

# Route Planning Algorithms in Transportation Networks

7th International Network Optimization Conference

Dorothea Wagner | May 18, 2015, Warsaw, Poland

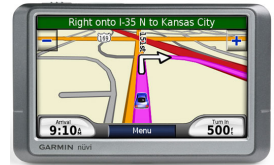
FACULTY FOR INFORMATICS · INSTITUTE FOR THEORETICAL INFORMATICS · CHAIR ALGORITHMICS





## Important application, e. g.,

- Navigation systems for cars
- Google Maps, Bing Maps, ...
- Timetable information



## Many commercial systems

- Use **heuristic** methods
- Consider “reasonable” part of the network
- Have no quality **guarantees**

Find methods for **route planning** in transportation networks with **provably optimal** solutions regarding the quality of the routes.



# Problem

## Request:

- Find the **best** connection in a transportation network

## Idea:

- Network as graph  $G = (V, E)$
- Edge weights are **travel times**
- **Shortest** paths in  $G$  equal **quickest** connections
- Classic problem (Dijkstra)

## Problems:

- Transport networks are **huge**
- Dijkstra too **slow** ( $> 1$  second)



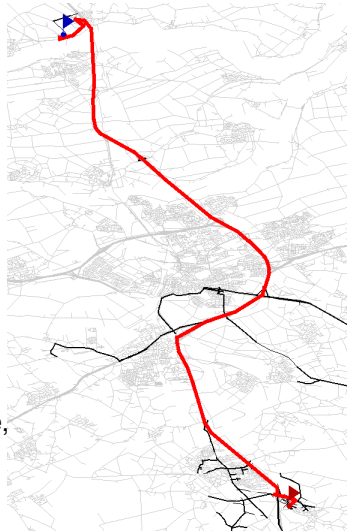


## Observations:

- Dijkstra visits **all** nodes closer than the target
- **Unnecessary** computations
- Many requests in a hardly changing network

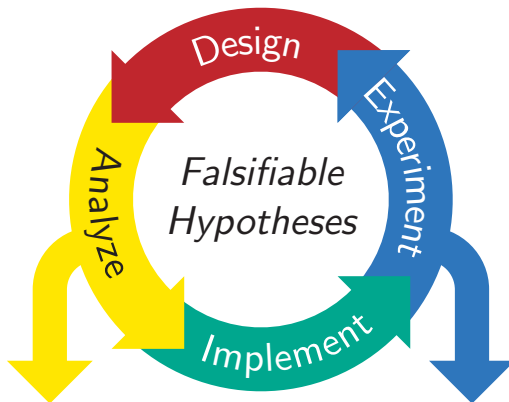
## Idea:

- Two-phase algorithm:
  - Offline: compute additional data during **preprocessing**
  - Online: **speed-up** query with this data
- 3 criteria: preprocessing time and space, speed-up over Dijkstra



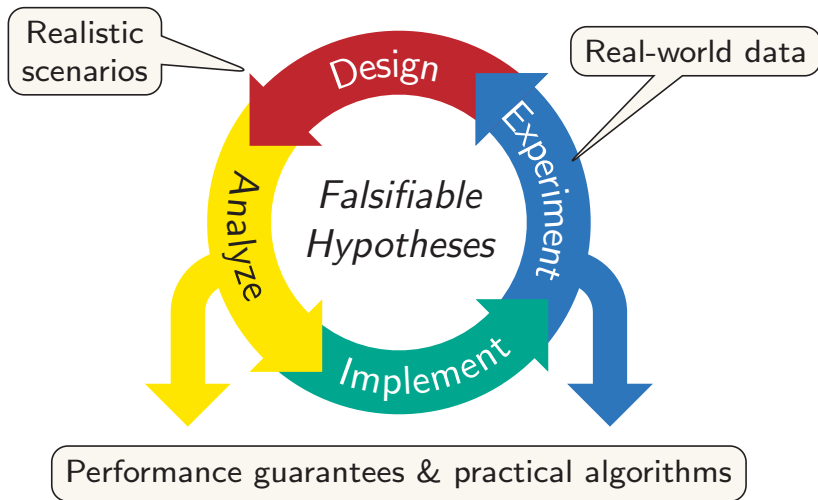


# Showpiece of Algorithm Engineering





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## Phase I: Theory (1959 - 1999):

- Improve theoretical worst-case running time
- By introduction of better data structures
- Bidirectional search,  $A^*$ -search (goal-directed)

## Phase II: Speed-up techniques (1999 - 2005):

- Two approaches: goal-directed and hierarchical approach
- Improvement on this for several inputs

## Phase III: Road networks (2005 - 2008):

- Focus on continent-sized road networks
- DIMACS challenge in 2006
- Speed-up factors in range of several millions over Dijkstra



## **Phase IV: Towards more realistic scenarios (2008-2012):**

- Time-dependency, multicriteria, alternative routes, . . .
- Timetable information
- Back to theory: why do things work?

## **Now: New challenges (since 2012):**

- Other metrics, e. g., energy consumption
- Customizability (supporting user-centric route planning)
- Multimodal



## Many techniques:

- Arc-Flags [[Lau04](#)]
- Multi-Level Dijkstra [[SWW00](#), [HSW08](#)]
  - Customizable Route Planning (CRP) [[DGPW11](#)]
- ALT: A\*, Landmarks, Triangle Inequality [[GH05](#), [GW05](#)]
- Reach [[GKW07](#)]
- Contraction Hierarchies (CH) [[GSSD08](#)]
- Transit Node Routing (TNR) [[ALS13](#)]
- Hub Labeling (HL) [[ADGW12](#)]
- ...



## Observation:

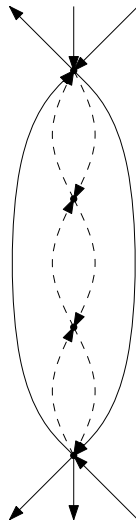
- Nodes with low degree are **not** important

## Contract graph

- **Iteratively** remove such nodes
- Add **shortcuts** to preserve distances between non-removed nodes

## Query:

- Bidirectional
- Prune edges heading to **less** important nodes





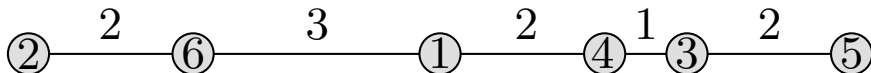
**Idea:** solely use contraction

## Approach:

- Heuristically order nodes by “importance”
- Contract nodes in that order
- Node  $v$  contracted by
  - 1 **forall the edges**  $(u, v)$  **and**  $(v, w)$  **do**
  - 2     **if**  $(u, v, w)$  *unique shortest path* **then**
  - 3         add shortcut  $(u, w)$  with weight  $\text{len}(u, v) + \text{len}(v, w)$ ;
- Query only looks at edges to more important nodes

