Time-Expanded vs Time-Dependent Models for Timetable Information

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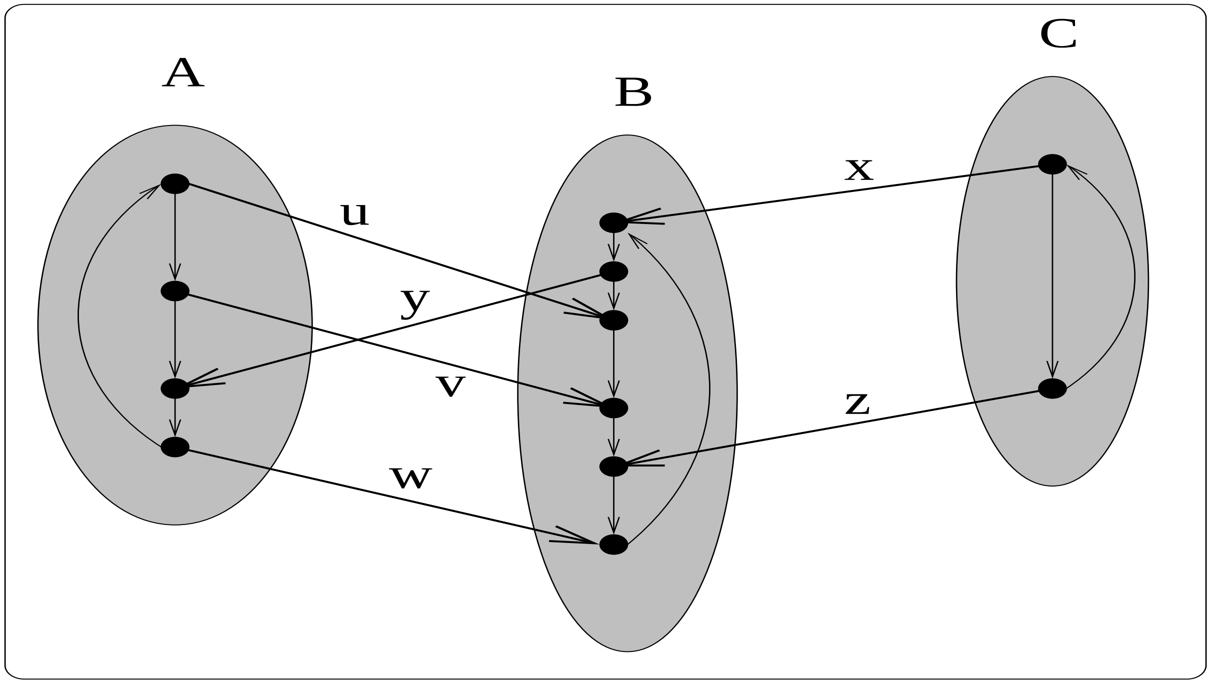
Timetable Information

- Reduces to computing shortest paths in a specific graph model

- Two main models:
  - Time-Expanded
  - Time-Dependent

- Main approach: Dijkstra-like algorithms

Time-Expanded Model



- Vertices $\longleftrightarrow$ Events (e.g., departures, arrivals) at stations

- Edges $\longleftrightarrow$ elementary connections between two events

- Weight of edge (a, b) : Time(b) $-$ Time(a)

