Algorithmic Challenges in Power Grids

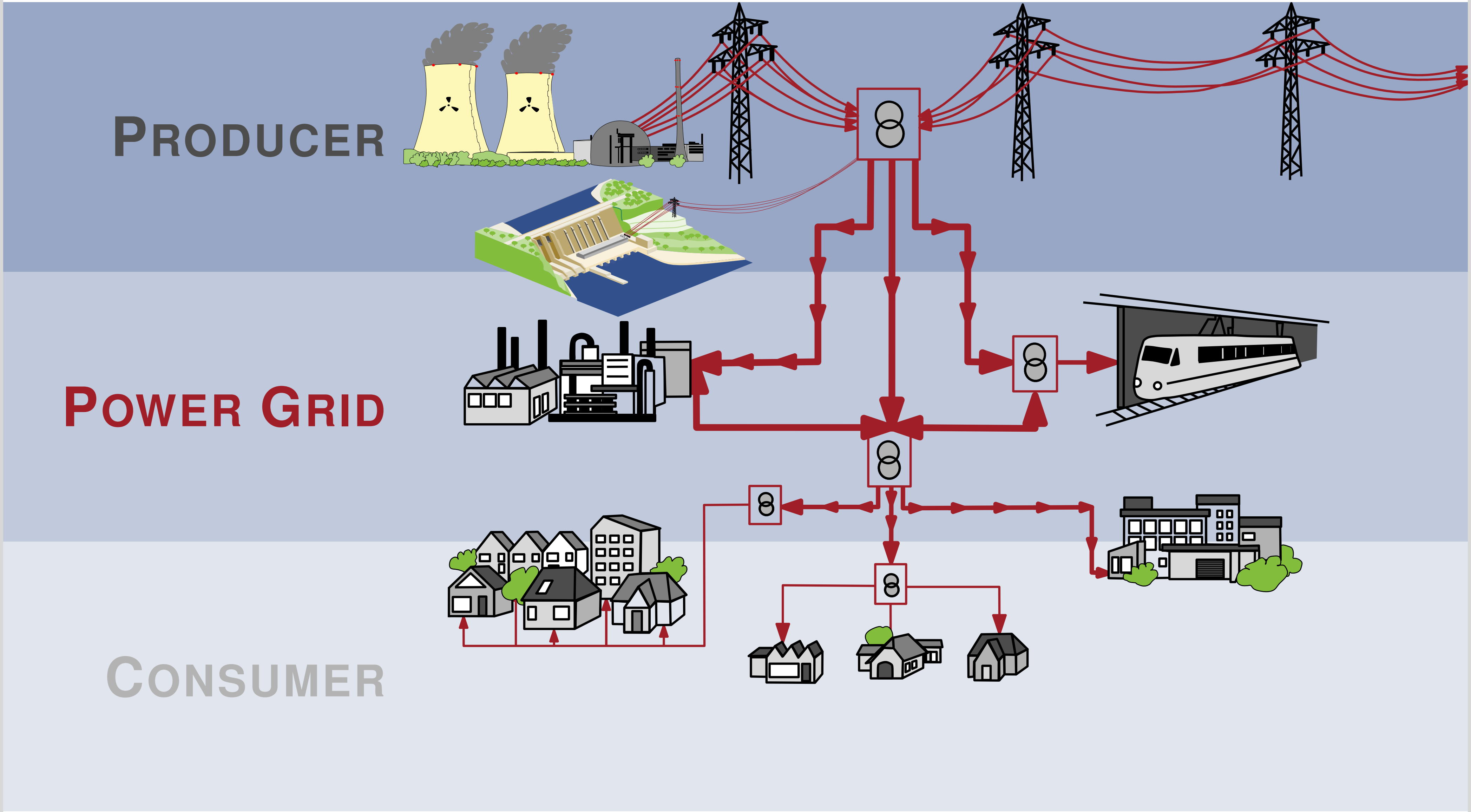
Algorithms for Big Data · Indo-German Spring School · February 19th, 2019

Dorothea Wagner

INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP

Karlsruhe Institute of Technology

Recent Development in Power Grids



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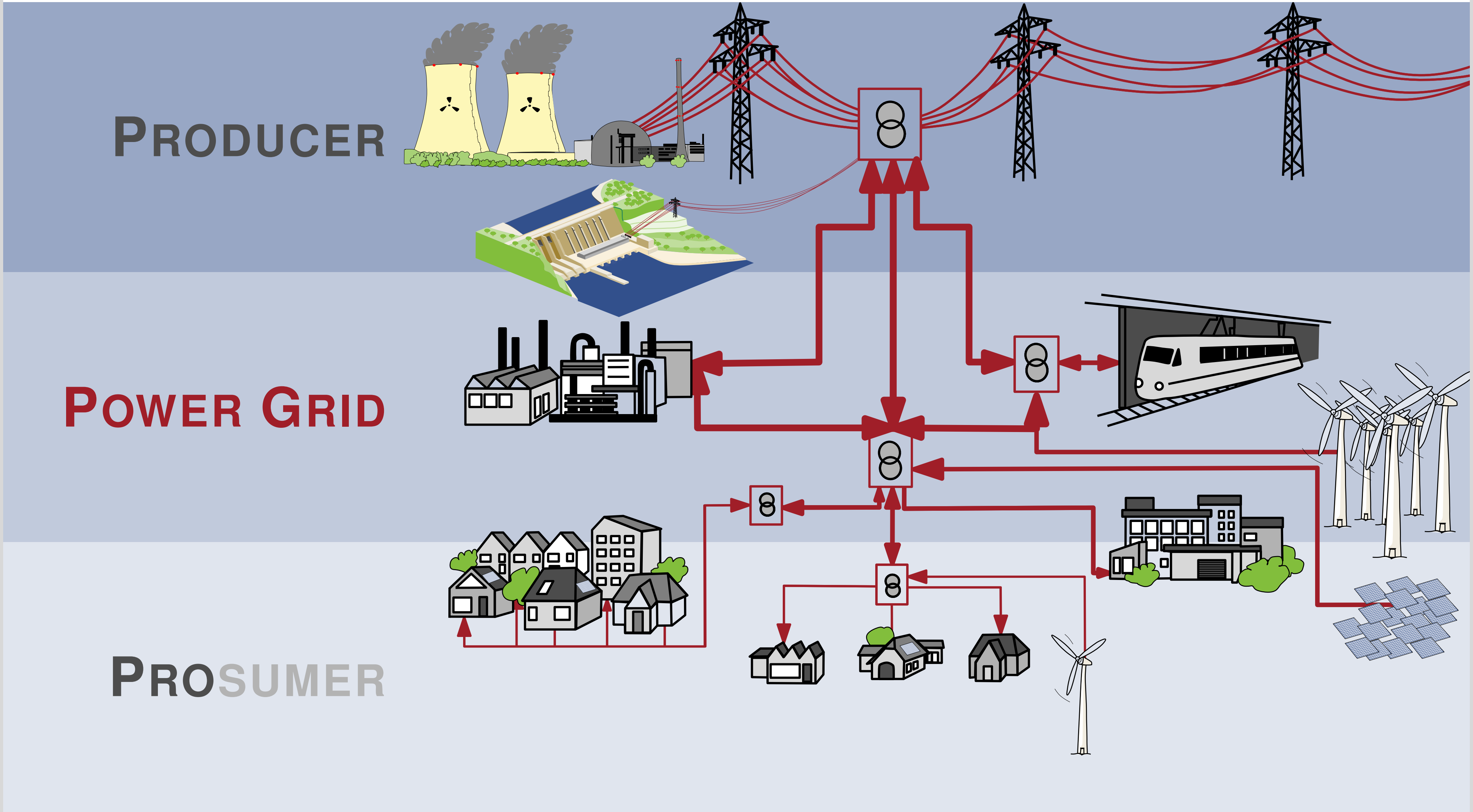

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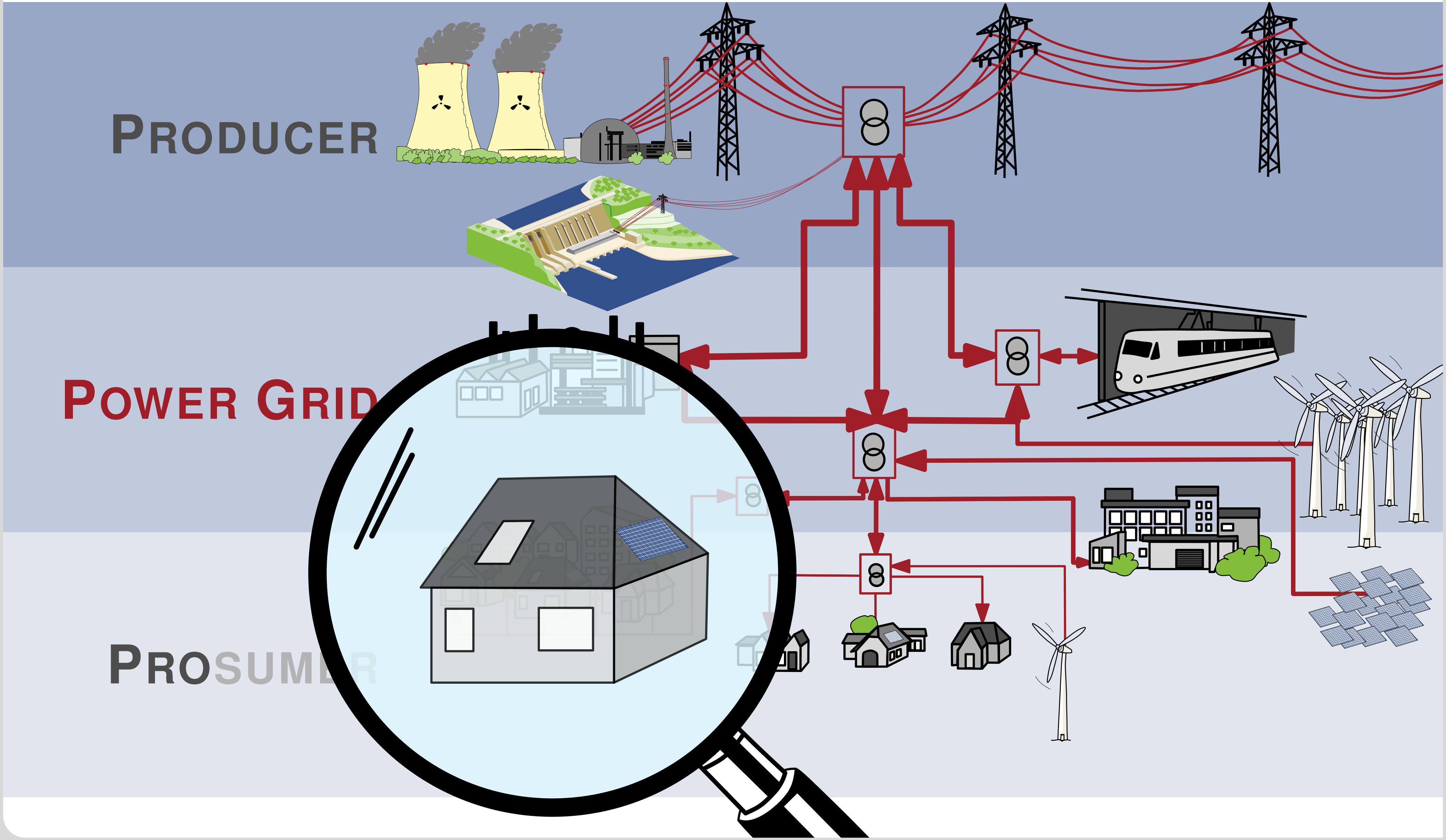

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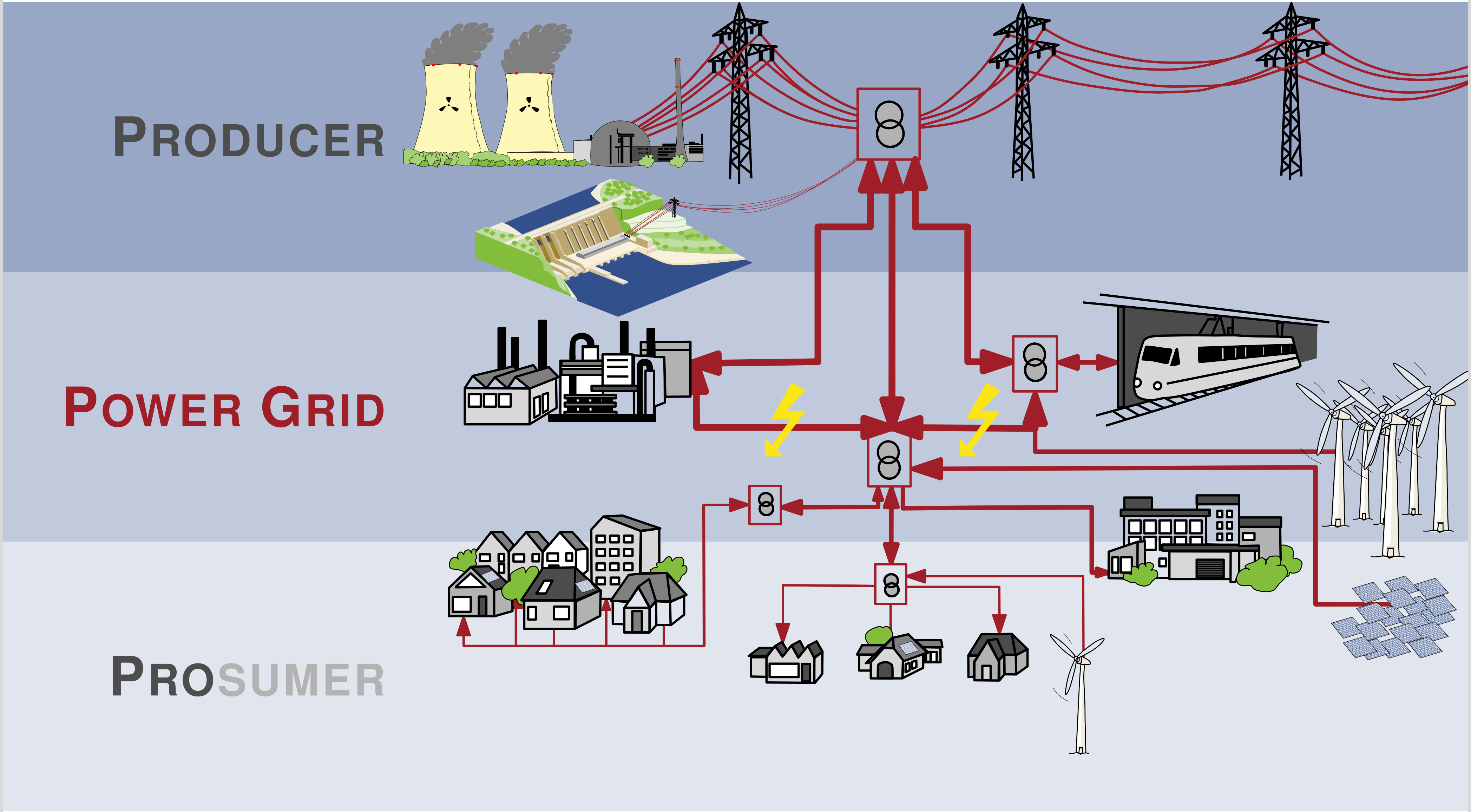

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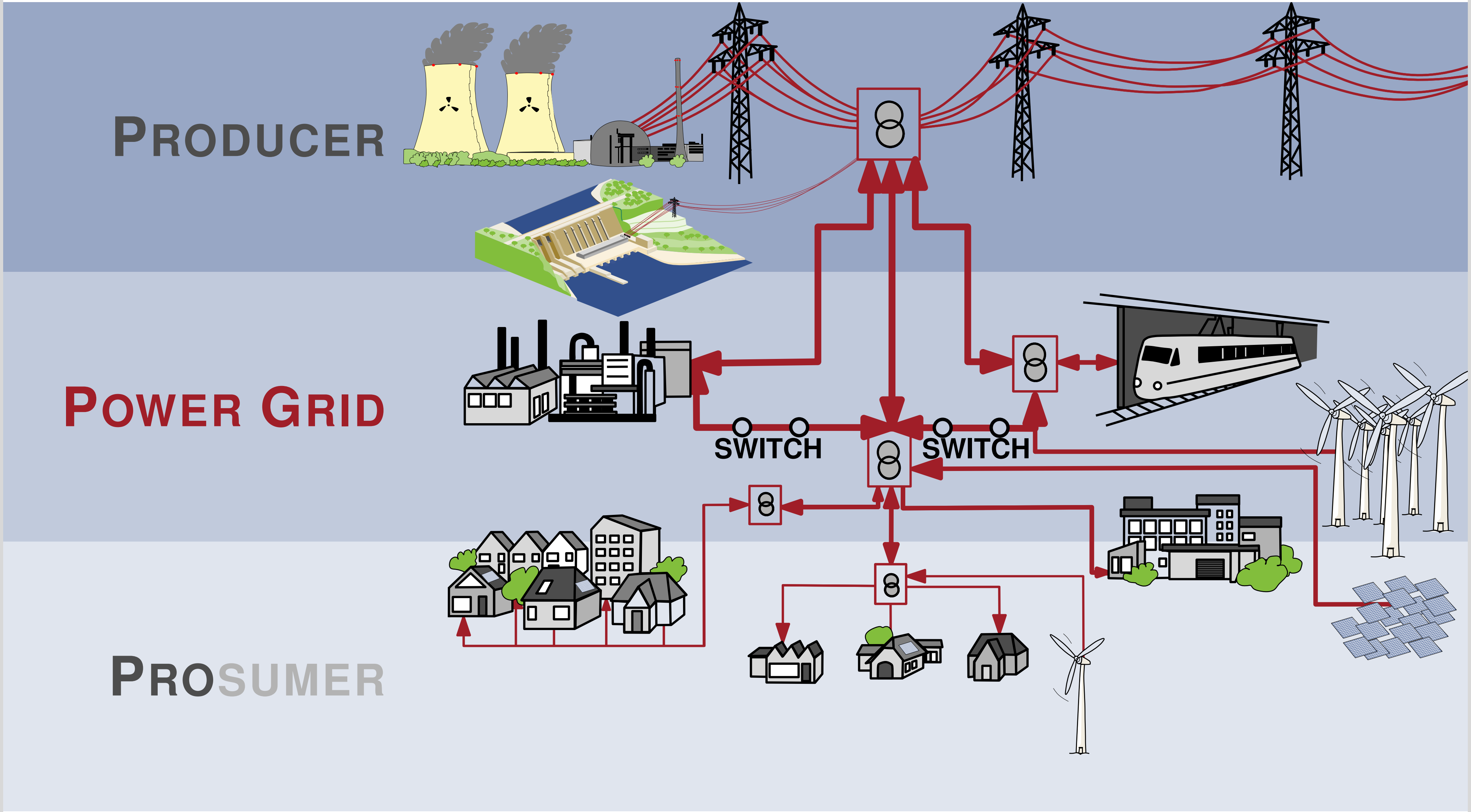

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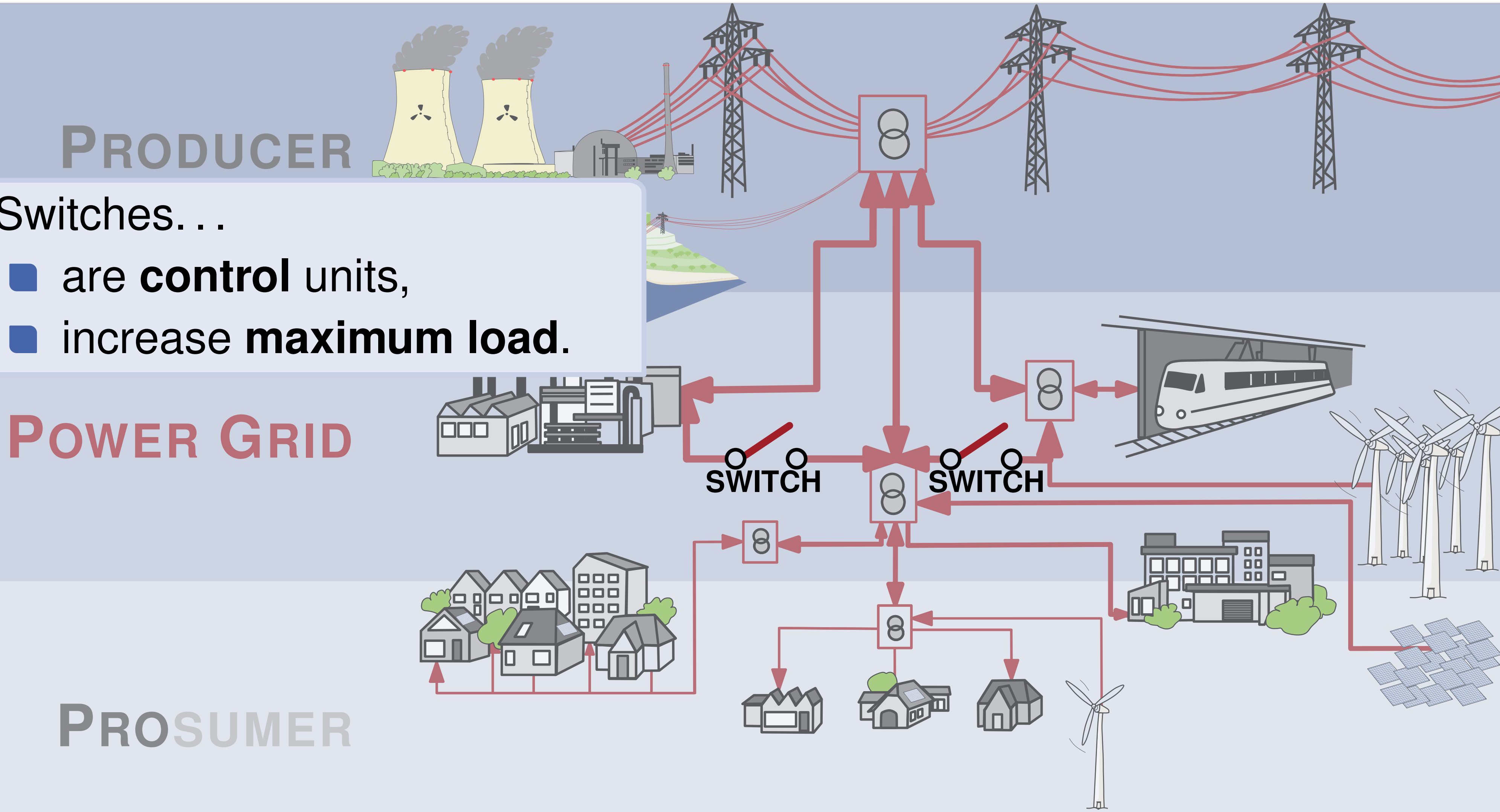
Recent Development in Power Grids

Switches. . .

- are **control** units,
- increase **maximum load**.



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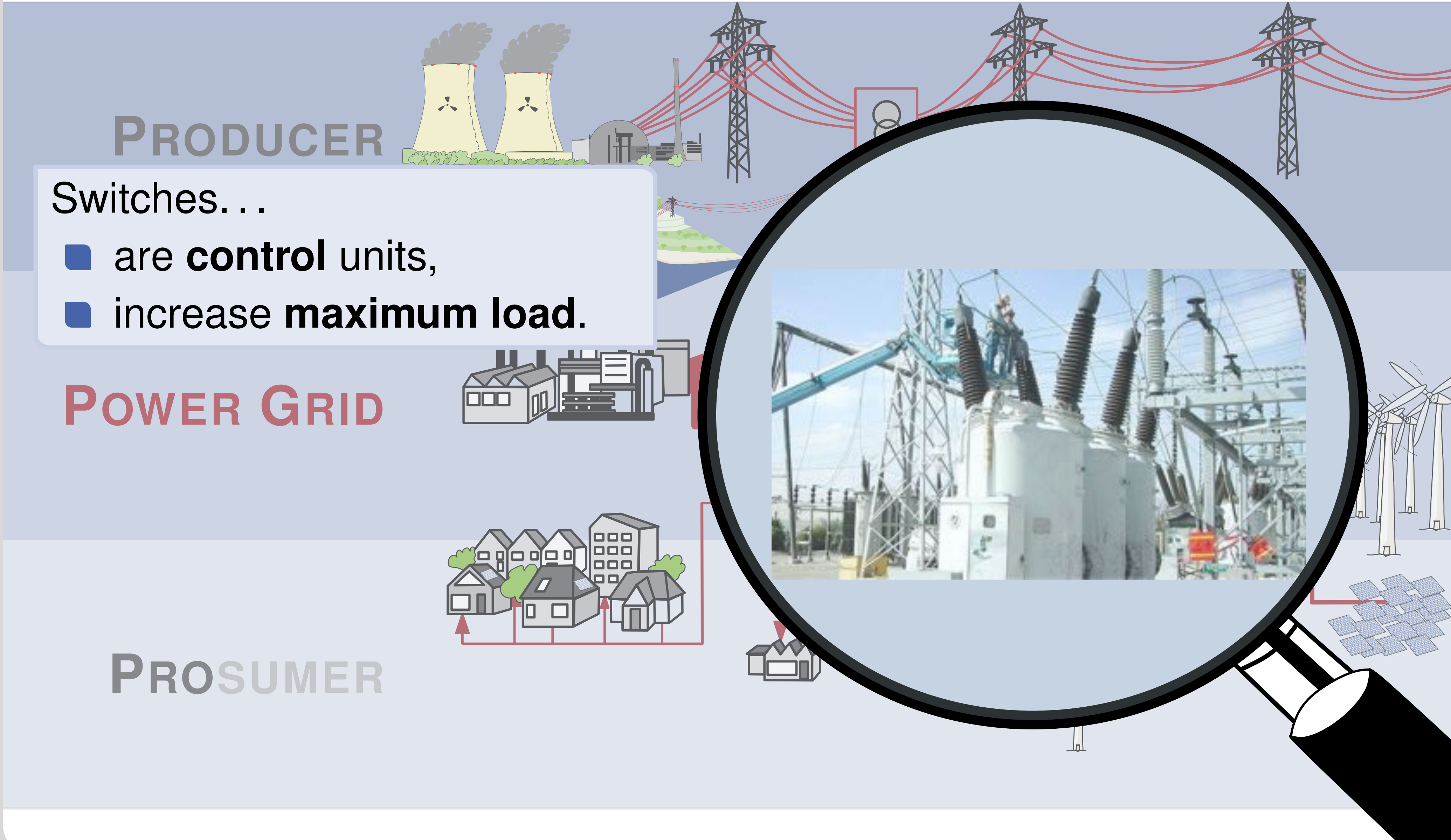

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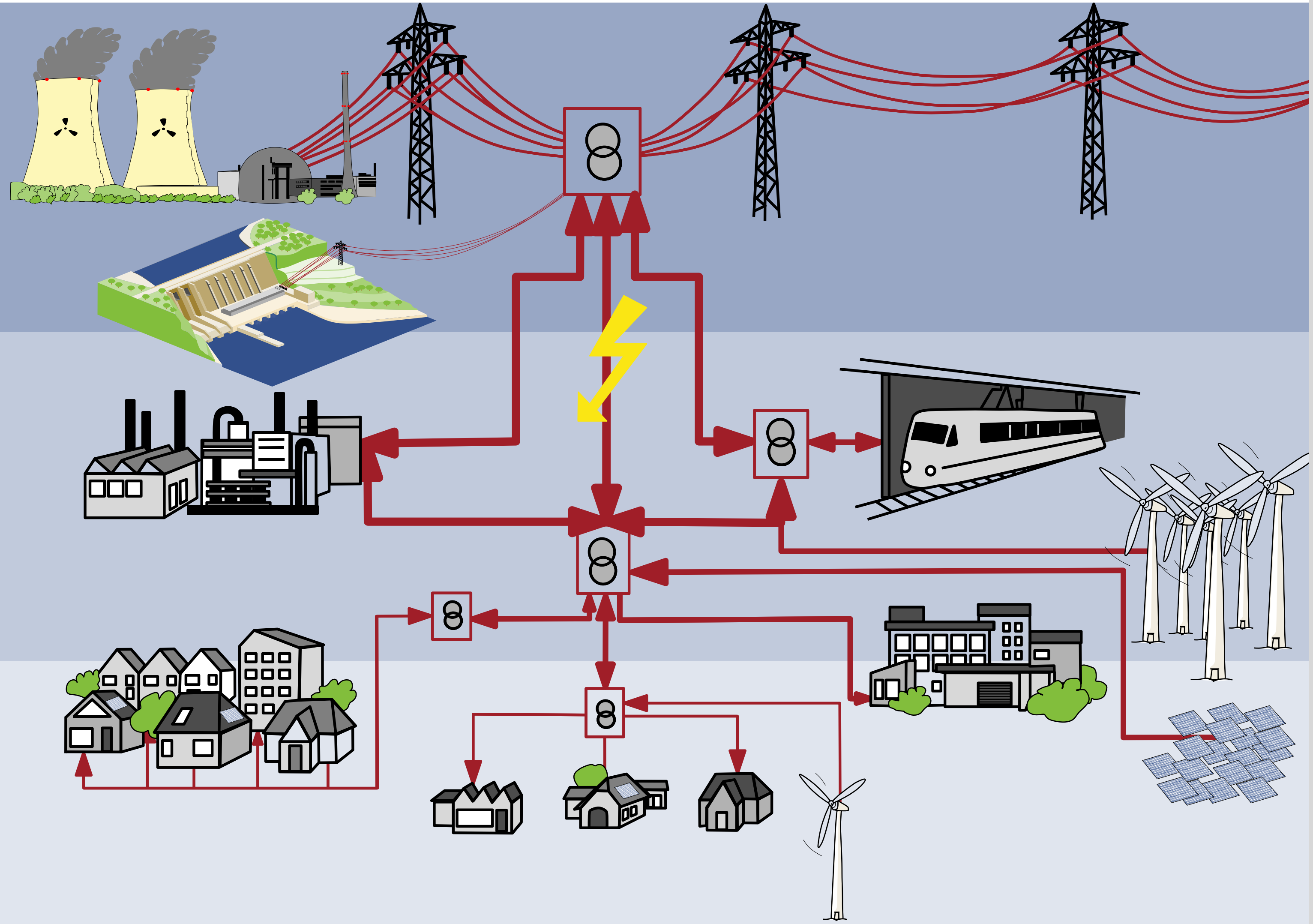
Recent Development in Power Grids

PRODUCER

POWER GRID

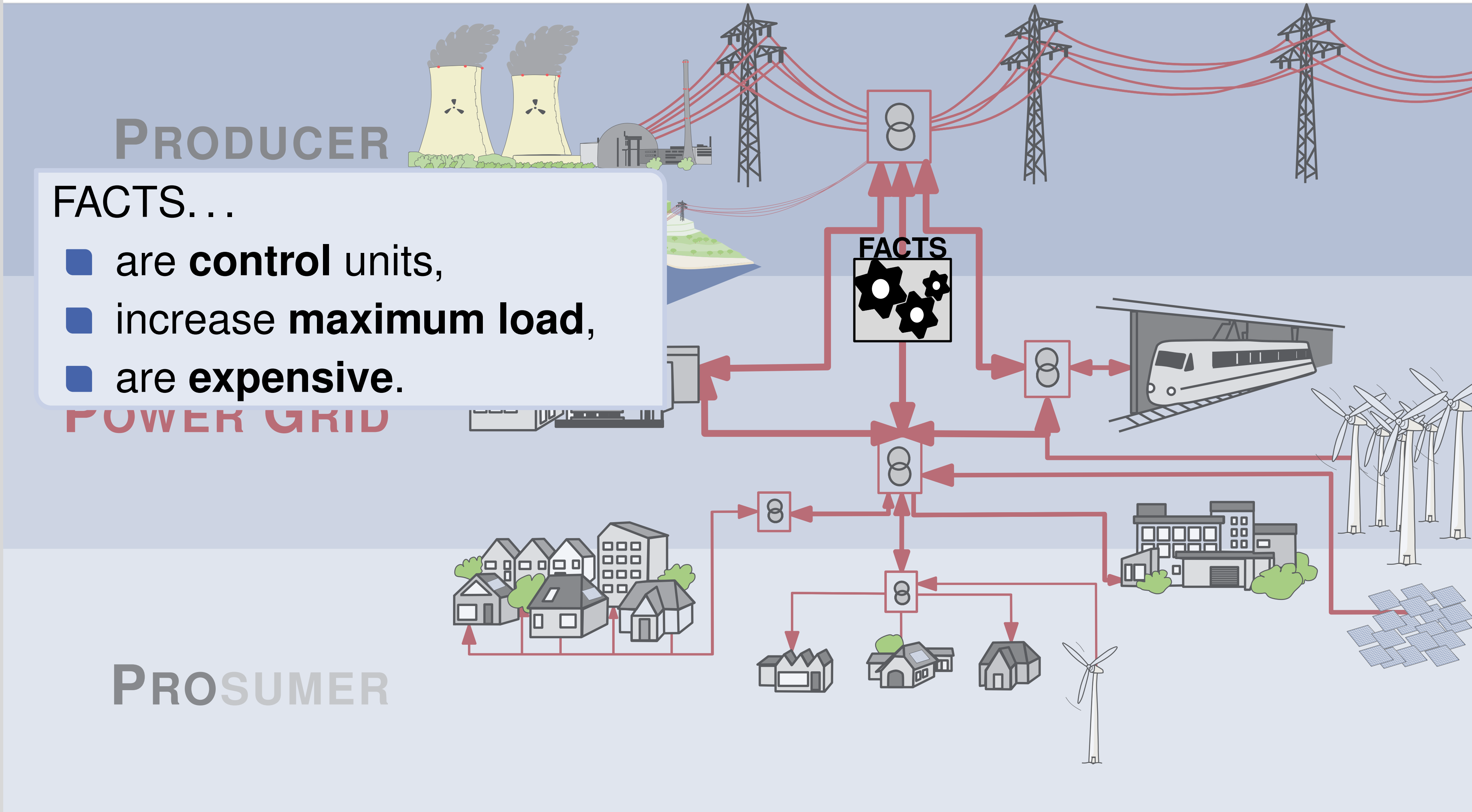
PROSUMER



Recent Development in Power Grids

FACTS…

- are **control** units,
- increase **maximum load**,
- are **expensive**.



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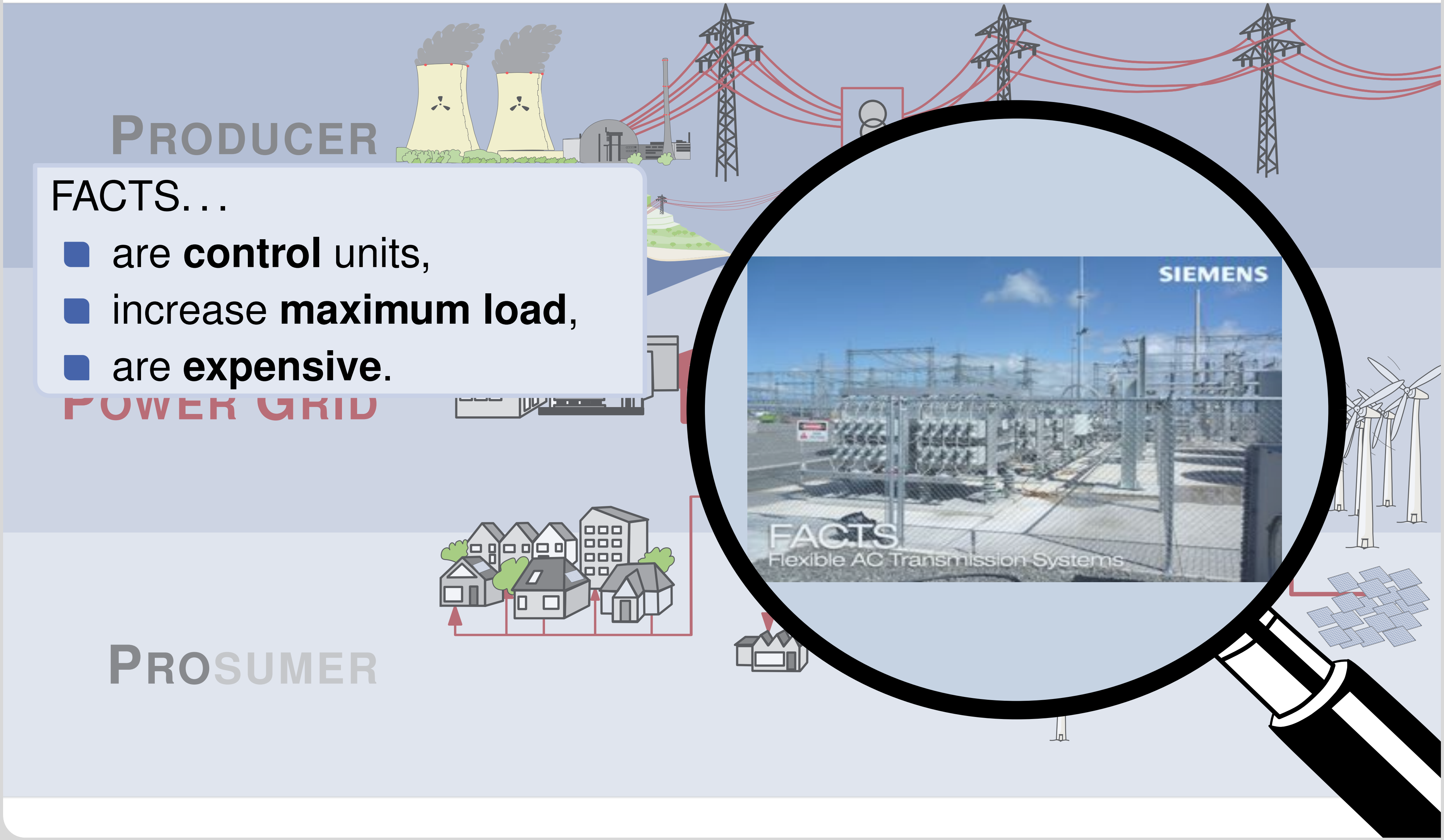

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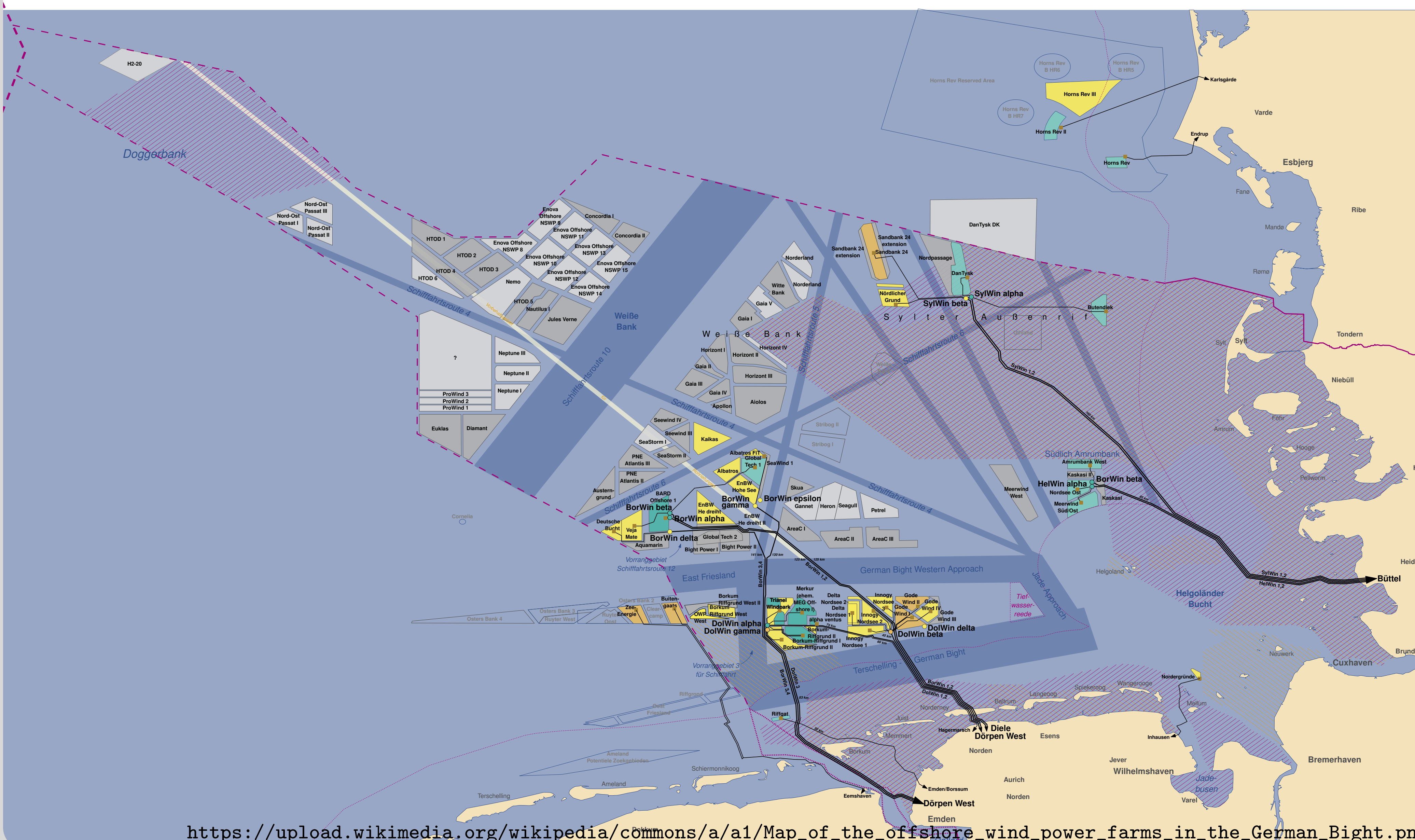
Recent Development in Power Grids and Offshore



https://upload.wikimedia.org/wikipedia/commons/a/a1/Map_of_the_offshore_wind_power_farms_in_the_German_Bight.png

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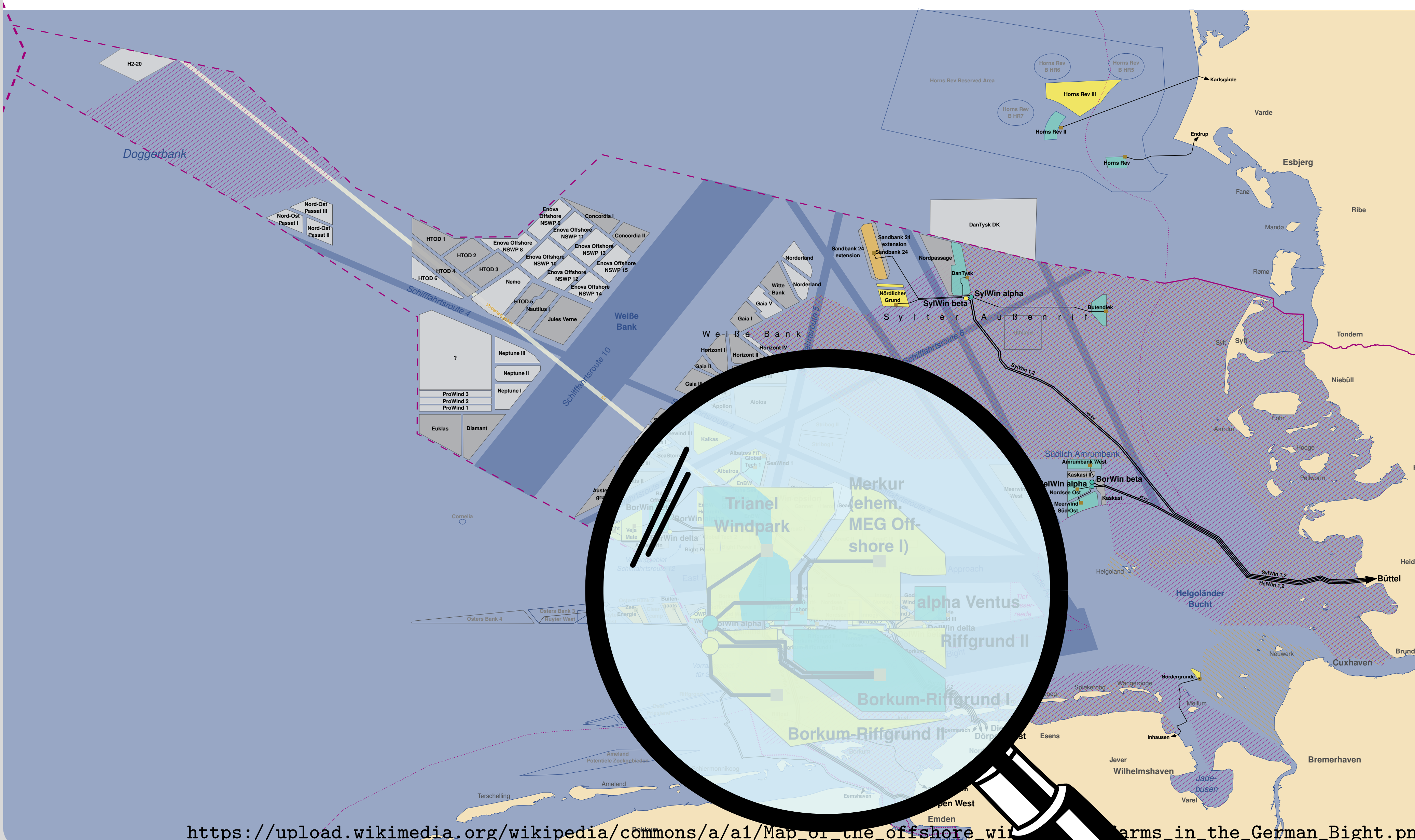
Recent Development in Power Grids and Offshore



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Recent Development in Power Grids and Offshore

Challenges

- Increasingly distributed energy production
- Independent power producers
- Volatile power flows and flow directions

$\Rightarrow$ Operating the power grid gets more demanding

Strategies to cope with the challenges

- Network expansion
- Investment in advanced control units (e.g. FACTS, Switches) for better utilization of existing grid

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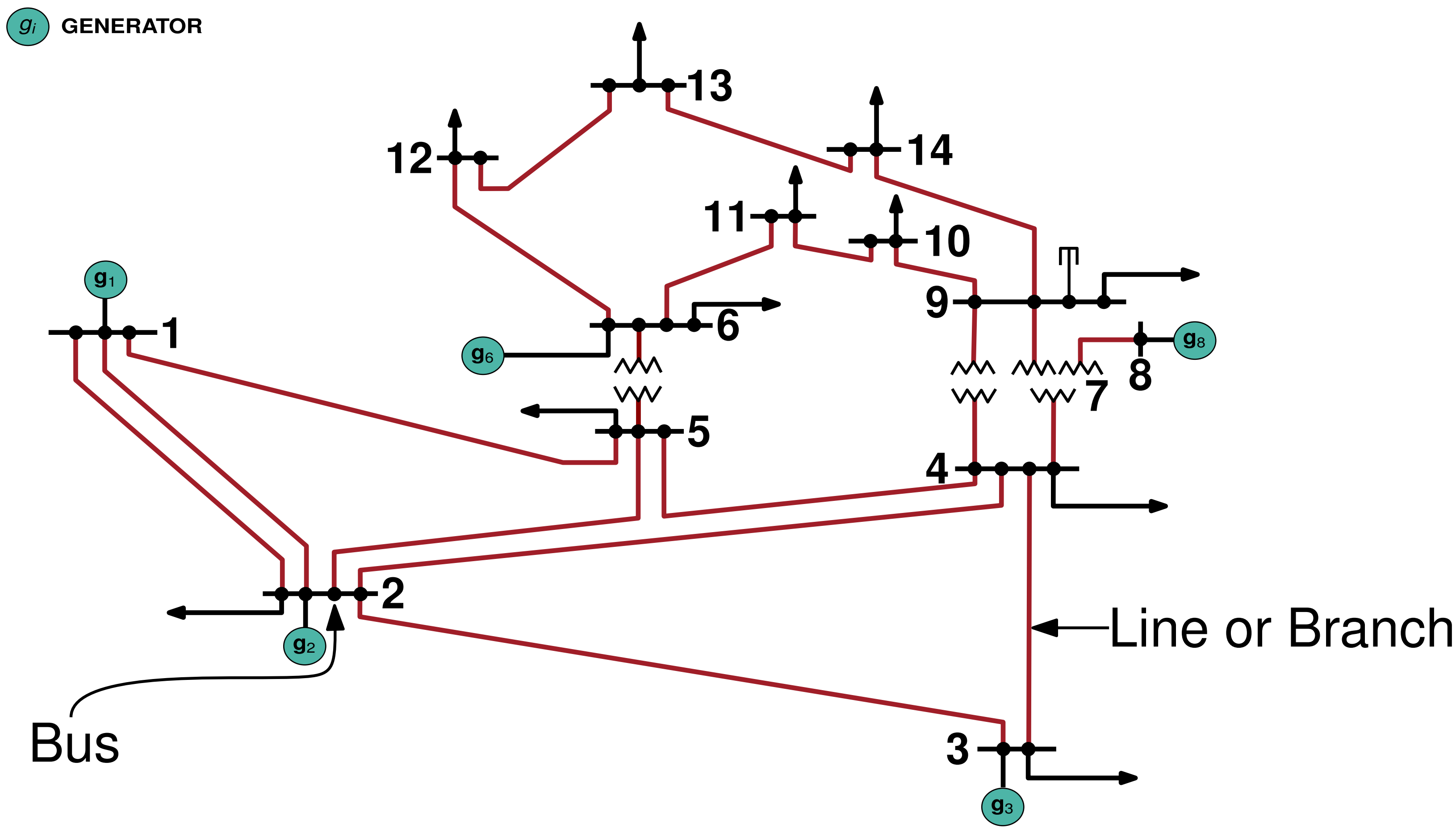
From a Transmission Network to a Graph

[University of Washington, 1999]

Graph $G = (V, E)$



AEP 14 BUS TEST SYSTEM BUS CODE DIAGRAM

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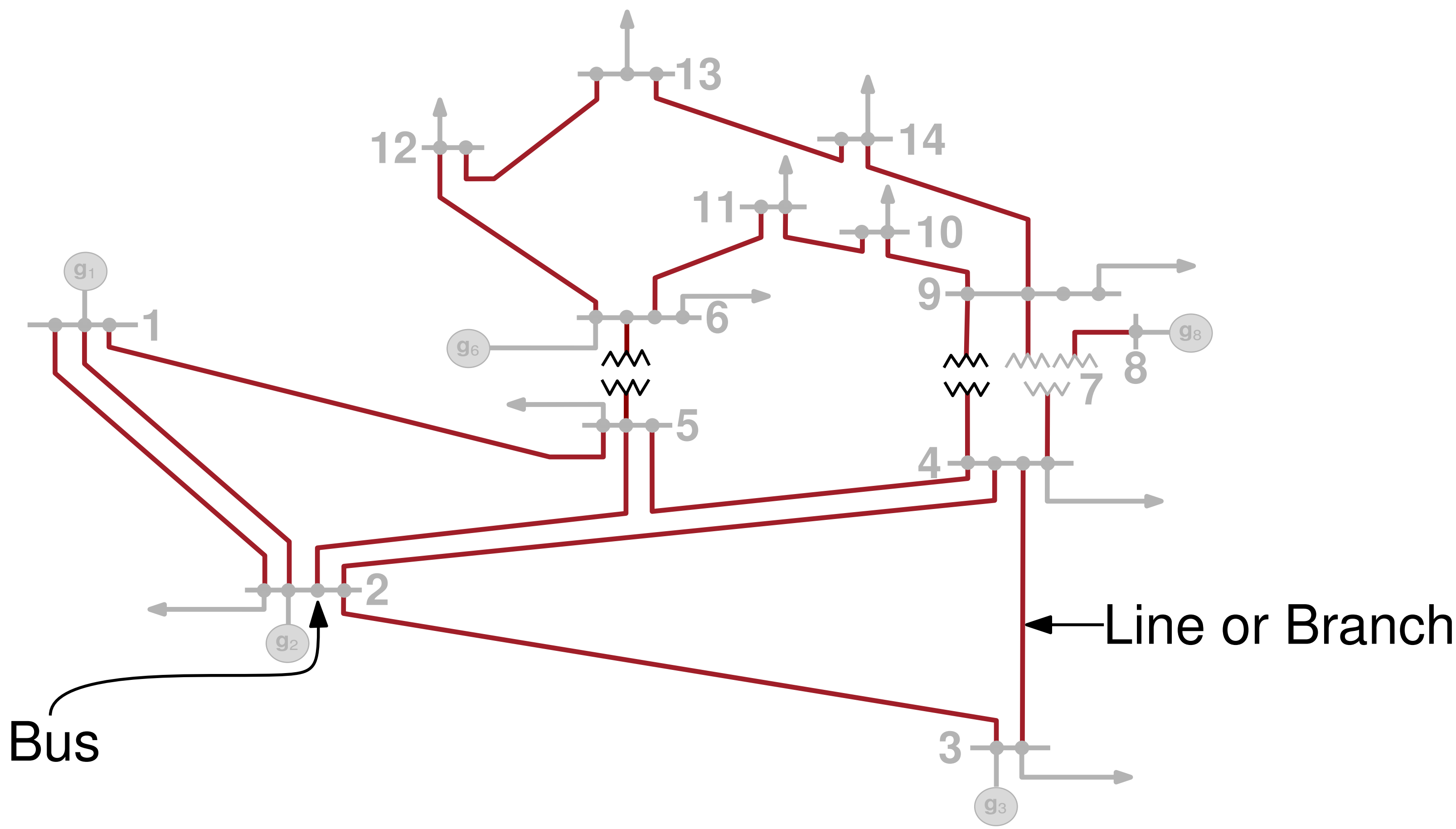