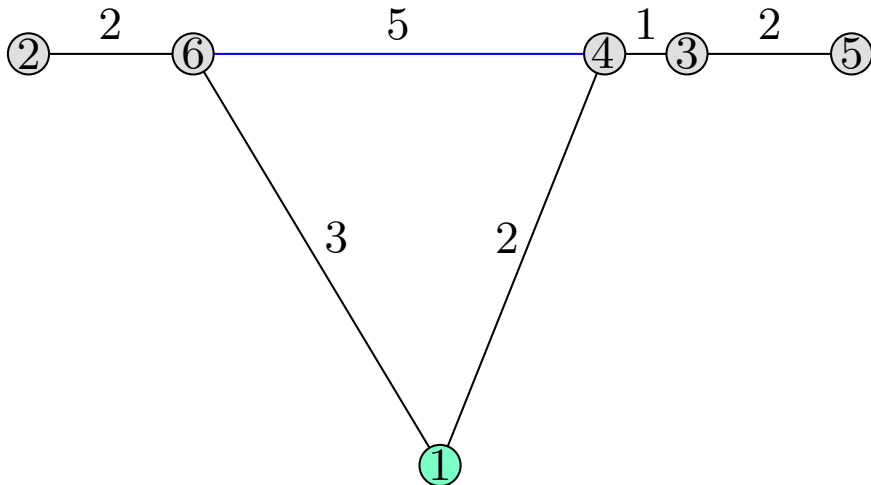


# Example: CH Preprocessing



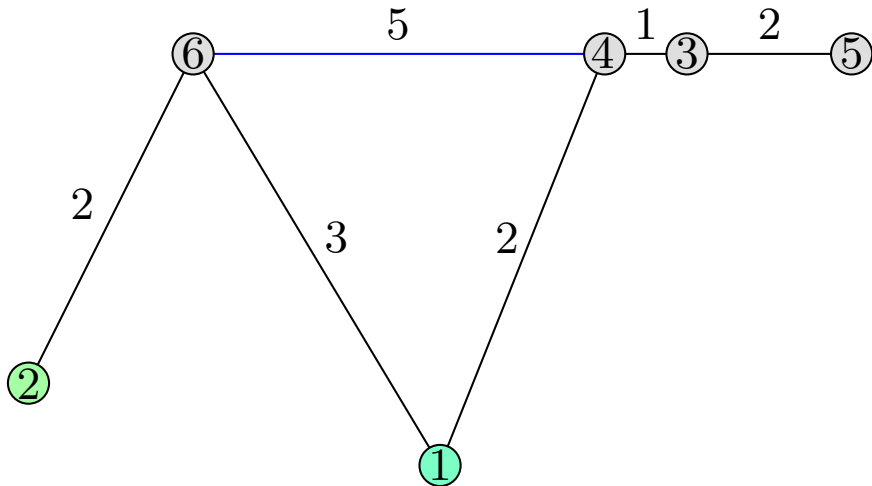


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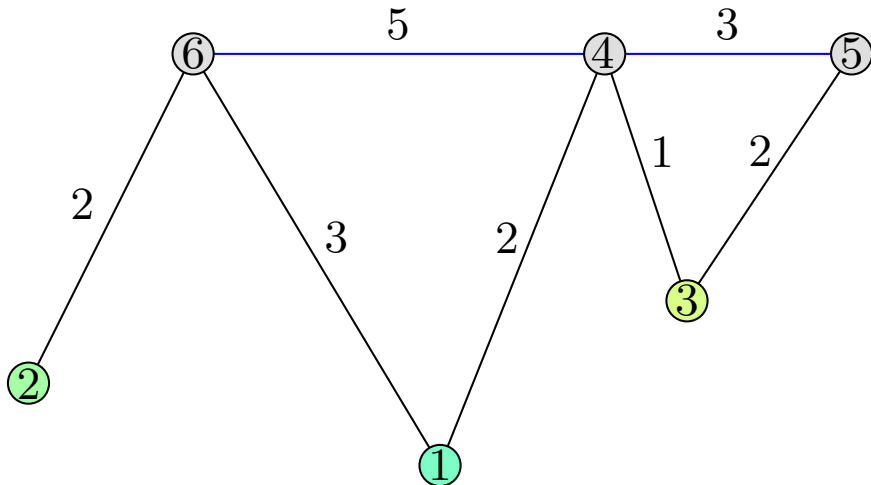


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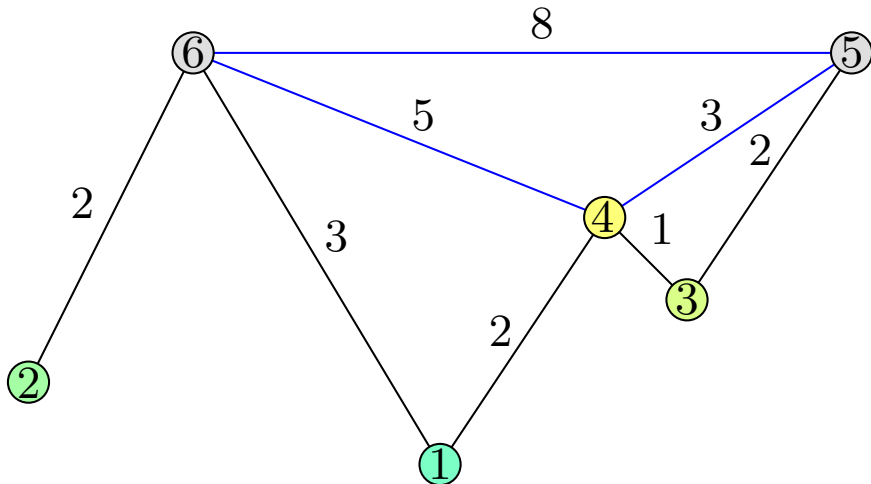


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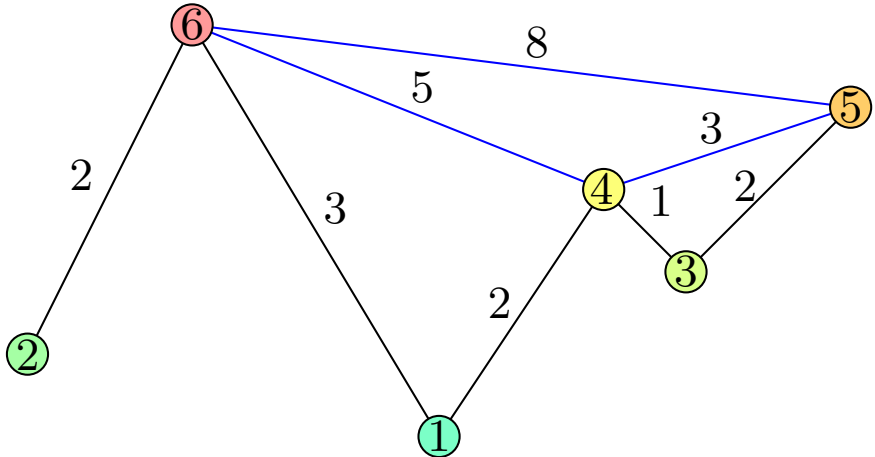