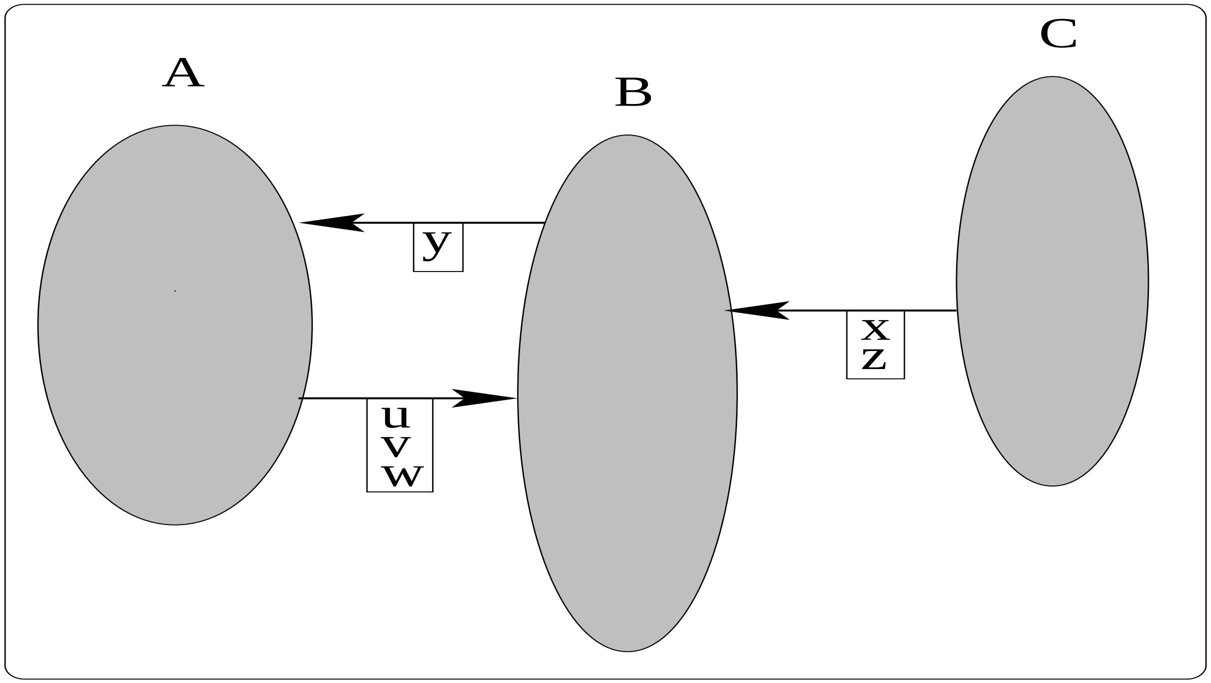
Time-Expanded Model (cntd)

- Large size

- Earliest Arrival problem:

  size can be reduced by half (all “arrival” vertices, except for those of the target station, can be ignored).

Time-Dependent Model



- Vertices $\longleftrightarrow$ Stations
- Edges $\longleftrightarrow$ elementary connections between two stations
- Weight of edge (a, b) : depends on the arrival time at station a

Time-Dependent Model (cntd)

- During Dijkstra-like computation:

  weight of edge (a, b) is computed on the fly.

- Simple method:

  binary search on an array associated with (a, b) and sorted w.r.t. departure time from a.

Size Comparison between Models

- Data: DB Timetable of Winter 1996/97

Model	#Vertices	#Edges	Average Out-degree	Avg #Elem. Connect./edge
T-Exp. (full)	931746	1397619	1.5	0.67
T-Exp. (half)	465888	931776	2	1
T-Dep.	6961	18664	5.36	23.53

Time-Expanded Model: Speedup Heuristics

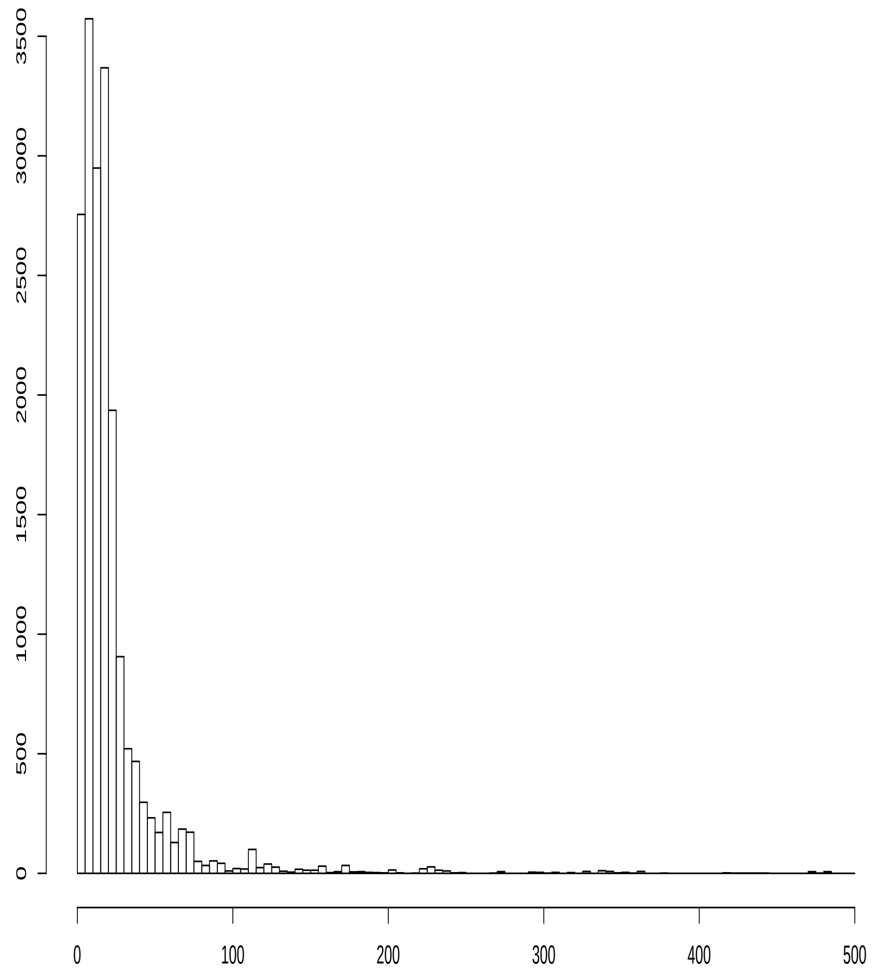
- [Schulz,Wagner,Weihe,WAE99]

