Algorithmen zur Visualisierung von Graphen

Rückblick und gitterbasiertes Graphenzeichnen mit ILP

INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

Tamara Mchedlidze · **Martin Nöllenburg**

11.02.2015

Rückblick der VL-Themen

1. Einführung
2. Baumlayouts
3. Serien-parallele Graphen
4. Geradlinige planare Zeichnungen: Shift- und Realizer-Methode
5. Datenstrukturen für planare Einbettungen
6. Kontaktrepräsentationen planarer Graphen
7. Lagenlayouts (Sugiyama-Framework)
8. Metro Map Layout
9. Knickminimierung und Kompaktierung in orthogonalen Layouts (TSM-Framework)
10. Aufwärtsplanarität
11. Kräftebasierte Algorithmen
12. Inkrementelle orthogonale Zeichnungen

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Welche Aspekte haben wir jeweils behandelt? Welche Querbezüge gibt es?

Prüfungsmodalitäten (Master)

Mündliche Prüfung (20 Minuten)

- Termin 1: 25.2.2015 ab 16:15 Uhr
- Termin 2: 25.3.2015 ab 16:15 Uhr
- Termin 3: 15.4.2015 ab 16:15 Uhr
- Anmeldung im Sekretariat (lilian.beckert@kit.edu)

Inhalt: Stoff aus Vorlesung und Übung (Skript, Literatur & Folien/Mitschrieb nutzen – nur was besprochen wurde)

Ziele:

- vorgestellte Algorithmen und Layoutprobleme kennen, erklären und analysieren
- Problemmodellierung/Ästhetikkriterien, Einordnung, Vor-/Nachteile, Beweisideen, ...

Prüfer: beide Dozenten gemeinsam, Tamara: Englisch

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außer Blatt 6

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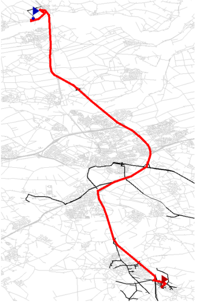
Werbung für Sommersemester 2015

- Praktikum Graphenvisualisierung (Master)

  aufbauend auf diese VL
  Teilnahme am Graph Drawing Contest 2015

- Algorithmische Kartografie (Master)

  vermutlich dienstags 9:45 Uhr



- Algorithmen für Routenplanung (Master)

  Montag 14:00 – 15:30
  Mittwoch 11:30 – 13:00

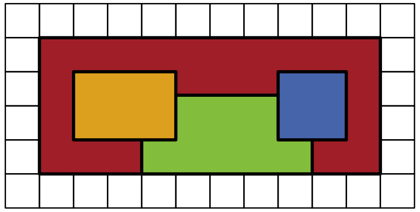
- Masterarbeiten und HiWi-Jobs

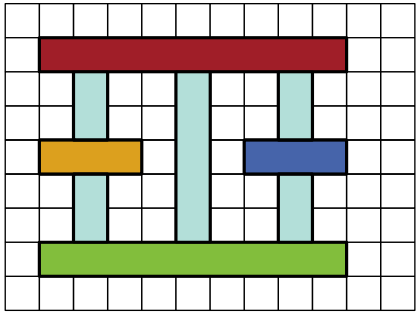
  Themen in allen Forschungsfeldern des Lehrstuhls
  einfach Mitarbeiter ansprechen

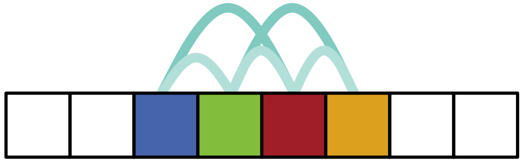
`i11www.iti.kit.edu`

Gitterbasiertes Graphenzeichnen mit ILP

basiert auf [Biedl et al. ’14]

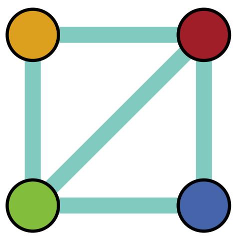


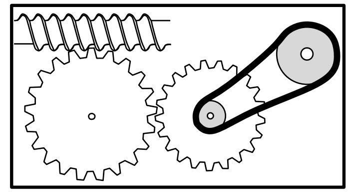




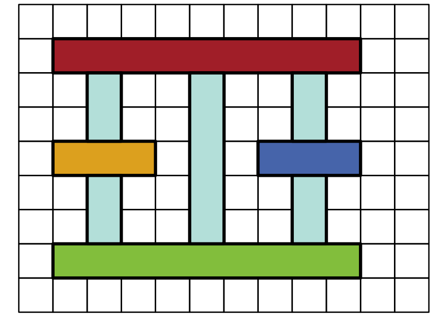
Motivation

Solve grid-based graph drawing problems in a unified framework:

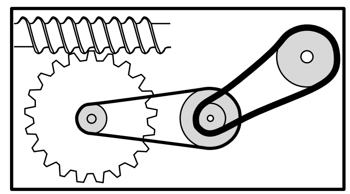




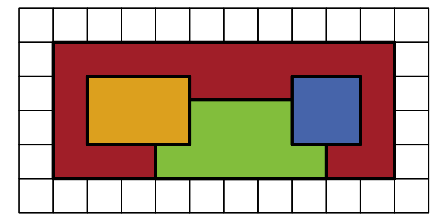
visibility representation



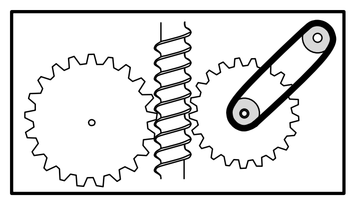
minimize
width
NP-hard



boxicity



recognition
NP-hard



st-orientation

st-orientation
minimize longest *st*-path
NP-hard

pathwidth,
bandwidth,
...

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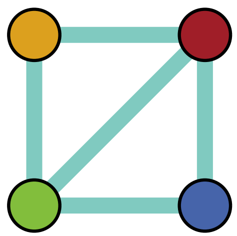

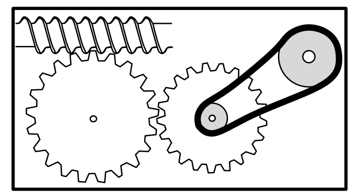

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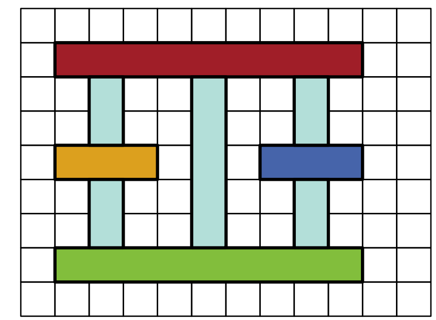
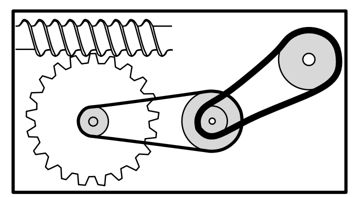


minimize width NP-hard



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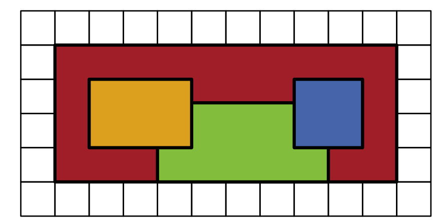
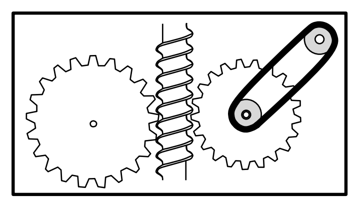


recognition NP-hard



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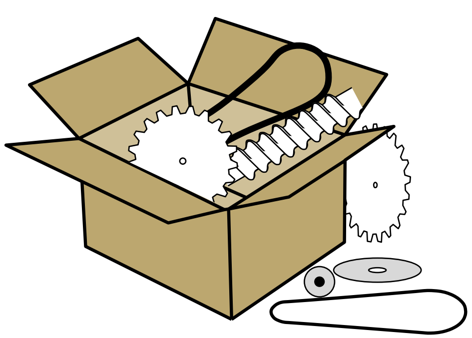


Apply Integer Linear Programming (ILP):

- Collection of general constraints.
- Solving problem = assemble constraints.

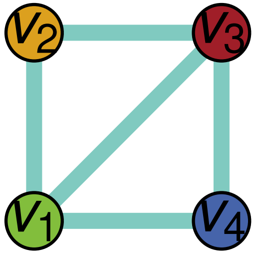
Approach

Given: Grid R, objects, constraints

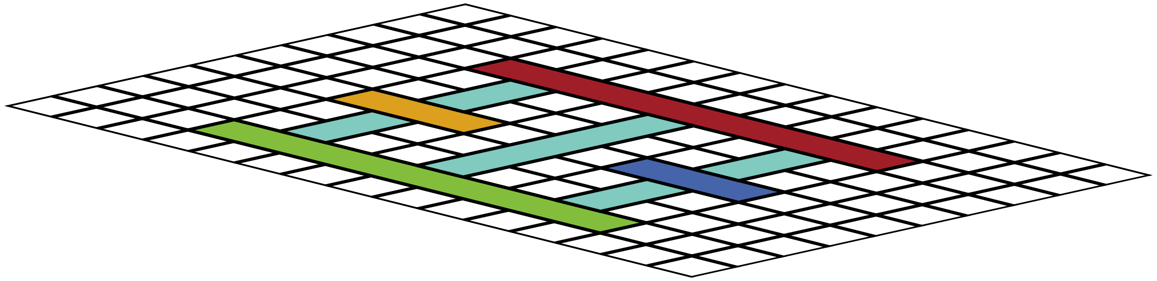
Find: Box on R for each object such that constraints are satisfied.

Example:

Visibility Representation



Vertices = horizontal boxes.
Edges = vertical boxes.



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Given: Grid R, objects, constraints

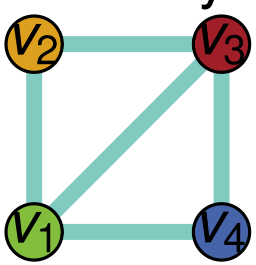
Find: Box on R for each object such that constraints are satisfied.

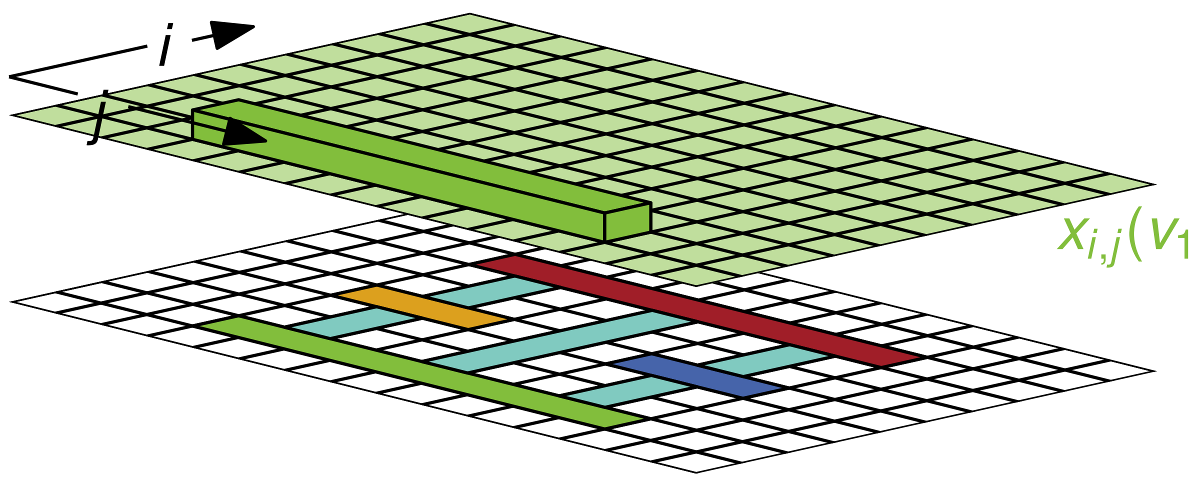
Representation of single box B:

- Grid of binary variables $x(B)_{i,j}$
- Meaning: $x(B)_{i,j} = 1$ iff grid point (i,j) belongs to B.

