Karlsruher Institut für Technologie
Fakultät für Informatik
ITI Sanders/Wagner

Computational Geometry
Winterterm 2015/2016

Exercises 5 – Quadtrees[1]

Discussion: Wednesday, 16. December 2015

Lecture 10 – Voronoi-Diagrams (14.12.2015)

Exercise 1 – Delaunay Triangulation - Meshing. In the lecture we have seen a meshing algorithm that only produces *non-obtuse* triangles, i.e., triangles that have no angle larger than 90°. Let $\mathcal{T}$ be a triangulation of a finite set $P \subset \mathbb{R}^2$ of points that only contains non-obtuse triangles.

Show that $\mathcal{T}$ is a Delaunay-triangulation of P.

Exercise 2 – Compressed Quadtrees. Let $P \subseteq \mathbb{R}^2$ be a set of n points and let $\mathcal{Q}$ be a quadtree with depth d on P. We now show how to reduce the storage consumption of $\mathcal{Q}$ from $O((d+1)n)$ to $O(n)$. To that end we remove any node v from $\mathcal{Q}$ that has only one child node under which points are stored. The node is removed by replacing the pointer from the parent of v to v by the pointer from the parent to the only interesting child of v.

1. Show that the described procedure actually reduces the storage consumption of $\mathcal{Q}$ to $O(n)$.
2. What is the running-time of that procedure?
3. Is it possible to reduce the construction time of a compressed quadtree?

Exercise 3 – Balancing. In the lecture we have defined that two adjacent rectangles in a quadtree are *balanced* if their size differ no more than by a factor two. We now require that adjacent rectangles must have the same size. Assume we transform a quadtree $\mathcal{Q}$ with m nodes into a balanced quadtree $\mathcal{Q}'$ satisfying that adapted definition. Is the number of nodes in Q' still linear in the number of nodes in Q? If not, what is an upper bound for that number?

Exercise 4 – Range Queries Using Quadtrees. Describe an algorithm for range queries that is based on quadtrees. You may assume that the quadtree is already given.

1. Analyze the worst-case running time for the case that R is an axis-aligned rectangle.
2. How changes the worst-case running time, if R is a half-plane bounded by a vertical line.

[1] Based on: M. de Berg, O. Cheong, M. van Kreveld, M. Overmars: Computational Geometry, 3rd ed., Springer-Verlag, 2008.