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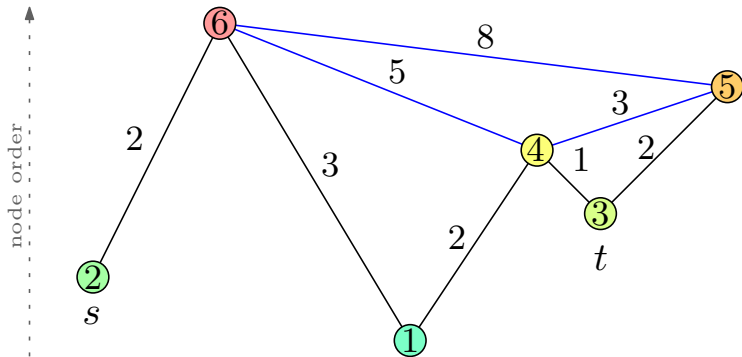


# Example: CH Preprocessing





- Modified bidirectional Dijkstra
- Upward graph  $G_{\uparrow} := (V, E_{\uparrow})$  with  $E_{\uparrow} := \{(u, v) \in E : u < v\}$   
downward graph  $G_{\downarrow} := (V, E_{\downarrow})$  with  $E_{\downarrow} := \{(u, v) \in E : u > v\}$
- Forward search in  $G_{\uparrow}$  and backward search in  $G_{\downarrow}$

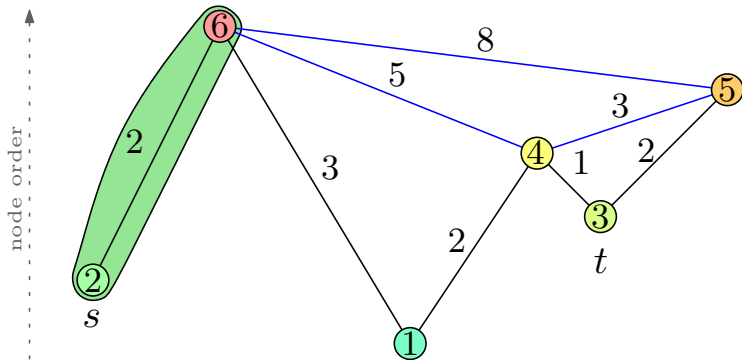




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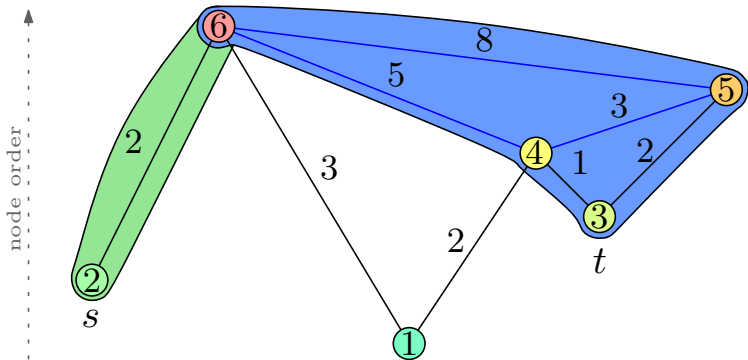
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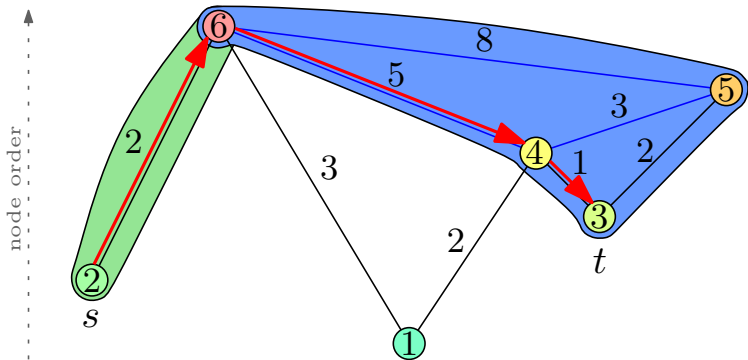


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**Question:** What is a good contraction order?

- No guarantees on search space [GSSD08]



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**WeakCH** [BCRW13]

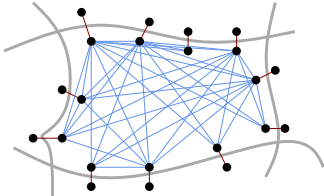
- Balanced separator nodes are important  
→ resulting CH is called *weak*
- $O(n^\alpha)$  separators  $\rightarrow O(n^\alpha)$  nodes in the search space
- Order is independent of metric



# (Multi-Level) Overlays [SWW00, HSW08]

**Observation:** many (long-distance) paths share large subpaths

**Idea:** precompute partial solutions

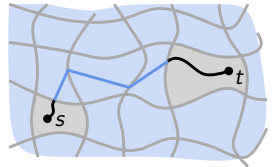


**Overlay graph:**

- Select **important nodes** (separators, path coverage, heuristic)
- Compute **shortcut-edges**:
  - Skip unimportant nodes
  - **Conserve distances** to important nodes

**Queries:**

- **Multi-level Dijkstra** variant
- Ignore edges towards **less** important nodes



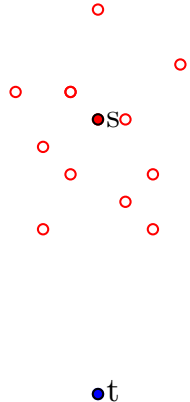
analogous: hierarchies with several levels of nodes of varying importances



# Hub Labeling

## Preprocessing:

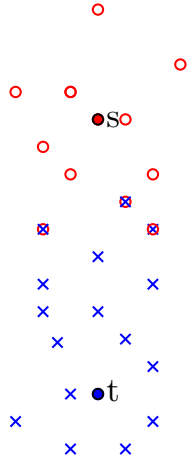
- For each node  $u$ , compute **label**  $L(u)$ 
  - A set of **hub** nodes  $v$  and their **distance**  $\text{dist}(u, v)$  to  $u$





## Preprocessing:

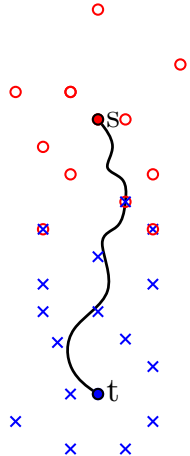
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for every  $s, t$ -pair, the shortest path goes through the intersection of  $L(s) \cap L(t)$





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- Find node  $v \in L(s) \cap L(t) \dots$



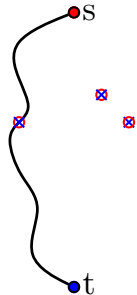


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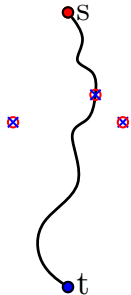