Example:

Visibility Representation





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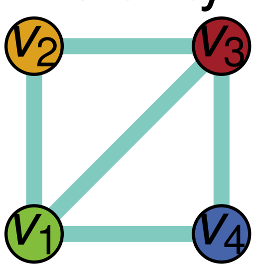

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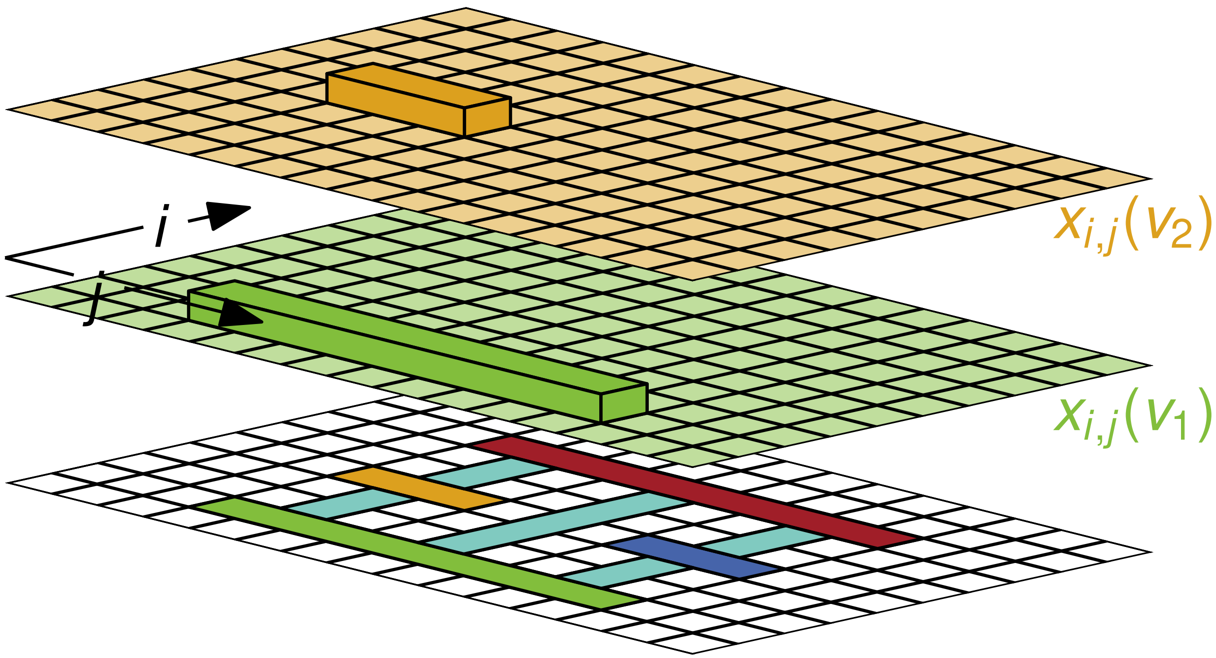

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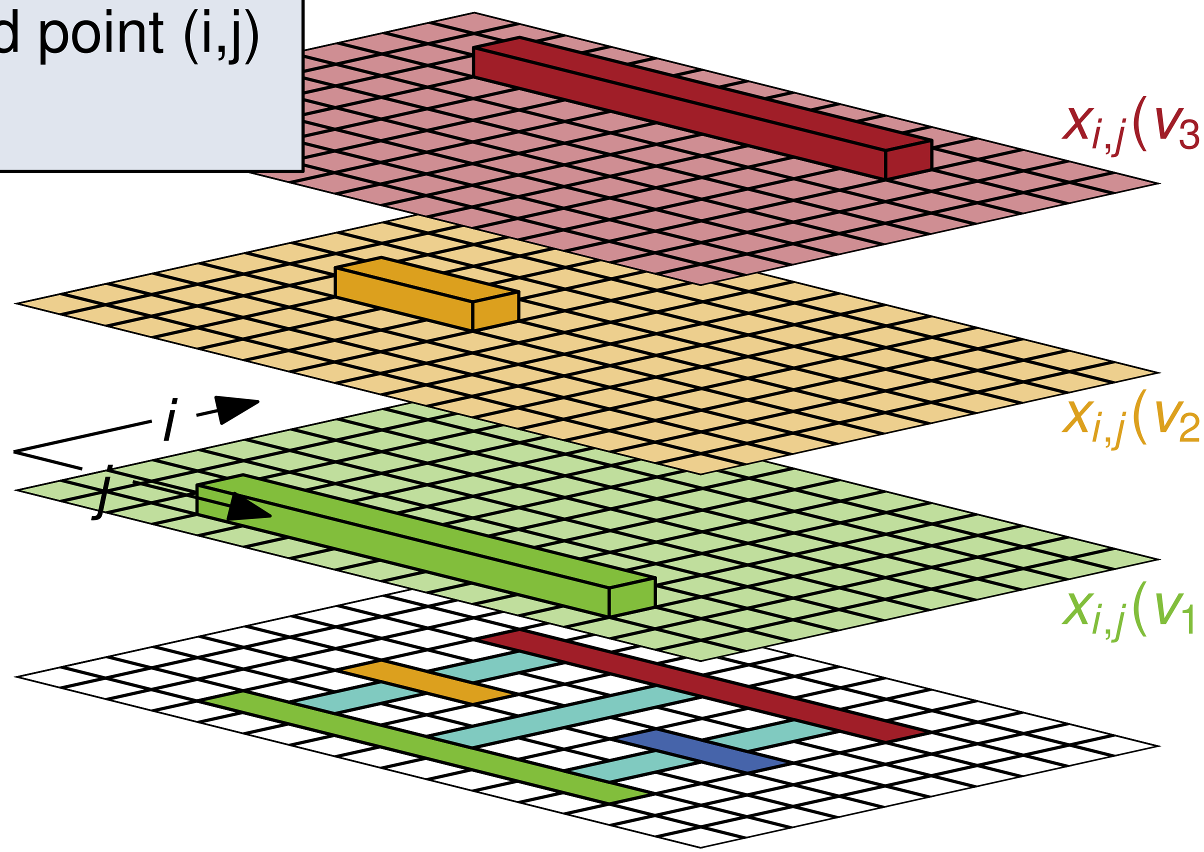

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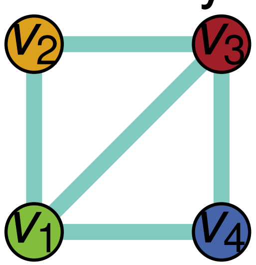

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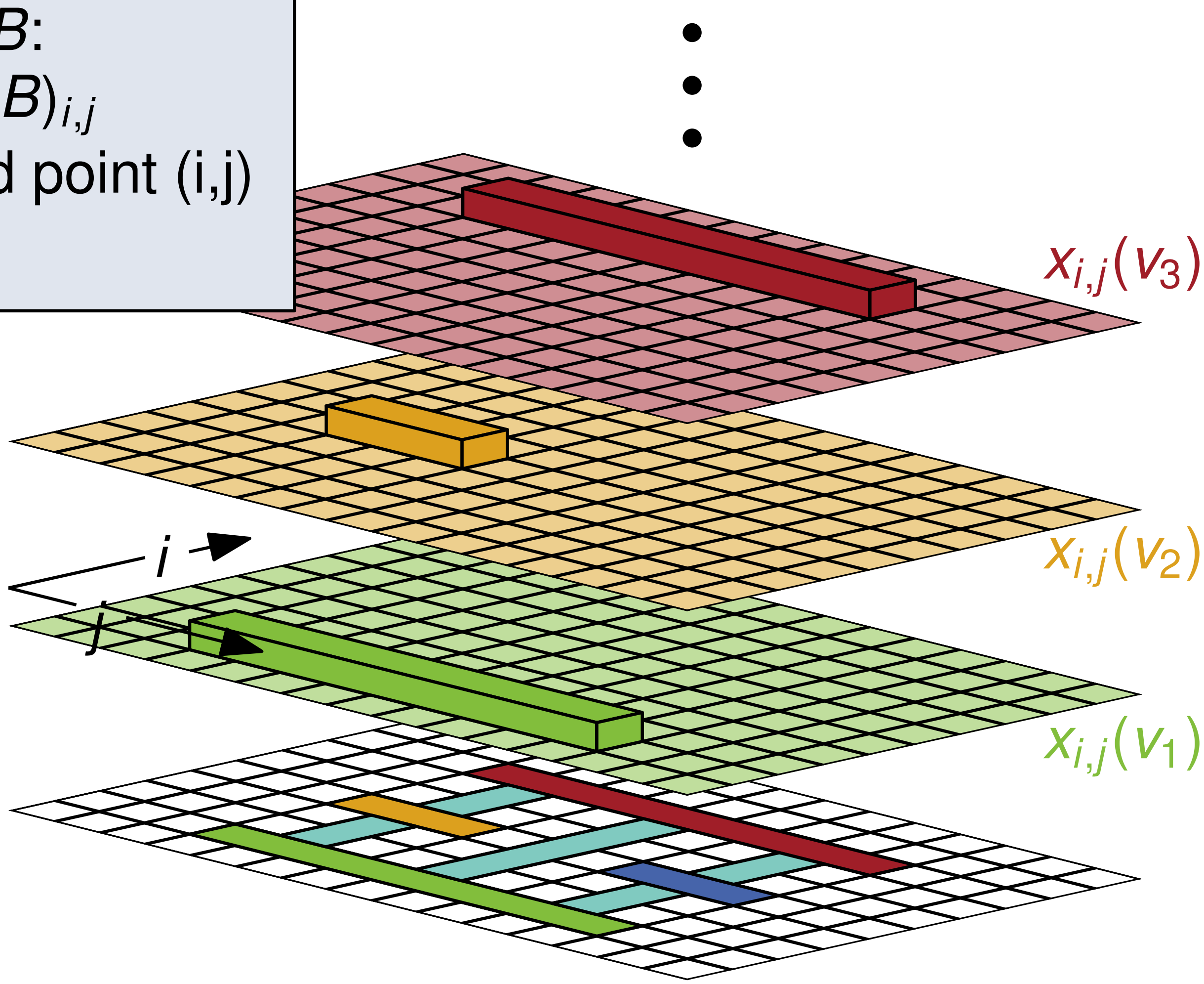

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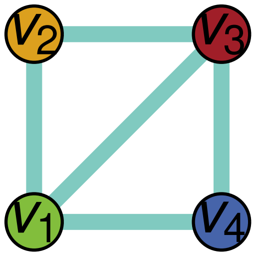

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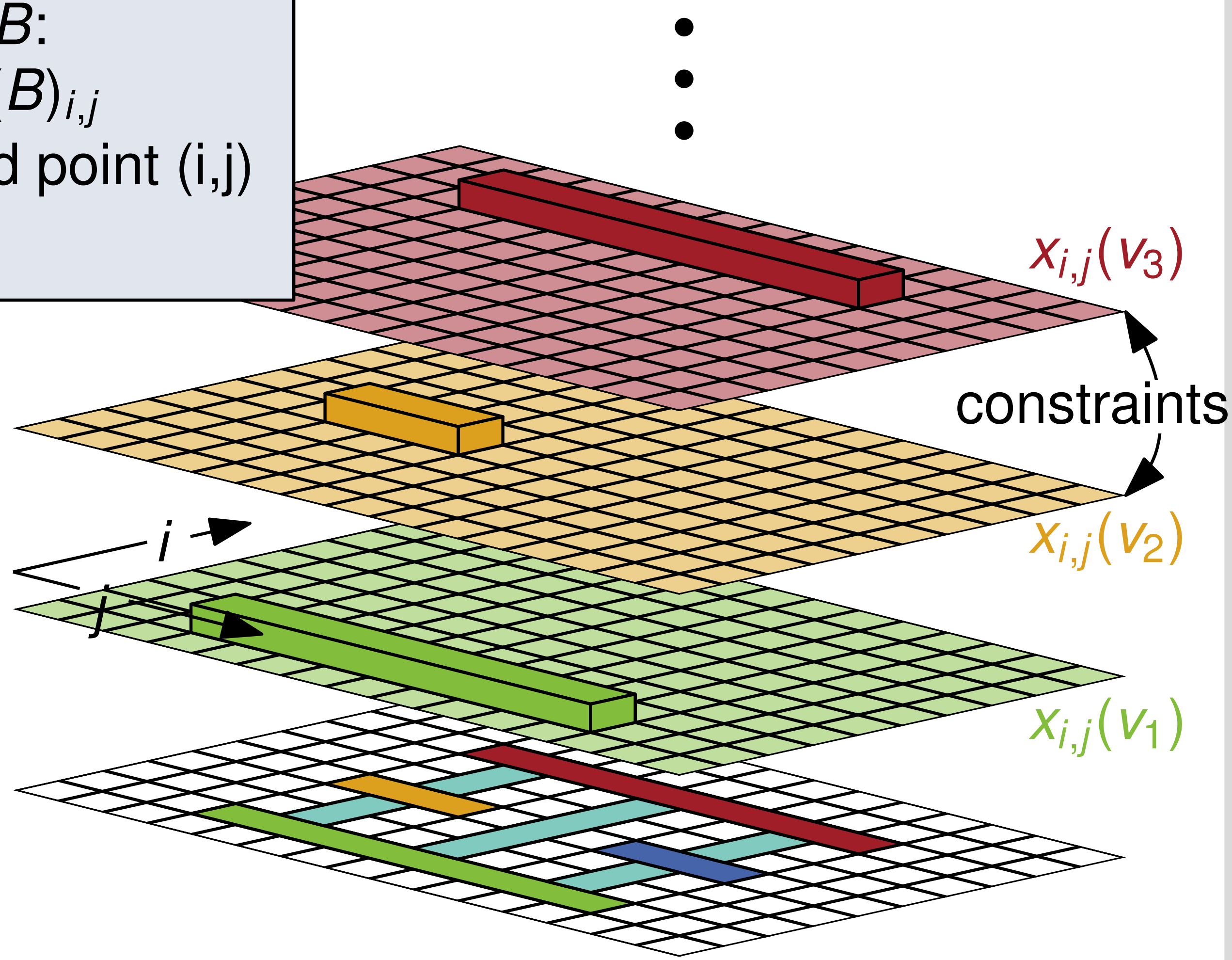

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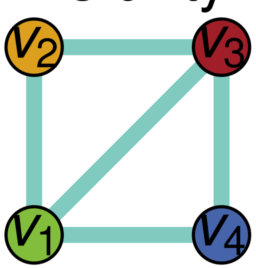

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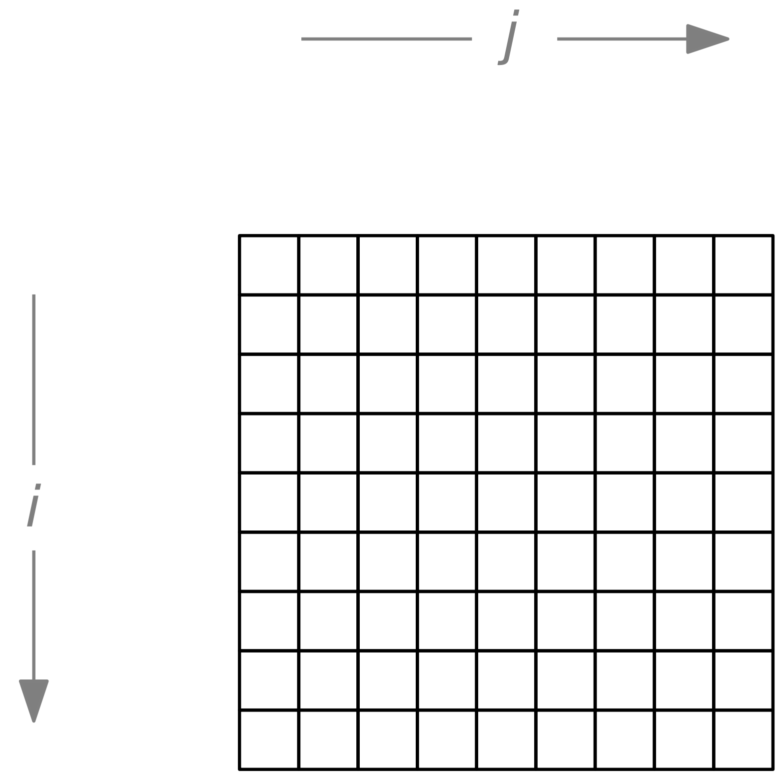
Representing Boxes

Represent box B by ILP constraints (here in 2D).

