Variable trip times for cyclic railway timetabling

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AMORE Research Seminar
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Why study variable trip times?

- Basic model: fixed trip times
- Larger solution space
- Possibly better solutions
- Used in practice!
Fixed trip times (1)

\[
\begin{align*}
\[h + r_1 - r_2, T-h]\_T & \quad [h,T-h]\_T \\
[d_1, a_1] & \quad [r_1]_T \\
[d_2, a_2] & \quad [r_2]_T \\
\end{align*}
\]

\[
d_2 - d_1 \notin X_T \\
= [0, h + r_1 - r_2]_T
\]
Fixed trip times (2)

Theorem (Lindner)

Suppose

\[ d_2 - d_1 + Tp_d \in [h, T - h] \]
\[ a_2 - a_1 + Tp_a \in [h, T - h] \]

Trains 1 and 2 do not overtake each other if and only if

\[ p_d = p_a \]
Problems with variable trip time

- Trip times straightforward
  \[ a_i - d_i \in [r_i, R_i] \]

- Safety constraints
  \[ d_2 - d_1 \not\in X_T = [-h, h + \rho_1 - \rho_2]_T \]

...but \( \rho_1, \rho_2 = ? \)

- Lindners’ result requires JPESP (Joined constraints) for expressing \( p_d = p_a \)
Variable trip times

**Theorem**

The constraints

\[ d_2 - d_1 \notin (-h, h)_T \]
\[ a_2 - a_1 \notin (-h, h)_T \]

are necessary to guarantee
- non-overtaking of 1 and 2
- respecting headways

If

\[ R_1 - r_2 < 2h \]
\[ R_2 - r_1 < 2h \]

then also sufficient.

\[ \rho_1 > \rho_2 + 2h \]
\[ \Rightarrow \max\{ \rho_1 - \rho_2 \} < 2h \]
\[ \Rightarrow R_1 - r_2 < 2h \]
Disjoint trip time windows

Assume \( r_1 > R_2 \)

Theorem
\[
d_2 - d_1 \notin (-h, h + r_1 - R_2)_T \\
\frac{a_1 - a_2}{h + r_1 - R_2}_T
\]

are necessary to guarantee
- non-overtaking of 1 and 2
- respecting headways
If
\[
(R_1 - r_1) + (R_2 - r_2) < 2h + (r_1 - R_2)
\]
then also sufficient.
Conclusion & Practical experience

- Similar result for opposite trains on single track
- General case: trip window at most 5 wide
- Disjoint case: more space
- Uses PESP constraints only
- Works well in practice
  - First without variable trip times
  - If not feasible, use variable trip times
Computational experiments for the Netherlands’ Intercity network

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**Model - cycles**

- Consider a cycle:

- **Cycle directions** \( c = c^+ \cup c^- \)
- **Decision variables** \( d_a \in [l_a, u_a] \)
- **Integer variable** \( q_c \)
- **For every cycle** \( c \in C : \)

\[
\sum_{a \in c^+} d_a - \sum_{a \in c^-} d_a = T q_c
\]
Objective function: \( d_a \)-variables

- Minimize \( \sum_{a \in A_{obj}} f_a(d_a) \)

- Shape of functions \( f_a \):

\[ f_a \]

\[ 0 \quad l_a \quad 0 \quad l_a \]

connection

\[ u_a \quad T \]

frequency
Alternative objective function - rolling stock

- Minimize number of rolling stock compositions
- Consider train cycle:

\[ \min \sum_{c \in C_t} q_c , \quad C_t \subseteq C \] set of train cycles
Preprocessing

- Contract nodes with degree 1 or 2
- Shrink subsequent safety constraints:

![Diagram]

- Substitute remaining equality constraints
Calculating a cycle basis

- Find a MST with respect to $w_a = u_a - l_a$
- Widest arcs each in only one cycle
- Bounds on the $q_c$ -variables:

\[
\sum_{a \in c} l_a \leq \sum_{a \in c} d_a = Tq_c \leq \sum_{a \in c} u_a
\]

\[
\Downarrow
\]

\[
\left[ \frac{1}{T} \sum_{a \in c} l_a \right] \leq q_c \leq \left[ \frac{1}{T} \sum_{a \in c} u_a \right]
\]
### Characteristics IC-97

- Train lines: 25
- Stations: 50
- Original size: \( n=1475, m=3342 \)
- After pre-processing: \( n=207, m=525 \)
- # connections: 34
- # stops: 162
- # safety: 300

- MIP solver CPLEX 6.5
- Sun Enterprise/250, two 400MHz UltraSPARC processors, 1 Gb mem.
Minimizing waiting time

- Minimize $\sum_{a \in A_{\text{conn}} \cup A_{\text{stop}}} f_a(d_a)$

- Variants:

<table>
<thead>
<tr>
<th>Instance</th>
<th>$\Delta_{\text{conn}}$</th>
<th>$\Delta_{\text{stop}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

frequency $[l_\omega u_a]$, $z_{\text{opt}}$

- window: $[28,32]$, 114
- fixed: $[30]$, 154
Cycle basis performance

- Cycle bases
  - MST
  - Train cycles

- Variants:

<table>
<thead>
<tr>
<th>Instance</th>
<th>$l_{\text{turn}}$</th>
<th>$u_{\text{turn}}$</th>
<th>$z_{\text{opt}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>10</td>
<td>40</td>
<td>764</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>40</td>
<td>247</td>
</tr>
<tr>
<td>F</td>
<td>5</td>
<td>59</td>
<td>154</td>
</tr>
</tbody>
</table>
Minimizing rolling stock & waiting time

- Minimize \( \sum_{a \in A_{\text{conn}} \cup A_{\text{stop}}} f_a (d_a) + K \sum_{c \in C_t} q_c \)
- Train cycle basis
- Variants:

<table>
<thead>
<tr>
<th>Instance</th>
<th>( z_{\text{opt}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>62.765</td>
</tr>
<tr>
<td>E</td>
<td>57.247</td>
</tr>
<tr>
<td>F</td>
<td>55.171</td>
</tr>
</tbody>
</table>