

Exercises 7 – Point Location¹

Discussion: 10.02.2016

Exercise 1 – Containment. Let P be a simple polygon consisting of n vertices and let q be a query point. The following algorithm determines whether q lies in P . Let $\rho := \{(q_x + \lambda, q_y) : \lambda > 0\}$ be a horizontal ray starting at q and going rightwards. Determine for every edge e of P , whether it intersects ρ . If the number of intersecting edges is odd, then q lies in P and otherwise not.

1. Prove the correctness of the algorithm.
2. Explain how to deal with degenerated cases.
3. What is the running time of the algorithm?

Exercise 2 – Special Polygons. Let P be a *convex* polygon consisting of n vertices. The vertices are given in sorted order along the boundary of P .

1. Show that one can determine in $O(\log n)$ time, whether a point q lies in P .
2. Can you generalize the solution of 1. for the case that P is not convex, but y -monotone?

Exercise 3 – Special Polygons II. A polygon P is *star-shaped*, if there is a point p in the interior of P , such that for any other point q in P the segment pq is completely contained in P . Assume that we are given such a star-shaped polygon and the point p .

1. Describe an algorithm that computes in $O(\log n)$ time, whether a point q lies in P or not.
2. Does your algorithm also work, if p is not given?

Exercise 4 – Ray-Shooting Problem. For a set of S of n non-intersecting segments we want to answer the following query: “Given a query ray ρ (a half-line starting at some point), find the first segment in S intersected by ρ .”

In the following assume that ρ is a *vertical* ray pointing upwards. Hence, only the starting point need be specified in such a query. We call it the *vertical ray shooting problem*.

1. Describe a data structure for the vertical ray shooting problem. Bound the query time and storage requirement.
2. Can you modify your data structure such that it also supports intersecting segments in S ?

¹Based on: M. de Berg, O. Cheong, M. van Kreveld, M. Overmars: Computational Geometry, 3rd ed., Springer-Verlag, 2008.