Grid of binary variables $x_{i,j}$



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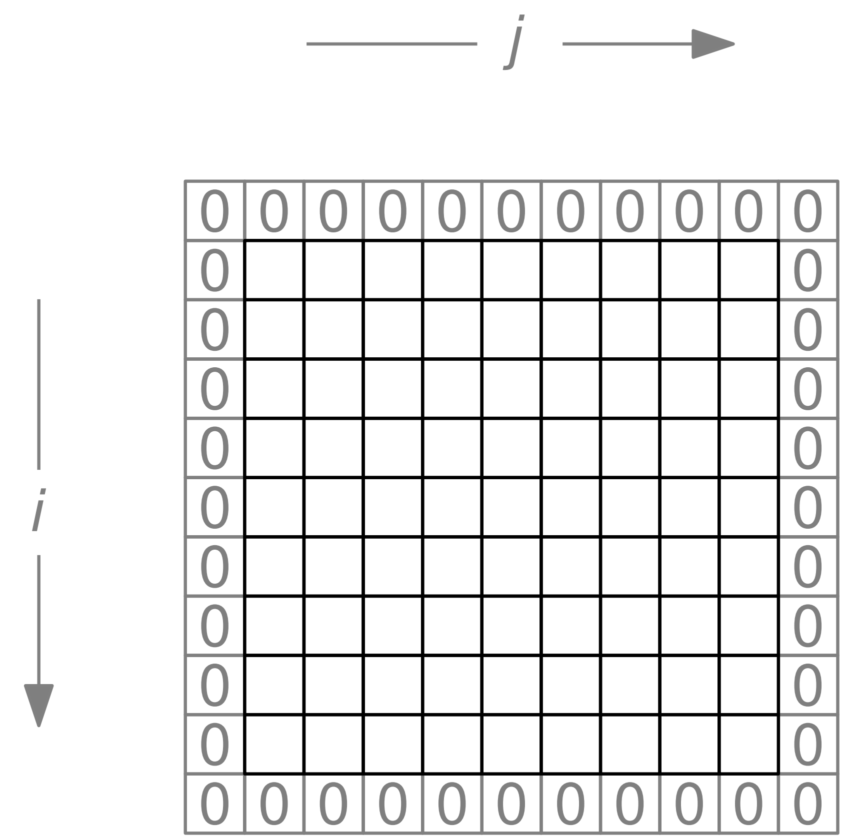

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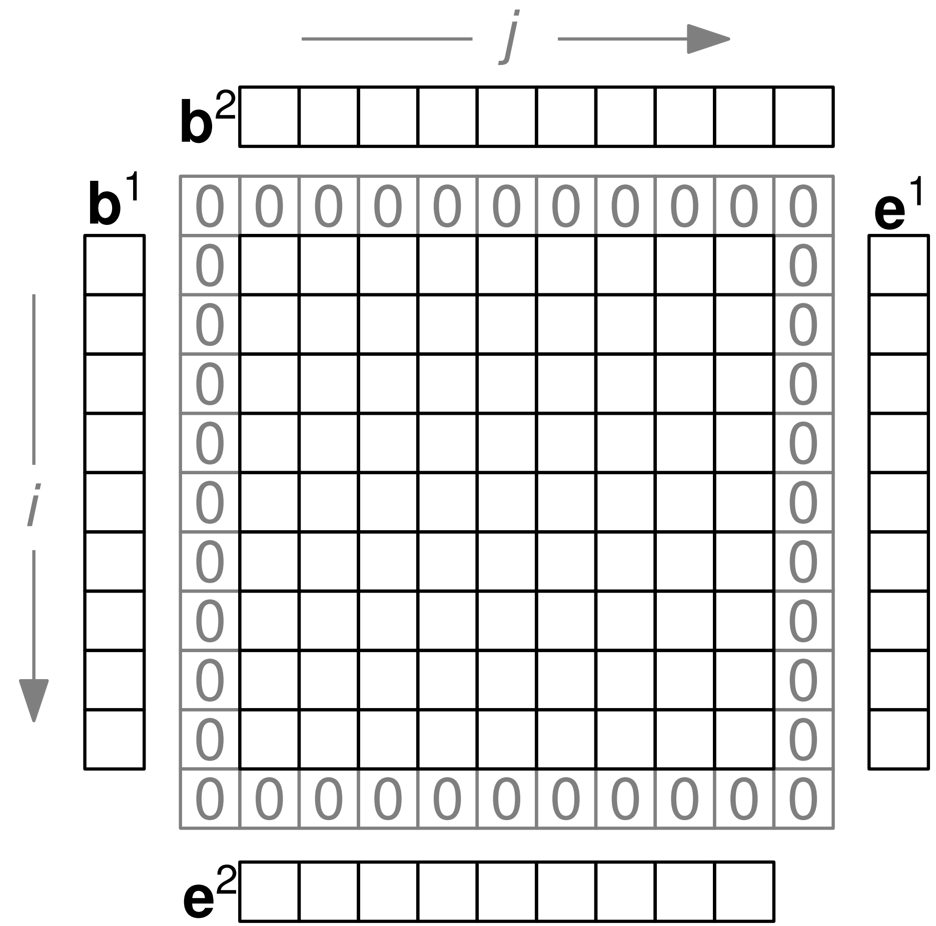

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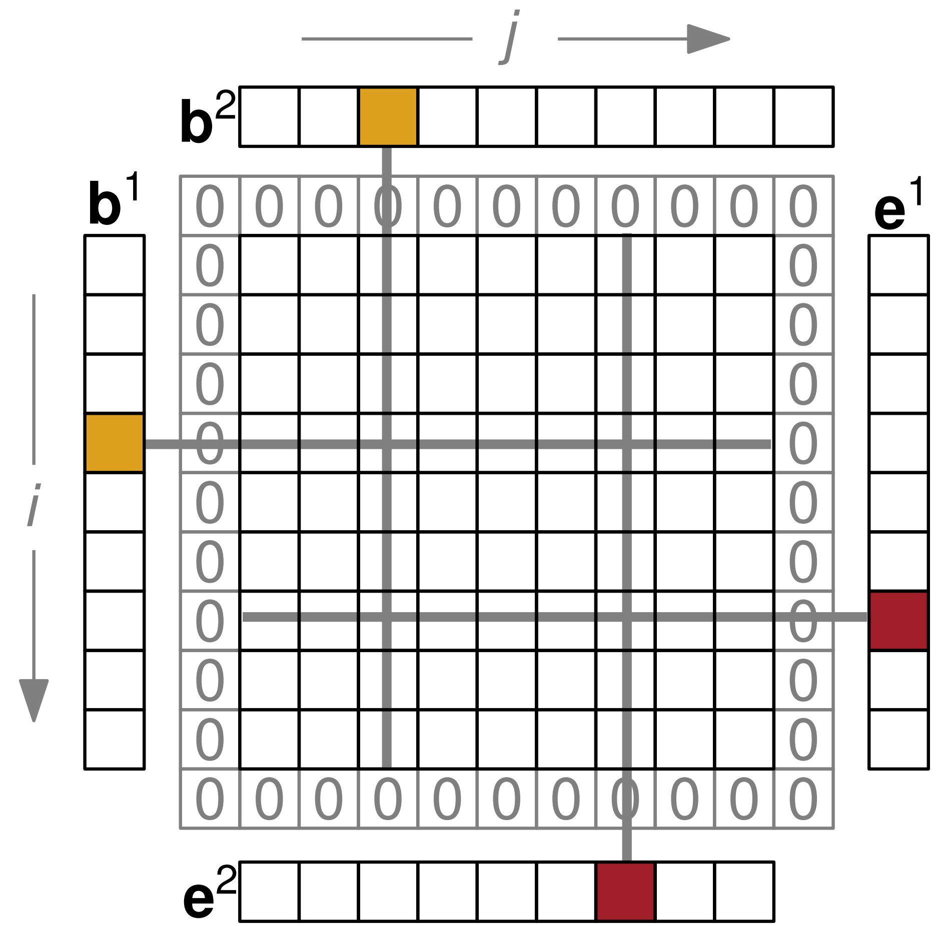
$\sum_i b_i^d = 1 \quad \sum_i e_i^d = 1$ for $d \in \{1, 2\}$



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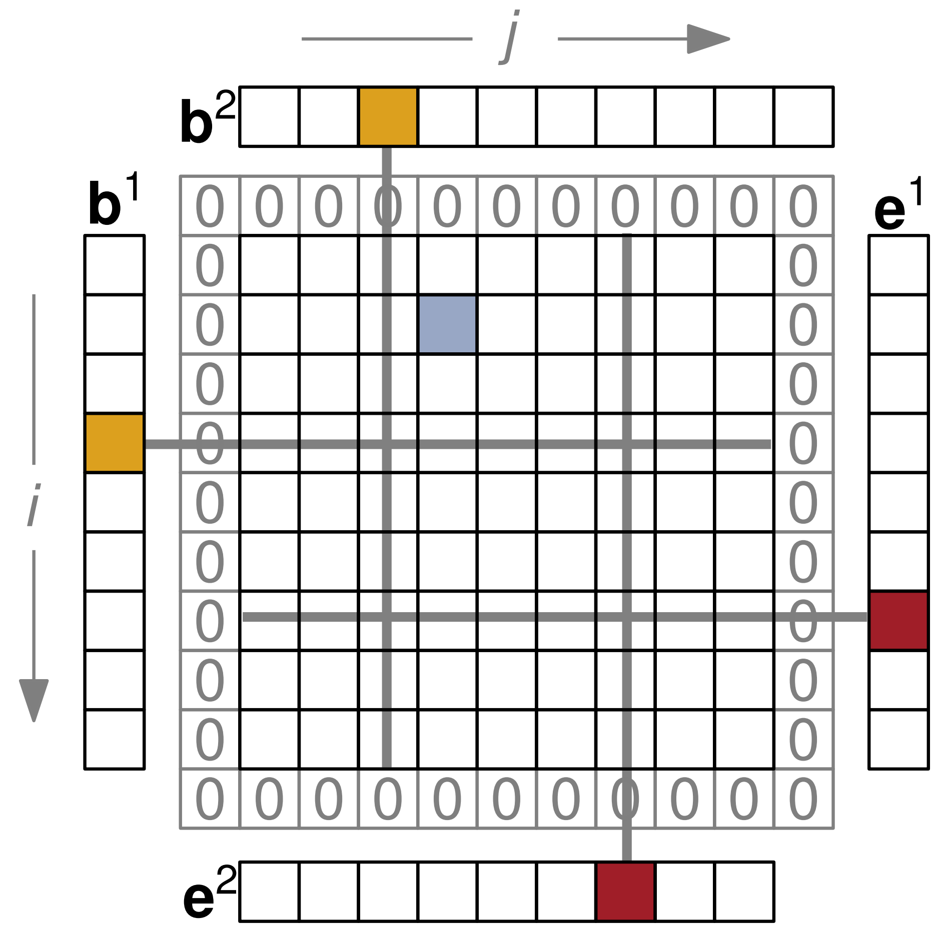
$\sum_{i,j} x_{i,j} \geq 1$