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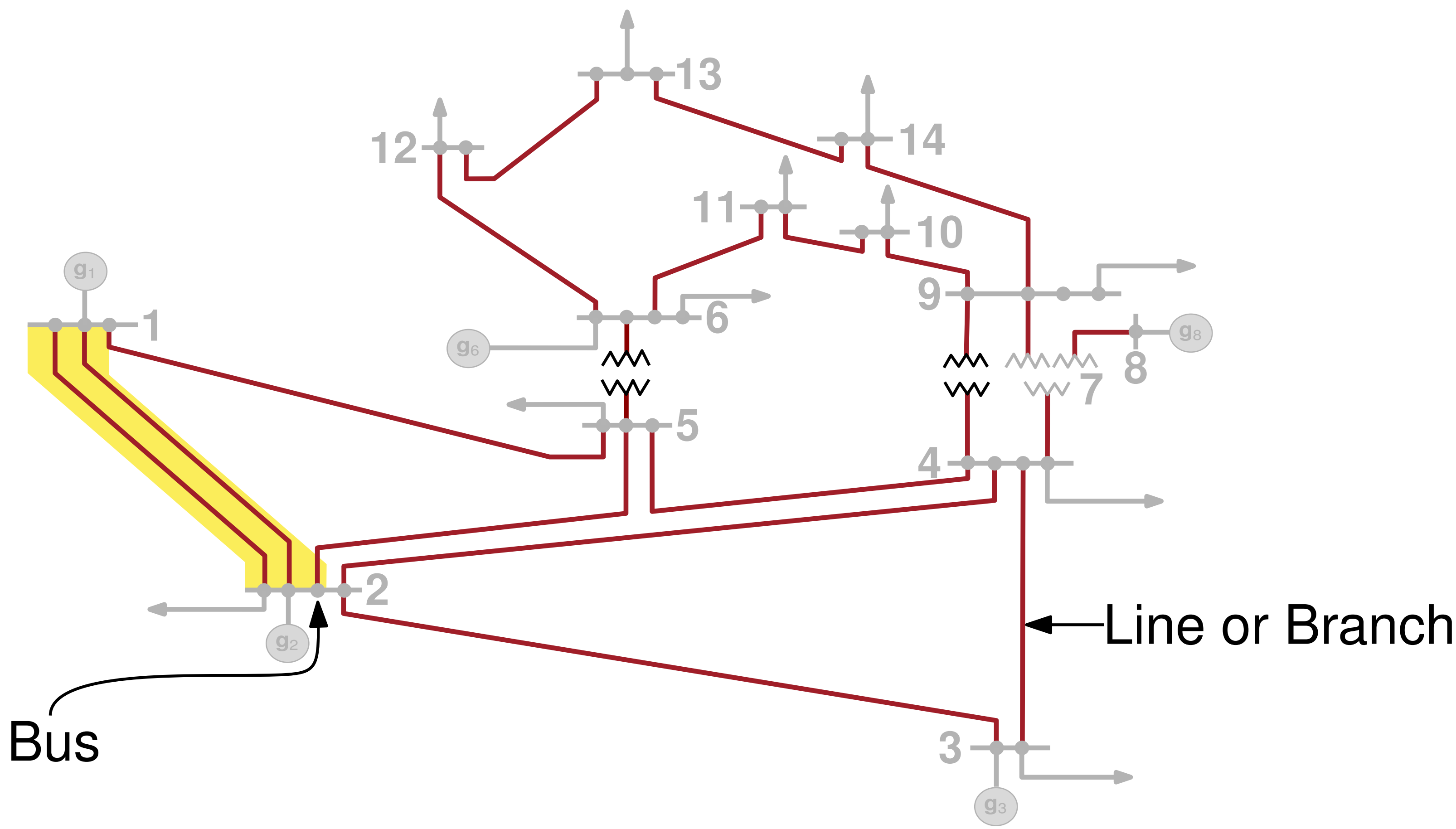

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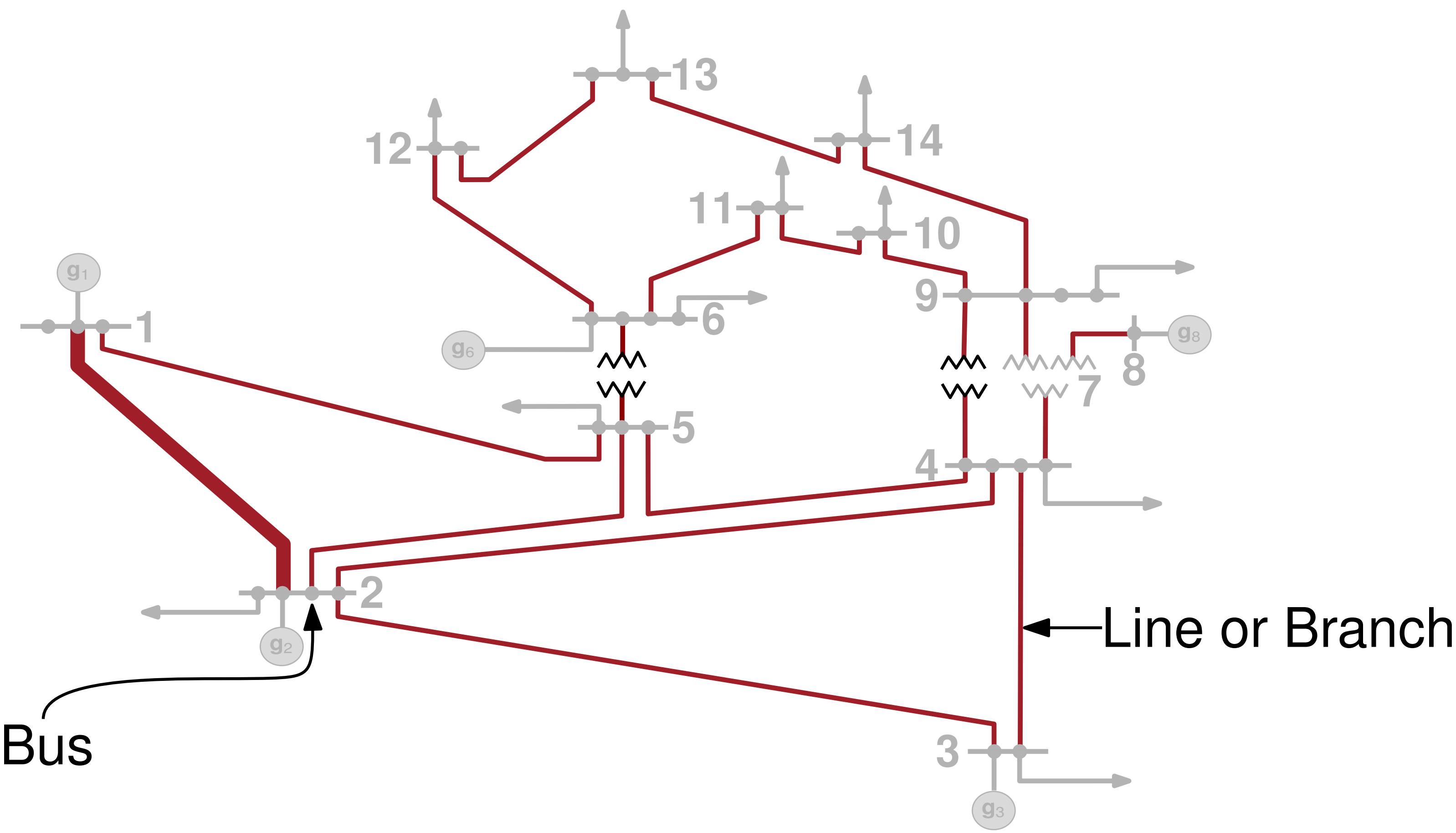

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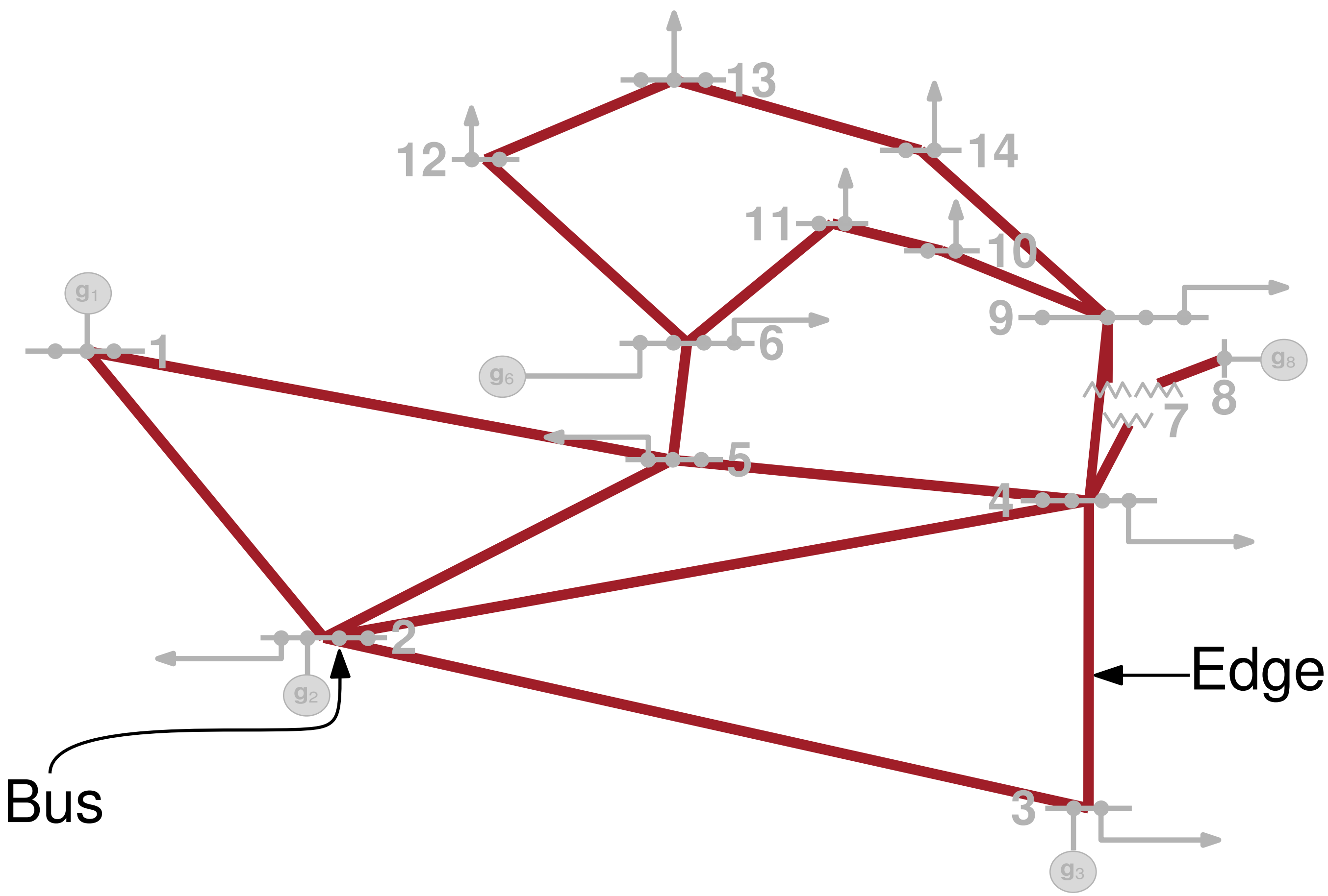

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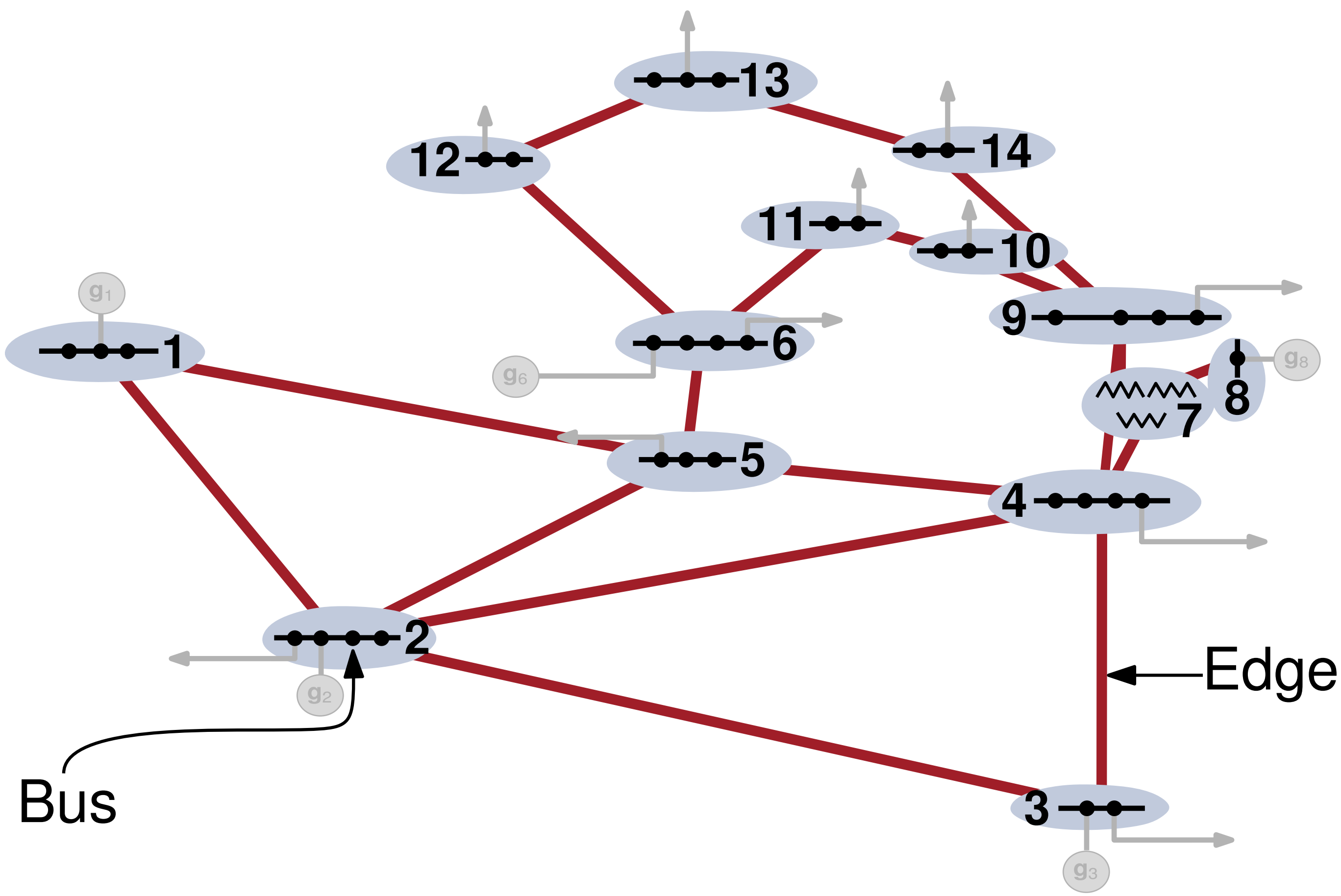

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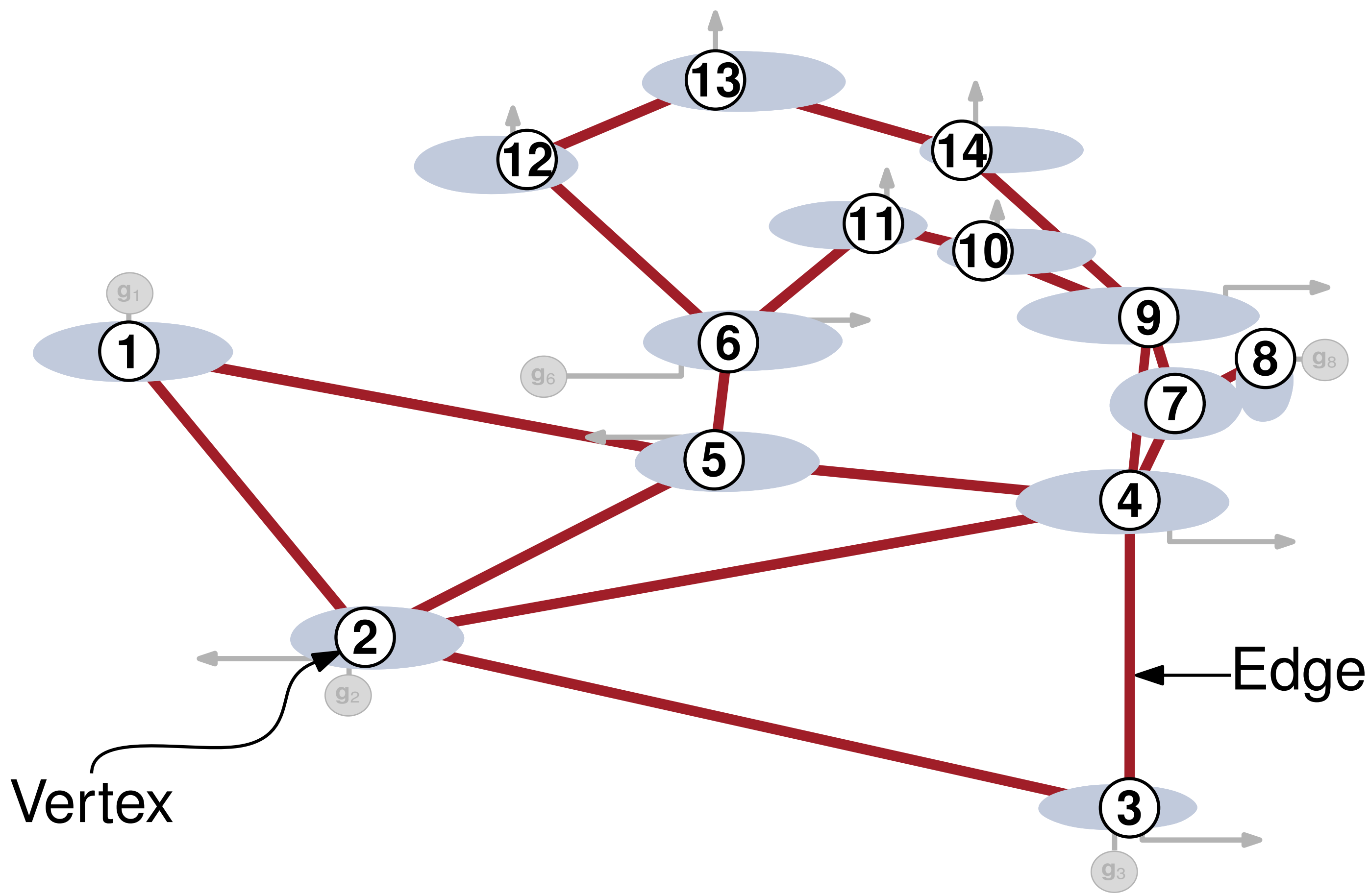

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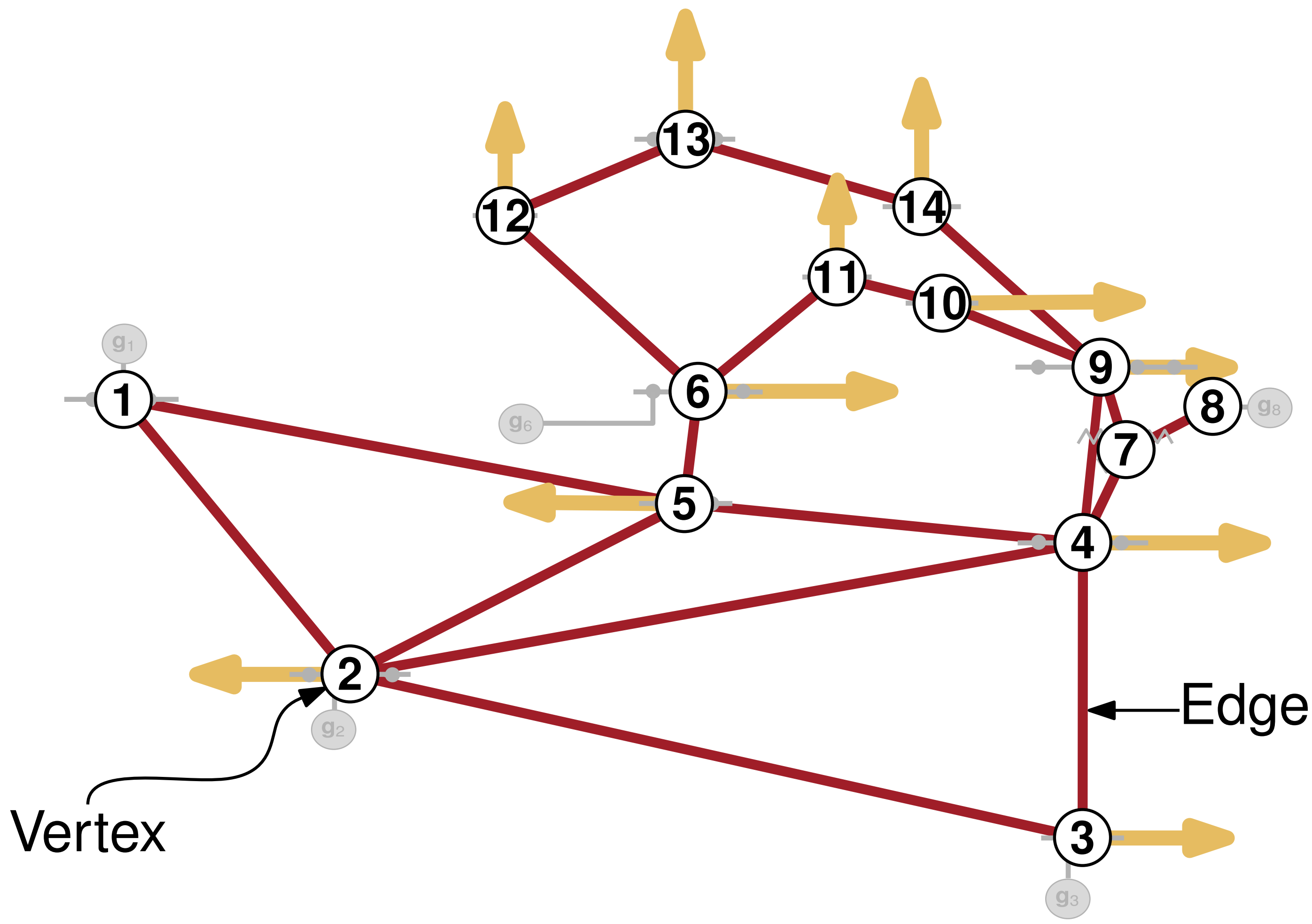

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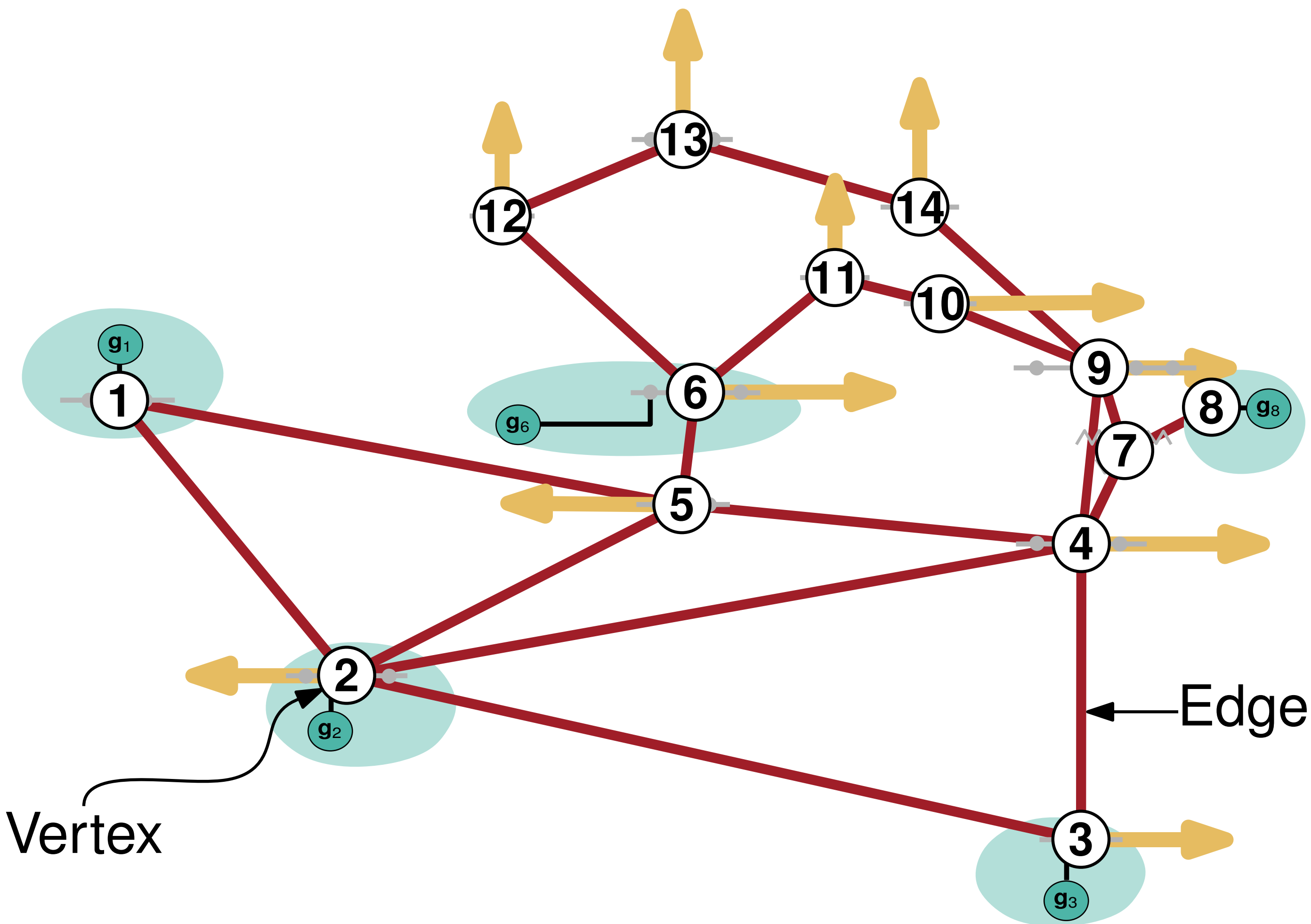

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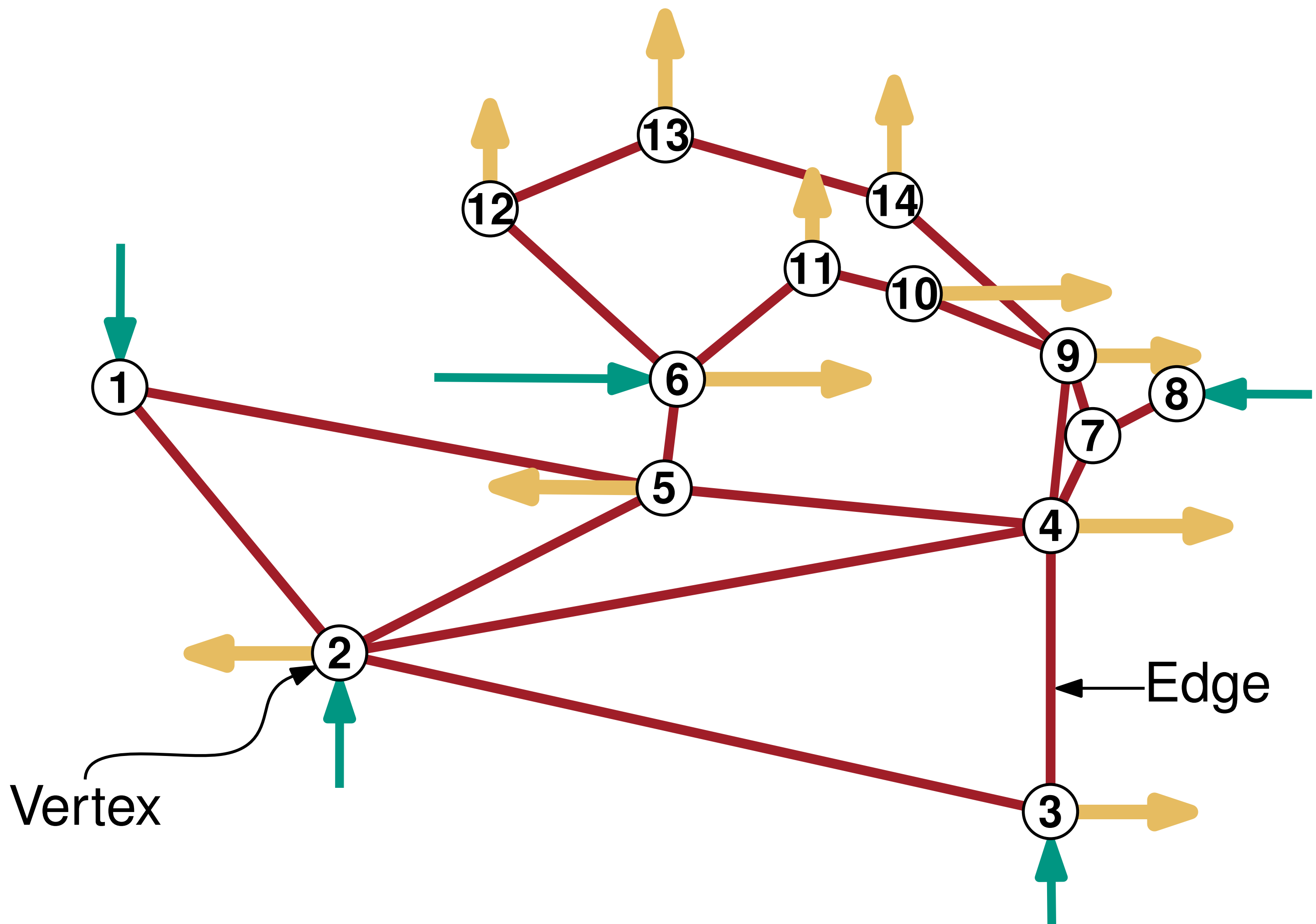
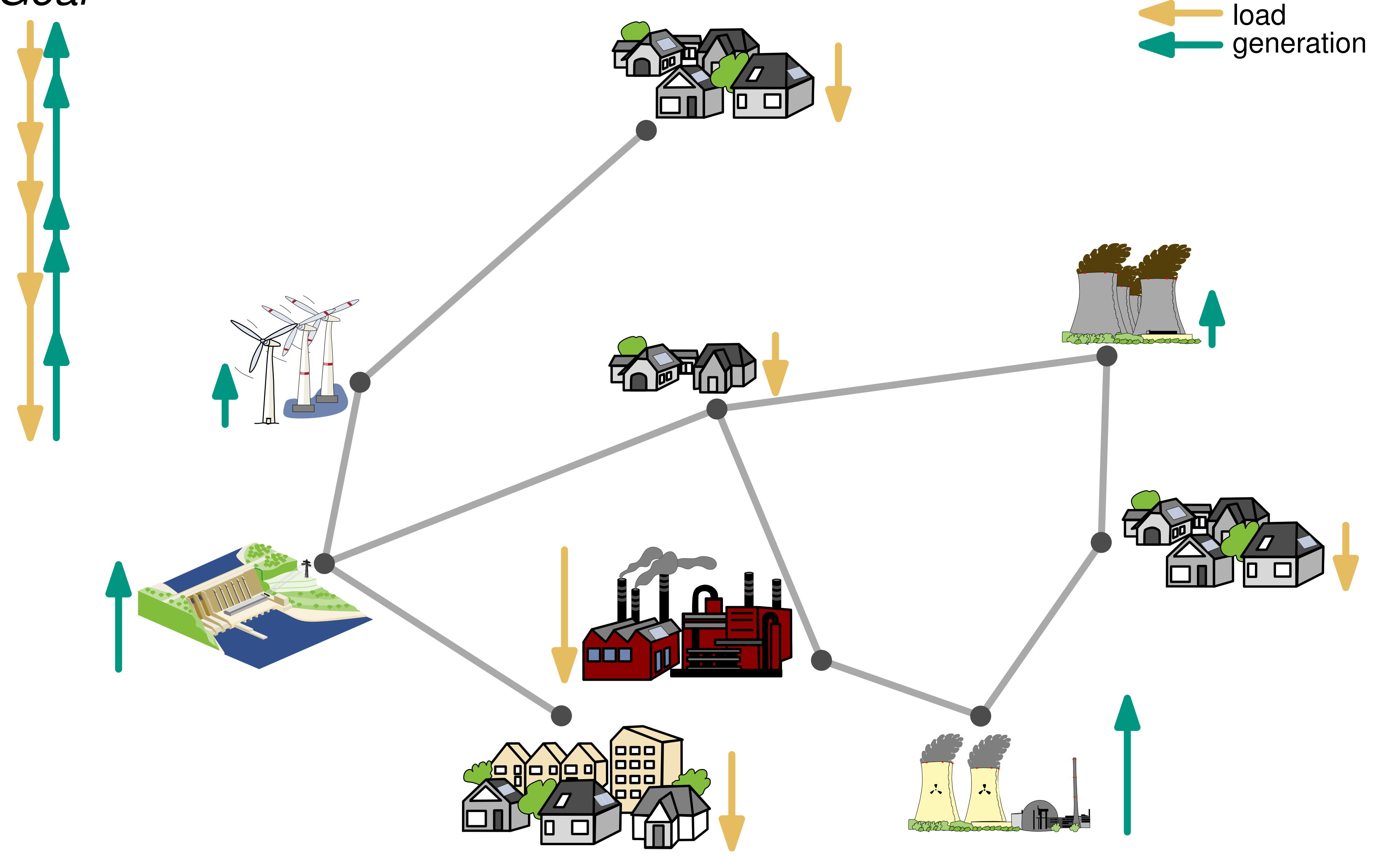


AEP 14 BUS TEST SYSTEM BUS CODE DIAGRAM

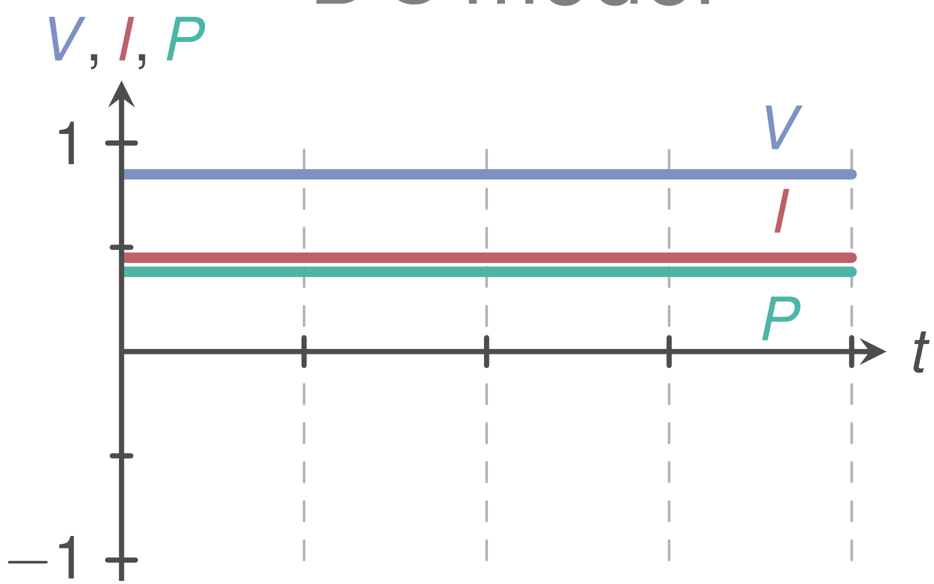
Conservation of Flow

Goal



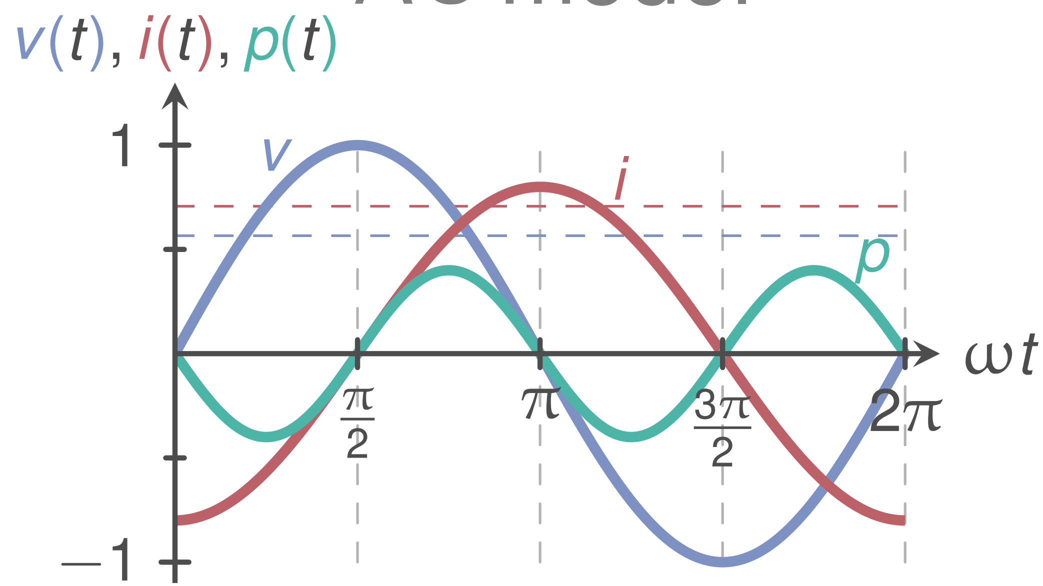
AC vs. DC Conservation of Flow

DC model



- Same polarity
- Linear
- Convex
- Most digital devices use DC
- Allows the connection of different AC systems

AC model



- Periodically changing polarity
- Complex
- Non-convex
- Most homes are wired for AC
- AC voltage levels conversion easier ⇒ easier to distribute

AC vs. DC Conservation of Flow

Constraints	Polar PQV	Rectangular PQV	Rectangular IV
Network	non-linear equations with quadratic terms, sin and cos functions ✗	quadratic equations	linear constraints ✓
Voltage angle differences	linear ✓	non-convex (arctan) ✗	linear (with additional constraints) ✓
Vertices	linear ✓	non-convex quadratic inequalities ✗	local quadratic, some are non-convex, some convex ✗

AC vs. DC Conservation of Flow

Constraints	Polar PQV	Rectangular PQV	Rectangular IV
[...] ferences	non-linear equations [...] linear ✓	quadratic [...] (arctan) ✗	[...] constraints) ✓
Vertices	linear ✓	non-convex quadratic inequalities ✗	local quadratic, some are non-convex, some convex ✗

AC conservation of flow is **subproblem** of most power grid problems.

AC conservation of flow is already **NP-hard** on **trees**.
[Lehmann et al., 2015]

→**Linearized AC** conservation of flow is **easy** to solve.

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AC vs. DC Conservation of Flow

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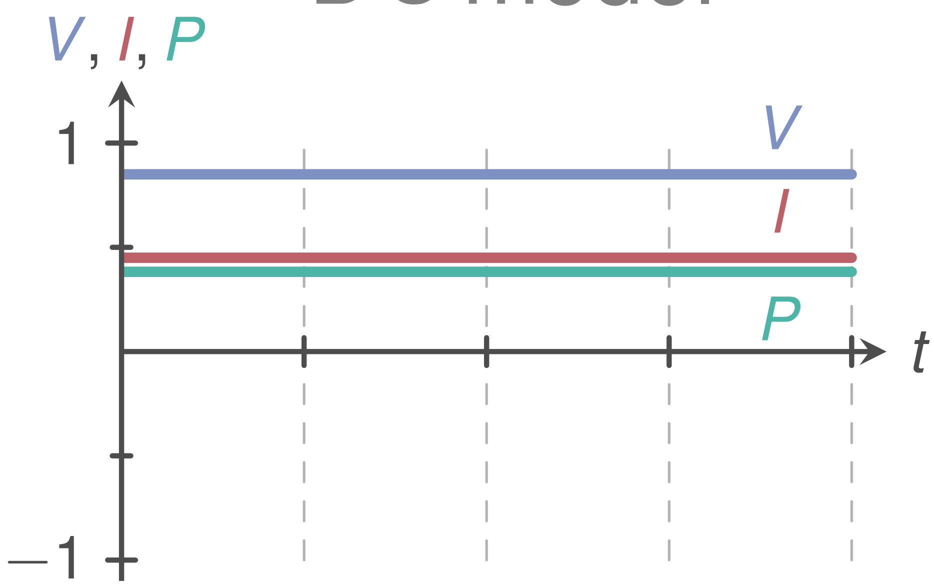
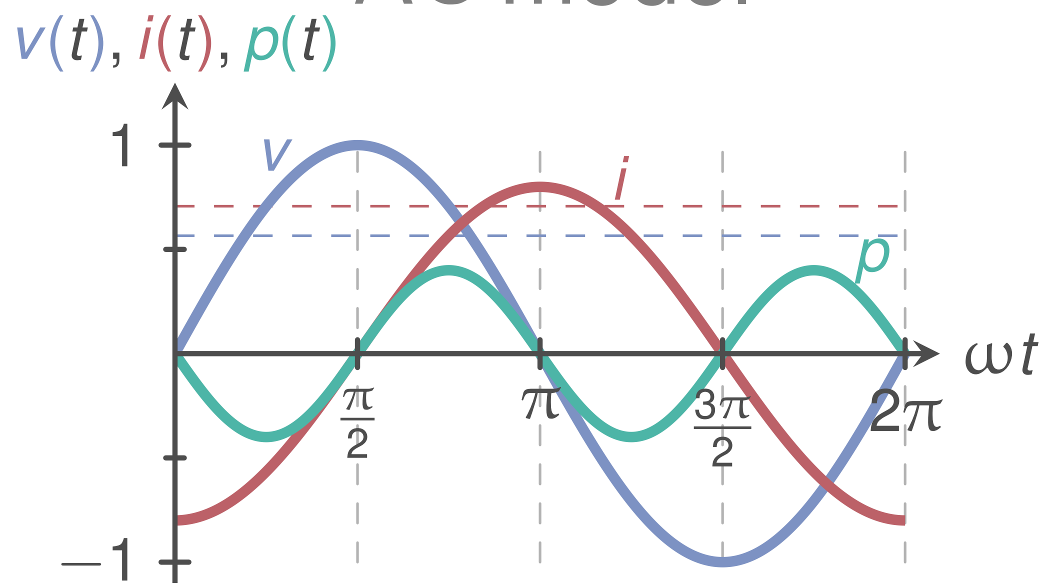


AC model



- No fast and robust solving techniques
- AC model has to be solved weekly; every 8 h, and 2 h; every 15 min, 5 min, 1 min, and 30 sec

$\Rightarrow$ Different model simplifications

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AC vs. DC Conservation of Flow

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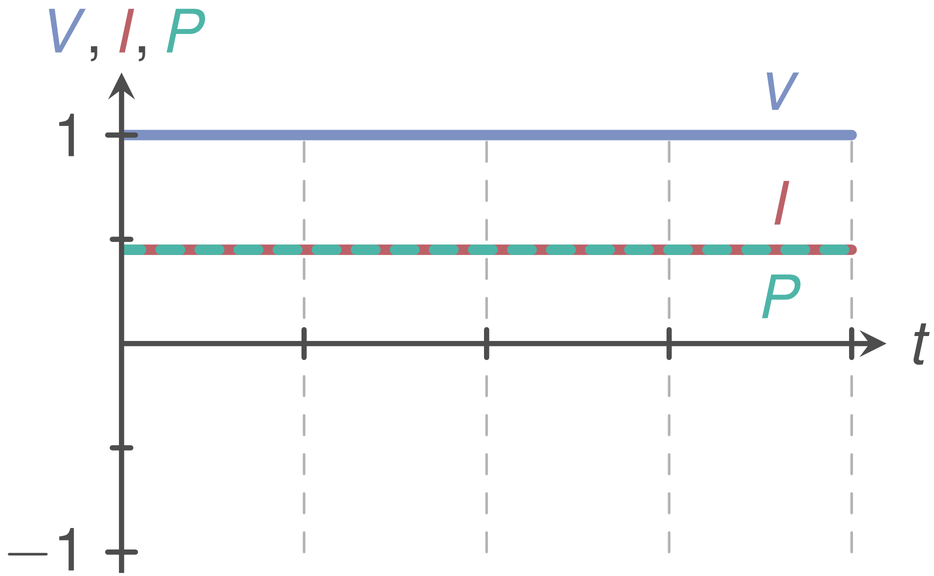
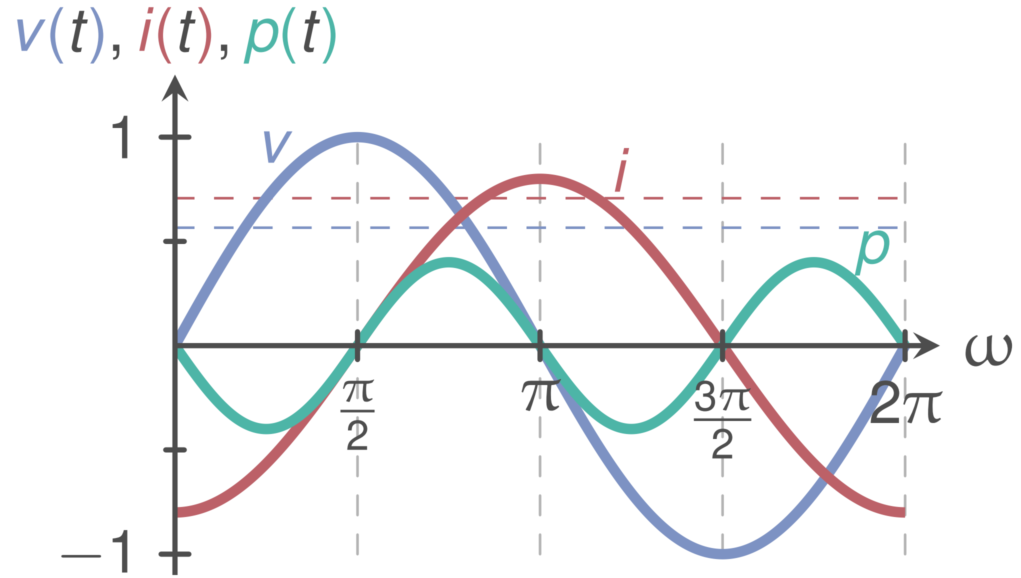
linearized AC model



AC model



- Normalization of the system
- Neglection of resistance, reactive power and other elements
- Linear equation system

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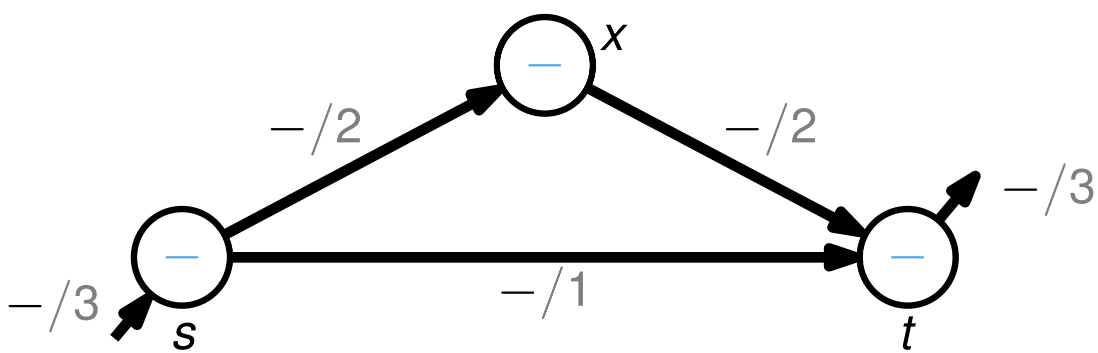
The MAXIMUM FLOW (MF) Problem



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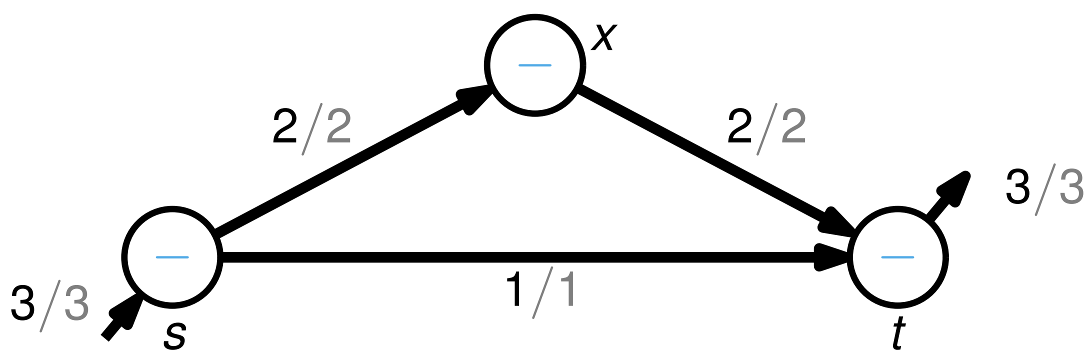

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The MAXIMUM FLOW (MF) Problem

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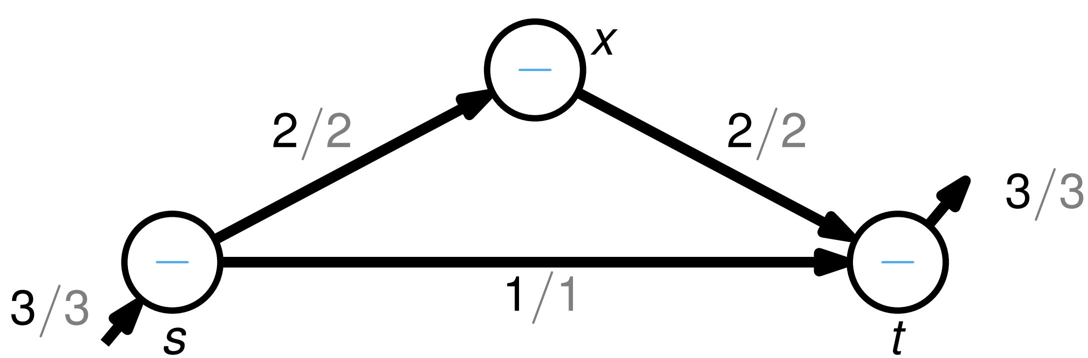

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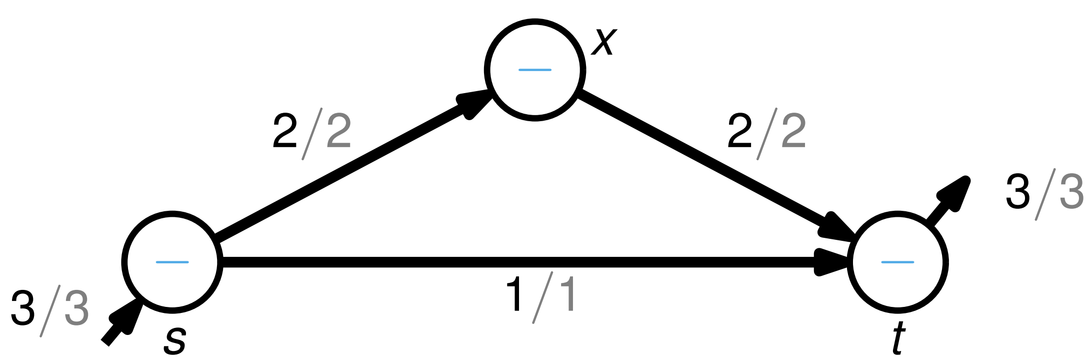
  $$\text{MF}(\mathcal{N}) = \max F(\mathcal{N}, f)$$

  with f being a feasible flow meaning



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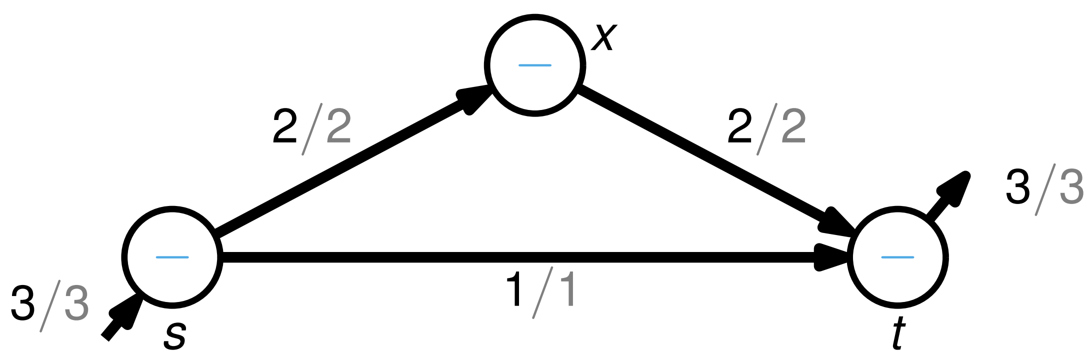

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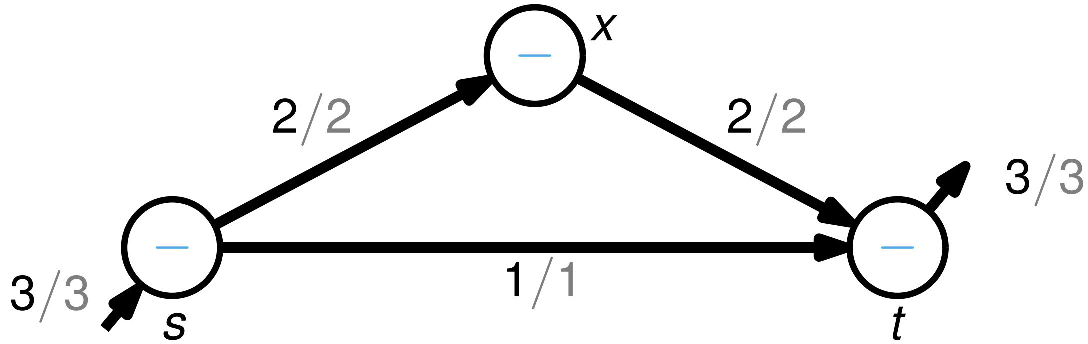

