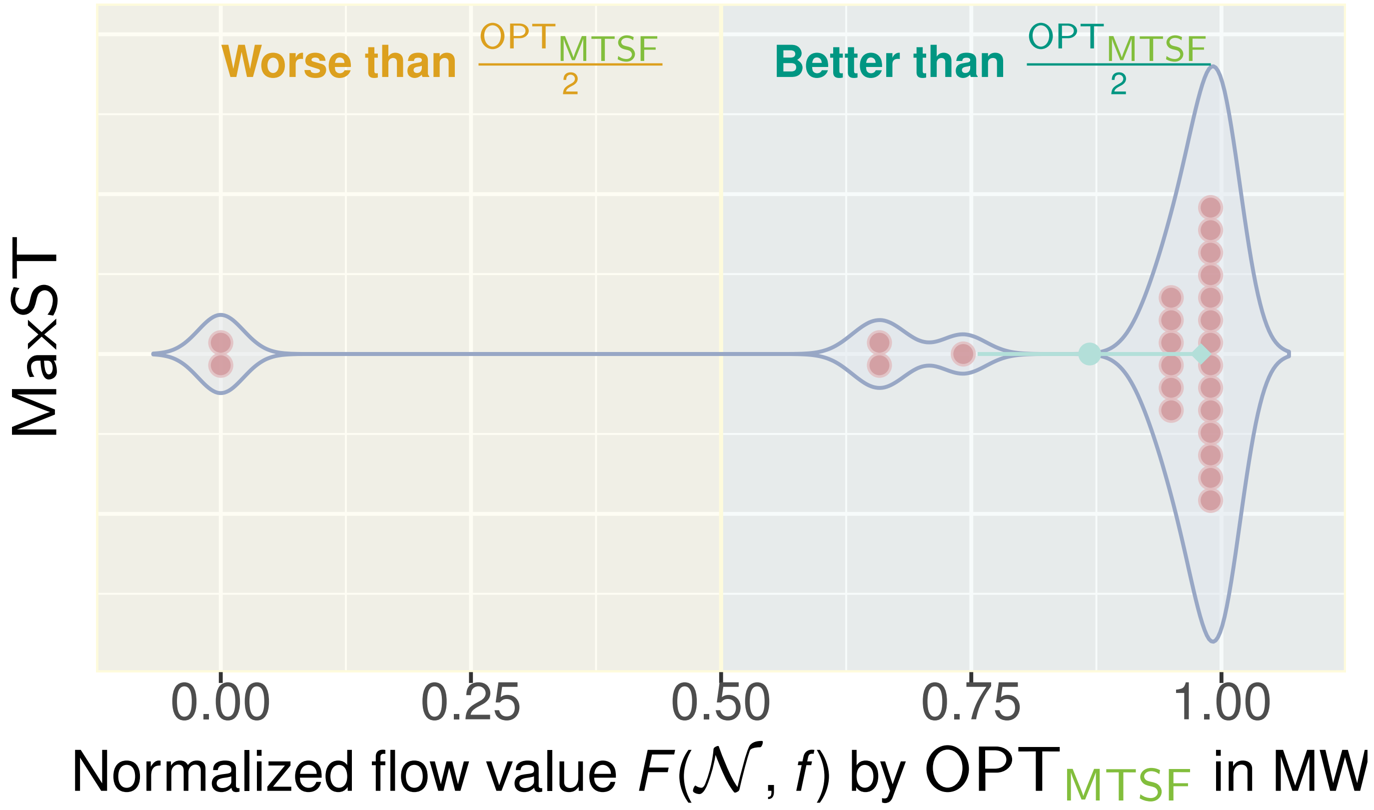


Theorem 1

MaxST is a factor 2-approximation algorithm for the MF and MTSF problem on cacti.

Simulations

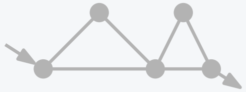
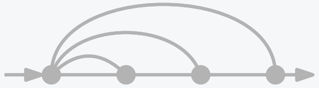
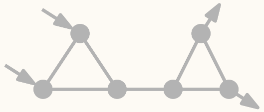
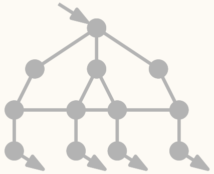
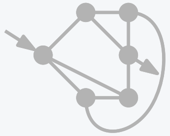
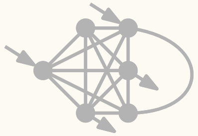
- Simulations on NESTA benchmark sets that are more realistic than the IEEE benchmark sets, e.g., with regards to thermal line limits



MaxST on **general graphs** is in most cases very close to the OPT_{MTSF}.

Overview of the MTSF Results

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	cacti		✗	✗
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G\rvert = 2$, $\lvert V_C\rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Overview of the MTSF Results

complexity

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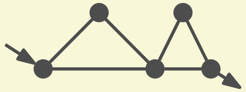
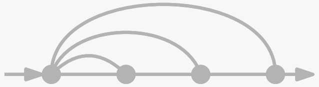
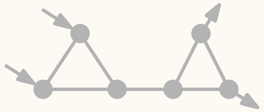
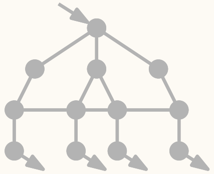
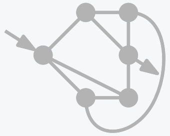
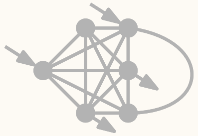
one generator, one load	**cacti**		✗	✗
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\|V_G\| = 2$, $\|V_C\| = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Overview of the MTSF Results

complexity

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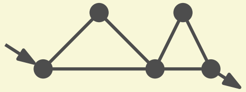
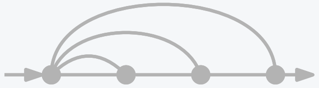
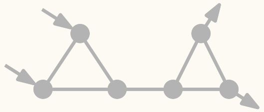
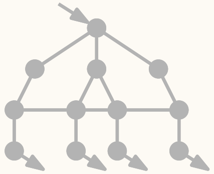
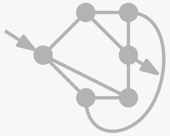
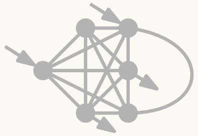
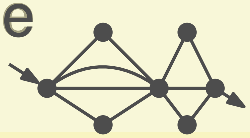
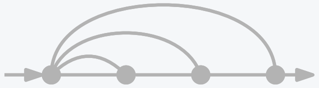
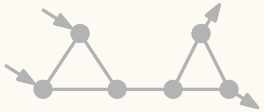
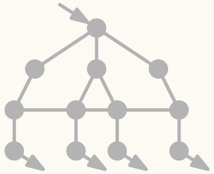
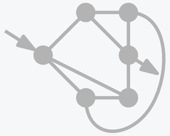
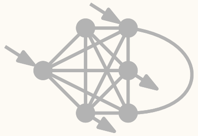
one generator, one load	cacti		polynomial-time solvable	DTP ✓
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G\rvert = 2$, $\lvert V_C\rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Overview of the MTSF Results

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\|V_G\|=2$, $\|V_C\|=2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Dominating Theta Path

Fix $u, v \in V$ and a u-v-path π.