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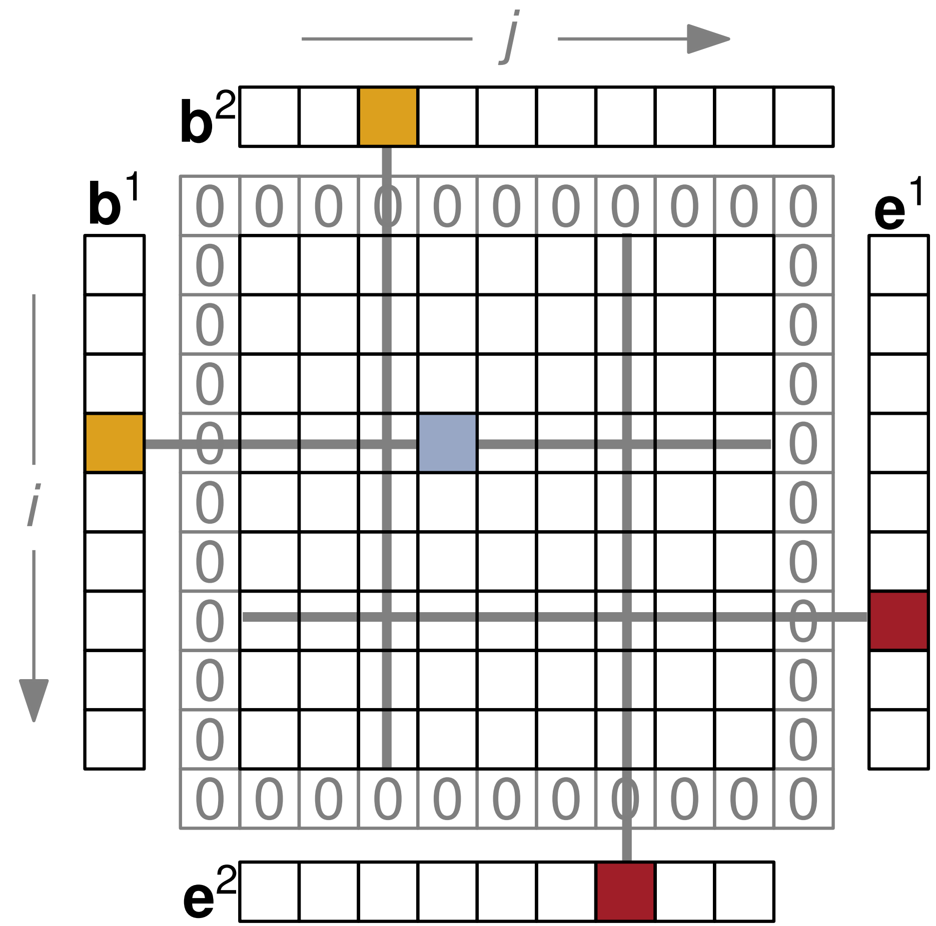

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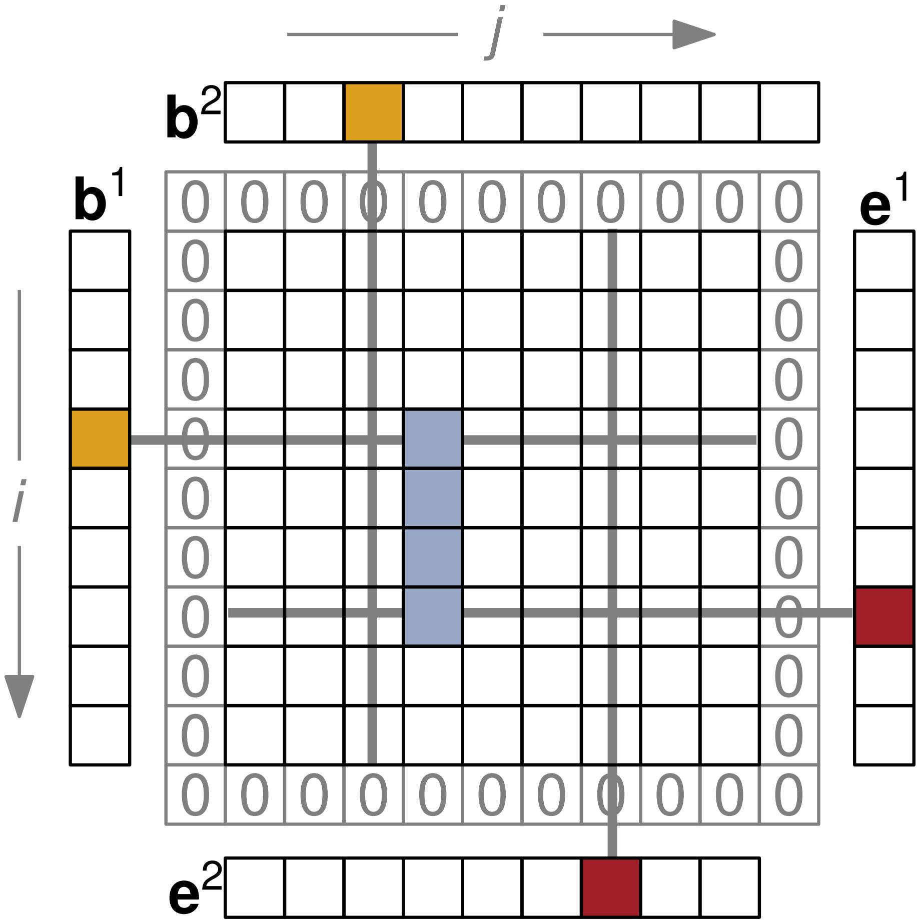

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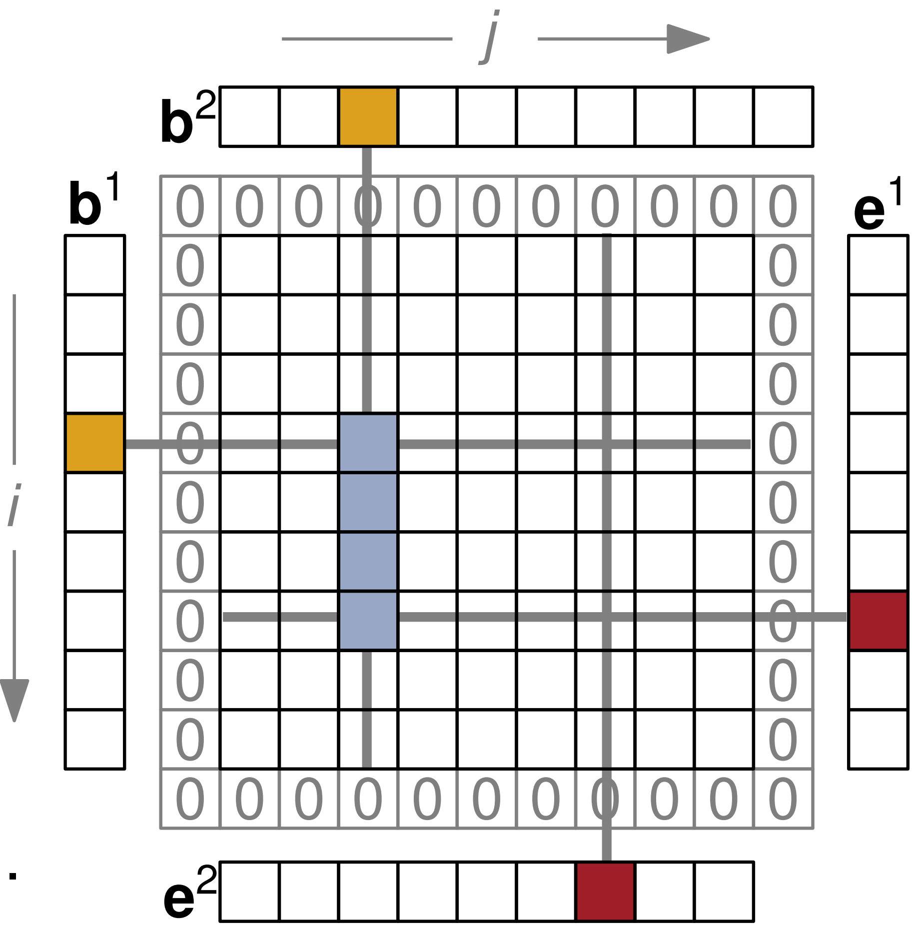

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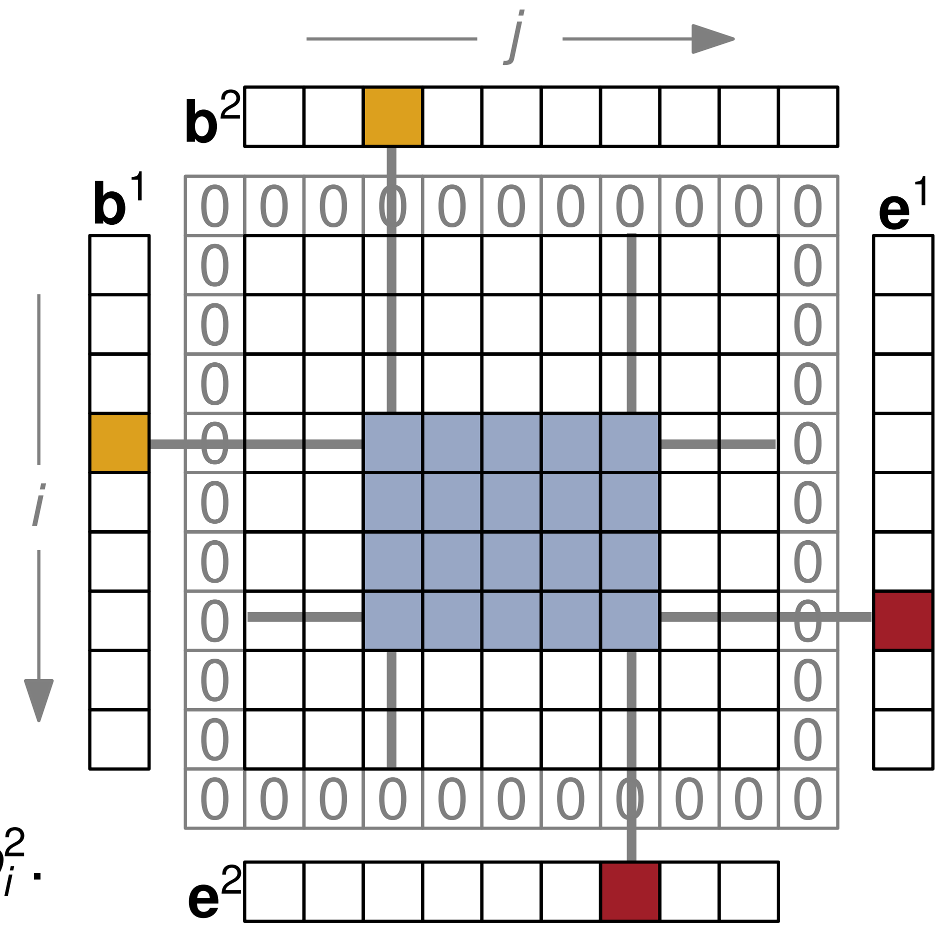

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Aufgaben:

- modelliere Höhe(B) = 1
- modelliere Höhe(B) = k
- modelliere $B_1 \cap B_2 = \emptyset$
- modelliere $B_1 \subset B_2$