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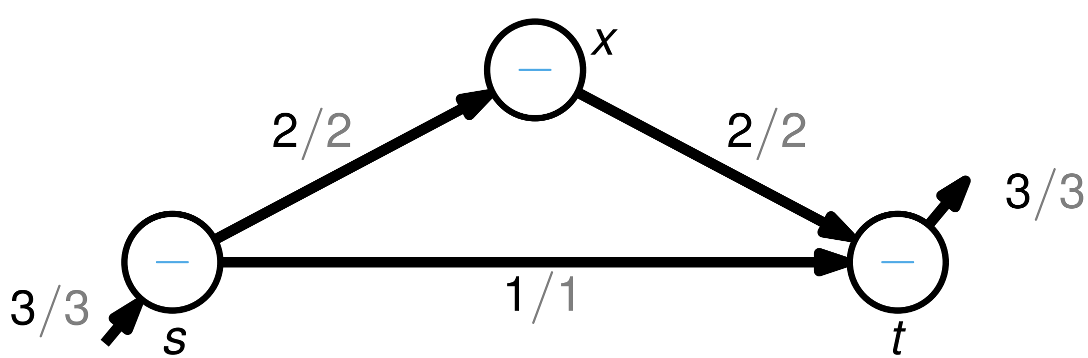

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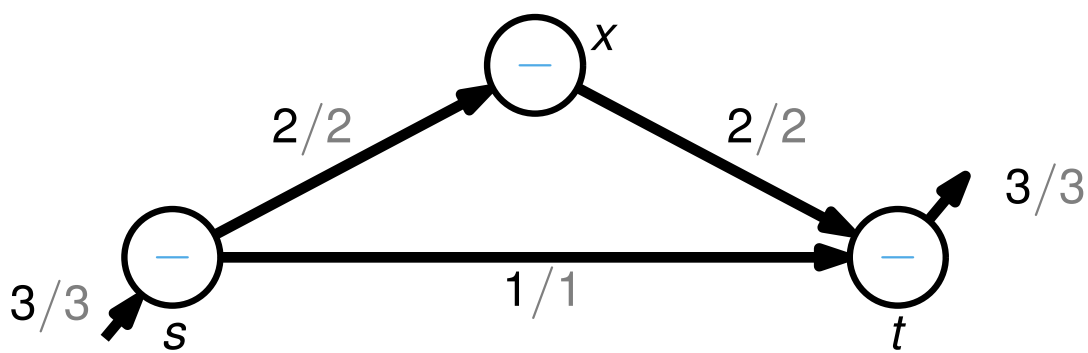

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Conservation of Flow

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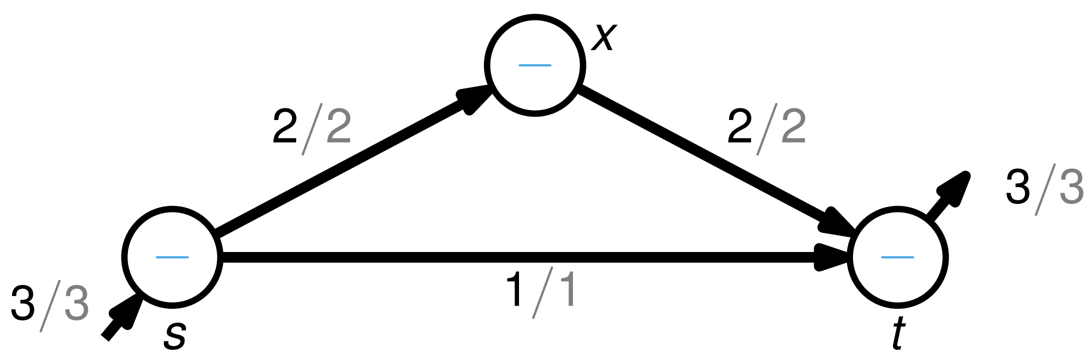

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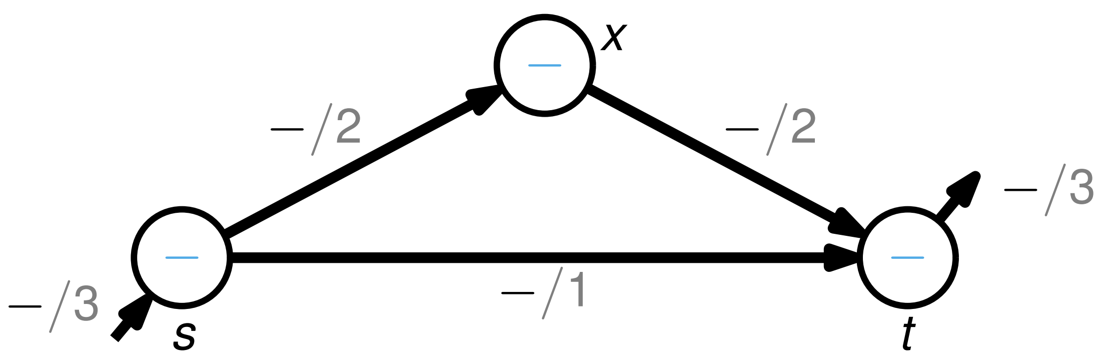
Maximum Power Flow (MPF)

[Zimmerman et al., 2011]

A feasible power flow has to satisfy (additional) physical constraints:

- The Kirchhoff's Current Law (KCL) which relates to flow conservation, i.e., $f_{\text{net}}(u) = 0$ for all $V \setminus (V_G \cup V_C)$



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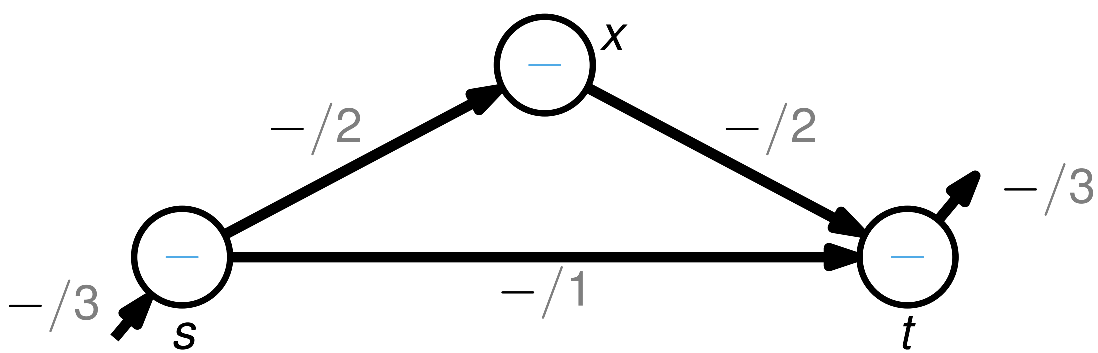

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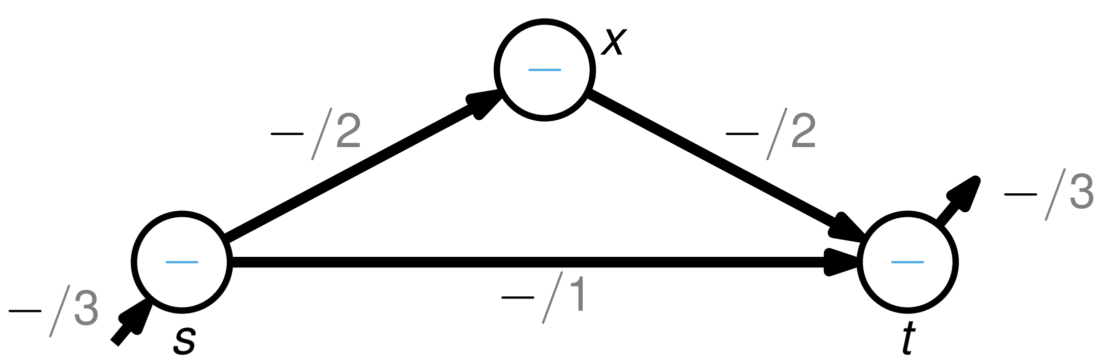

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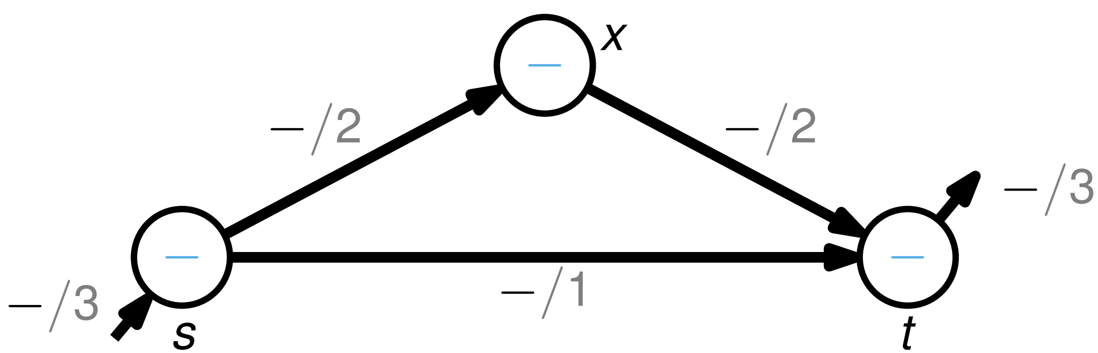
$$\theta(v) - \theta(u) = f(u, v) \qquad \forall (u, v) \in E$$
$$\theta_{\min}(u) \leq \theta(u) \leq \theta_{\max}(u) \qquad \forall u \in V$$



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Maximum Power Flow (MPF)

[Zimmerman et al., 2011]

A feasible power flow has to satisfy (additional) physical constraints:

$$(\theta(x) - \theta(s)) = f(s, x)$$

$$(\theta(t) - \theta(x)) = f(x, t)$$

- In addition, the Kirchhoff's Voltage Law (KVL) with assignment of potentials (voltage angles) $\theta : V \to \mathbb{R}$

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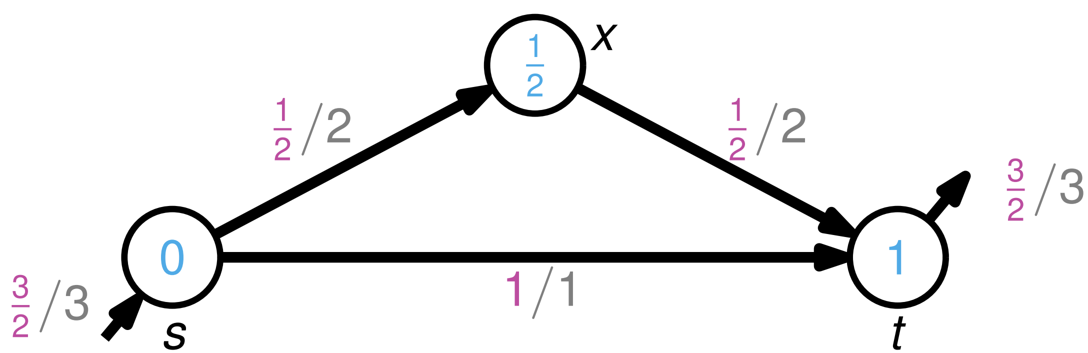

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A feasible power flow has to satisfy (additional) physical constraints:

$$\left.\begin{aligned}(\theta(x) - \theta(s)) &= f(s,x)\\ (\theta(t) - \theta(x)) &= f(x,t)\end{aligned}\right\} \quad (\theta(x) - \theta(s) + \theta(t) - \theta(x)) = f(s,x) + f(x,t)$$

$$(\theta(t) - \theta(s)) = f(s,t)$$

- In addition, the Kirchhoff's Voltage Law (KVL) with assignment of potentials (voltage angles) $\theta : V \to \mathbb{R}$

$$\theta(v) - \theta(u) = f(u,v) \qquad \forall (u,v) \in E$$

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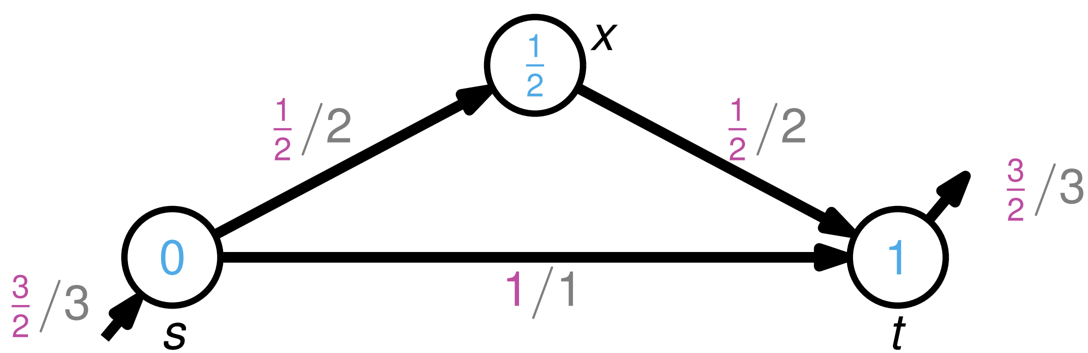

Maximum Power Flow (MPF)

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A feasible power flow has to satisfy (additional) physical constraints:

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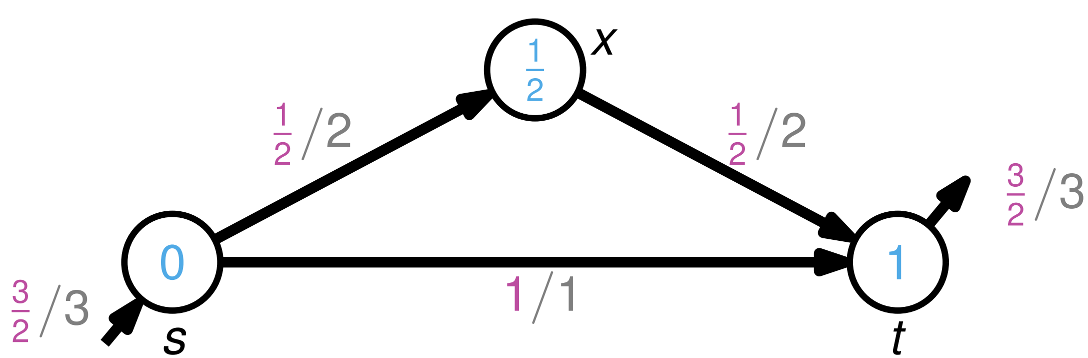

Maximum Power Flow (MPF)

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A feasible power flow has to satisfy (additional) physical constraints:

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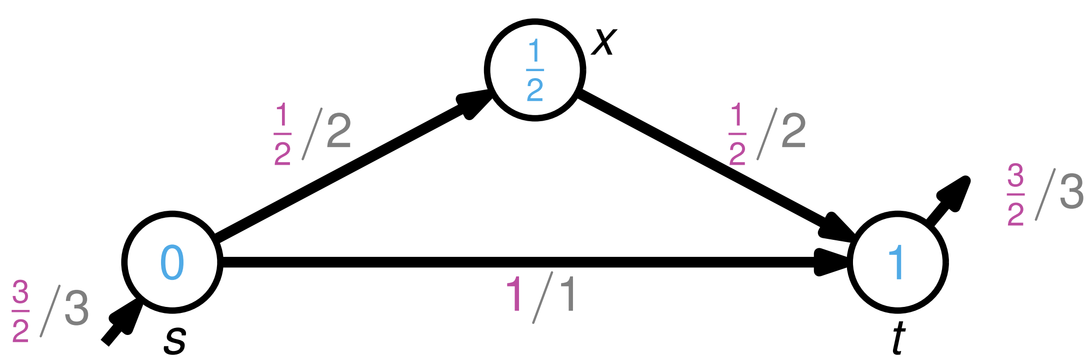
$$\theta(v) - \theta(u) = f(u,v) \qquad \forall (u,v) \in E$$

$$\theta_{\min}(u) \leq \theta(u) \leq \theta_{\max}(u) \qquad \forall u \in V$$



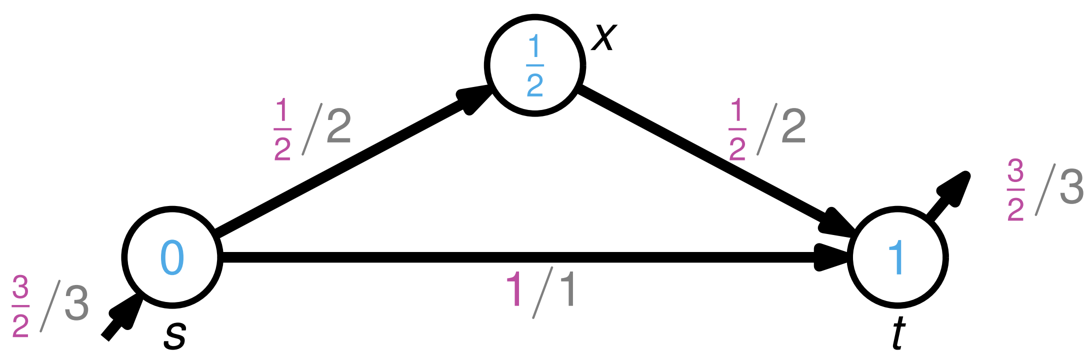
Maximum Power Flow (MPF)

[Zimmerman et al., 2011]

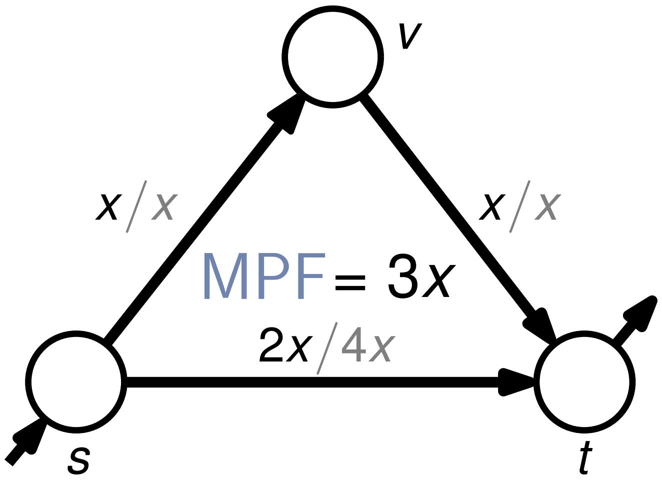
- The value of the Maximum Power Flow is defined as
$$\mathrm{MPF}(\mathcal{N}) = \max F(\mathcal{N}, f)$$
with f being a feasible power flow meaning

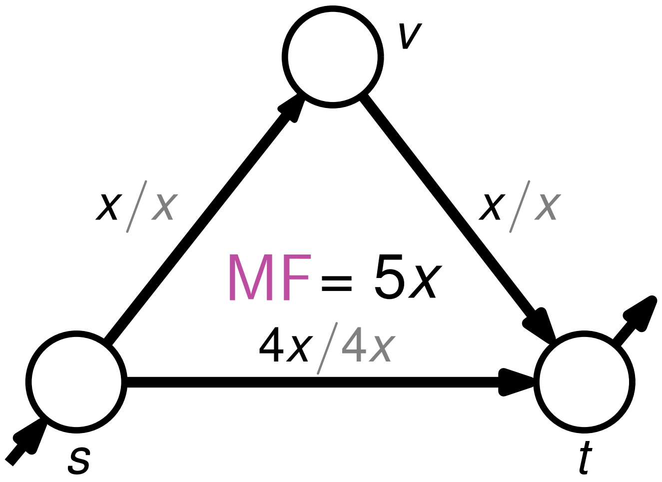
- The Kirchhoff's Current Law (KCL) which relates to flow conservation, i.e., $f_{\text{net}}(u) = 0$ for all $V \setminus (V_G \cup V_C)$

- In addition, the Kirchhoff's Voltage Law (KVL) with assignment of potentials (voltage angles) $\theta\colon V \to \mathbb{R}$
$$\theta(v) - \theta(u) = f(u, v) \qquad \forall (u, v) \in E$$
$$\theta_{\min}(u) \leq \theta(u) \leq \theta_{\max}(u) \qquad \forall u \in V$$



Maximum Power Flow vs. Maximum Flow





physical model
(AC linearization)

lower bound

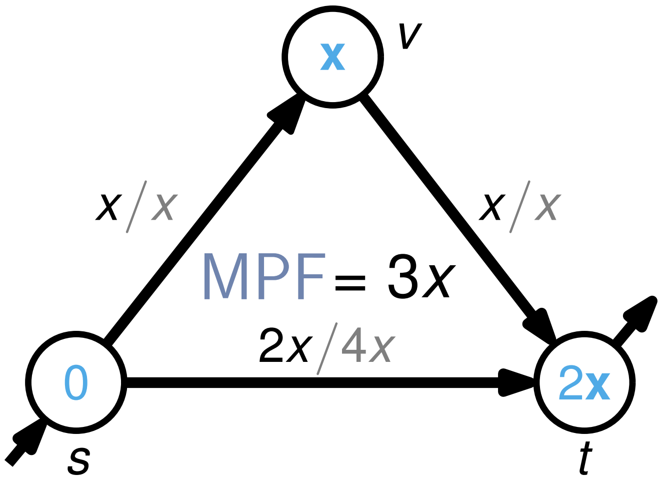
flow model

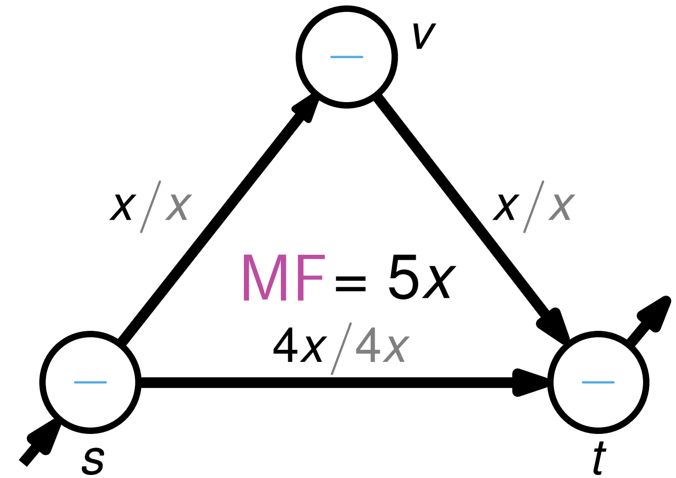
upper bound

capacity constraints

Kirchhoff's Current Law (KCL)

Maximum Power Flow vs. Maximum Flow





physical model
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capacity constraints

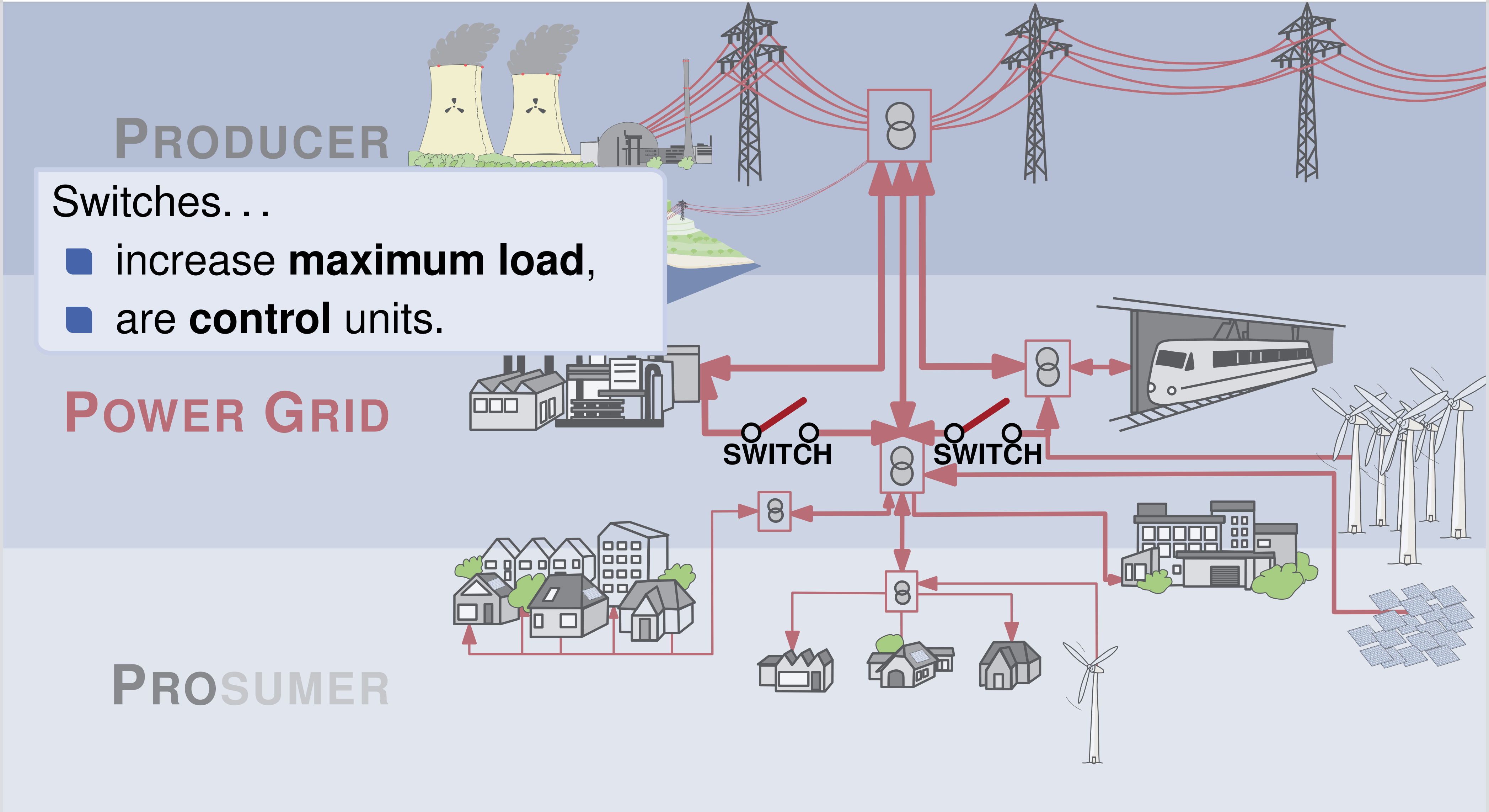
Kirchhoff’s Current Law (KCL)

Kirchhoff’s Voltage Law: $f(u, v) = \qquad \theta(v) - \theta(u)$ for all $(u, v) \in E$

Switching

Switches. . .

- increase **maximum load**,
- are **control** units.



The Maximum Transmission Switching Flow (MTSF) Problem [Fisher et al., 2008]

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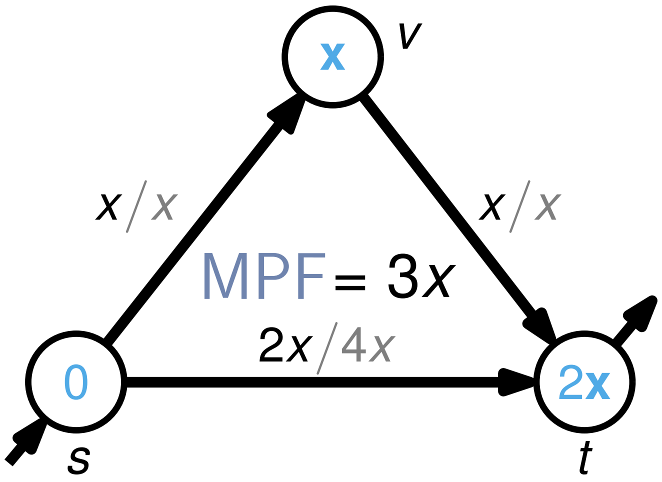
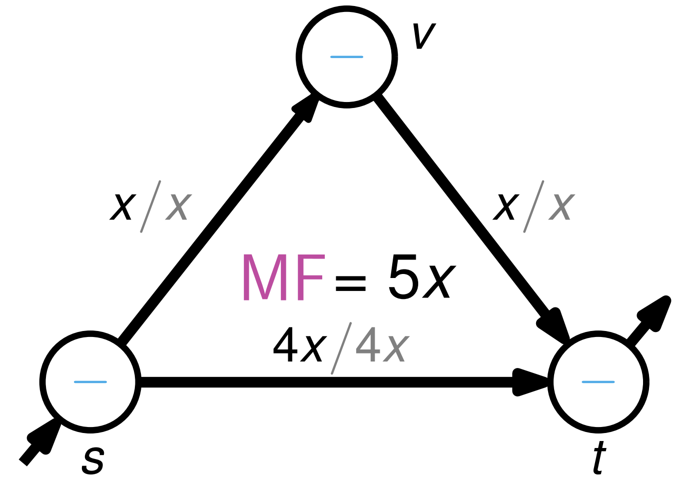
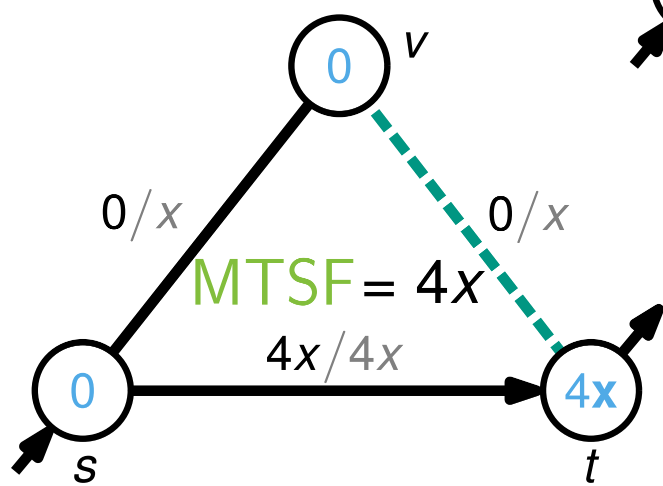




physical model

(AC linearization)

lower bound



flow model

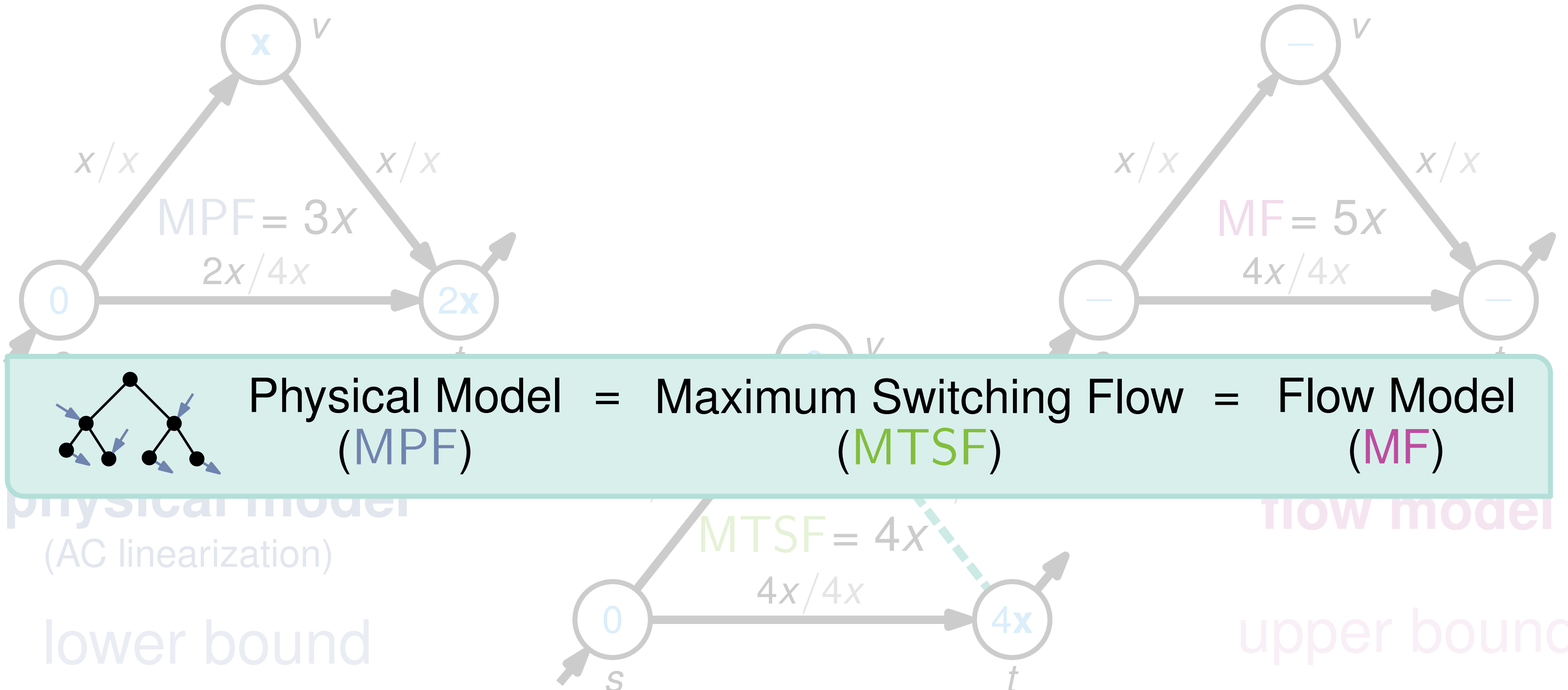
upper bound

capacity constraints

Kirchhoff's Current Law (KCL)

Kirchhoff's Voltage Law: $f(u, v) = z(u, v)(\theta(v) - \theta(u))$ for all $(u, v) \in E$

The Maximum Transmission Switching Flow (MTSF) Problem [Fisher et al., 2008]



Physical Model (MPF) = Maximum Switching Flow (MTSF) = Flow Model (MF)

capacity constraints

Kirchhoff's Current Law (KCL)

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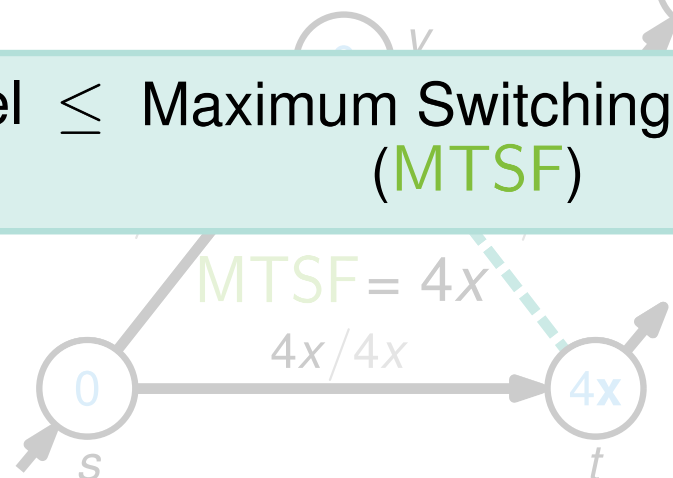

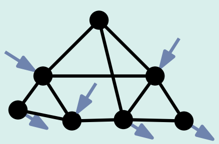

Physical Model (MPF) $\leq$ Maximum Switching Flow (MTSF) $\leq$ Flow Model (MF)

physical model
(AC linearization)

lower bound

flow model

upper bound

Kirchhoff's Voltage Law: $f(u, v) = z(u, v)(\theta(v) - \theta(u))$ for all $(u, v) \in E$

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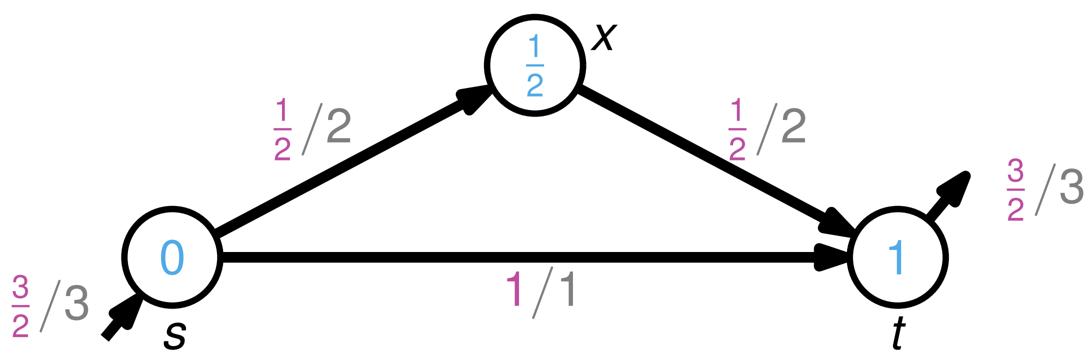
The Maximum Transmission Switching Flow (MTSF) Problem [Fisher et al., 2008]

- The value of the Maximum Transmission Switching Flow is defined as

$$\text{MTSF}(\mathcal{N}) := \max_{S \subseteq E} \text{MPF}(\mathcal{N} - S)$$

with f being a feasible power flow meaning

$$f_{\text{net}}(u) = 0 \qquad \forall u \in V \setminus (V_G \cup V_C)$$



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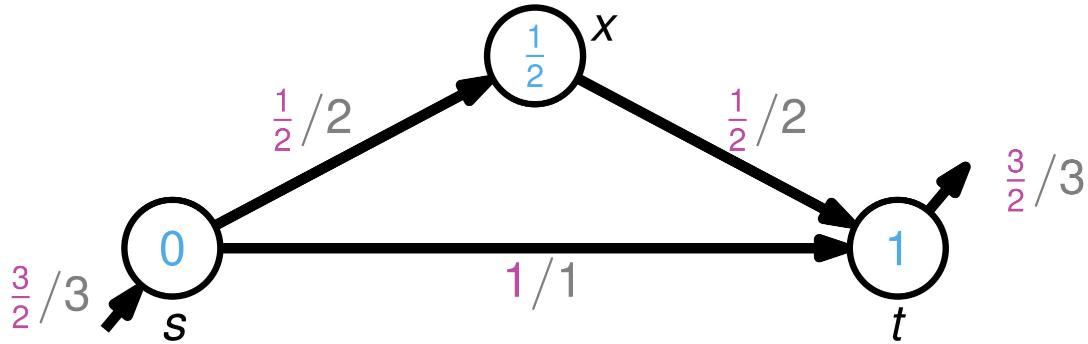
$$z(u,v) \in \{0,1\}$$



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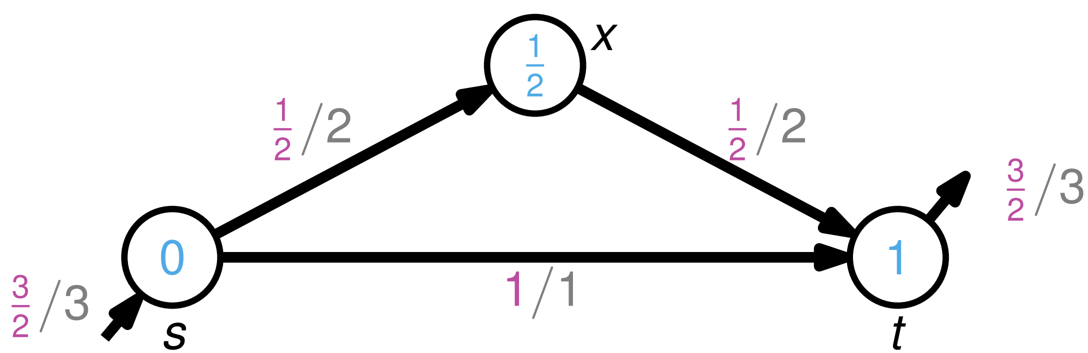

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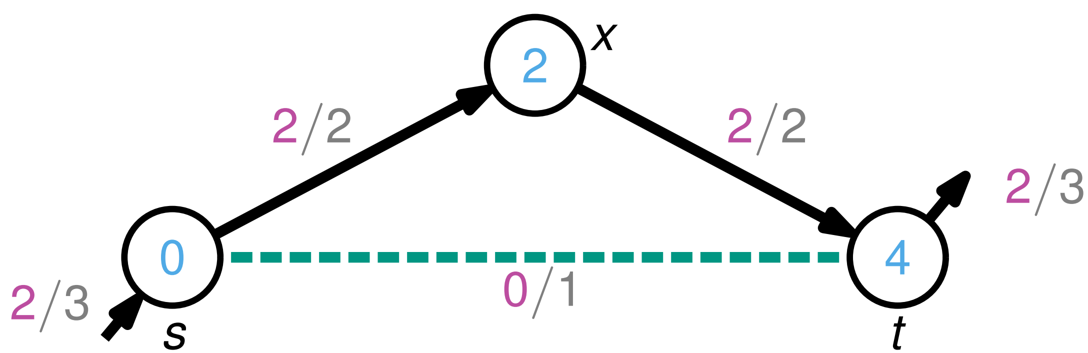

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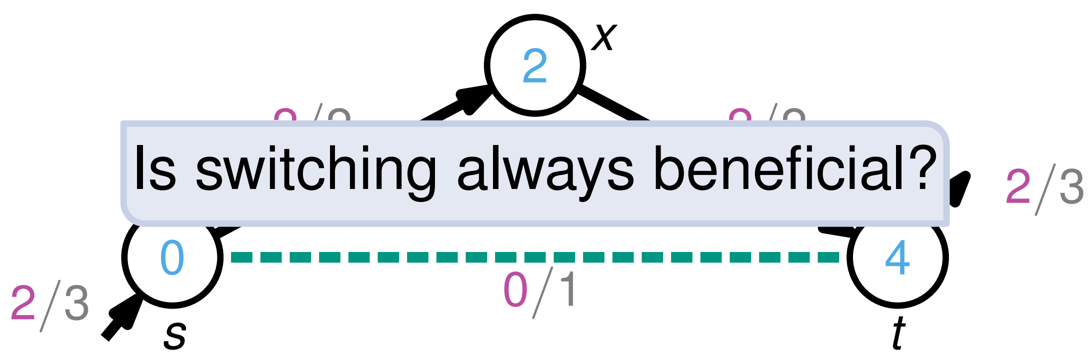

Is switching always beneficial?

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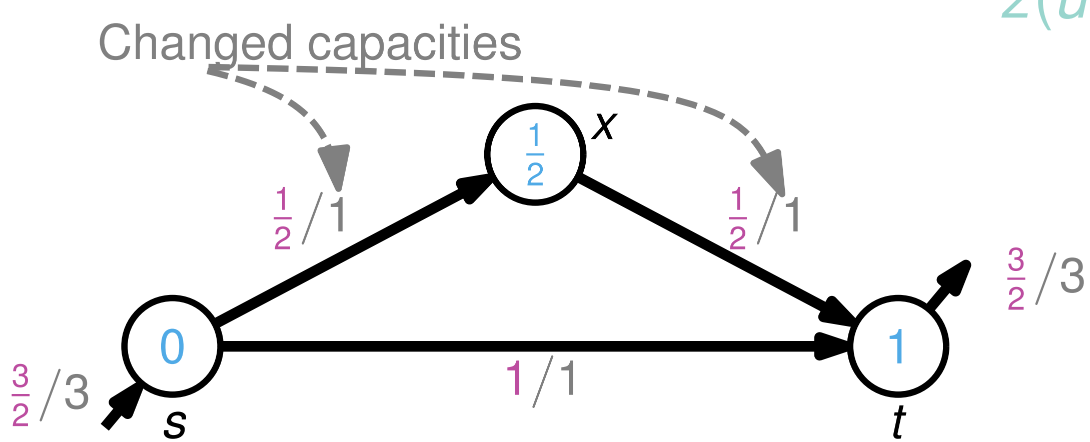

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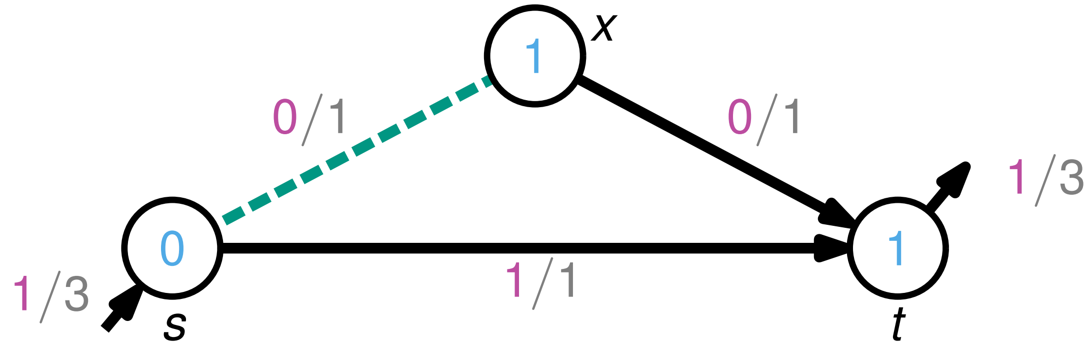

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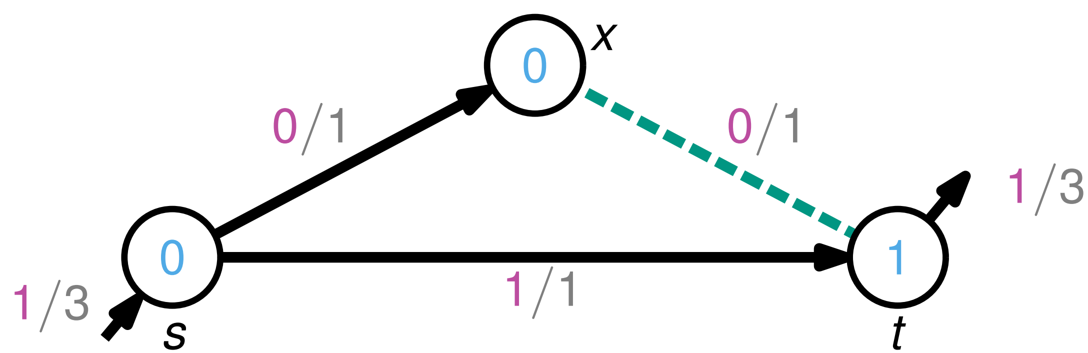

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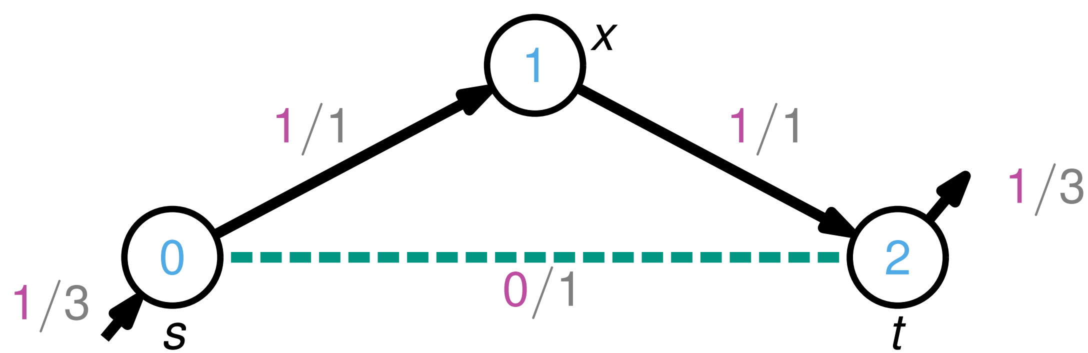

The Maximum Transmission Switching Flow (MTSF) Problem [Fisher et al., 2008]

Optimization Problem MTSF

Instance: A power grid $\mathcal{N}$.

Objective: Find a set $S \subseteq E$ of switched edges such that MPF($\mathcal{N} - S$) is maximum among all choices of switched edges S.

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Overview of the MTSF Results

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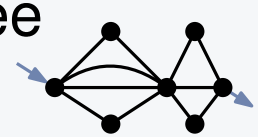
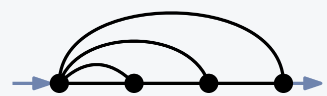
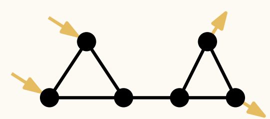
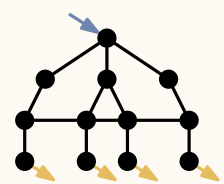
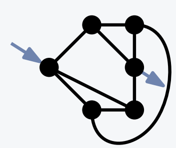
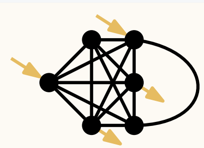
complexity

	Graph Structure		Complexity	Algorithm
one generator, one load	penrose-minor-free graphs		polynomial-time solvable	DTP [Grastien et al., 2018]
	series-parallel graphs		NP-hard [Grastien et al., 2018]	✗
arbitrary generators, arbitrary loads	cacti with max degree of 3		NP-hard [Lehmann et al., 2014]	2-approx. [Grastien et al., 2018]
	2-level trees		NP-hard [Lehmann et al., 2014]	✗
	planar graphs with max degree of 3		strongly NP-hard [Lehmann et al., 2014]	✗
$\|V_G\| = 2$, $\|V_C\| = 2$	arbitrary graphs		non-APX [Lehmann et al., 2014]	✗

Dominating Theta Path (DTP)

[Section 5; Grastien et al., 2018]

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Fix $u, v \in V$ and a u-v-path π.