Susceptance Norm:

$$\|\pi\|_b := \sum_{e \in E(\pi)} \frac{1}{b(e)}$$

Minimum Capacity:

$$\underline{\text{cap}}(\pi) := \min\{\text{cap}(e) \mid e \in \pi\}$$

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Angle Difference of π:

$$\Delta\Theta(\pi) := \|\pi\|_b \cdot \underline{\mathrm{cap}}(\pi)$$

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Dominating Theta Path (DTP):

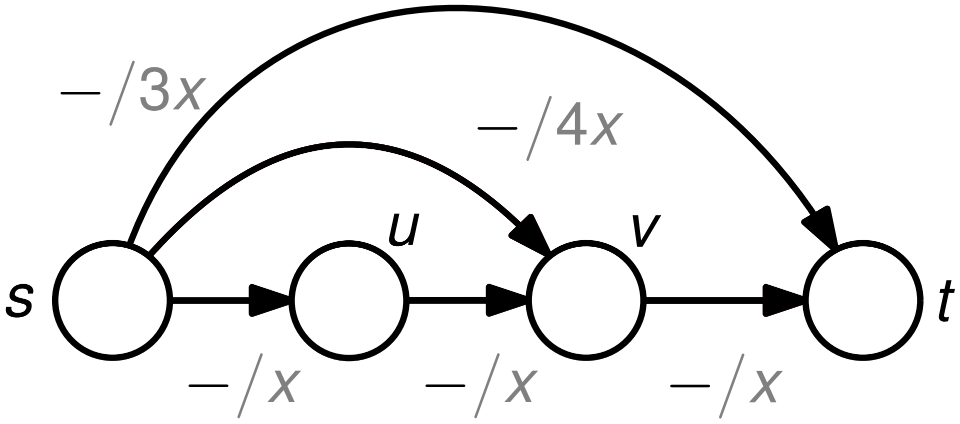
$$\Delta\Theta_{\text{min}}(u, v) := \min\{\Delta\Theta(\pi) \mid \pi \text{ is a } u\text{-}v\text{-path}\}$$

Computing **DTP**

Description:

- Bicriterial Dijkstra with labels $(\|\pi\|_b, \underline{\text{cap}}(\pi))$
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$



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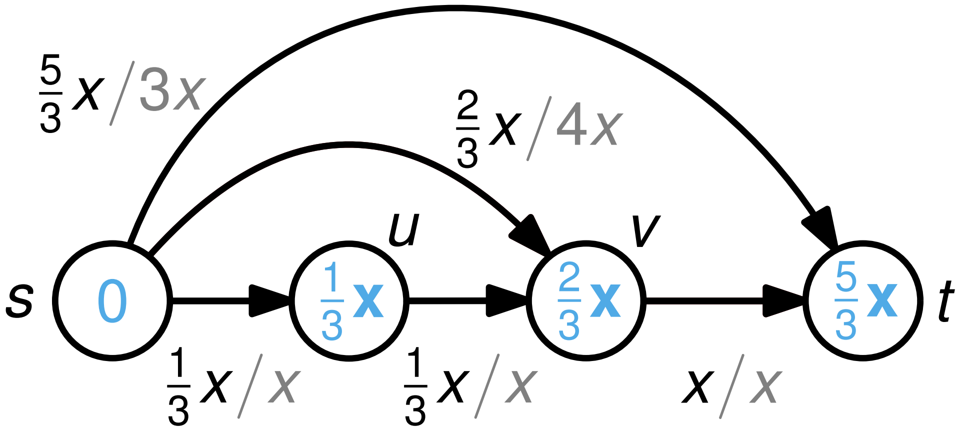

$\text{OPT}_{\text{MPF}} = \frac{8}{3}x$

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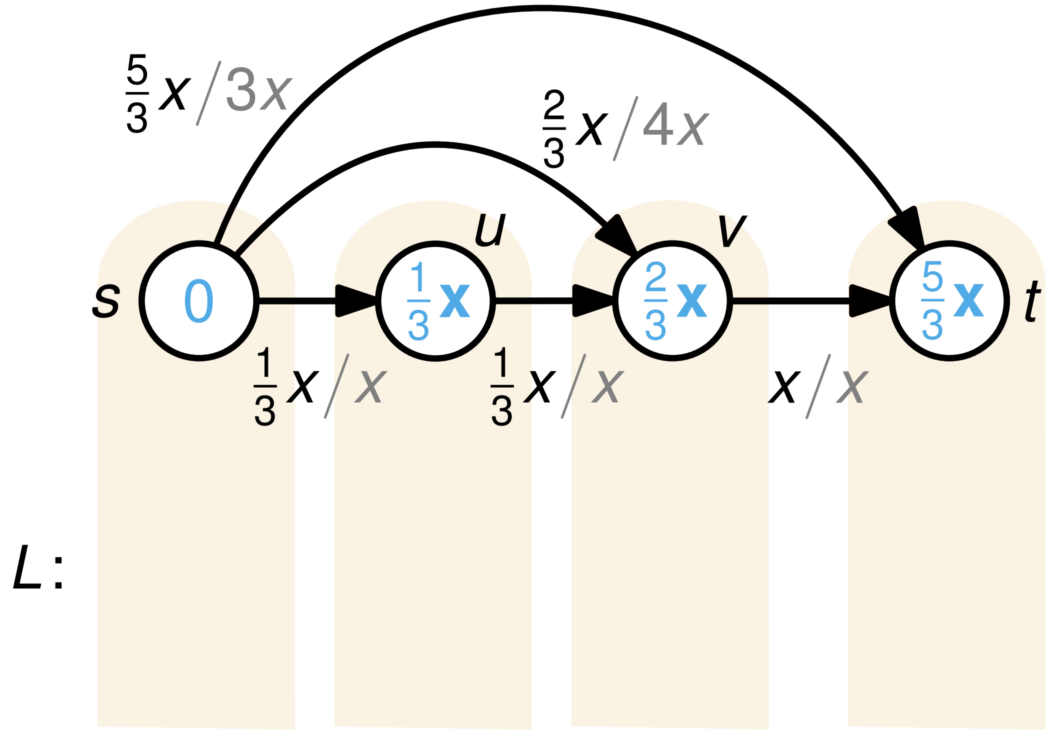

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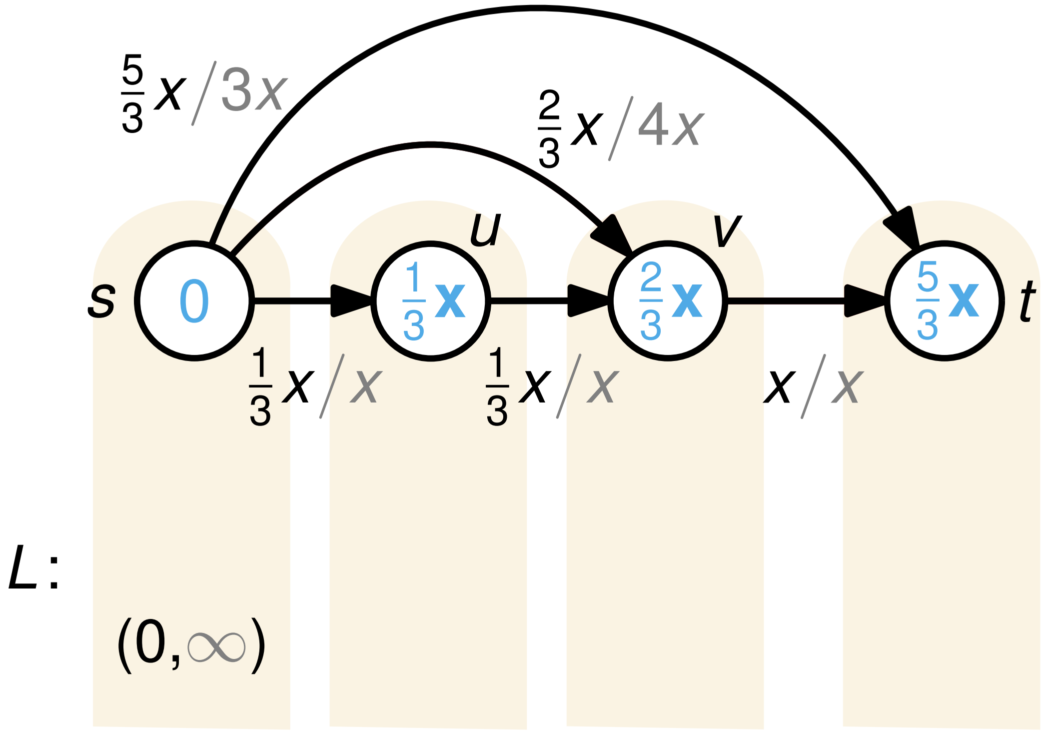

