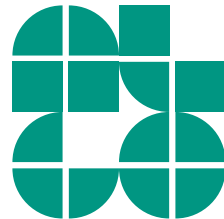


Computational Geometry – Exercise

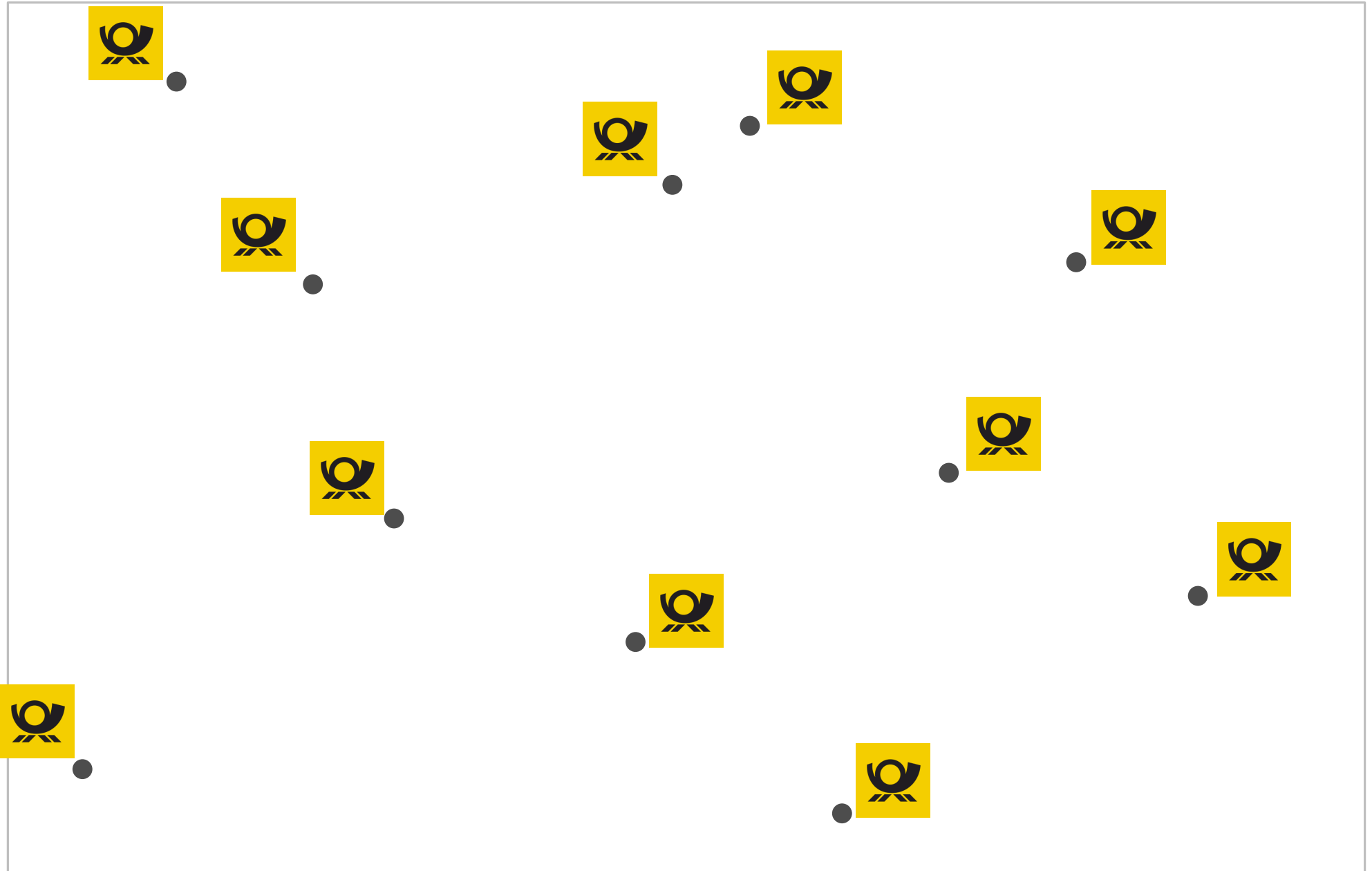
Voronoi-Diagrams

LEHRSTUHL FÜR ALGORITHMIK I · INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