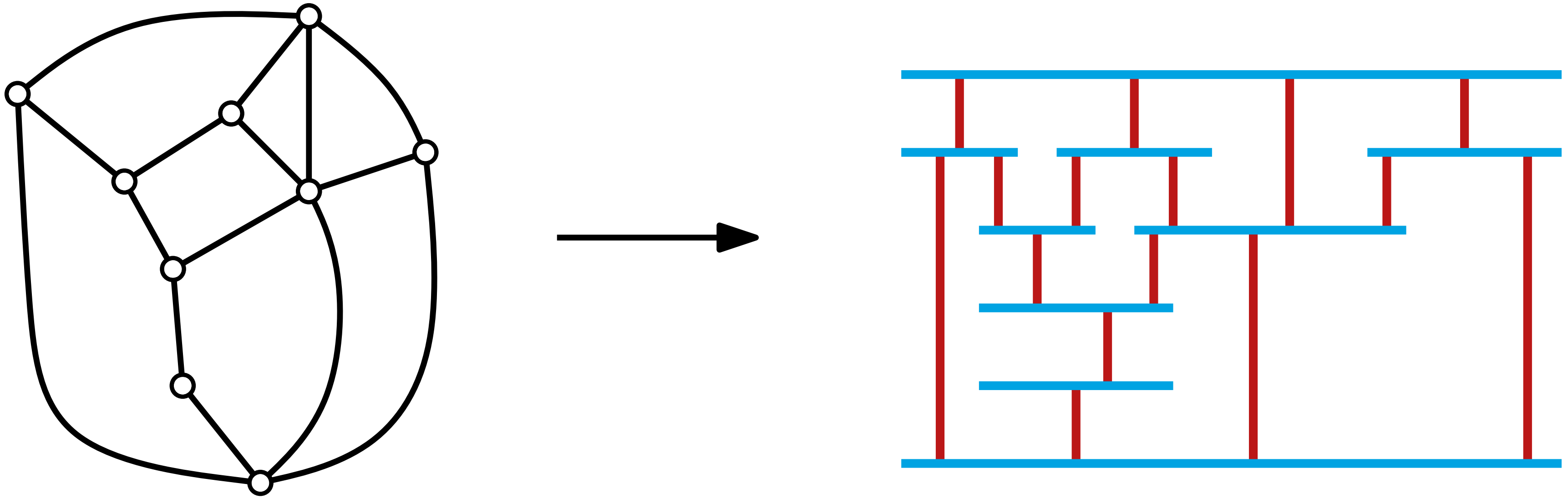
Visibility Representations

Def: A **visibility representation** of a graph $G = (V, E)$ draws

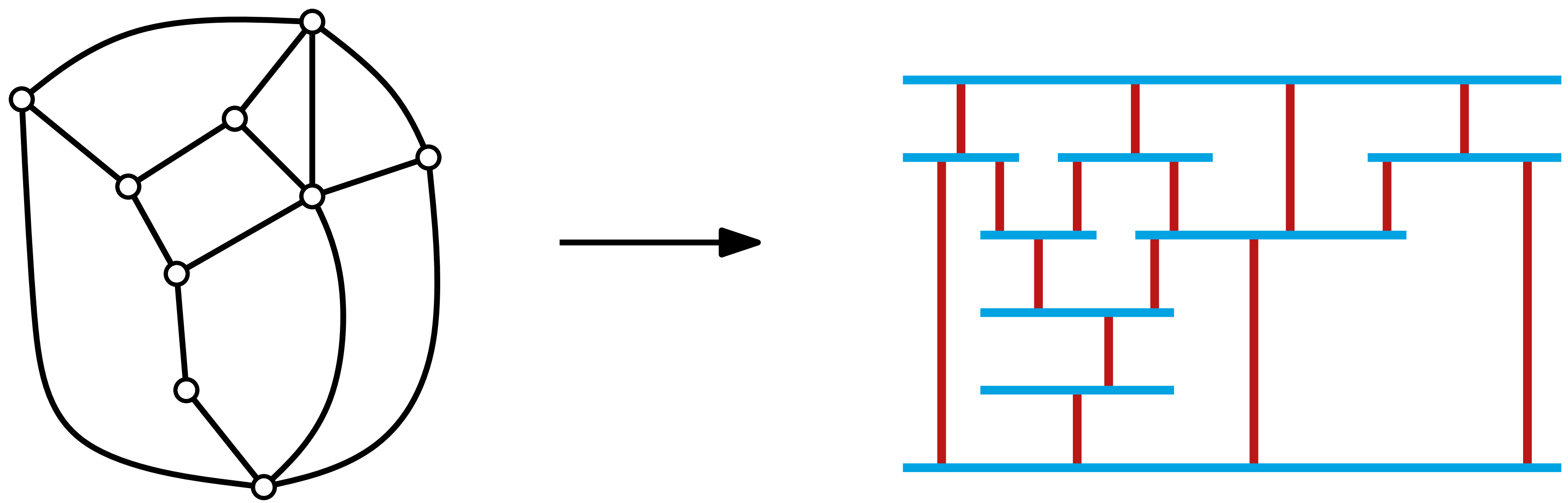
- each vertex v as a horizontal line segment $\Gamma(v)$
- each edge $e = (u, v)$ as a vertical line segment $\Gamma(e)$

such that

- no two vertex segments intersect
- no two edge segments intersect
- each edge segment $\Gamma(u, v)$ has its endpoints on $\Gamma(u)$ and $\Gamma(v)$ and intersects no other vertex segment



Properties of Visibility Representations



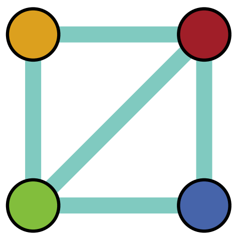
- graph G must be planar (obviously)
- every planar graph has a visibility representation

  [Wismath '85], [Tamassia, Tollis '86]

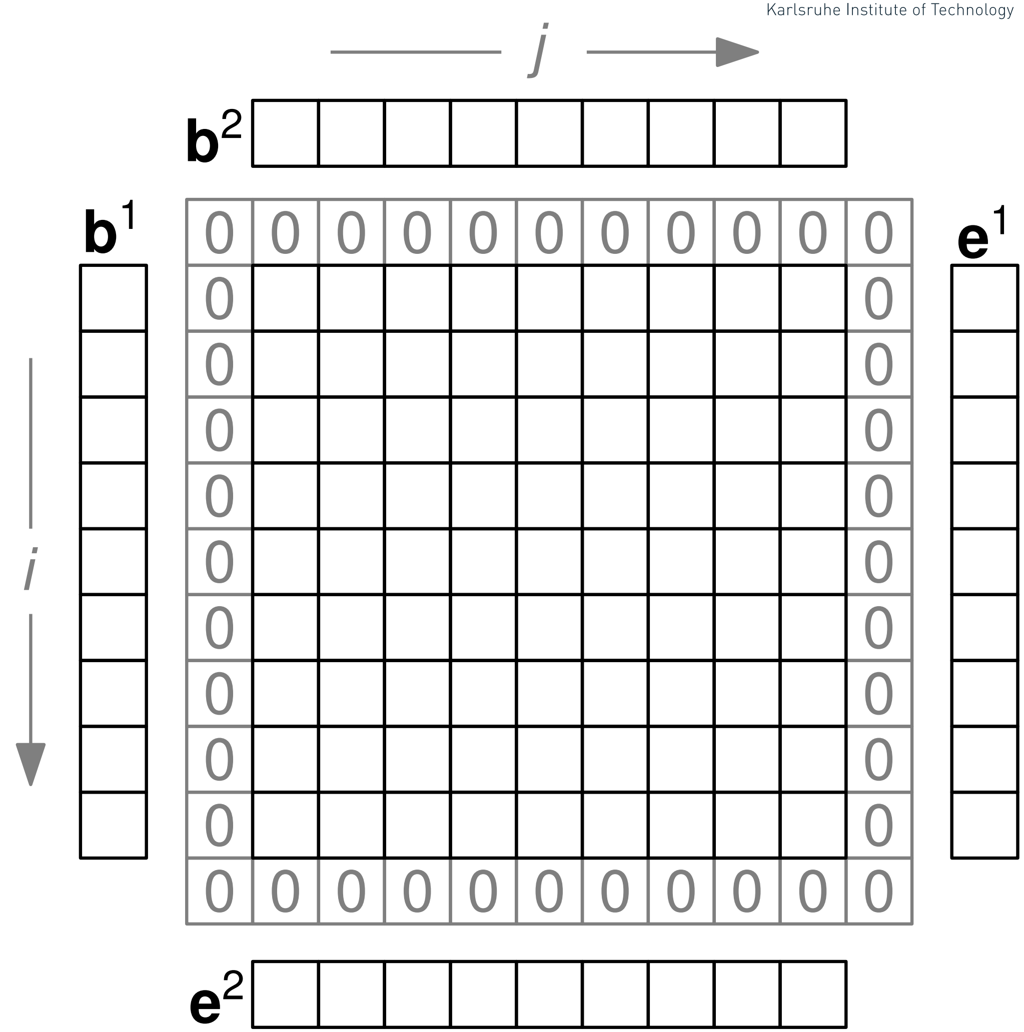
- minimizing the area of a visibility representation is NP-hard

  [Lin, Eades '03]

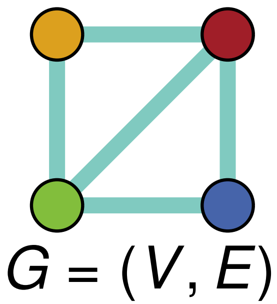
Modeling Visibility Representation



$G = (V, E)$



Modeling Visibility Representation



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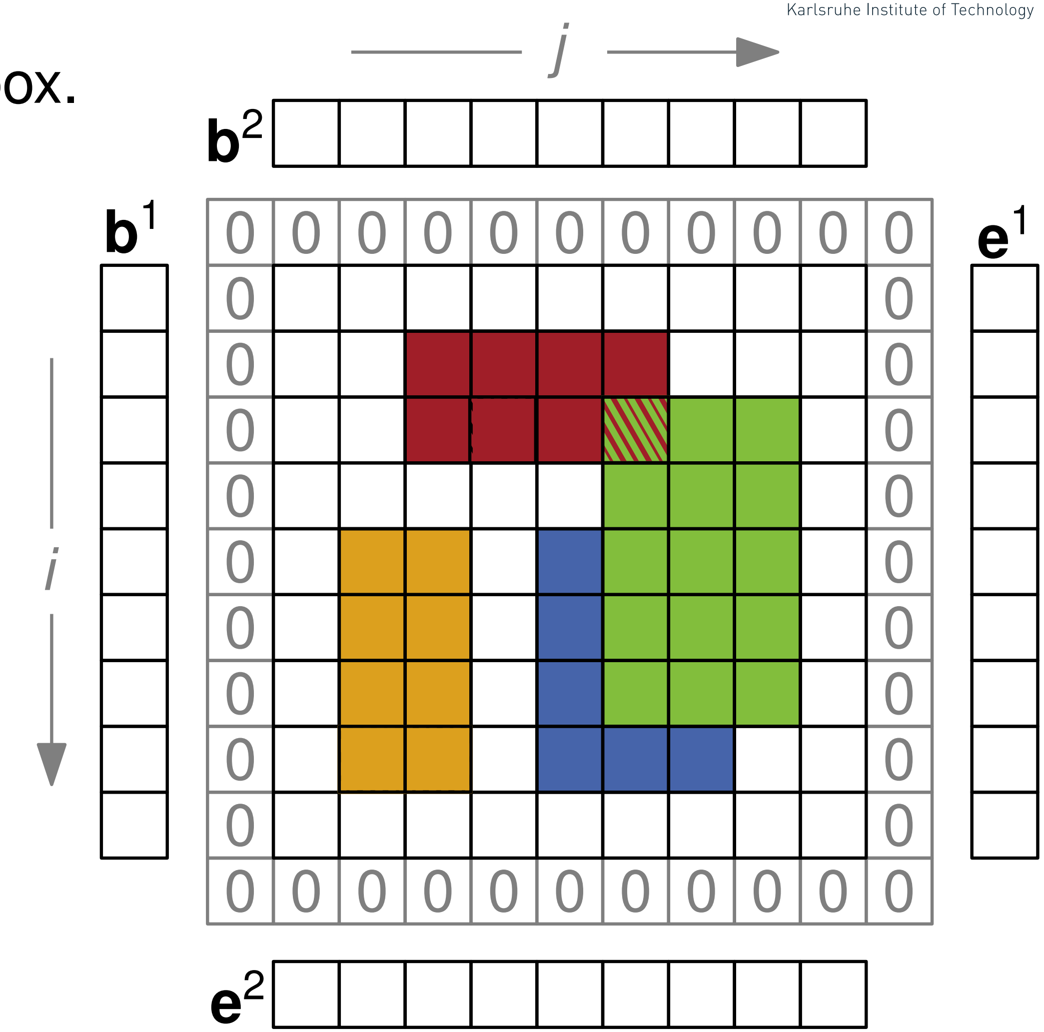

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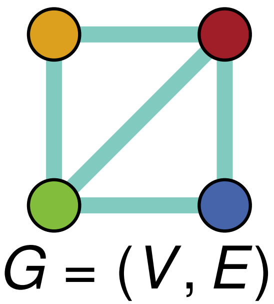

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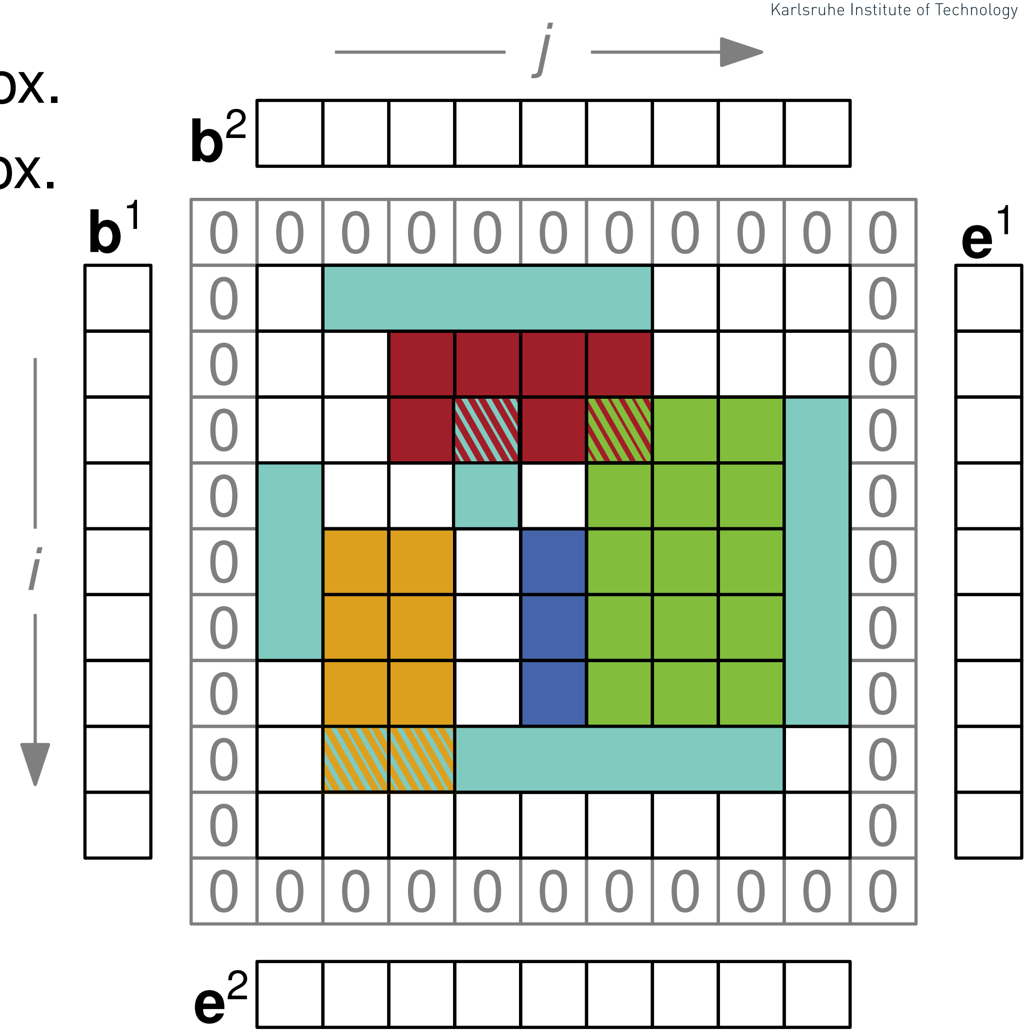