Susceptance Norm:

$$\|\pi\| := \text{length of } \pi$$

Minimum Capacity:

$$\text{cap}_{\min}(\pi) := \min\{\text{cap}(e) \mid e \in \pi\}$$

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Minimum Capacity:

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Angle Difference of π:

$$\Delta\theta(\pi) := \|\pi\| \cdot \text{cap}_{\text{min}}(\pi)$$

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Angle Difference of π:

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Dominating Theta Path (DTP):

$$\Delta\theta_{\min}(u, v) := \min\{\Delta\theta(\pi) \mid \pi \text{ is a } u\text{-}v\text{-path}\}$$

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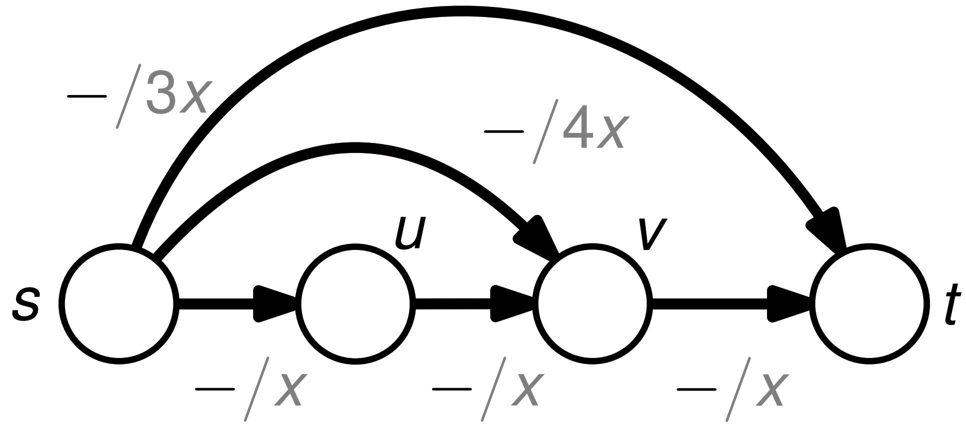
Computing DTP [Section 5; Grastien et al., 2018]

Description:

- Bicriterial Dijkstra with labels ($\|\pi\|$, $\mathrm{cap}_{\min}(\pi)$)
- at most $|E|$ labels per vertex



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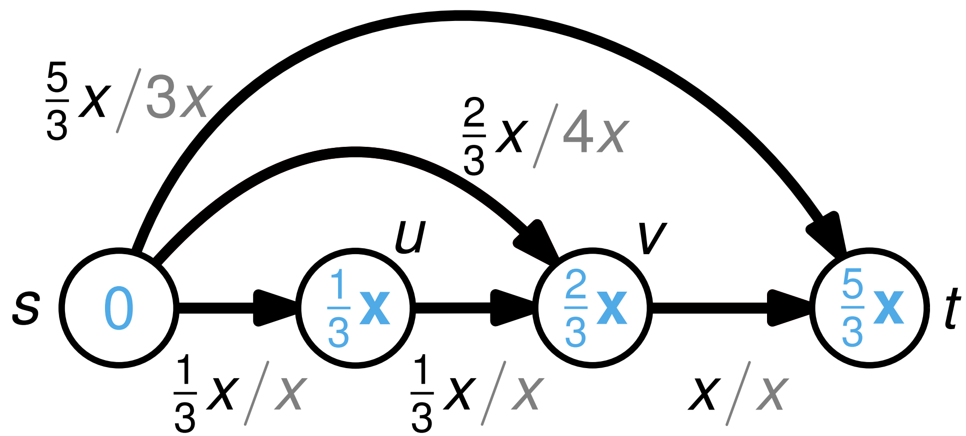

MPF = $\frac{8}{3}x$

Computing DTP [Section 5; Grastien et al., 2018]

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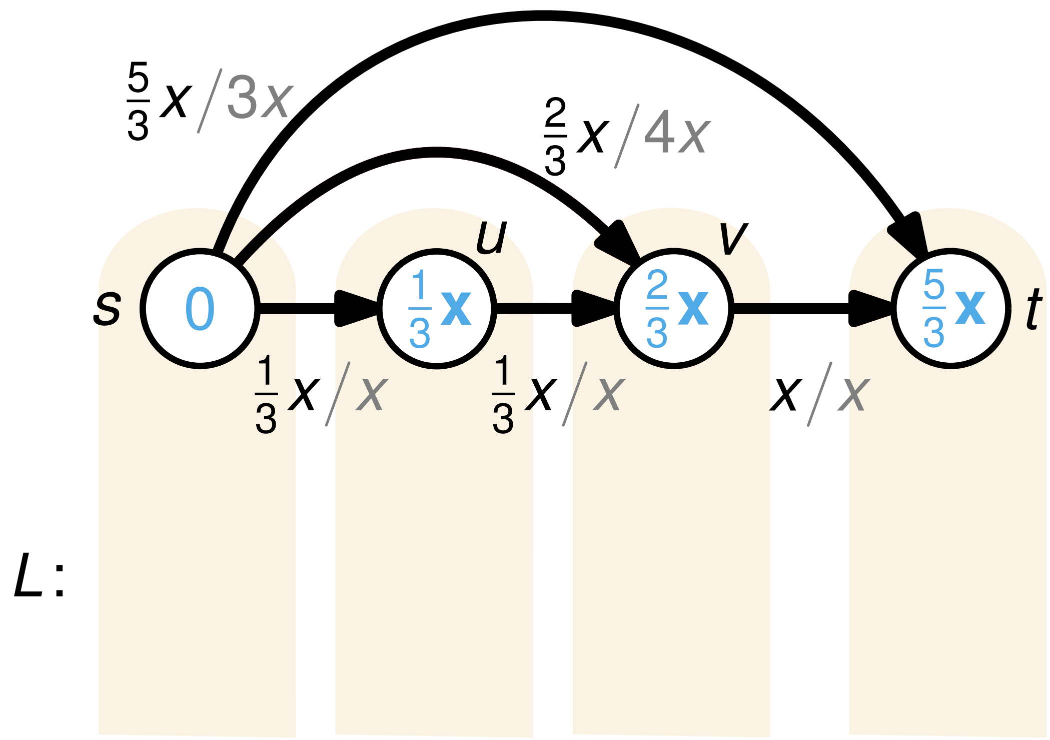

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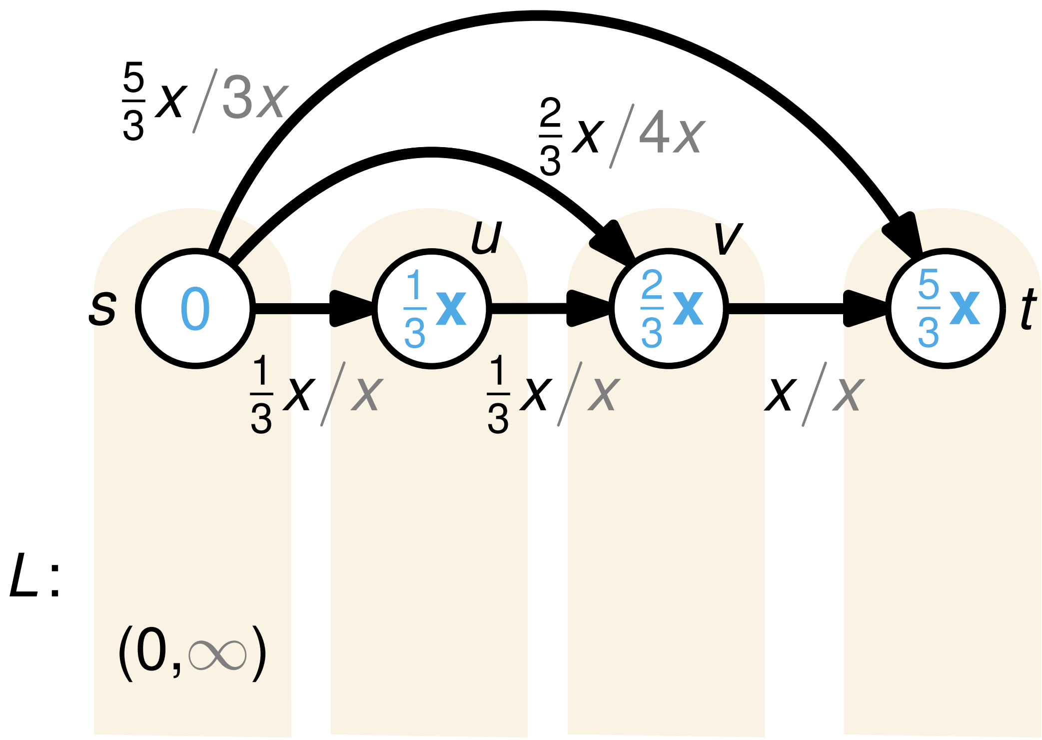

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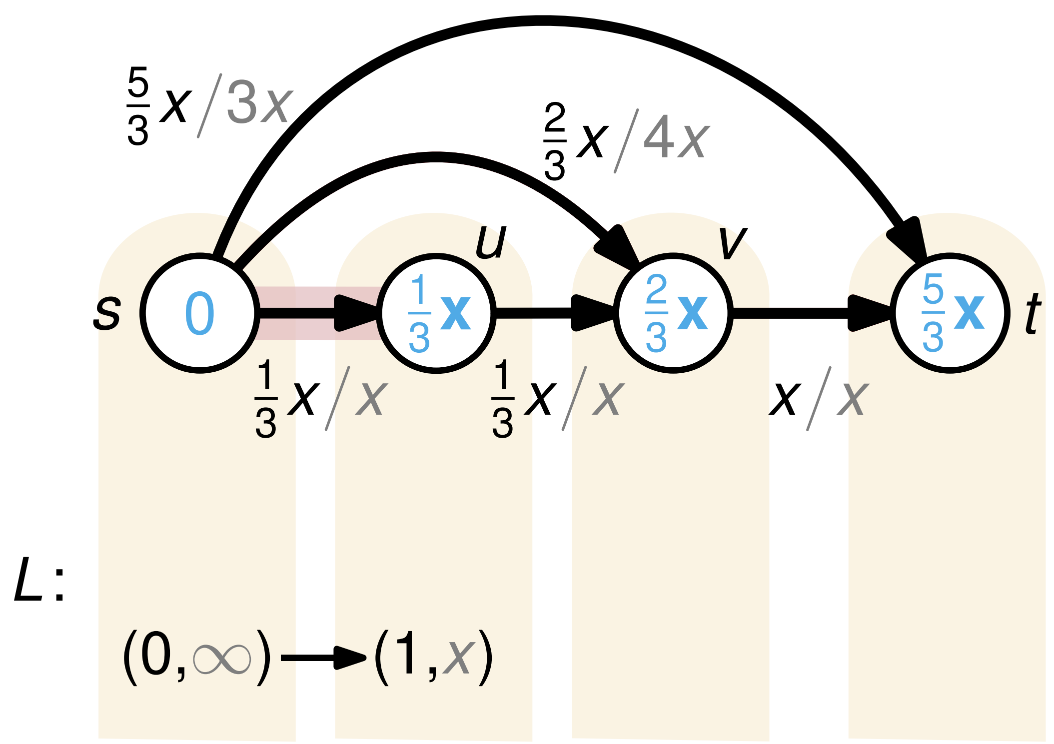

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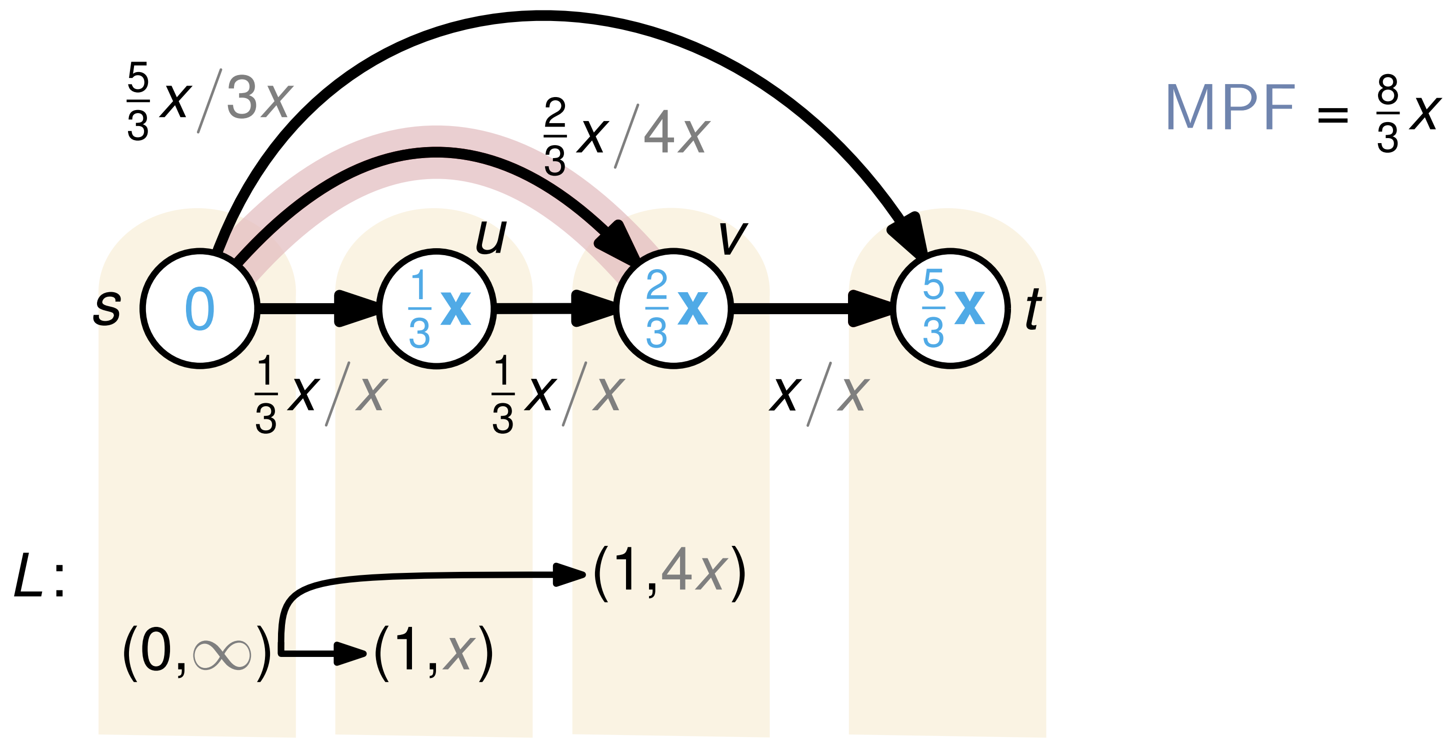

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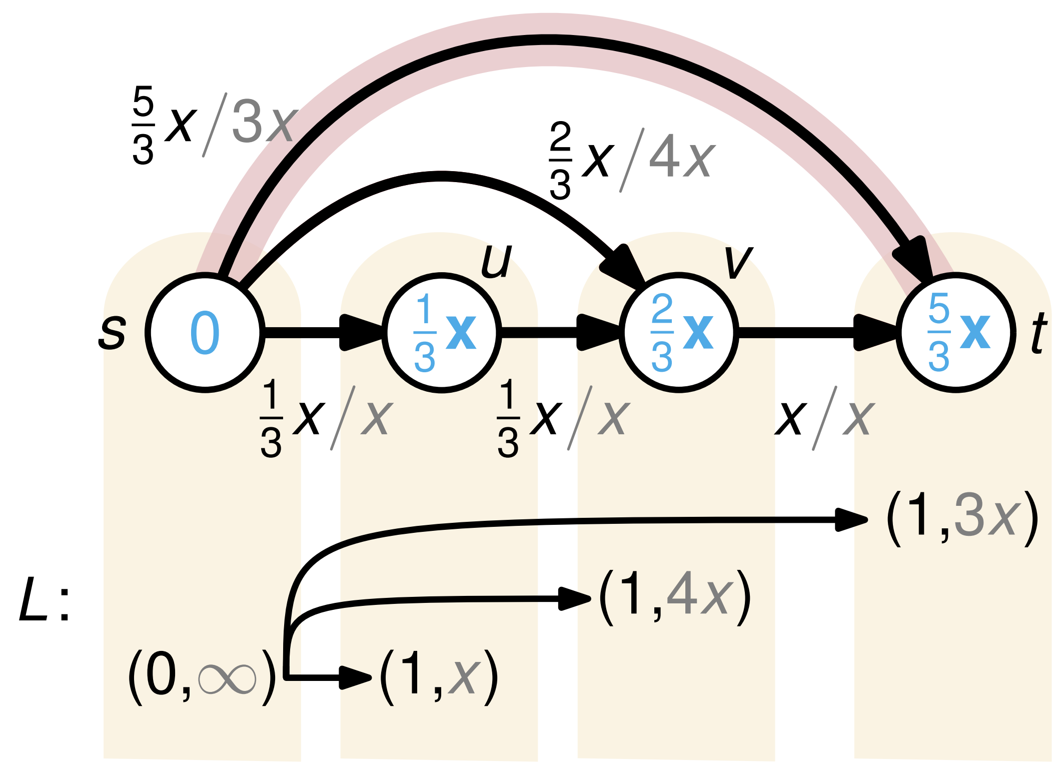

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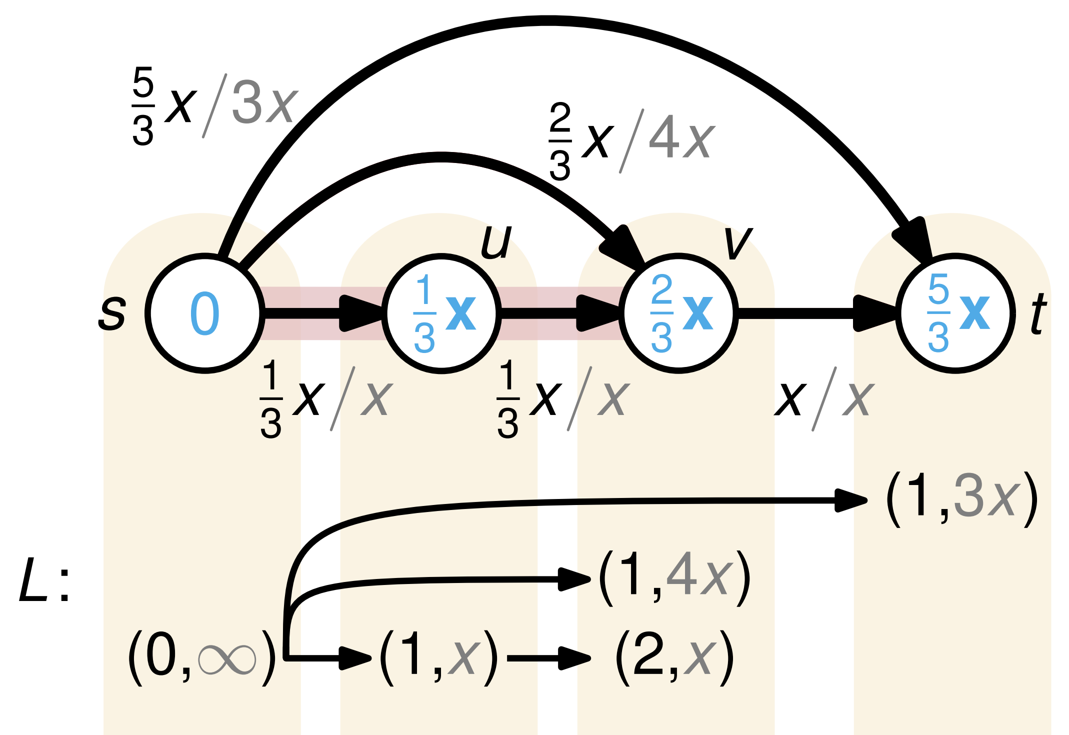

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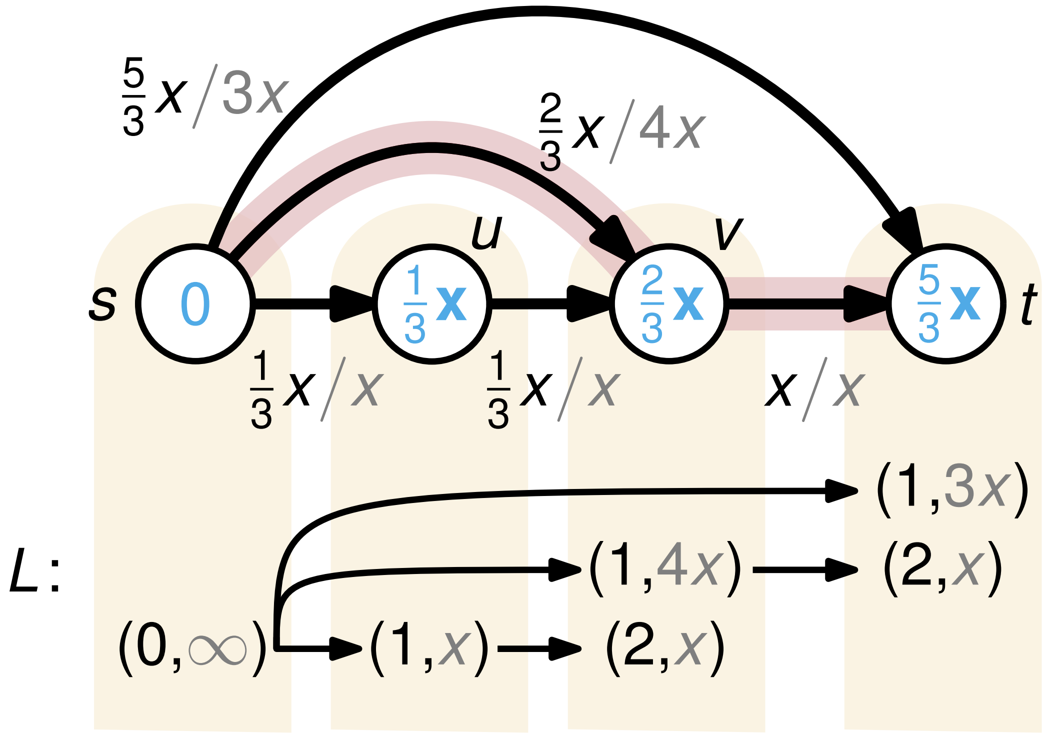

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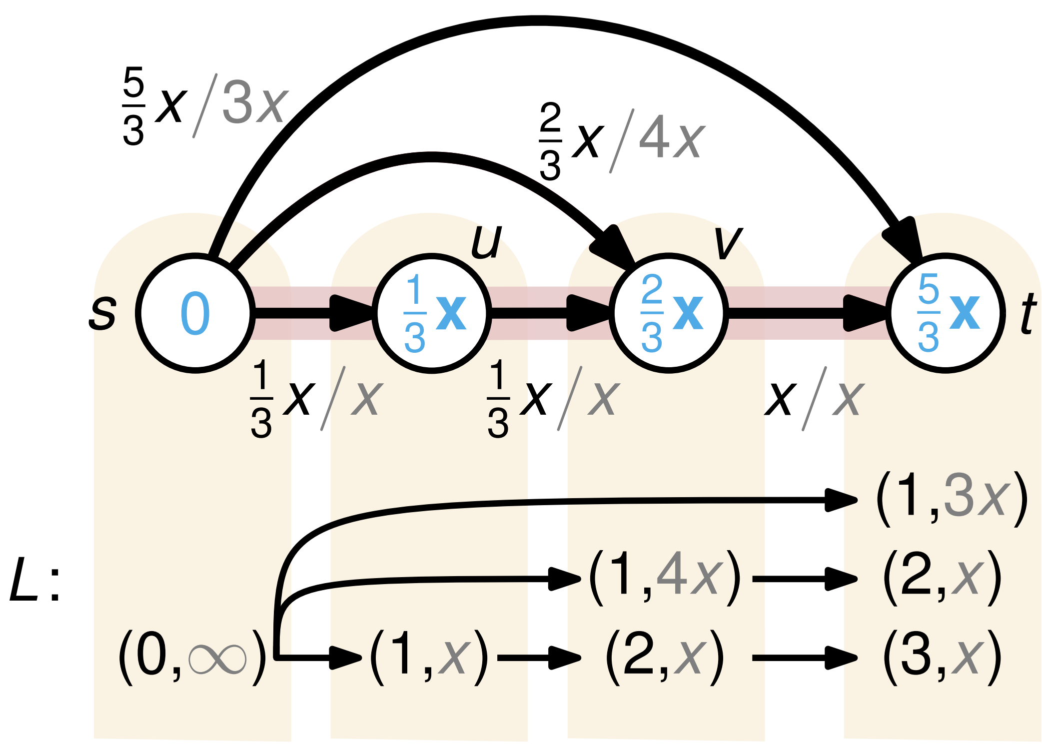

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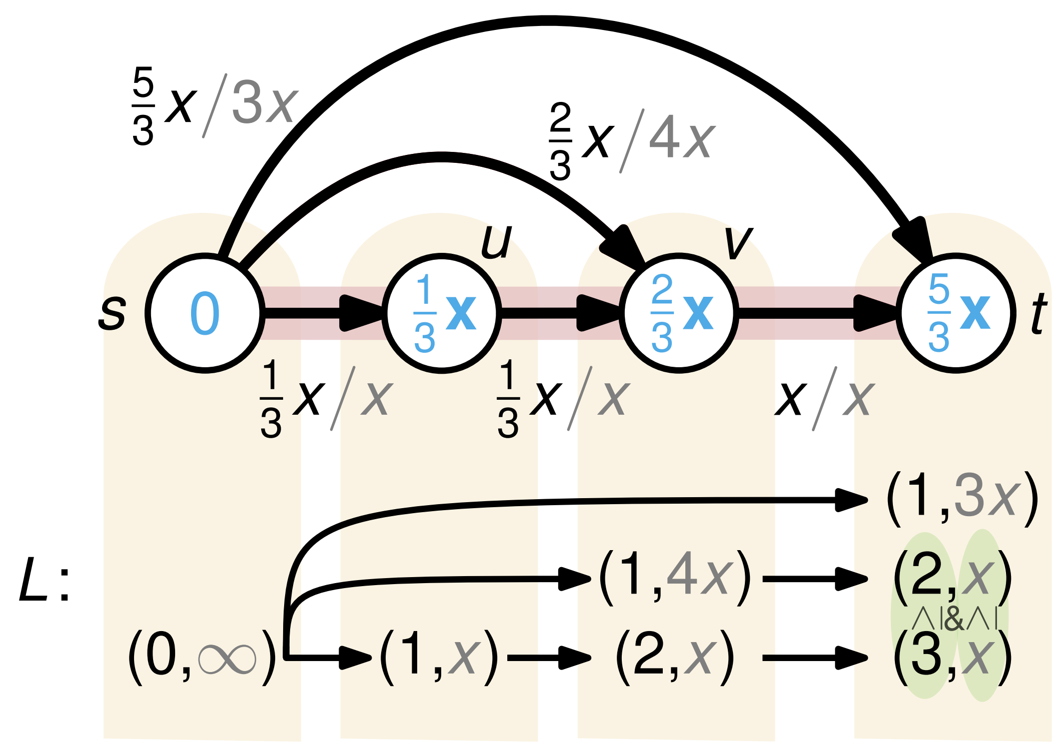

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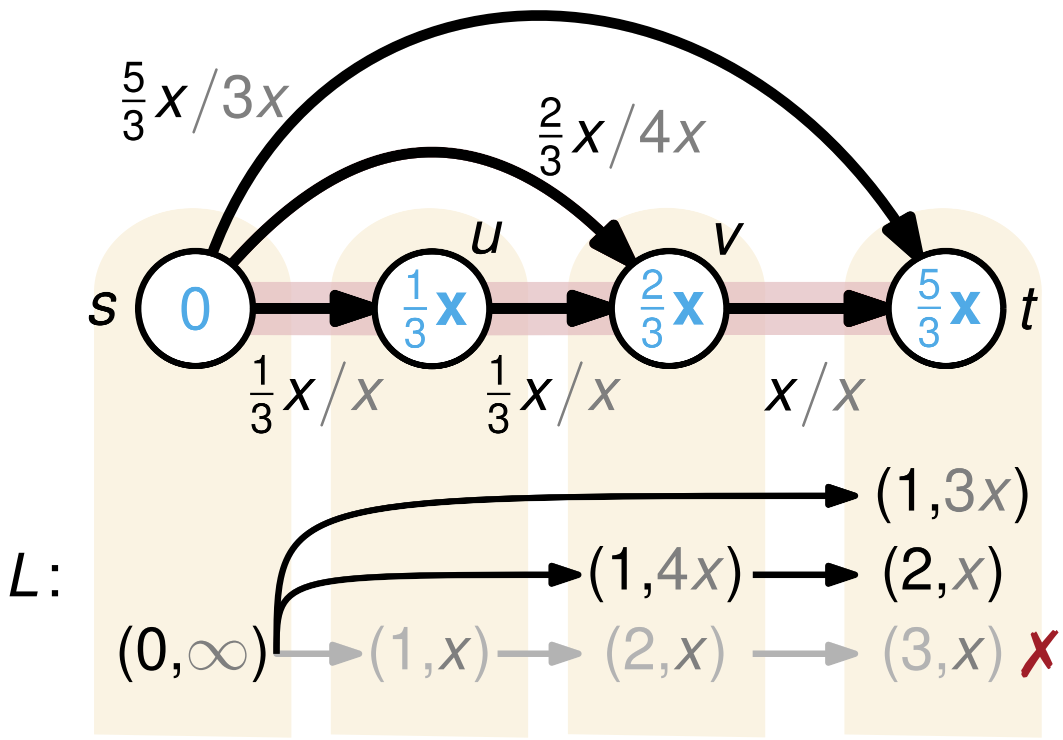

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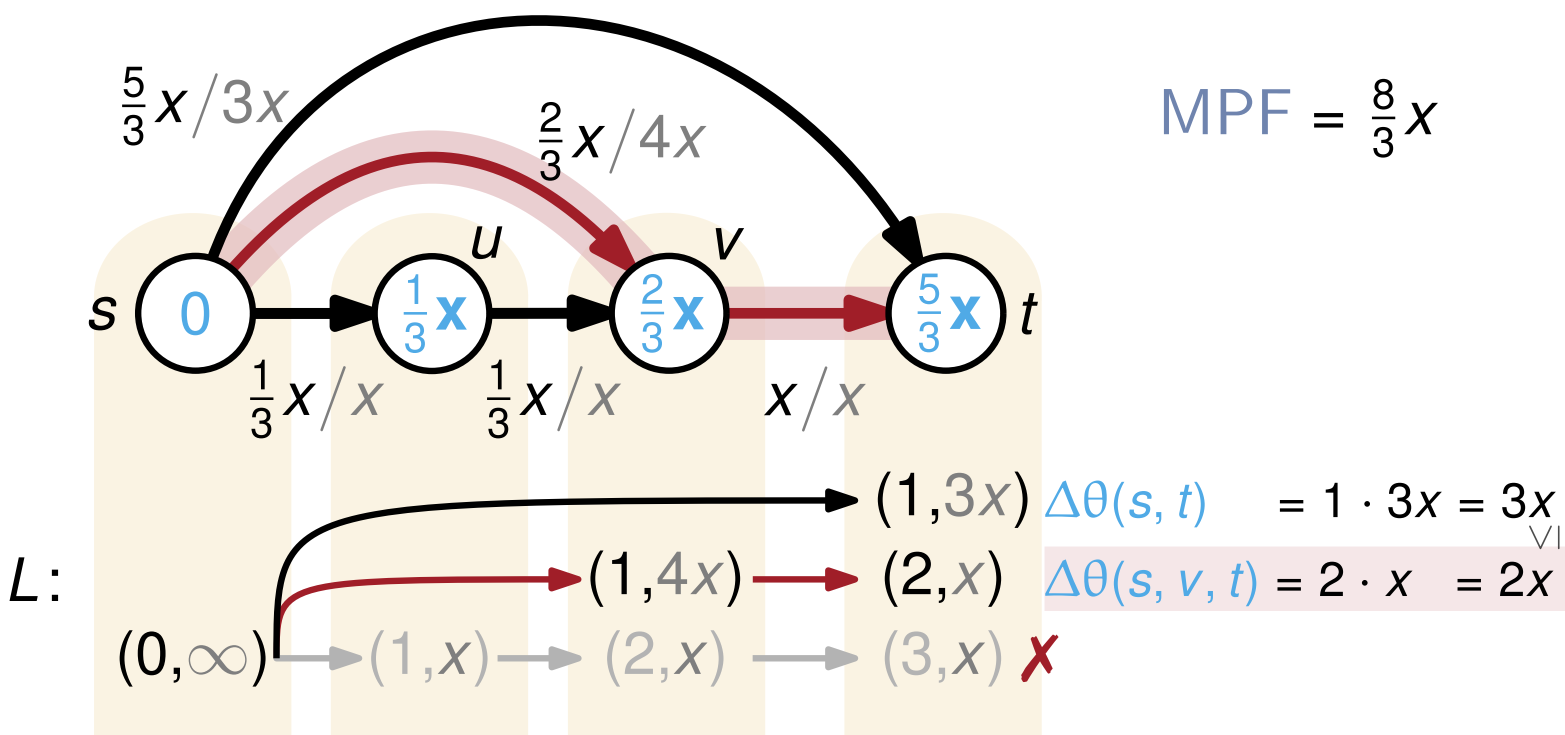

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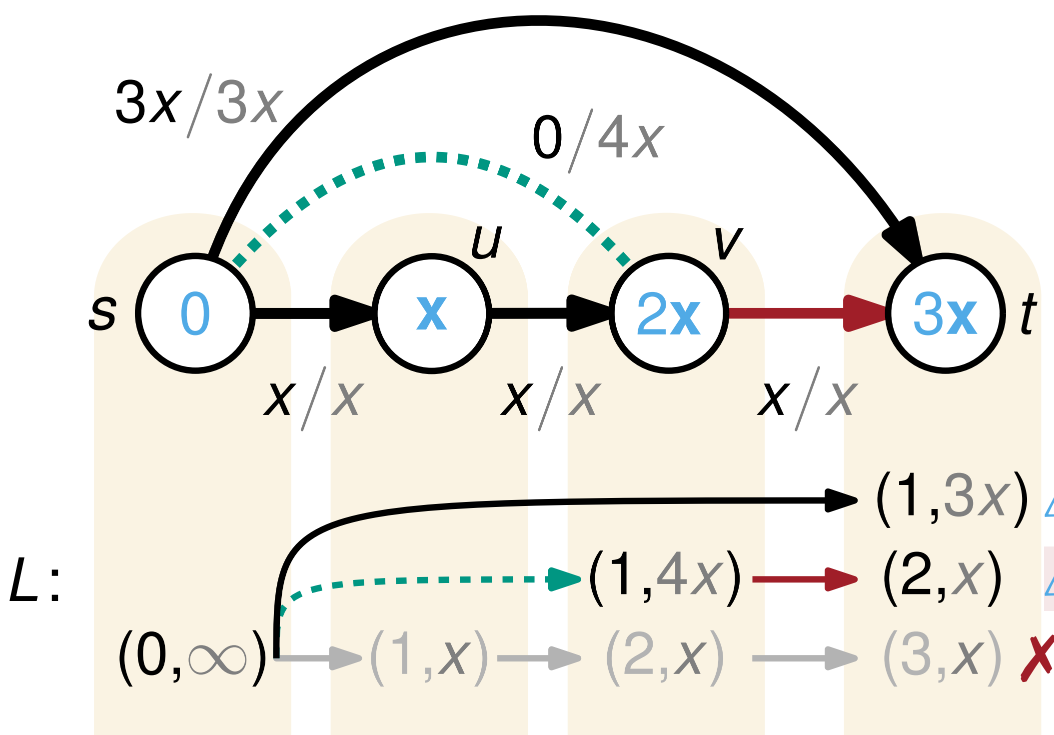

MPF = $4x$
= MTSF

$\Delta\theta(s,t) = 1 \cdot 3x = 3x$

$\geq$

$\Delta\theta(s,v,t) = 2 \cdot x = 2x$

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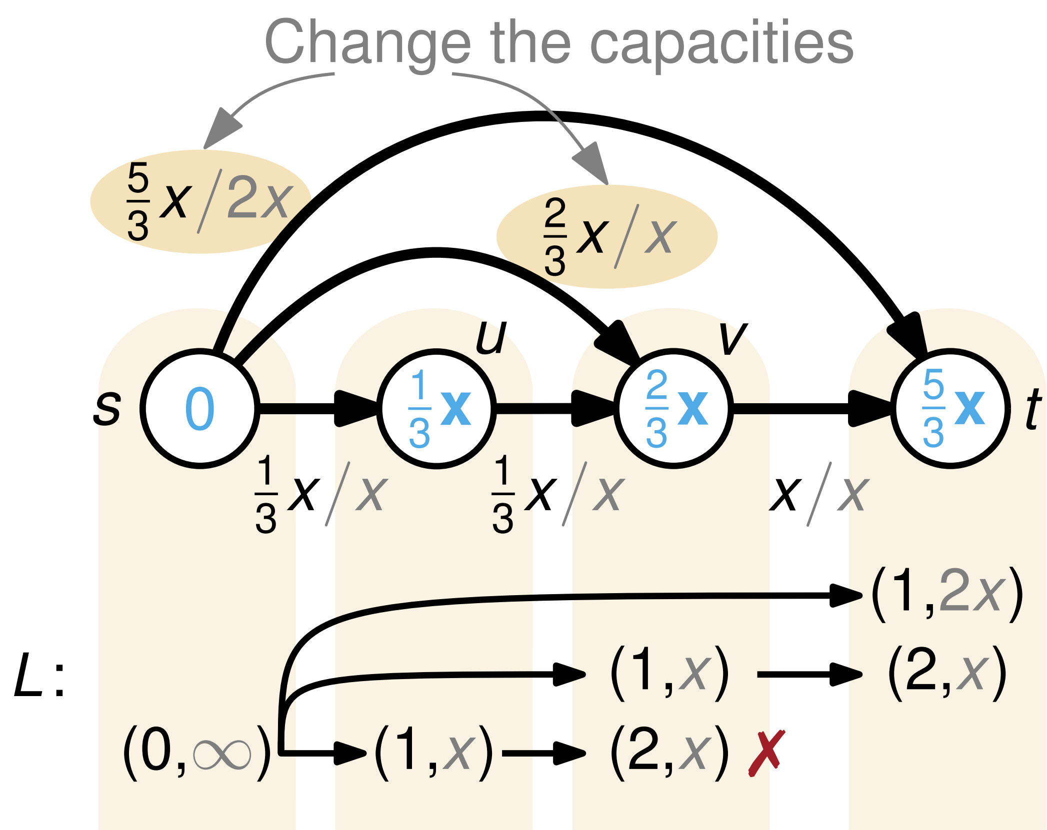

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Computing DTP [Section 5; Grastien et al., 2018]

Description:

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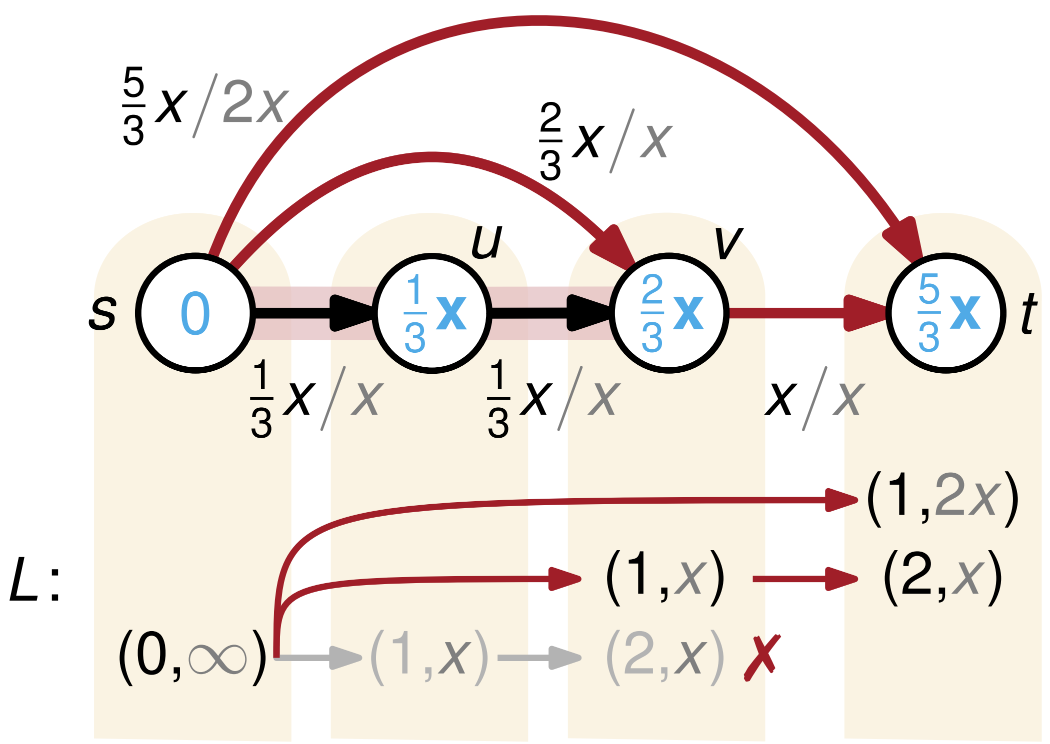
- DTPs from s do not have to form a tree



MPF $= \frac{8}{3}x$

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Computing DTP [Section 5; Grastien et al., 2018]

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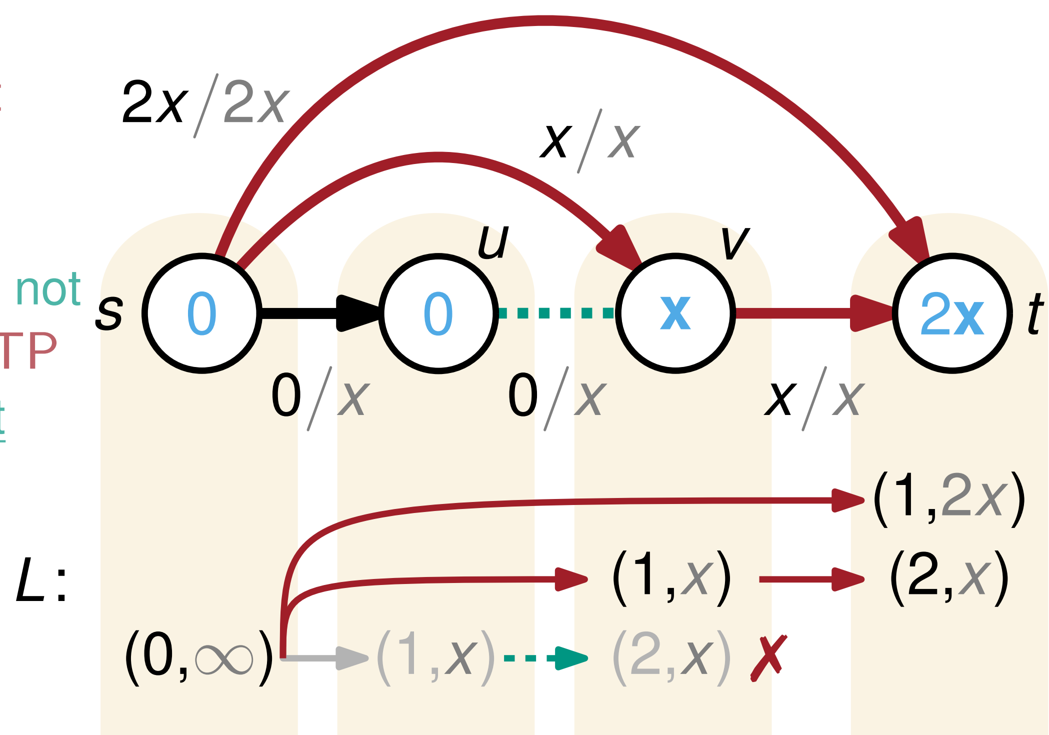
- DTPs from s do not have to form a tree
- Optimal switches do not have to lie on the DTP if the structure is not penrose-minor free



MPF = $3x$
= MTSF

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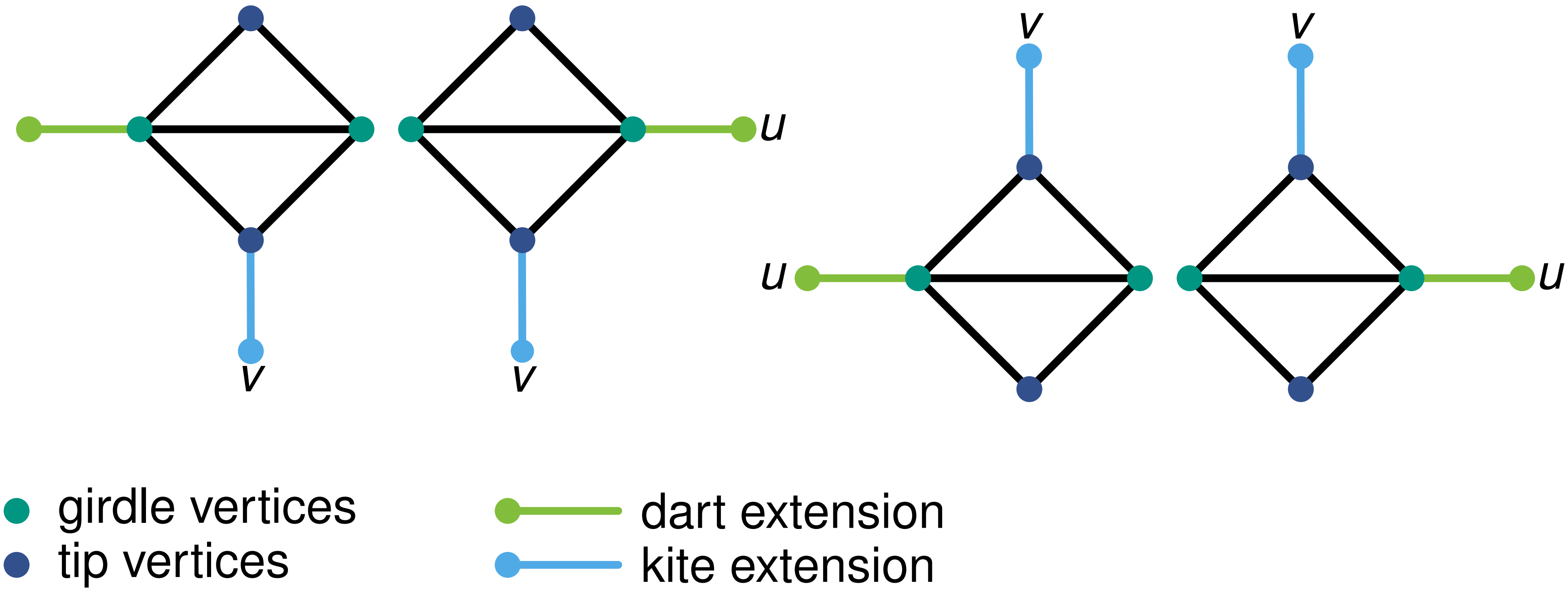
Penrose Graphs

[Section 5; Grastien et al., 2018]



All cases show penrose graphs, where u and v are either generators or consumers, but not both the same. They are a combination of a kite graph (i.e., diamond graph with an additional edge on one of the tip vertices) and a dart graph (i.e., diamond graph with an additional edge on one of the girdle vertices).

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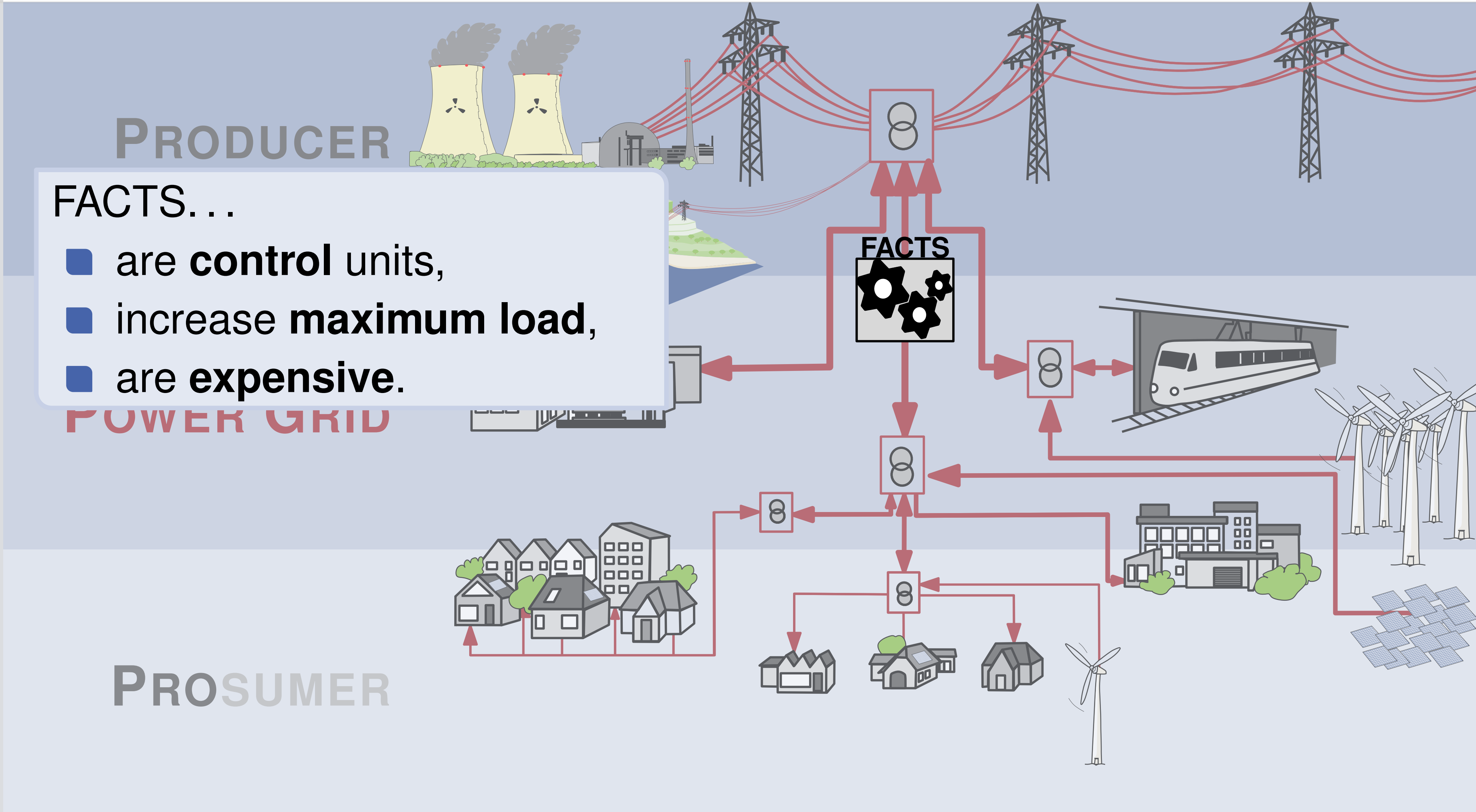
Flexible AC Transmission Systems (FACTS)

FACTS...

- are **control** units,
- increase **maximum load**,
- are **expensive**.



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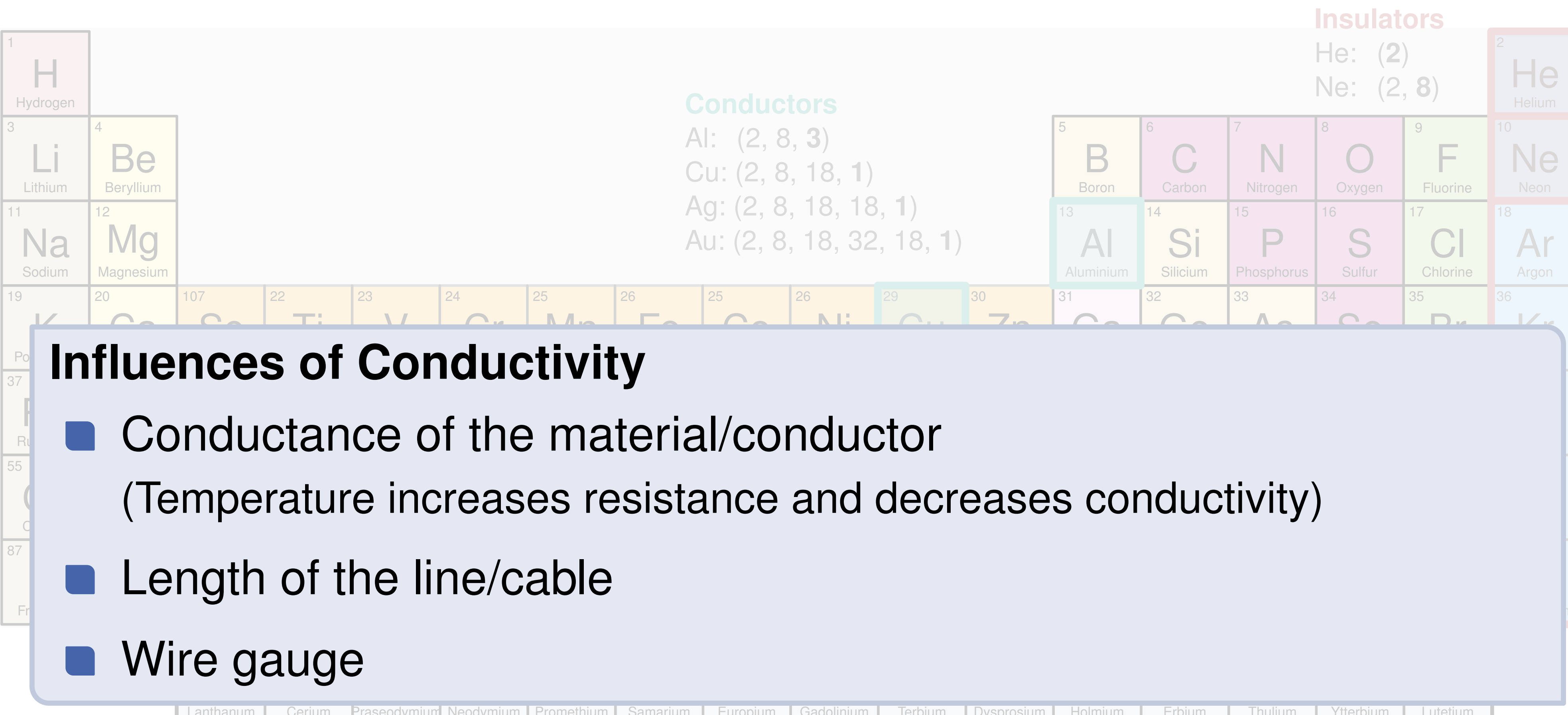
Influence of Conductivity



Influences of Conductivity

- Conductance of the material/conductor
  (Temperature increases resistance and decreases conductivity)
- Length of the line/cable
- Wire gauge

In the linear AC-Model the conductivity can be changed by the susceptance $b(u, v) \in \mathbb{R}$. To change the susceptance we use FACTS.