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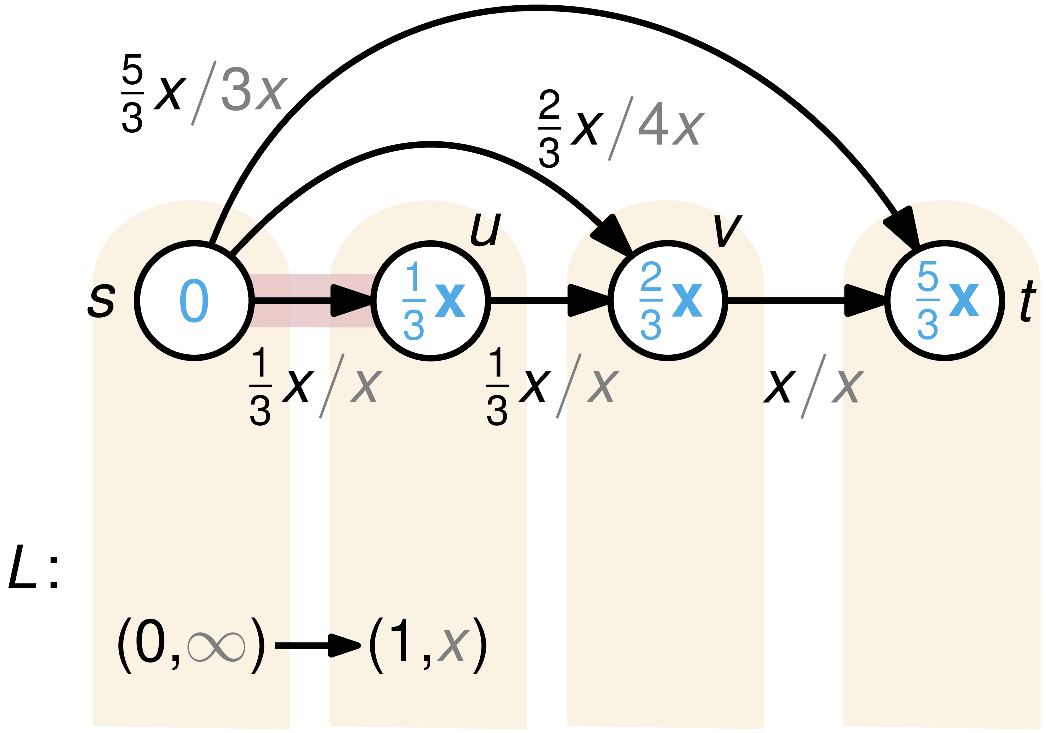

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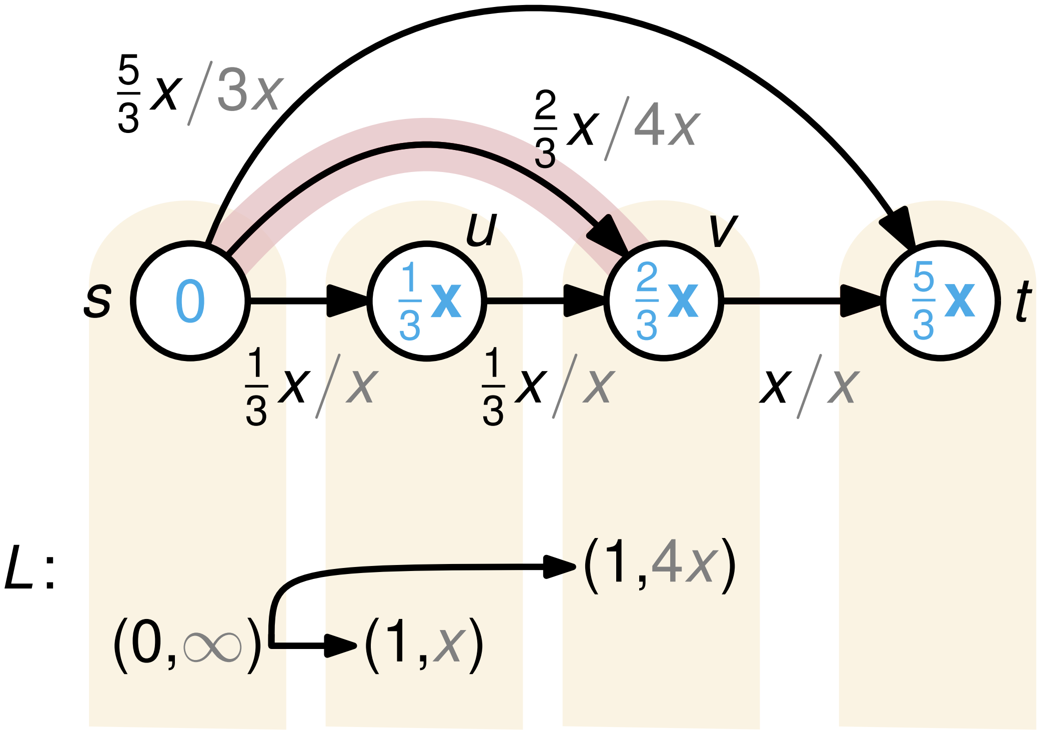

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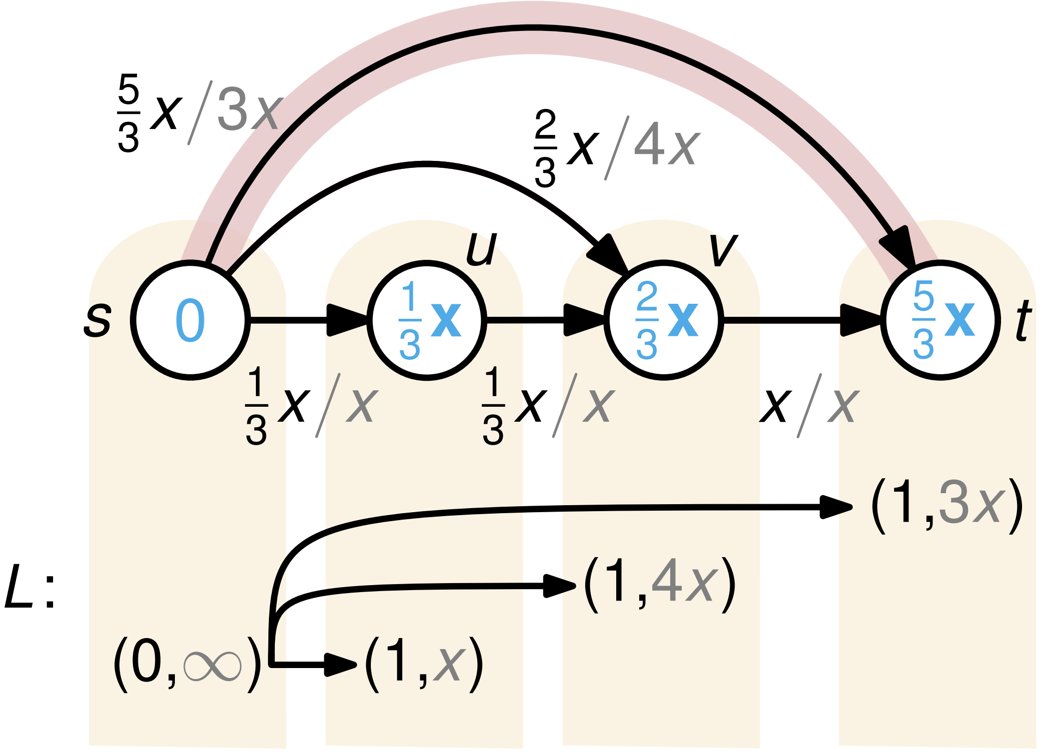