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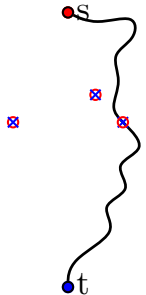


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## Observations:

- **Very simple query** (can even be implemented in SQL)
- Query performance depends only on label sizes
- The “magic” lies in computing a small labeling efficiently

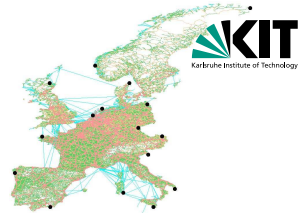




# Experimental Evaluation

**Input:** Road network of Europe

- Approx. 18M nodes
- Approx. 42M edges



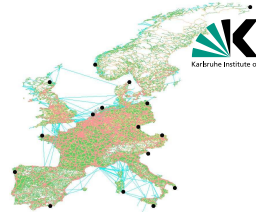
Algorithm	Preprocessing		Query	
	Time [h:m]	Space [GiB]	Time [ $\mu$ s]	Speedup
Dijkstra [Dij59]	—	—	2 550 000	—
ALT [GH05, GW05]	0:42	2.2	24 521	104
CRP [DGPW11]	1:00	0.5	1 650	1 545
Arc-Flags [Lau04]	0:20	0.3	408	6 250
CH [GSSD08]	0:05	0.2	110	23 181
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In use at Bing, Google, TomTom, ...



# New Challenges

## More realistic metrics:

- Turn costs, electro mobility
- Points of interests  
(nearest POIs, shortest via-POIs)
- User customizable metrics  
e.g., height restrictions, avoid freeways,  
eco-friendliness, . . .
- Fast customization time per metric
- Very small space overhead



## Multimodal networks:

- Change the type of transportation during the journey
- Allow only “reasonable” transfers
- Several constraints to the shortest path
- Multicriteria





## Electric vehicles:

- Future means of transportation
- Run on regenerative energy sources

## But:

- Restricted battery capacity
- Long recharging times
- “Range anxiety”



⇒ Consider energy consumption in route planning applications

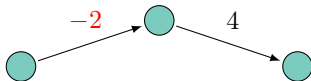
**Task:** Given start and destination in a road network,  
find the route that minimizes energy consumption.



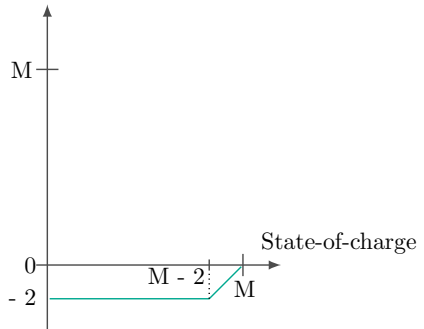
## Challenges:

- Negative edge weights (recuperation)
- Battery constraints (no over-, undercharging)

Energy consumption depends on battery state-of-charge (at the start):



Consumption

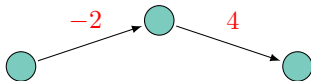




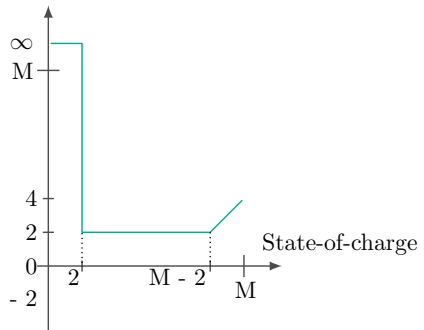
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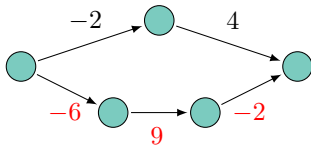




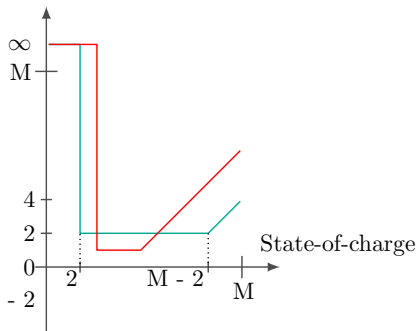
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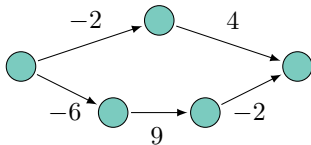




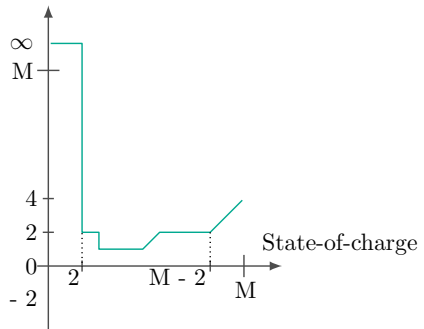
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## Requirements for speedup techniques:

- Shortcuts are functions, not scalar values
- User-dependent consumption profiles ( $\Rightarrow$  custom metrics)

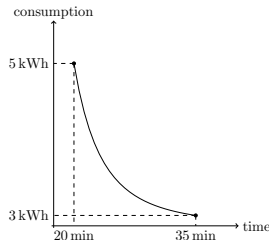
## Experiments:

- Energy-optimal paths: 63 % extra time
- Fastest paths: 62 % extra energy

$\Rightarrow$  Energy-optimal routes: follow slow roads

## Trading travel time for energy consumption:

- Consider constrained paths
  - E.g., find the fastest path such that the battery does not run out
  - $\mathcal{NP}$ -hard
- Energy can be saved driving below speed limit
- Additional instructions to the driver



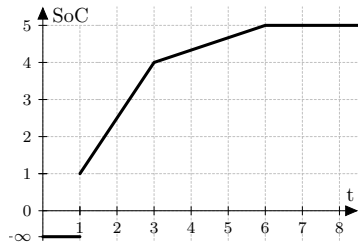


**Task:** Find the **fastest** path such that the battery does not run out.

- Recharging allowed at some nodes (but requires charging time).
- Realistic models of charging stations:
  - Charging power varies
  - Super chargers
  - Battery swapping stations

## Approach:

- Extension of bicriteria search
- Propagates charging functions
- CHArge: Combination with CH and A\*
  - Optimal routes in seconds / minutes
- Heuristic approaches (based on CHArge)
  - Near-optimal solutions in well below a second





## Problem

- Preprocessing is metric-dependent
- State-of-the-art algorithms tailored to travel time  
heavily exploit ‘hierarchy’ of road categories

## Naive solution

- Compute preprocessing for each metric, e. g.
  - Distance
  - Pedestrian
  - Travel time, but don't use toll roads
  - Travel time, avoid left turns, height restrictions, avoid tolls, ...
- Preprocessing and query time increase significantly
- Higher space overhead



# From Theory to Practice: Customizable Contraction Hierarchies [DSW14]

## Idea:

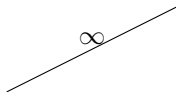
- CH topology is the same regardless of metric
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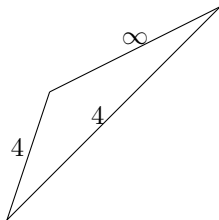
an edge in the CH