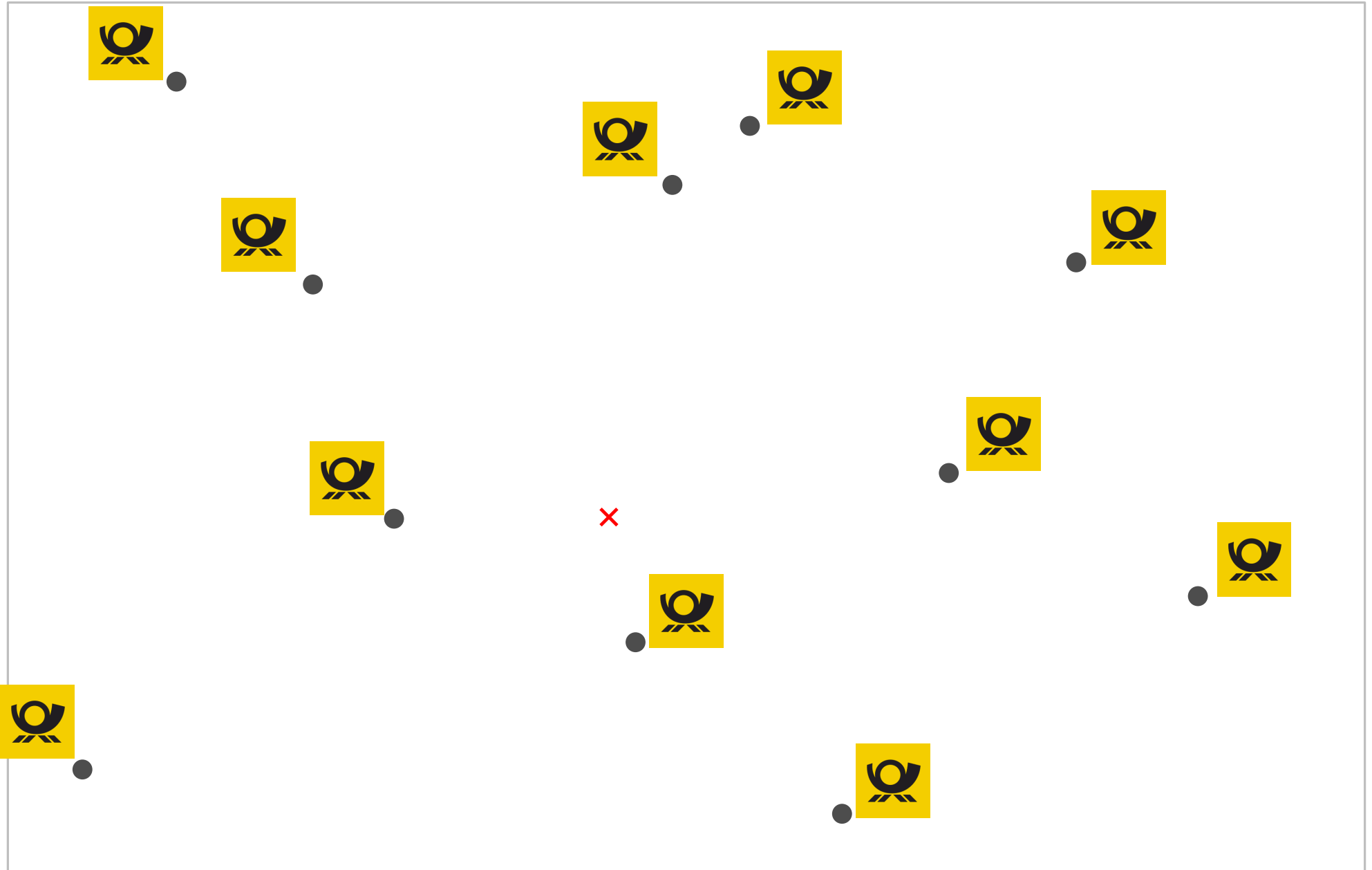
Benjamin Niedermann
09.12.2015



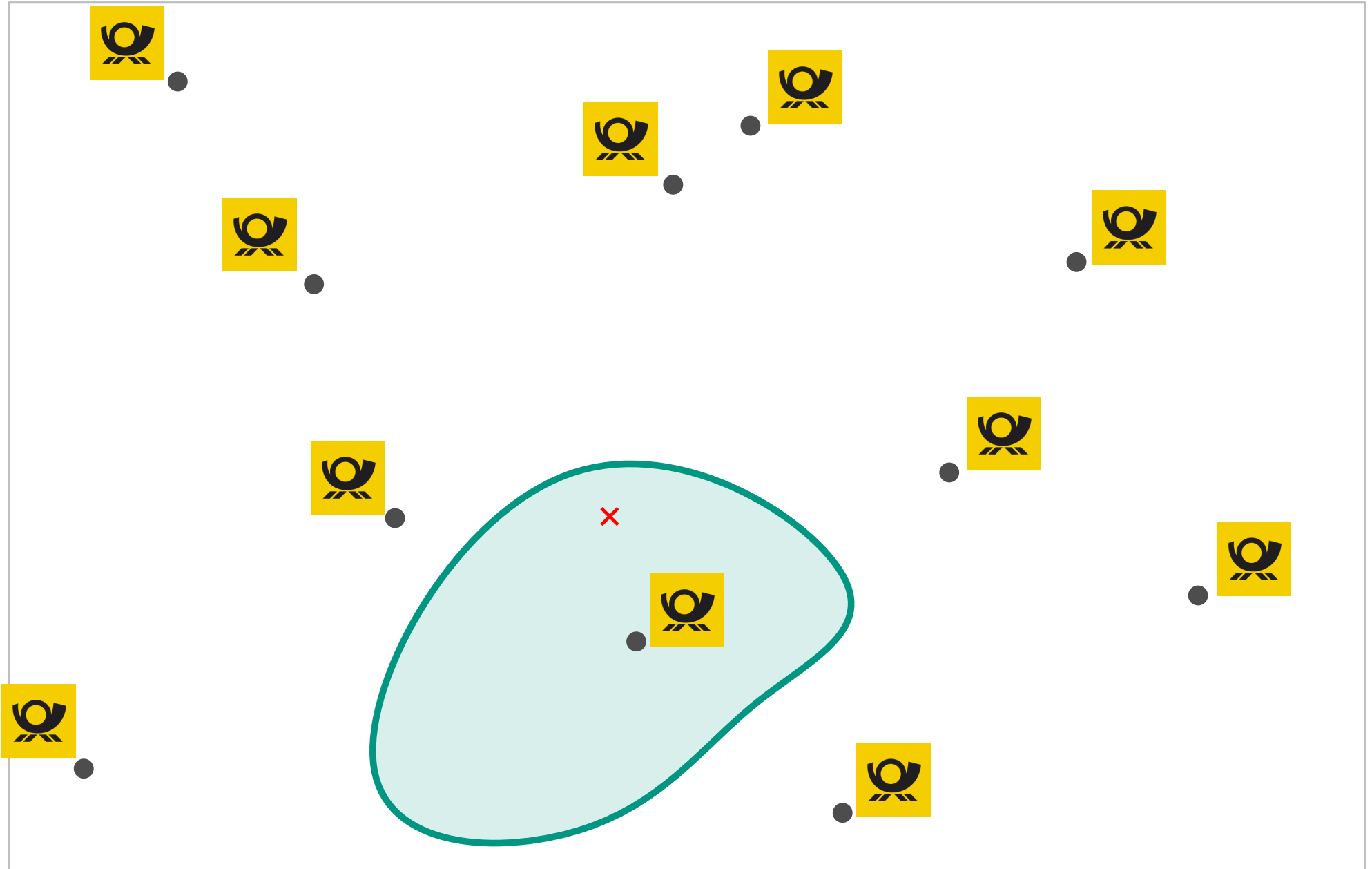
The Post Office Problem



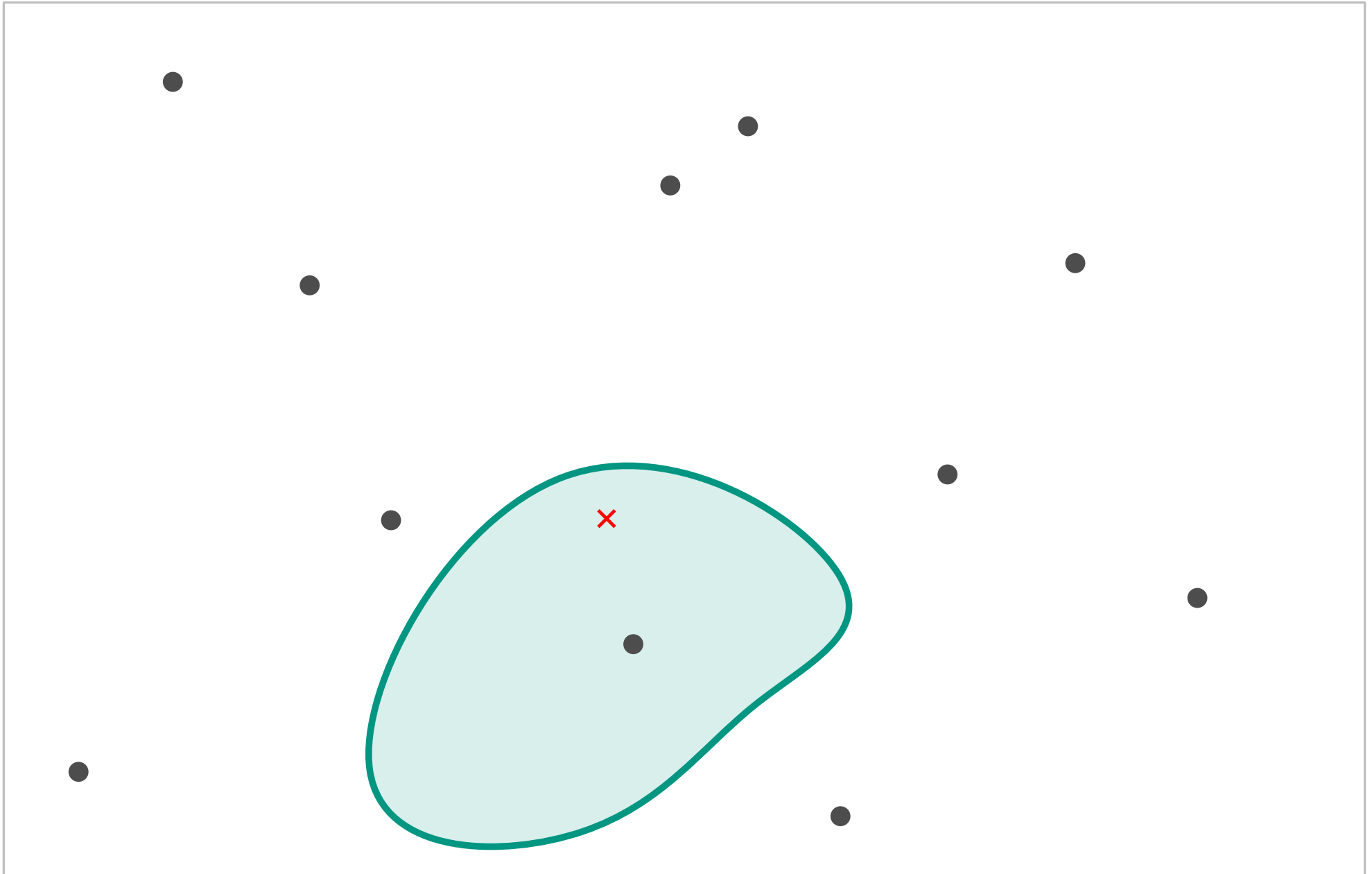
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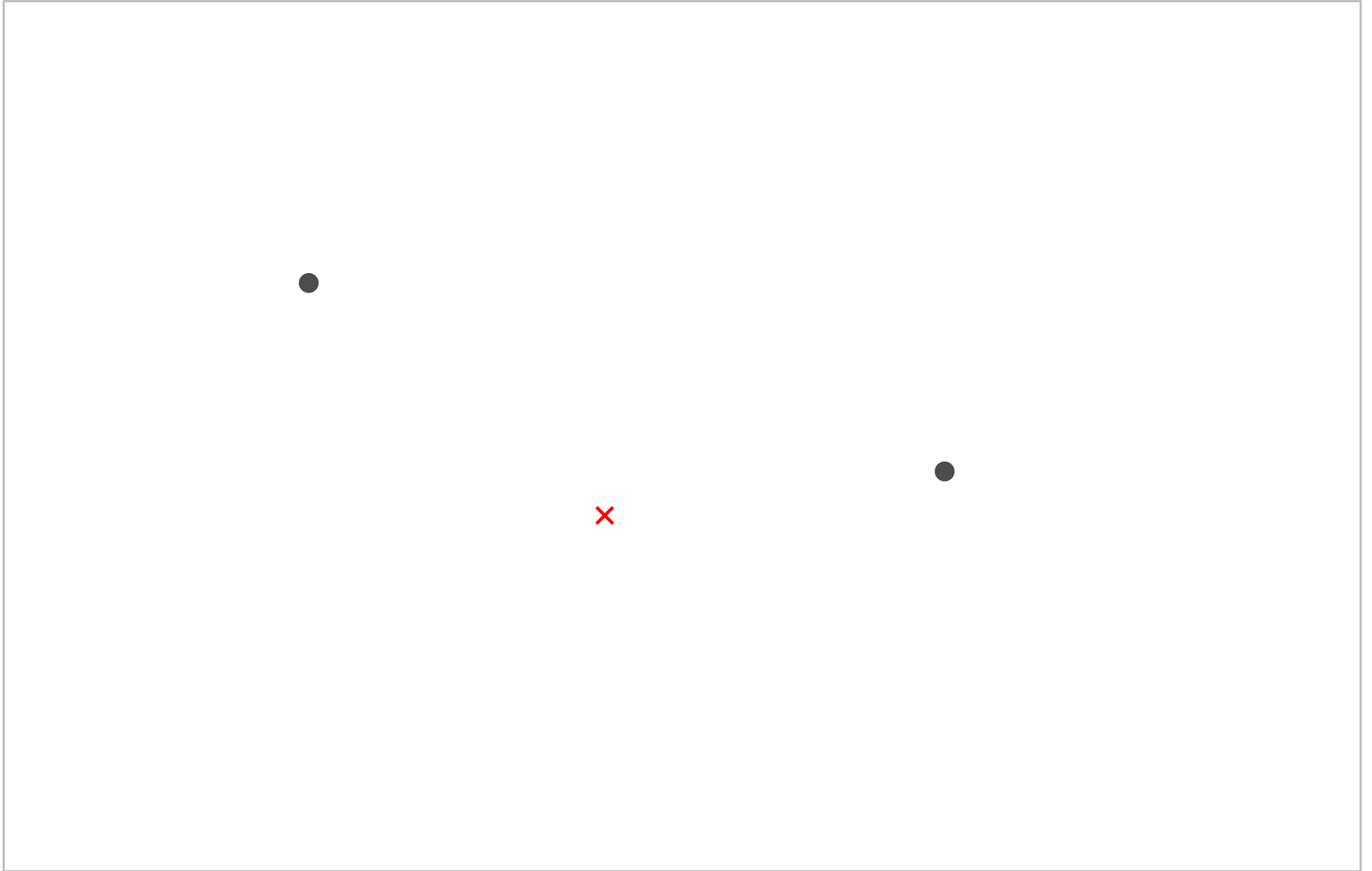
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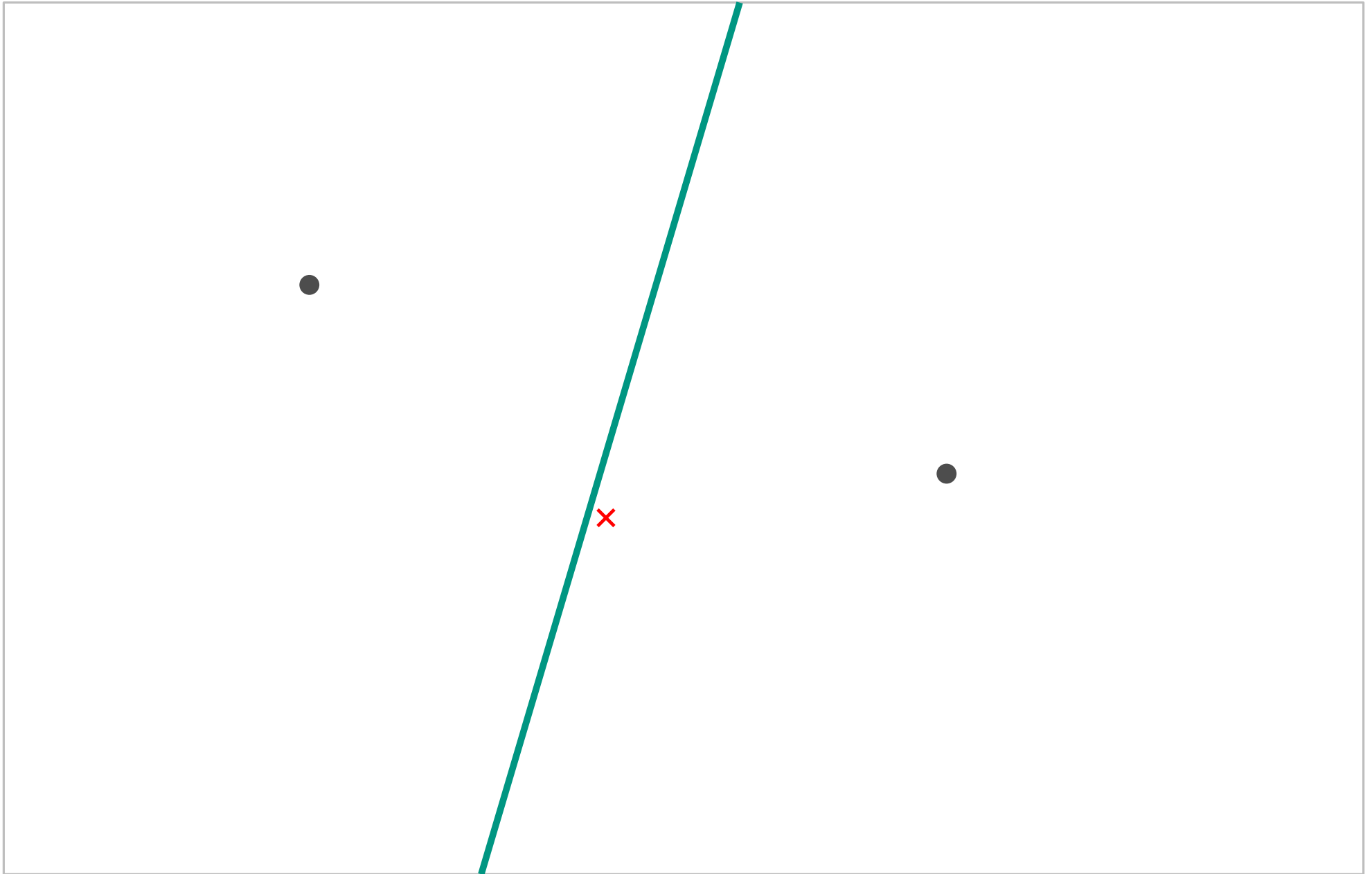
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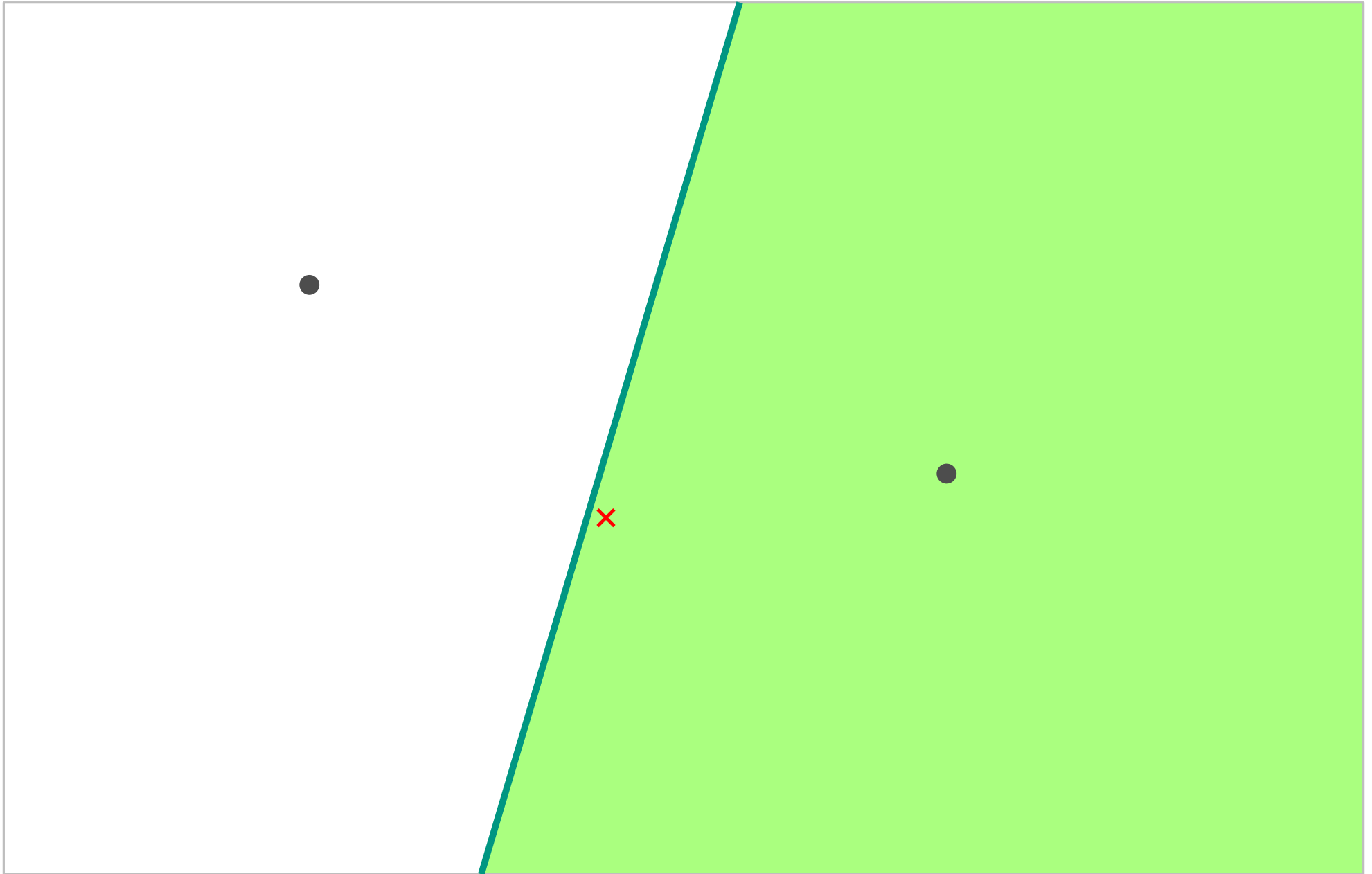
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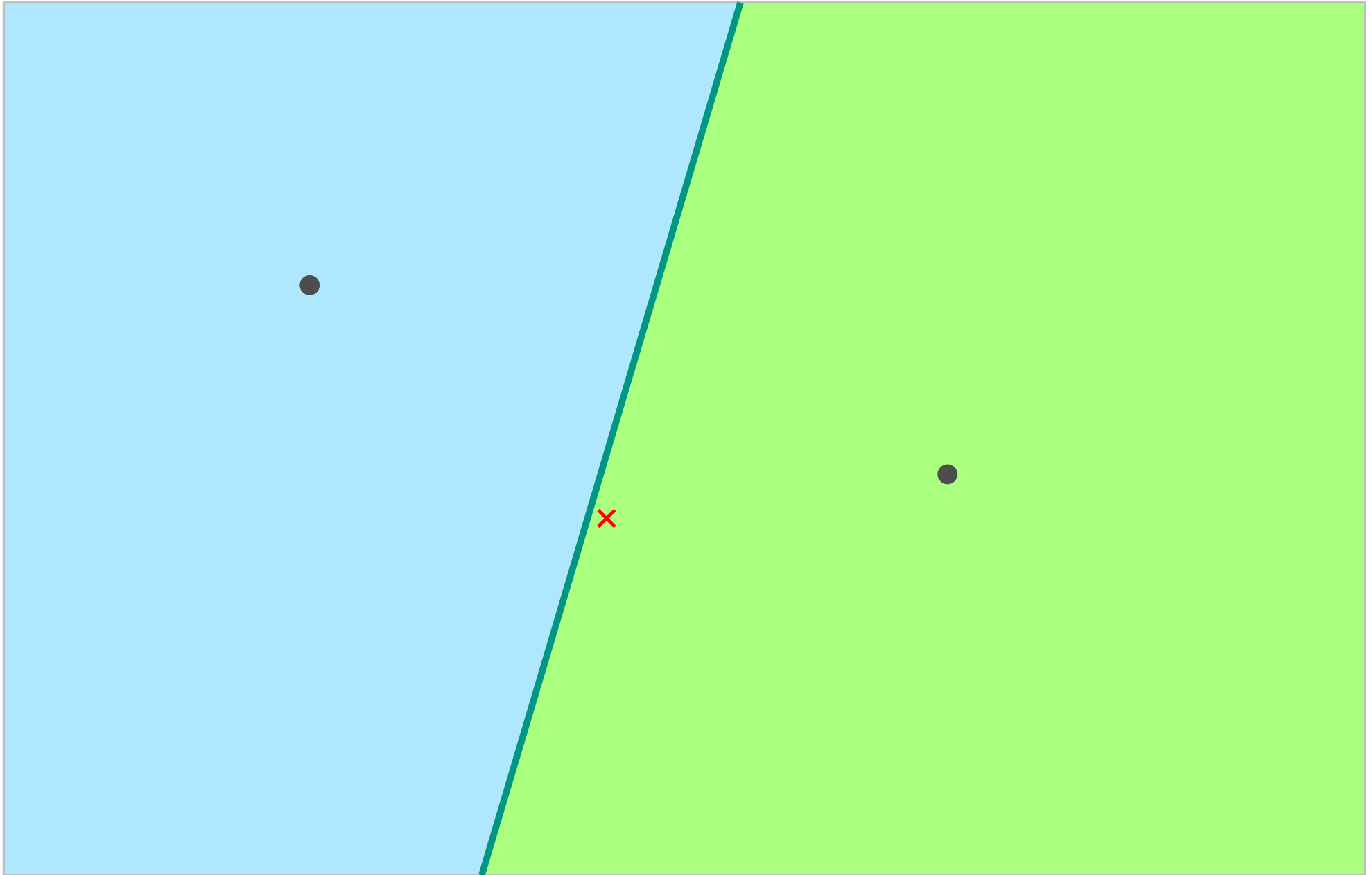
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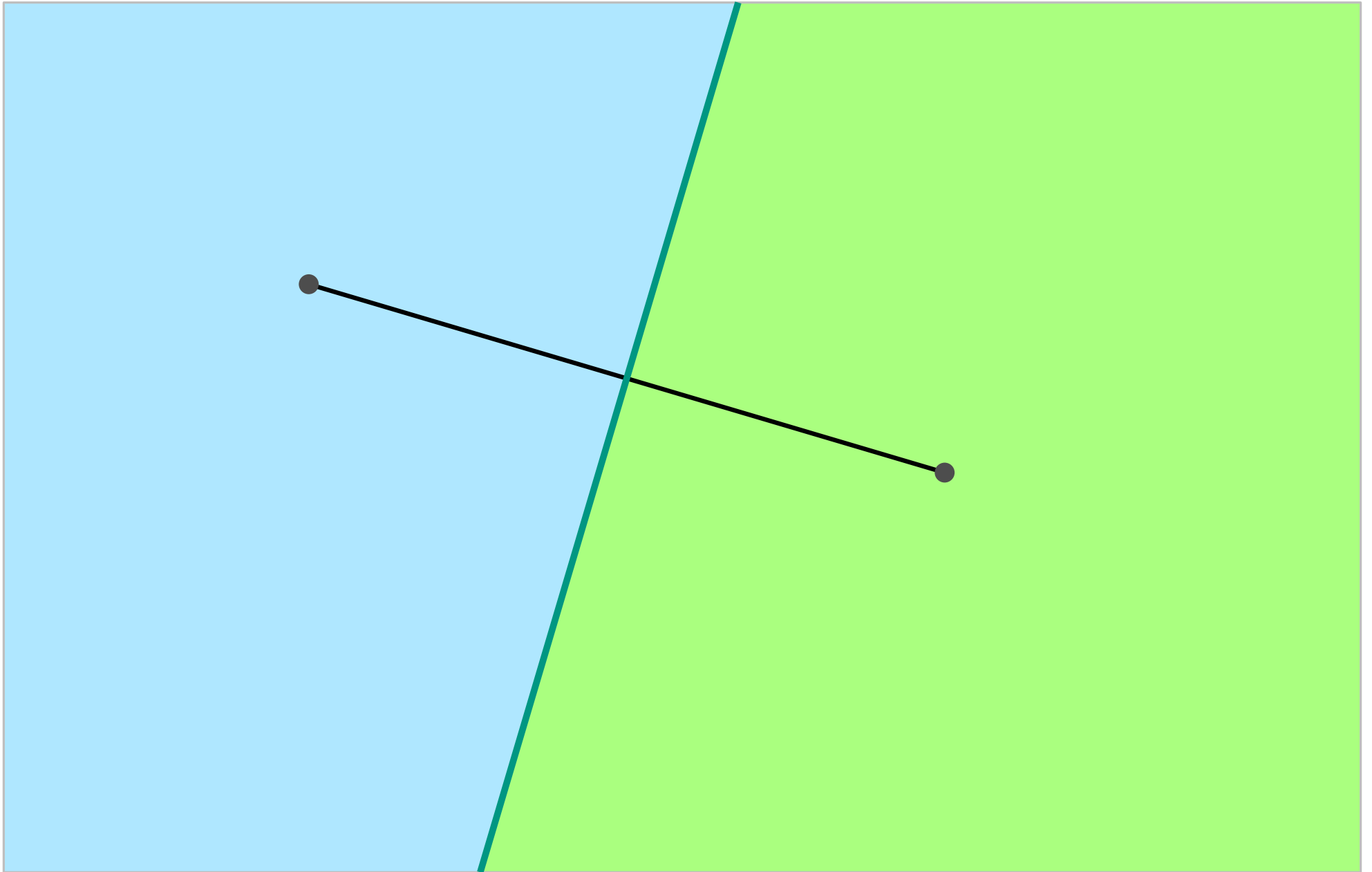
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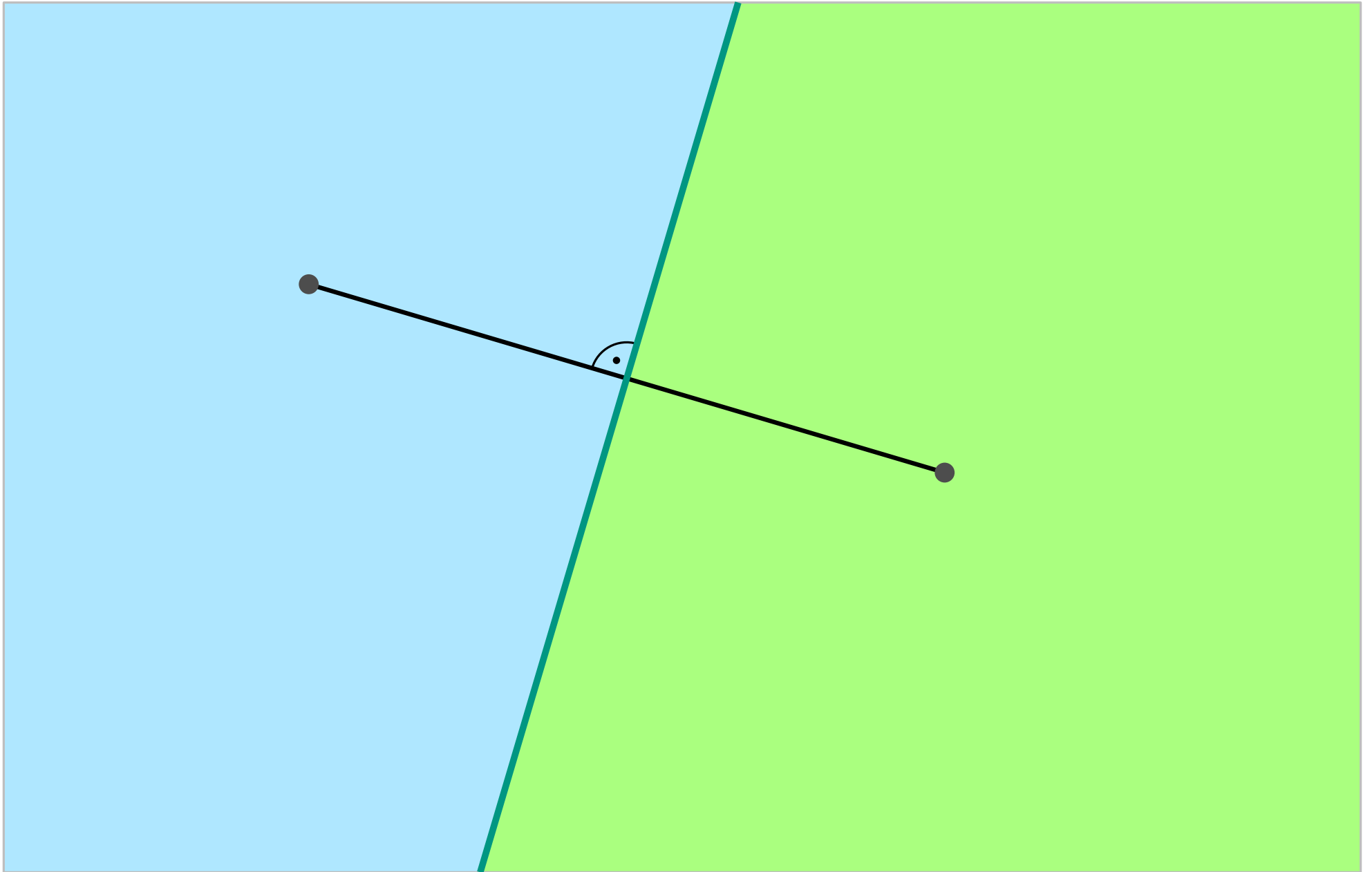
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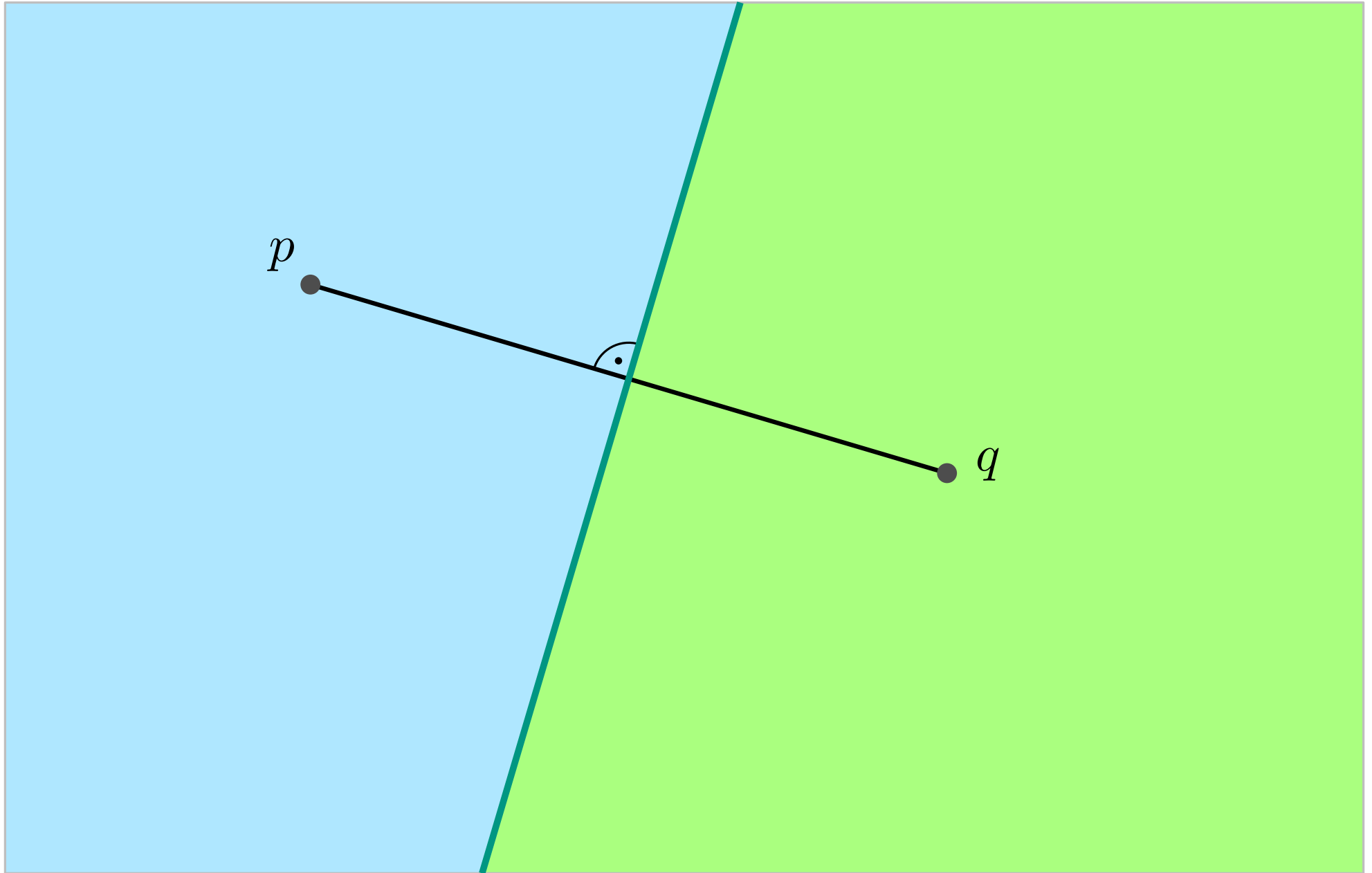
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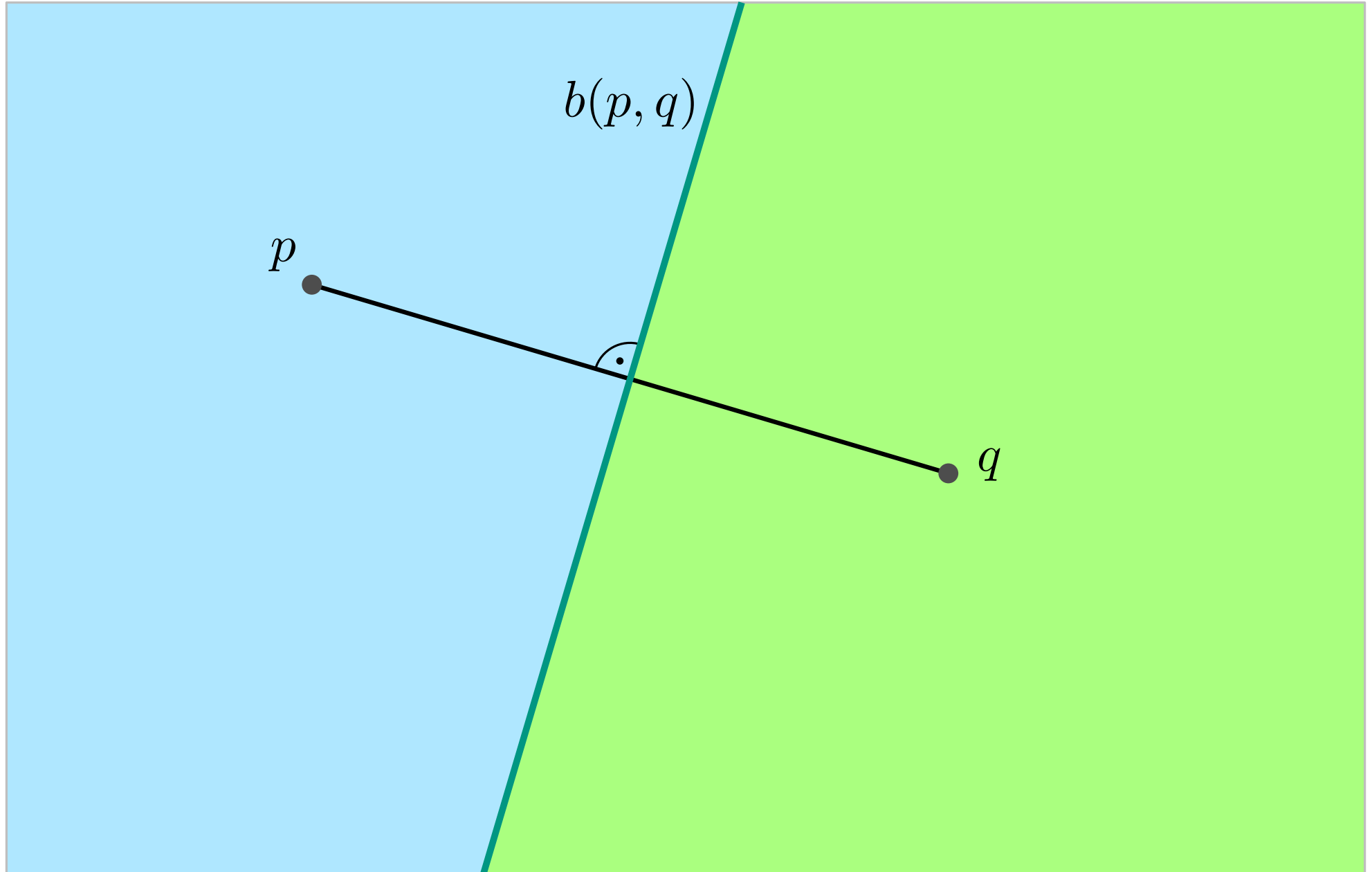
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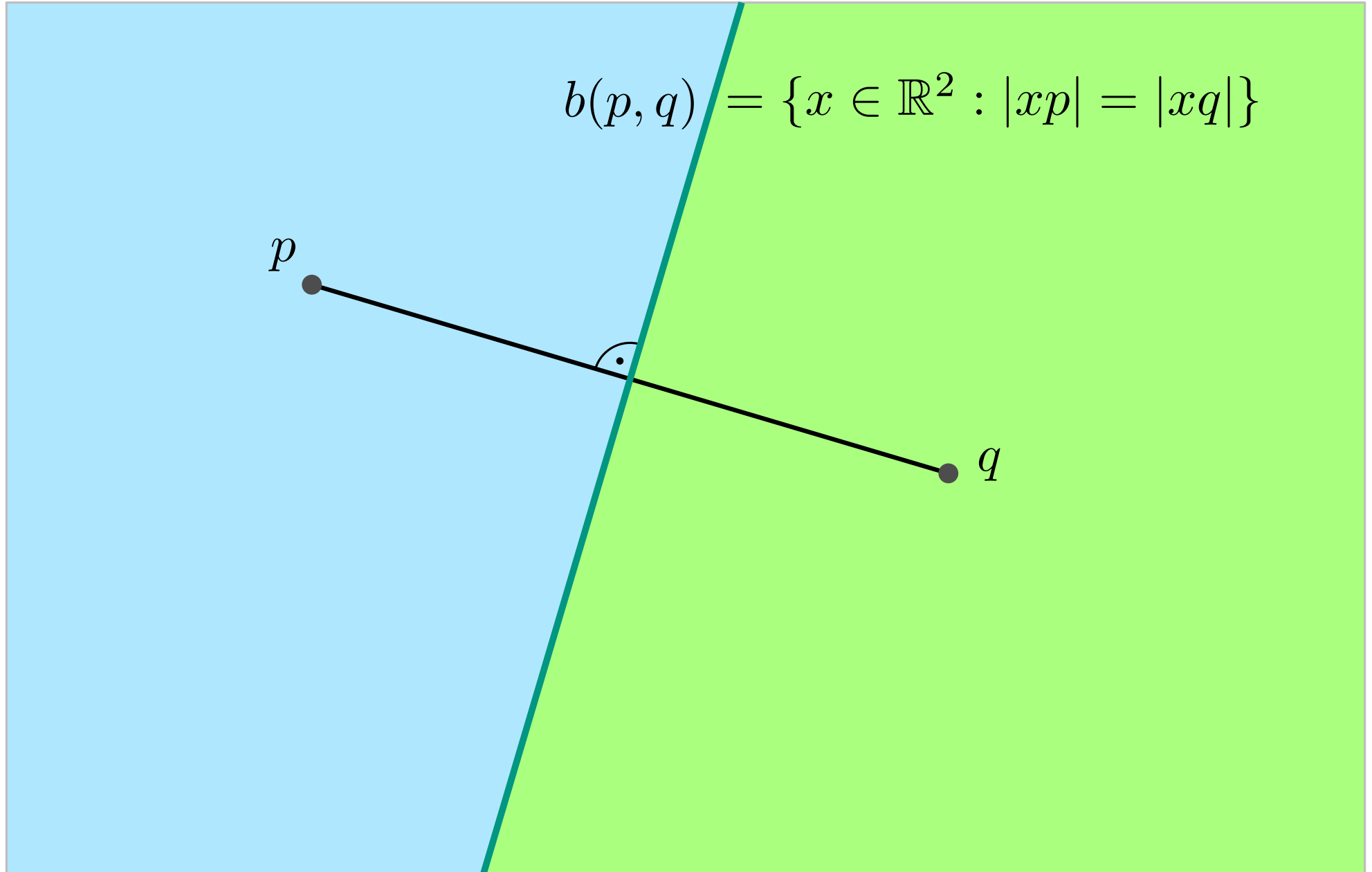