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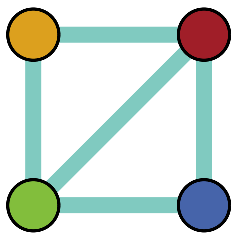
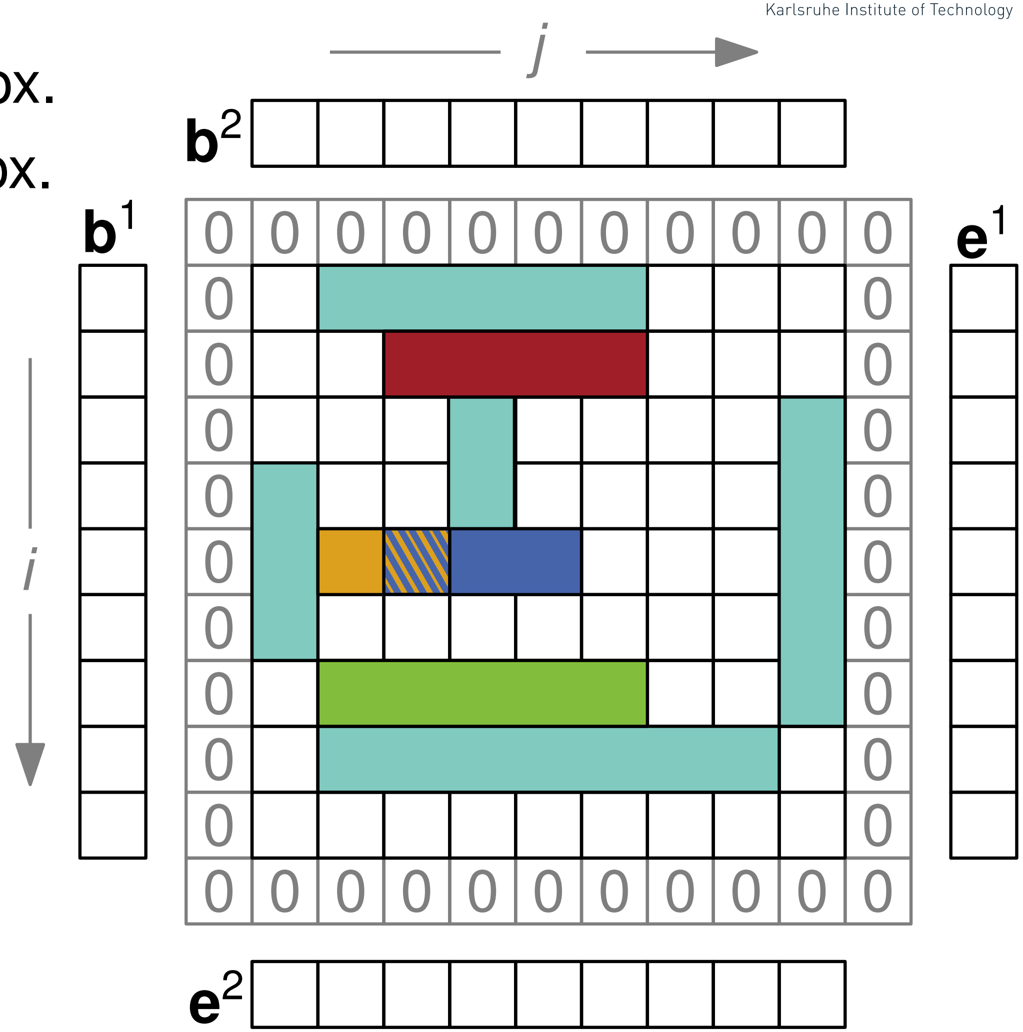


$G = (V, E)$

1.) $\forall v \in V$ introduce box.

2.) $\forall e \in E$ introduce box.

3.) Vertices are horizontal segments:



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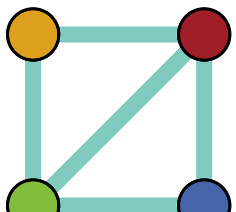

$G = (V, E)$

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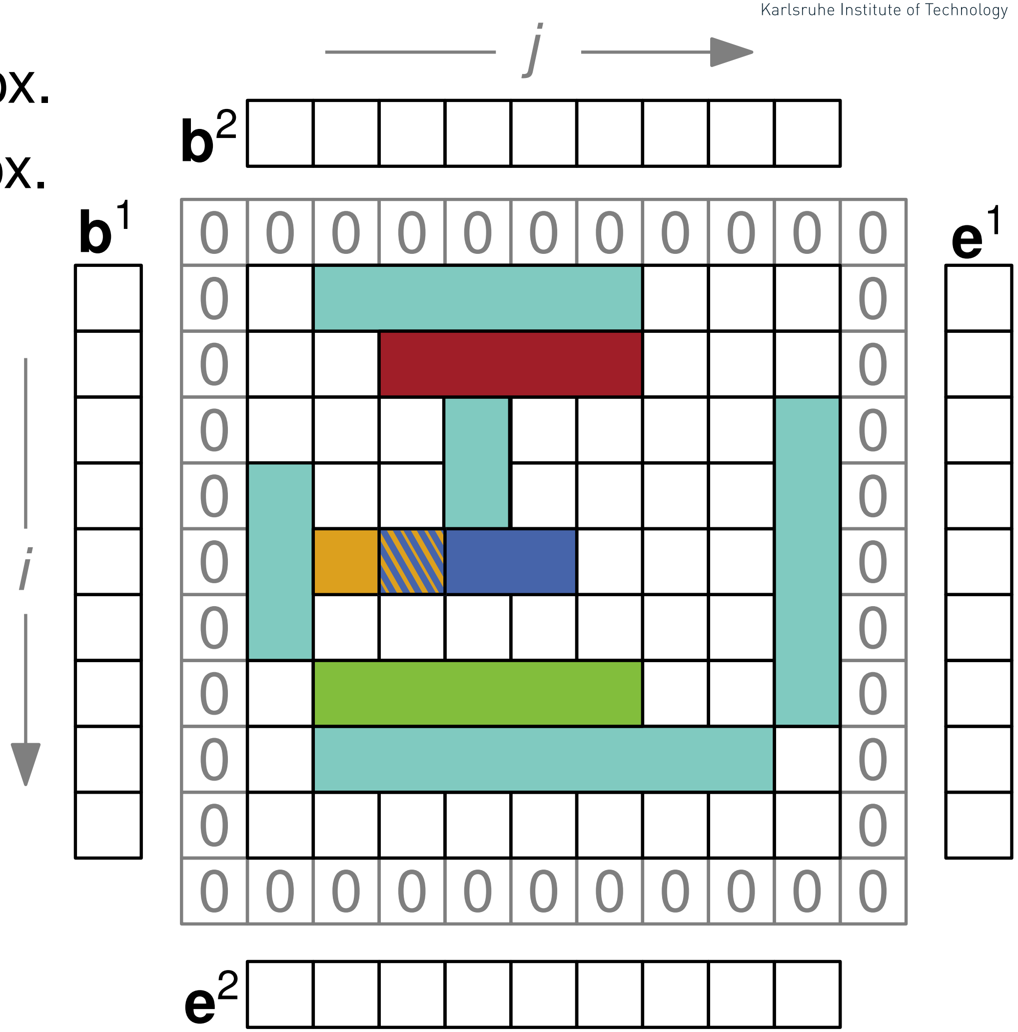
$$b_i^1(v) = e_i^1(v)$$



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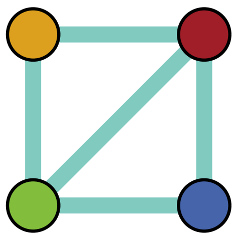

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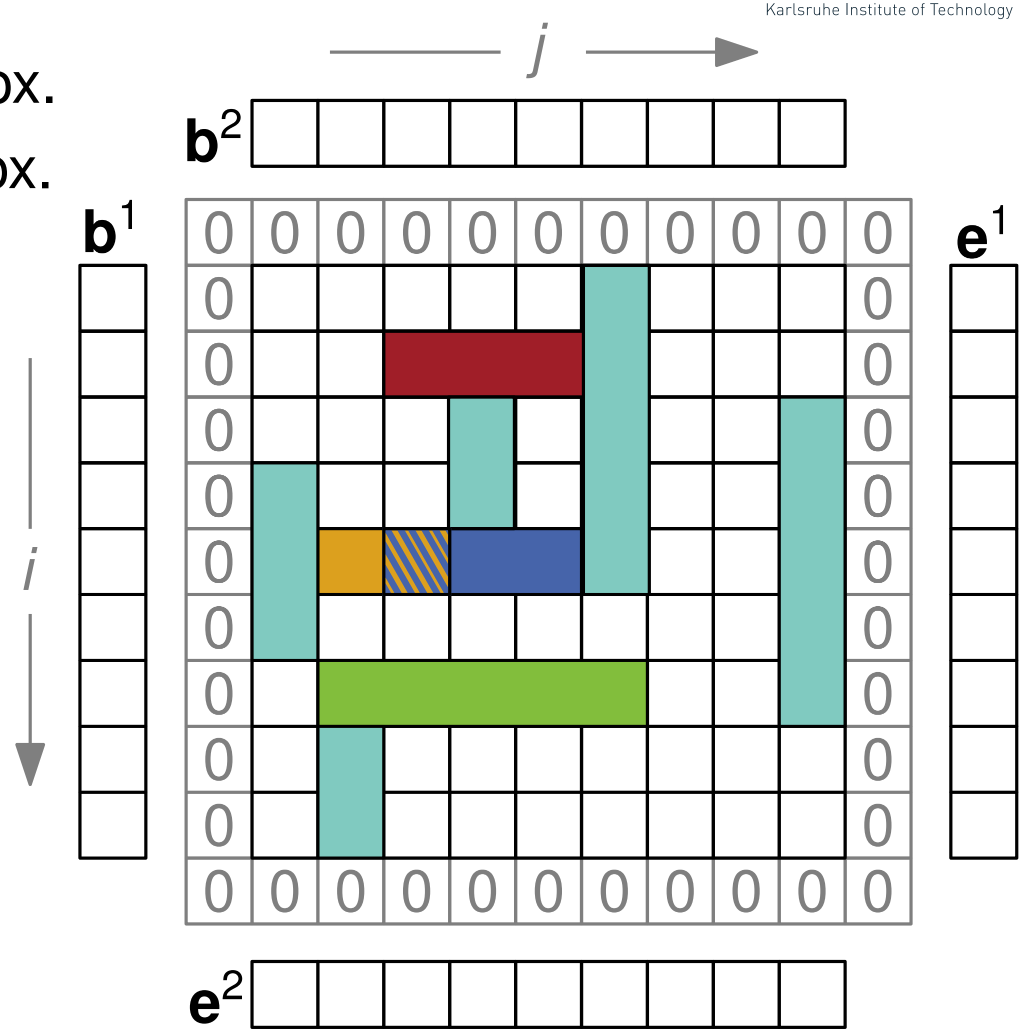

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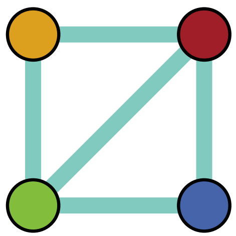

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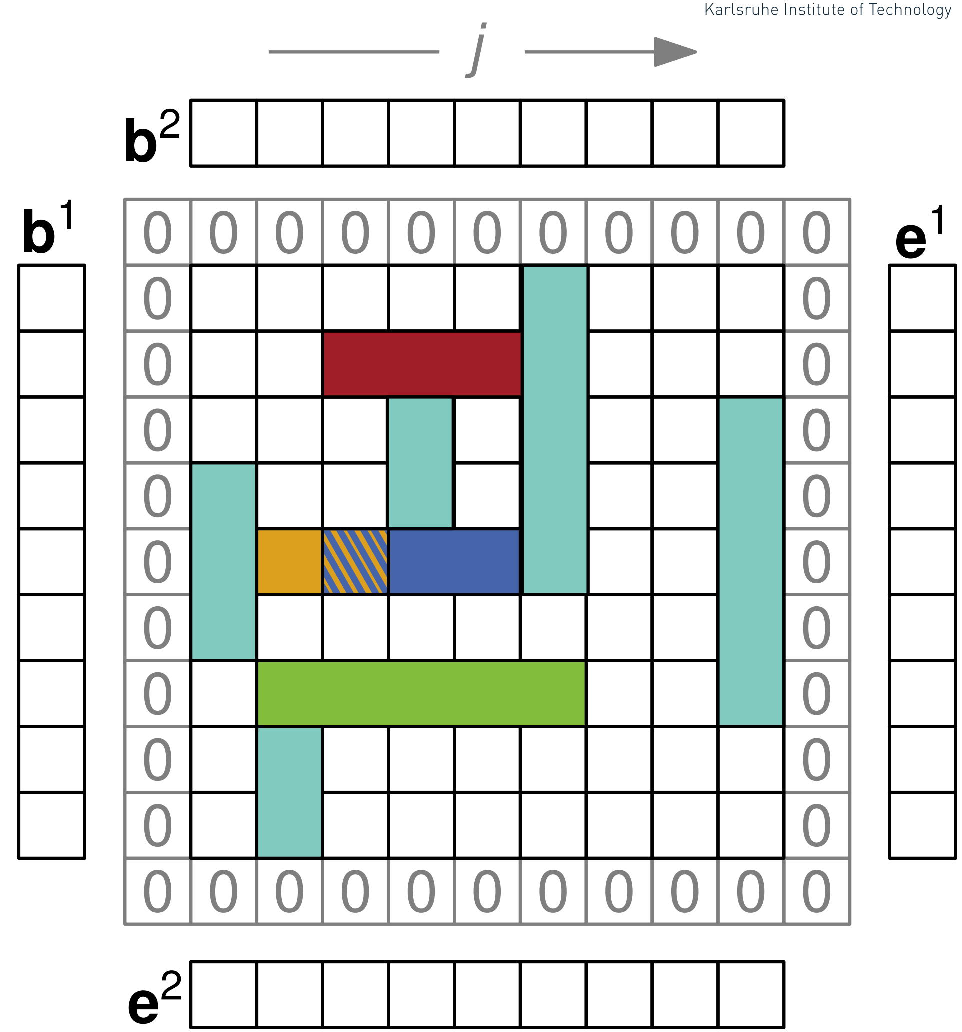