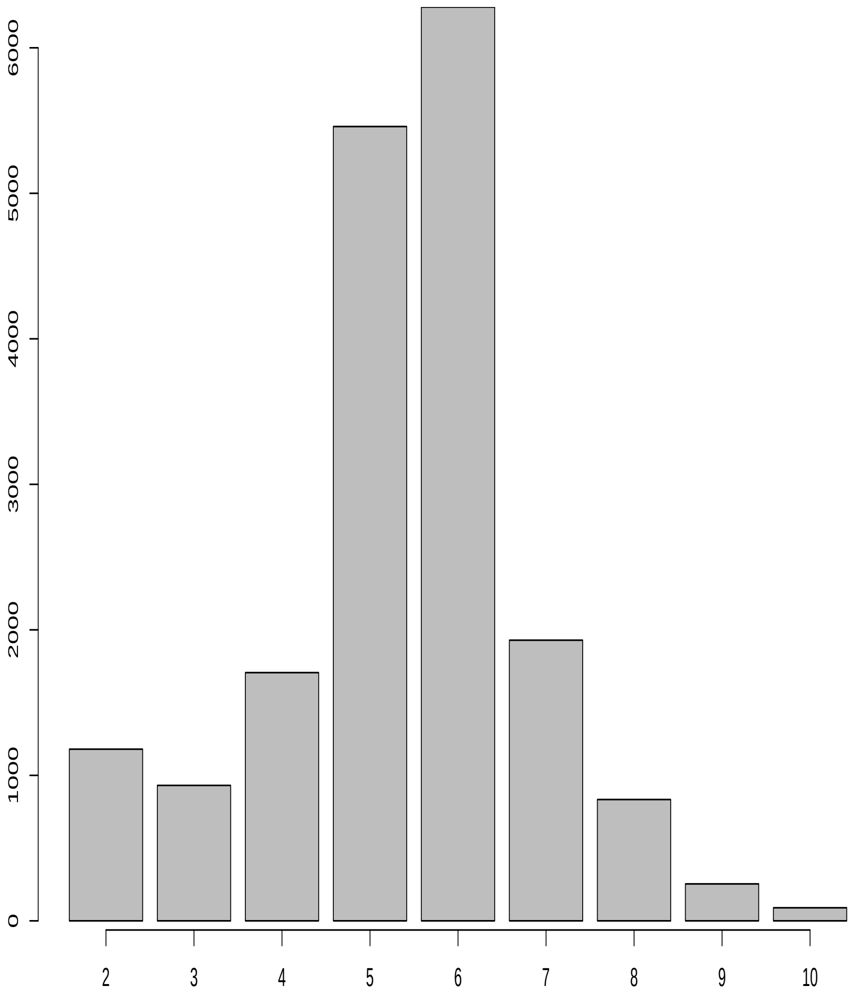
Heuristic	Speedup (over Dijkstra)
A: angle restriction	5.7
S: selection of stations	8.6
G: goal-directed search	1.4
A + S + G	30.0

- [Schulz,Wagner,Zaroliagis, ALENEX2002]

Heuristic	Speedup (over Dijkstra)
Multi-level Graphs	11.2

Time-Dependent Model: Simple method (binary search)





↑ # Edges
→ # Elementary Connections

↑ # Edges
→ $\log_2$(# Elementary Connections)