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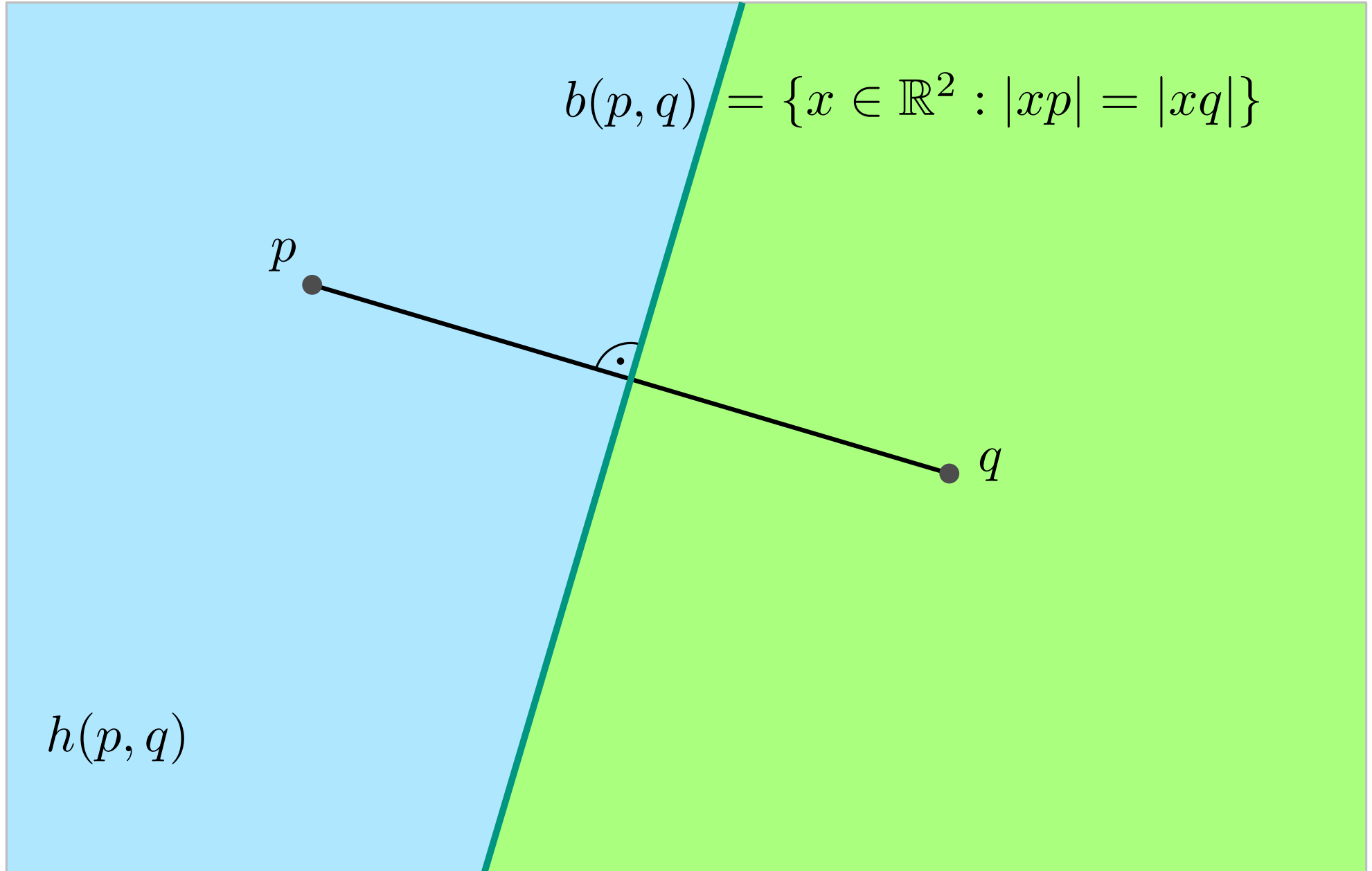
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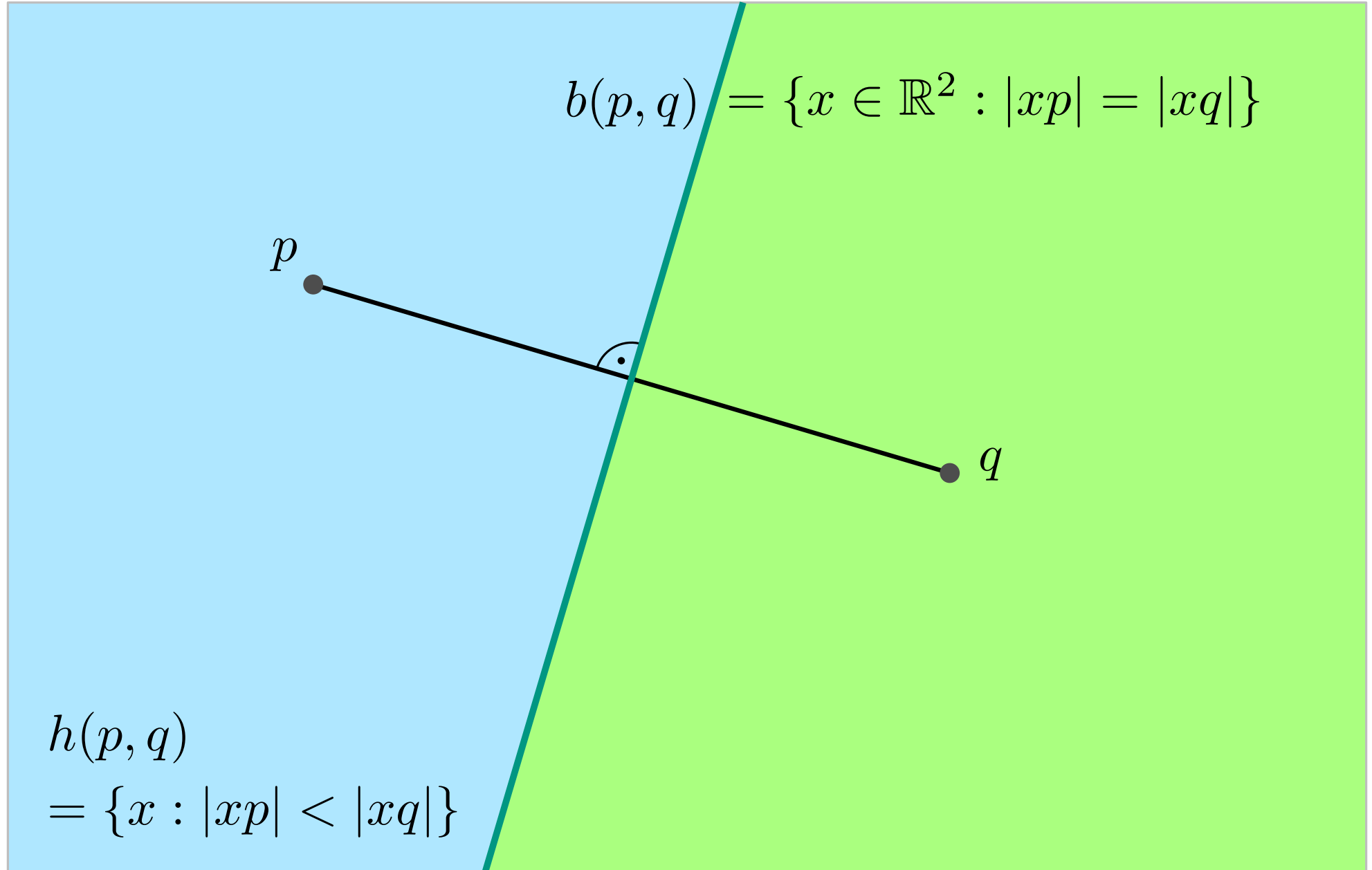
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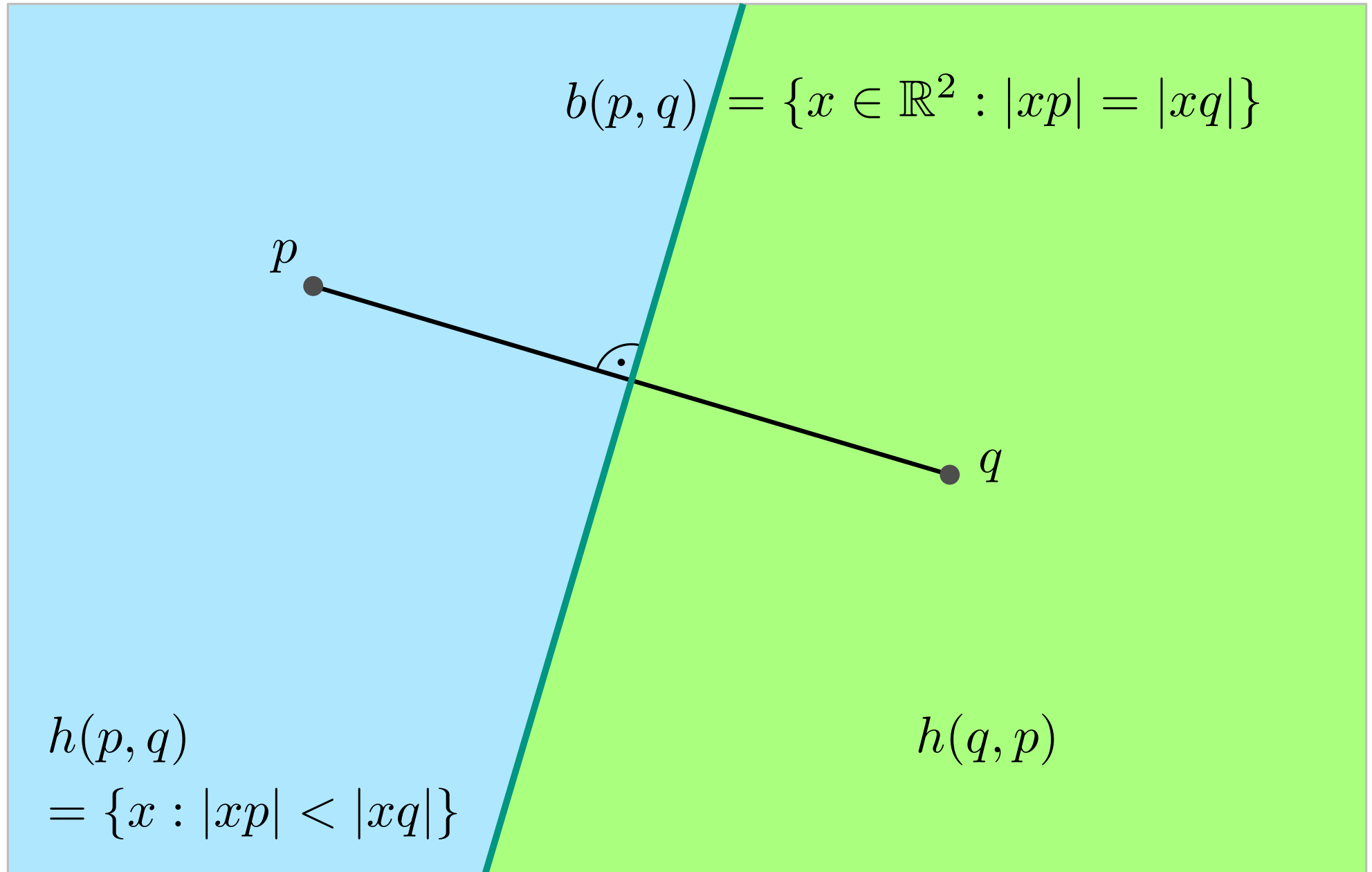
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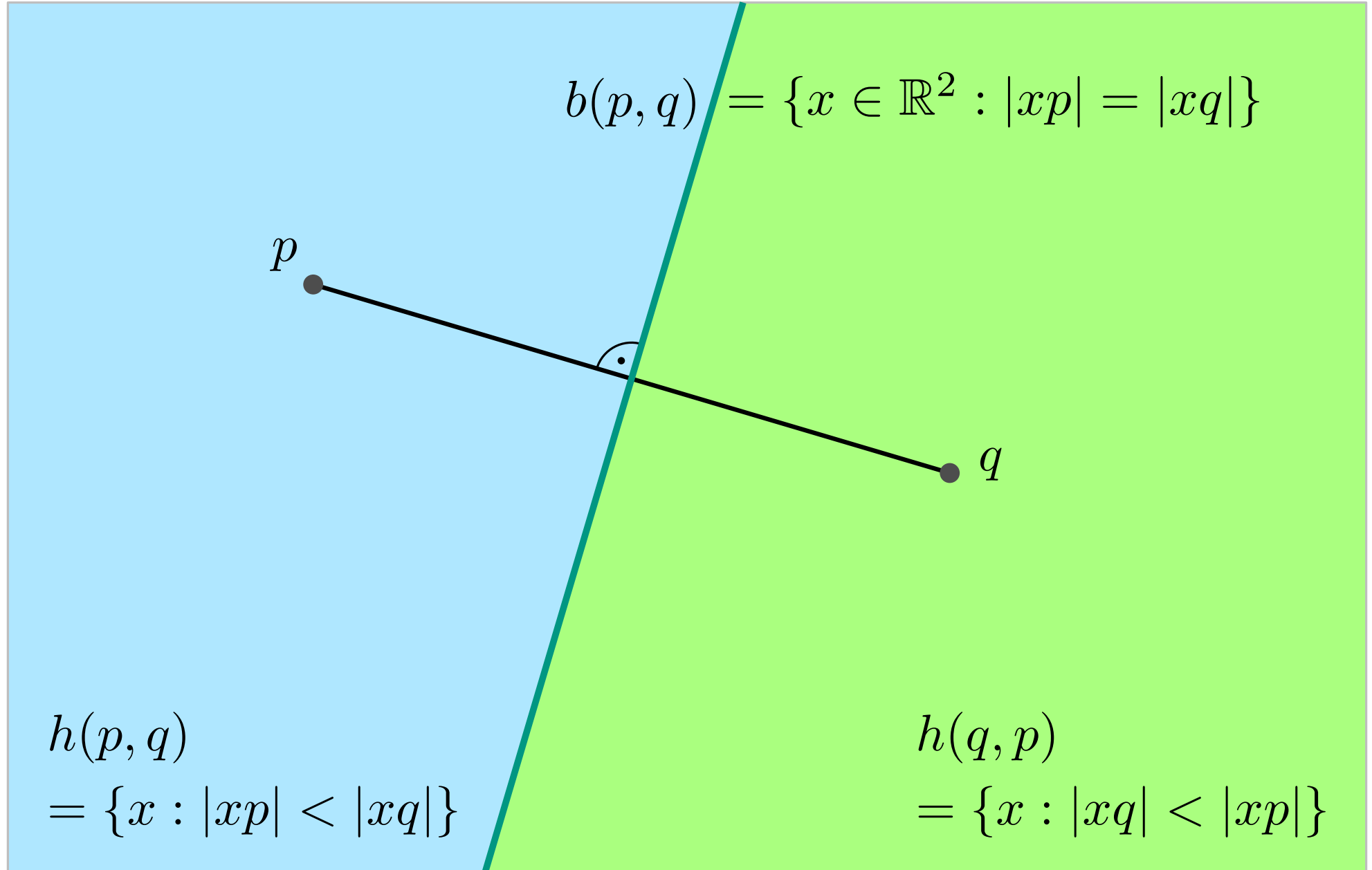
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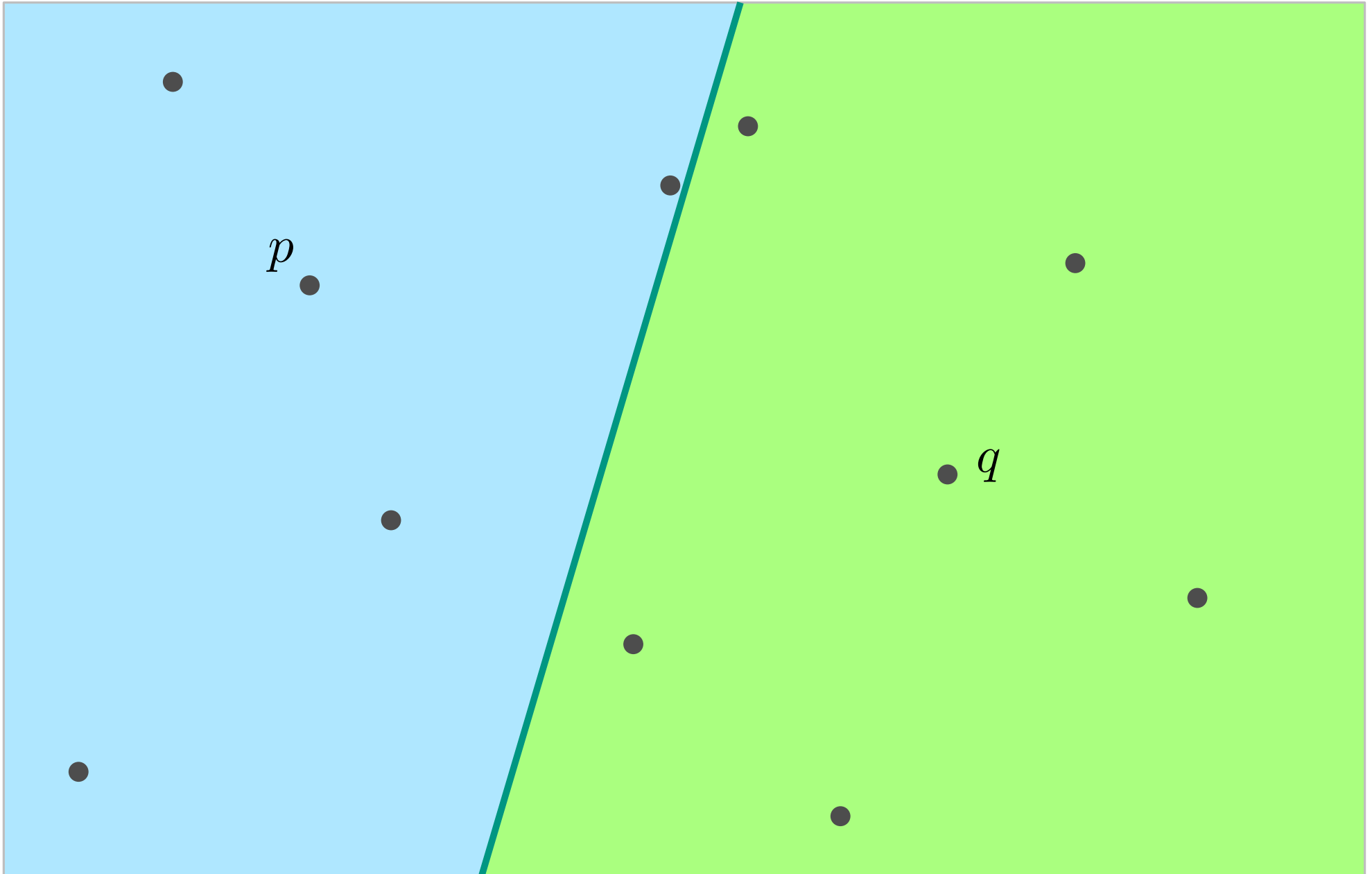
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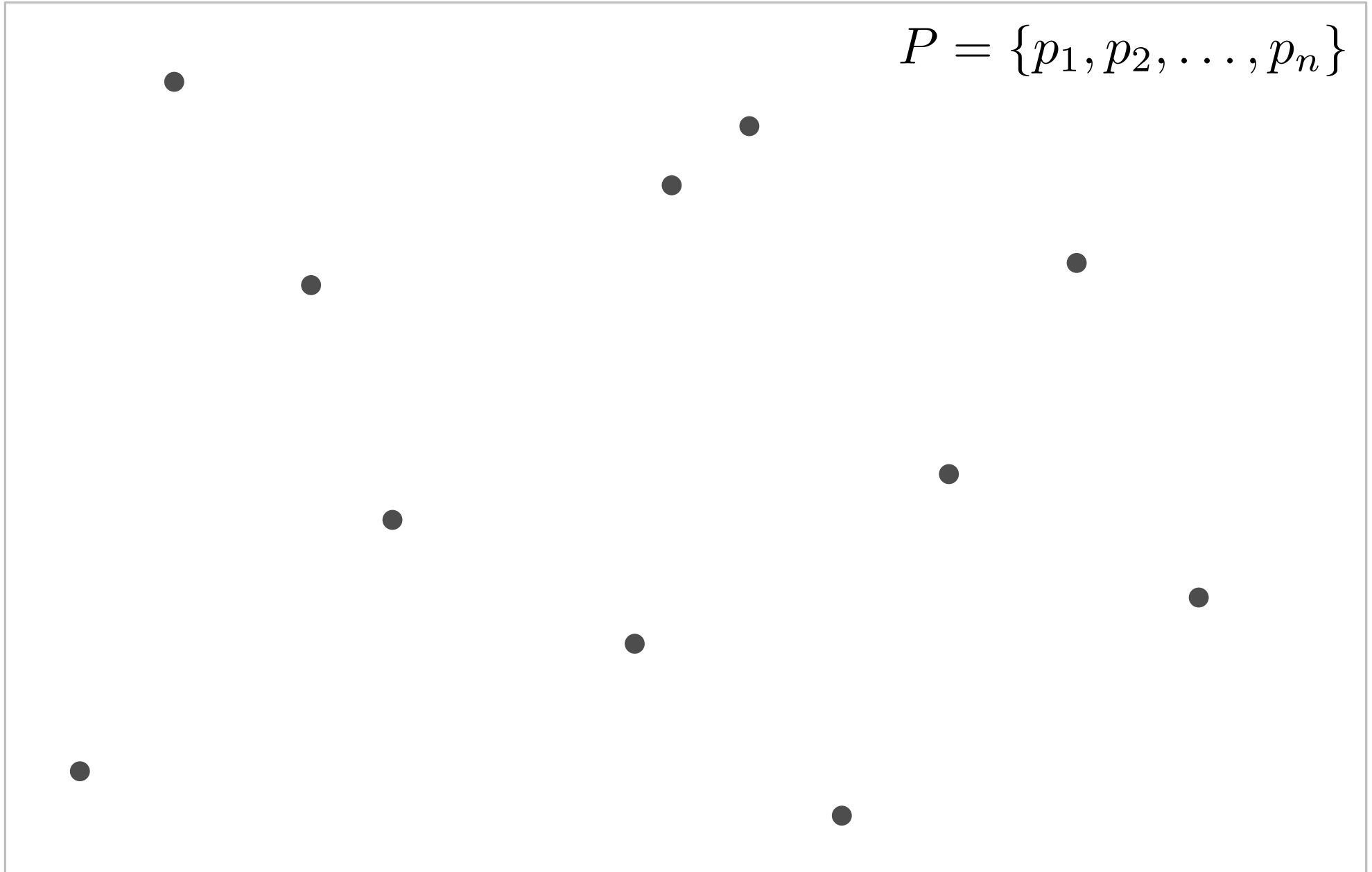
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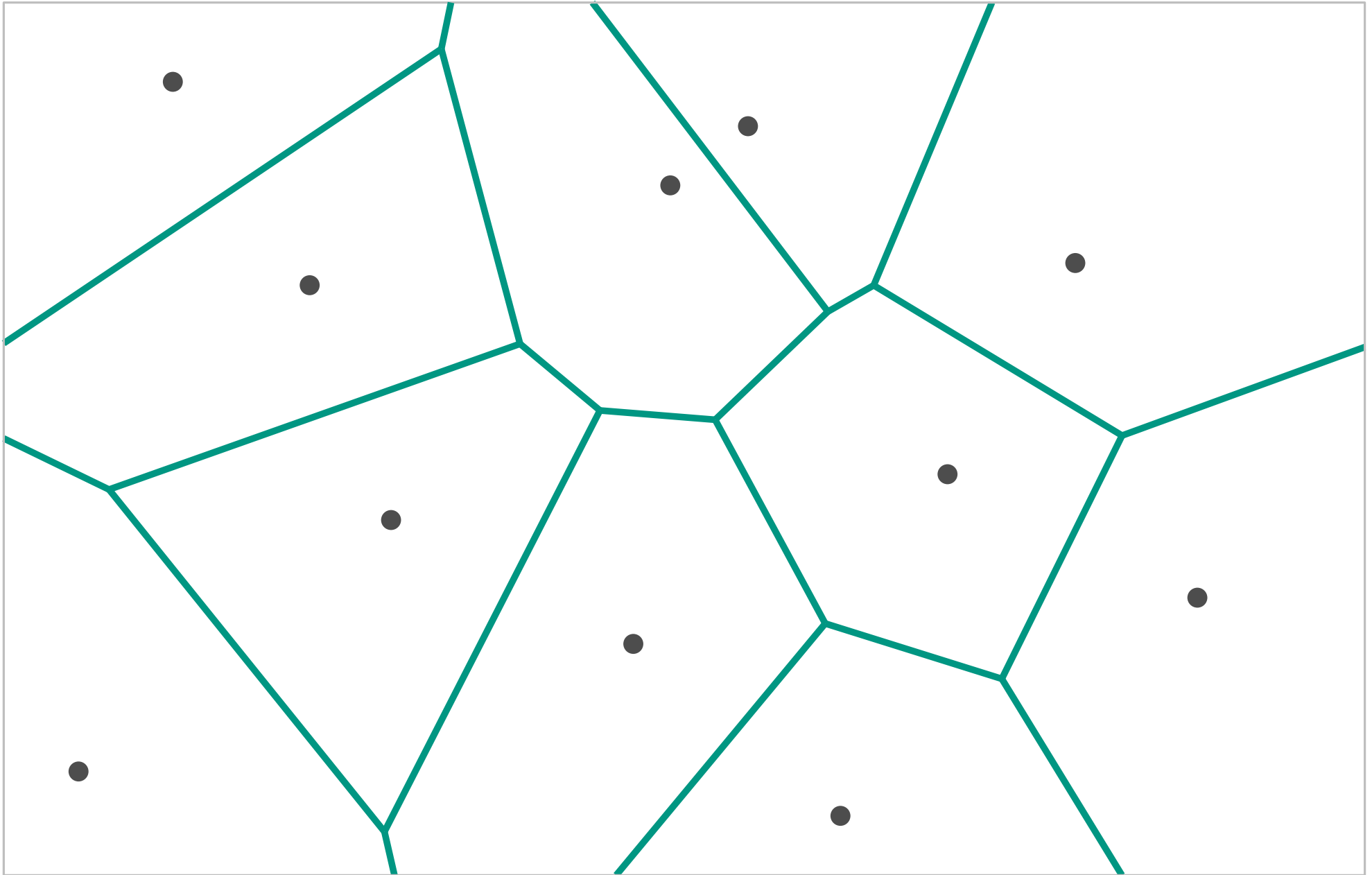
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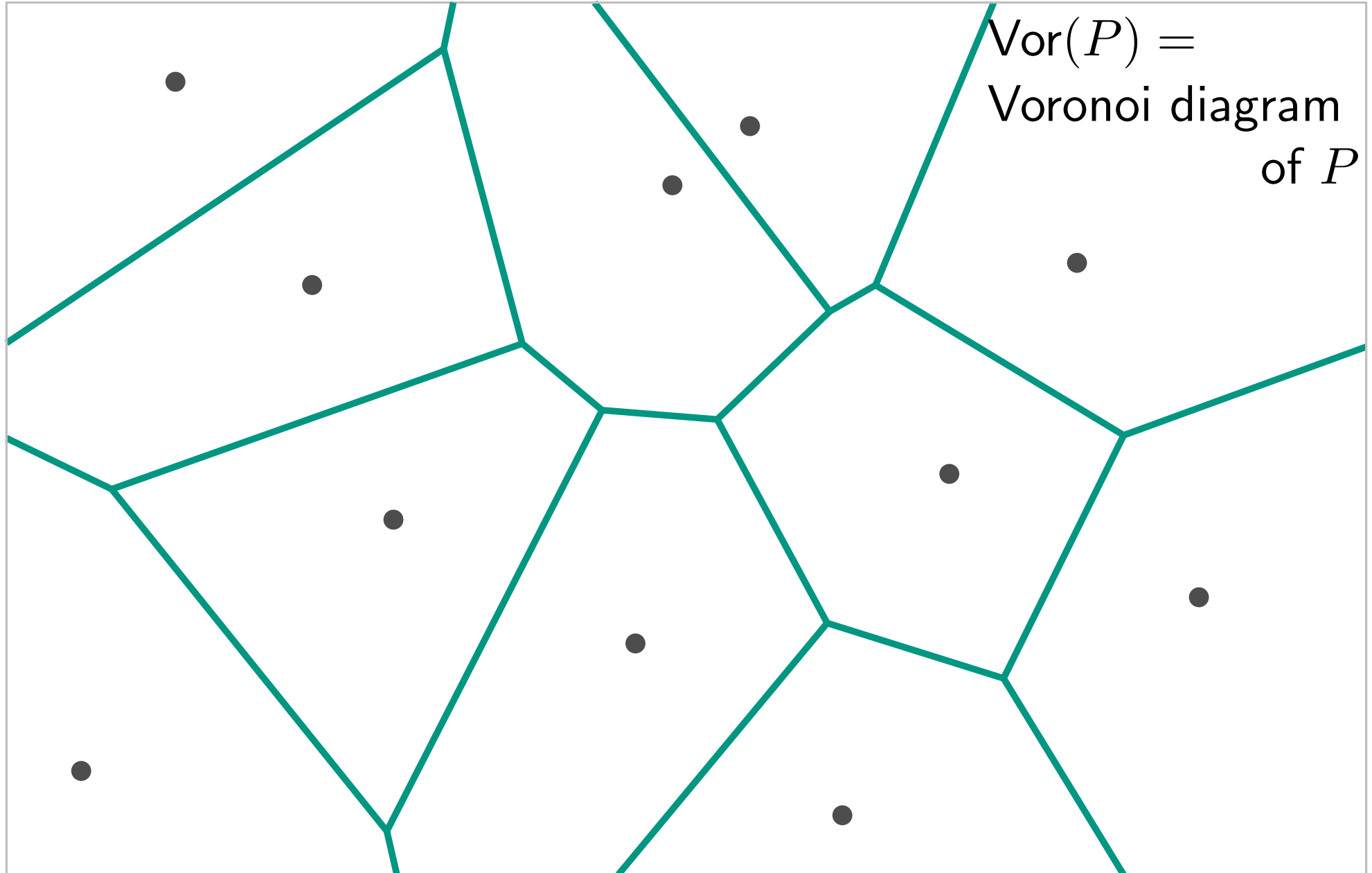
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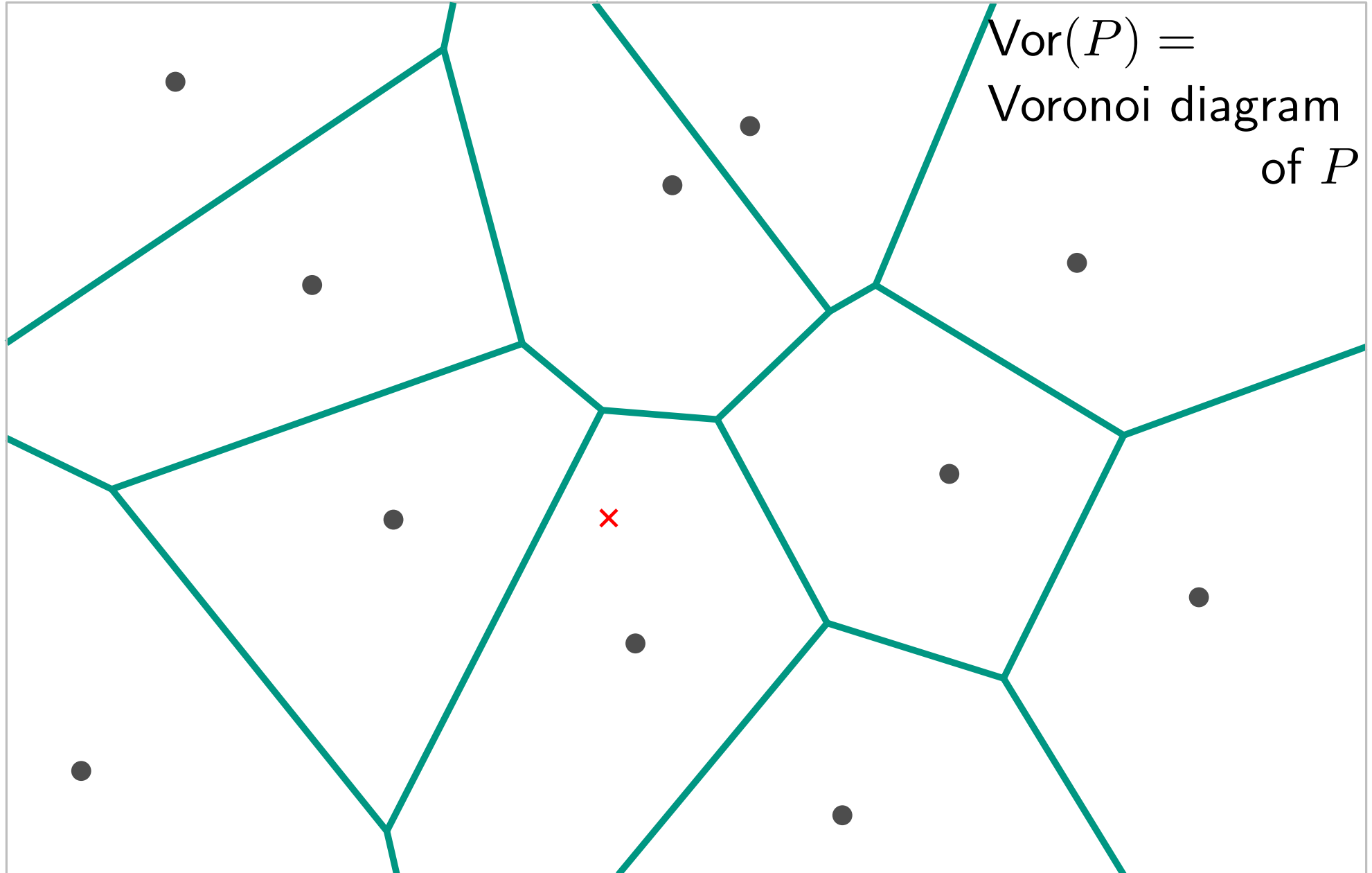
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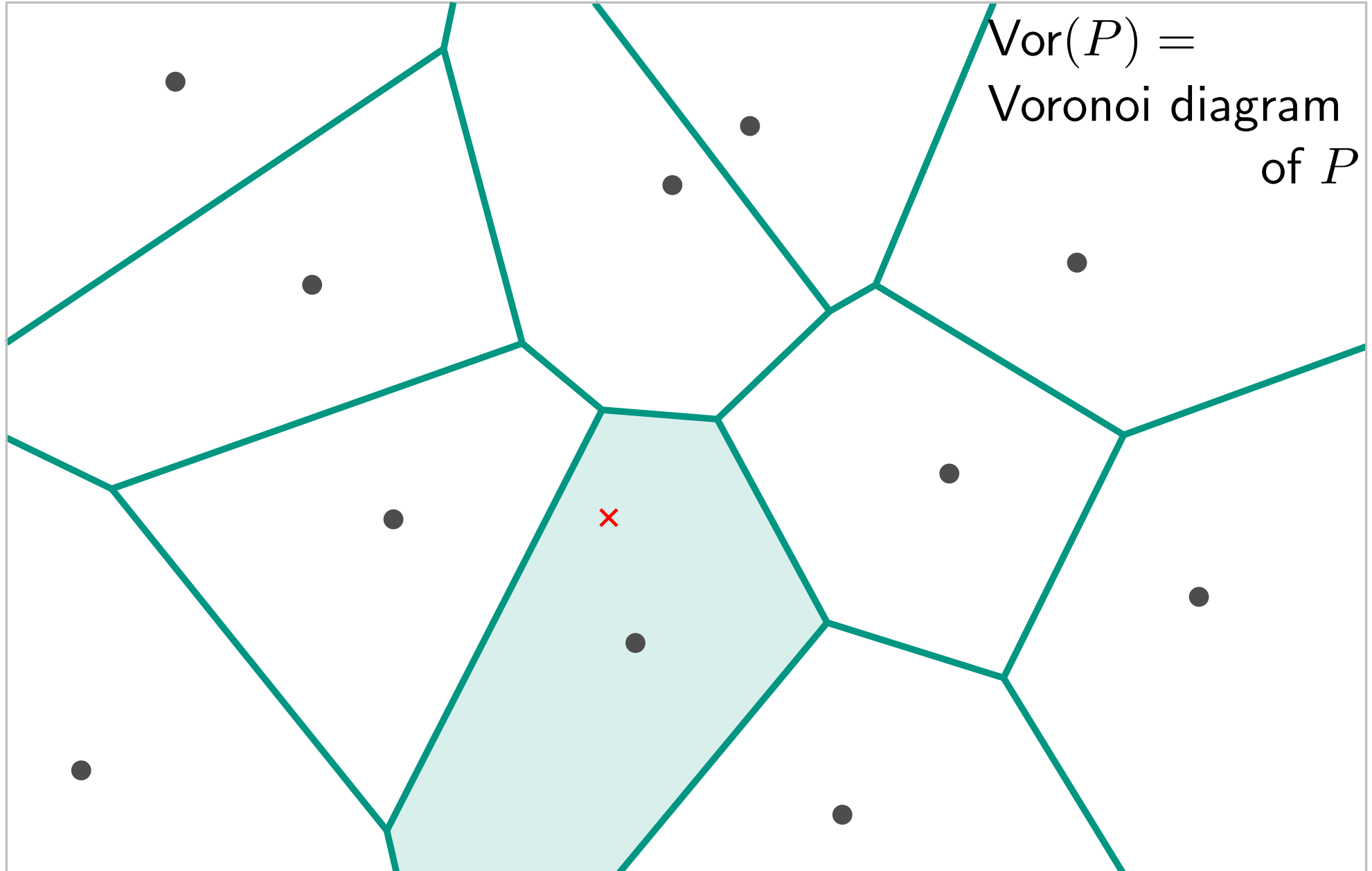
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The Post Office Problem