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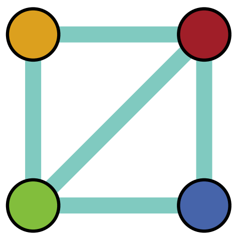

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3.) Vertices are horizontal segments:

$$b^1_i(v) = e^1_i(v)$$

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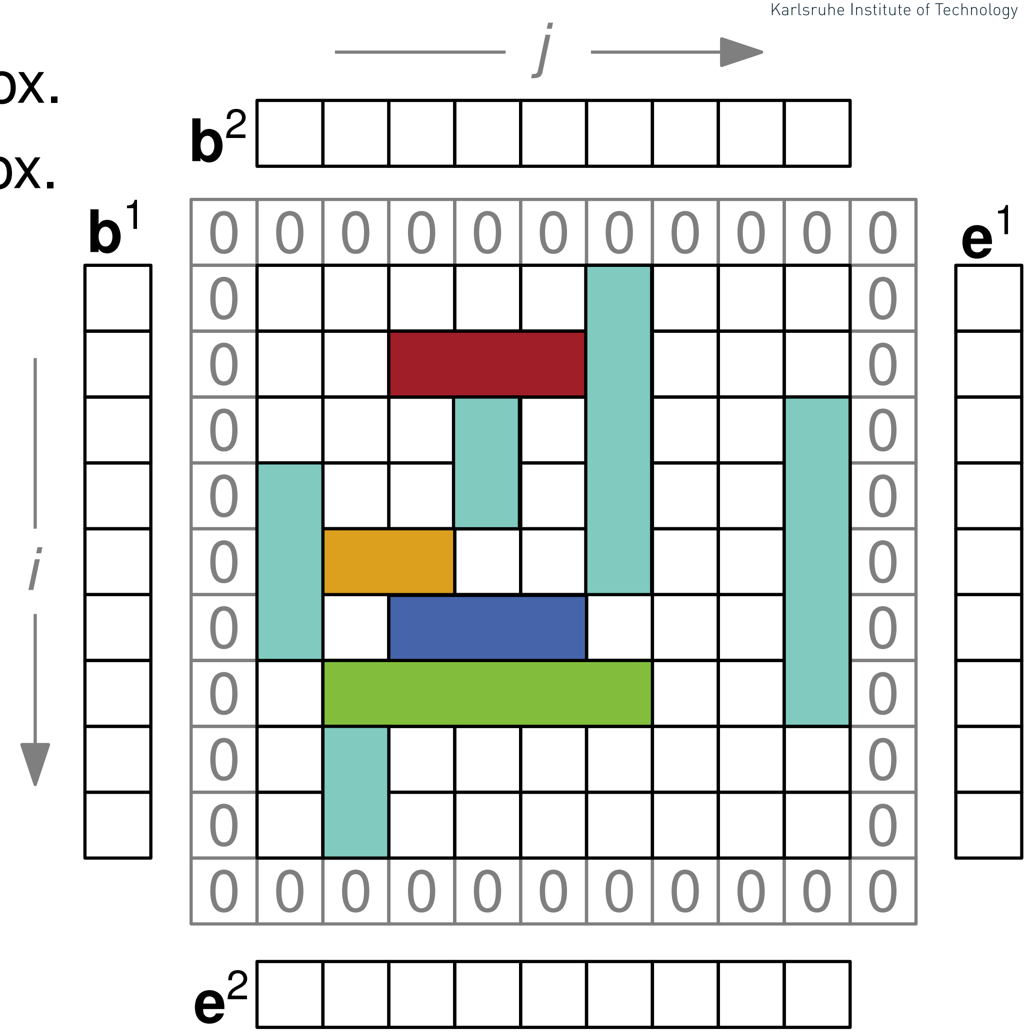

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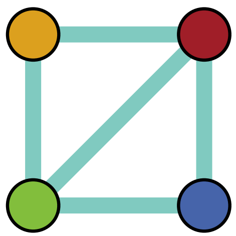

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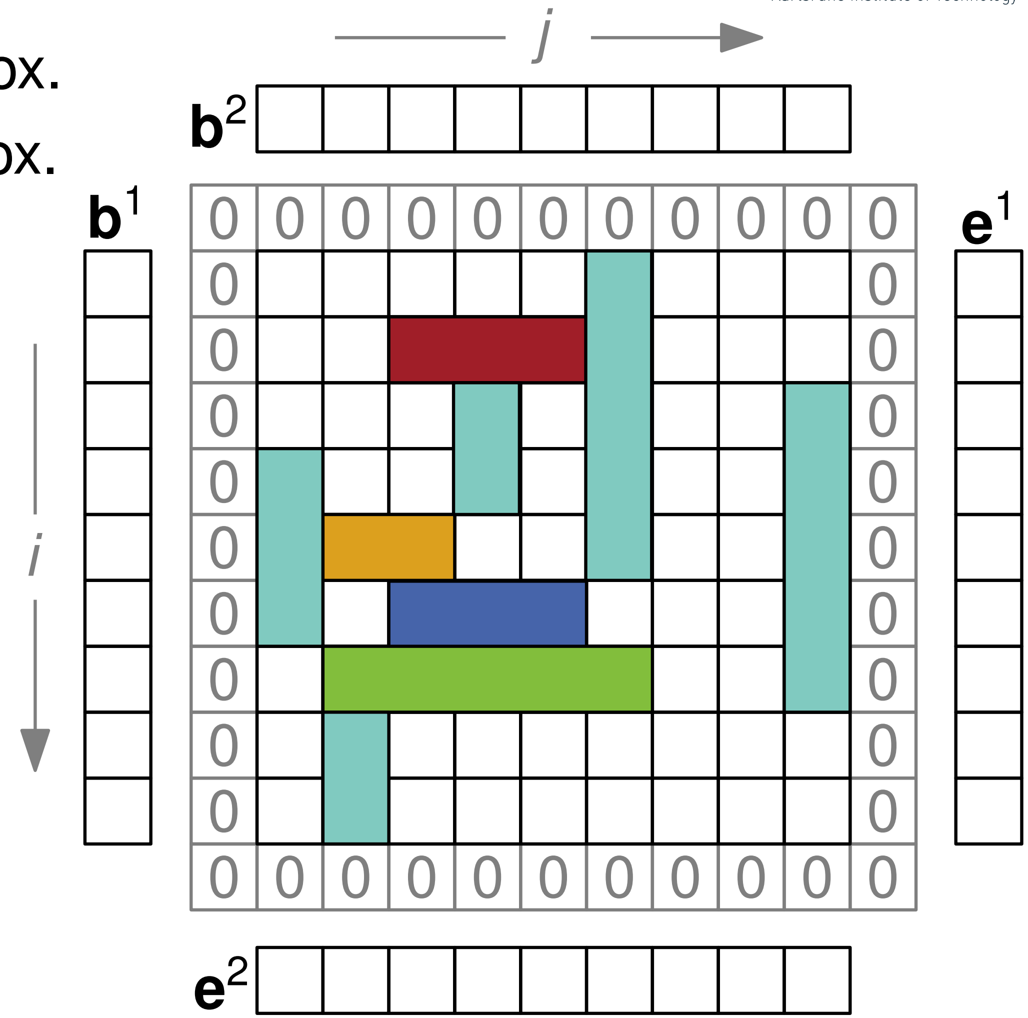

3.) Vertices are horizontal segments:

$$b_i^1(v) = e_i^1(v)$$

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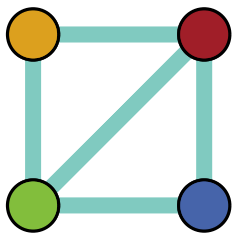
5.) Vertices do not overlap:

$$\sum_{v \in V} x_{i,j}(v) \leq 1$$

Modeling Visibility Representation



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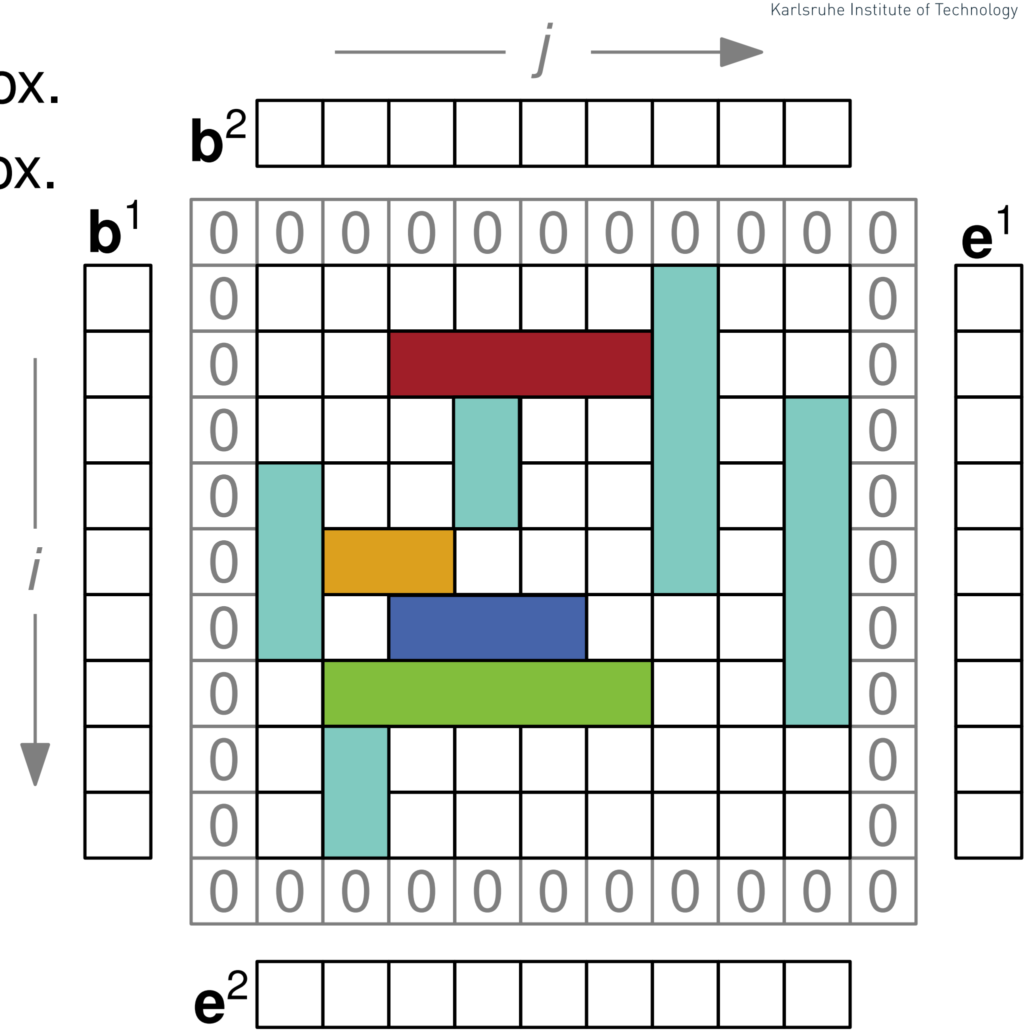

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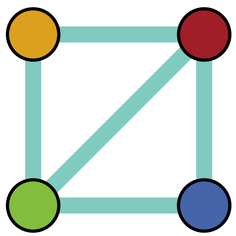