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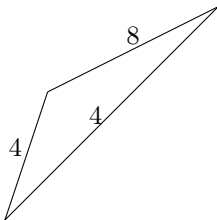
establish lower triangle inequality



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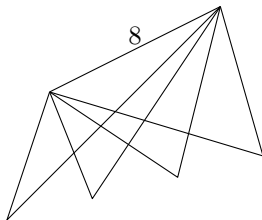
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do this for all lower triangles

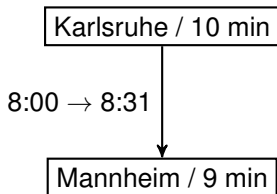


# What is a Timetable?

Karlsruhe / 10 min

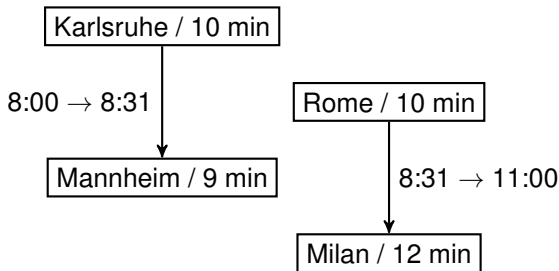


# What is a Timetable?



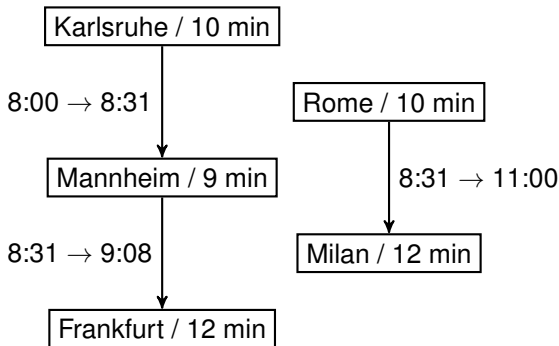


# What is a Timetable?





# What is a Timetable?





# Existing Approaches

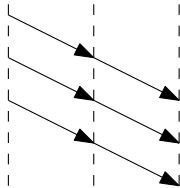
list of connections and stops

query



# Existing Approaches

list of connections and stops

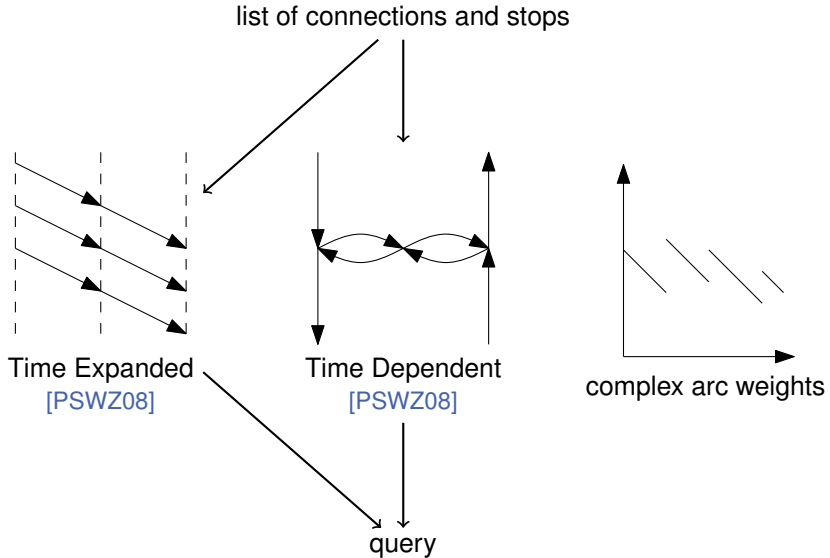


Time Expanded  
[PSWZ08]

query

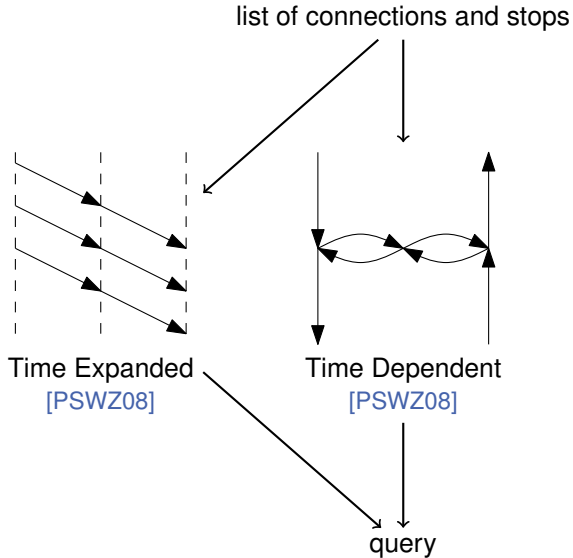


# Existing Approaches



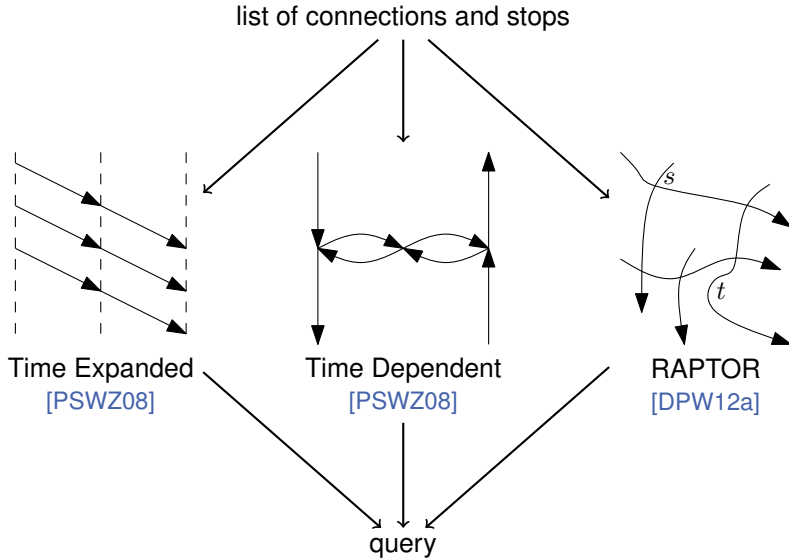


# Existing Approaches



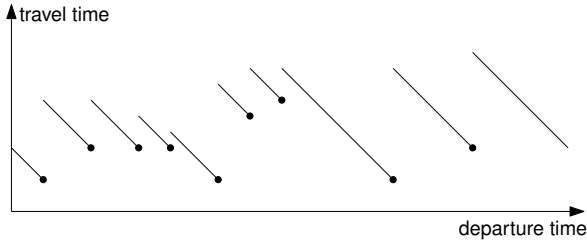


# Existing Approaches





- **Inherently time-dependent:** discrete departure times
- **More query scenarios:**
  - Depart now: earliest arrival time?
  - Depart later: shortest travel time?
  - Profile queries: set of journeys with varying departure times
  - Multicriteria: number of transfers, price, ...
- **Different network structure:** less hierarchical, less well-separated, very different schedules at night, ...