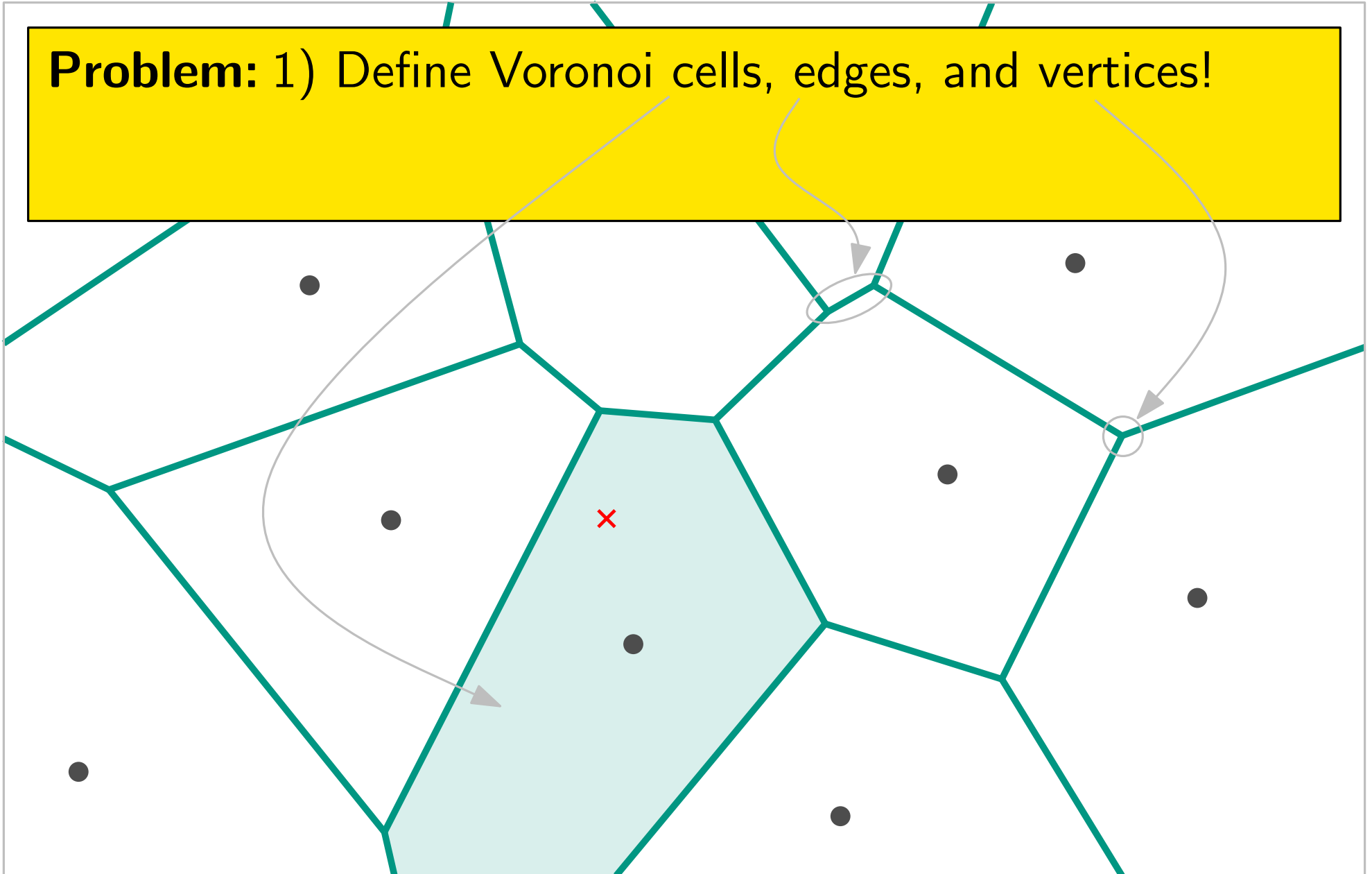


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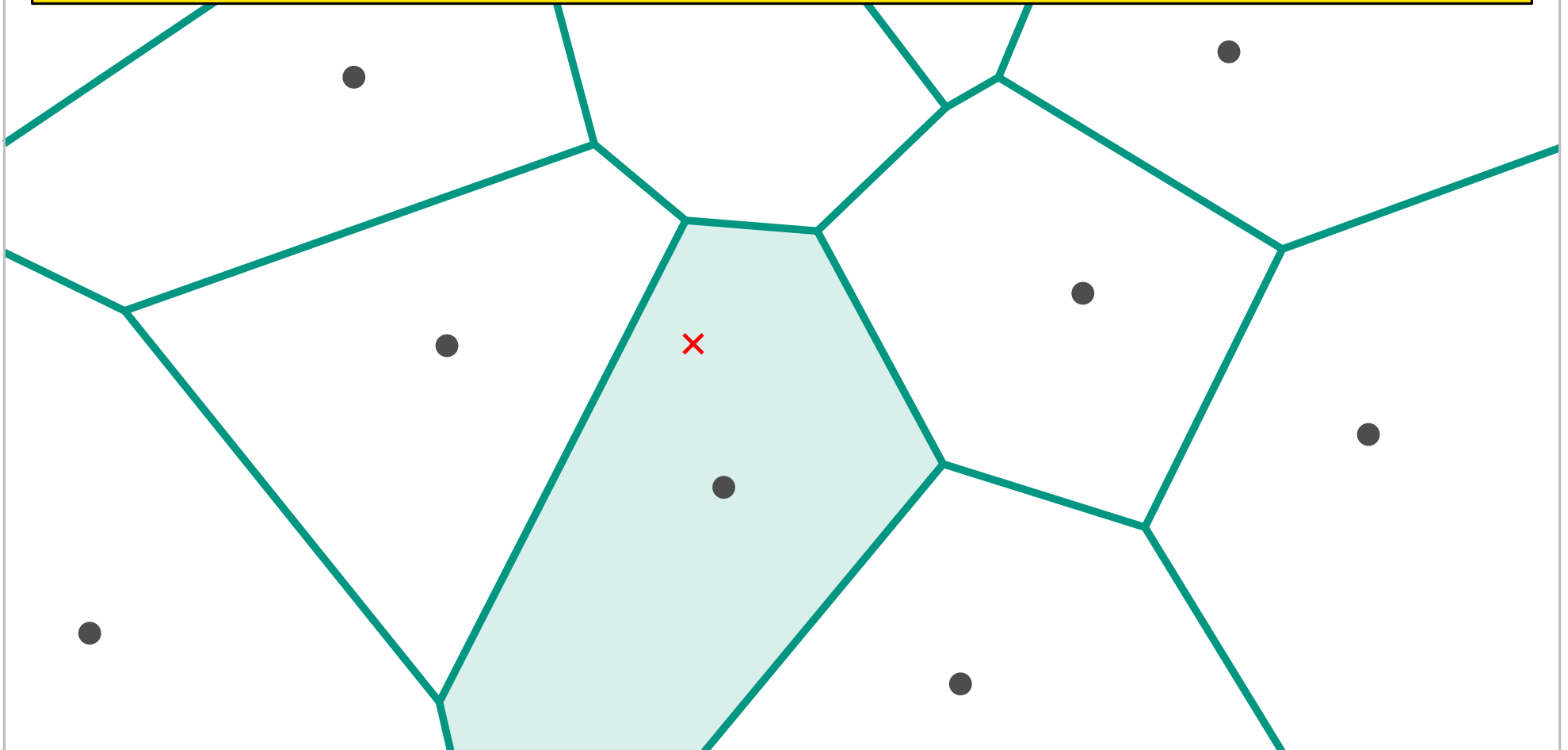
The Post Office Problem

Problem: 1) Define Voronoi cells, edges, and vertices!



The Post Office Problem

Problem: 1) Define Voronoi cells, edges, and vertices!
2) Are Voronoi cells convex?



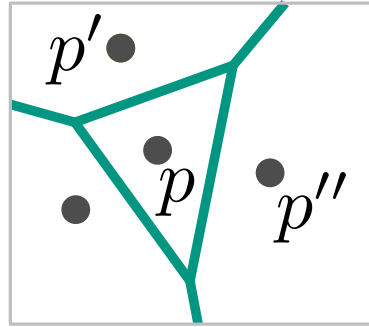
The Voronoi Diagram

Let P be a set of points in the plane and let $p, p', p'' \in P$.

