

Exercise Sheet 1

Assignment: October 26
Discussion: November 9

1 Tree Layouts

Let $T = (V, E)$ be a binary tree. For a vertex $v \in V$, we denote its x -coordinate by $x(v)$ and its y -coordinate by $y(v)$.

- (a) We draw the tree T as follows. For each vertex v of T , we set $x(v)$ equal to the rank of v in a postorder traversal of T , and $y(v)$ equal to its depth in T .
 - (i) Show that the resulting straight-line drawing is planar.
 - (ii) What is the area of the drawing?
 - (iii) What happens if instead of a postorder traversal we use a preorder traversal?
 - (iv) Can the algorithm be extended to rooted ordered trees?
- (b) We draw the tree T as follows. For each vertex v of T , we set $x(v)$ equal to the rank of v in a preorder traversal of T , and $y(v)$ equal to the rank of v in a postorder traversal of T .
 - (i) Show that the resulting straight-line drawing is planar and *strictly downward* (for each edge (u, v) , with $\text{depth}(u) < \text{depth}(v)$, it holds that $y(u) > y(v)$).
 - (ii) Show that a vertex v is in the subtree rooted at vertex u if and only if $x(v) > x(u)$ and $y(v) < y(u)$.
 - (iii) Does the drawing display isomorphism of the subtrees?

2 HV-Layouts

Give an algorithm that for a given n -vertex binary tree constructs an HV-layout with minimum area in $O(n^2)$ time. Consider both ordered and non-ordered trees.

3 Outerplanar and Series-Parallel Graphs

A graph G is called *outerplanar* if it has a planar drawing where all vertices lie on the boundary of the external face. Show that every biconnected outerplanar graph is series-parallel.

4 Visibility Representation

In a *visibility representation* of a graph $G = (V, E)$ the vertices are represented by horizontal segments. We say that two vertices u and v *see* each other, if they can be connected by a vertical rectangle of non-zero width that does not cross any other vertex-segment. Thus, in a visibility representation of G , two vertices u, v see each other iff $(u, v) \in E$. The bottom figure on the left shows a visibility representation of the graph on top.

Show that each series-parallel graph has a visibility representation.

