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Exercise Sheet 4

Discussion: 20. December 2017

Exercise 1: Forces

In the following we consider force-based layout methods.

- (a) Define forces that ensure that
 - (i) vertices closely stay to pre-defined positions,
 - (ii) vertices are closely placed to the x-axis,
 - (iii) edges are aligned parallel to the y-axis,
 - (iv) directed edges are drawn upwards (similar to layered drawings).
- (b) Let G = (V, E) be a graph and assume that we are given a clustering C for G, i.e., a partition of V in pairwise disjoint subsets C_1, \ldots, C_k with $\bigcup_{C \in C} C = V$. Define forces that make sure that vertices of the same cluster lie closely together, while vertices of different clusters lie apart from each other.
- (c) So far we have assumed for the spring-embedder algorithm that vertices are represented by points. How can the approach be adapted if the vertices are represented by circles that should not overlap? What about rectangles?

Exercise 2: Feedback Arc Set

In the lecture we introduced the MINIMUM FEEDBACK ARC SET and the MINIMUM FEEDBACK SET problems. Let D = (V, A) be a directed graph and A' be a subset of A. The set A' is a *feedback* arc set of D if $D_f = (V, A \setminus A')$ is acyclic. If $D_r = (V, (A \setminus A') \cup \{vu \mid uv \in A'\})$ is acyclic, then A'is a *feedback set of* D. Every feedback set is a feedback arc set, the reverse is not necessarily true. Prove the following.

Lemma 1 A set $A' \subset A$ is a minimum feedback arc set of D if and only if A' is a minimum feedback set of D.

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Exercise 3: Homeomorphic Drawings

Let Γ be planar straight-line drawing of a graph G = (V, E). For our purposes we say that two drawings Γ an Γ' are *home-omorphic* if for every $v \in V$ there is a continuous movement of v starting from its position in Γ and ending at its position in Γ' , such that during the movement no vertex passes over an edge.

Lemma 2 There is an arrangement A of lines with respect to Γ such that moving a vertex $v \in V$ within a cell of Ayields a drawing Γ' homeomorphic to Γ .

Use this Lemma to design a spring-embedder that computes a drawing homeomorphic to a given input drawing.