Time-Dependent vs Time-Expanded: Simple approaches

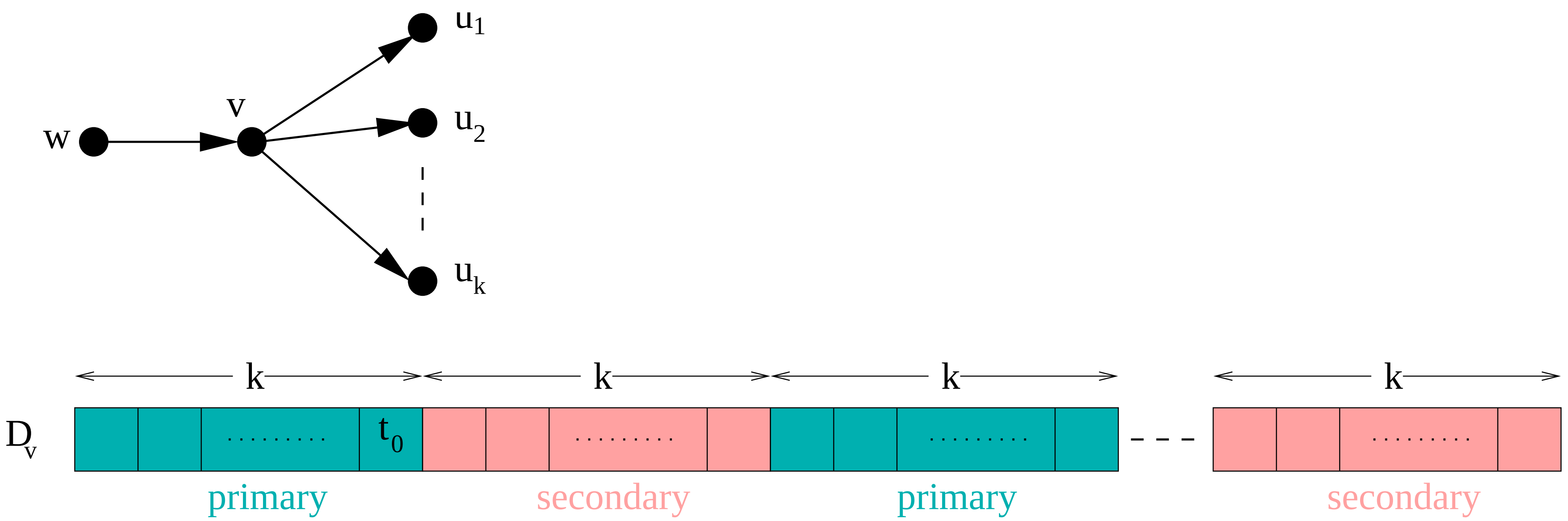
Model	Algorithm	Avg Query Time [ms]	#Elem. Connect.	#Nodes touched	#Edges touched
T-Exp.(f)	Dijkstra	44.17	18406	33653	47807
T-Exp.(h)	Dijkstra	29.2	18406	18406	33377
T-Dep	Bin. Search	5.61	14033	1515	4463

Time-Dependent Model: Speedup Heuristics

- Avoiding Binary Search

- Goal-directed Search (potentials)

Time-Dependent Model: Avoiding Binary Search



- Sort all events of (v, u_i), $1 \leq i \leq k$.

- $\forall (v, u_i)$, find first event with $t \geq t_0$ and put it into the next secondary segment.

- Let t_1 be the arrival time of a primary event $P^w \in D_w$. Create a pointer from P^w to the very next primary event $P^v \in D_v$ with timestamp $t_2 \geq t_1$.

Time-Dependent Model: Potentials

- For edge (u, v) and τ the arrival time at u:

$$wt'(u, v, \tau) = wt(u, v, \tau) - p[u] + p[v]$$

- $p[\ ]$ must be *valid*, i.e., $wt'(u, v, \tau) \geq 0$.