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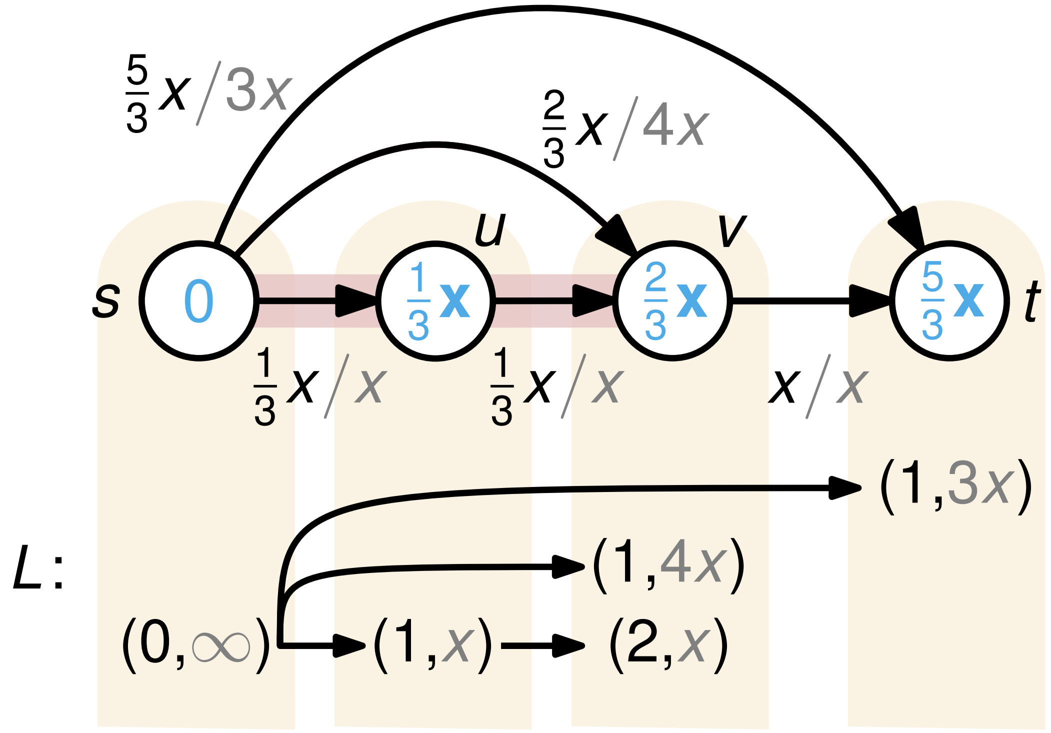

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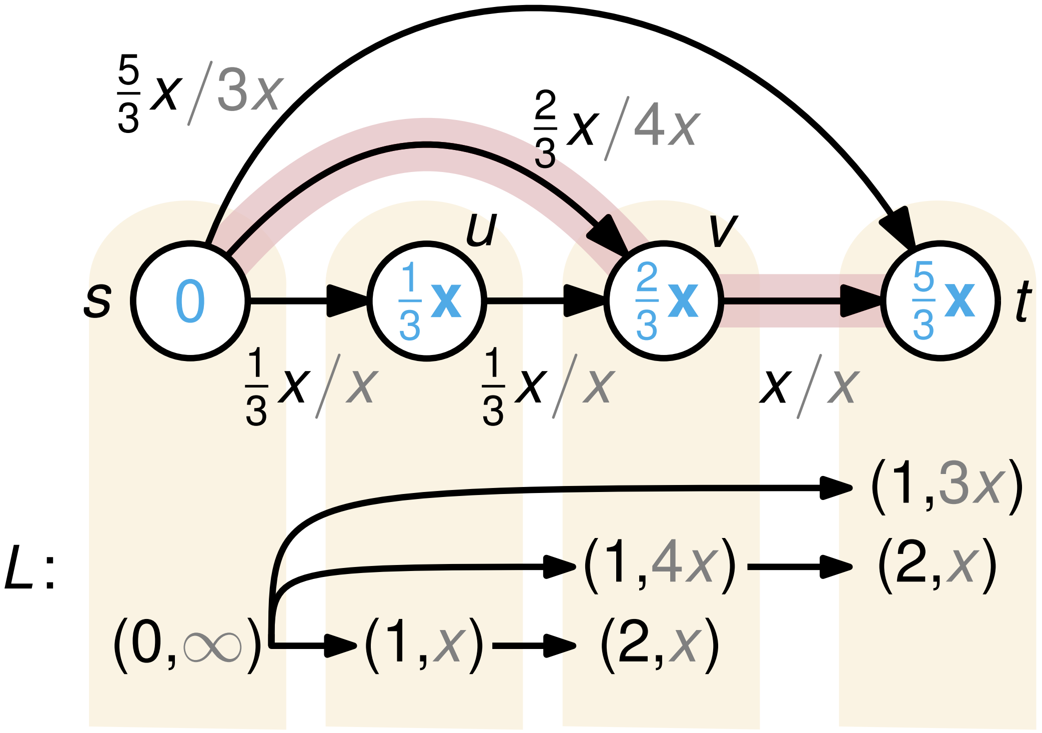

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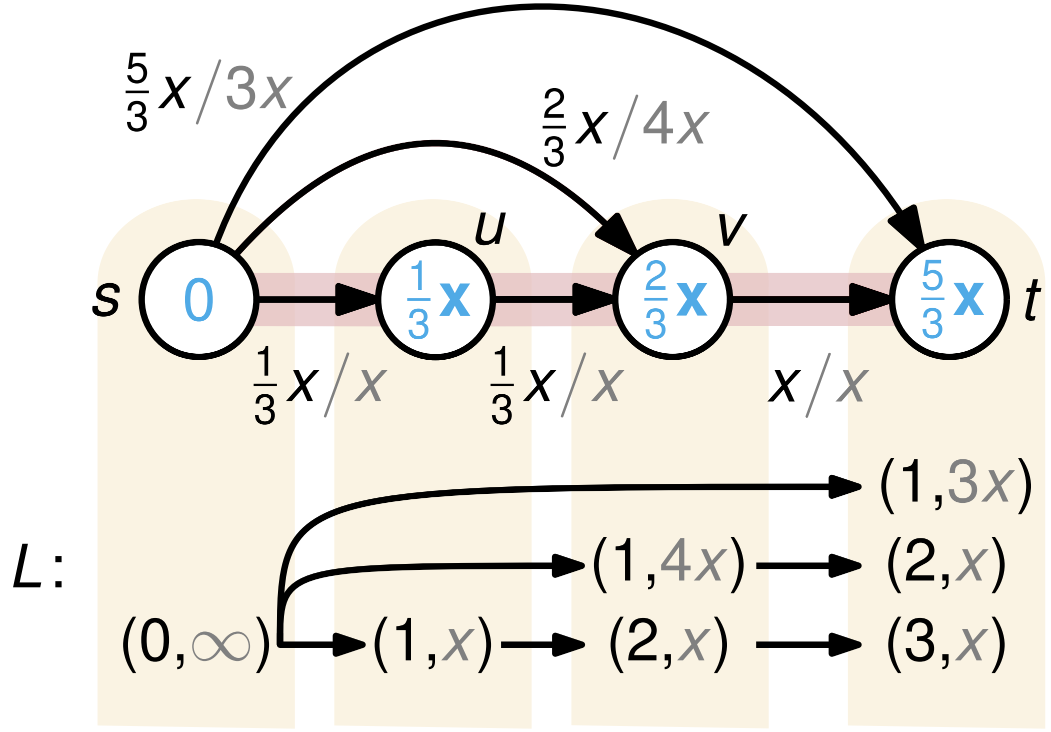

