Karlsruher Institut für Technologie
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Computational Geometry
Winterterm 2015/2016

Exercises 6 – Duality & Well-Separated Pair Composition[1]

Discussion: To be announced.

Duality (11.01.2016)

Exercise 1 – Duality I. In the lecture we have seen that the dual of a line segment is a double wedge, with a wedge left and right to the point that is dual to the line containing the line segment.

1. What is the dual of a triangle with vertices p, q and r?
2. What is the dual of a circle through the points p, q and r?

Exercise 2 – Duality II. Let L be a set of n lines in the plane. We want to find an axis-aligned rectangle $\mathcal{B}(L)$ that contains all vertices of the arrangement $\mathcal{A}(L)$. Describe an algorithm that computes $\mathcal{B}(L)$ in $O(n \log n)$ time.

Exercise 3 – Duality III. Let R be a set of n red points in the plane, and let B be a set of n blue points in the plane. We call a line ℓ a *separator* of R and B, if all blue points lie on one side and all red points on the other side of ℓ.

1. Describe an algorithm that decides in $O(n \log n)$ time whether a separator of R and B exists.
2. Describe an randomized algorithm, that decides in $O(n)$ expected time whether a separator of R and B exists.

Exercise 4 – Duality IV. Let S be a set of n points in the plane. Describe an $O(n^2)$ algorithm that computes the line on which most points of S lie.

[1] Based on: M. de Berg, O. Cheong, M. van Kreveld, M. Overmars: Computational Geometry, 3rd ed., Springer-Verlag, 2008.

Well-Separated Pair Decomposition (13.01.2016)

Exercise 5 – Foundations. Let $s > 0$ and let $x := 2/s + 1$. Further, let $S := \{x^i \mid 0 \leq i \leq n-1, i \in \mathbb{N}\}$ and let $\{A_j, B_j\}$ $(1 \leq j \leq m)$ be an arbitrary s-WSPD for S. Show that

$$\sum_{j=1}^{m}(|A_j| + |B_j|) = \binom{n}{2} + m$$

Hint: For each j at least one of both sets A_j and B_j is a singleton.

Exercise 6 – Neighbor I. Let P be a set of n points in $\mathbb{R}^d$. Let $p \in P$ and let $q \in P$ be the next neighbor of p in P, i.e., $|pq| = \min\{|pr| : r \in P, r \neq p\}$. Consider an arbitrary s-WSPD for P with $s > 2$.

1. Let $\{A, B\}$ be a pair in this decomposition and assume that p lies in A and q lies in B. Show that A only contains p.
2. Show that the size of an arbitrary s-WSPD with $s > 2$ is at least $n/2$.

Exercise 7 – Neighbor II. Let P be a set of n points in $\mathbb{R}^d$. Further, let $p, q \in P$ be a pair of points with minimal distance to each other, i.e., $|pq| = \min\{|ab| : a \in P, b \in P\}$. Consider an arbitrary s-WSPD $\mathcal{W}$ for P with $s > 2$. Show that $\mathcal{W}$ contains the pair $\{\{p\}, \{q\}\}$.