- For destination node $t \in V$:

$$p[u] = d(u, t)\lambda_t, u \in V, \lambda_t \geq 0$$

where

$$0 \leq \lambda_t = \min_{\substack{(u,v) \in E \\ d(u,t) - d(v,t) > 0}} \frac{\min_\tau wt(u, v, \tau)}{d(u, t) - d(v, t)}$$

- $wt(u, v, \tau) - p[u] + p[v] \geq 0 \Rightarrow wt(u, v, \tau) - \lfloor p[u] \rfloor + \lfloor p[v] \rfloor \geq 0$

Time-Dependent Model: Comparison of Heuristics

Heuristic	Avg Query Time [ms]	#Nodes Touched	#Edges Consid.	#Edges Unneces.
Bin. Search	5.61	1515	2360	452
Avoid B.S.	6.41	1515	3446	829
Euclidean Pot.	6.75	984	1584	–
Eucl. Pot. & int PQ	6.54	988	1590	–
Manhattan Pot.	6.35	1025	1645	–
Manh. Pot. & int PQ	5.59	1030	1652	–
Avoid B.S.++	5.66	1515	2360	–
Avoid B.S.++ & Manh. Pot. & int PQ	5.38	1030	1652	–

- *Considered edges* : distance of tail vertex not yet known

- *Unnecessary edges* : those of considered for which the elementary connection departs later than the earliest arrival time at destination

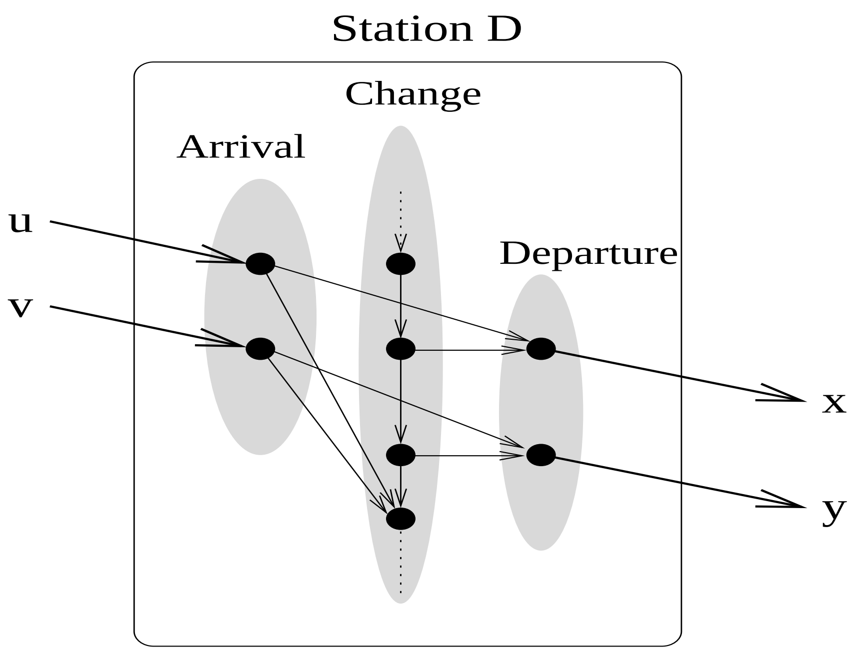
Extensions: Modeling Train Changes

1. Introduce minimum time required for changes (global or local per station)

2. Introduce exceptions when trains depart from the same platform

3. For every train arrival, maintain a list of connecting trains

Modeling Train Changes: Time-Expanded Model



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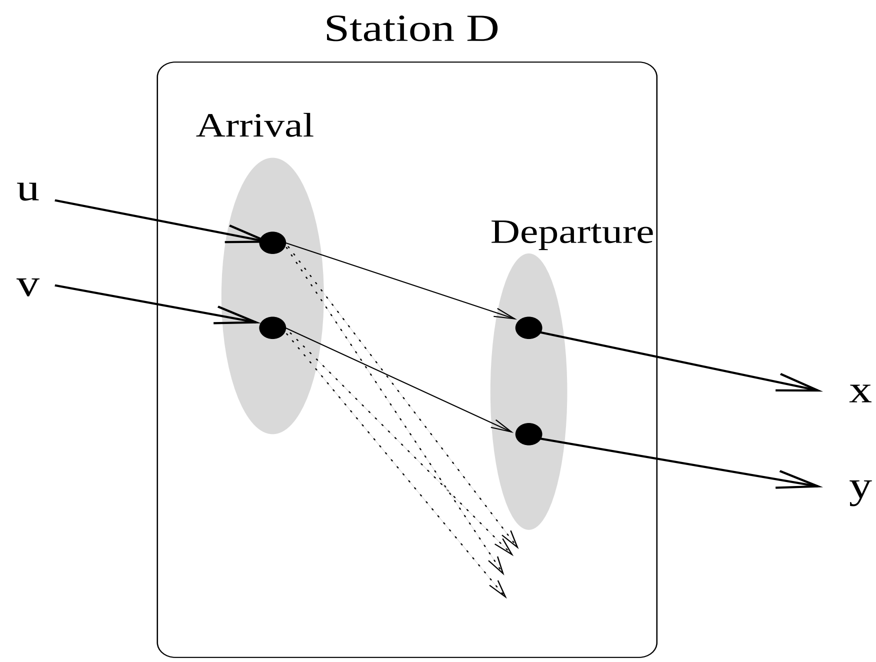