# Connection Scan (CSA) [DPSW13]

**Output:** earliest arrival time

**Input:** timetable, source stop, source time, target stop

stop ID	0	1	2	3	4	...
earliest arrival time	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$	...

elementary connections ordered by departure time	...	dep.stop	arr.stop	dep.time	arr.time		dep.stop	arr.stop	dep.time	arr.time		dep.stop	arr.stop	dep.time	arr.time		...
---	-----	----------	----------	----------	----------	--	----------	----------	----------	----------	--	----------	----------	----------	----------	--	-----

\*missing in the example: footpaths and minimum change times



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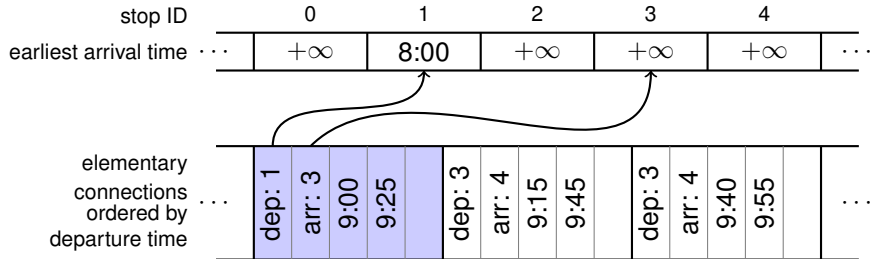
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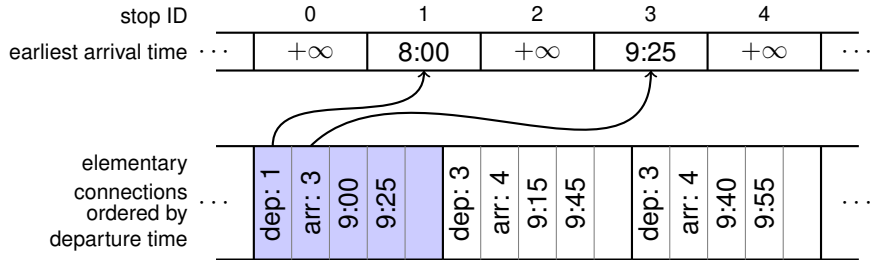
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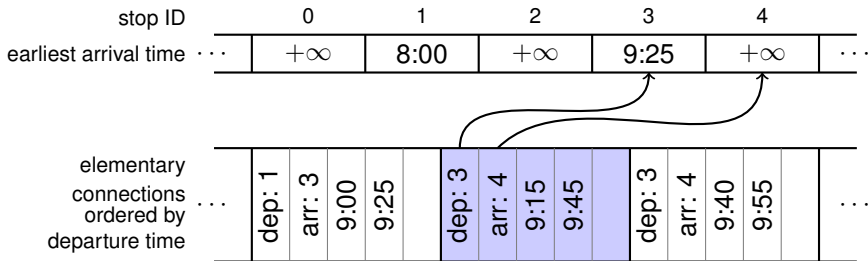


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**Output:** earliest arrival time

**Input:** timetable, source stop, source time, target stop



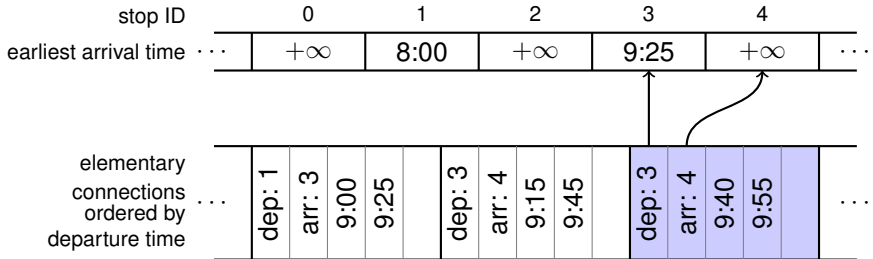
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**Output:** earliest arrival time

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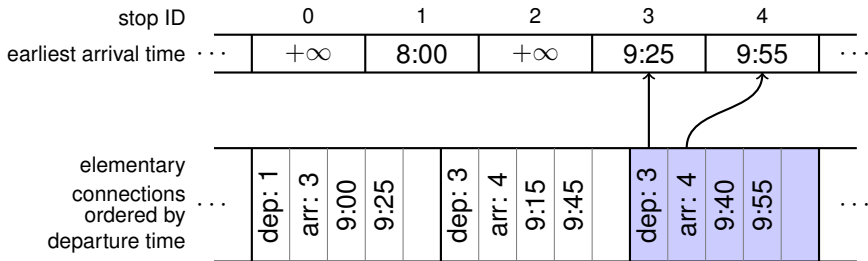
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---	--------	--------	------	------	--	--------	--------	------	------	--	--------	--------	------	------	--	-----

\*missing in the example: footpaths and minimum change times

time table graph is a DAG  
faster than Dijkstra, better use of modern processor architectures



## Input: timetable

- London: 5 M connections, 21 k stops
- Germany: 46 M connections, 252 k stops

	Algorithm	Time [ms]	speed-up.
London	TE Dijkstra [PSWZ08]	44.8	—
	TD Dijkstra [PSWZ08]	10.9	4.1
	CSA [DPSW13]	1.8	24.9
DE	TE Dijkstra [PSWZ08]	2960.2	—
	CSA [DPSW13]	298.6	9.9
	CSAccel [SW14]	8.7*	340.2

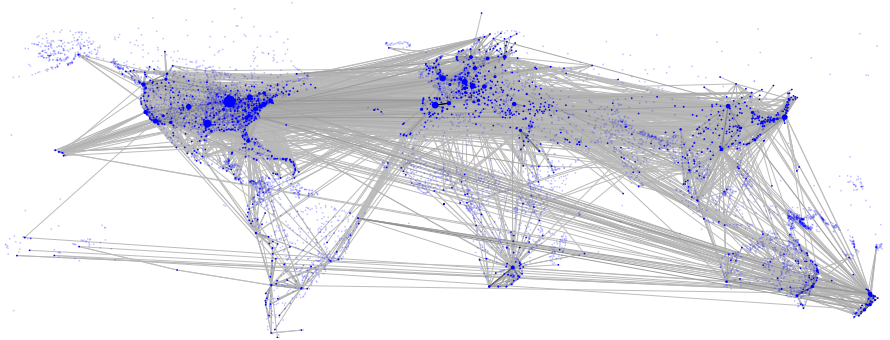
Intel Xeon E5-2670, 2.6 GHz, 64 GiB DDR3-1600 RAM, 20 MiB L2 cache

\*preprocessing: 30 min, 256.4 MiB



# Navigation Device for the World

Worldwide network composed of car, rail, flight, ...