The Voronoi Diagram

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Voronoi Diagram

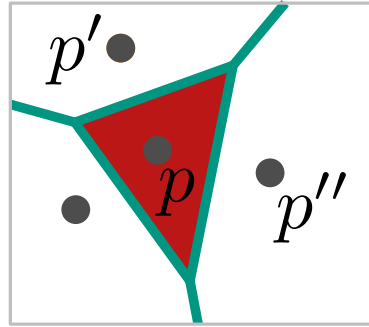


$\text{Vor}(P)$

The Voronoi Diagram

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Voronoi Diagram

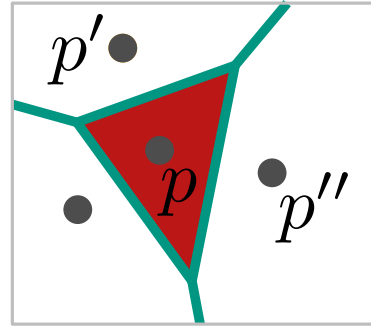


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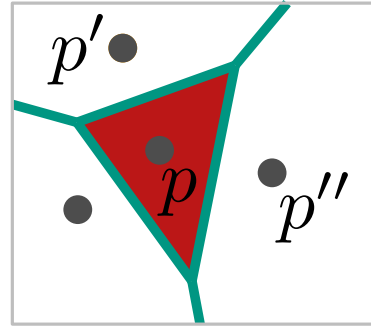
- Voronoi cell

$$\mathcal{V}(\{p\}) =$$

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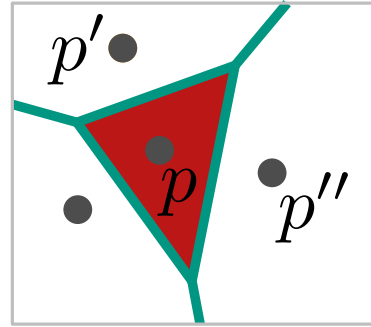
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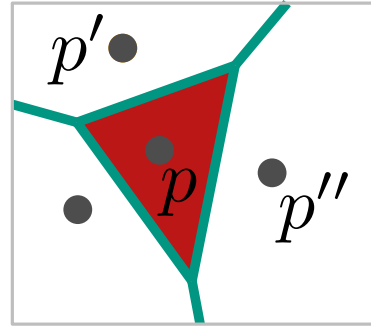
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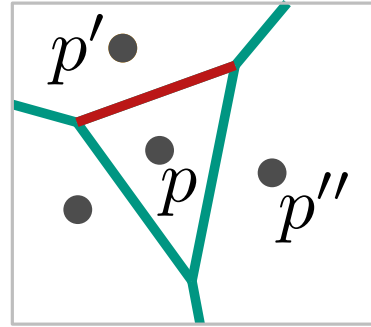
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$$\begin{aligned}\mathcal{V}(\{p\}) = \mathcal{V}(p) &= \{x \in \mathbb{R}^2 : |xp| < |xq| \forall q \in P \setminus \{p\}\} \\ &= \bigcap_{q \neq p} h(p, q)\end{aligned}$$

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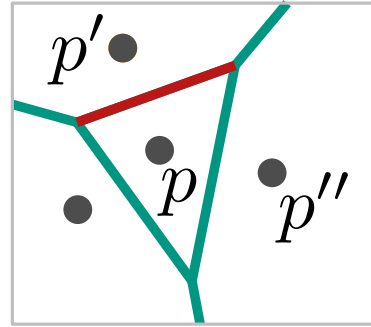
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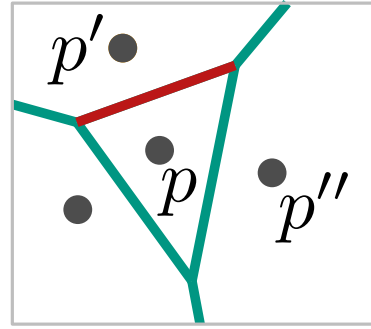
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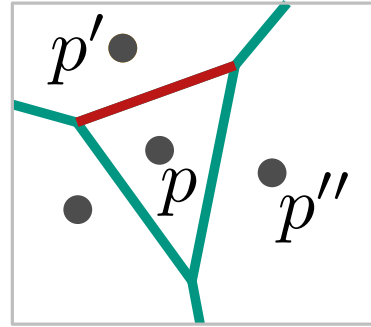
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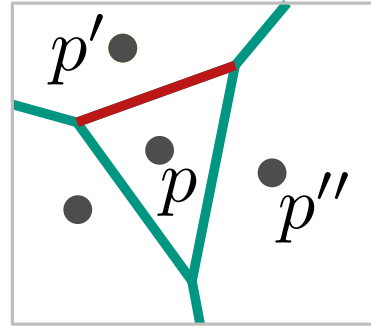
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$$\begin{aligned}\mathcal{V}(\{p, p'\}) &= \{x : |xp| = |xp'| \text{ and } |xp| < |xq| \forall q \neq p, p'\} \\ &= \partial\mathcal{V}(p) \cap \partial\mathcal{V}(p')\end{aligned}$$

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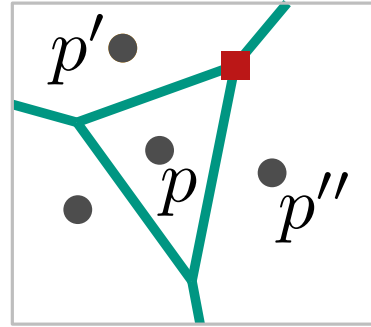
- Voronoi edges

$$\begin{aligned}\mathcal{V}(\{p, p'\}) &= \{x : |xp| = |xp'| \text{ and } |xp| < |xq| \forall q \neq p, p'\} \\ &= \text{rel-int}(\partial\mathcal{V}(p) \cap \partial\mathcal{V}(p')), \text{ d.h. without endpoints}\end{aligned}$$

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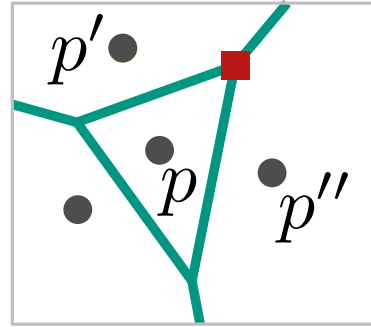
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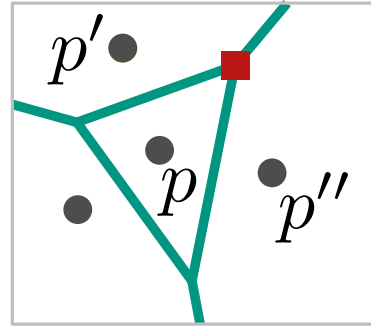
- Voronoi vertices

$$\mathcal{V}(\{p, p', p''\})$$

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Voronoi Diagram



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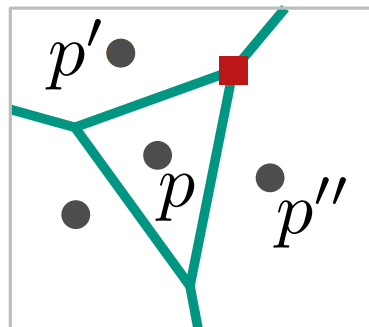
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$$\mathcal{V}(\{p, p', p''\}) = \partial\mathcal{V}(p) \cap \partial\mathcal{V}(p') \cap \partial\mathcal{V}(p'')$$

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Voronoi Diagram



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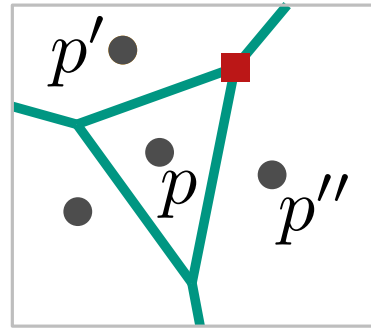
- Voronoi vertices

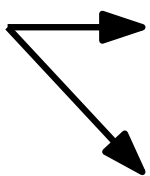
$$\mathcal{V}(\{p, p', p''\}) = \partial\mathcal{V}(p) \cap \partial\mathcal{V}(p') \cap \partial\mathcal{V}(p'')$$

The Voronoi Diagram

Let P be a set of points in the plane and let $p, p', p'' \in P$.

Voronoi Diagram



$\text{Vor}(P)$  subdivision

- Voronoi cell

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- Voronoi edges

$$\begin{aligned}\mathcal{V}(\{p, p'\}) &= \{x : |xp| = |xp'| \text{ and } |xp| < |xq| \forall q \neq p, p'\} \\ &= \text{rel-int}(\partial\mathcal{V}(p) \cap \partial\mathcal{V}(p')), \text{ d.h. without endpoints}\end{aligned}$$

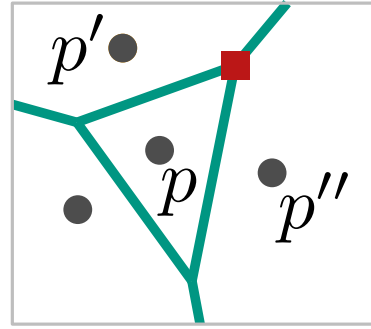
- Voronoi vertices

$$\mathcal{V}(\{p, p', p''\}) = \partial\mathcal{V}(p) \cap \partial\mathcal{V}(p') \cap \partial\mathcal{V}(p'')$$

The Voronoi Diagram

Let P be a set of points in the plane and let $p, p', p'' \in P$.

Voronoi Diagram



$\text{Vor}(P)$ $\begin{cases} \rightarrow \text{subdivision} \\ \rightarrow \text{geometric graph} \end{cases}$

- Voronoi cell

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Properties

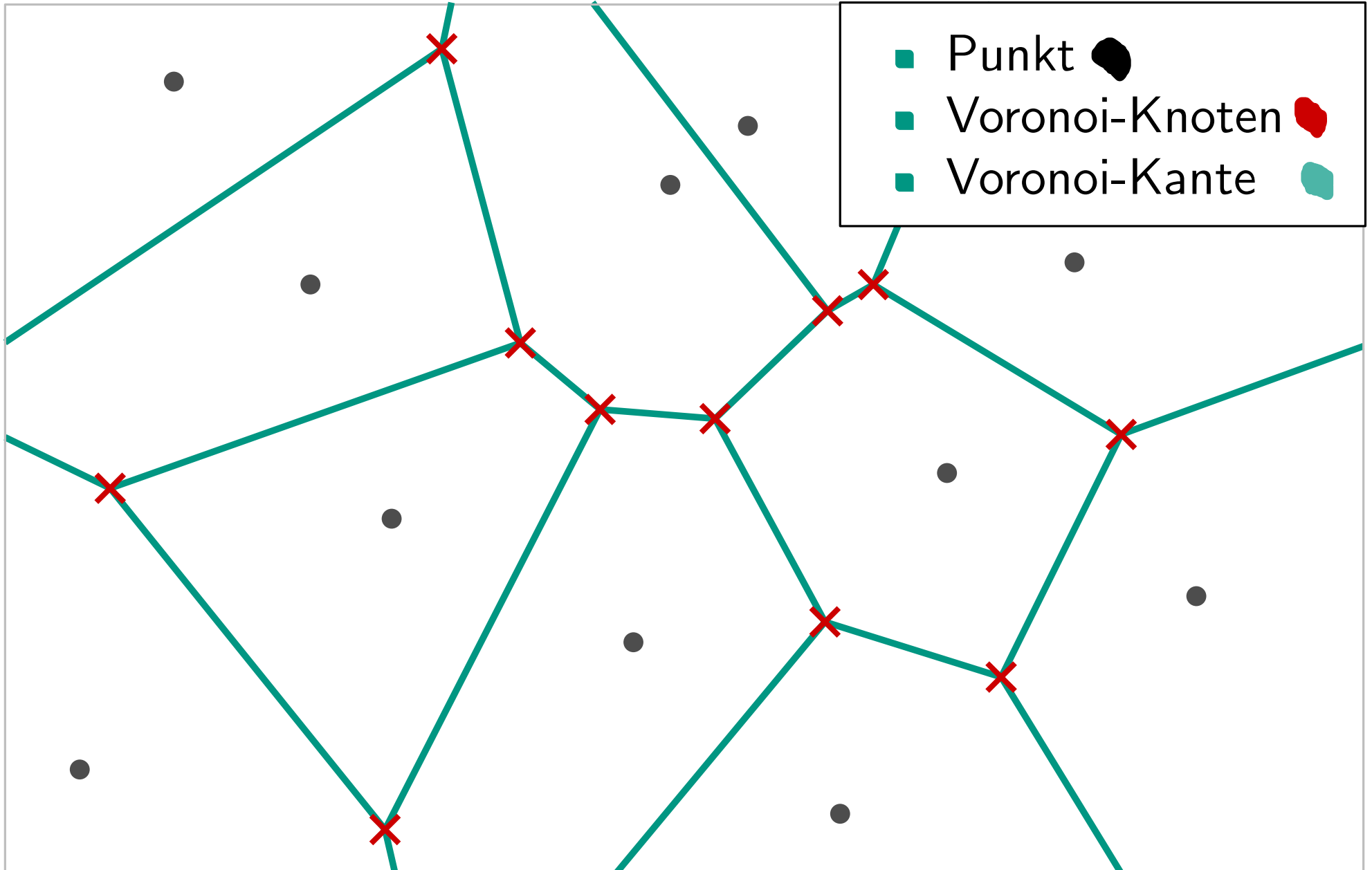
Theorem 1: Let $P \subset \mathbb{R}^2$ be a set of n points. If all points are collinear, then $\text{Vor}(P)$ consists of $n - 1$ parallel lines. Otherwise $\text{Vor}(P)$ is connected and its edges are either segments or half lines.

Find a set P so that a cell in $\text{Vor}(P)$ has linear complexity.

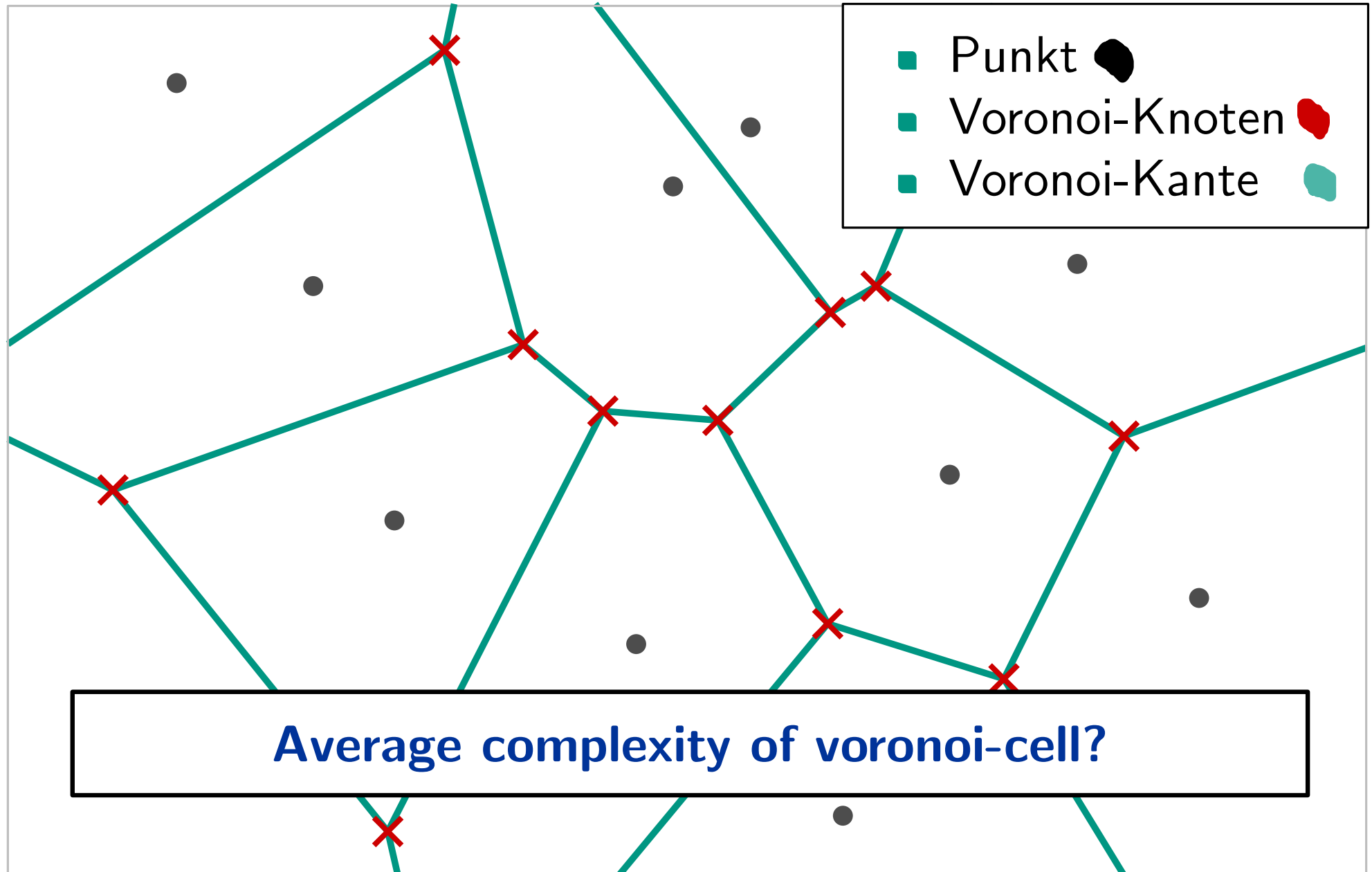
Can this happen with (almost) all cells?

Theorem 2: Let $P \subset \mathbb{R}^2$ be a set on n points. $\text{Vor}(P)$ has at most $2n - 5$ nodes and $3n - 6$ edges.

Exercise 1



Exercise 1



In the Direction of the Sweep Line

Obviously it's the intersection of $\text{Vor}(P)$ and sweep line ℓ at the current time point is not known yet.

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Instead, we view the part above ℓ as already fixed!

In the Direction of the Sweep Line

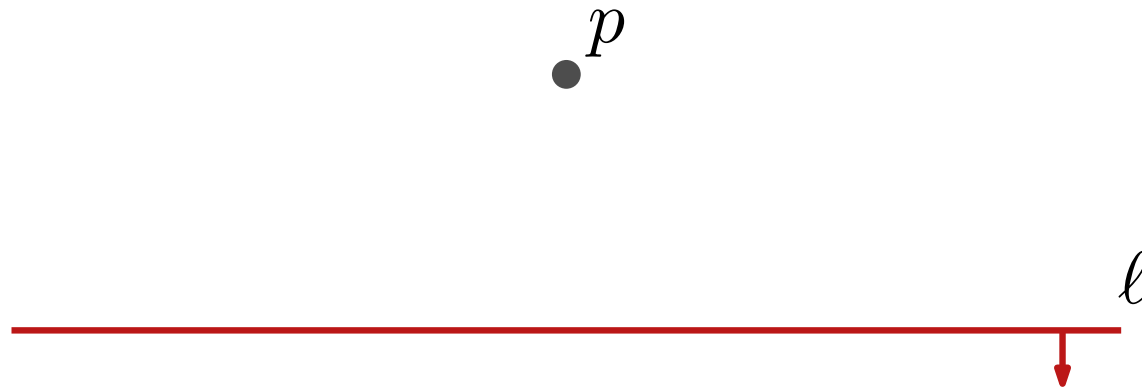
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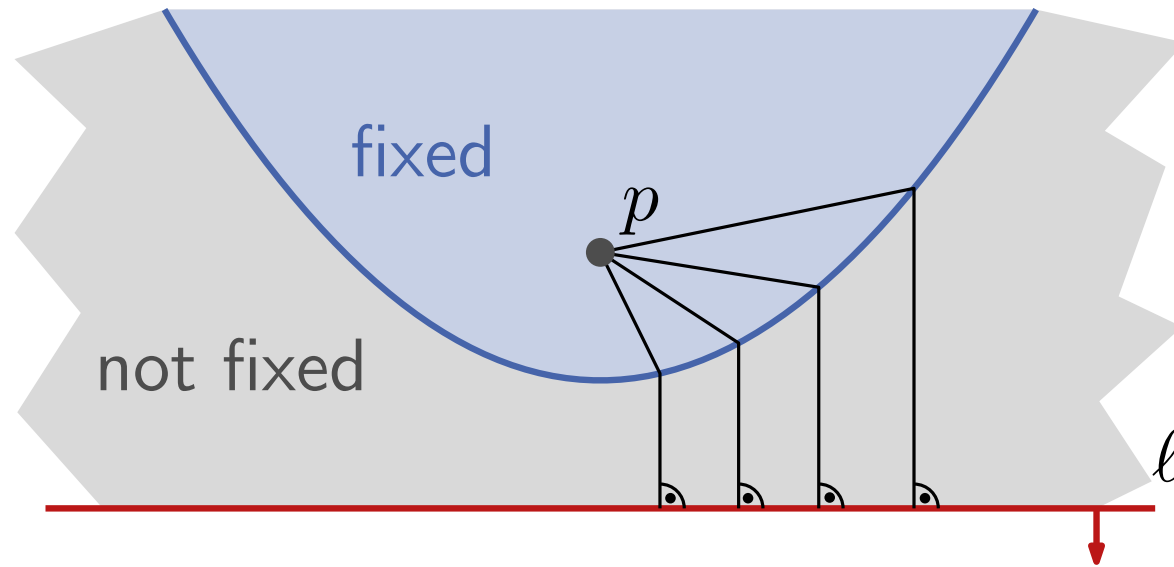
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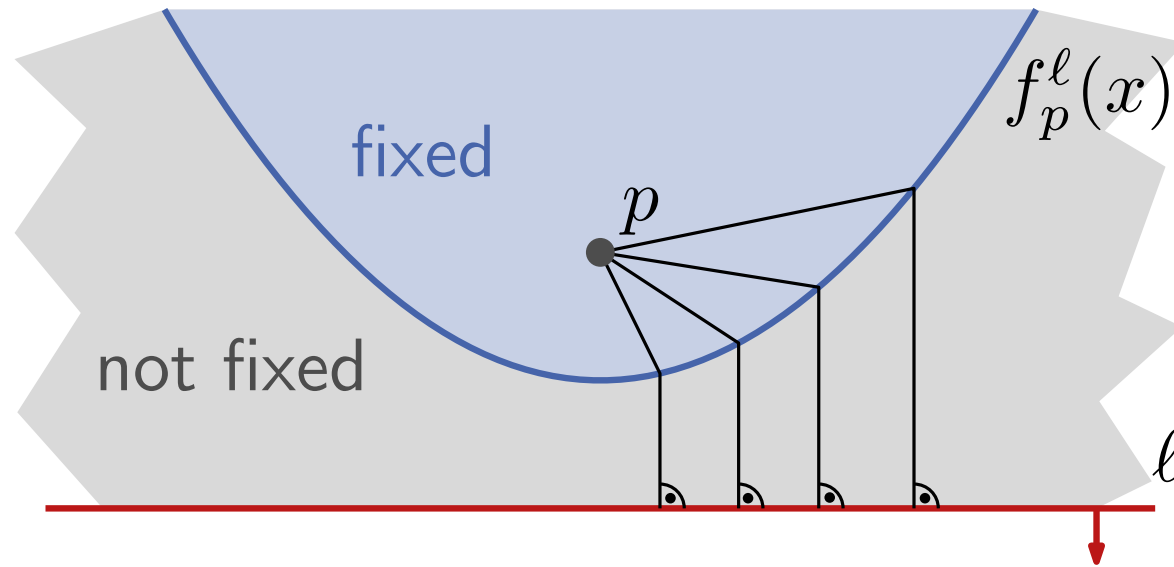
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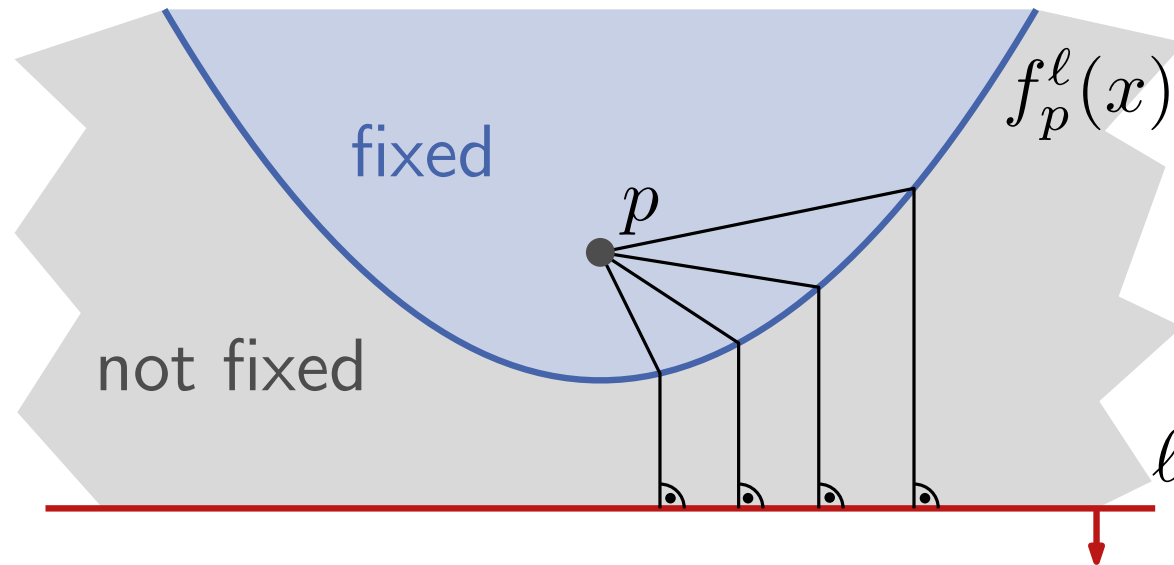
Instead, we view the part above ℓ as already fixed!