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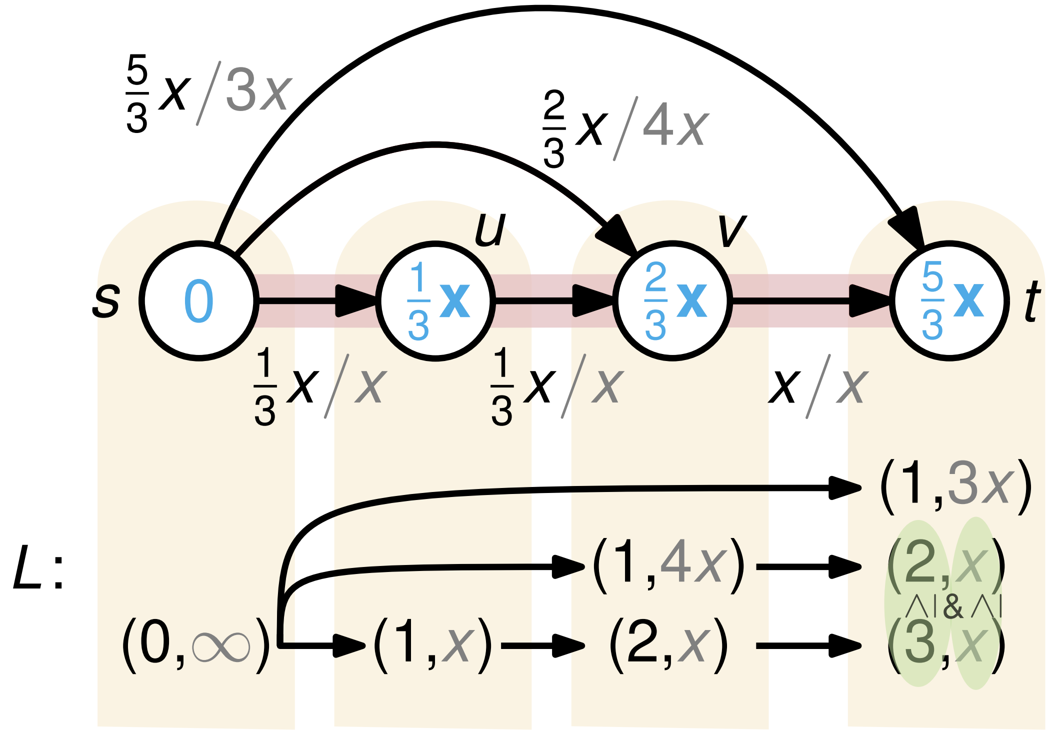

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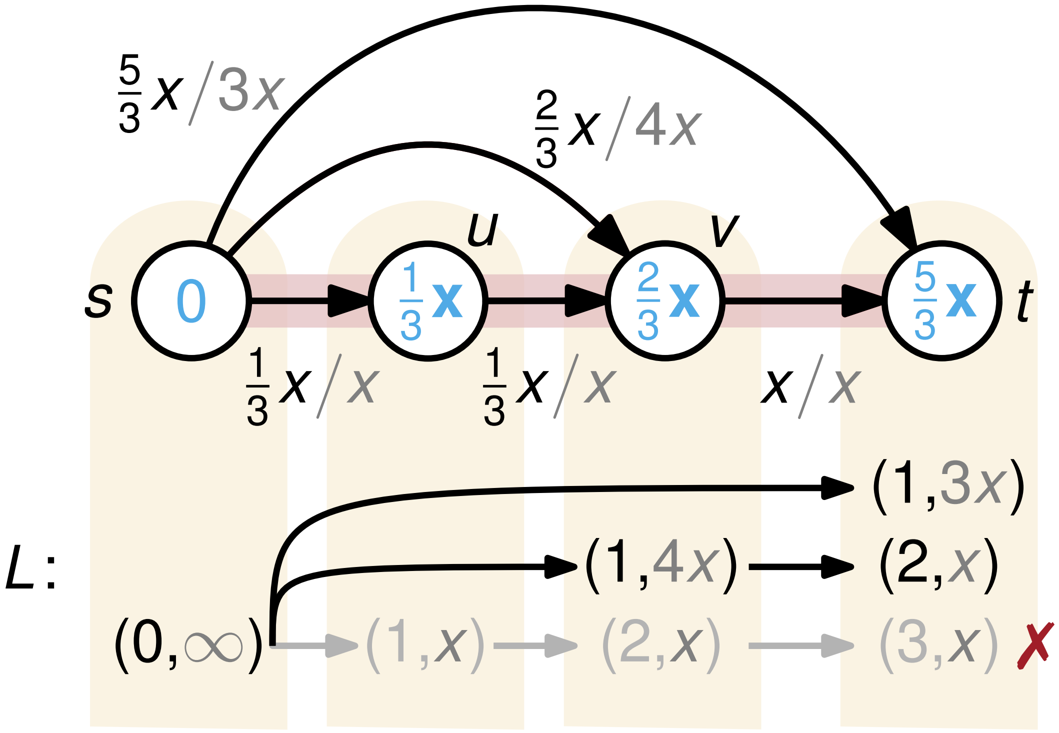

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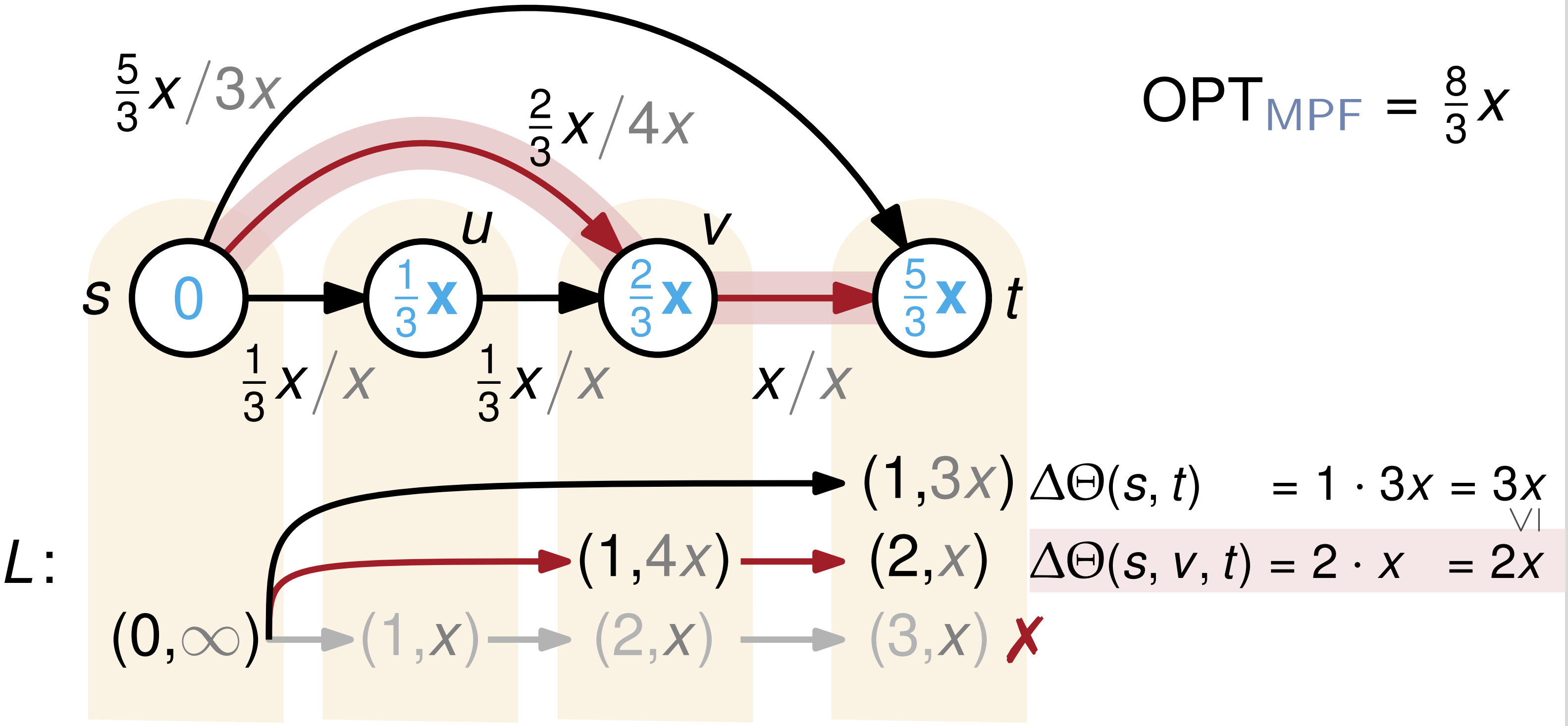