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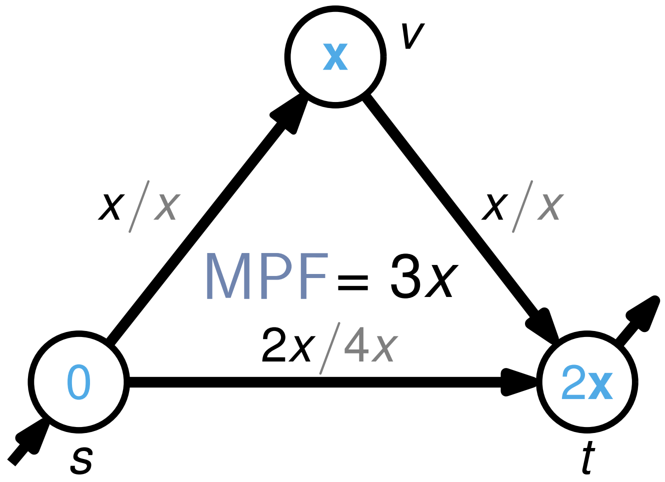
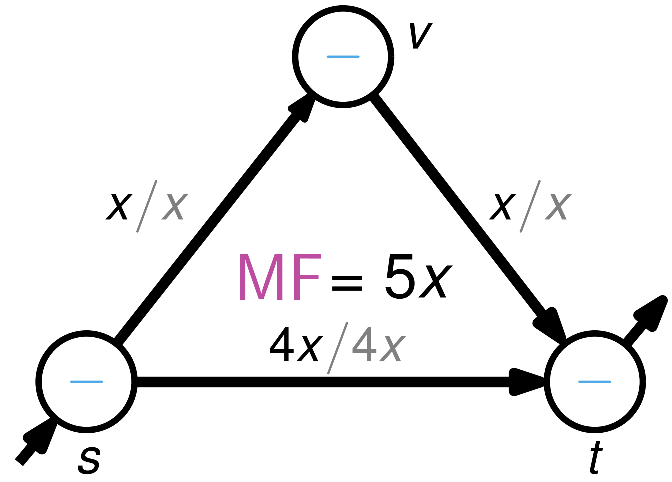
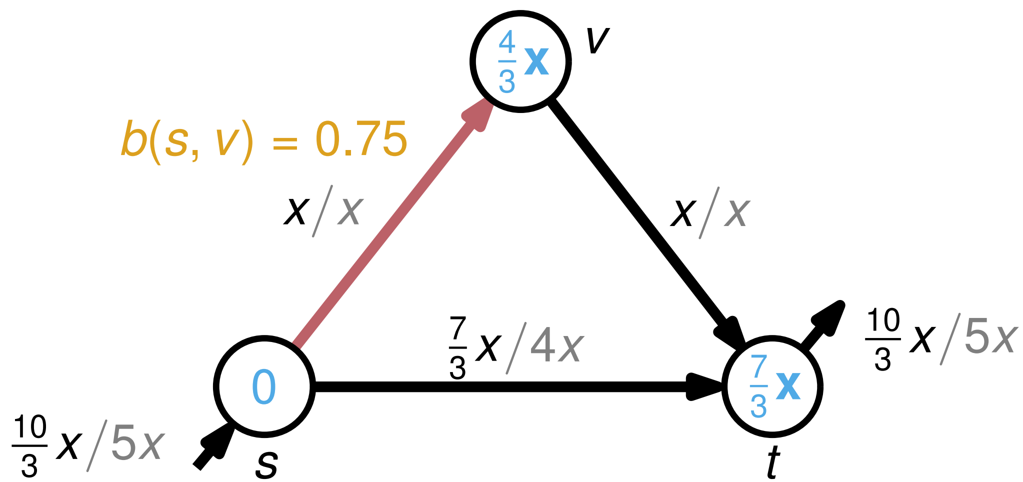
The Maximum FACTS Flow (MFF) Problem



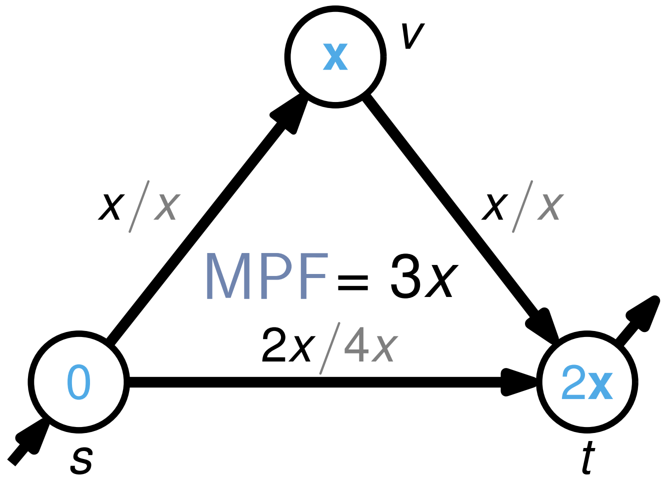


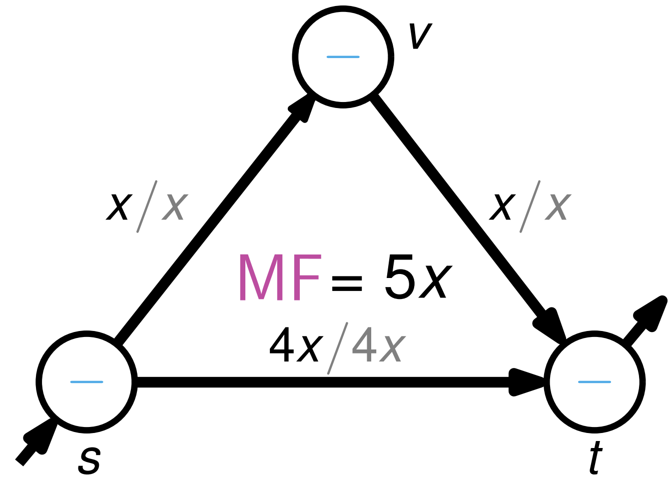


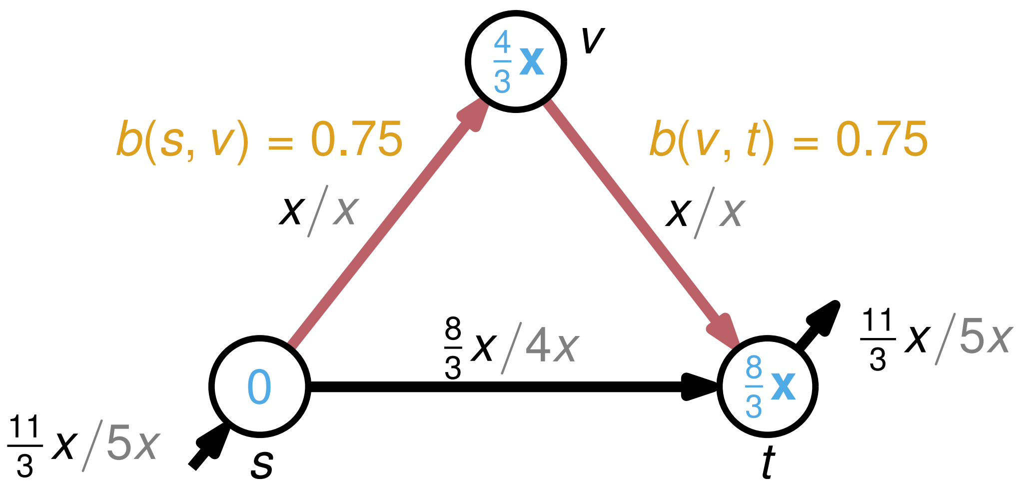
$b(i,j) := [0.75; 1.25]$
$\forall (i,j) \in E$

$$\forall (u,v) \in E\colon\; f(u,v) = b(u,v)\big(\theta(v) - \theta(u)\big)$$

The MAXIMUM FACTS FLOW (MFF) Problem







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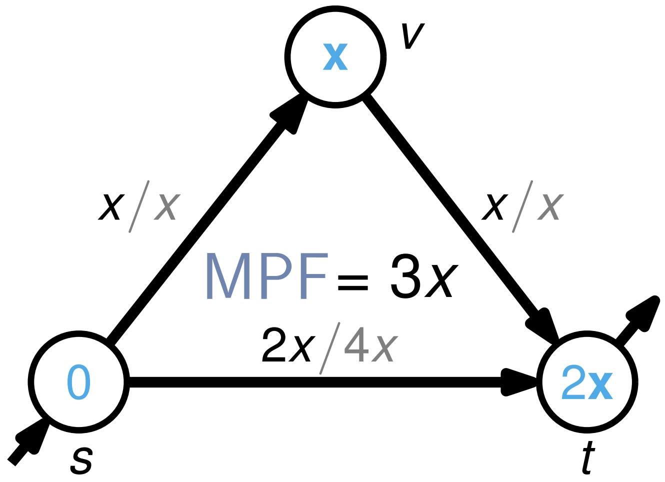

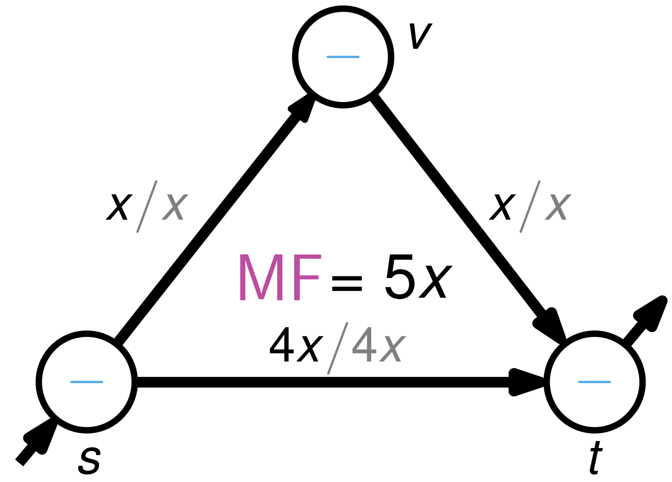

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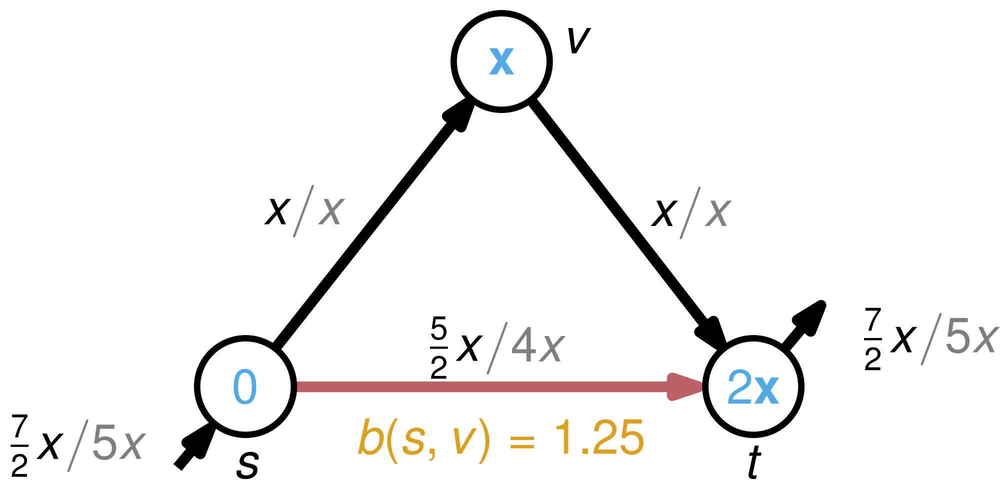

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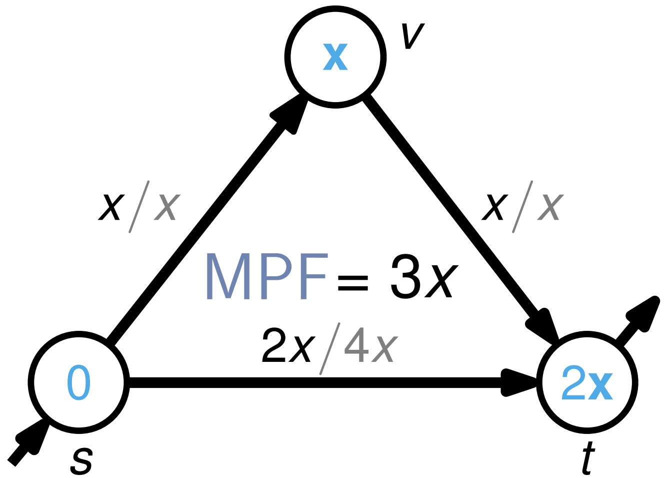

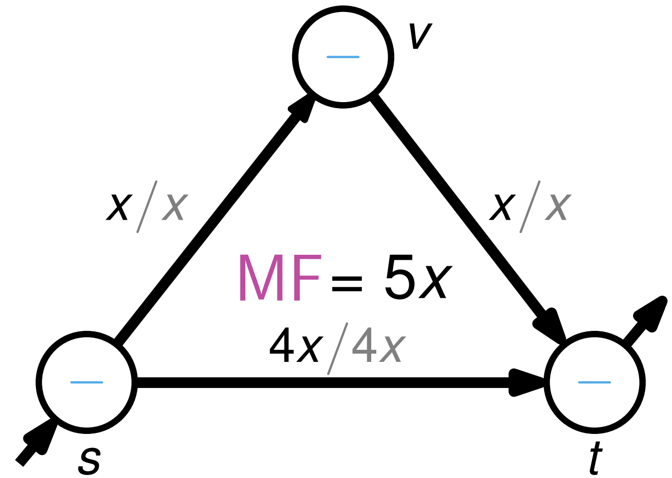

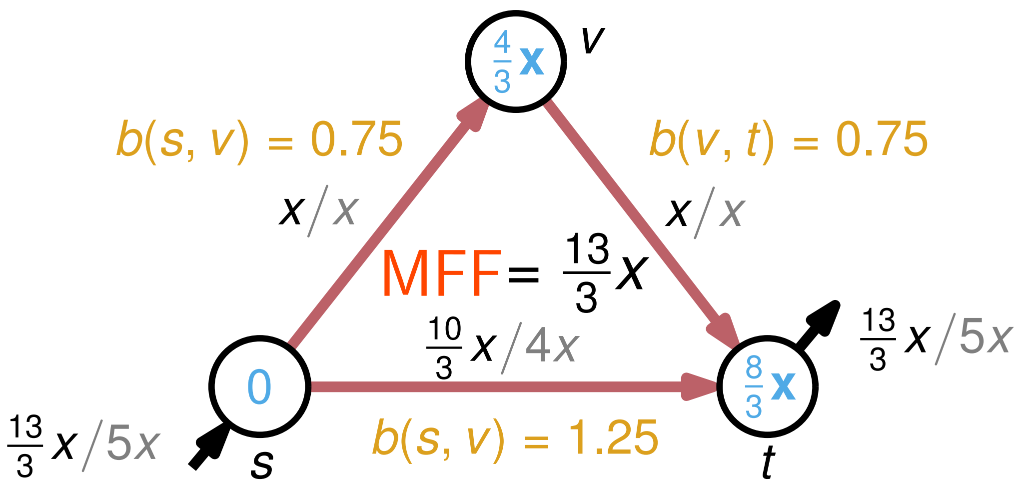

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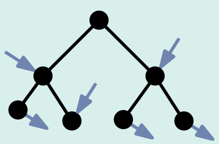

Physical Model (MPF) = Maximum FACTS Flow (MFF) = Flow Model (MF)

$\forall (i, j) \in E$

$\forall (u, v) \in E\colon f(u, v) = b(u, v)\big(\theta(v) - \theta(u)\big)$

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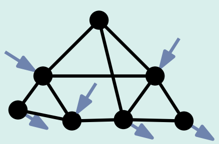

Physical Model (MPF) $\leq$ Maximum FACTS Flow (MFF) $\leq$ Flow Model (MF)

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The MAXIMUM FACTS FLOW (MFF) Problem

[Lehmann et al., 2015]

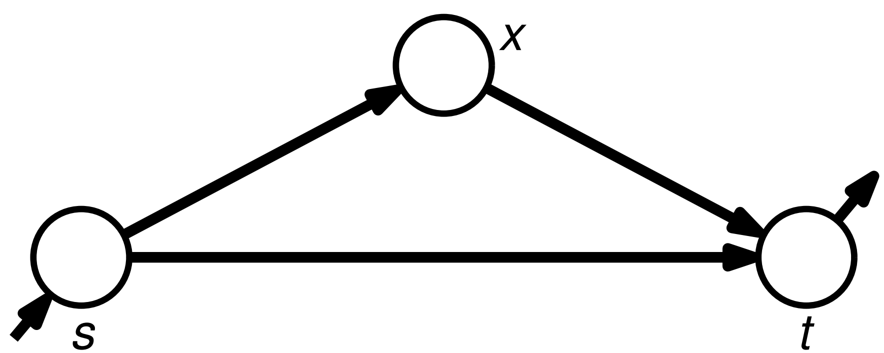
- The value of the Maximum Flexible AC Transmission Switching Flow (MFF) is defined as

$$\text{MFF}(\mathcal{N}, k) := \max_{E' \subseteq E, b} \text{MPF}(\mathcal{N}) \qquad |E'| \leq k$$

with f being a feasible power flow meaning

$$f_{\text{net}}(u) = 0 \qquad \forall u \in V \setminus (V_G \cup V_C)$$

$k = 1$



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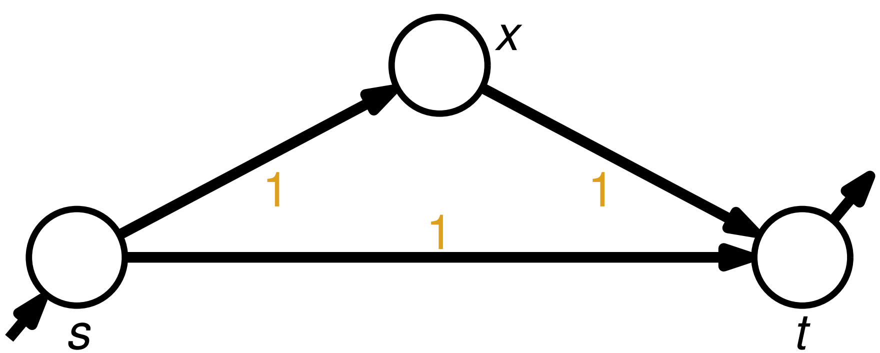

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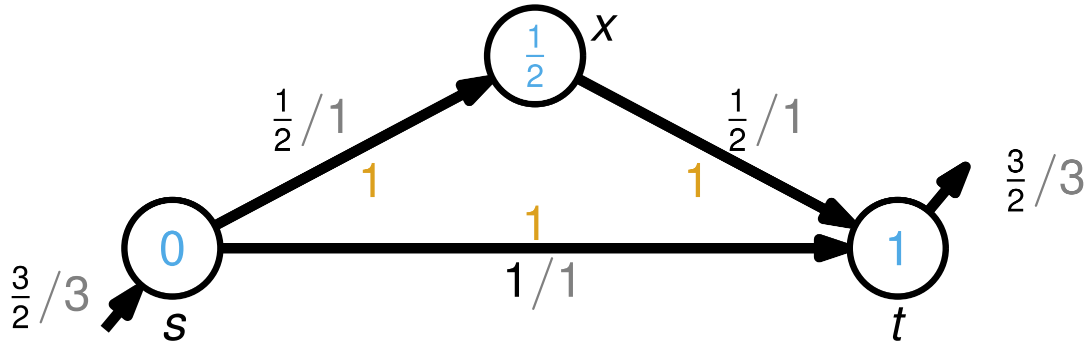

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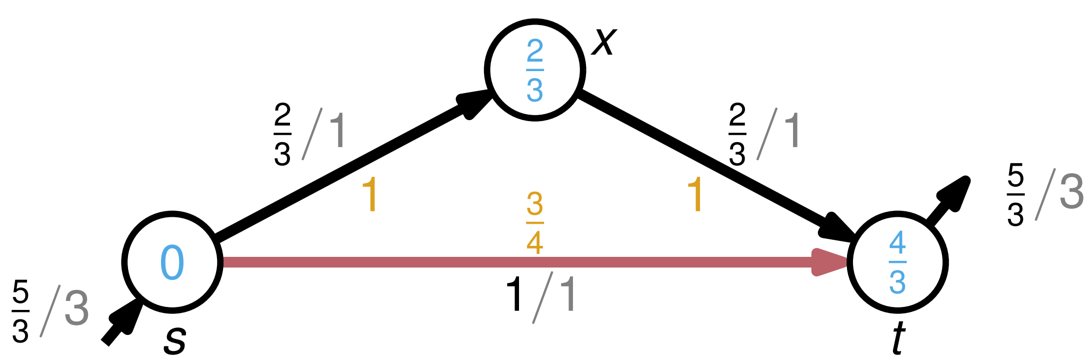

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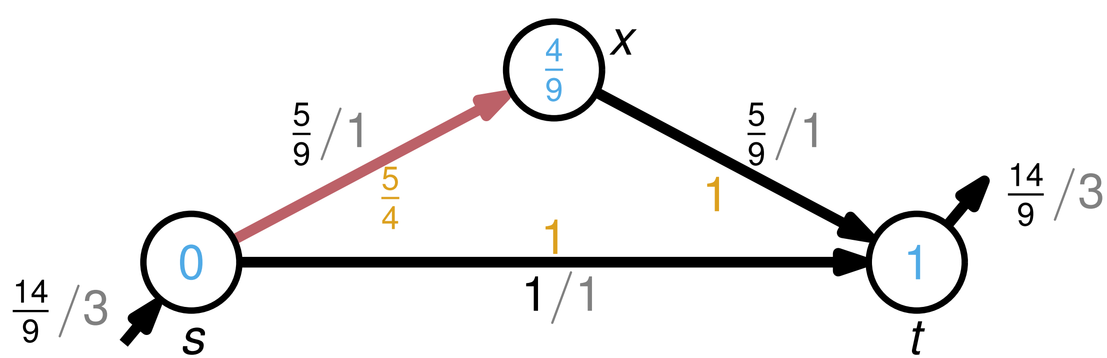

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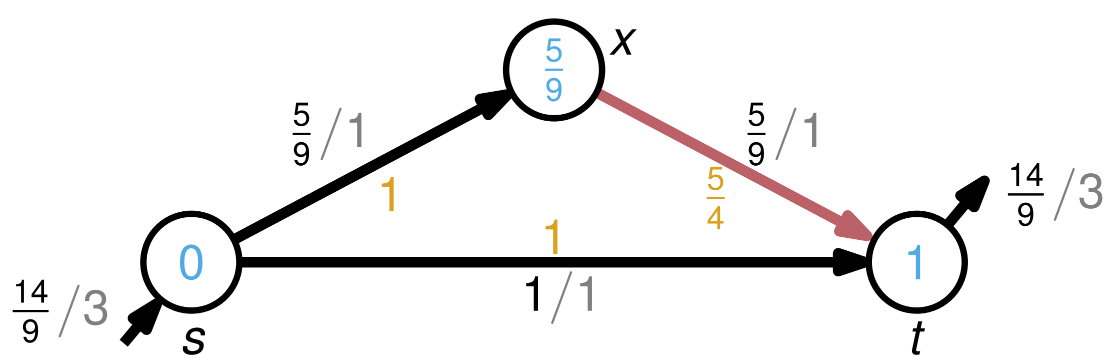

The Maximum FACTS Flow (MFF) Problem

[Lehmann et al., 2015]

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Optimization Problem MFF

Instance: A power grid $\mathcal{N}$.

Objective: Find a set $E' \subseteq E$ of edges with FACTS and a susceptance configuration $b(e)$ with $e \in E'$ such that MPF($\mathcal{N}$) is maximum among all choices of FACTS placements and susceptance configurations while complying with $|E'| \leq k$.

The OPTIMAL FACTS FLOW (OFF) Problem

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- The value of the OPTIMAL POWER FLOW (OPF) is defined as
$$\text{OPF}(\mathcal{N}) = \min \gamma(\mathcal{N}, f)$$
with f being a feasible power flow and the generator cost function γ.

Optimization Problem OFF

Instance: A power grid $\mathcal{N}$.

Objective: Find a set $E' \subseteq E$ of edges with FACTS and a susceptance configuration $b(e)$ with $e \in E'$ such that $\text{OPF}(\mathcal{N})$ is minimum among all choices of FACTS placements and susceptance configurations while complying with $|E'| \leq k$.

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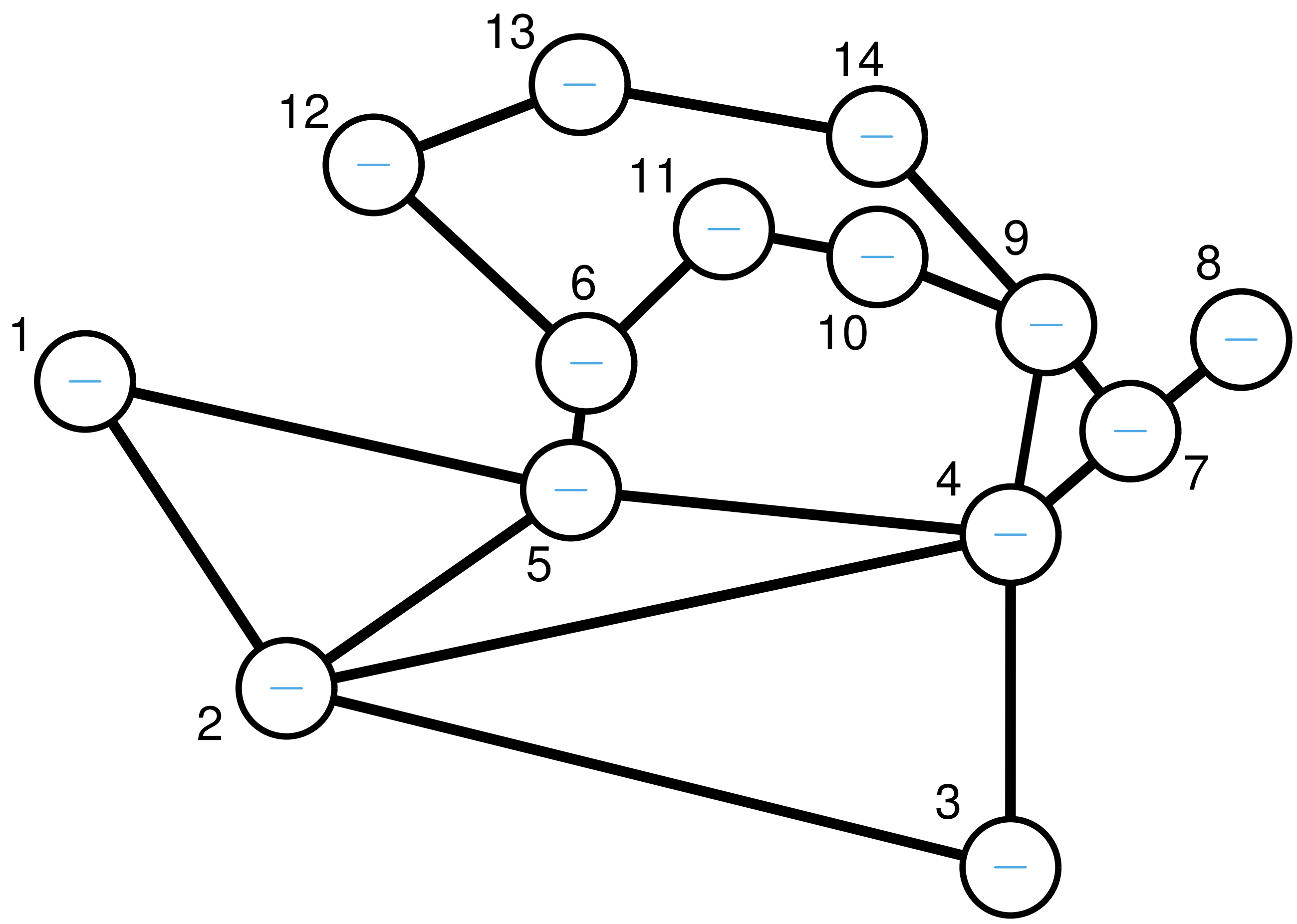
Optimal FACTS Flow (OFF)



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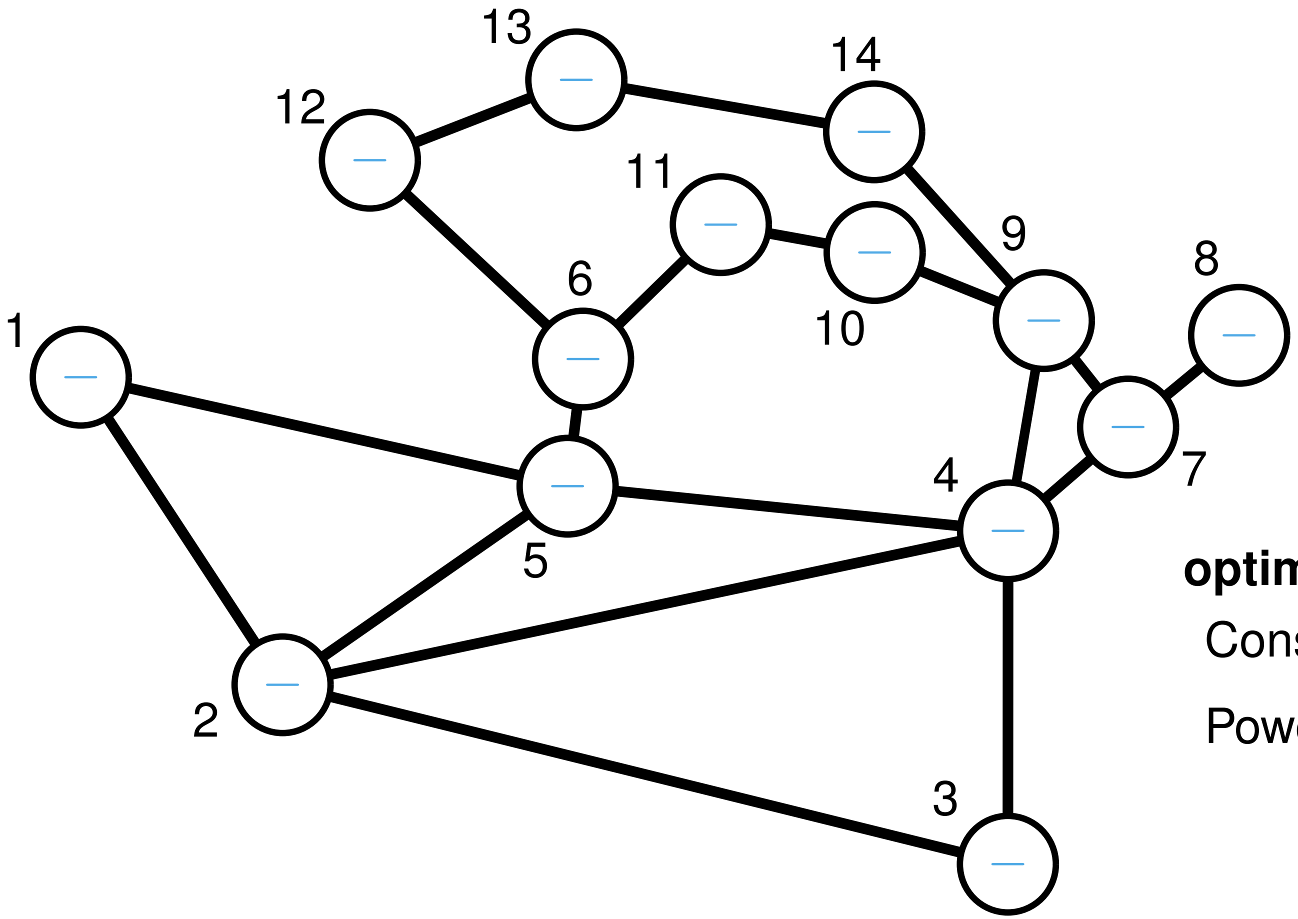
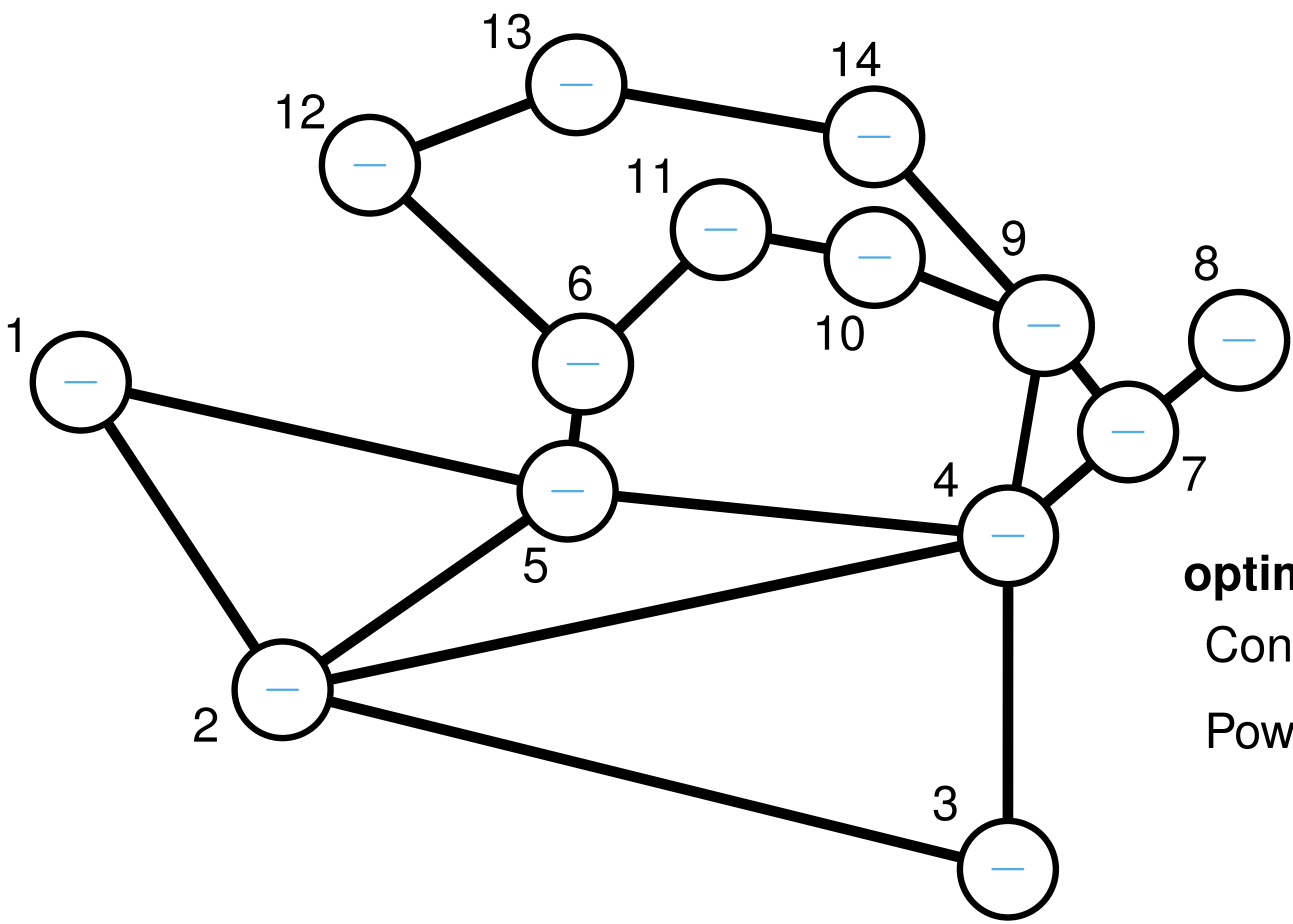


optimize with regards to:

Conservation of Flow

Power Flow Constraint

Optimal FACTS Flow (OFF)



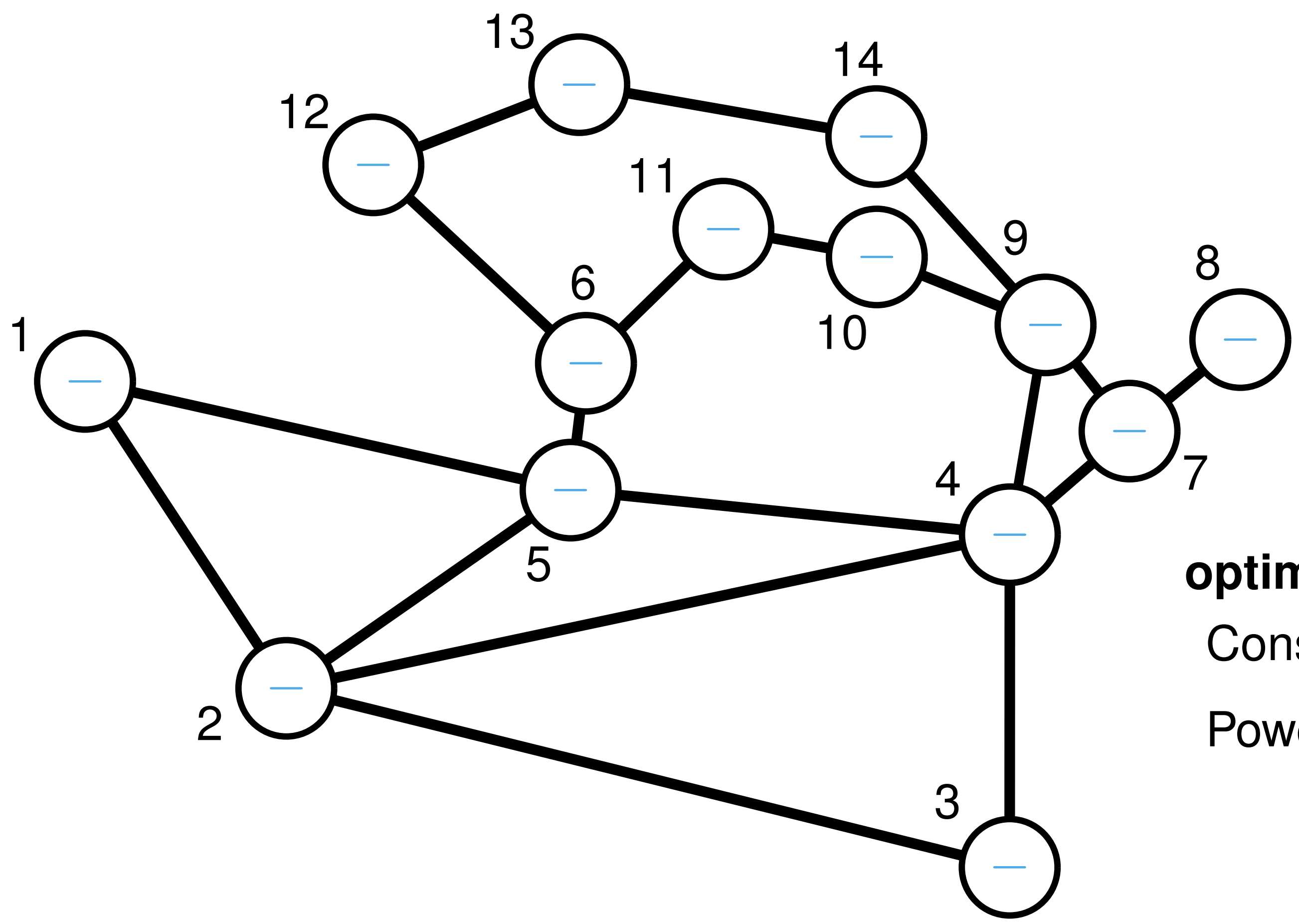
optimize with regards to:

Conservation of Flow

Power Flow Constraint

minimize Costs

Optimal FACTS Flow (OFF)



optimize with regards to:

Conservation of Flow ✓

Power Flow Constraint ✓

Physical Model

minimize Costs

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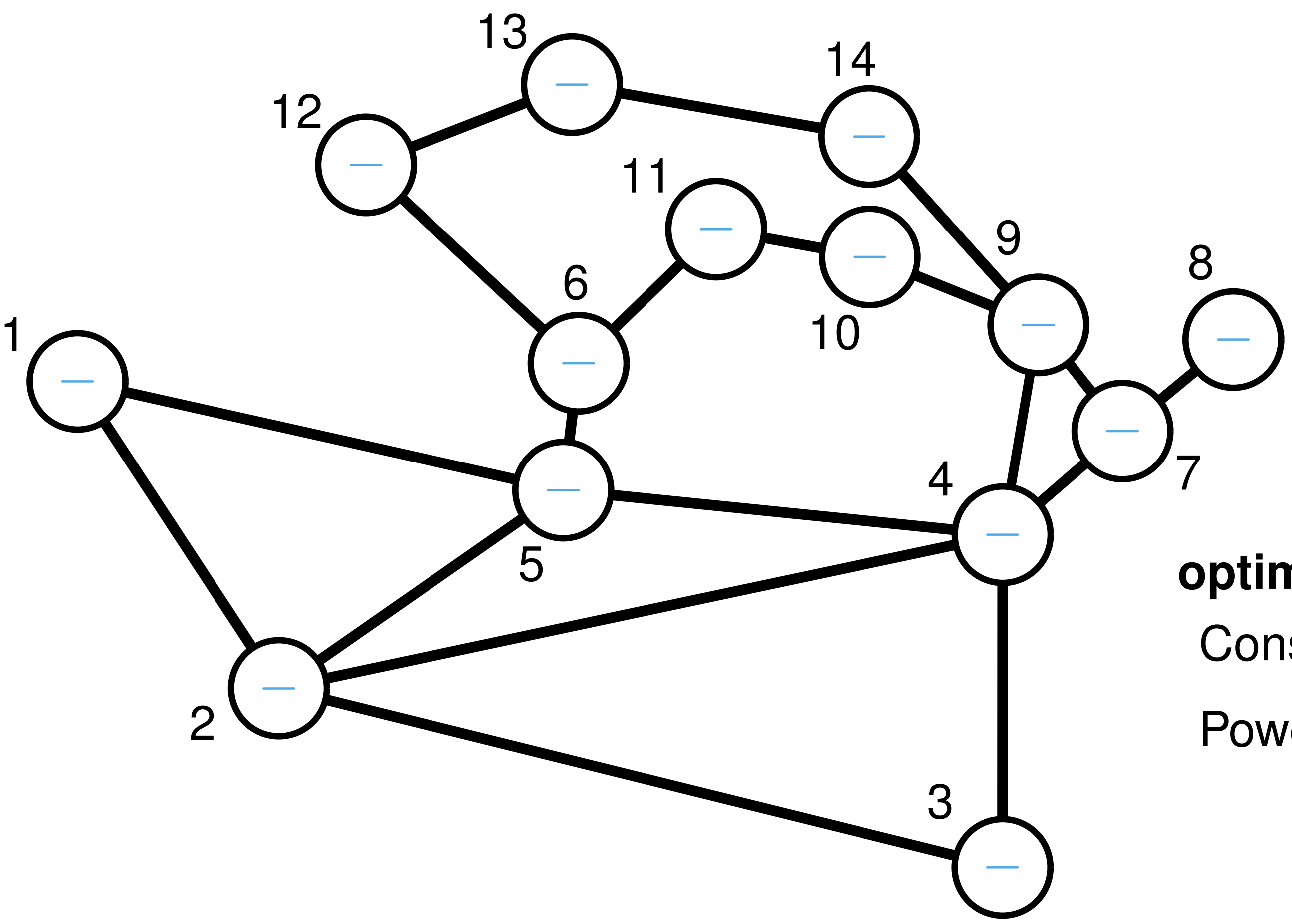
OPTIMAL FACTS FLOW (OFF)



optimize with regards to:

	Physical Model	Flow Model
Conservation of Flow	✓	✓
Power Flow Constraint	✓	✗

Physical Model

minimize Costs

Flow Model

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OPTIMAL FACTS FLOW (OFF)

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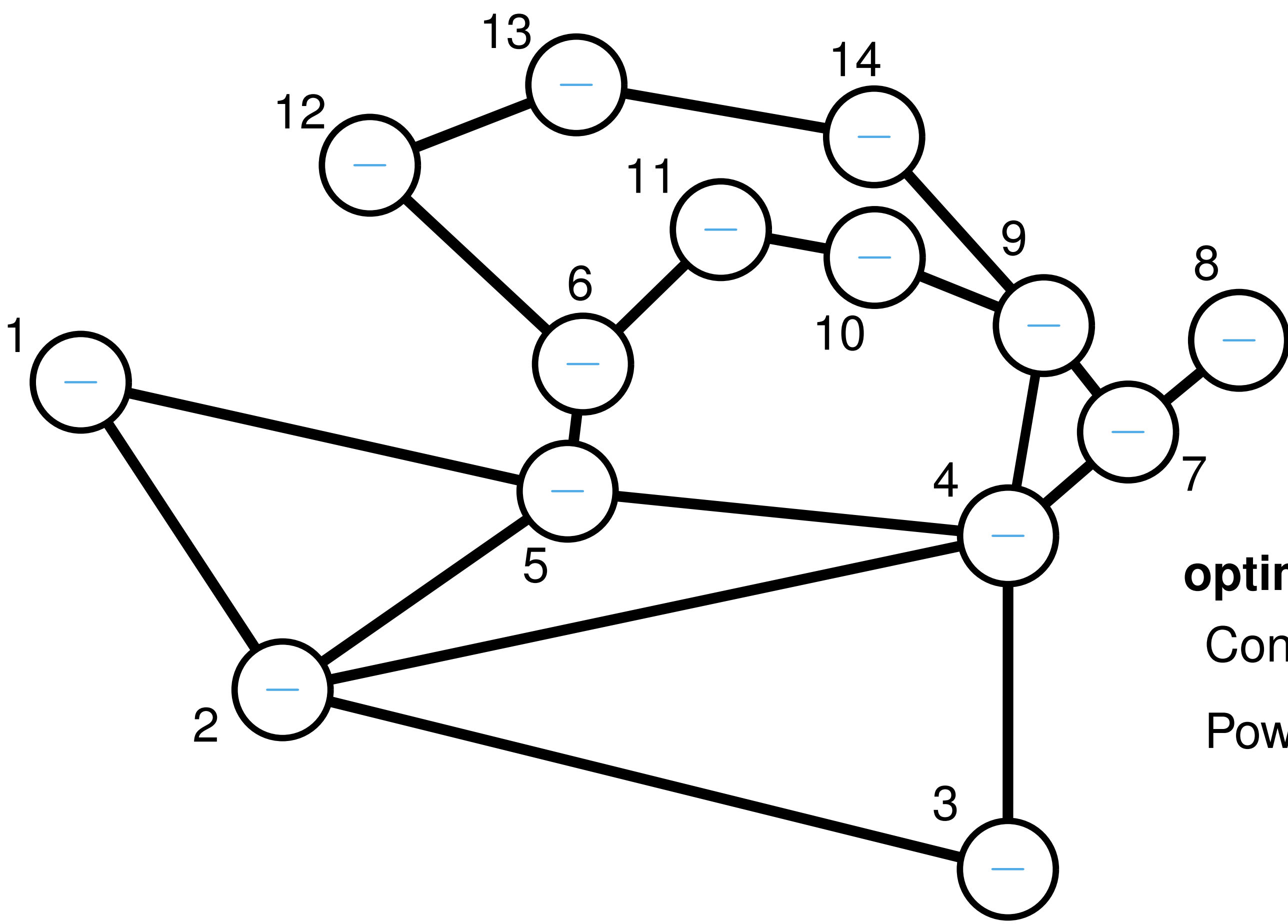

optimize with regards to:

Conservation of Flow ✓ ✓

Power Flow Constraint ✓ ✗

minimize Costs

Physical Model

upper bound

Flow Model

lower bound

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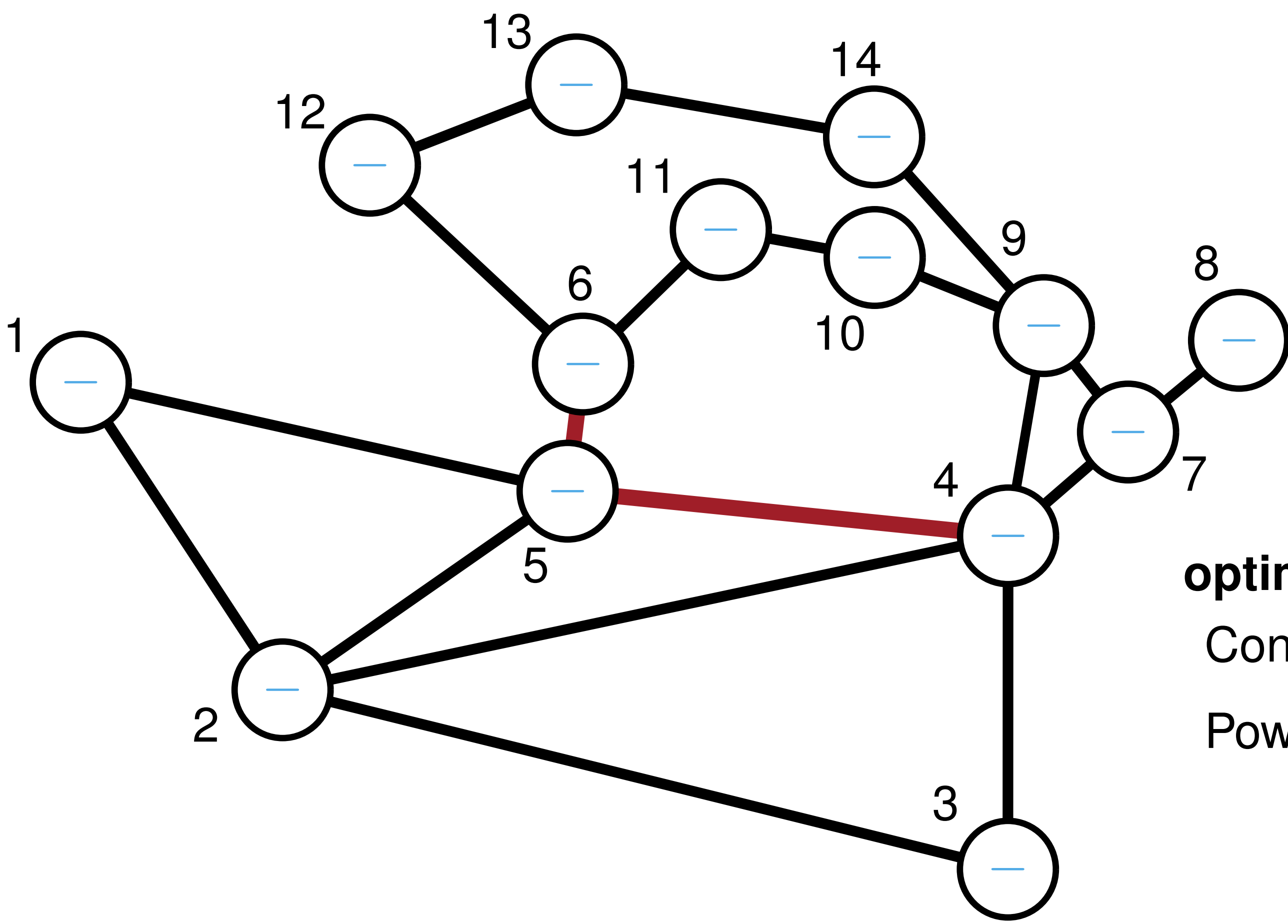
OPTIMAL FACTS FLOW (OFF)



FACTS

optimize with regards to:

Conservation of Flow	✓	✓	✓
Power Flow Constraint	✓	✗	✓

minimize Costs

Flow Model

lower bound

Physical Model

upper bound

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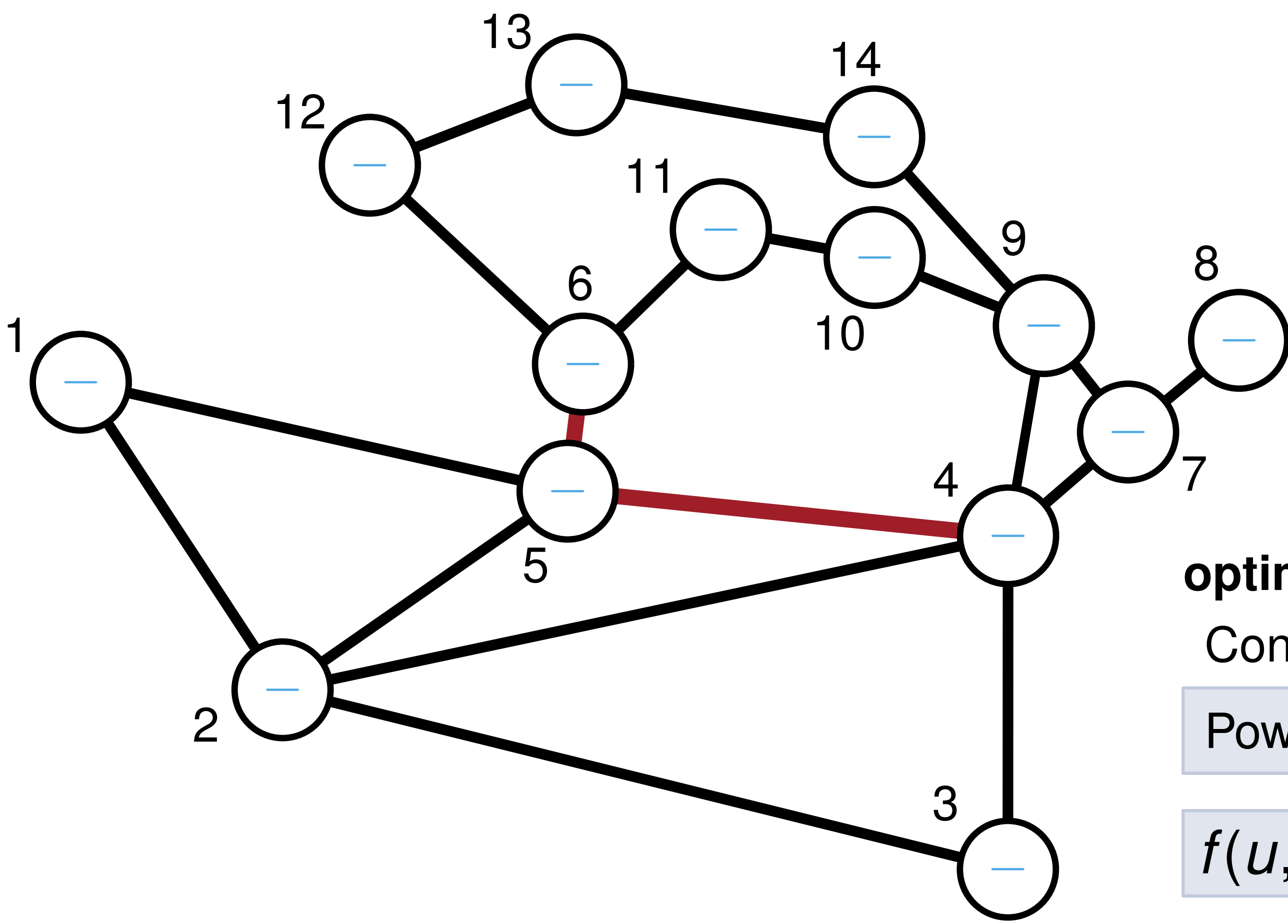
Optimal FACTS Flow (OFF)



FACTS

optimize with regards to:

Conservation of Flow	✓	✓	✓
Power Flow Constraint	✓	✗	✓

$$f(u, v) = b(u, v) \cdot (\theta(u) - \theta(v))$$

Flow Model

lower bound

minimize Costs

Physical Model

upper bound

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Matching the Flow Model [Leibfried et al.& Mchedlidze et al., 2015]

FACTS are expensive – how many do we need?

1. How many FACTS are necessary for globally optimal power flows? Which edges need to have a FACTS?
2. For a given number of available FACTS, is there a positive effect on flow costs and operability when approaching grid capacity limits?