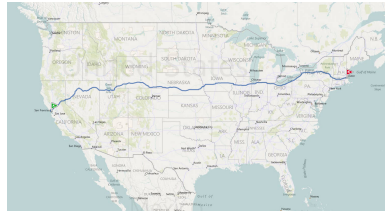


Up to now:

- Restricted to **one** transportation network
- Time-independent and time-dependent (separately)

What we **really** want is planning a journey by

- Choosing **source** and **destination**
- **Desired means** of transportation (car, train, flight, . . .)
- . . . in a **mixed network**





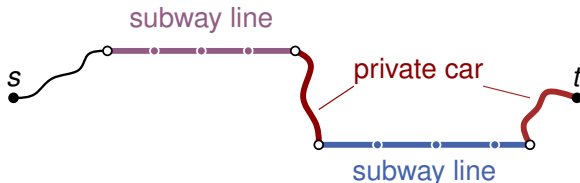
- **Bidirectional search**  
easily adaptable (time-dependency is hard)
- **Goal-directed search**  
ALT adaptable but low speed-ups,  
Arc-Flags turns out difficult
- **Contraction**  
adaptable with some restrictions
  - Contracted graph is called the **Core**

two promising approaches:

- **Access-node routing (ANR)**  
adapting ideas from transit-node routing (table lookups)
- **User-constrained CH (UCCH)**  
augmenting contraction hierarchies

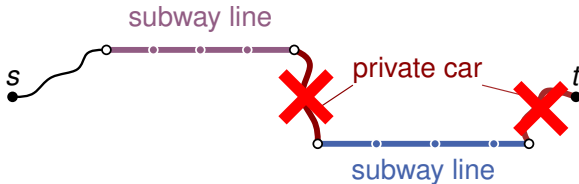


**Problem:** Unrestricted journeys allow arbitrary transfers





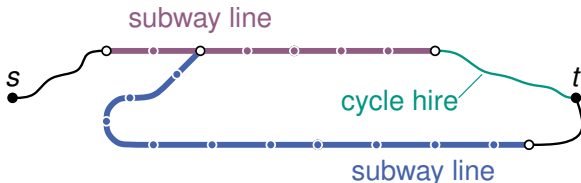
**Problem:** Unrestricted journeys allow arbitrary transfers



- Not all sequences of transportation modes are reasonable



**Problem:** Unrestricted journeys allow arbitrary transfers

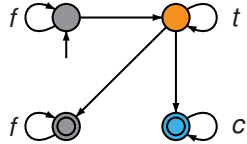


- Not all sequences of transportation modes are reasonable
- Preferred mode of transport varies between users



## “Label Constrained Shortest Path Problem” (LCSP)

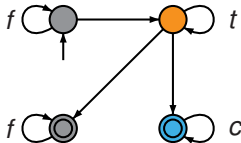
- Define alphabet of transportation mode
- Finite-state automaton describes sequences of vehicles
- Every path must fulfill the requirements imposed by the automaton





## “Label Constrained Shortest Path Problem” (LCSP)

- Define alphabet of transportation mode
- Finite-state automaton describes sequences of vehicles
- Every path must fulfill the requirements imposed by the automaton



## Algorithms for LCSP

- Dijkstra on the product graph with the automaton works but is slow [BJM00]
- Speed-up techniques: ANR [DPW09], SDALT [KLPC11]
- Automaton as input during the query: UCCH [DPW12b]



## Multimodal CH:

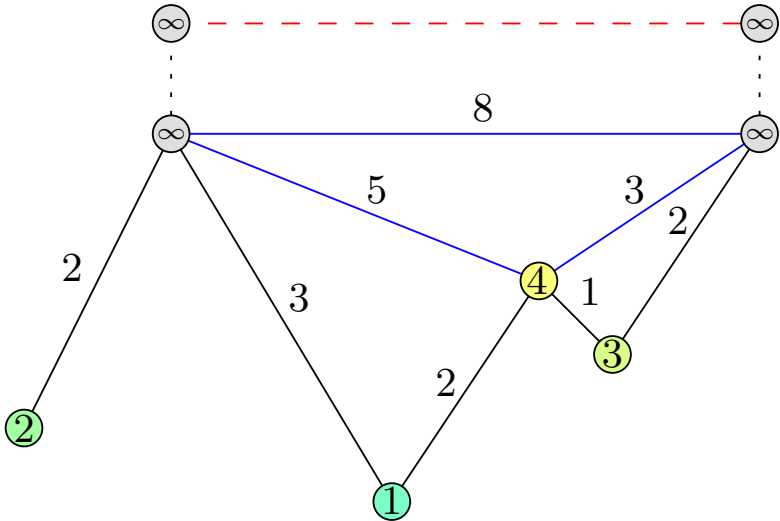
- Contraction introduces shortcuts with label sequences
- Witness search depends on constraints  
requires a-priori knowledge of the constraint automata

**Idea:** do not contract nodes with incident link-edges.

- Contraction and witness search are limited to each modality
- ⇒ Preprocessing independent of mode sequence constraints



# Example: UCCH Preprocessing





## Preprocessing

- Linked nodes are not contracted thus contained in the core
- Shortcuts between core nodes preserve distances  
allows using the road network between rail stations

## Query

- CH search on the component
- Label constrained search on the core
- Engineering yields further improvement



# Experimental Evaluation

## Networks:

road: europe & north america (50 M nodes, 125 M edges)

train: europe (31 k stops, 1.6 M connections)

flight: Star Alliance (1 172 airports, 28 k connections)

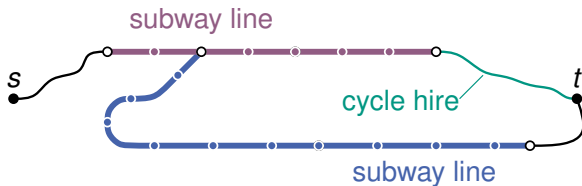
		Preprocessing		Query	
	Algorithm	Time [h:m]	Space [MiB]	Time [ms]	Speedup
road & flight	Dijkstra	—	—	33 862	1
	ANR <small>[DPW09]</small>	3:04	14 050	1.07	31 551
	UCCH <small>[DPW12b]</small>	1:18	542	0.67	50 540
all three	Dijkstra	—	—	35 261	1
	ANR <small>[DPW09]</small>	—	—	—	—
	UCCH <small>[DPW12b]</small>	1:27	558	70.52	500

Intel Xeon E5430, 2.66 GHz, 32 GiB RAM, 12 MiB L2 cache



# Solution?

## Problems of LCSP





# Solution?

## Problems of LCSP

$s$   
•

?

$t$   
•



## Problems of LCSP

$s$   
•

?