$\mathrm{OPT}_{\mathrm{MPF}} = \frac{8}{3}x$

$\Delta\Theta(s,t) = 1 \cdot 3x = 3x$

$\geq$

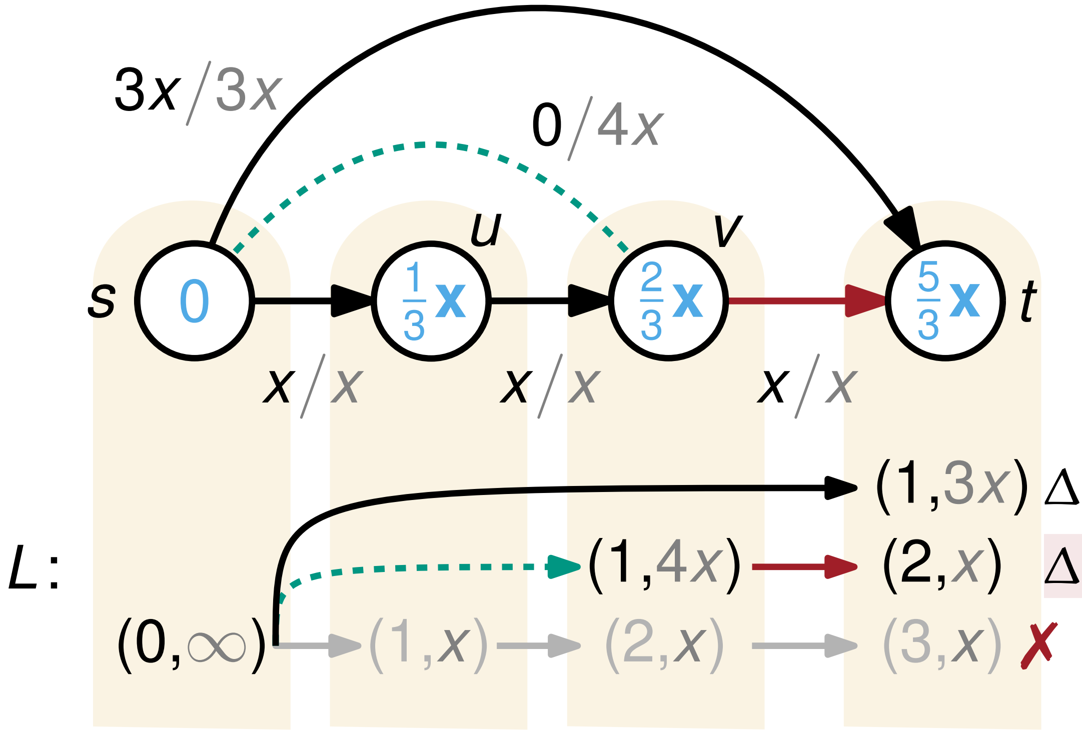
$\Delta\Theta(s,v,t) = 2 \cdot x = 2x$

Computing **DTP**

Description:

- Bicriterial Dijkstra with labels ($\|\pi\|_b$, $\underline{\text{cap}}(\pi)$)
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$



$\text{OPT}_{\text{MPF}} = 4x$
$= \text{OPT}_{\text{MTSF}}$

$\Delta\Theta(s,t) = 1 \cdot 3x = 3x$

$\geq$

$\Delta\Theta(s,v,t) = 2 \cdot x = 2x$

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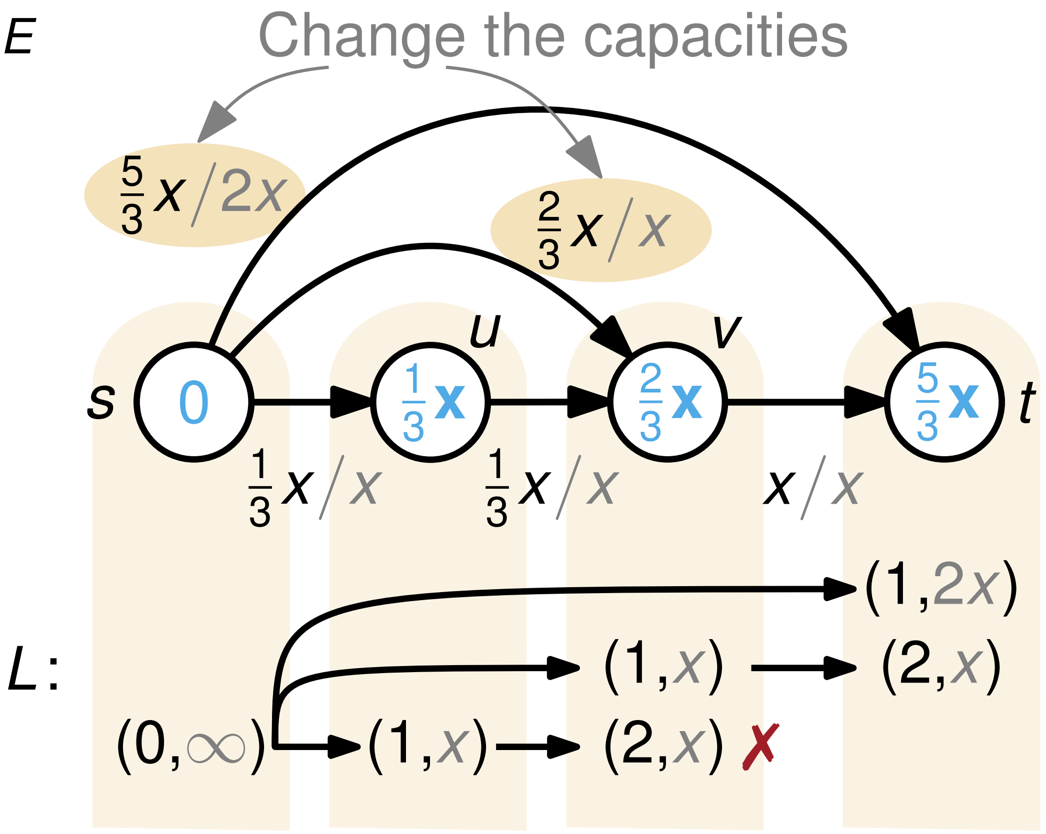
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$



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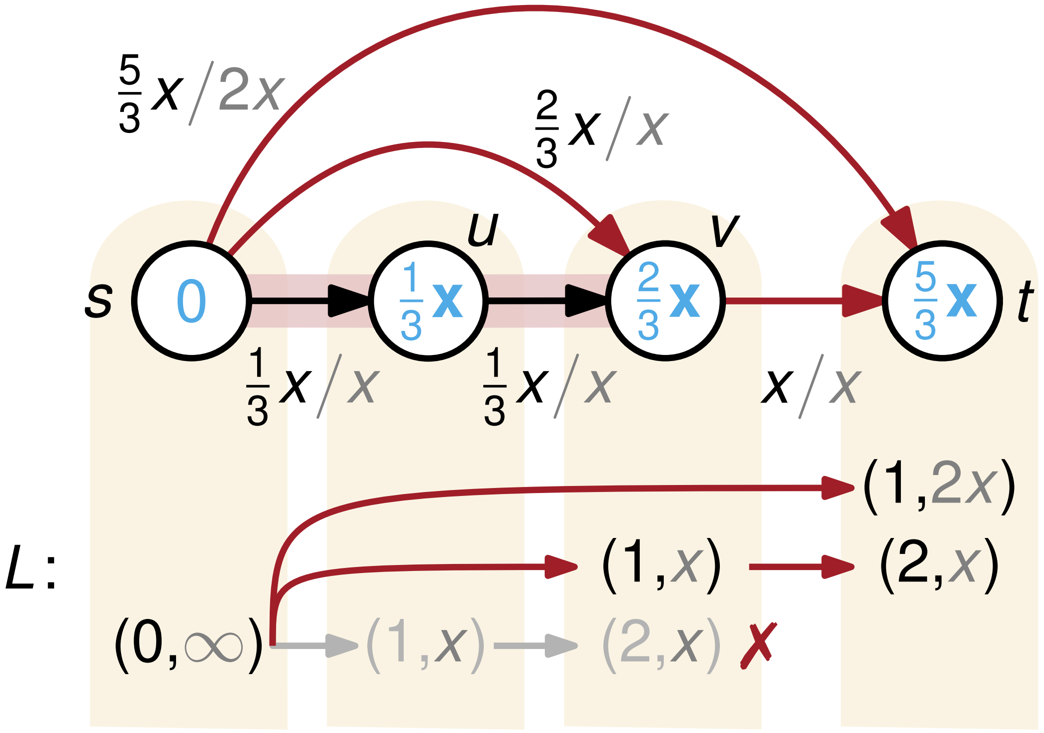
- DTPs from s do not have to form a tree



