Exercise Sheet 1

Assignment: October 26
Discussion: November 9

1 Tree Layouts

Let $T = (V, E)$ be a binary tree. For a vertex $v \in V$, we denote its $x$-coordinate by $x(v)$ and its $y$-coordinate by $y(v)$.

(a) We draw the tree $T$ as follows. For each vertex $v$ of $T$, we set $x(v)$ equal to the rank of $v$ in a postorder traversal of $T$, and $y(v)$ equal to its depth in $T$.

(i) Show that the resulting straight-line drawing is planar.
(ii) What is the area of the drawing?
(iii) What happens if instead of a postorder traversal we use a preorder traversal?
(iv) Can the algorithm be extended to rooted ordered trees?

(b) We draw the tree $T$ as follows. For each vertex $v$ of $T$, we set $x(v)$ equal to the rank of $v$ in a preorder traversal of $T$, and $y(v)$ equal to the rank of $v$ in a postorder traversal of $T$.

(i) Show that the resulting straight-line drawing is planar and strictly downward (for each edge $(u, v)$, with depth$(u) < depth(v)$, it holds that $y(u) > y(v)$).
(ii) Show that a vertex $v$ is in the subtree rooted at vertex $u$ if and only if $x(v) > x(u)$ and $y(v) < y(u)$.
(iii) Does the drawing display isomorphism of the subtrees?

2 HV-Layouts

Give an algorithm that for a given $n$-vertex binary tree constructs an HV-layout with minimum area in $O(n^2)$ time. Consider both ordered an non-ordered trees.

3 Outerplanar and Series-Parallel Graphs

A graph $G$ is called outerplanar if it has a planar drawing where all vertices lie on the boundary of the external face. Show that every biconnected outerplanar graph is series-parallel.
4 Visibility Representation

In a visibility representation of a graph $G = (V, E)$ the vertices are represented by horizontal segments. We say that two vertices $u$ and $v$ see each other, if they can be connected by a vertical rectangle of non-zero width that does not cross any other vertex-segment. Thus, in a visibility representation of $G$, two vertices $u, v$ see each other iff $(u, v) \in E$. The bottom figure on the left shows a visibility representation of the graph on top.

Show that each series-parallel graph has a visibility representation.