Left Figure:
[2] http://www.lichtenwald-mentaltraining.de/files/bild_licht_im_wald.jpg

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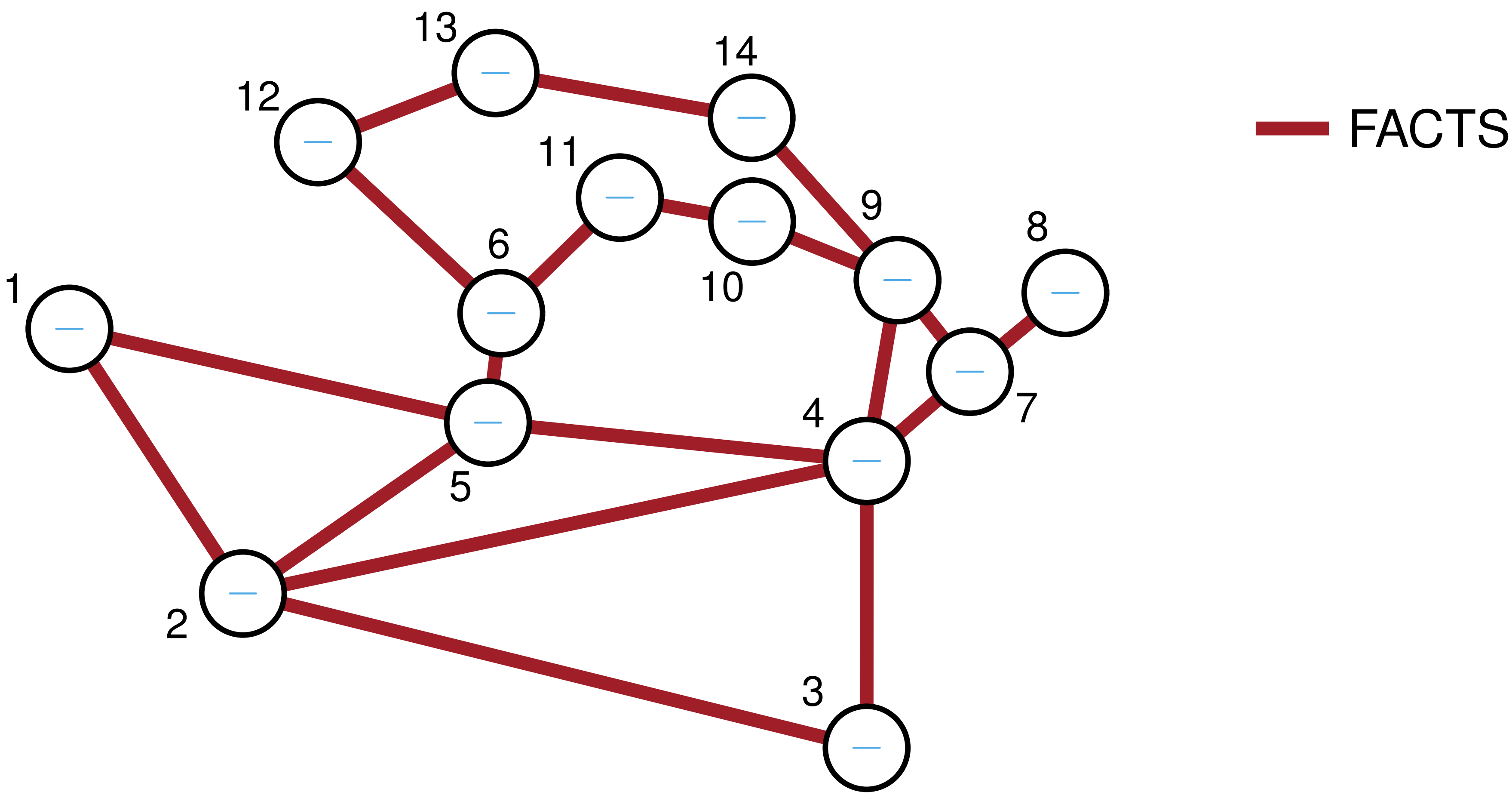
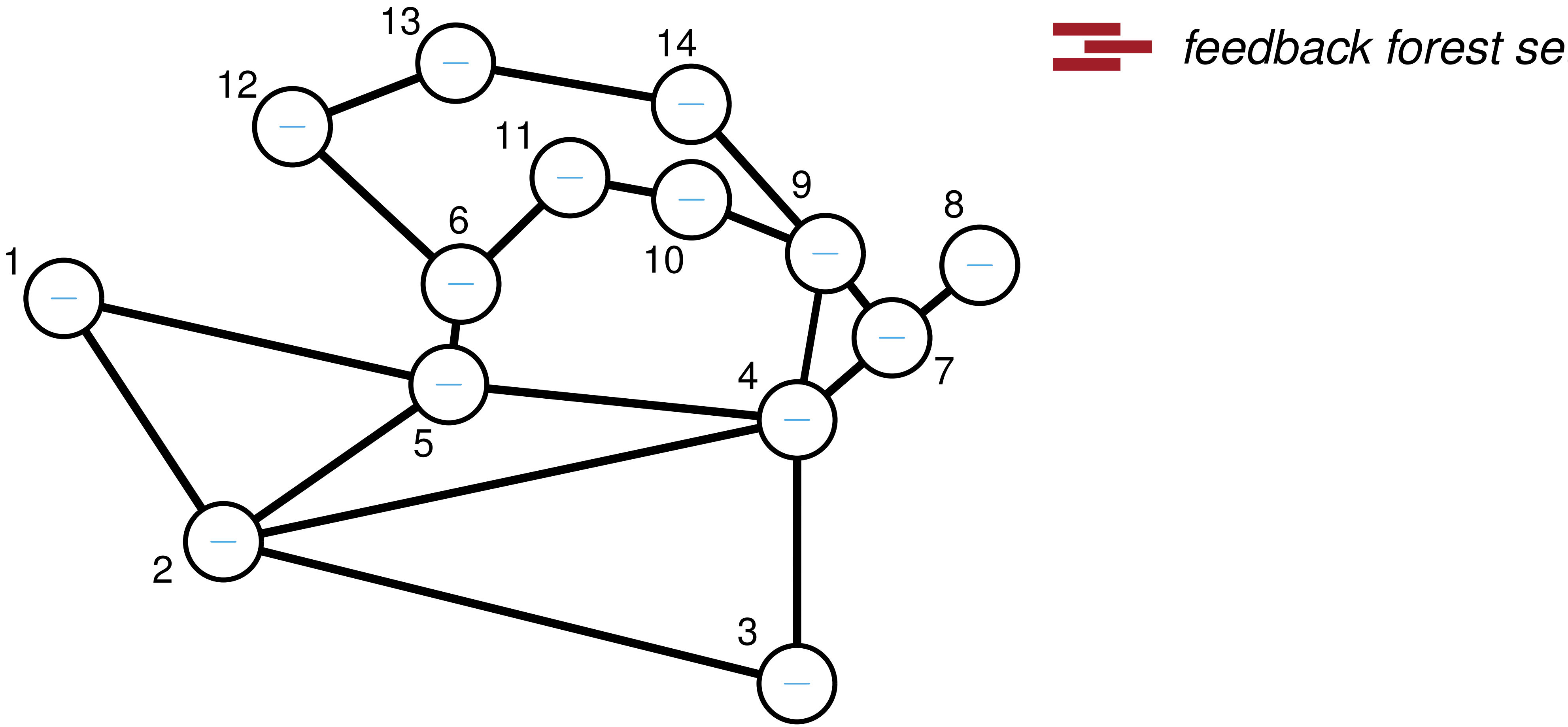
Globally Optimal Power Flows [Leibfried et al.& Mchedlidze et al., 2015]



Can we become as good as the Flow Model with fewer FACTS?

Feedback Forest Set [Leibfried et al.& Mchedlidze et al., 2015]



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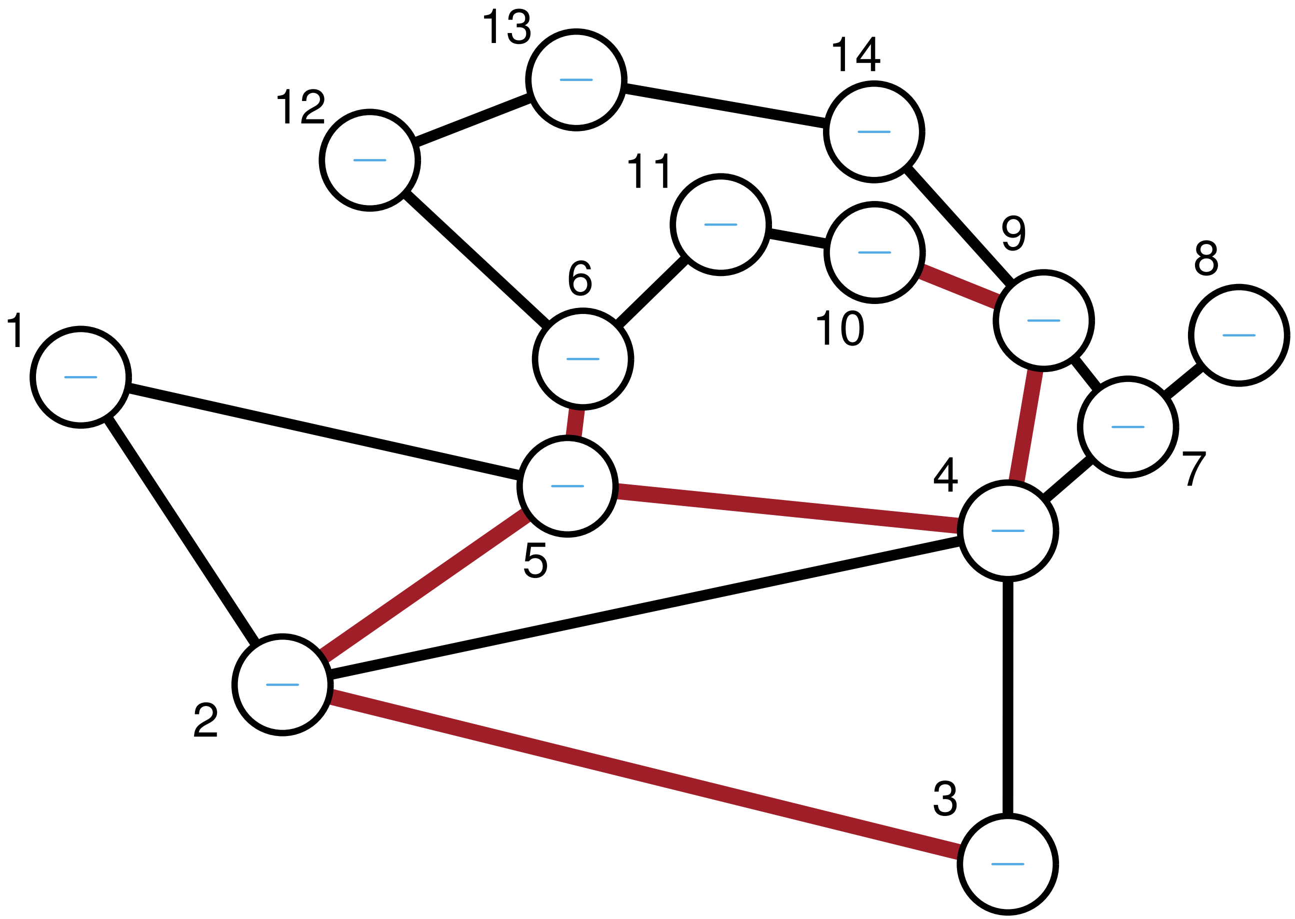
Feedback Forest Set [Leibfried et al.& Mchedlidze et al., 2015]



A set of trees (*forest*) remains!

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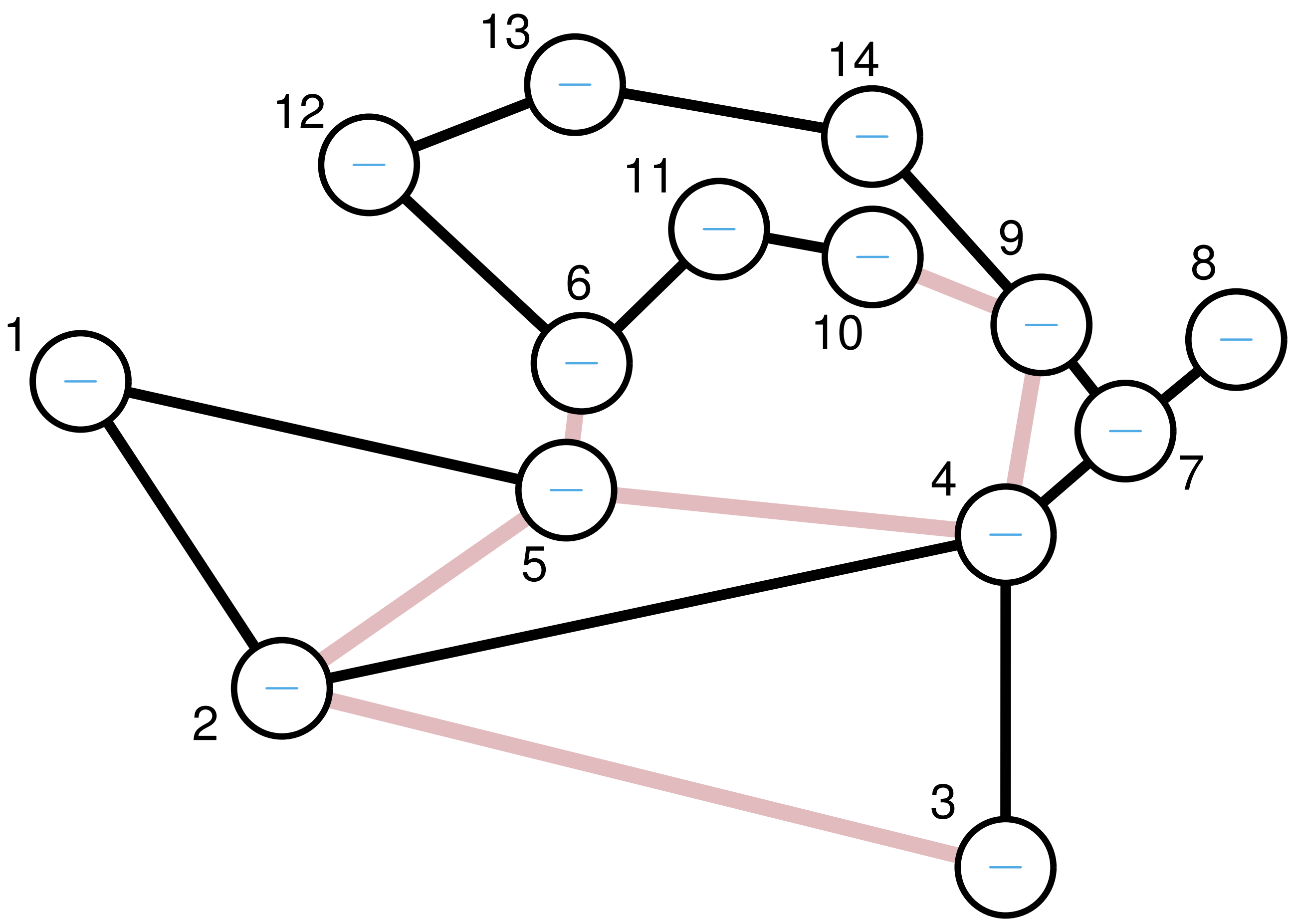
Feedback Forest Set [Leibfried et al.& Mchedlidze et al., 2015]



feedback forest set

⇓

A set of trees (*forest*) remains!

If the graph without FACTS represents a forest all flows represent feasible power flows.

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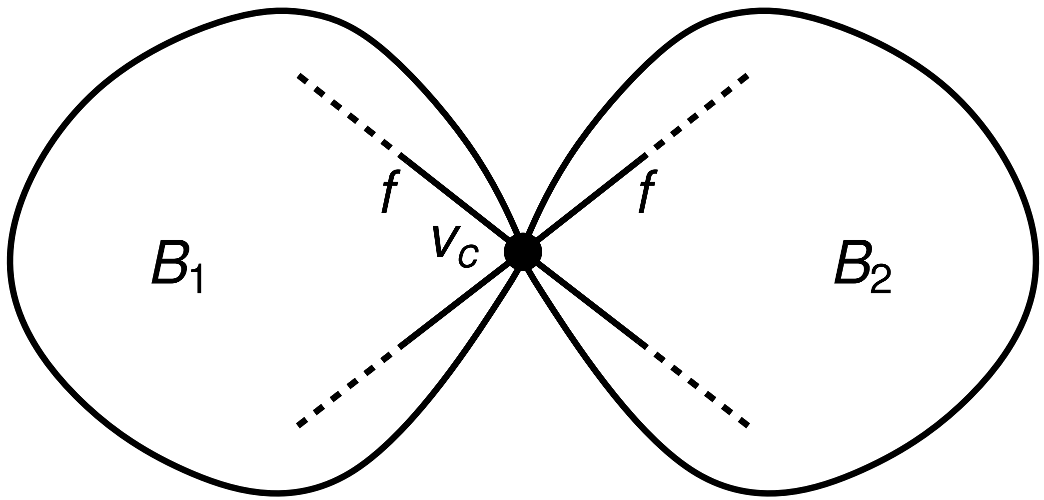
Generalized Algorithmic Idea [Leibfried et al.& Mchedlidze et al., 2015]

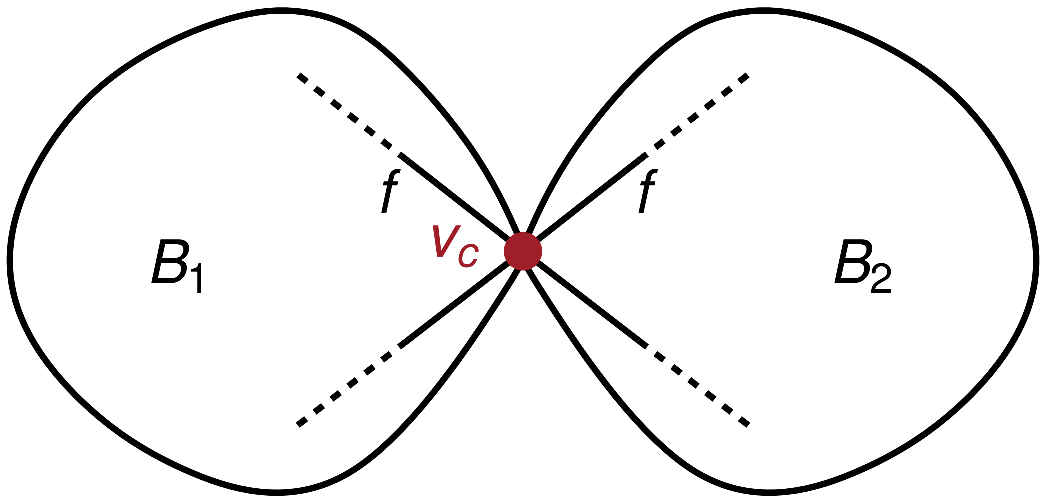


Idea

- Decompose the graph G at the cut-vertex v_c into subgraphs B_i
- The feasible power flows f does not change for the subgraphs B_i
- If we have a feasible power flows for each block B_i and combine the subgraphs at v_c this leads to a feasible power flows again

Generalized Algorithmic Idea

[Leibfried et al.& Mchedlidze et al., 2015]



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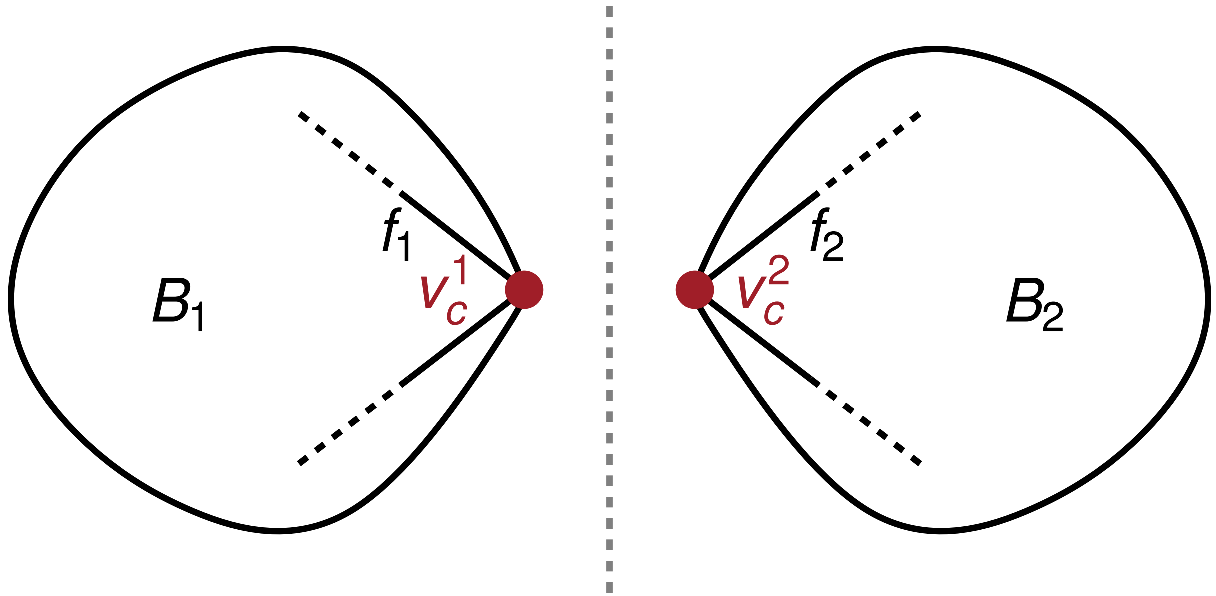

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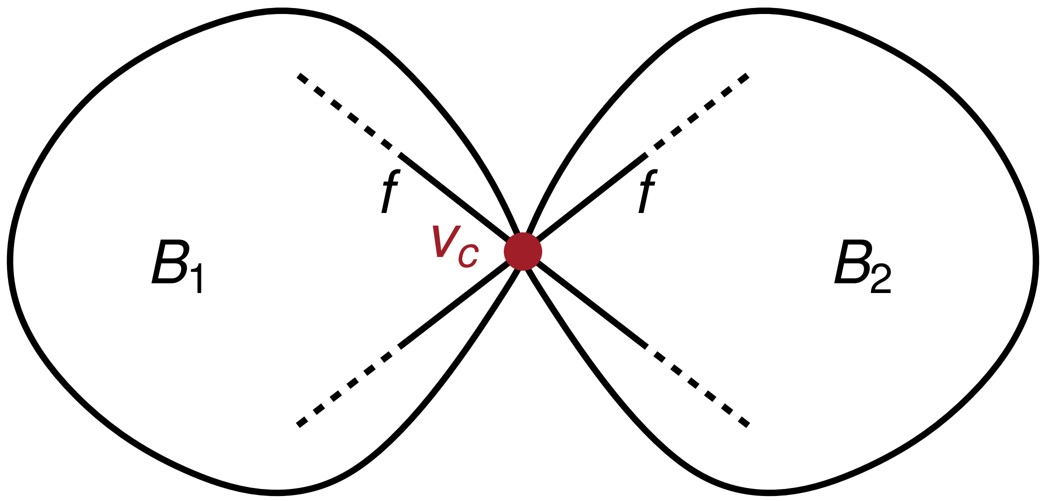

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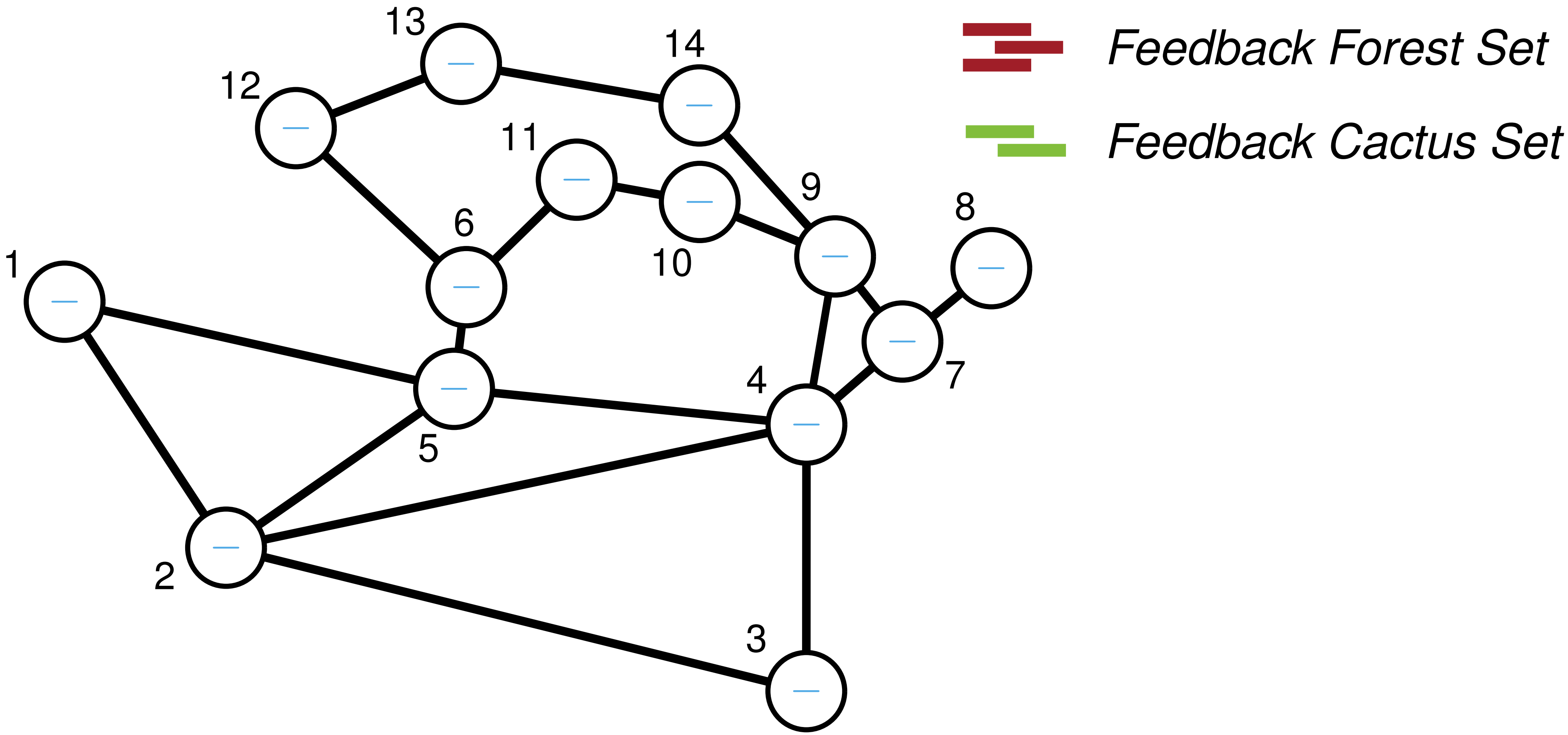
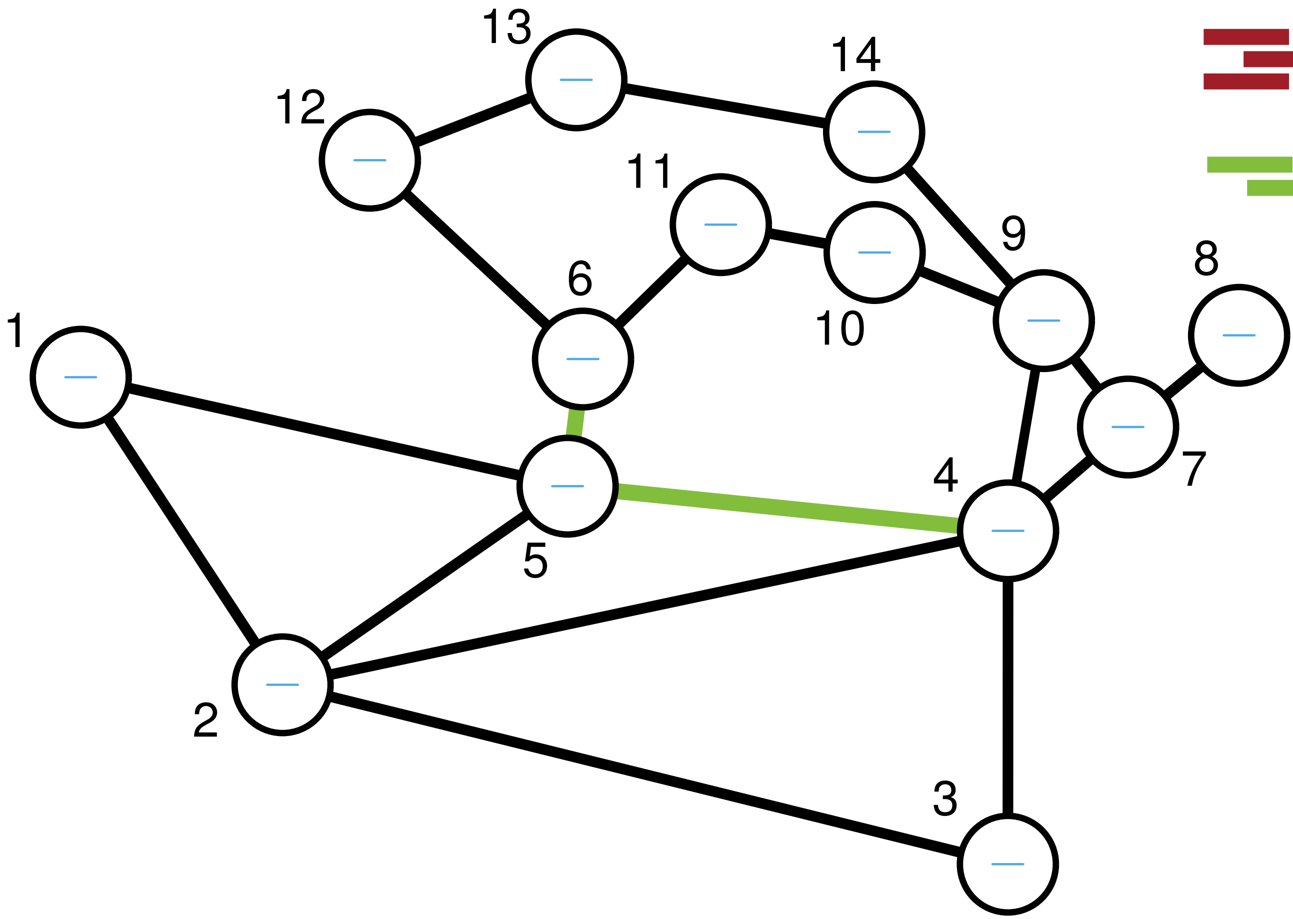
Feedback Cactus Set [Leibfried et al.& Mchedlidze et al., 2015]



Feedback **Cactus** Set [Leibfried et al.& Mchedlidze et al., 2015]



Feedback Forest Set

Feedback Cactus Set

⇓

A set of *Cacti* remains!

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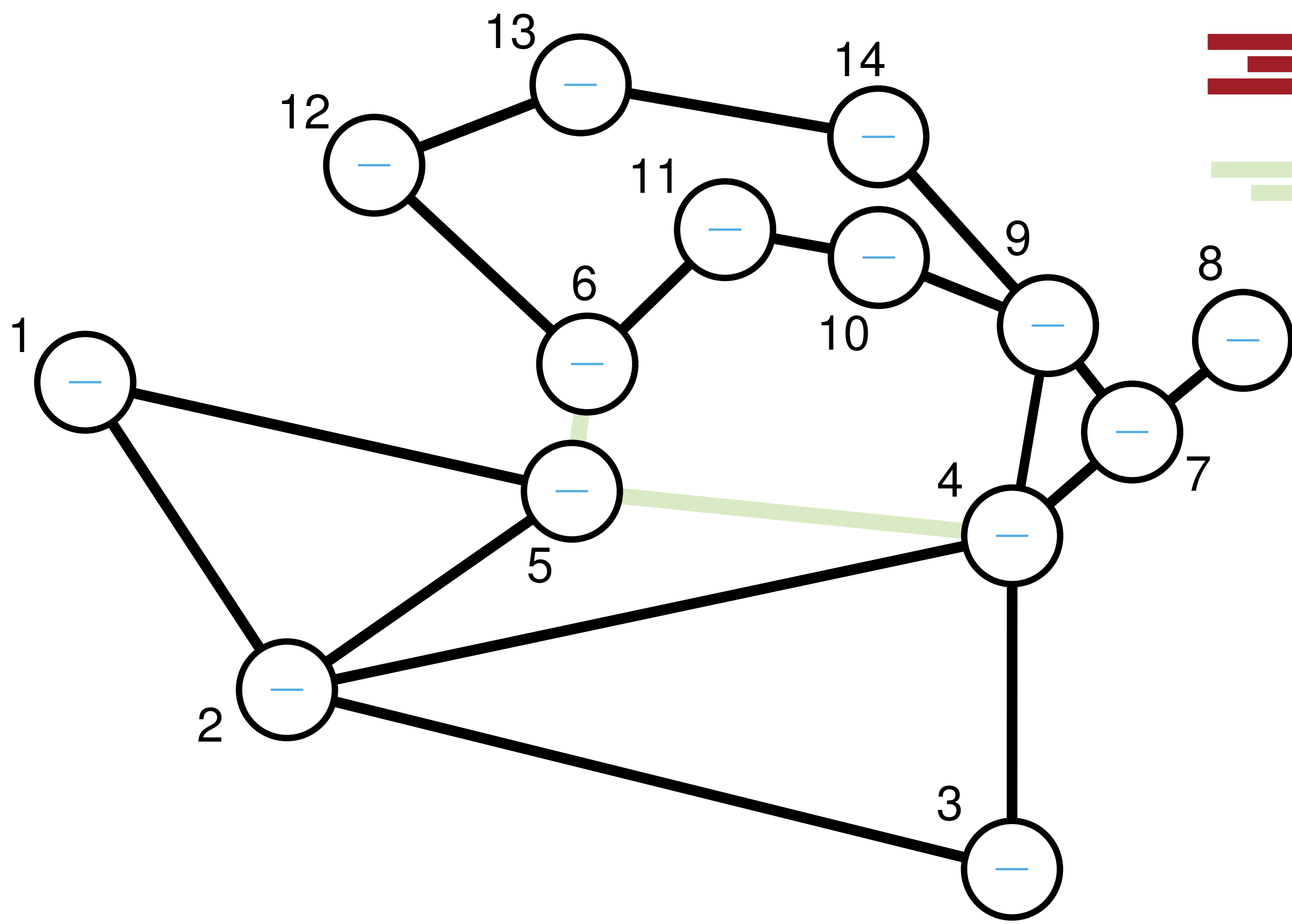
Feedback Cactus Set [Leibfried et al.& Mchedlidze et al., 2015]



Feedback Forest Set

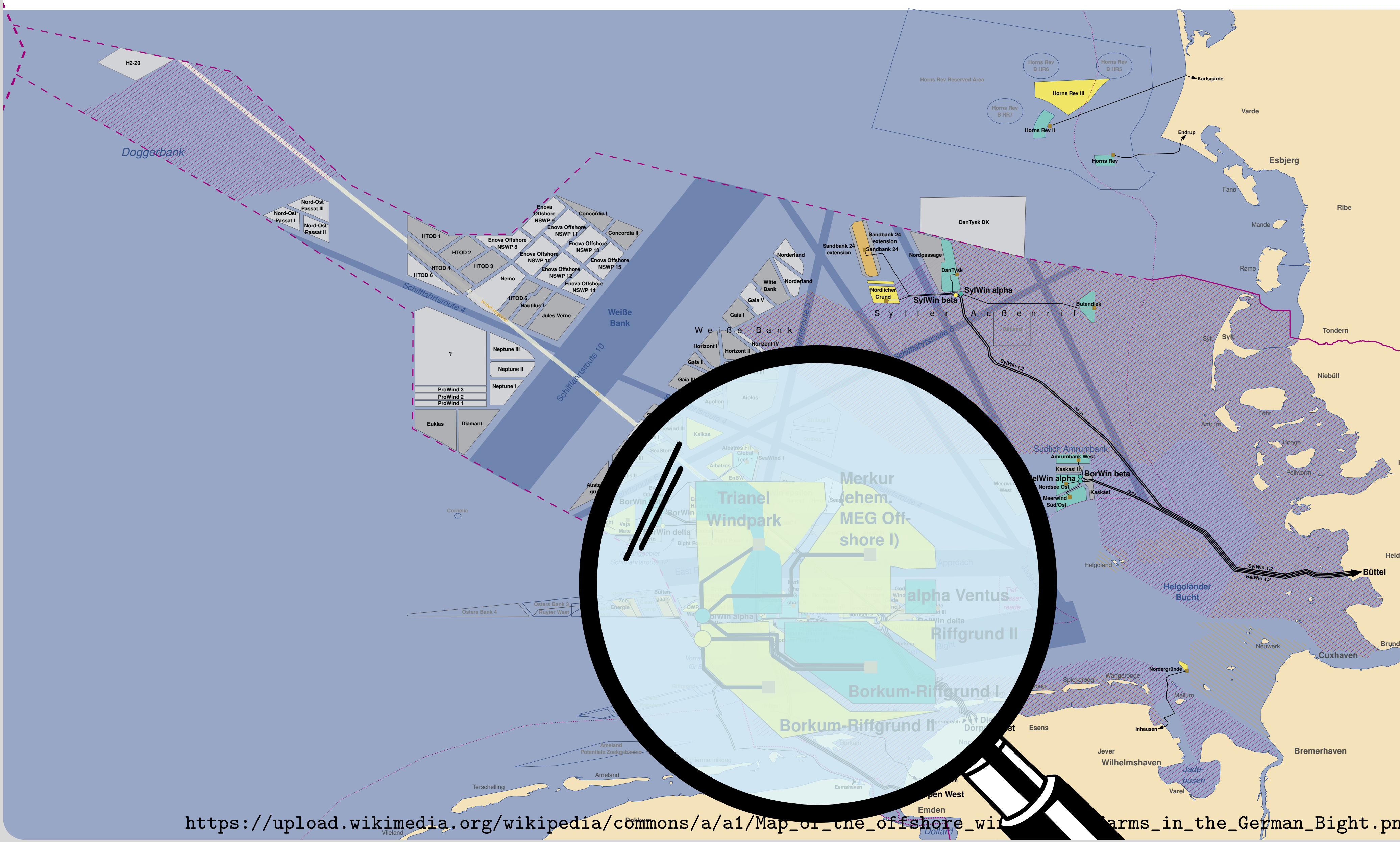
Feedback Cactus Set

⇓

A set of *Cacti* remains!

If the remaining graph is a cactus and the capacities on the cycles are suitably bounded then there is for every flow a cost-equivalent feasible power flow.

The Wind Farm Cable Layout Problem



https://upload.wikimedia.org/wikipedia/commons/a/a1/Map_of_the_offshore_wi[...]arms_in_the_German_Bight.png

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The Wind Farm Cable Layout Problem

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The Wind Farm Cable Layout Problem [Lehmann et al., 2017]

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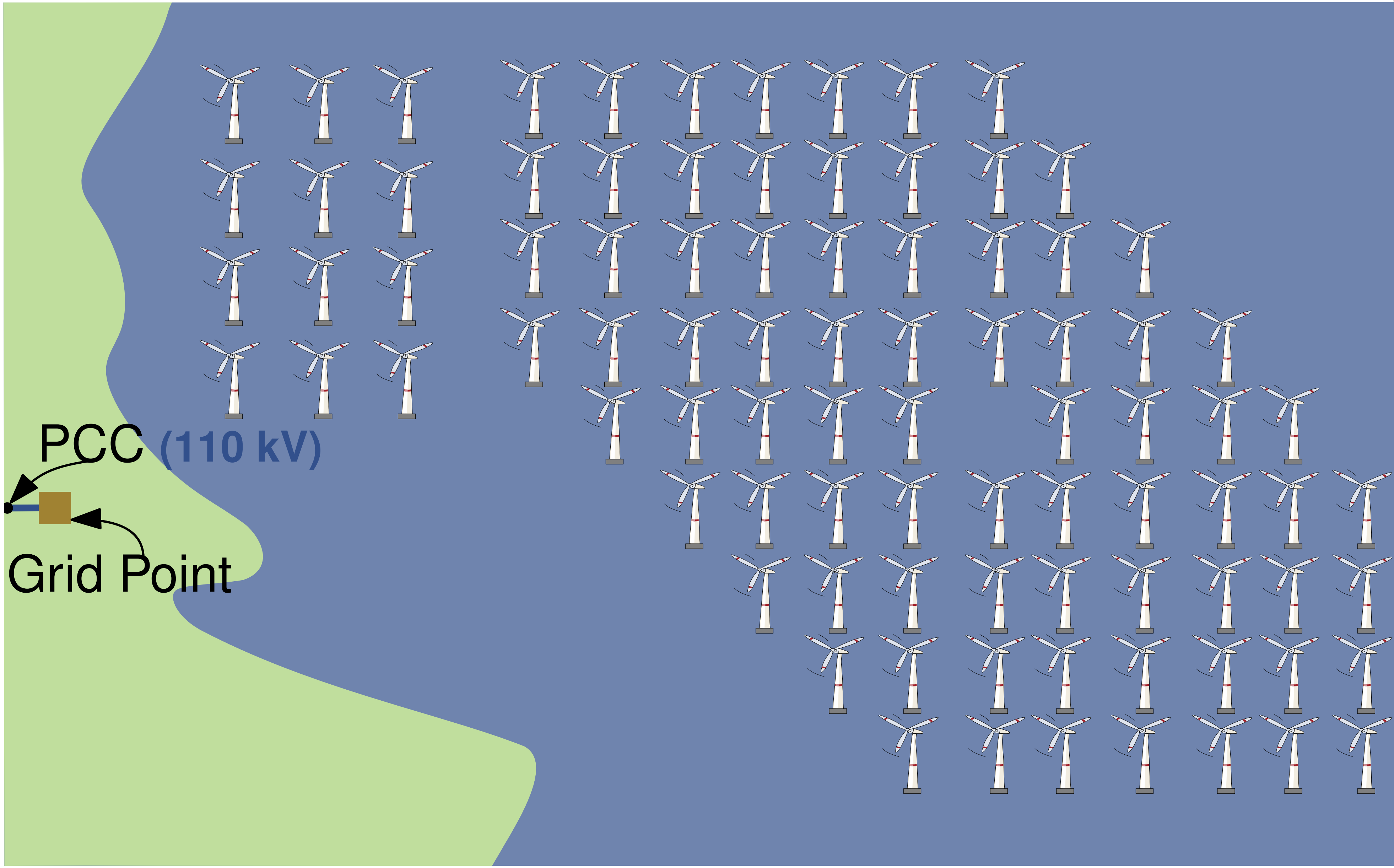

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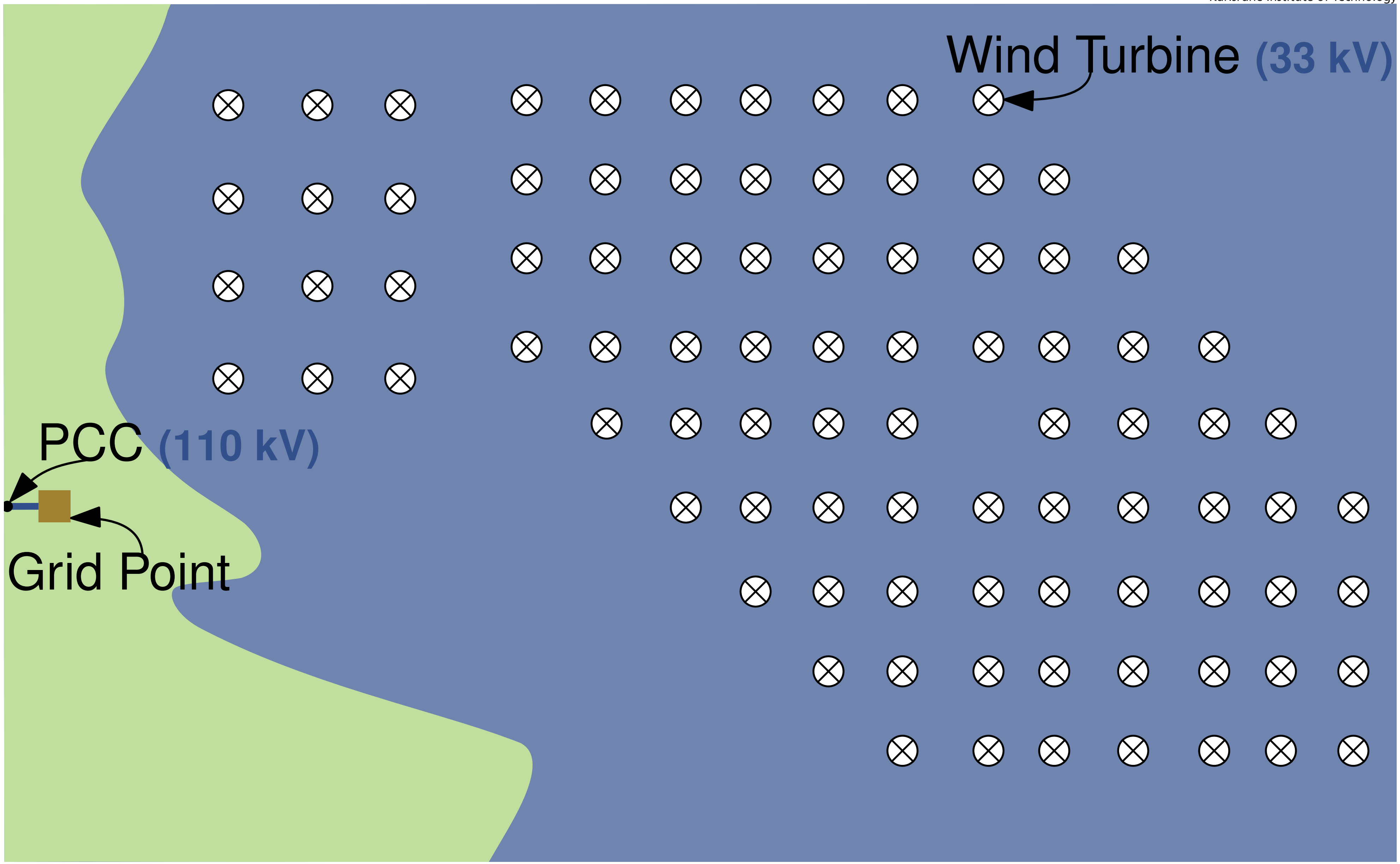
The Wind Farm Cable Layout Problem [Lehmann et al., 2017]



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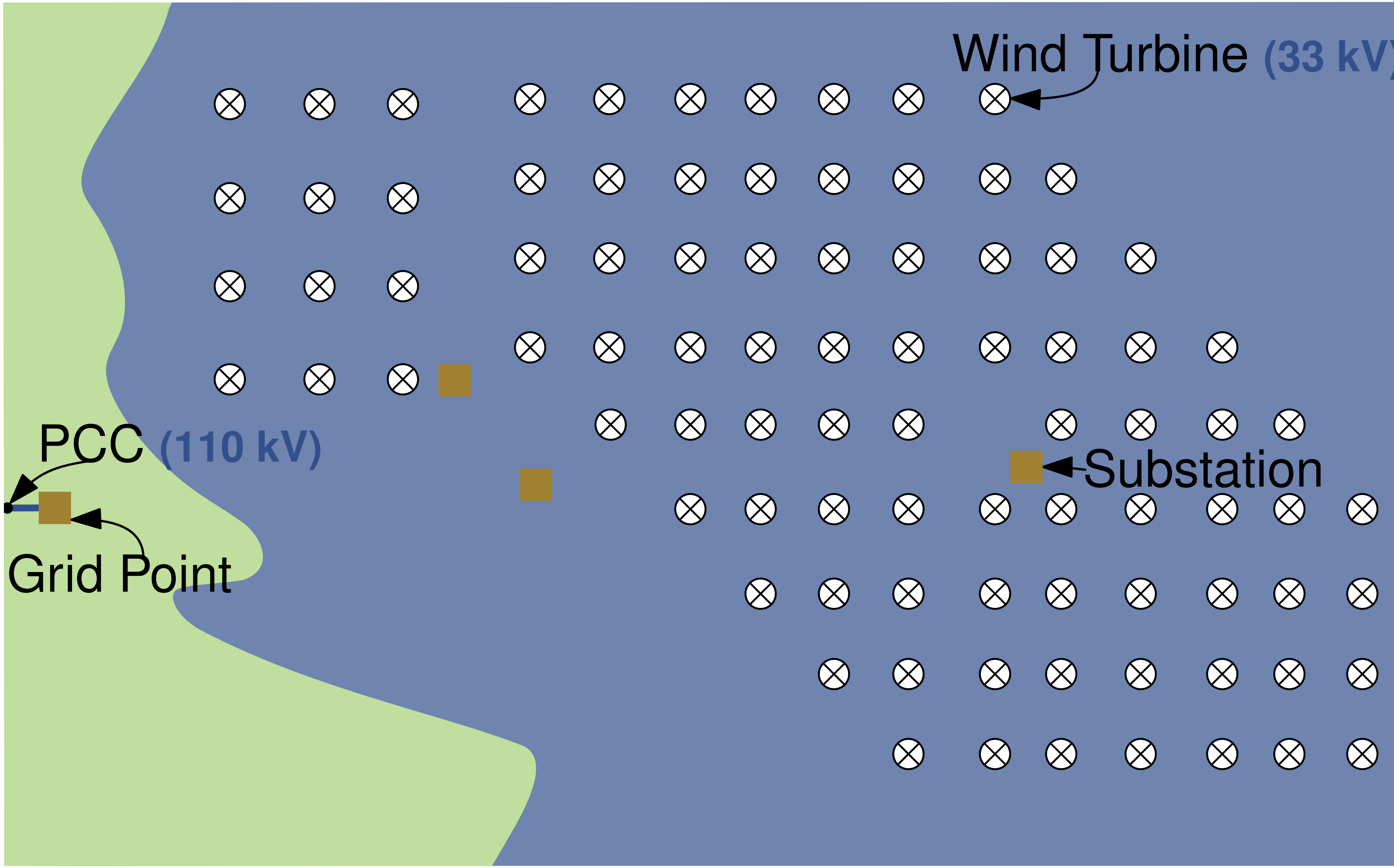
The Wind Farm Cable Layout Problem [Lehmann et al., 2017]



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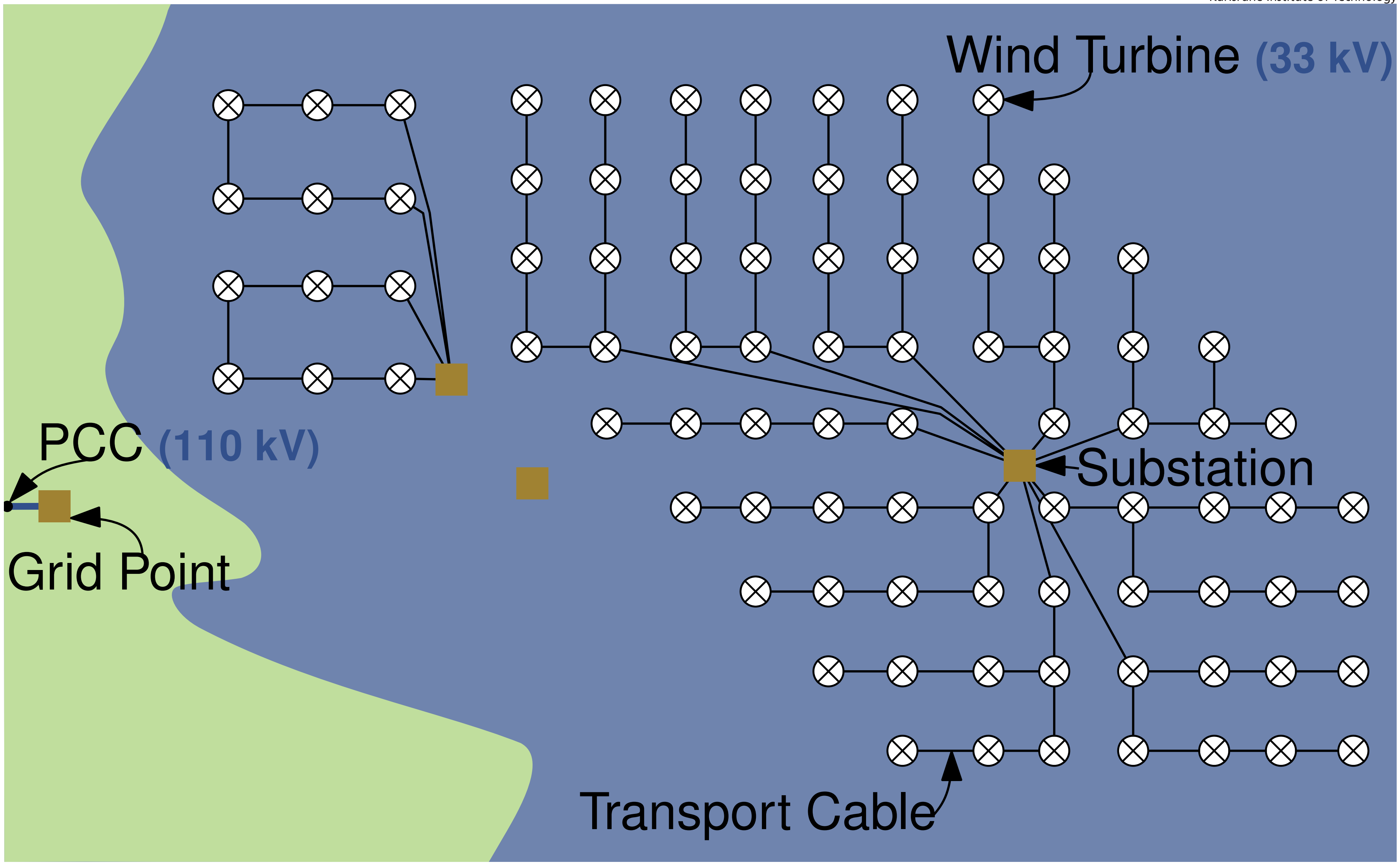

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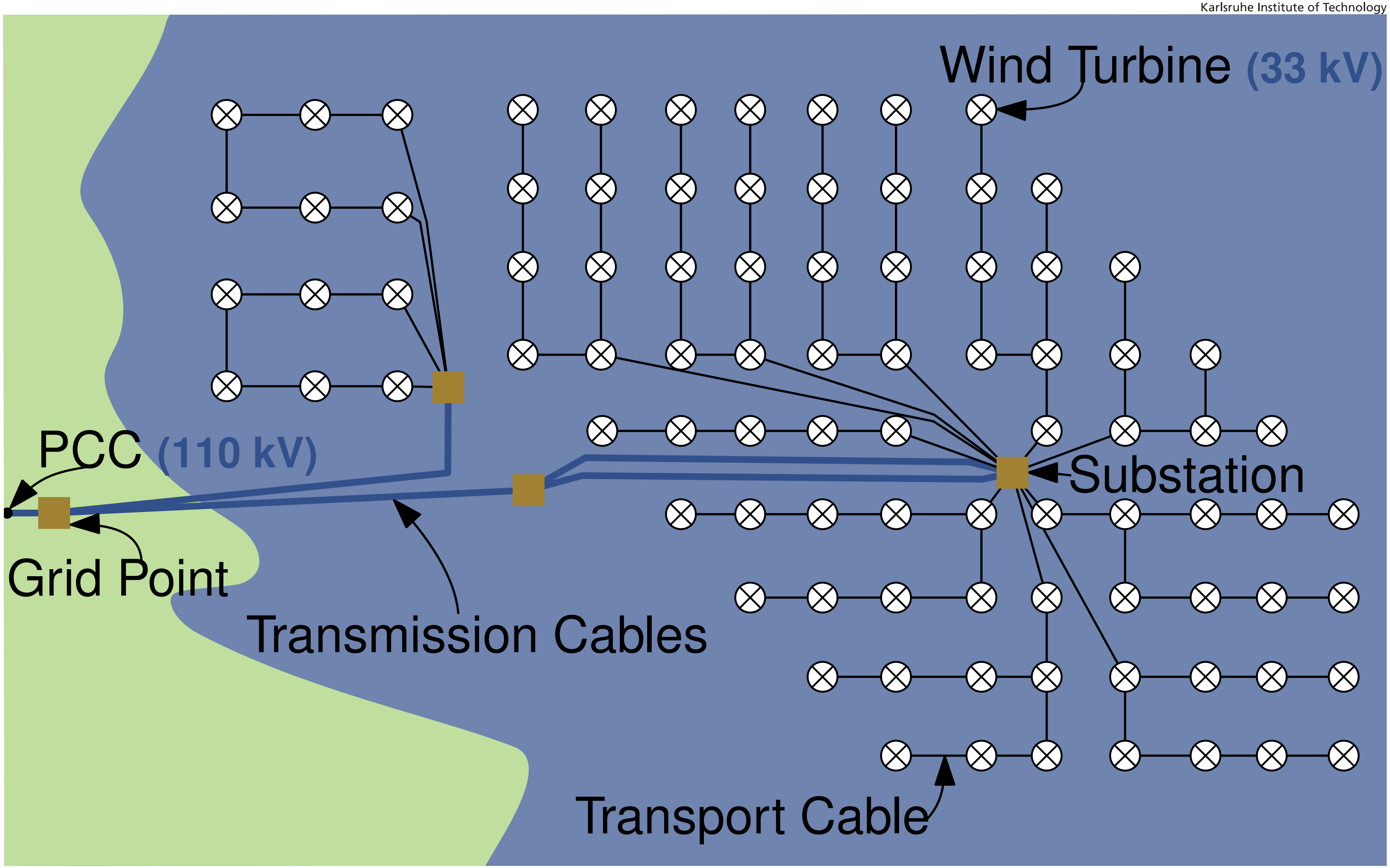