$\text{OPT}_{\text{MPF}} = \frac{8}{3}x$

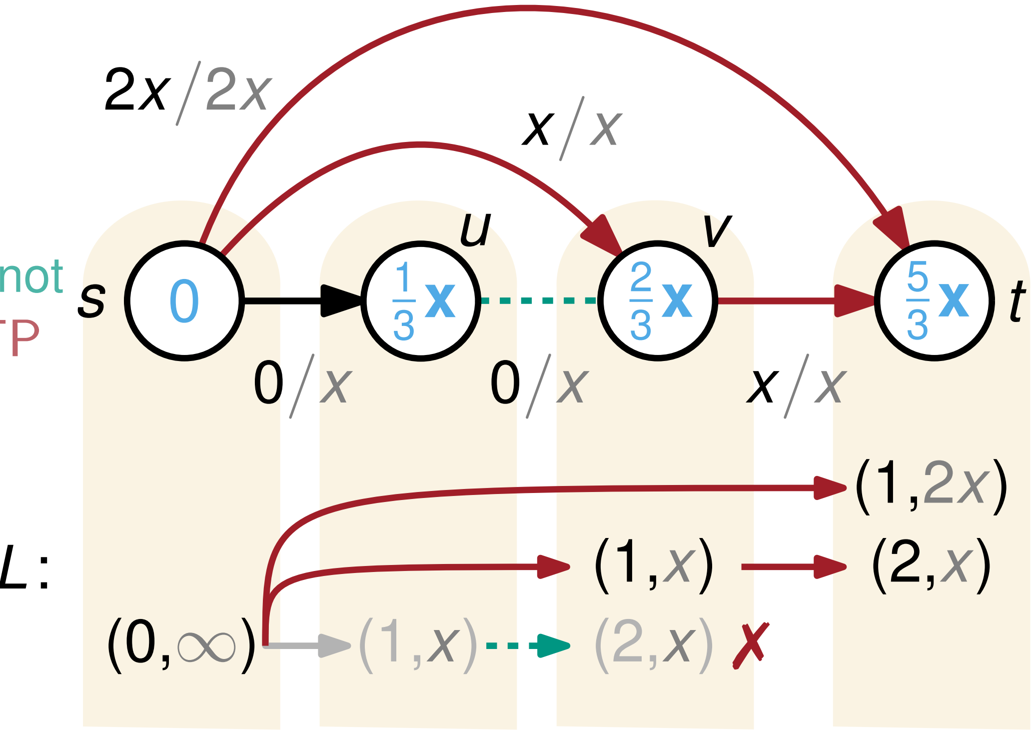
Computing DTP

Description:

- Bicriterial Dijkstra with labels $(\|\pi\|_b, \underline{\text{cap}}(\pi))$
- at most $|E|$ labels per vertex

$b(i,j) := 1 \quad \forall (i,j) \in E$

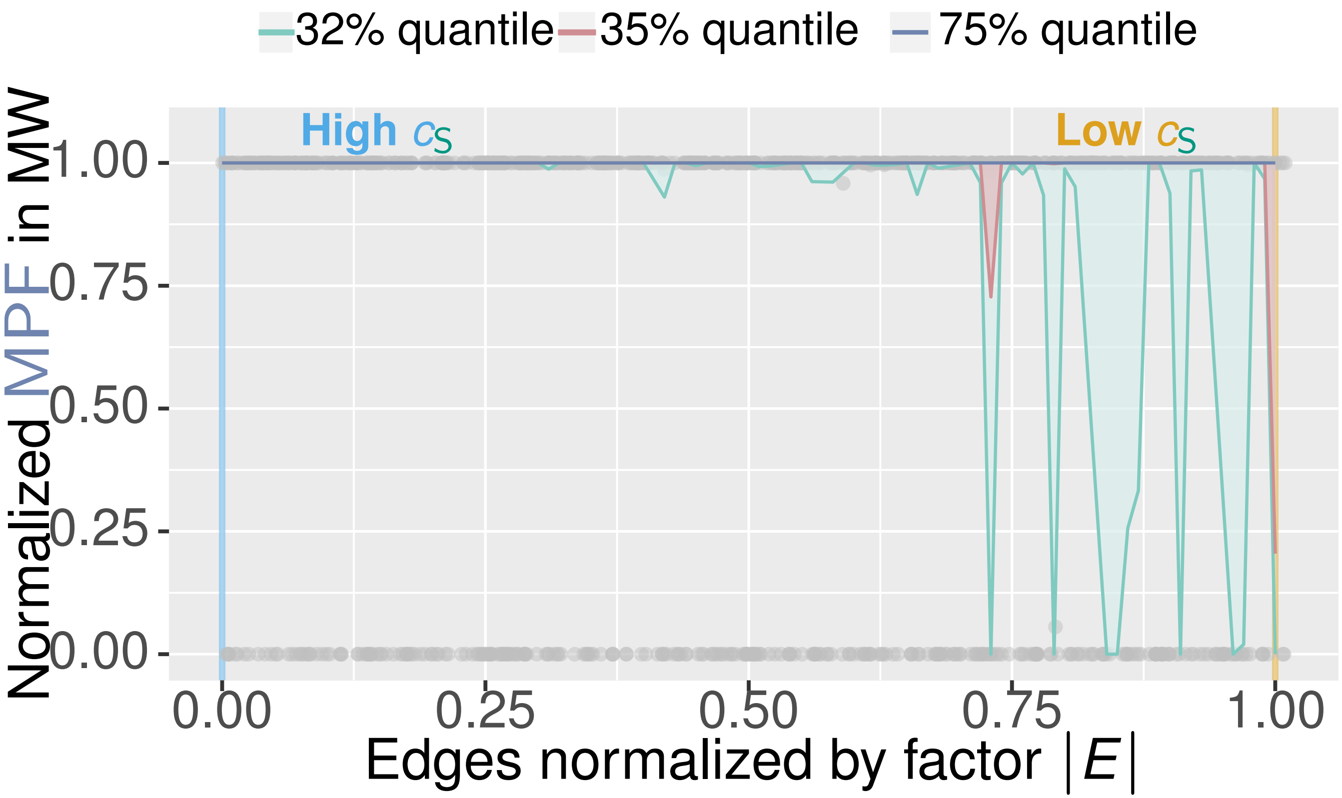
- DTPs from *s* do not have to form a tree
- Optimal switches do not have to lie on the DTP if the structure is not penrose-minor free