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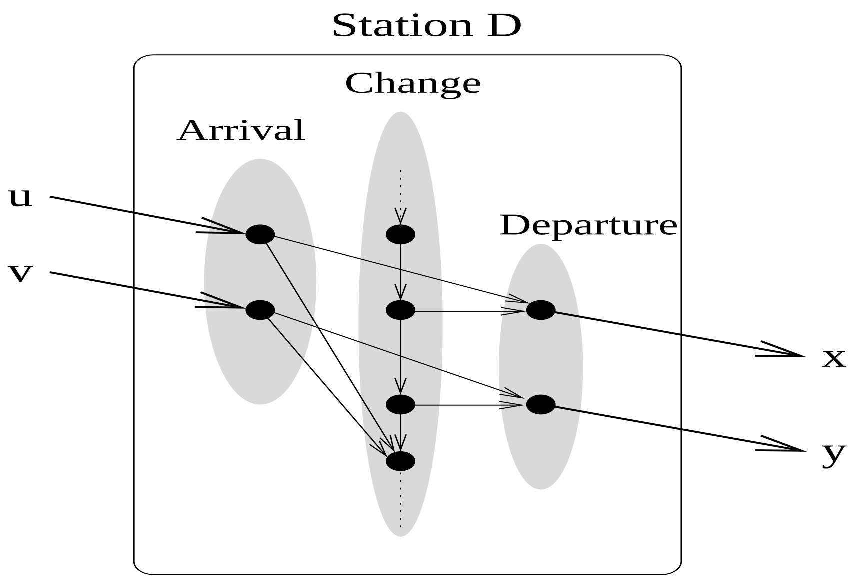
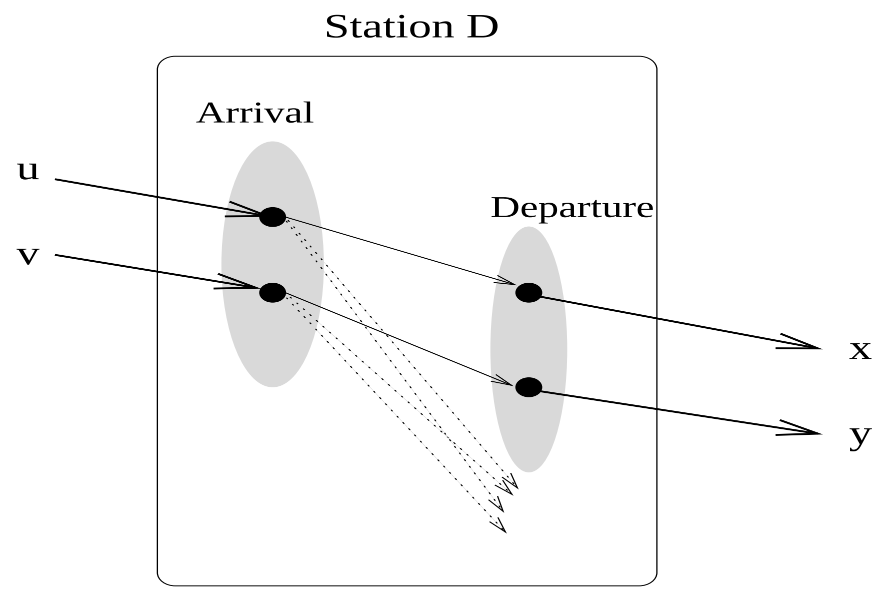
Modeling Train Changes: Time-Dependent Model





- Above modeling doesn't work

  Assume

  — trains $T_1 = (u, x)$ and $T_2 = (v, y)$

  — earliest arrival at D is through u

  — T_1 does not reach destination, but T_2 does

  $$\Downarrow$$

  May miss earlier connection through T_2 to destination

Conclusions & Further Research

- Time-dependent models
  - require less space
  - do not necessarily provide faster query times
  - not trivial to model other optimization criteria
- Extensive comparative study among all heuristics in both models
- Investigate extension of both models towards other optimization criteria