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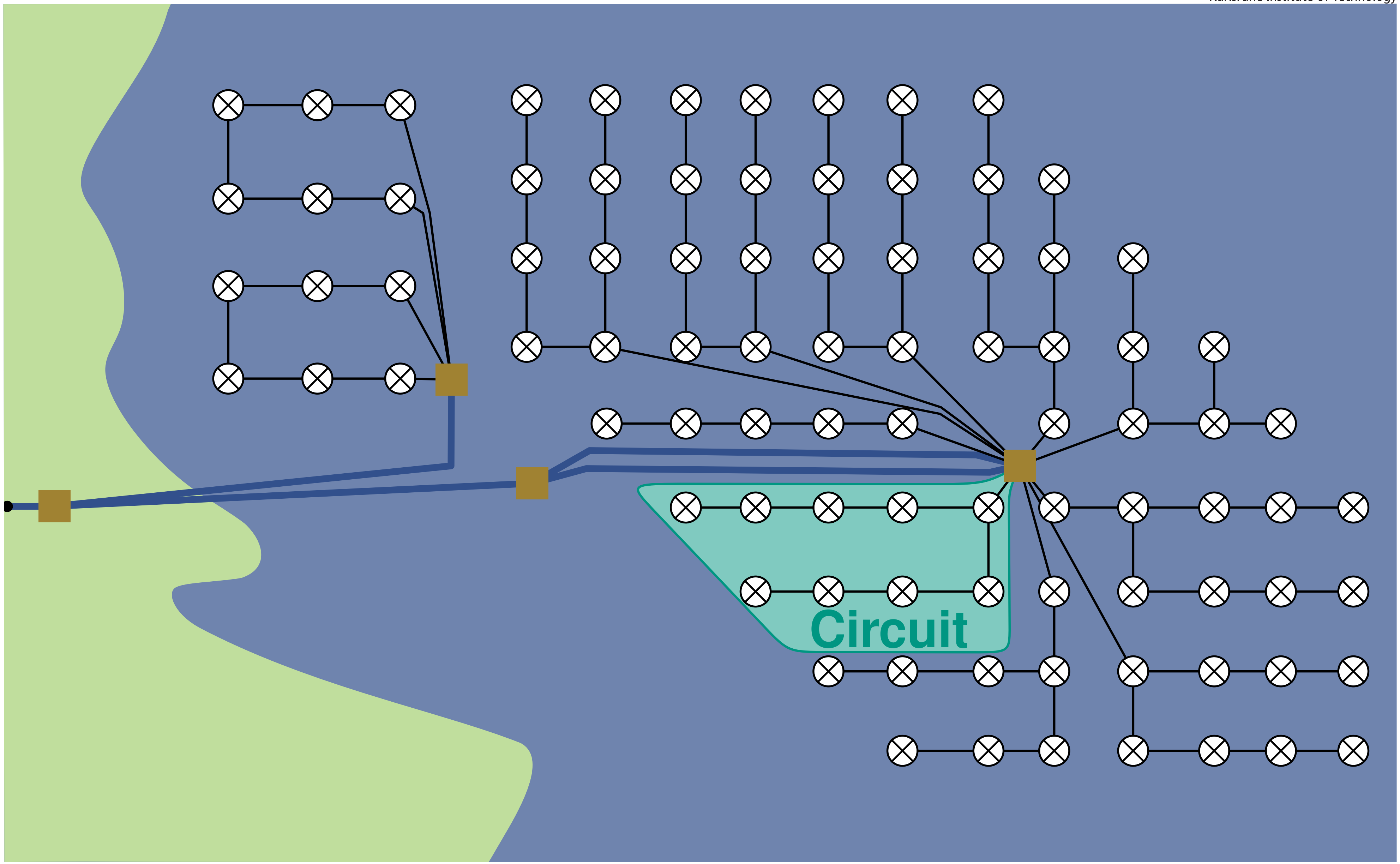

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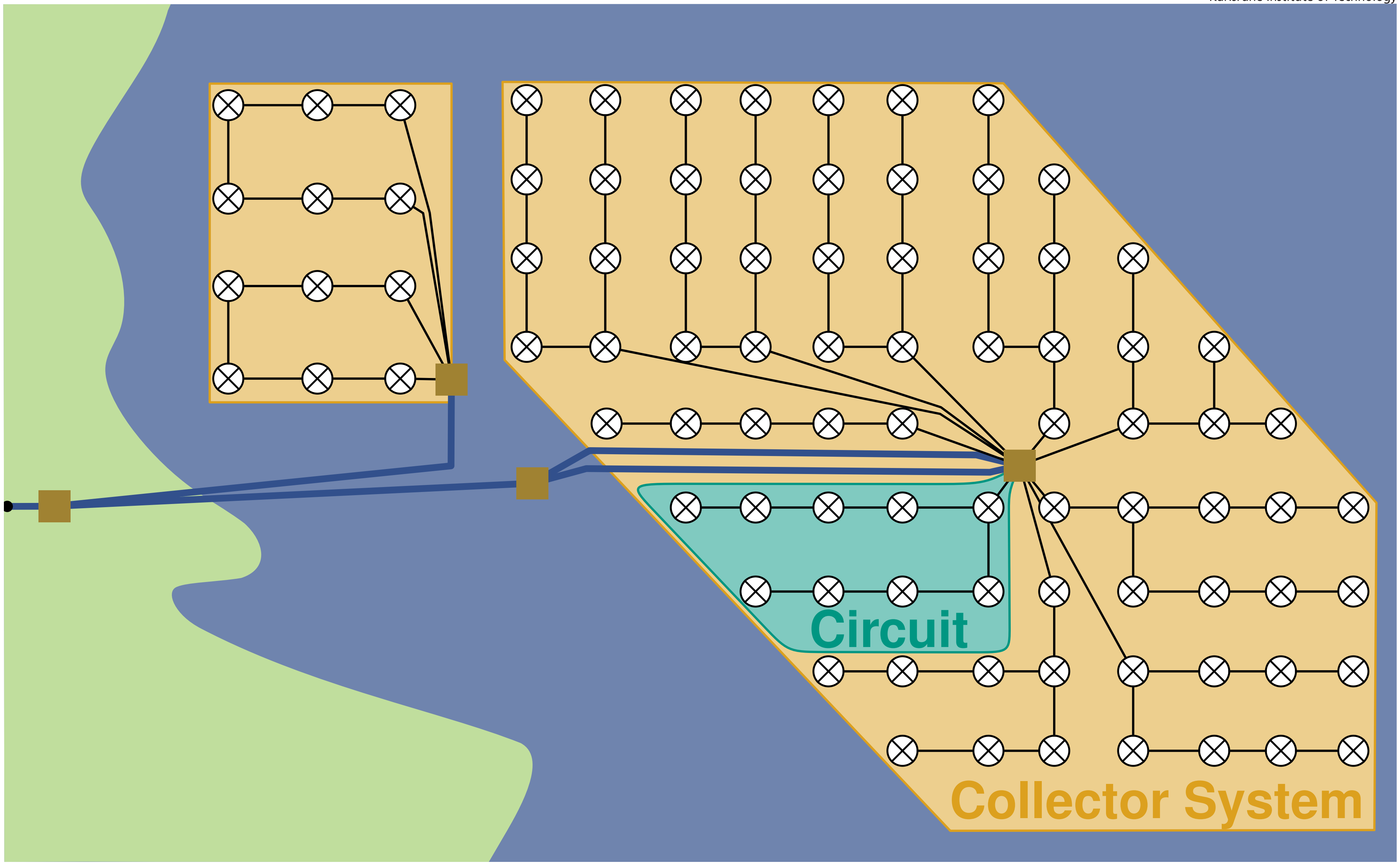

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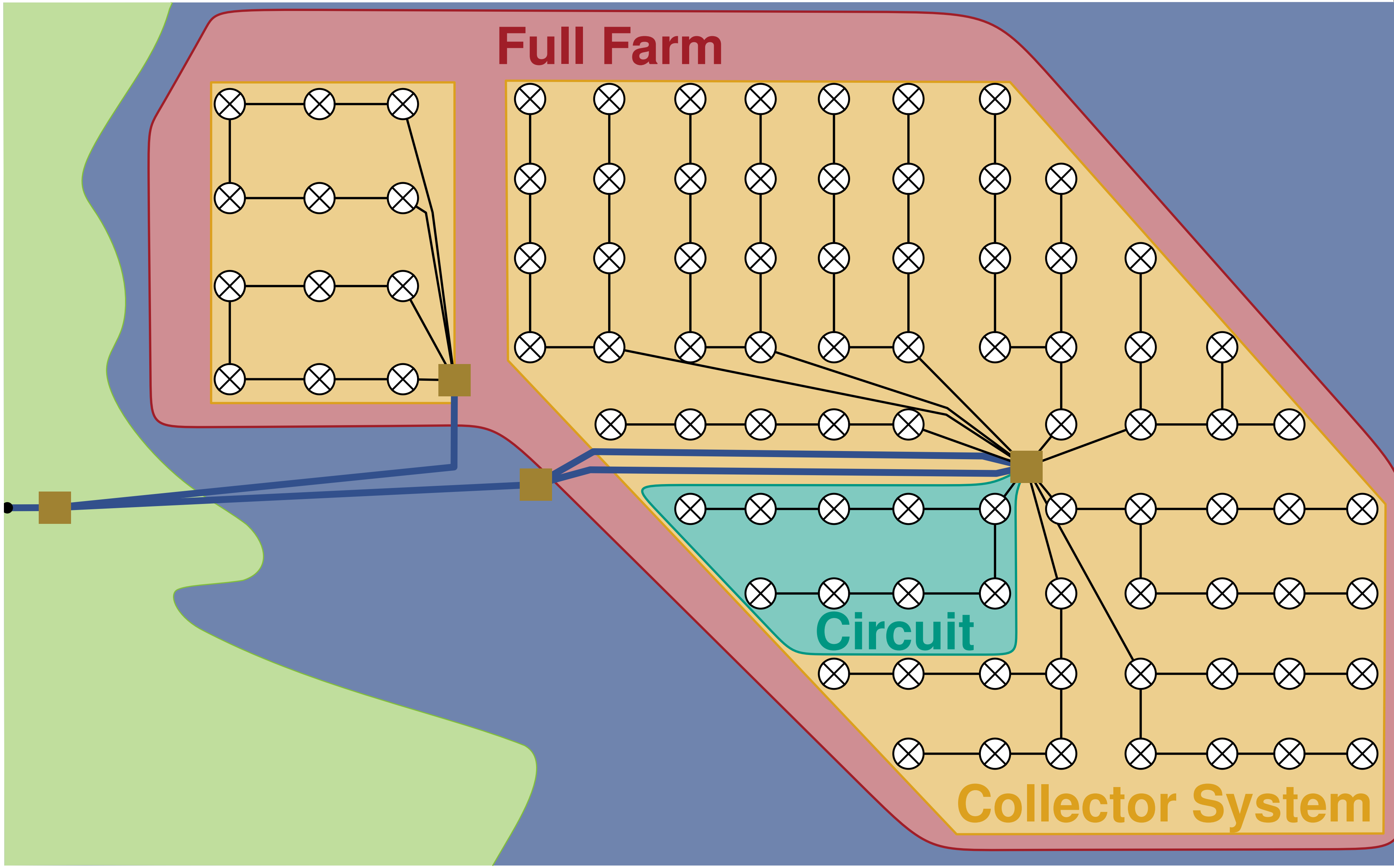

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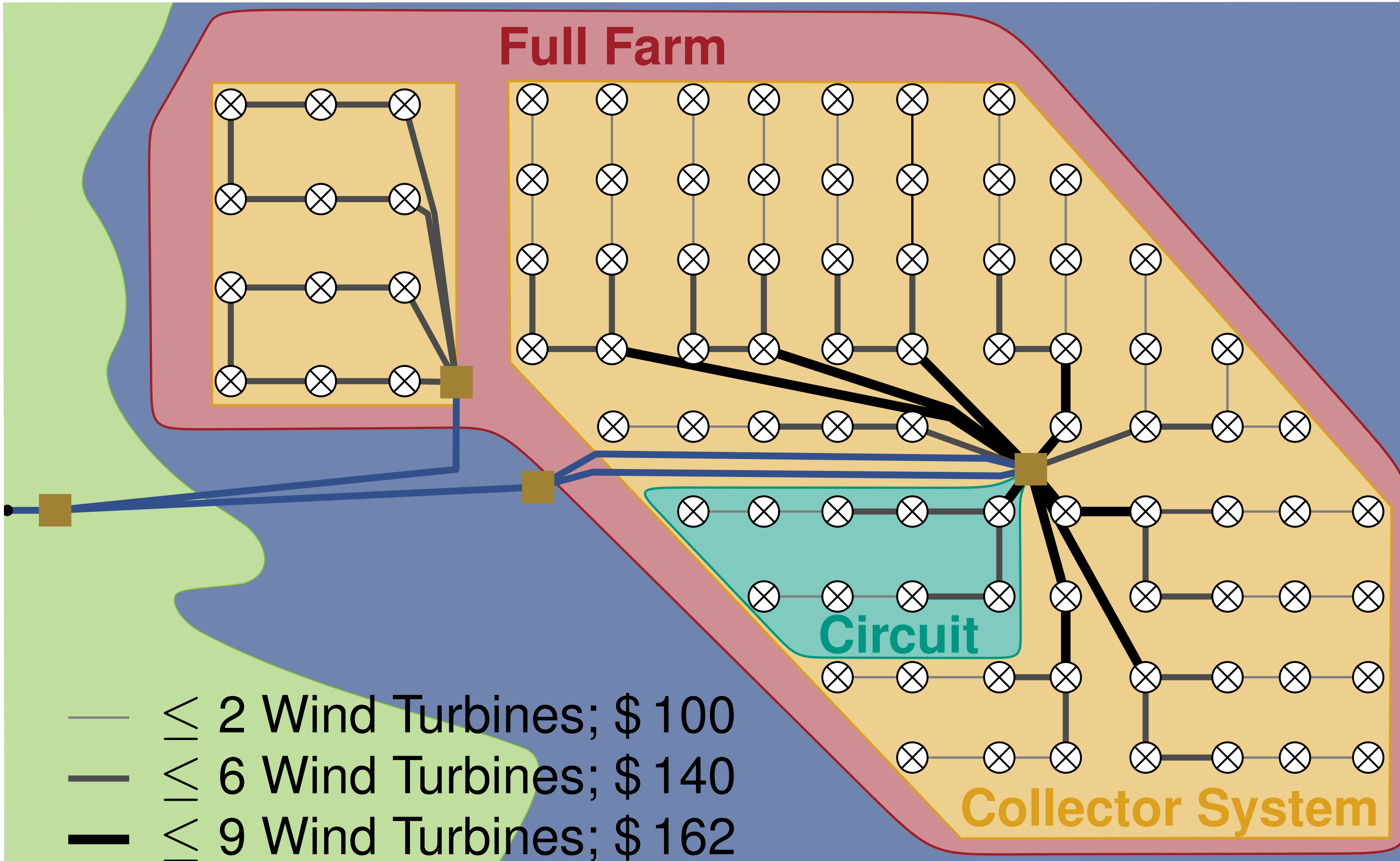
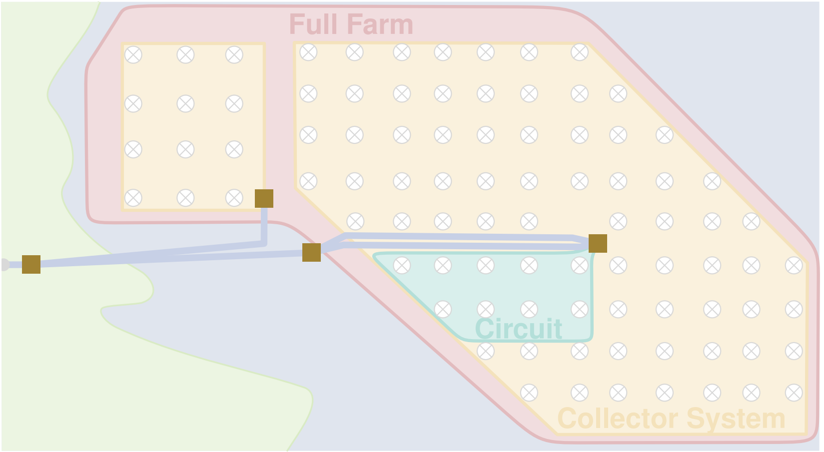


The Wind Farm Cable Layout Problem [Lehmann et al., 2017]

Given

- V_S set of substations (each with **capacity**),
  V_T set of turbines (each with **unit production**),
  E set of edges (possible connections),
  for each edge: cable types (each with **cost** and **capacity**)



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The Wind Farm Cable Layout Problem [Lehmann et al., 2017]

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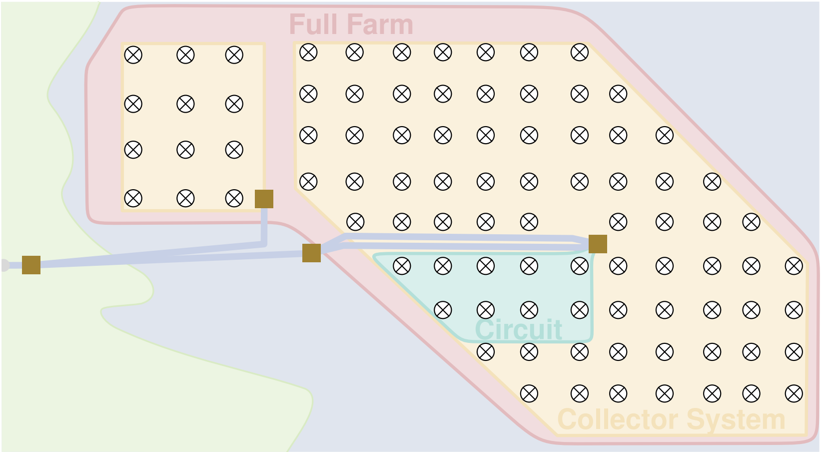

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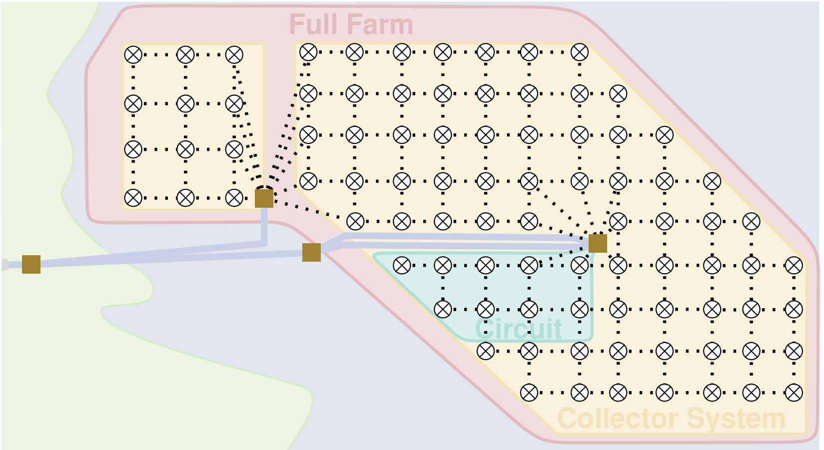

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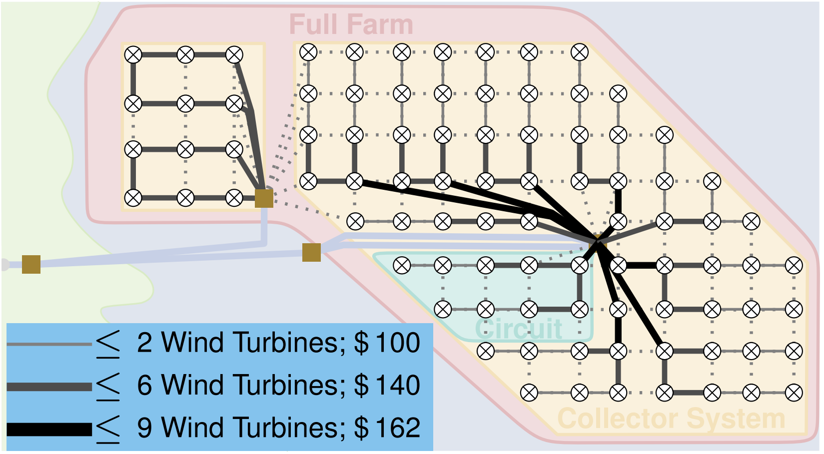

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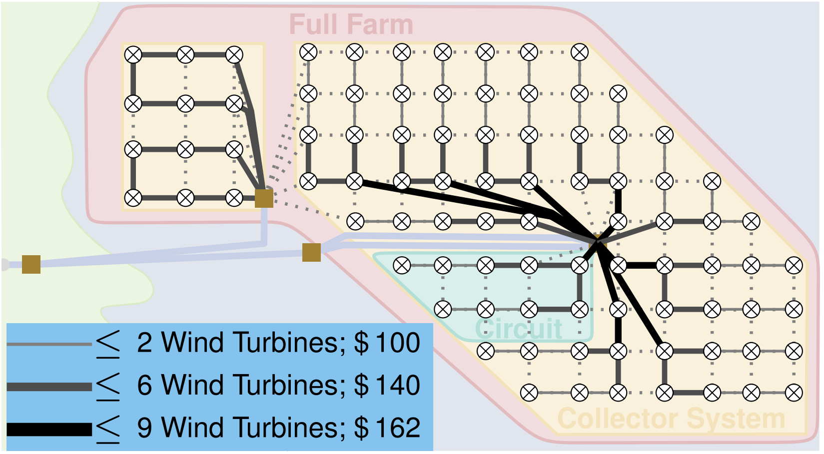

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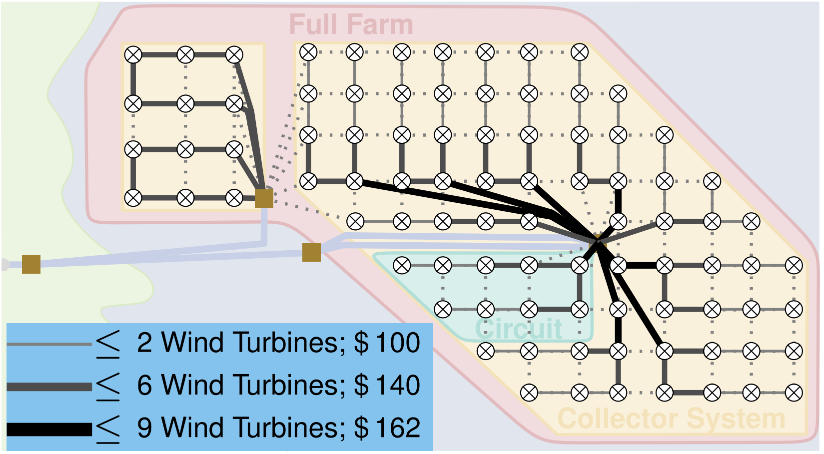
minimizing **total cable cost**



The Wind Farm Cable Layout Problem [Lehmann et al., 2017]

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Given
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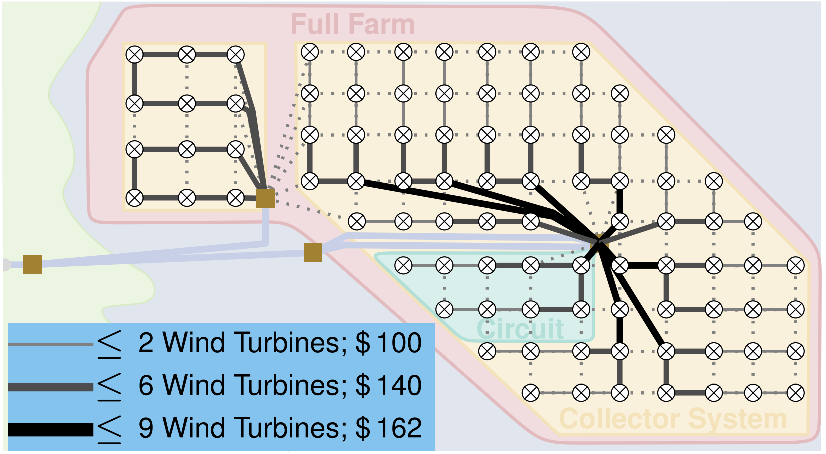
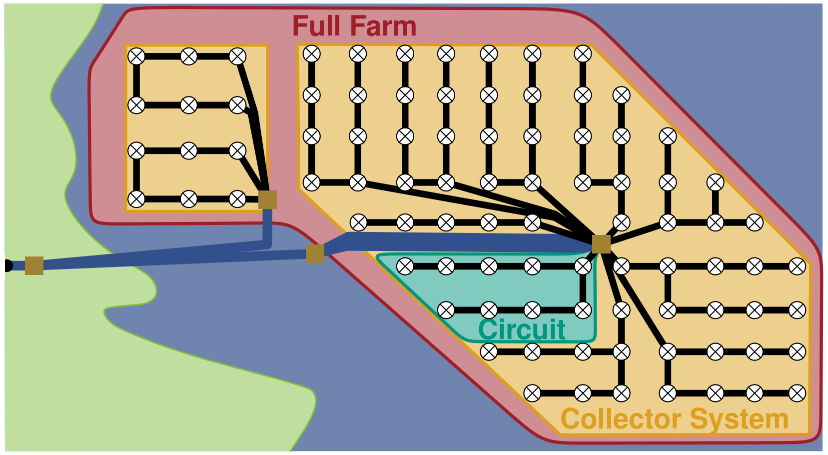
find for each edge: the **cable type**

minimizing **total cable cost**

subject to
cable capacity constraints
substation capacity constraints
flow conservation constraints

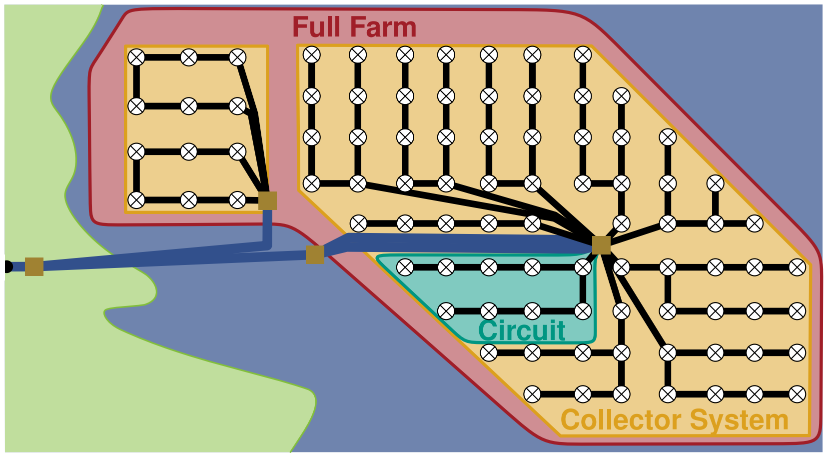


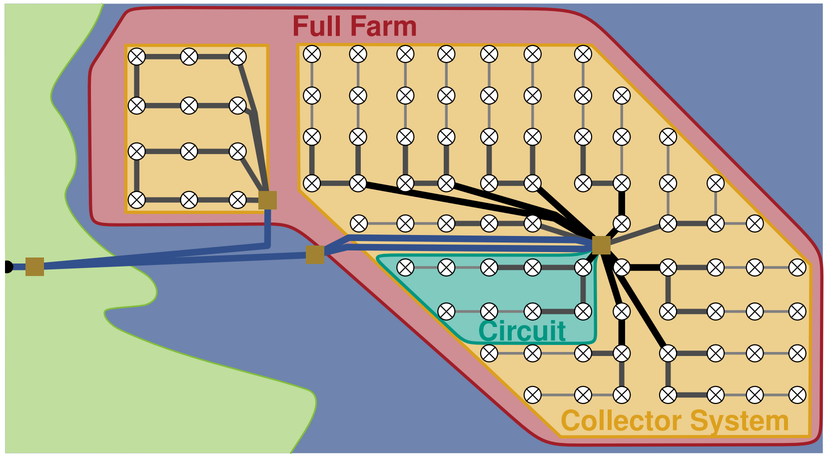
Problem Classification [Lehmann et al., 2017]



P (MST)	Circuit Problem
NP-hard (CMST)	Substation Problem
NP-hard (Heuristics)	Full Farm Problem

Problem Classification [Lehmann et al., 2017]





P (MST)	Circuit Problem	NP-hard
NP-hard (CMST)	Substation Problem	NP-hard
NP-hard (Heuristics)	Full Farm Problem	NP-hard

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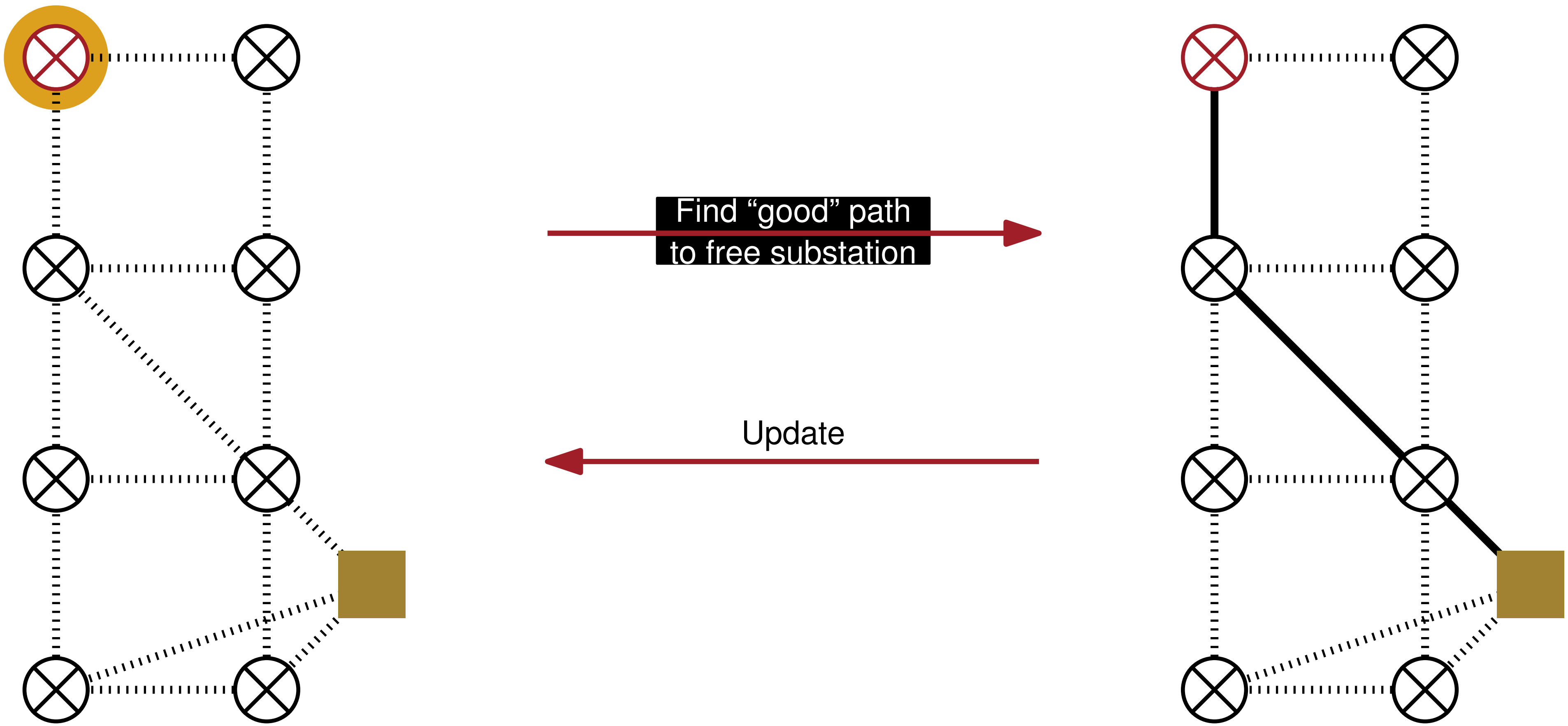
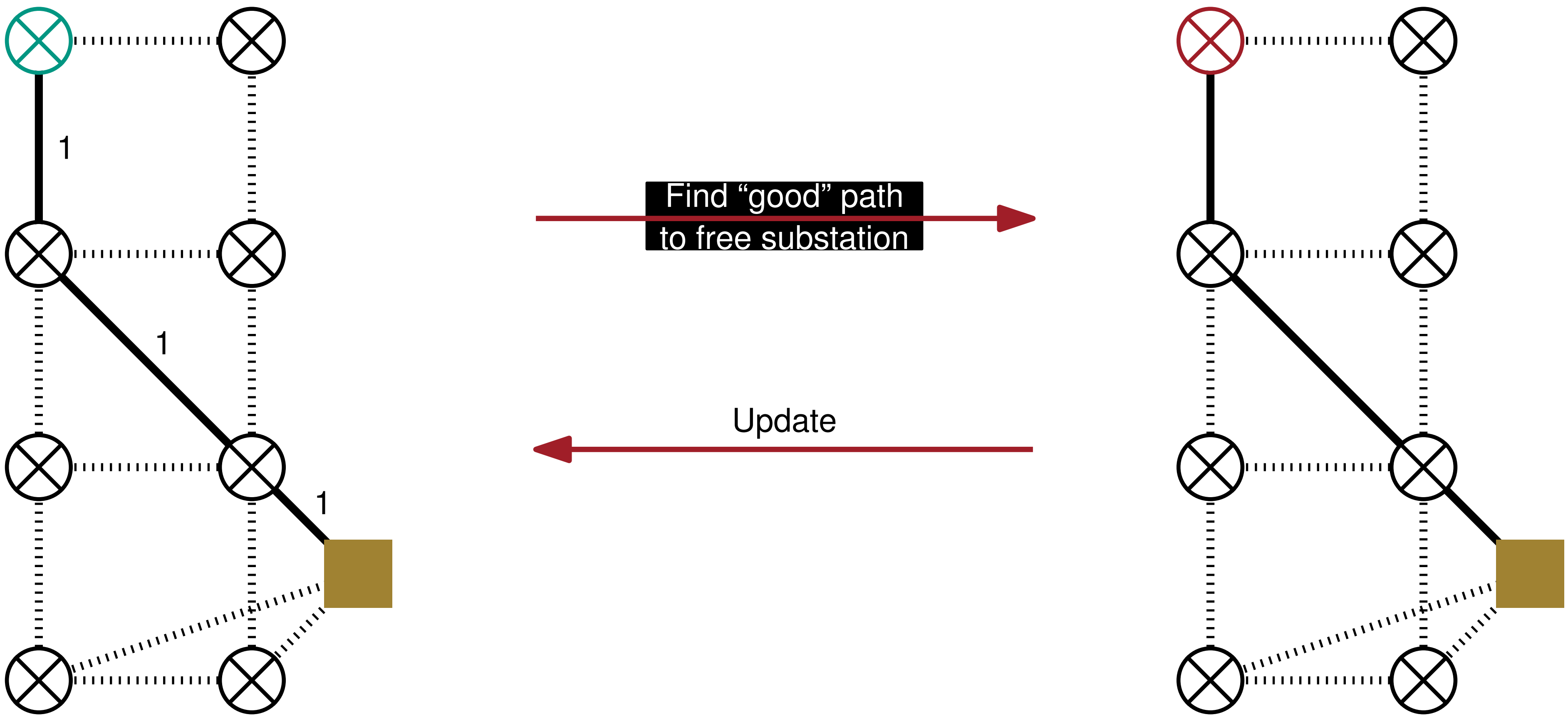
Network Flows and Wind Farm Cabling

[Gritzbach et al., 2018]



Network Flows and Wind Farm Cabling

[Gritzbach et al., 2018]



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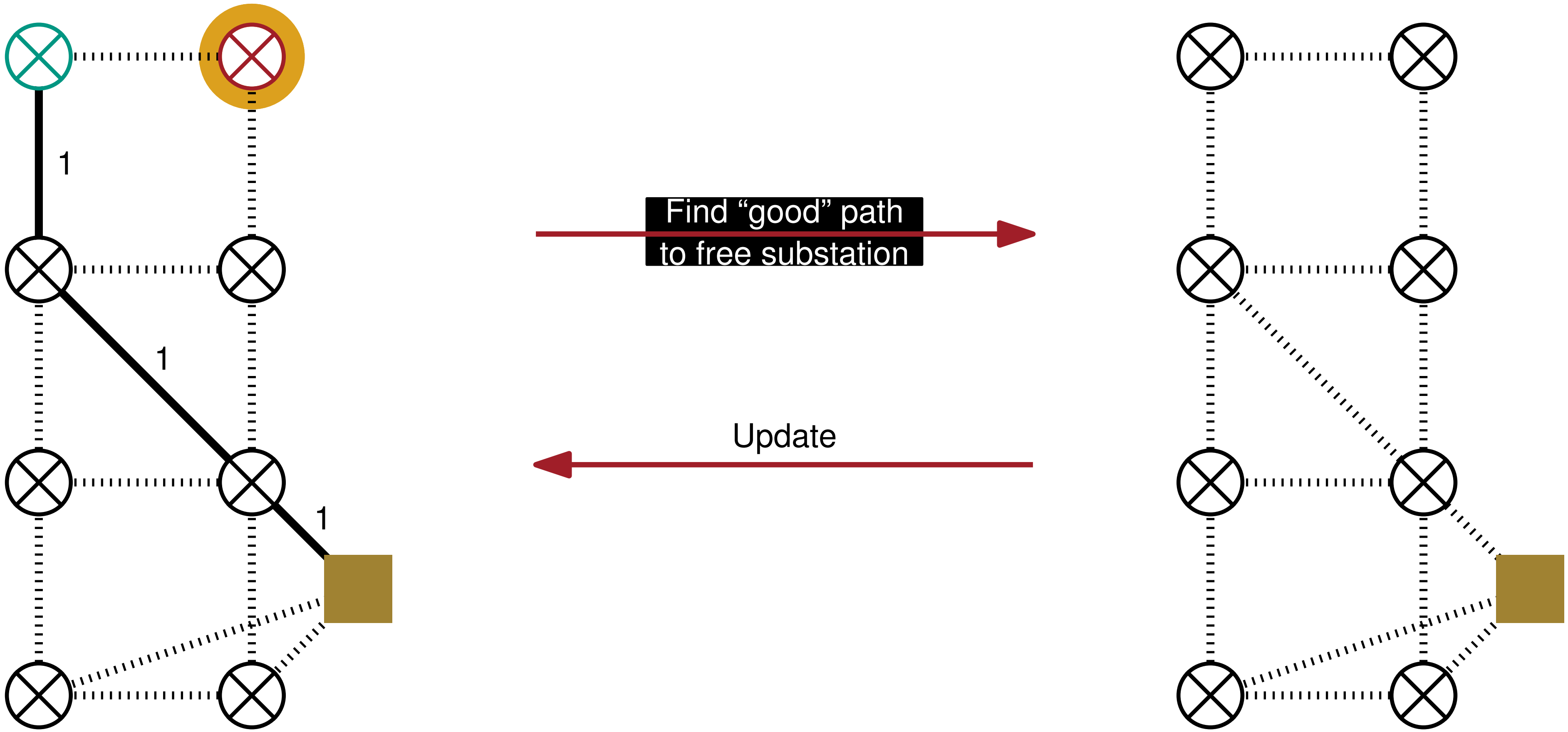

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Network Flows and Wind Farm Cabling

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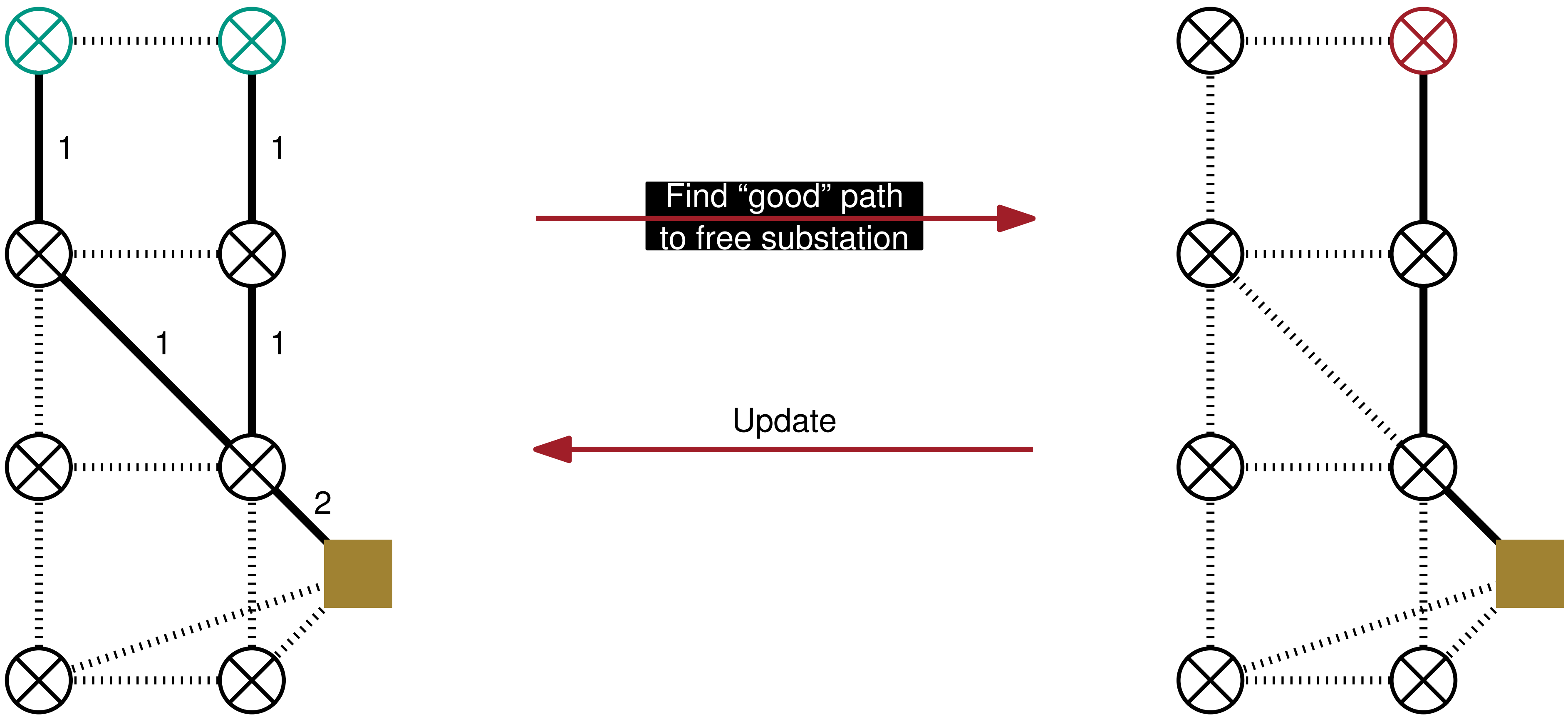

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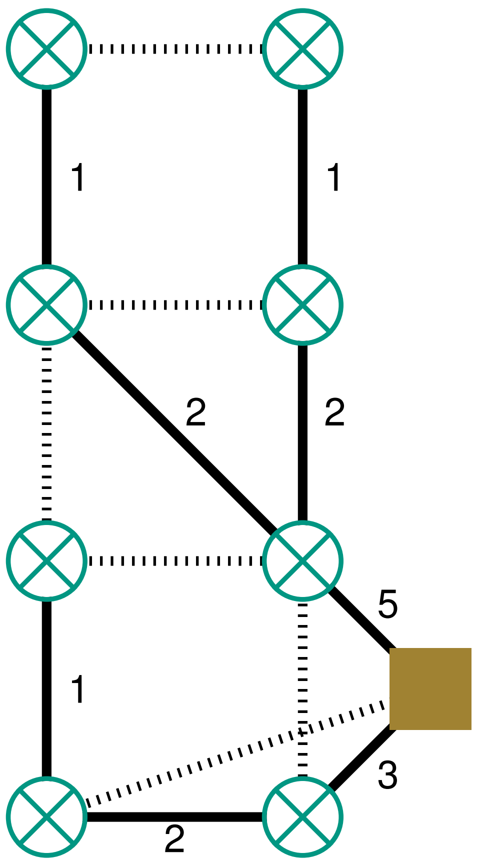

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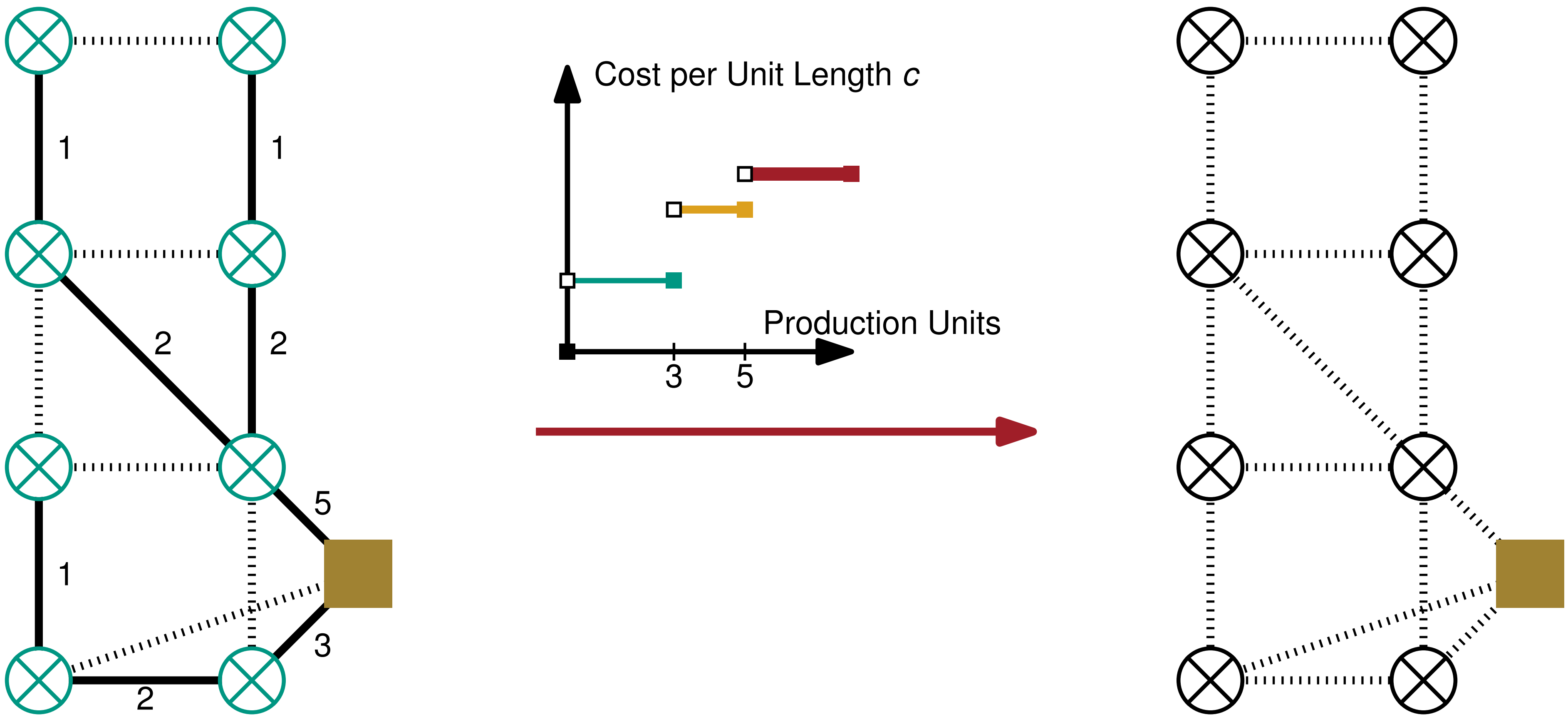
Network Flows and Wind Farm Cabling

[Gritzbach et al., 2018]



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Network Flows and Wind Farm Cabling

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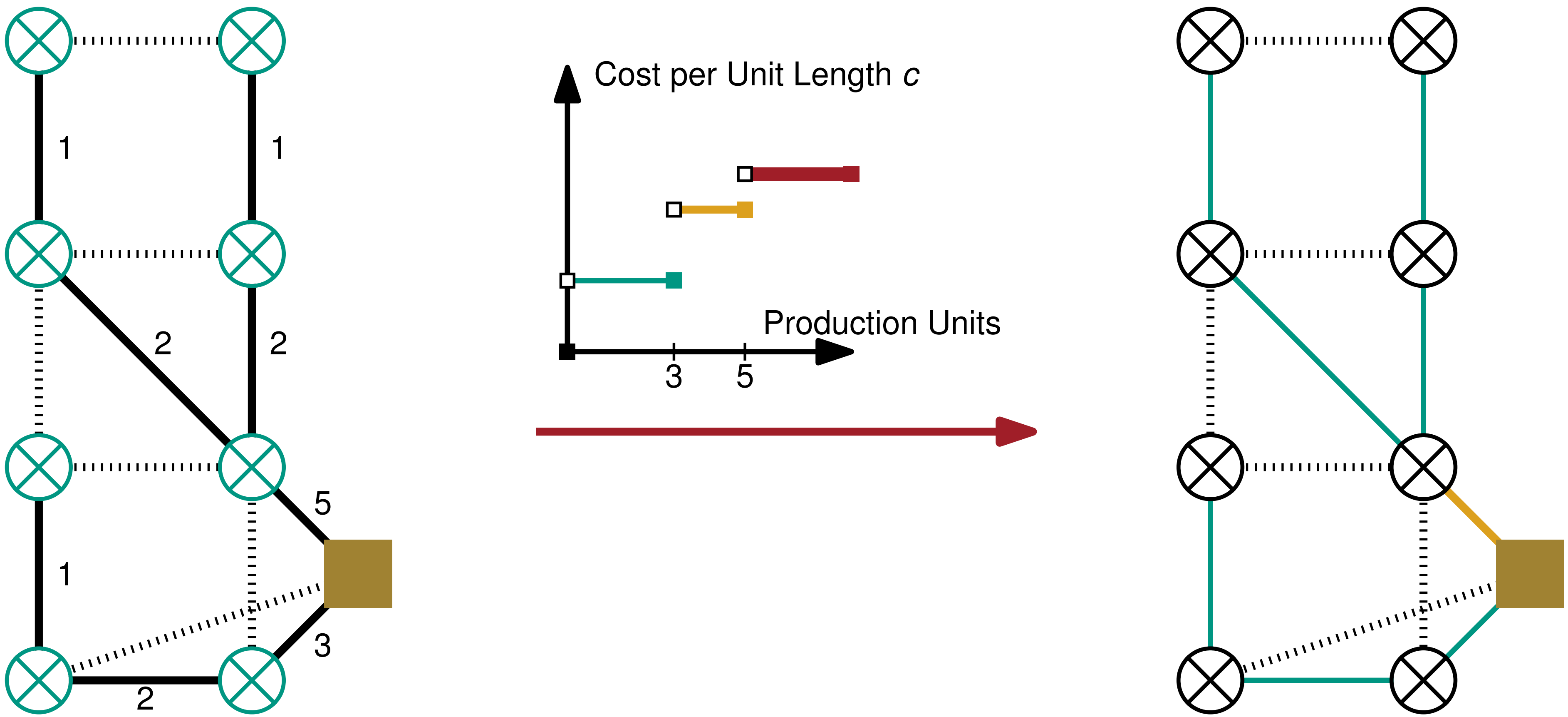
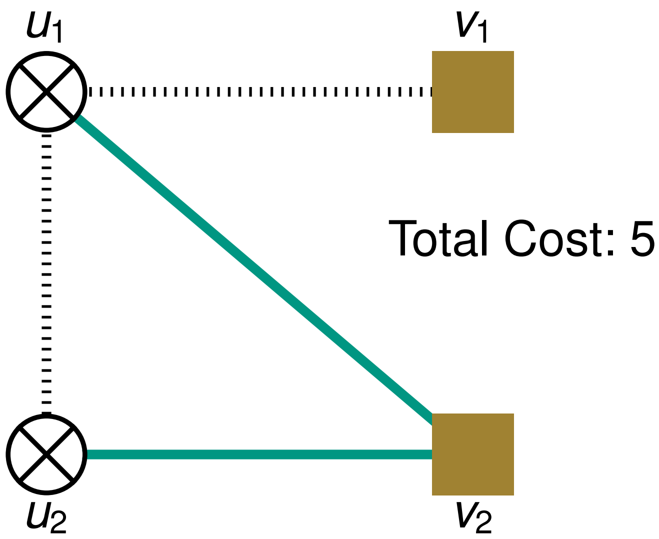
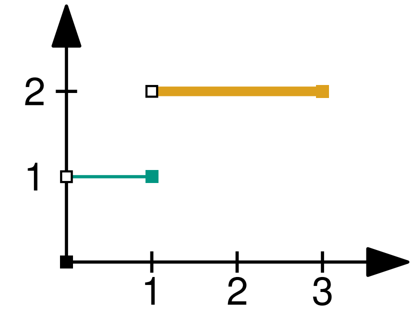