In the Direction of the Sweep Line

Obviously it's the intersection of $\text{Vor}(P)$ and sweep line ℓ at the current time point is not known yet.

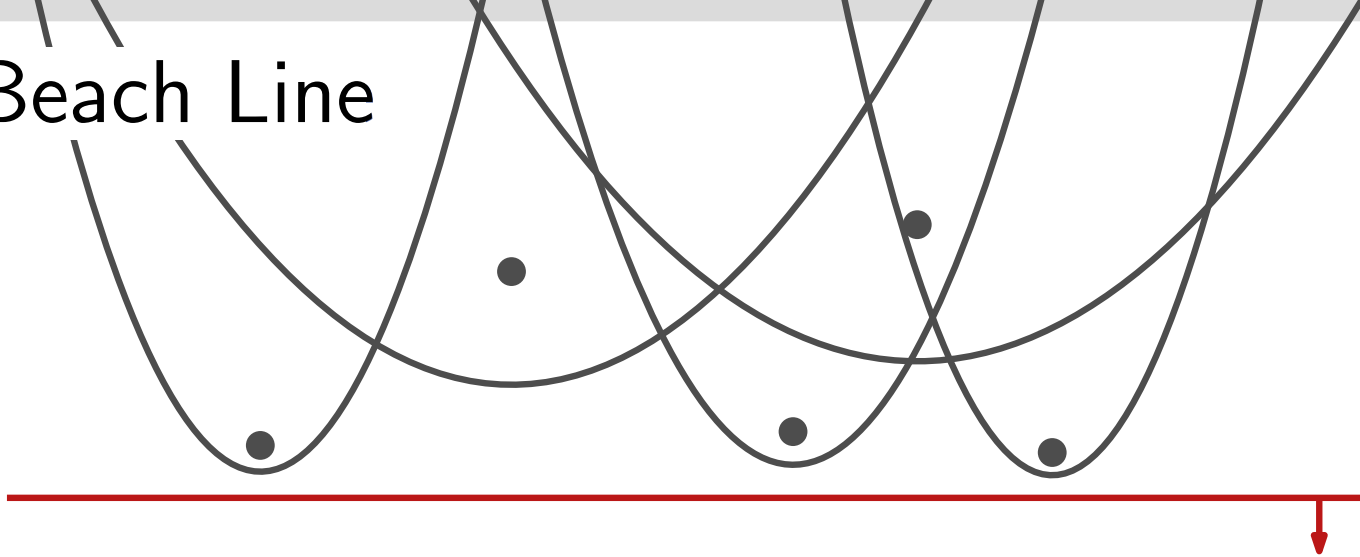
Instead, we view the part above ℓ as already fixed!



Enforcing the equality $|pq| = |q\ell|$ gives

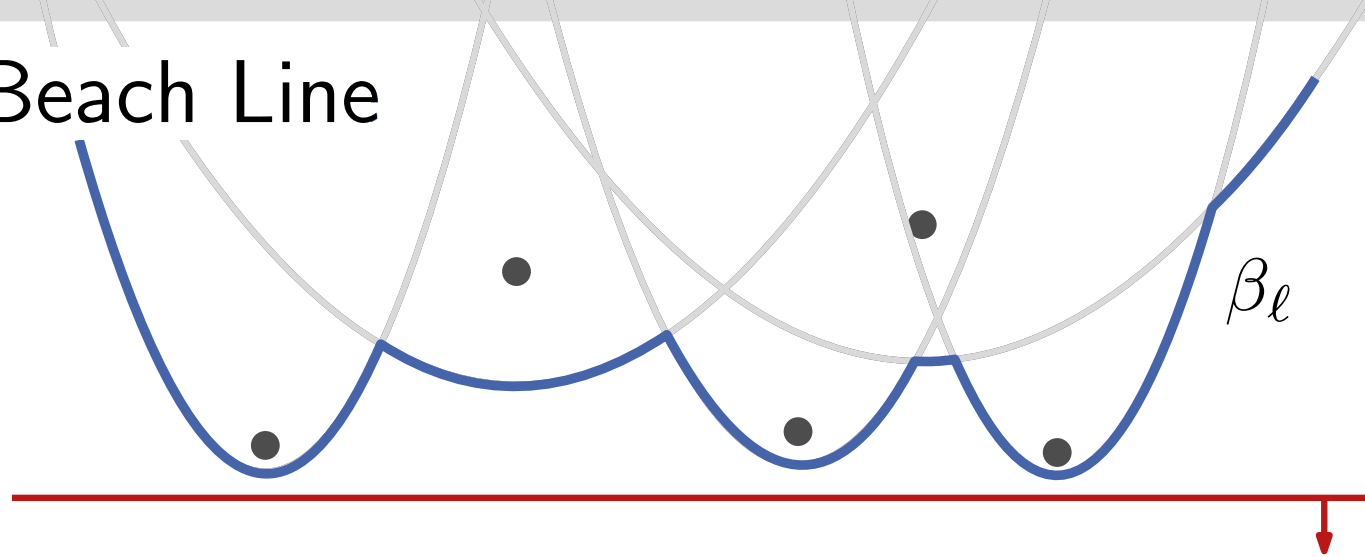
$$f_p^\ell(x) = \frac{1}{2(p_y - \ell_y)} (x - p_x)^2 + \frac{p_y + \ell_y}{2}$$

The Beach Line



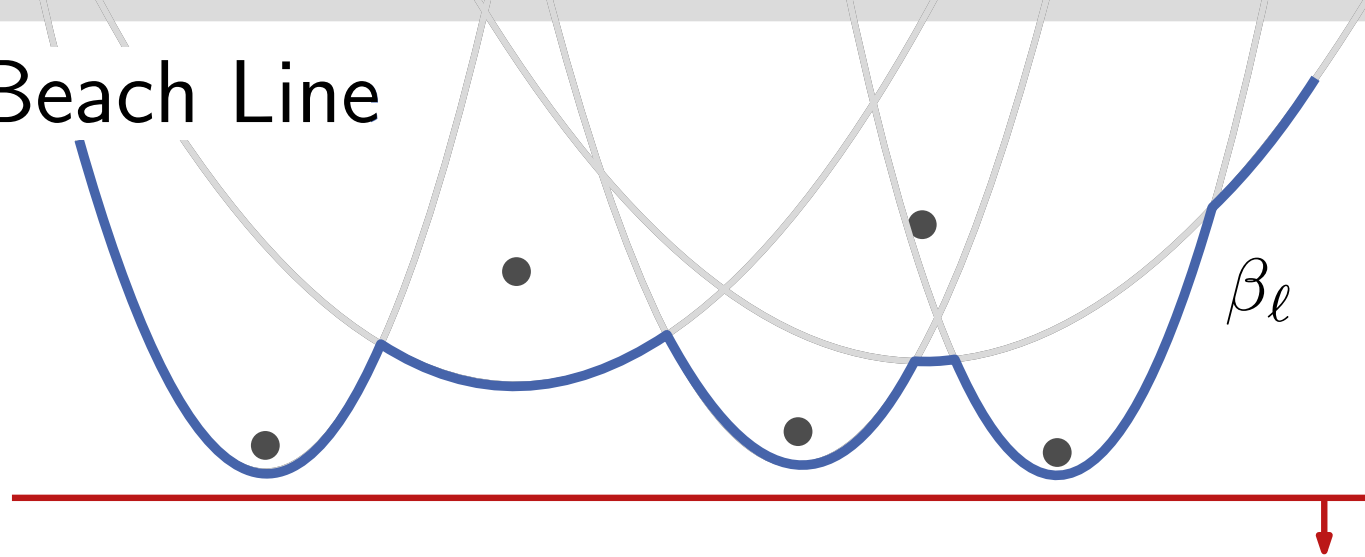
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The Beach Line



Definition: The **beach line** β_l is the lower envelope of parabolas f_p^l for the points already found.

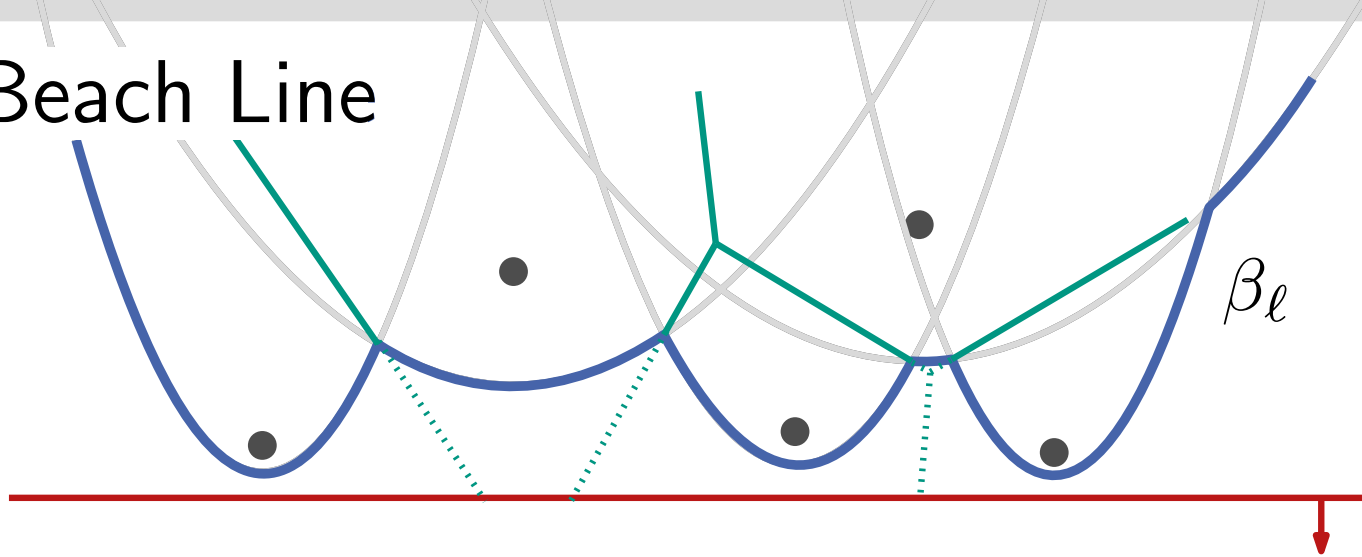
The Beach Line



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What does it have to do with $\text{Vor}(P)$?

The Beach Line

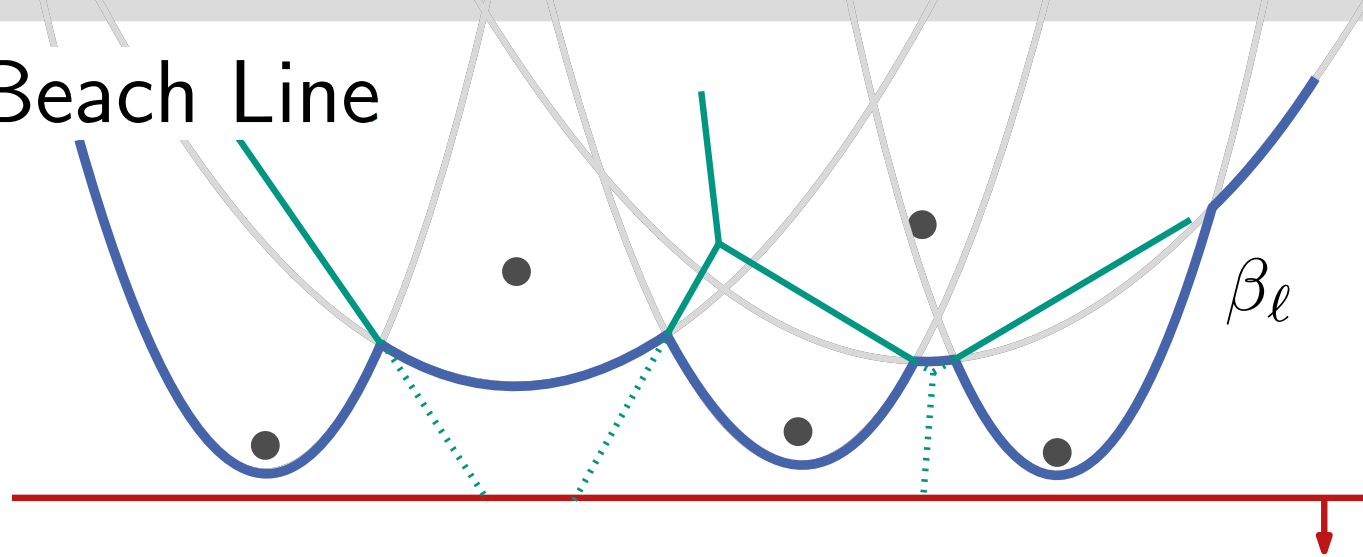


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- The beach line is x -monotone
 - Intersection points in the beach line lie on Voronoi edges

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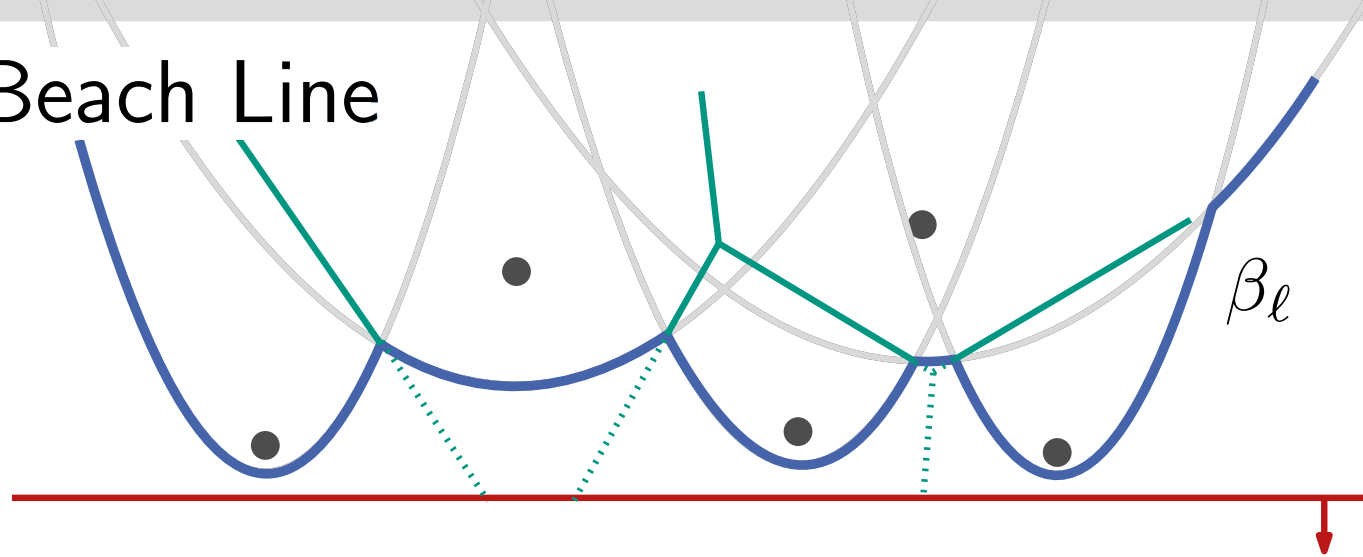


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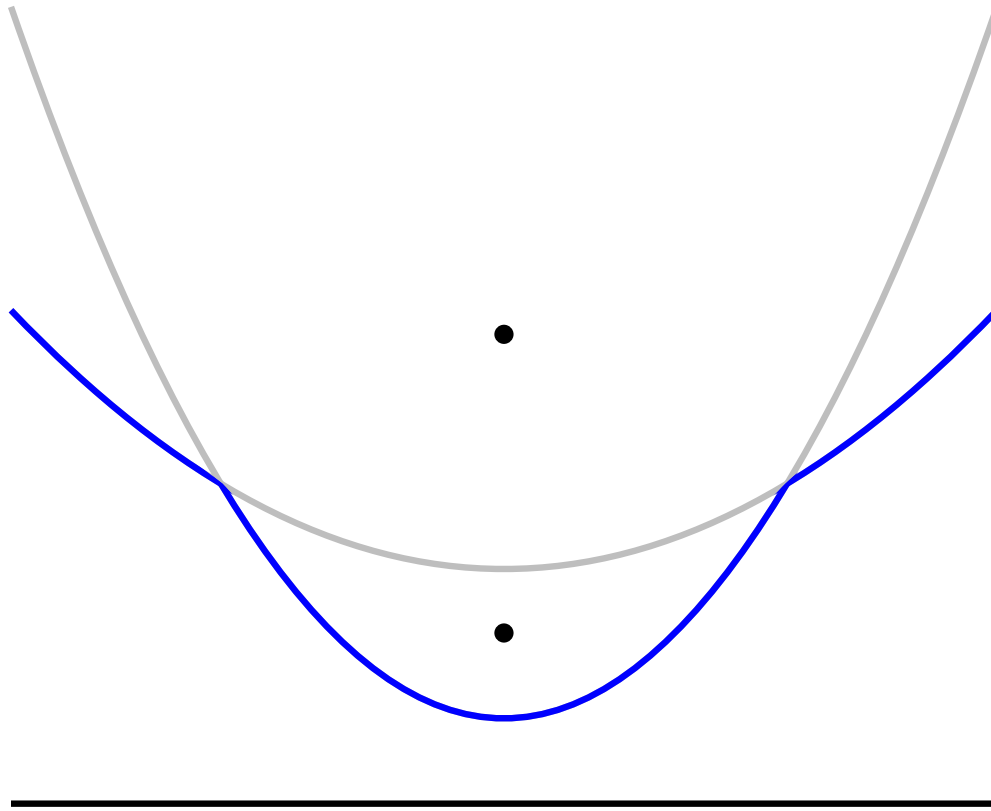
Goal: Store (implicit) contour β_ℓ instead of $\text{Vor}(P) \cap \ell$

Exercise 2

Give an example where a parabola contributes more than one arc to the beach line.

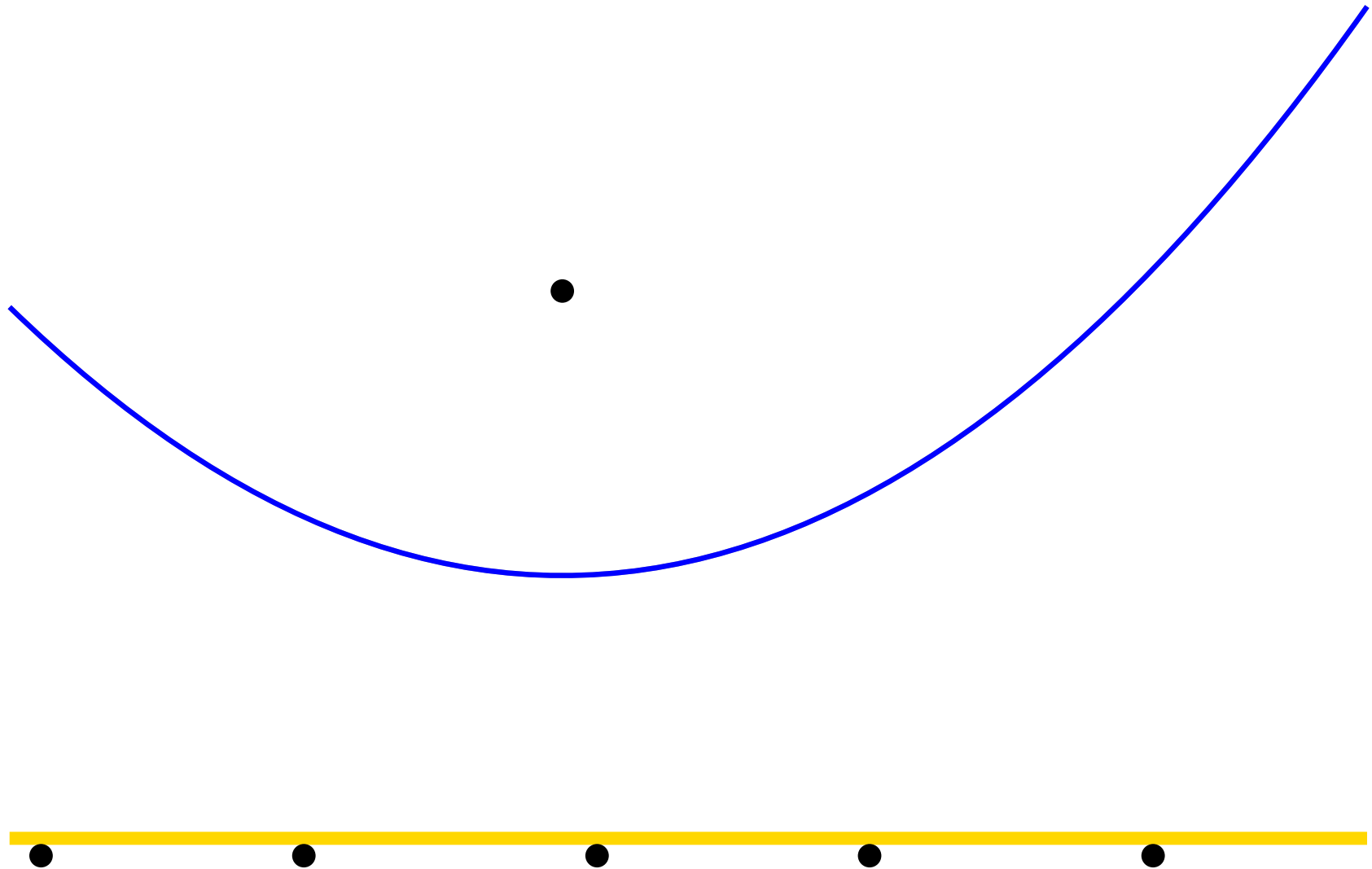
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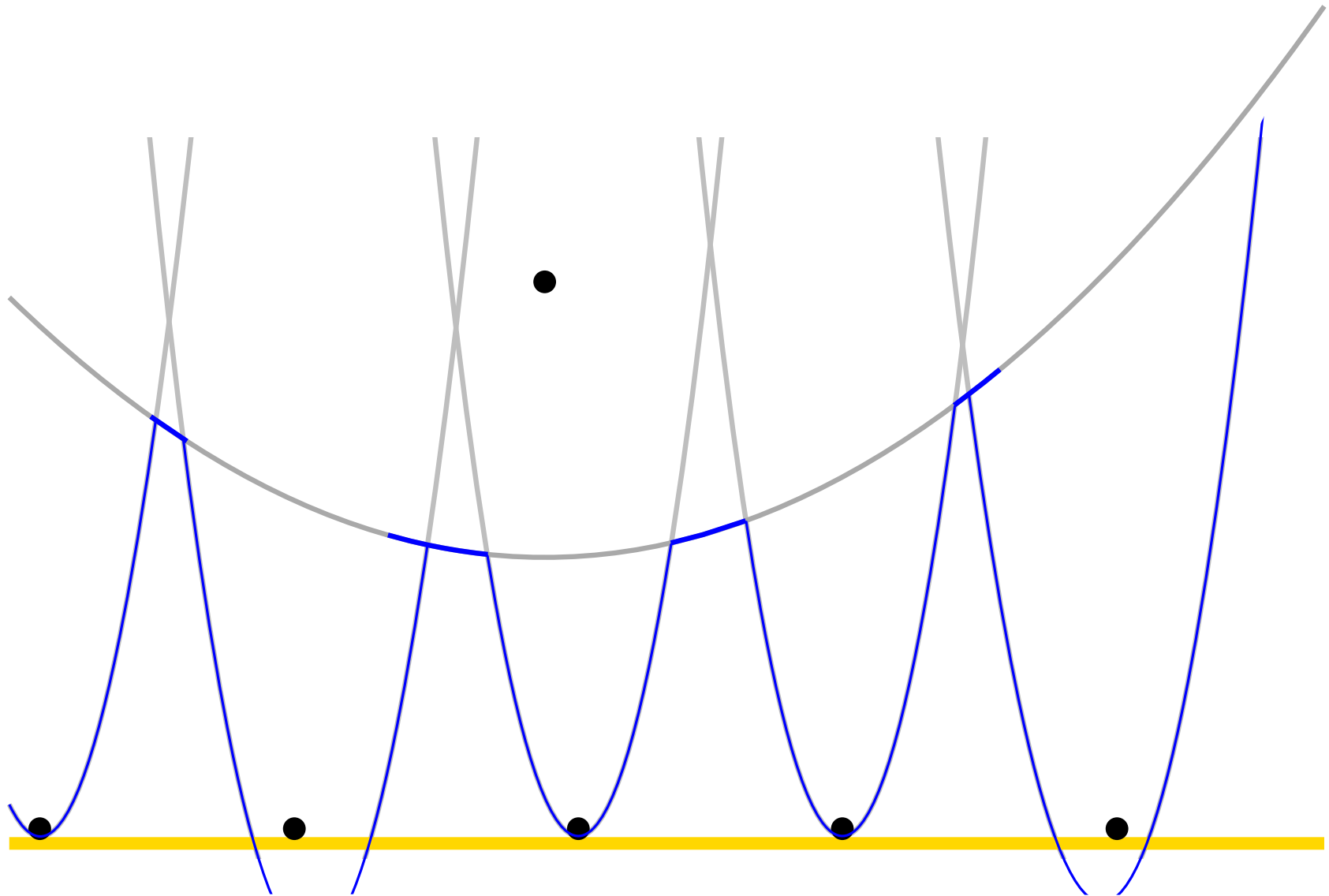
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A parabola, that contributes linear many arcs to the beach line.