$\text{OPT}_{\text{MPF}} = 3x$
$= \text{OPT}_{\text{MTSF}}$

Simulations

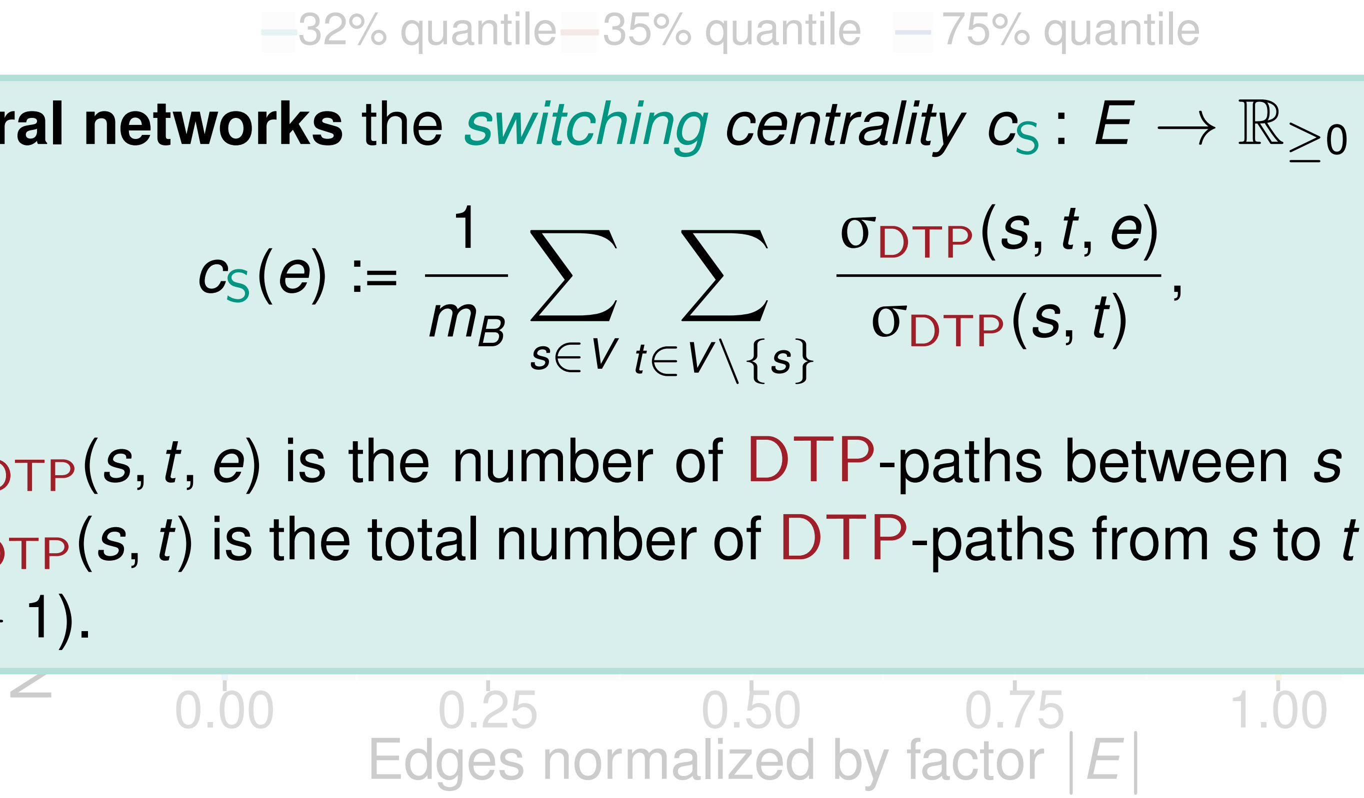
- Simulations on NESTA benchmark sets that are more realistic than the IEEE benchmark sets, e.g., with regards to thermal line limits



The MPF decreases mainly for edges having a small centrality c_S.

Simulations

- Simulations on NESTA benchmark sets that are more realistic than the IEEE benchmark sets, e.g., with regards to thermal line limits



On **general networks** the *switching centrality* $c_S : E \to \mathbb{R}_{\geq 0}$ is defined by

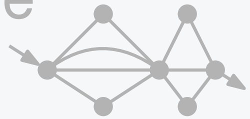
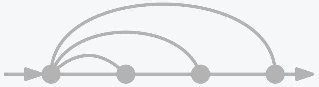
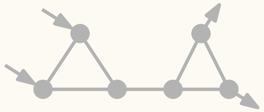
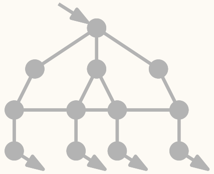
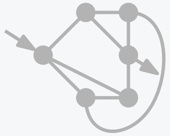
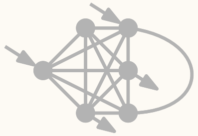
$$c_S(e) := \frac{1}{m_B} \sum_{s \in V} \sum_{t \in V \setminus \{s\}} \frac{\sigma_{\mathrm{DTP}}(s,t,e)}{\sigma_{\mathrm{DTP}}(s,t)},$$

where $\sigma_{\mathrm{DTP}}(s,t,e)$ is the number of DTP-paths between s and t that use e, $\sigma_{\mathrm{DTP}}(s,t)$ is the total number of DTP-paths from s to t and $m_B = |V|(|V|-1)$.

The MPF decreases mainly for edges having a small centrality c_S.

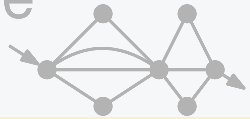
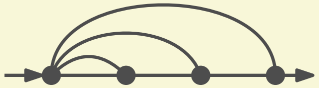
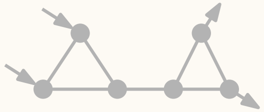
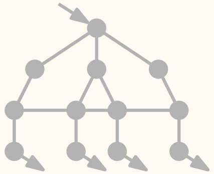
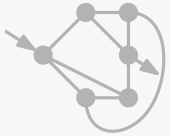
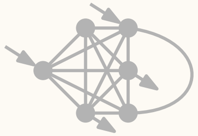
Complexity of the MTSF

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G \rvert = 2$, $\lvert V_C \rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

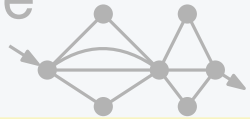
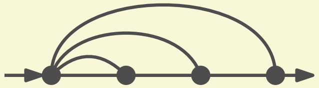
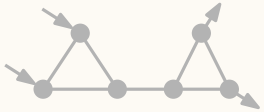
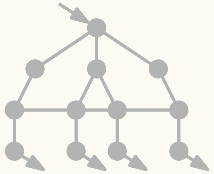
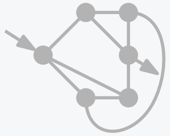
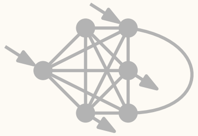
Complexity of the MTSF

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
	series-parallel graphs		✗	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G \rvert = 2$, $\lvert V_C \rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

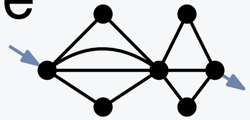
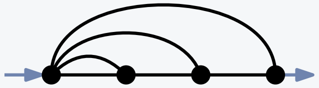
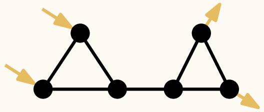
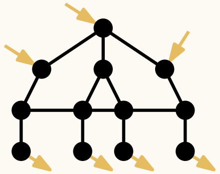
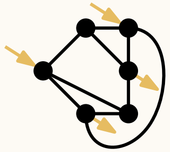
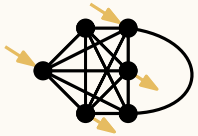
Complexity of the MTSF

complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP ✓
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. ✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\lvert V_G \rvert = 2$, $\lvert V_C \rvert = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Summary & Future Work

complexity

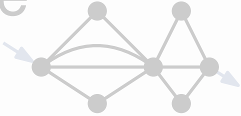
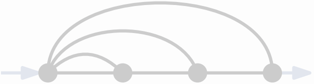
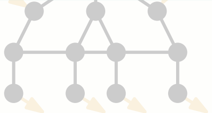
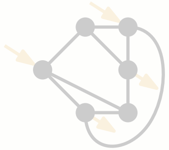
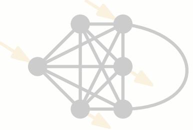
	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	✓
	series-parallel graphs		NP-hard	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	✓
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Summary & Future Work

one generator, one load

arbitrary generators, arbitrary loads

complexity

Graph Structure		Complexity	Algorithm
penrose-minor-free graphs		polynomial-time solvable	✓
series-parallel graphs		NP-hard	✗
cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	✓
2-level trees		NP-hard [Lehmann et al., 2014]	✗
planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

- What happens if we minimize the number of switches or fix a set of non-switchable edges?
- Is there a PTAS on cacti for MTSF?