Negative Cycle Canceling [Gritzbach et al., 2018]



- Substation capacity: 2
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:



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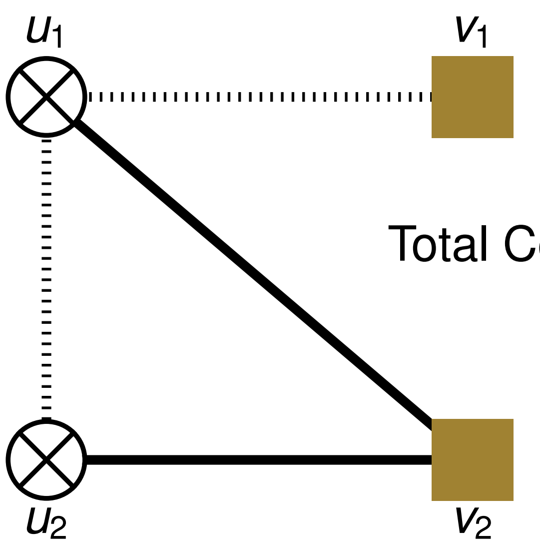
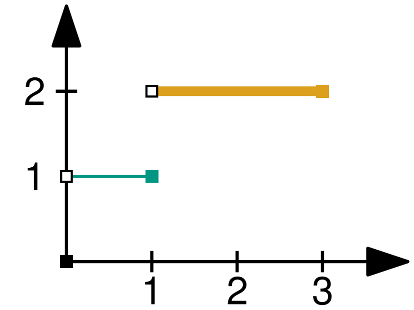
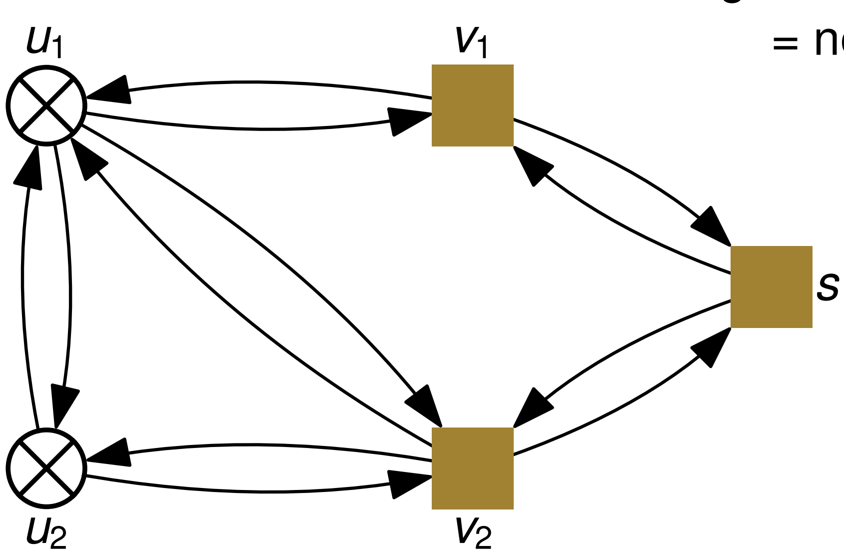


Total Cost: 5

- Substation capacity: 2
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:



Find edge weights for change of flow $\Delta = 1$
= new cost - old cost



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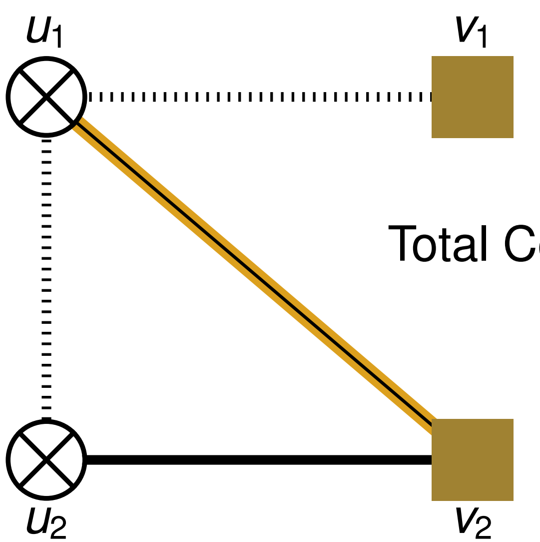

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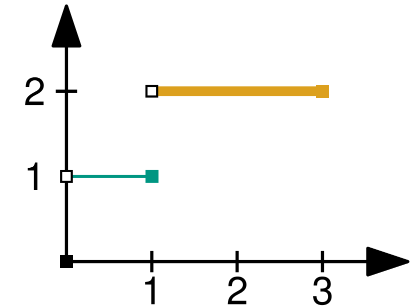
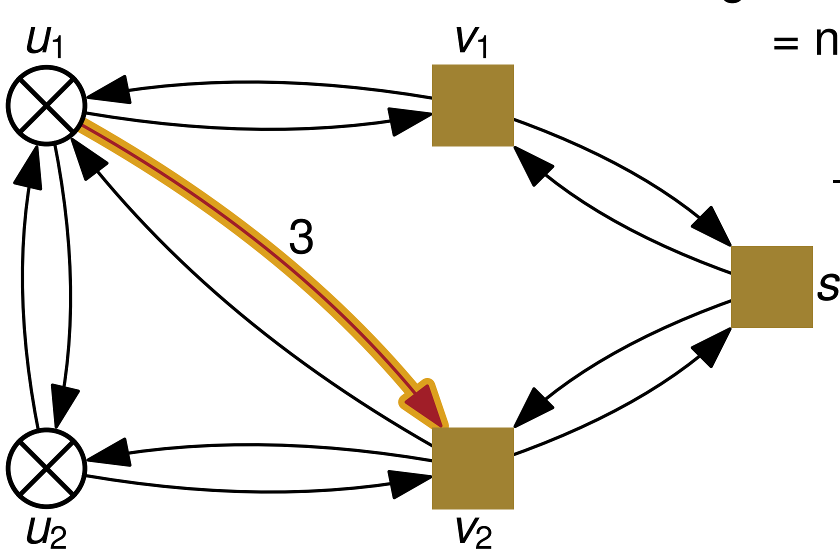
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:



Find edge weights for change of flow $\Delta = 1$

= new cost - old cost

old flow	old cost
1	3



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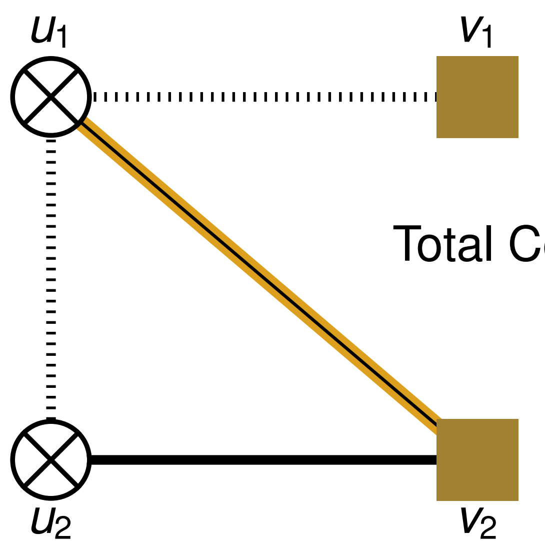
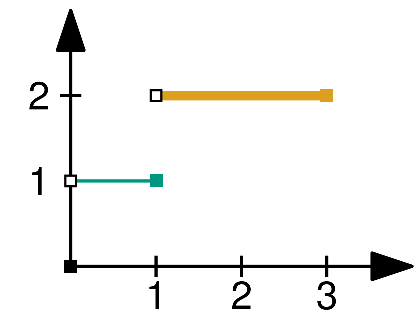
Negative Cycle Canceling [Gritzbach et al., 2018]

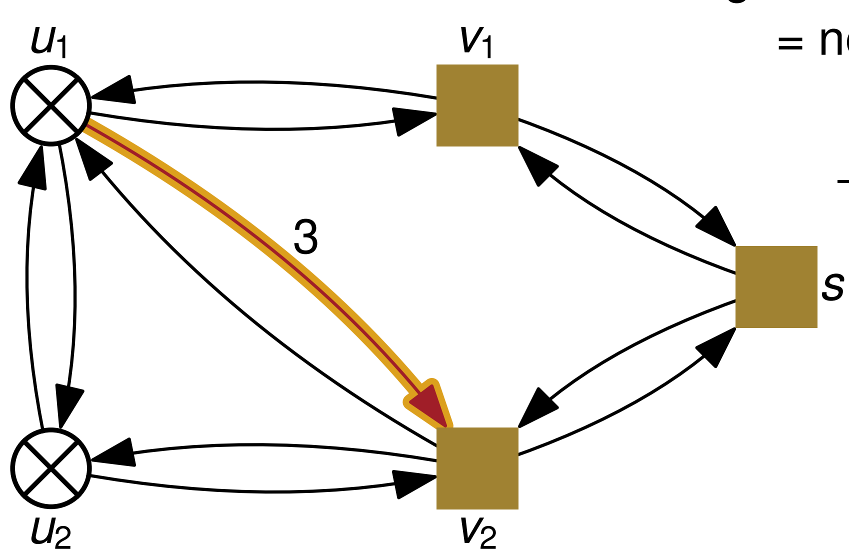


Total Cost: 5

- Substation capacity: 2
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:



Find edge weights for change of flow $\Delta = 1$
= new cost - old cost



old flow	old cost	new flow	new cost
1	3	2	6

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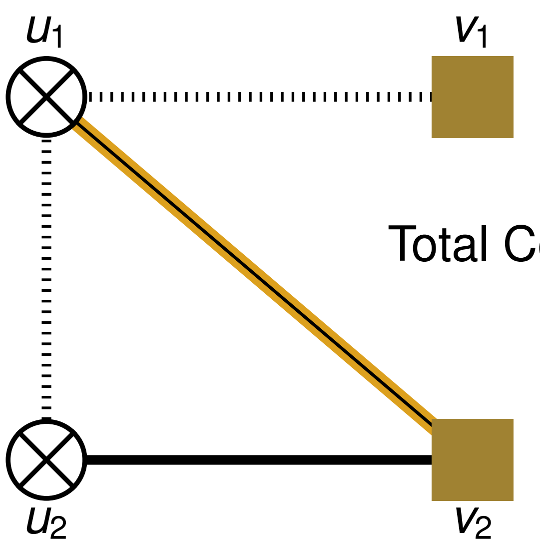
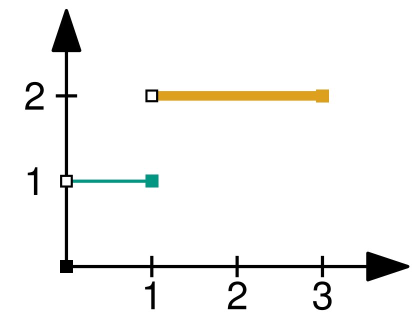
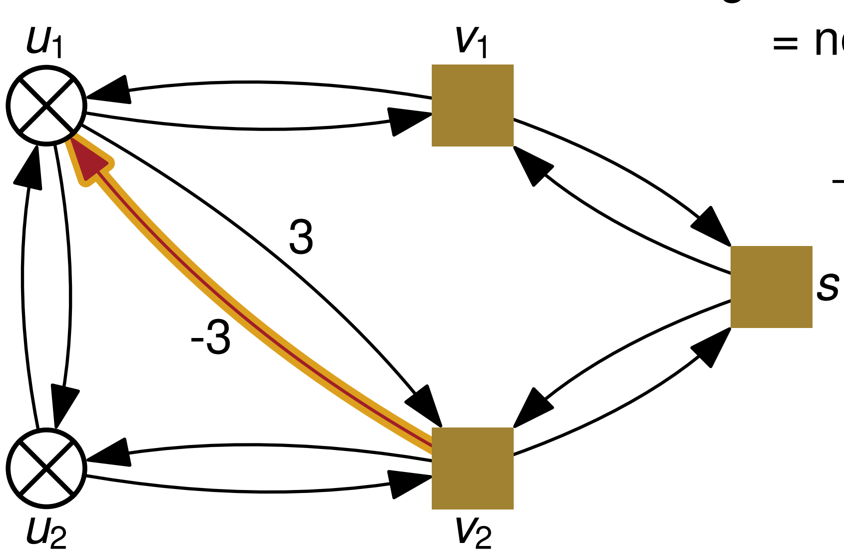
Negative Cycle Canceling [Gritzbach et al., 2018]



Total Cost: 5

- Substation capacity: 2
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:

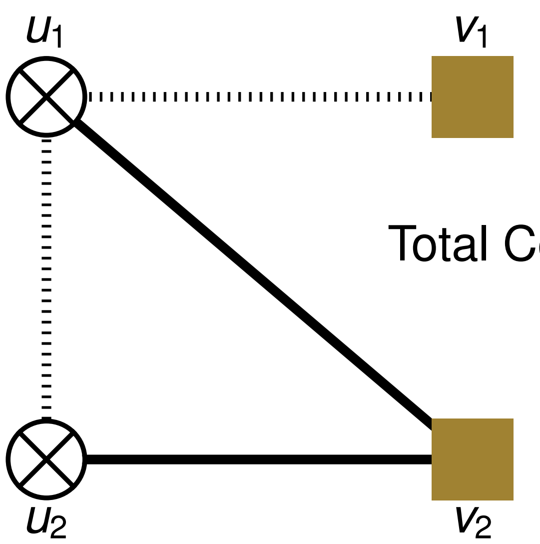




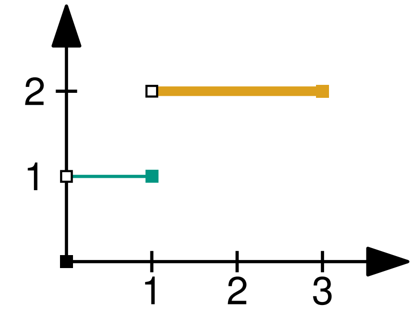
Find edge weights for change of flow $\Delta = 1$
= new cost - old cost

old flow	old cost	new flow	new cost
1	3	2	6
1	3	0	0

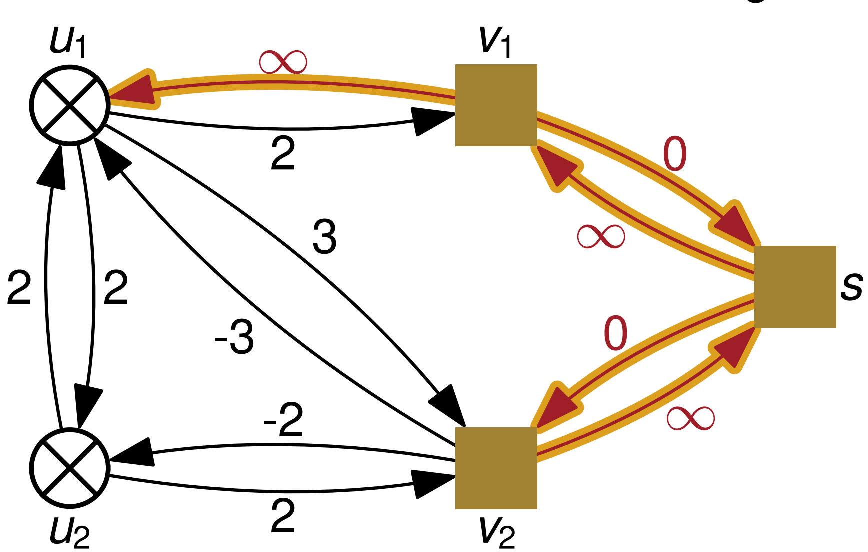
Negative Cycle Canceling [Gritzbach et al., 2018]



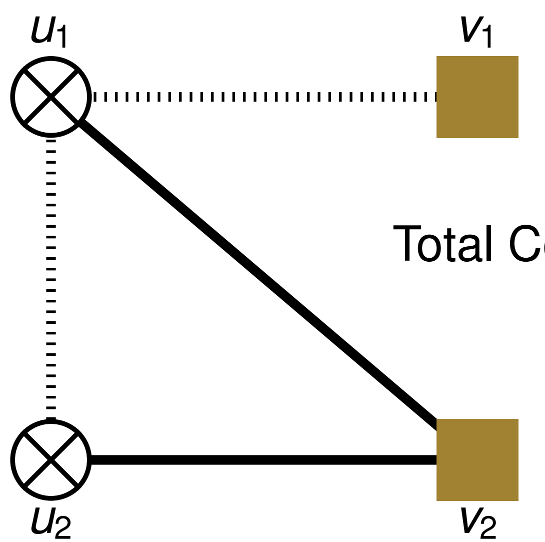
- Substation capacity: 2
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:



Find edge weights for change of flow $\Delta = 1$

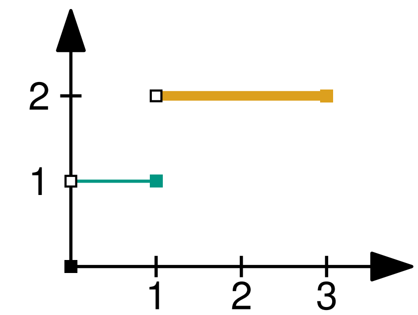


Negative Cycle Canceling [Gritzbach et al., 2018]

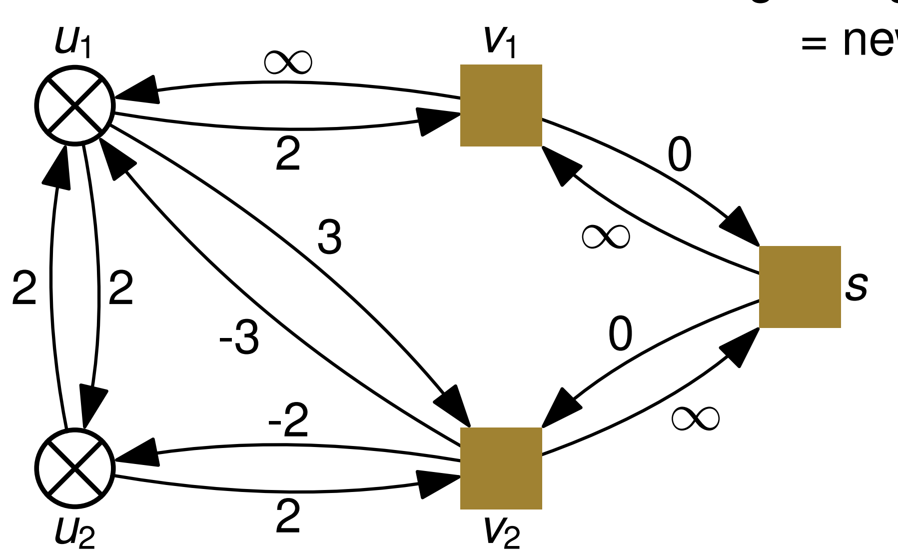


Total Cost: 5

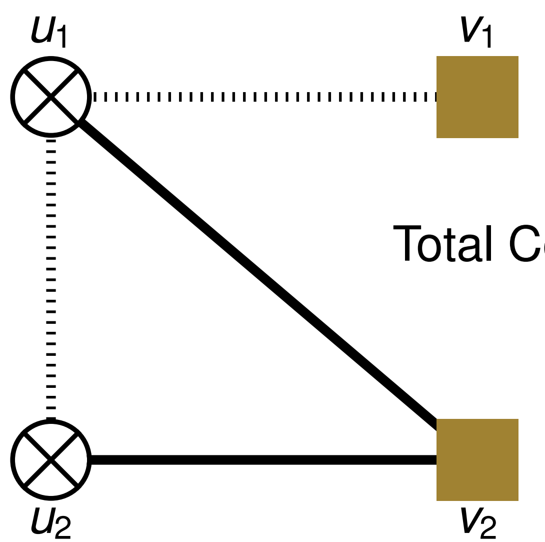
- Substation capacity: 2
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:



edge weights for change of flow $\Delta = 1$
$= \text{new cost} - \text{old cost}$

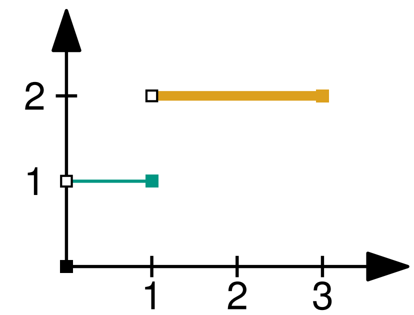


Negative Cycle Canceling [Gritzbach et al., 2018]

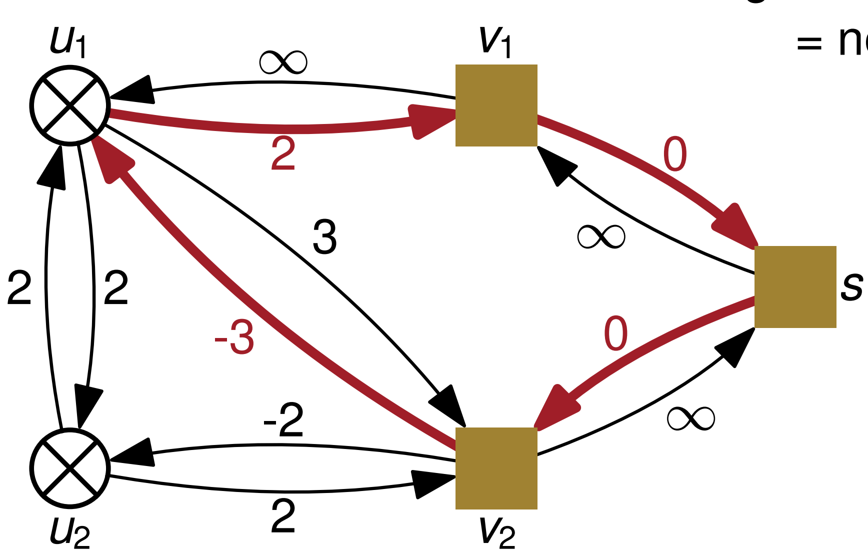


Total Cost: 5

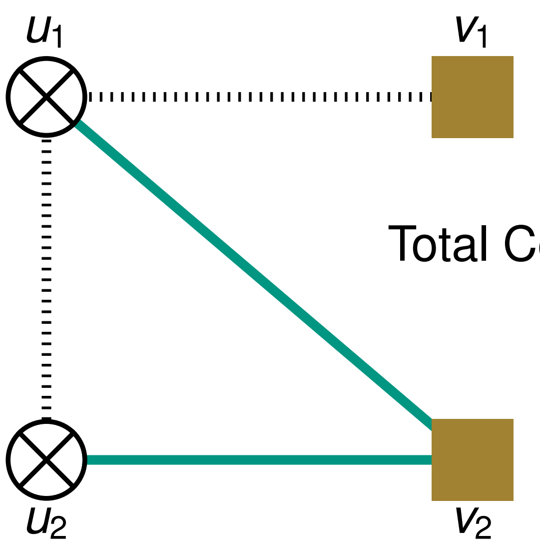
- Substation capacity: 2
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:



edge weights for change of flow $\Delta = 1$
= new cost - old cost

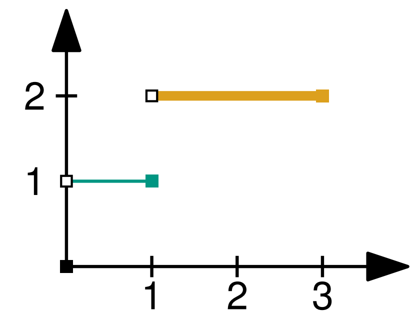


Negative Cycle Canceling [Gritzbach et al., 2018]

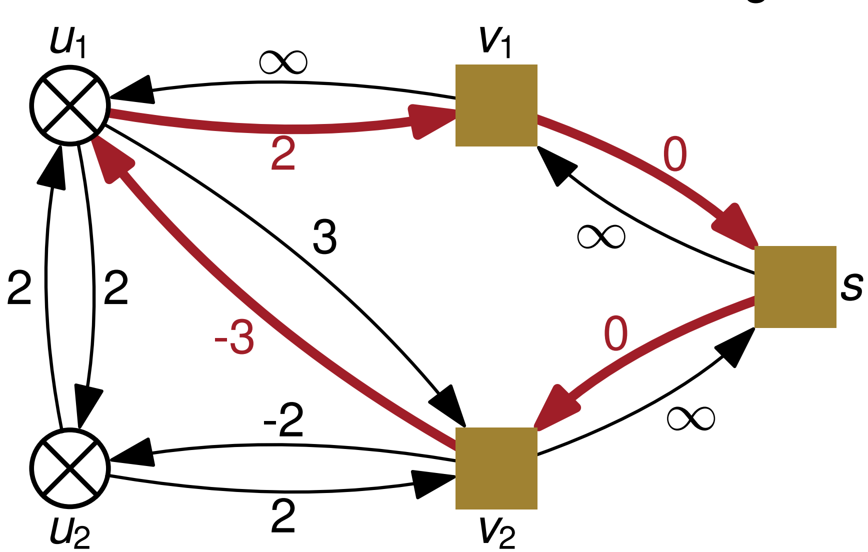


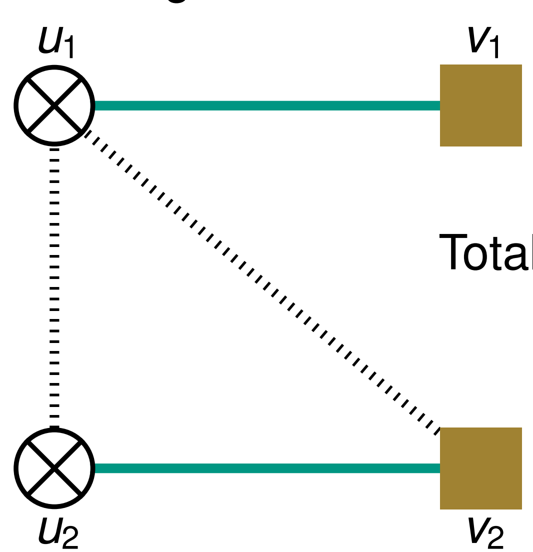
Total Cost: 5

- Substation capacity: 2
- Edge lengths: 2 (edge $u_1 v_2$: 3)
- Cable types:



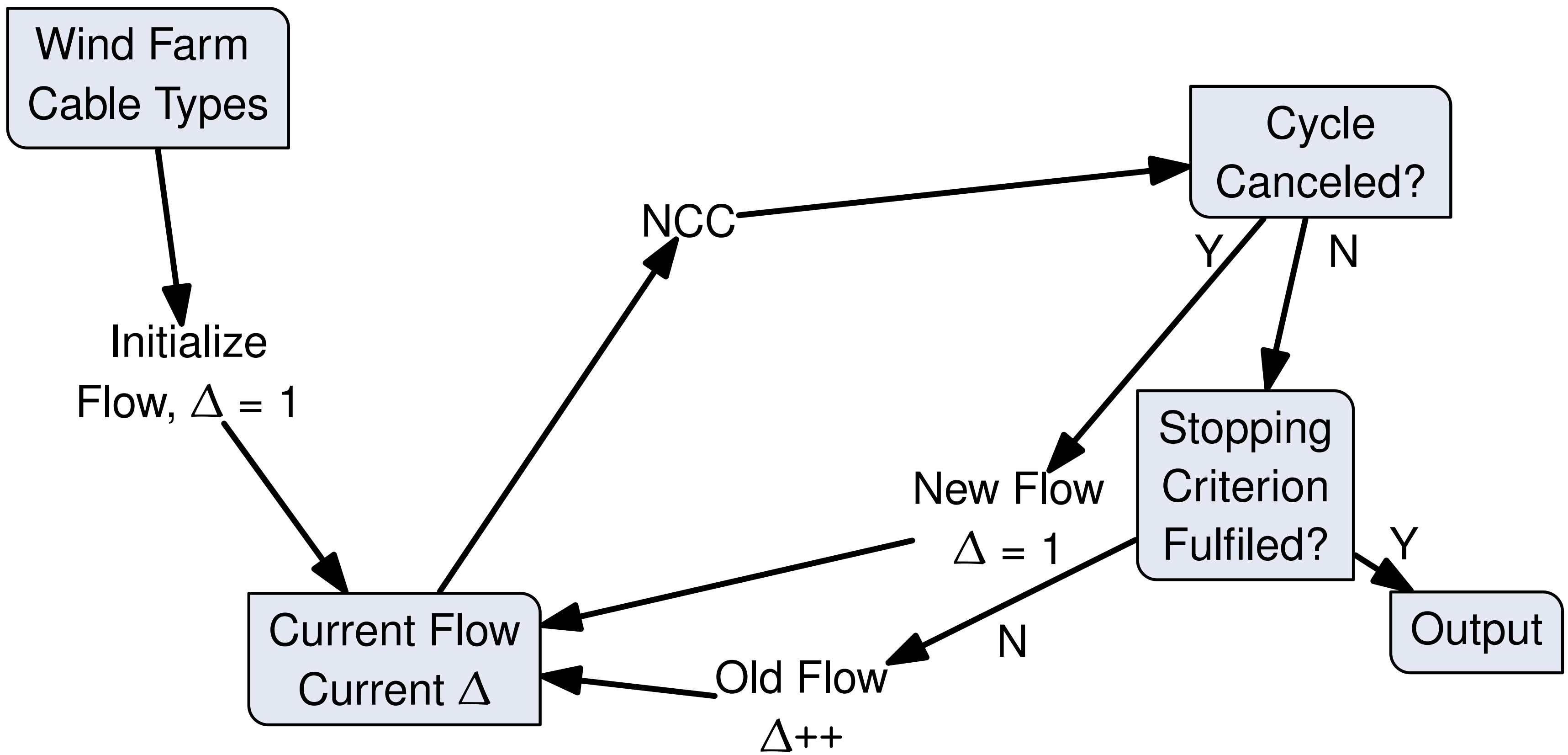
edge weights for change of flow $\Delta = 1$



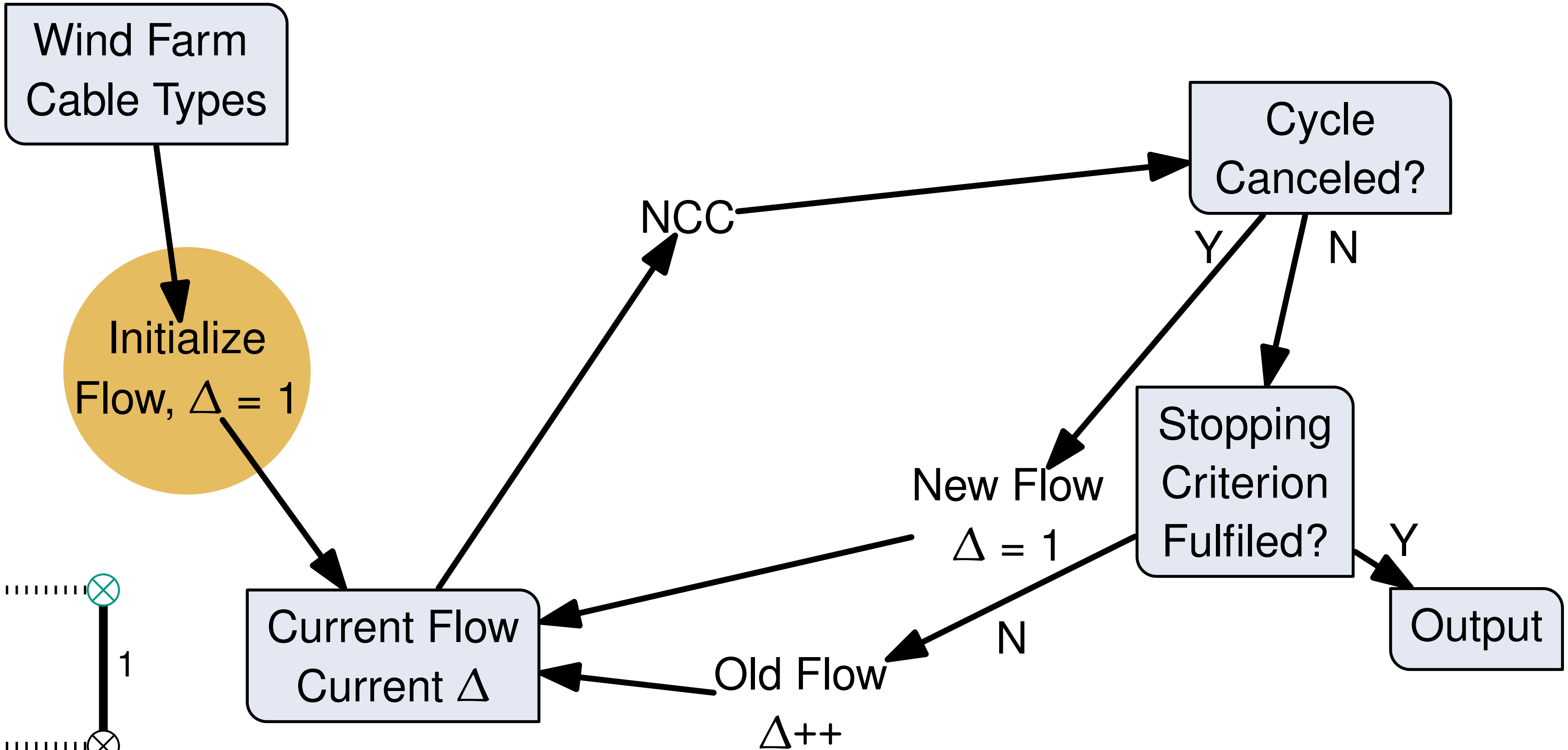


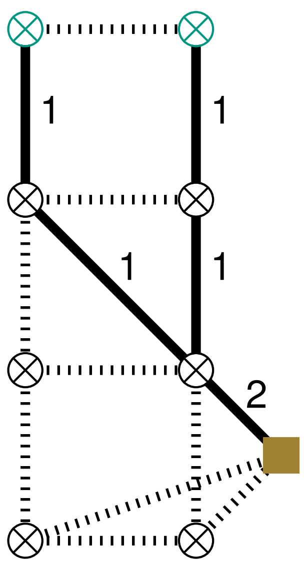
Total Cost: 4

Algorithmic Overview [Gritzbach et al., 2018]



Algorithmic Overview [Gritzbach et al., 2018]





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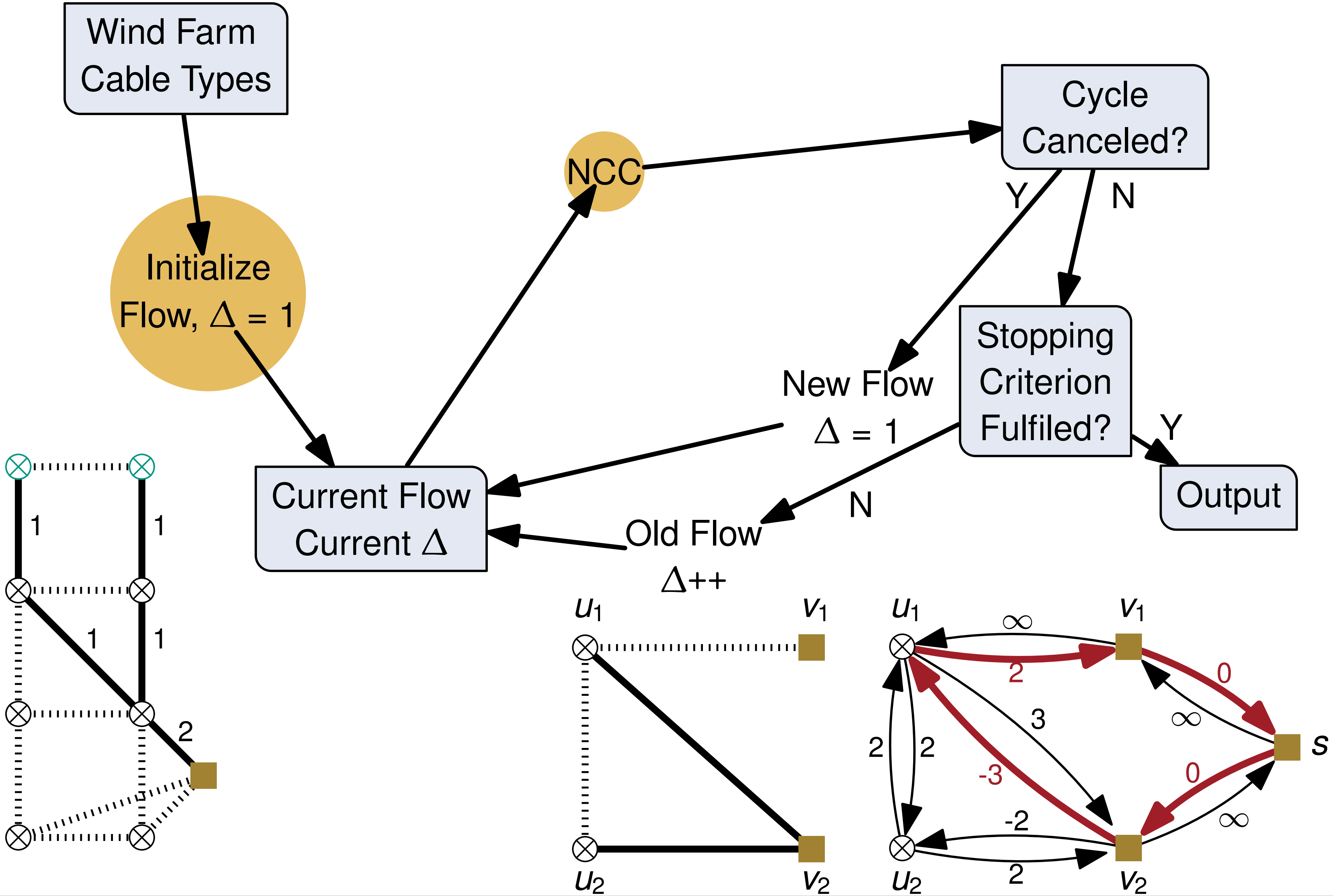
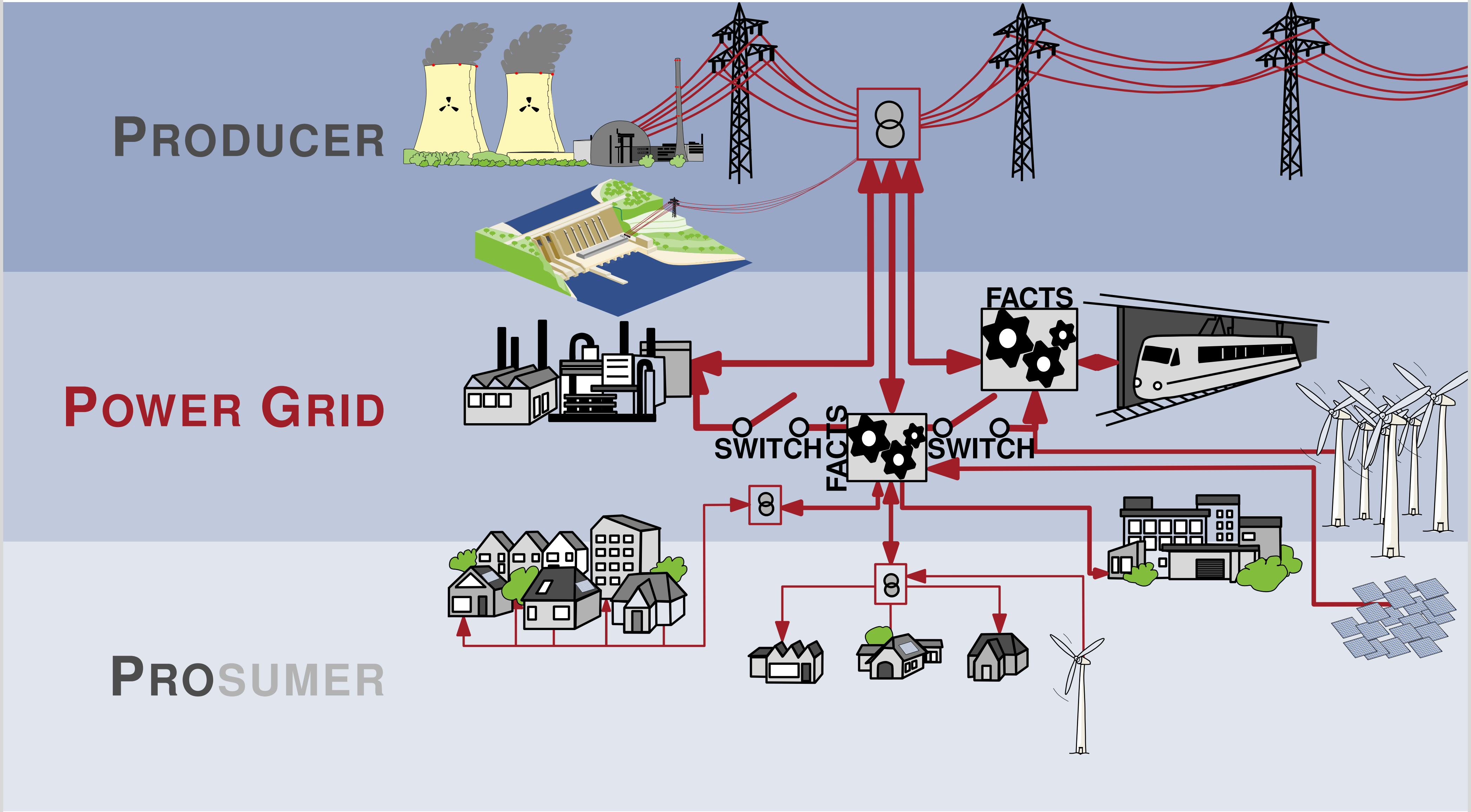
Algorithmic Overview [Gritzbach et al., 2018]



Summary



Summary

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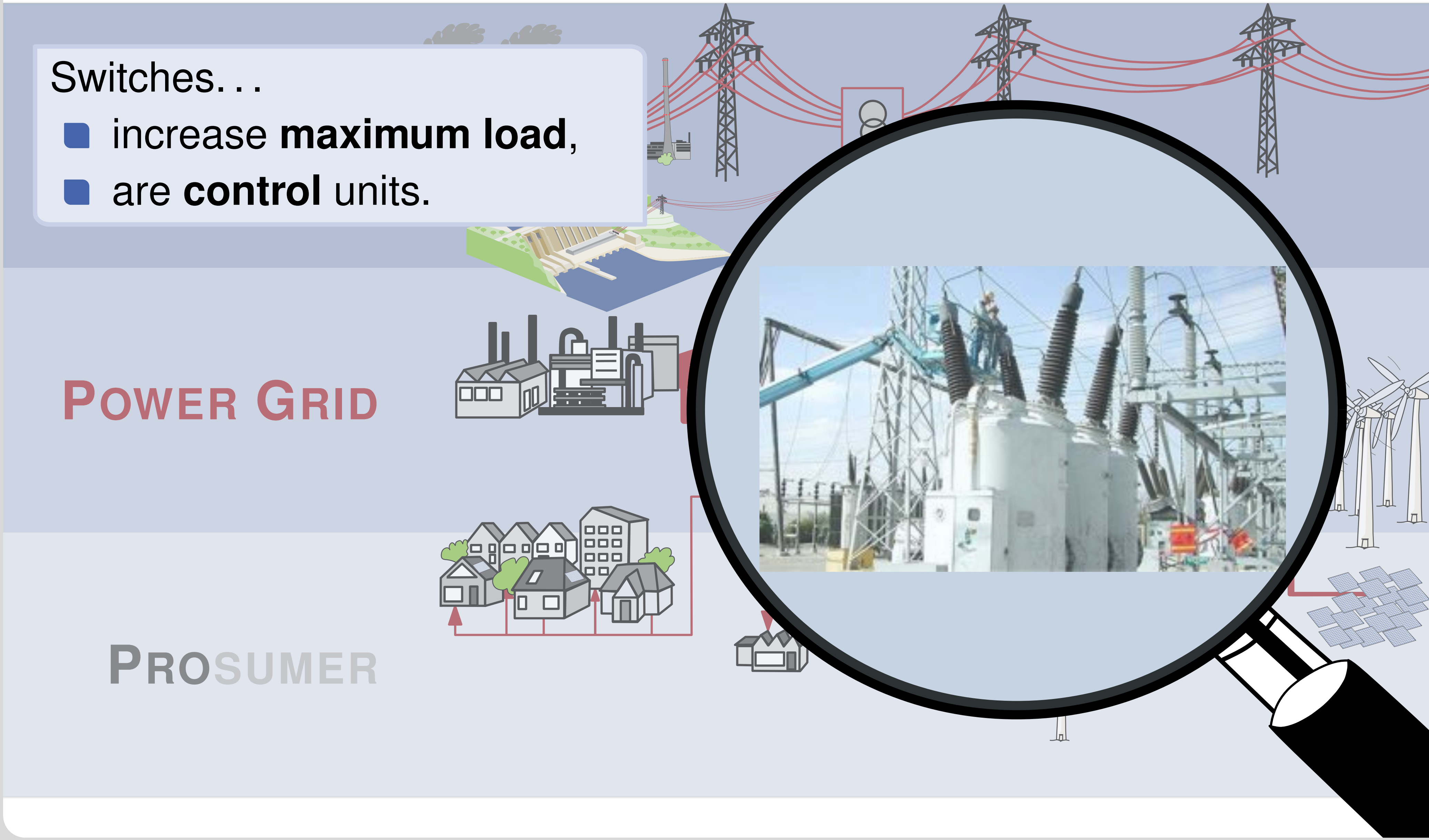
Switches. . .

- increase **maximum load**,
- are **control** units.



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Summary

Switches. . .

- increase **maximum load**,
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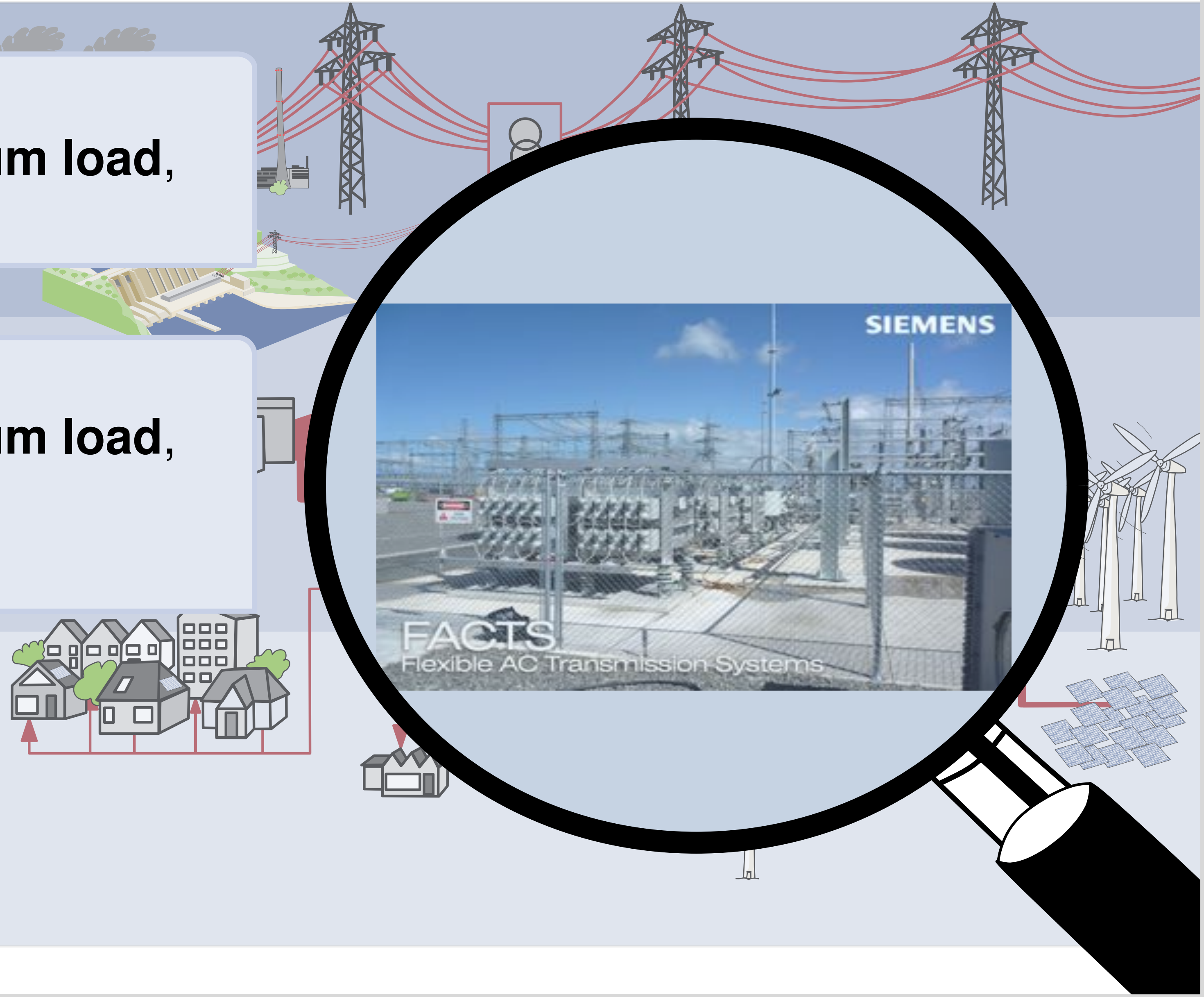
FACTS. . .

- increase **maximum load**,
- are **control** units,
- are **expensive**.



Summary

Karlsruhe Institute of Technology

Switches. . .

- increase **maximum load**,
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FACTS. . .

- increase **maximum load**,
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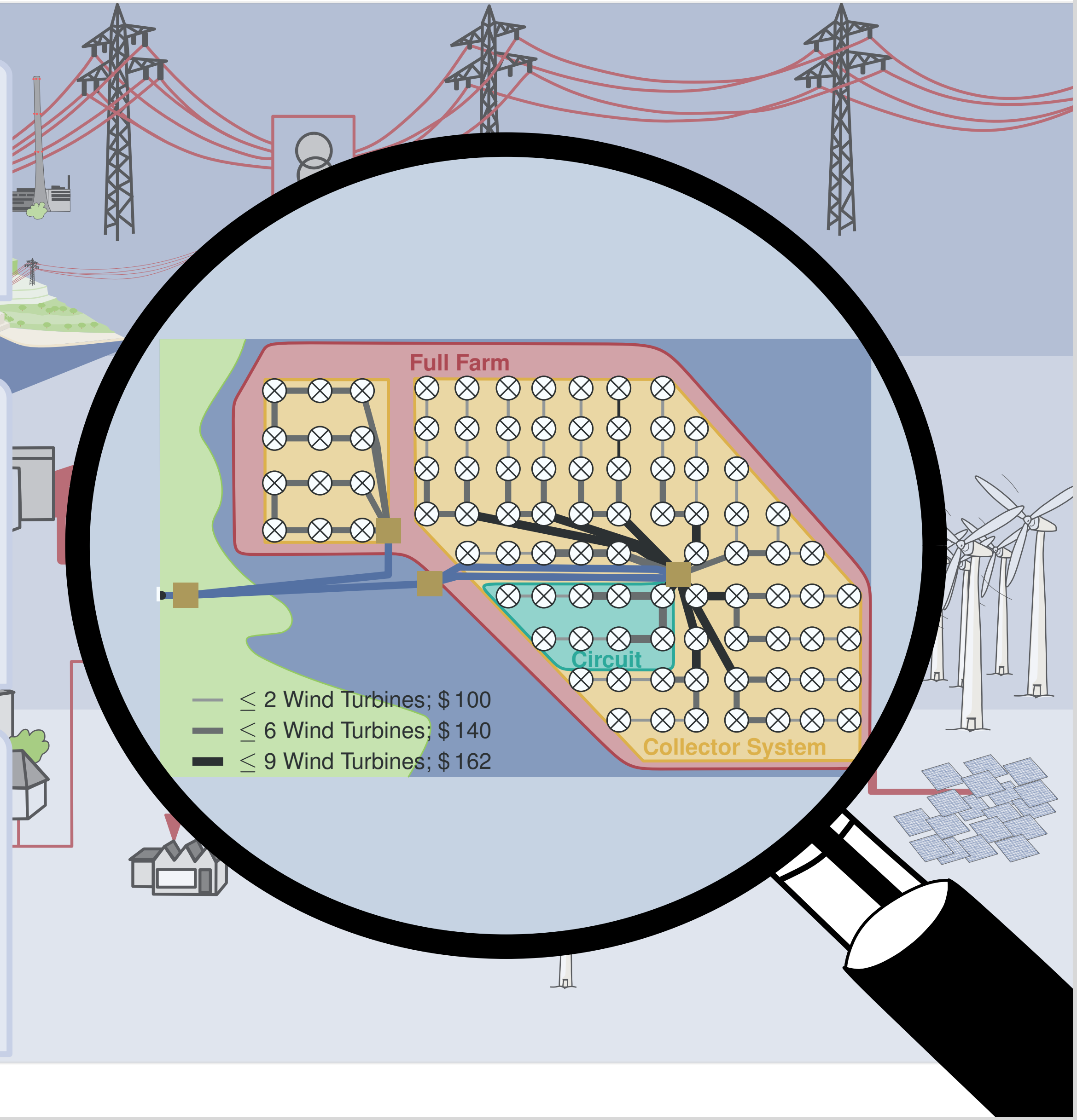
- are **expensive**.

Optimal Windfarm Cabling. . .

- significantly **decreases** the overall building costs,
- allows **multiple** cable types.



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