$t$   
•

- Restrictions must be known in advance
- User might not know them
- Only a single (best?) journey is computed (no alternatives)

**Goal:** compute a *useful set* of multimodal journeys



# Multicriteria Multimodal Routing [DDP<sup>+</sup>13]

**Idea:** compute **multicriteria**, multimodal Pareto sets

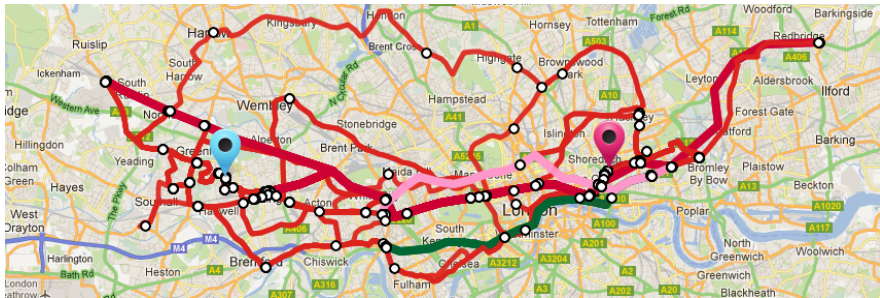
- Optimize arrival time plus
- Various (per mode of transport) „**convenience criteria**“  
for example # transfers (trains), walking time, taxi costs, etc.



# Multicriteria Multimodal Routing [DDP+13]

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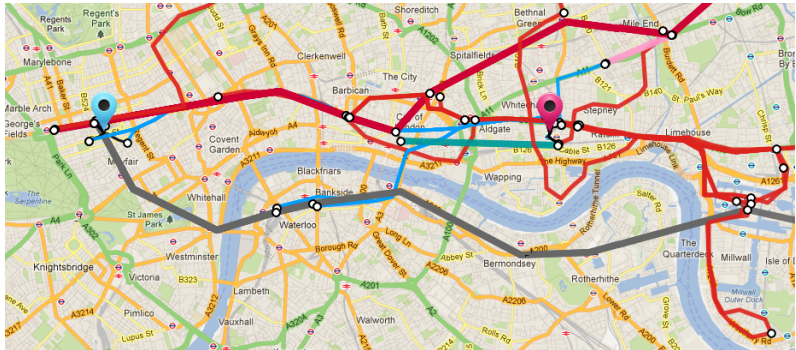


criteria: arrival time, # transfers, walking time 69 journeys.

**Known problem:** Pareto set sizes explode in the number of criteria



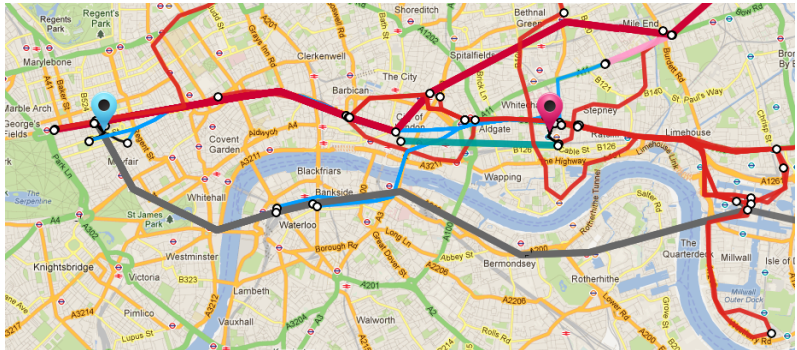
# Relevant Journeys



- 10 min of walking to arrive 10 sec earlier?
- 1 hour of bus drive to walk 10 sec less?



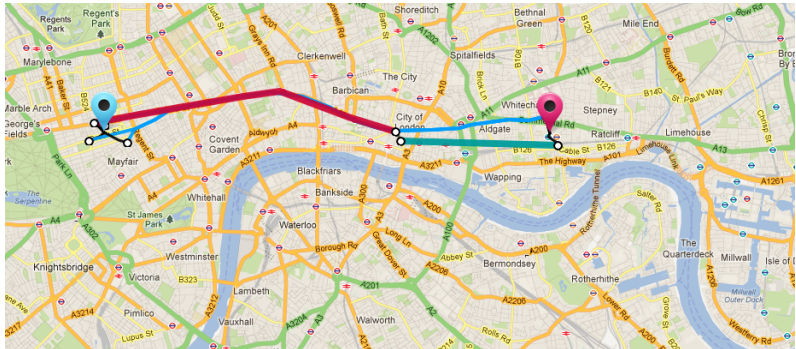
# Relevant Journeys



- 10 min of walking to arrive 10 sec earlier?
- 1 hour of bus drive to walk 10 sec less?
- Rate the journeys using fuzzy logic [FA04]
- Journeys with a higher rating are more relevant



# Relevant Journeys



the three top-rated journeys.

- 10 min of walking to arrive 10 sec earlier?
- 1 hour of bus drive to walk 10 sec less?
- Rate the journeys using fuzzy logic [FA04]
- Journeys with a higher rating are more relevant



**Problem:** queries are slow ( $> 1$  s)

many irrelevant journeys  $\Rightarrow$  can we avoid computing them?

## Filter already during the algorithm

- **MCR-hf:** fuzzy filter
- **MCR-hb:** Pareto filter, but discrete criteria

## Restricted walking (arbitrary heuristic)

- **MCR-tx-ry:** max  $x$  minutes of walking between vehicles and max.  $y$  at source/target

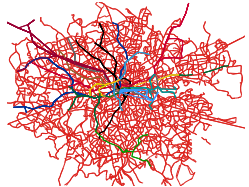
## Reduce the dimension/number of criteria

- **MR-x:** increase for every  $x$  minutes of walking the #transfers by +1



## London, multimodal:

- Roads: 260 k nodes, 1.4 M edges
- Subway, bus, tram, ...  
21 k stops, 5 M connections
- 564 cycle hire station



**Criteria:** arrival time, # transfers, walking time

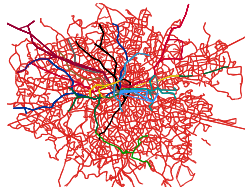
Algorithm	# Sol.	Time [ms]	Quality-6	
			Avg.	Sd.
MCR	29.1	1 438.7	100 %	0 %
MCR-hf	10.9	699.4	89 %	11 %
MCR-hb	9.0	456.7	91 %	10 %
MCR-t10-r15	13.2	885.0	30 %	31 %
MR-10	4.3	39.4	45 %	29 %

Intel Xeon E5-2670, 2.6 GHz, 64 GiB DDR3-1600 RAM, 20 MiB L2 cache



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## Summary

- **Algorithm Engineering**: combination of theory and practice
- (Very) **fast route planning** on road and timetable networks
- Considered **metric** matters
- **Multimodal route planning** is more expensive
  - Network offers **many** interesting **trade-offs** between criteria
  - **Multicriteria optimization** useful, to allow the user to choose his journey

## Outlook

- Formalization of **quality** for multimodal journeys done?
- **Scalability**: multimodal multicriteria for worldwide routing?
- Additional questions: **delay-robustness**, **park & ride**, ... ?



# Thank you for your attention!







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