Crew Scheduling

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Overview

- Crew Scheduling Problem Definition
- CSL Prototypes & Experience
- Airline Crew Scheduling
- Bus & Driver Shift Scheduling
- OR Modeling & Solution Approach
- Column Generation Approach
- Discussion
Crew Scheduling Problem

Definition

- Assignment of well-defined tasks (pairing & shift construction) to a group of people while respecting a set of complicated legality rules and resource constraints.
- Most of the legality rules are non-linear and evolving through time.
CSL Prototypes & Experience

- Airline Crew Scheduling (Pairing Construction & Crew Assignment)
- Bus & Driver Shift Scheduling
Airline Crew Scheduling Problem

- **Flight Legs**
- **Pairing Legality Rules**
- **Other Activities** (training, vacation, etc)
- **Crew Pairing**
  
- **Roster Legality Rules**
- **Crew Assignment**
  
- **Crew Members**
  
Schedule
Crew Pairing Solution
Methodology

- Crew Pairing and Crew assignment are too big to be solved together.
- A good solution for Crew Pairing is a must for the efficient and productive use of the airline crews.
Airline Crew Scheduling

- Entities of the problem
  - Flight Leg: A non-stop flight with its crew complement and fleet requirements
  - Duty: A legal sequence of legs for one day
  - Pairing (Trip): A legal sequence of duties
    - Pairings start and end at the same crew base
  - Roster: A set of pairings and other activities assigned to a specific crew member
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Hierarchy of activities

Duty #1

LH 137

MUC 11:00
HAM 13:00
HAM 14:00
STR 15:00

Briefing (60 min)

Debriefing (15 min)

Trip

Duty #2

Night Stop

STR 8:00
MUC 9:00
MUC 10:00
ATH 14:45
ATH 16:00
FRA 19:30

Briefing

Debriefing
Bus & Driver Shift Scheduling

- Solved every afternoon for the work load of the next day
- Shift: a set of routes that will be performed by a bus and its associated driver in a day
- Shifts must be legal according to a complex set of rules while respecting previous bus-ending points
- A good solution for the problem is a legal set of shifts that efficiently covers the work load
  - (more later)
Solution Approaches for the Crew Pairing Problem

- Generate and Optimize
  - Select sub-problems (Heuristic filtering)
  - Phase 1. Generate a large set of legal pairings (Generate)
  - Phase 2. Select the best pairings (Optimize)
  - Iterate

- Column Generation
Generate and Optimize in Production (CARMEN)

- Initially used in CARMEN’s Pairing Construction System (PAC)
- In use since 1995 by most European Airlines
- Clever sub-problem selection filters and tools
- Day by Day (DbD) iteration process
- Efficient modeling of complex legality rules via a separate rule system
Time distribution of the Generate & Optimize Approach

- Trip Generation: 80%
- Optimization: 5%
- Sub-Problem Selection: 15%
Trip Generation Process

Connection matrix graph
(each leg appears only once)

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Trip Generation Algorithm

Depth first search algorithm

- For each starting node a separate search tree is defined
- The DFS process is controlled by:
  - Search width
  - Maximum number of total trips
  - Maximum number of trips per starting node
  - Legality rules
Basic Procedure for Crew Scheduling Problems OR(1)

- Formulated as a Set Covering (SCP) or Set Partitioning (SPP) problems

(SCP) \( \min \{cx : Ax \geq 1, x \in \{0,1\}^n\} \)
(SPP) \( \min \{cx : Ax = 1, x \in \{0,1\}^n\} \)
OR Modeling Approach (2)

- A binary variable (column) represents a **legal** schedule of a person that covers a set of tasks
- Each variable (column) embeds all non-linear legality rules
- Legality rules are external to the model
- Constraints ensure the covering of all tasks
The airline crew pairing problem involves the finding of a set of trips that covers a set of flights with minimum cost.

\[
\begin{align*}
\text{trip 1} & & \text{trip 2} & & \text{trip 3} & & \cdots & & \text{trip n} \\
\text{flight 1} & & 1 & & 0 & & 1 & & \cdots & & 1 \\
A = \text{flight 2} & & 1 & & 1 & & 0 & & \cdots & & 1 \\
\cdots & & \cdots & & \cdots & & \cdots & & \cdots & \cdots \\
\text{flight m} & & 0 & & 1 & & 1 & & \cdots & & 1 \\
\end{align*}
\]

\[m = 10^2 - 10^4\]
\[n = 3 \times 10^4 - 10^6\]
OR Solution Approach

- Generate and Optimize
  - Generate a large number of ‘good’ legal columns and select the best ones
  - Generation of ‘good’ columns is a time-consuming task
  - Selection of ‘good’ columns requires an efficient IP Solver
Solution Approach (2)