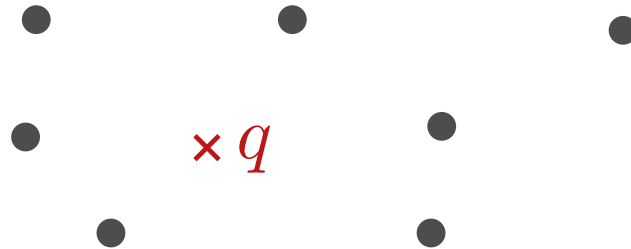
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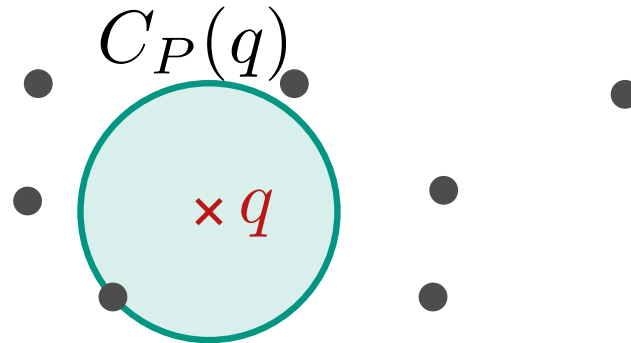
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Definition: Let q be a point. Define $C_P(q)$ to be the points in P that lie on the empty circle with center q .



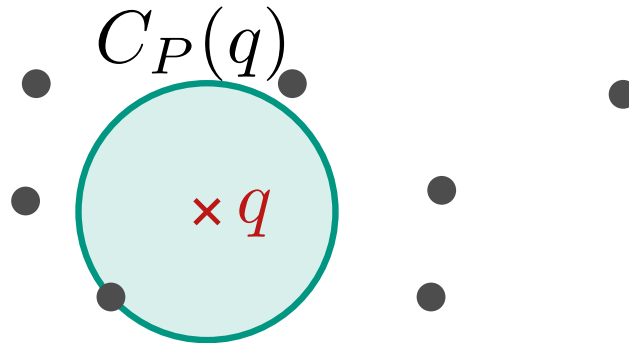
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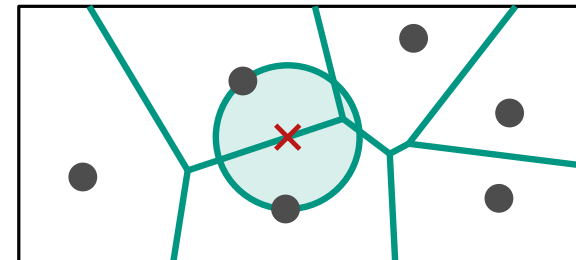
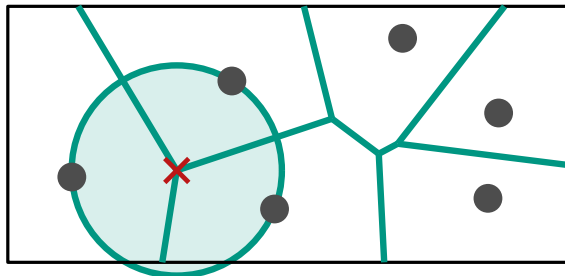
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Theorem 3:

- A point q is a Voronoi vertex
 $\Leftrightarrow |C_P(q) \cap P| \geq 3,$
- the bisector $b(p_i, p_j)$ defines a Voronoi edge
 $\Leftrightarrow \exists q \in b(p_i, p_j)$ with $C_P(q) \cap P = \{p_i, p_j\}.$



Exercise 3 – Next Neighbor

Given: Set P of n points.

Describe algorithm that computes for each point $p \in P$ its next neighbor $a(p)$ in P in $O(n \log n)$ time.

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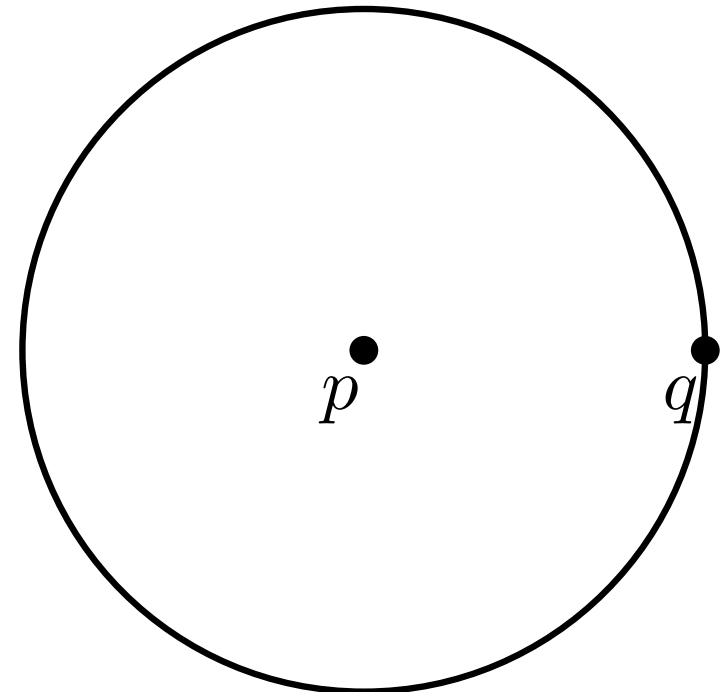
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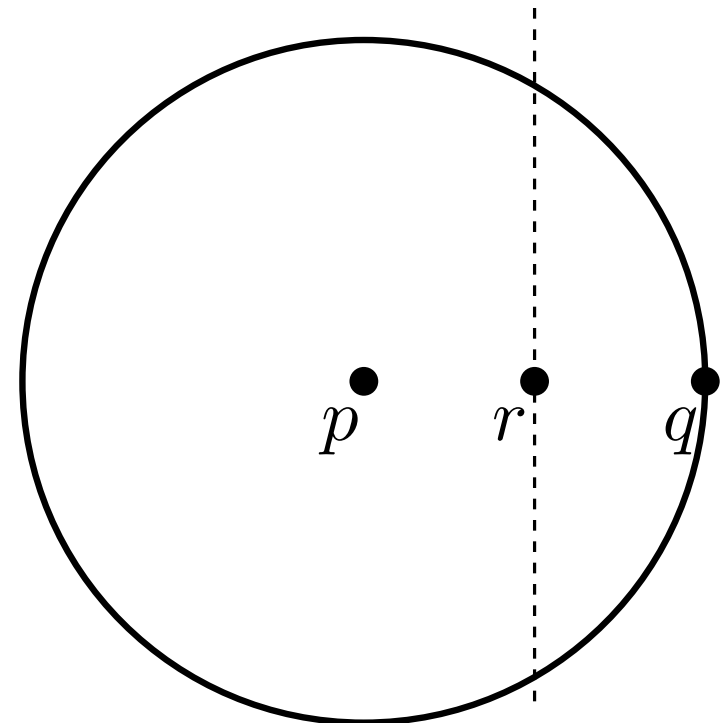
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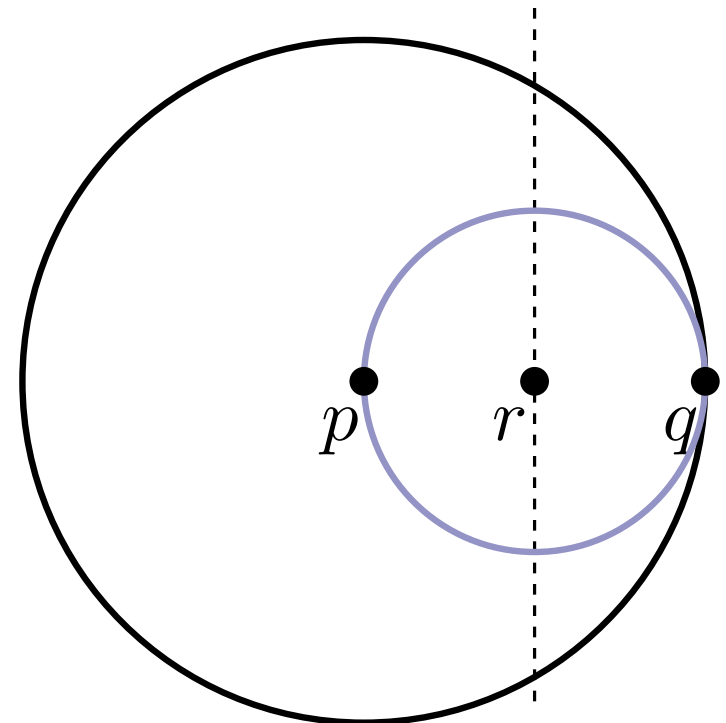
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Exercise 4 – Nuclear Power Plants

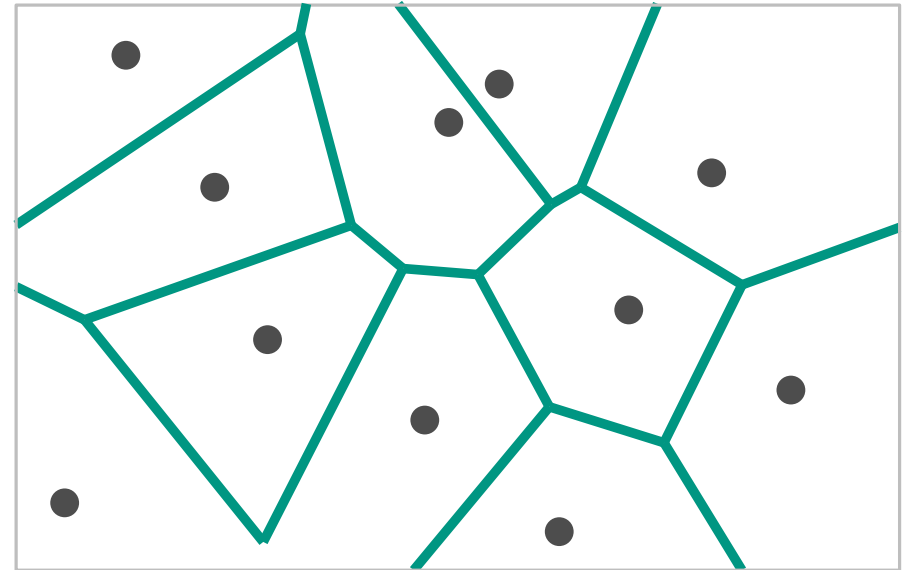
Given:

Find: Point $p \in R$ with maximum distance to nuclear power plants.

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Given: nuclear plants (set P of points) and preferred region (rectangle R).

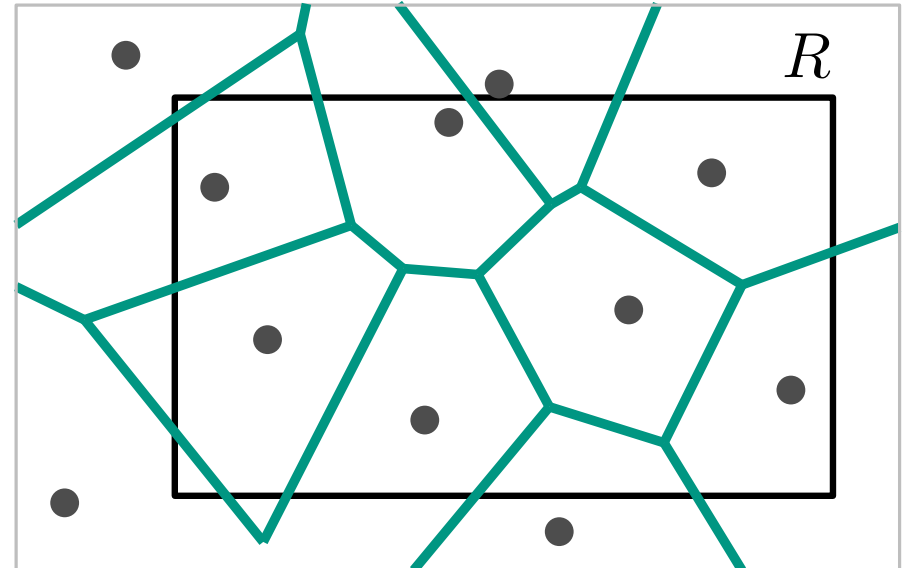
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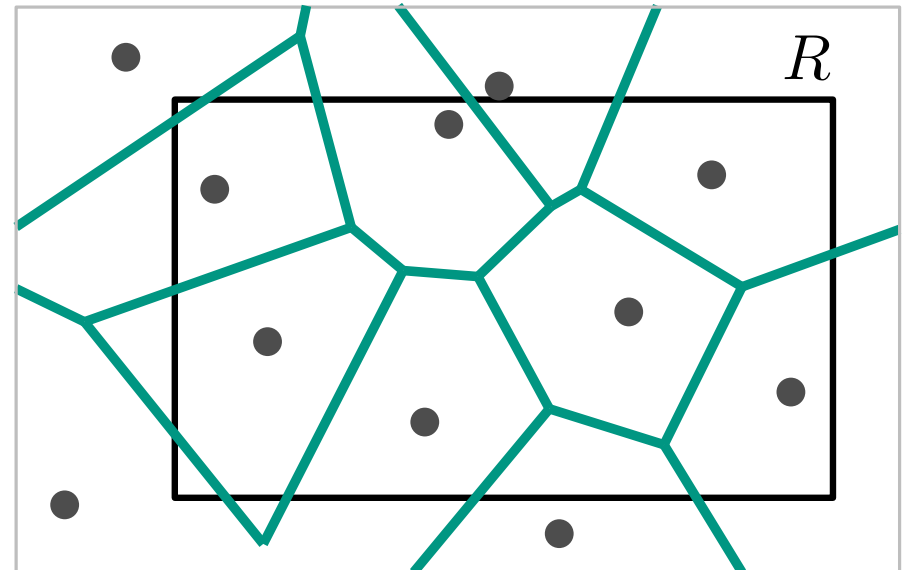
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Candidates:

- Voronoi vertices
- Corners of R
- Intersections of R & Voronoi-Kanten

a) Candidates are sufficient

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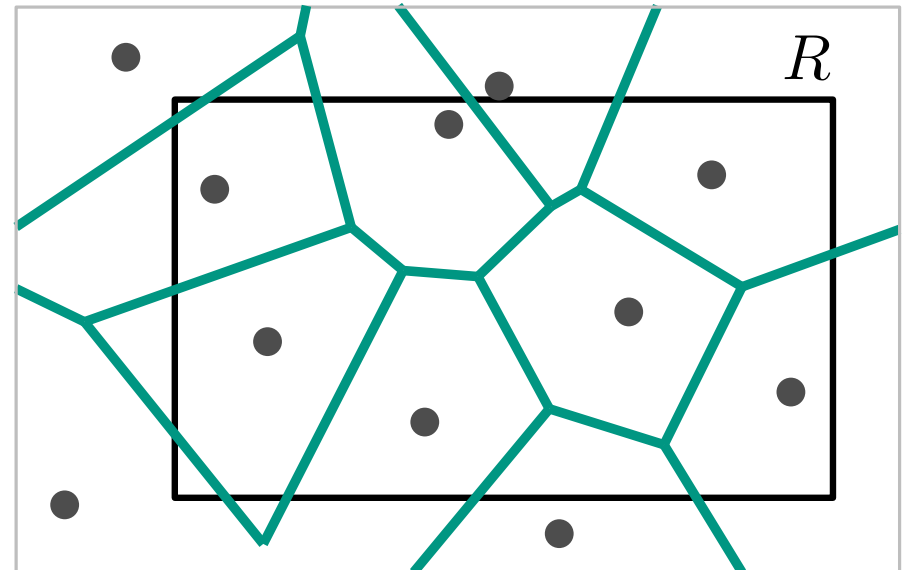
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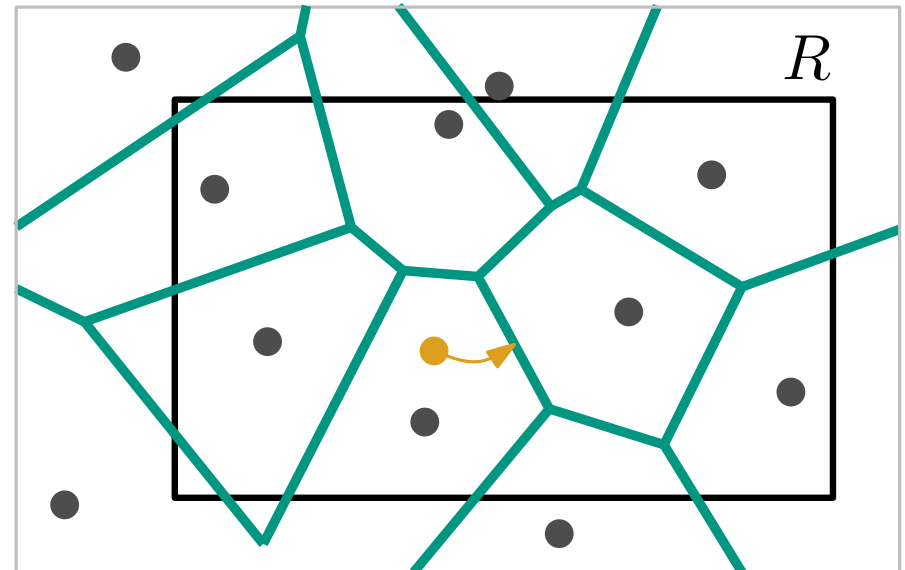
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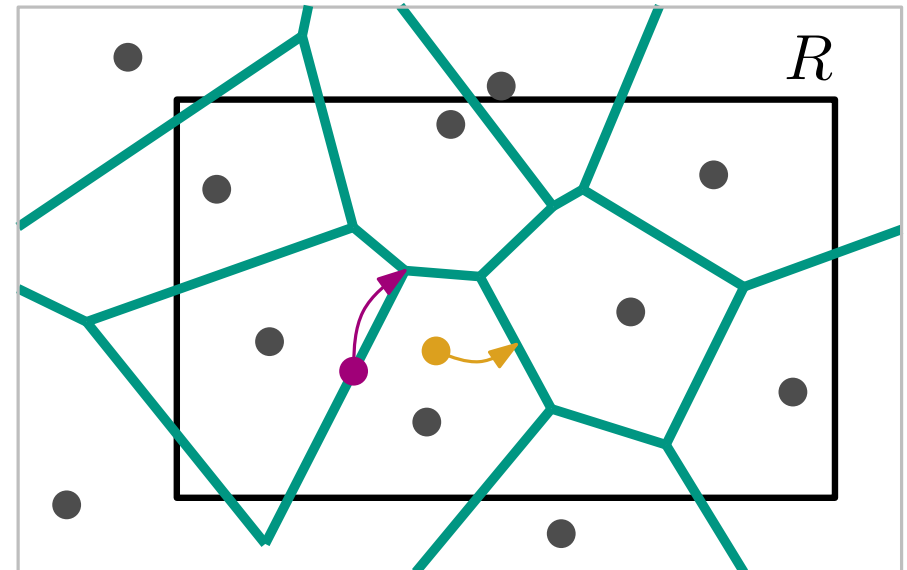
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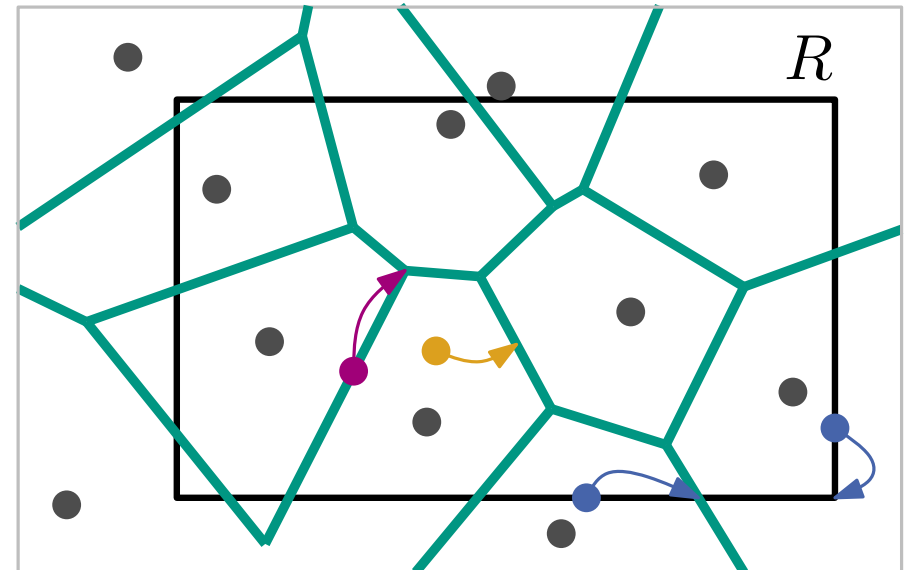
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r lies on edge of R → Push towards intersection of R & Vor.-edges, or

→ Push towards corner of R



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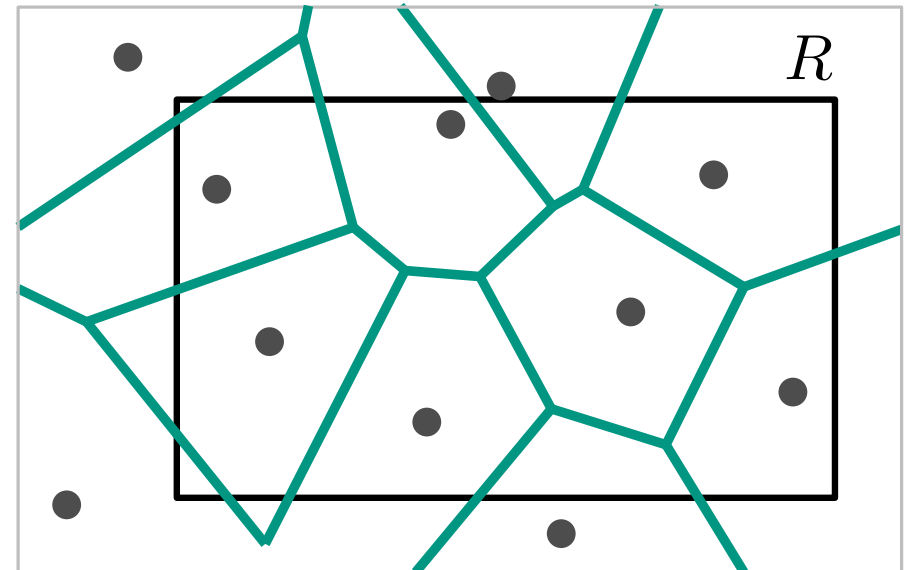
b) Computation in $\mathcal{O}(n)$ time

Idea: Check all candidates.