- Small amount of RAM required for the generation phase
- Clever problem specific heuristics for sub-problem selection & the (DbD) solution strategy
- Powerful IP Optimizer (able to identify reasonable solutions from ~1,000,000 columns)
Solution Approach (3)

- Used in the production environment for many years by several European airlines
- Computer generated solution were often inferior to the ones of human experts and/or users could further improve the solution!
- Need to solve larger problems with stable heuristic processes
Column Generation (CG)

- Known for many decades for the solution of large LP problems

- Main Idea of CG approach:
  - Consider only a small number of variables at a time
  - Solve a small LP (master problem) and get a primal and a dual solution
  - Generate new attractive columns (sub-problem), with negative reduced cost, by using the dual solution of the master problem in order to improve the previous LP solution
  - Repeat the procedure until no further improvement can be made
Column Generation

Requirements for LP & IP

- Efficient data structures for the implicit representation of all problem variables
  - Large amounts of RAM
- Fast algorithms for generation of new legal and promising columns
- LP Optimizer
- No need for strong IP optimizer!
Column Generation & Scheduling

- Master Problem ensures covering of tasks
- Sub problem usually has the structure of a graph
- Nodes are simple or composite activities (i.e. flights, duties)
- Arcs connect activities that are legal to be connected in pairs

\[
\text{Master Problem:}\quad \min c x \\
\text{s.t.}\quad Ax = 1
\]
Master Problem

- Relaxed IP model
- One constraint for each task
- In each step solve a problem that has the basis of previous iteration and the newly generated attractive columns
- Return primal and dual solution

<table>
<thead>
<tr>
<th>Schedule</th>
<th>1</th>
<th>Schedule</th>
<th>2</th>
<th>...</th>
<th>Schedule</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flight1</td>
<td>1</td>
<td>Flight2</td>
<td>1</td>
<td>0</td>
<td>...</td>
<td>0</td>
</tr>
<tr>
<td>A =</td>
<td></td>
<td>...</td>
<td></td>
<td></td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>Flight5</td>
<td>0</td>
<td>...</td>
<td></td>
<td></td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

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Sub-problem

- Basic structure is a graph or a connection matrix
- Nodes are the flights
- Arcs connect flights that can be legally connected as a pair
- Cost of a node is the cost of including the corresponding flight in some pairing
- Cost of an arc is the dual of the constraint of the source node flight “the source node is present for all possible pairings after this point”
Sub-problem (2)

- Legality Rules
- The reduced cost of a new pairing (start to end) is the cost of the path
- Pairing: Flight1, Flight4, Flight5, has reduced cost \((c_1+c_4+c_5) - (y_1+y_4+y_5)\)
- A k-shortest path type algorithm provides the best candidate pairings

**ASSUMPTION:** The cost of a schedule is the sum of the costs of all flights (additive function)
  - Often OK even if cost is non-linear!
Duty Based Sub-problem

- Embed legality of duties
- Nodes of the network are legal duties
- Two duties that can be legally followed are connected with an arc
- Dual of each node is the sum of duals of the legs of the corresponding duty
- Cost of each node is the cost of the corresponding duty
- Number of nodes increases
- Number of arcs decreases
- Network is smarter and is easier to look for legal pairings
Search for new attractive pairings

- Sub-problem network (flight or duty) cannot embed all legality rules
- k-shortest path algorithms may produce a large number of illegal pairings!
- DFS + shortest path always produces legal pairings
Search for new attractive pairings

(2)

- Build new legal pairings using a depth first search procedure
- DFS proceeds using the shortest path results for each node

(more in the next presentation)
IP Solution (1)

- An LP (fractional) solution is always known but an IP solution is actually required.

- Procedure for IP solution creation:
  - Reduce problem dimensions by freezing a part of the solution and re-applying the CG strategy on the remaining problem.
  - At a certain point when problem dimensions are small an IP solution can be located with some other IP optimization method.
IP Solution (2)

Start

Solve problem and find best LP Solution

Branchings are finished?

NO

Select Sub-problem (branching)

Resolve problem

is solution IP?

NO

Asses current problem status and prepare next trial

Finish

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Discussion

- Sub-problem identification and the iterative process that will lead us to a good solution is the key to success
  - Intelligent domain specific criteria for the selection of sub-problems (DbD)
  - Problem independent strategy via the use of LP and duality theory (CG)