There are $\mathcal{O}(n)$ many:

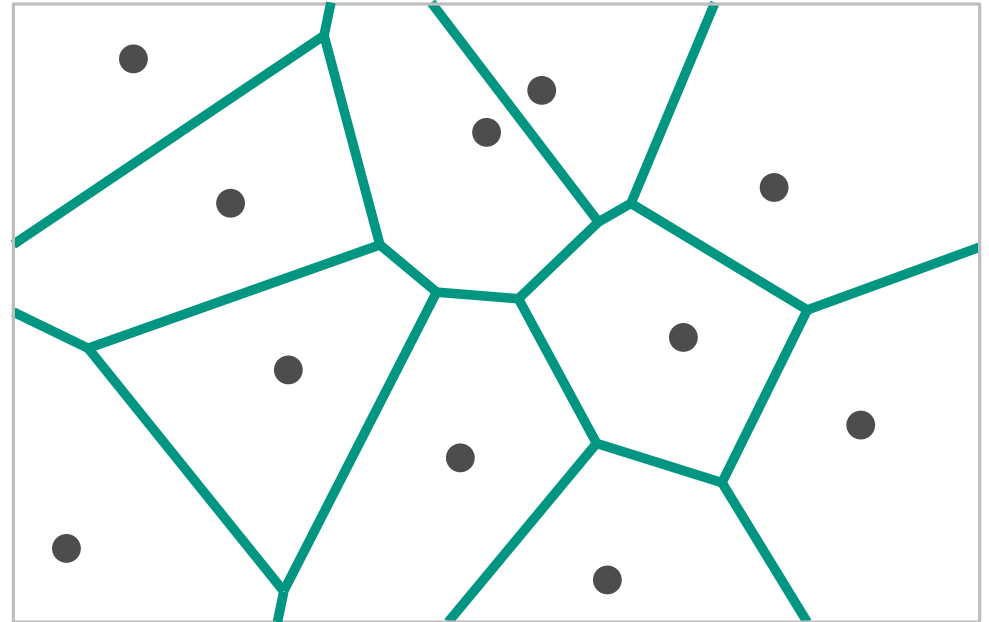
- $\mathcal{O}(n)$ many Vor.-edges & -vertices
- R intersects each edge in at most two intersection points.

—► Computation in $\mathcal{O}(n)$ linear time.



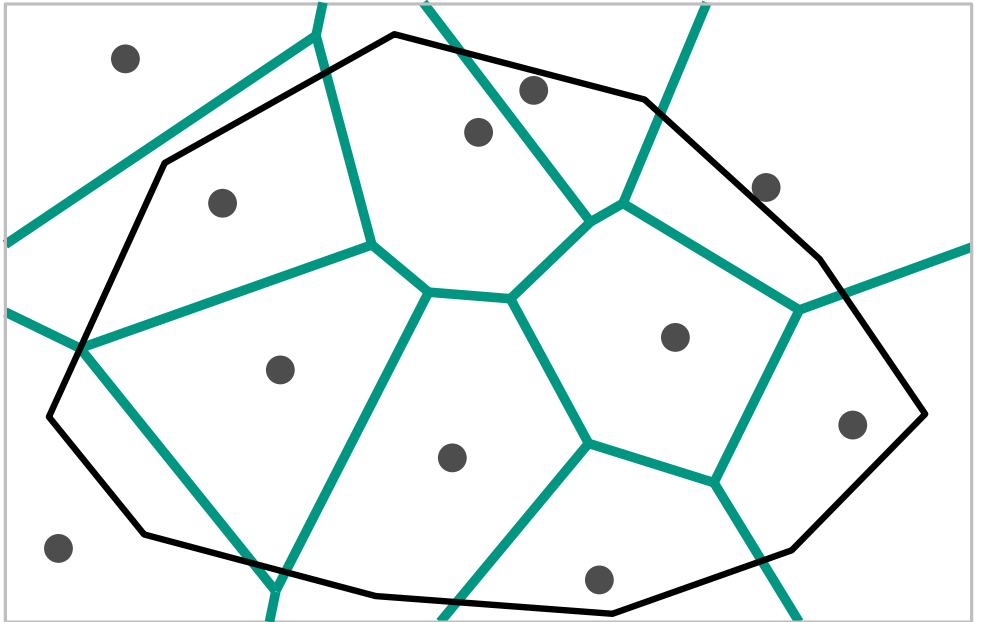
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Now: Convex polygon P with m nodes instead of rectangle R .



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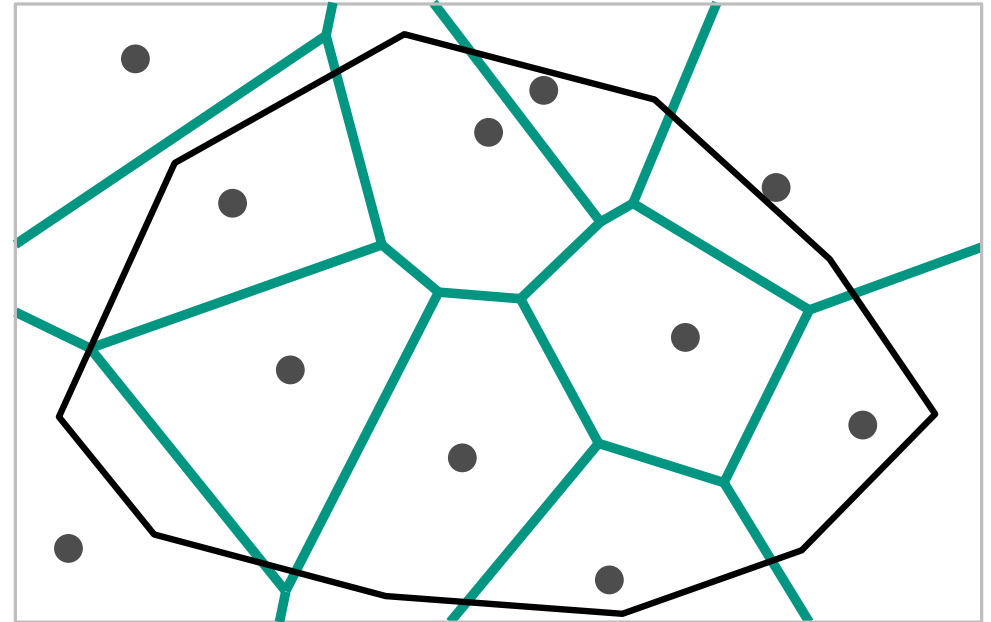
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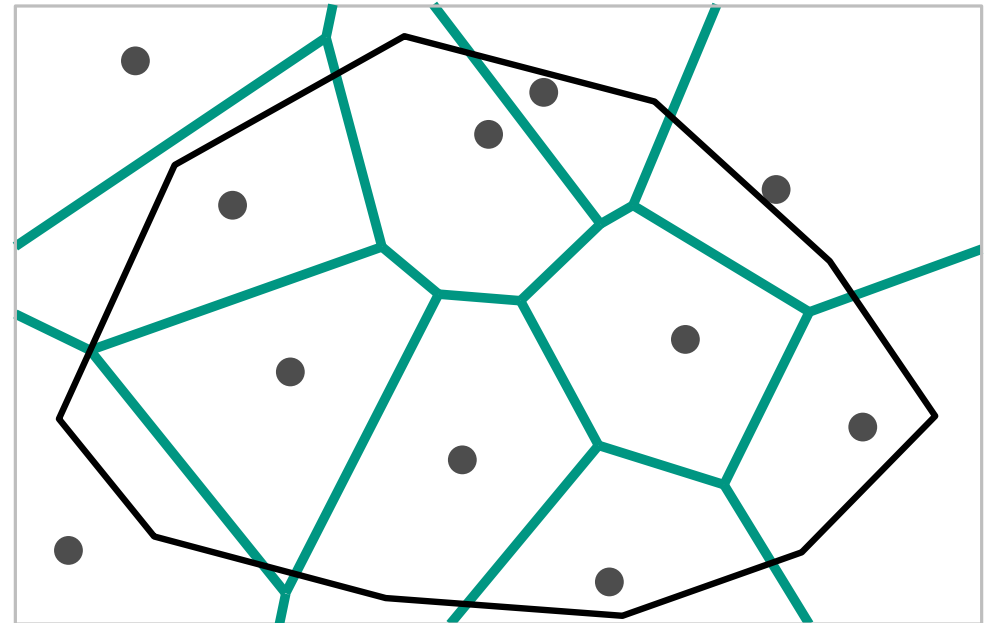
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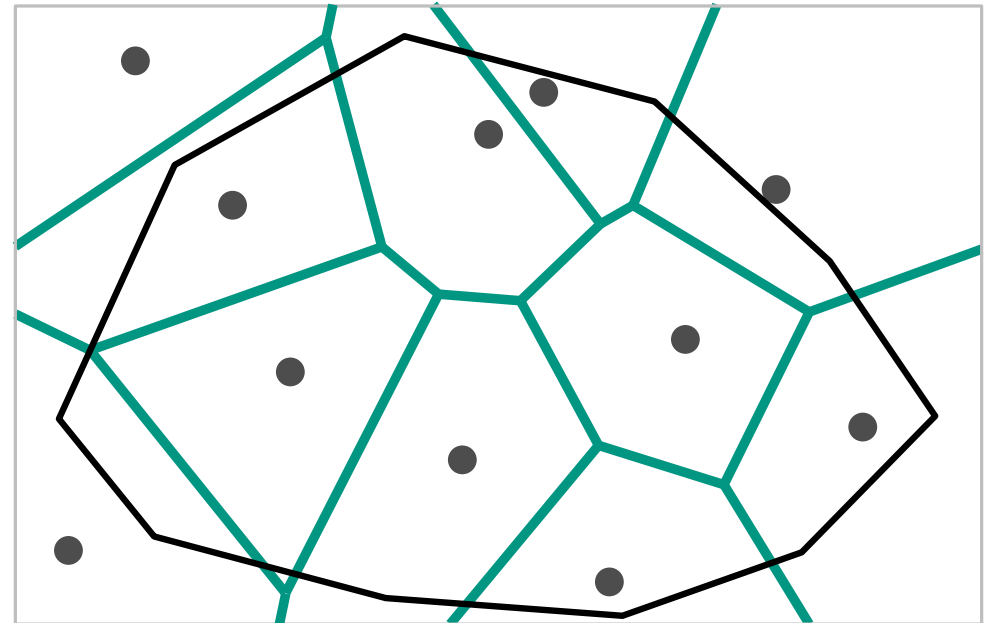
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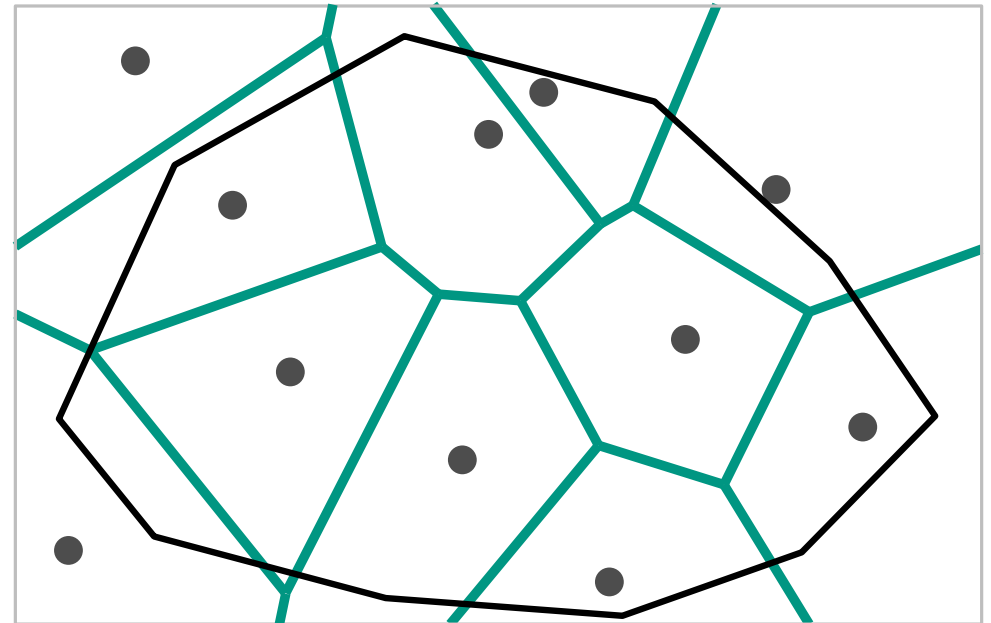
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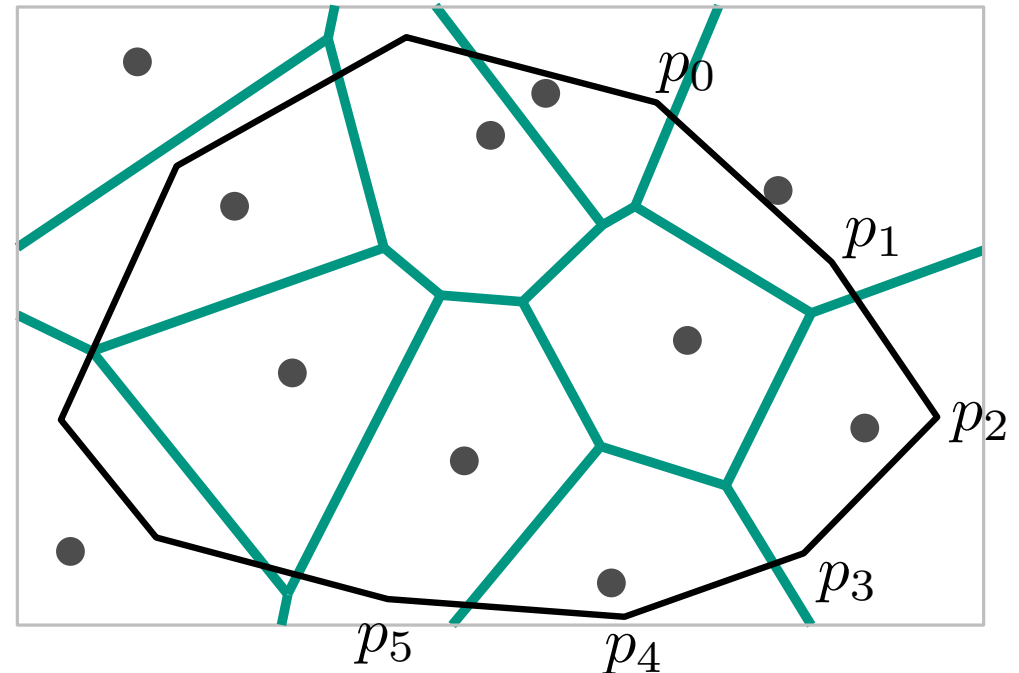
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1.) Same arguments as in a).

2.) Determine set \mathcal{C} of all cells that are intersected by P or contained in P in $\mathcal{O}(m + n)$ time.

→ set \mathcal{C} induces candidates.



Construct Set \mathcal{C}

Find cells intersected by P :

Init.: $C \leftarrow$ cell containing p_0 , $i \leftarrow 0$.

- 1) Mark C as visited
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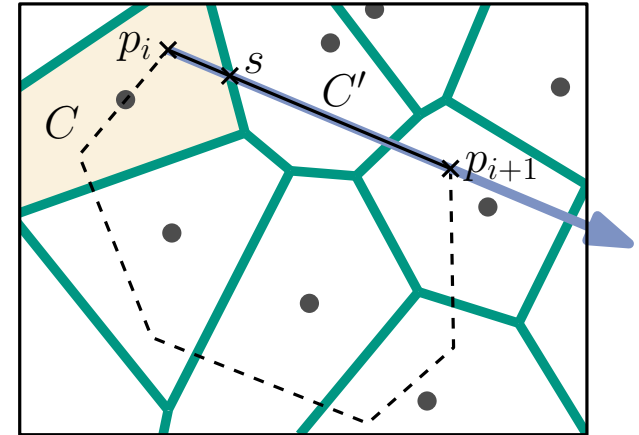
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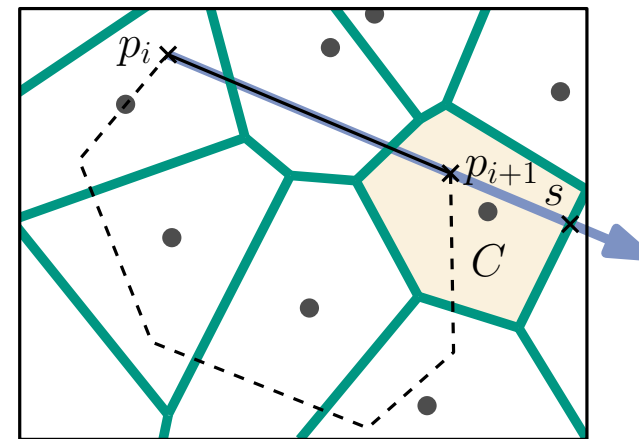
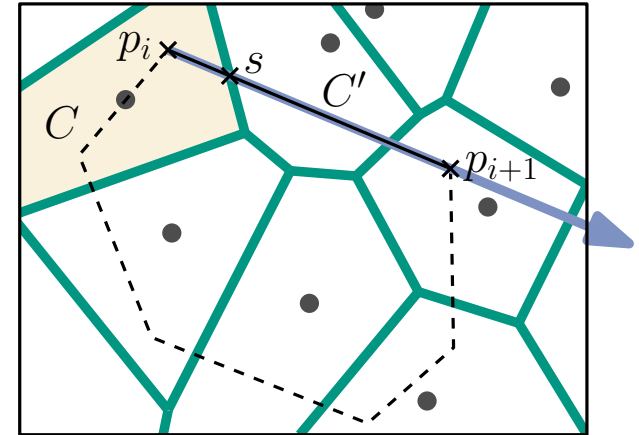
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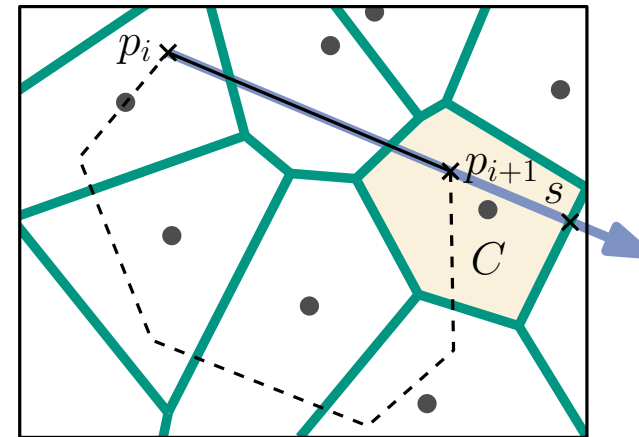
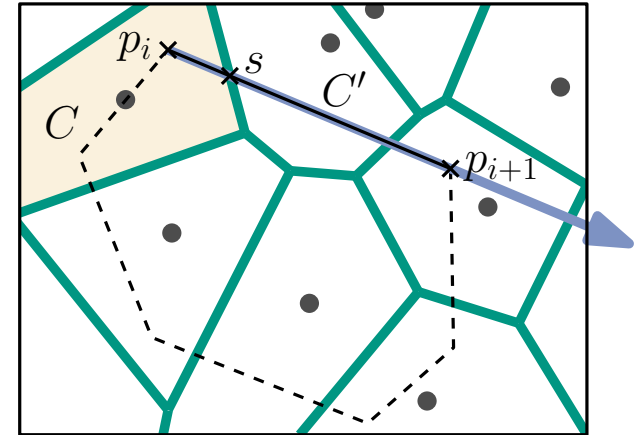
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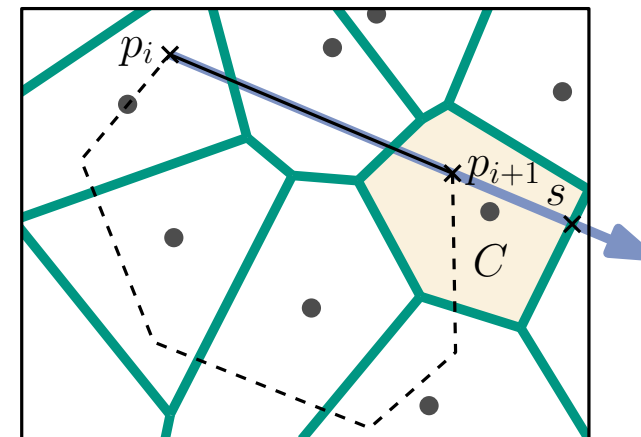
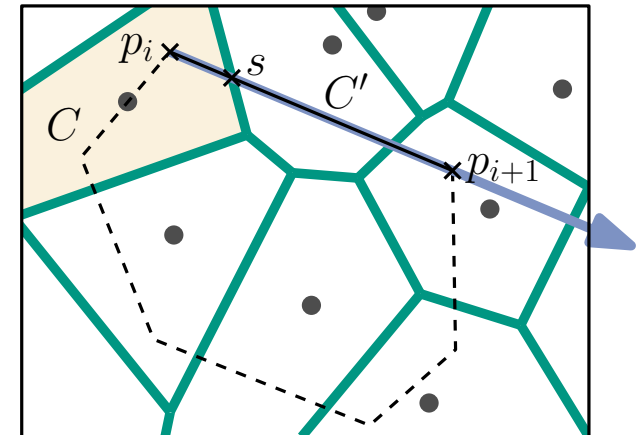
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Find cells contained in polygon.

depth-first search, bounded by the cells found in the first step.



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Find Average running time $O(1)$, because each cell has $O(1)$ complexity in average.

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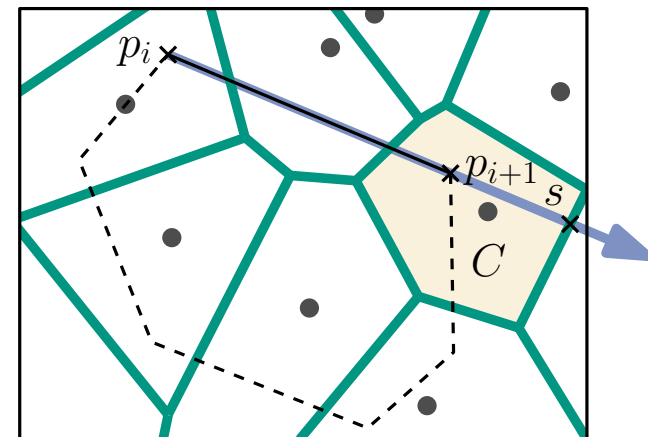
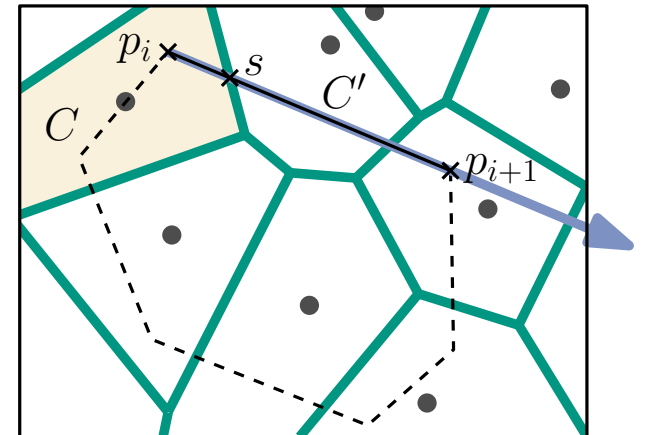
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Since P is convex, each Voronoi edge is intersected at most twice.

→ $O(m + n)$ running time.

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