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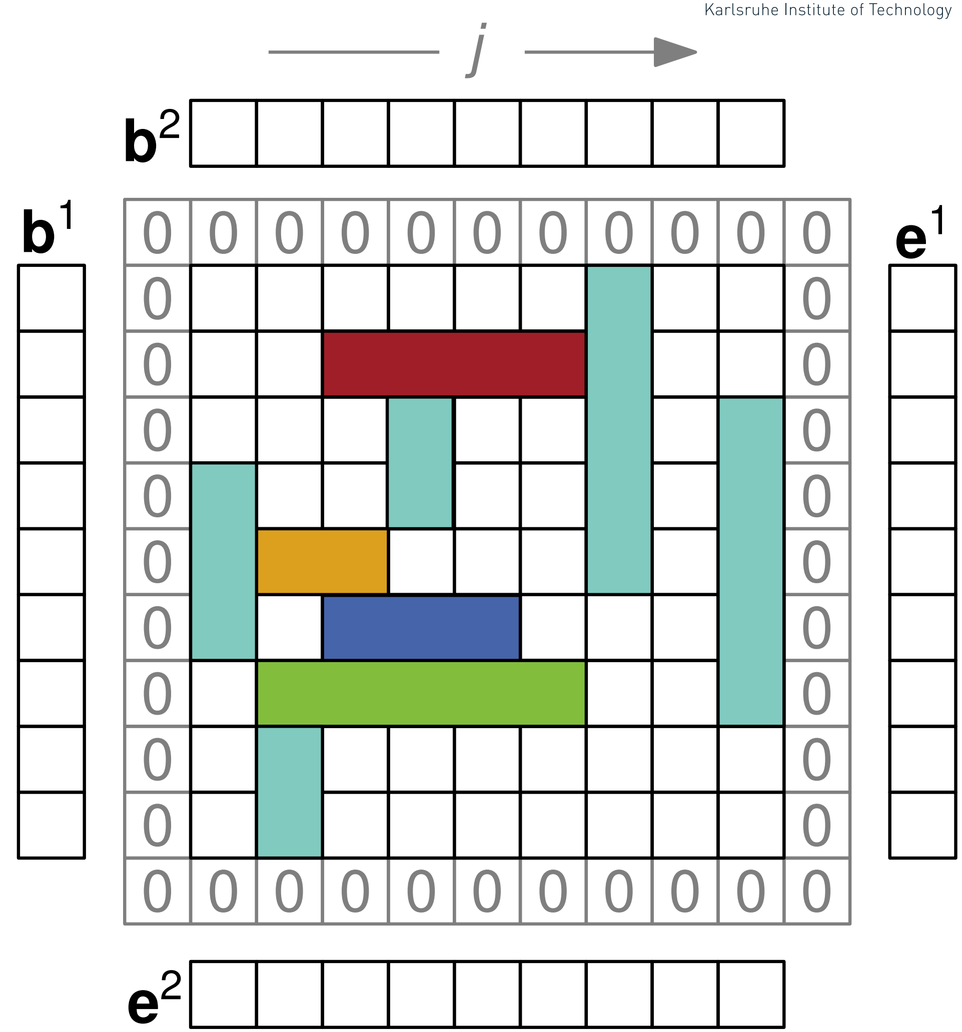

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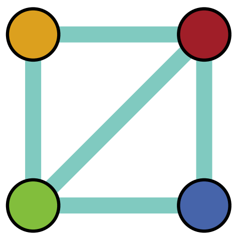
6.) Edges do not overlap non-incident vertices:

$$\sum\nolimits_{v \in V \setminus e} x_{i,j}(v) \leq (1 - x_{i,j}(e)) \text{ for all } e \in E$$

Modeling Visibility Representation



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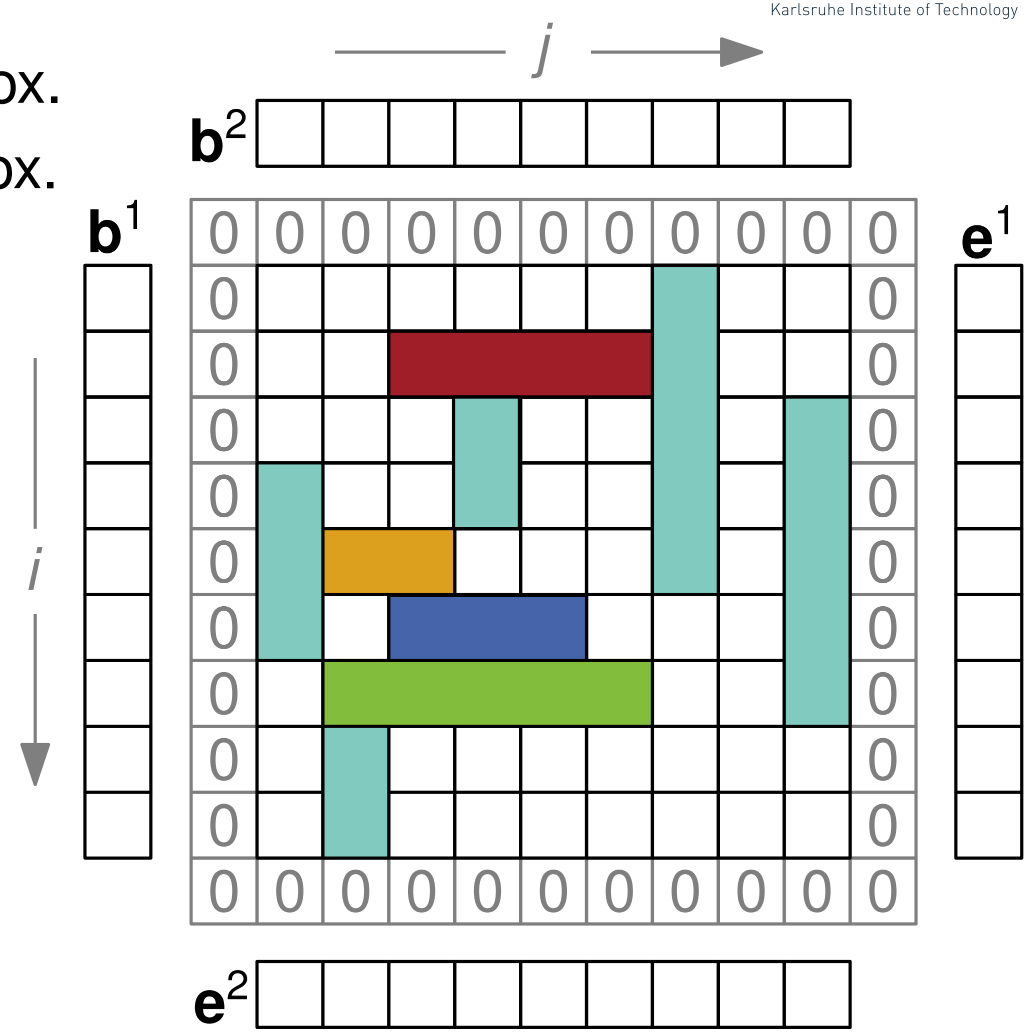

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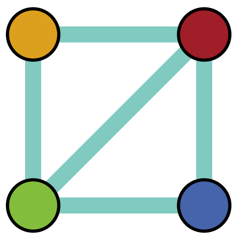
Modeling Visibility Representation



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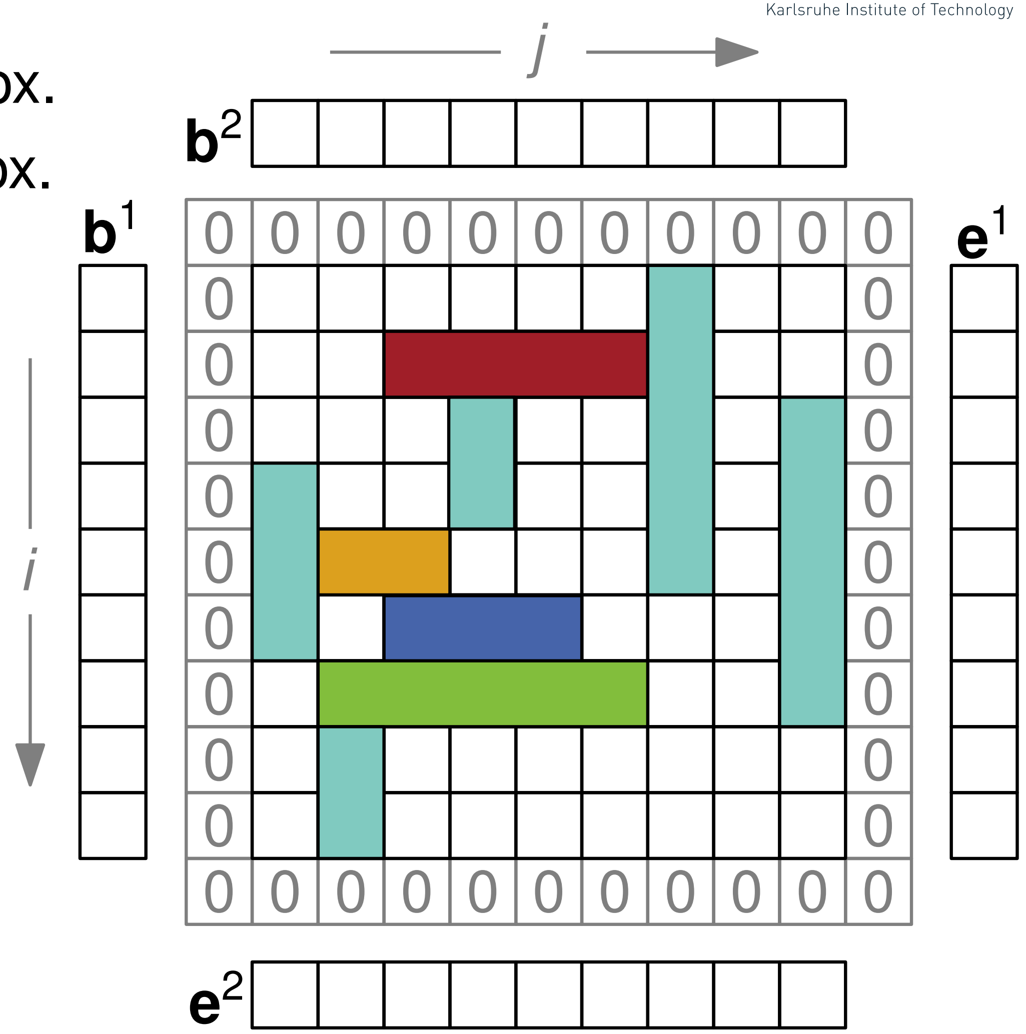
For all $e = \{u, v\} \in E$:

a.) Grid of binary variables

$x_{i,j}(e, u)$ $x_{i,j}(e, v)$



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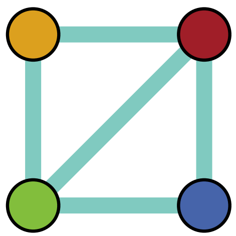

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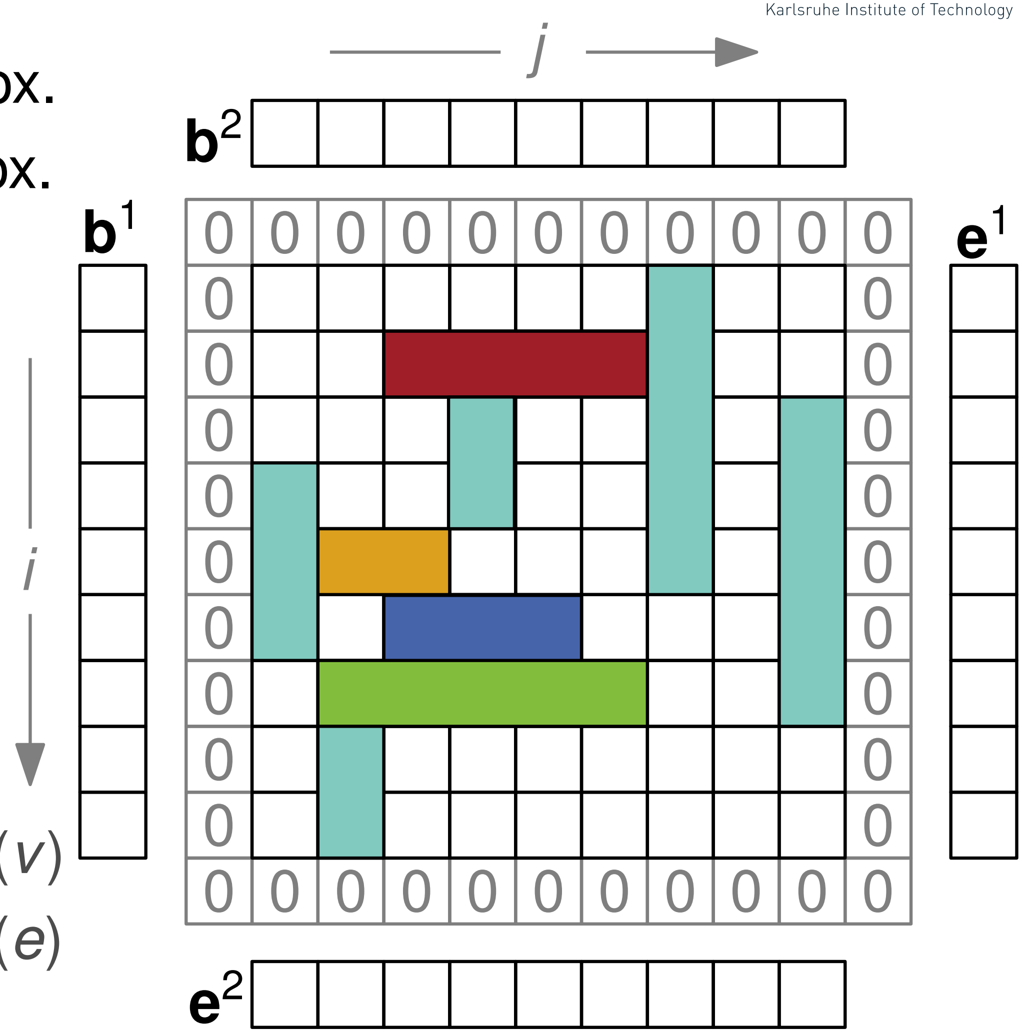
a.) Grid of binary variables

$x_{i,j}(e, u) \quad x_{i,j}(e, v)$

b.) Common point

$x_{i,j}(e, u) \leq x_{i,j}(u) \quad x_{i,j}(e, v) \leq x_{i,j}(v)$

$x_{i,j}(e, u) \leq x_{i,j}(e) \quad x_{i,j}(e, v) \leq x_{i,j}(e)$



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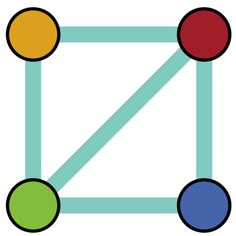

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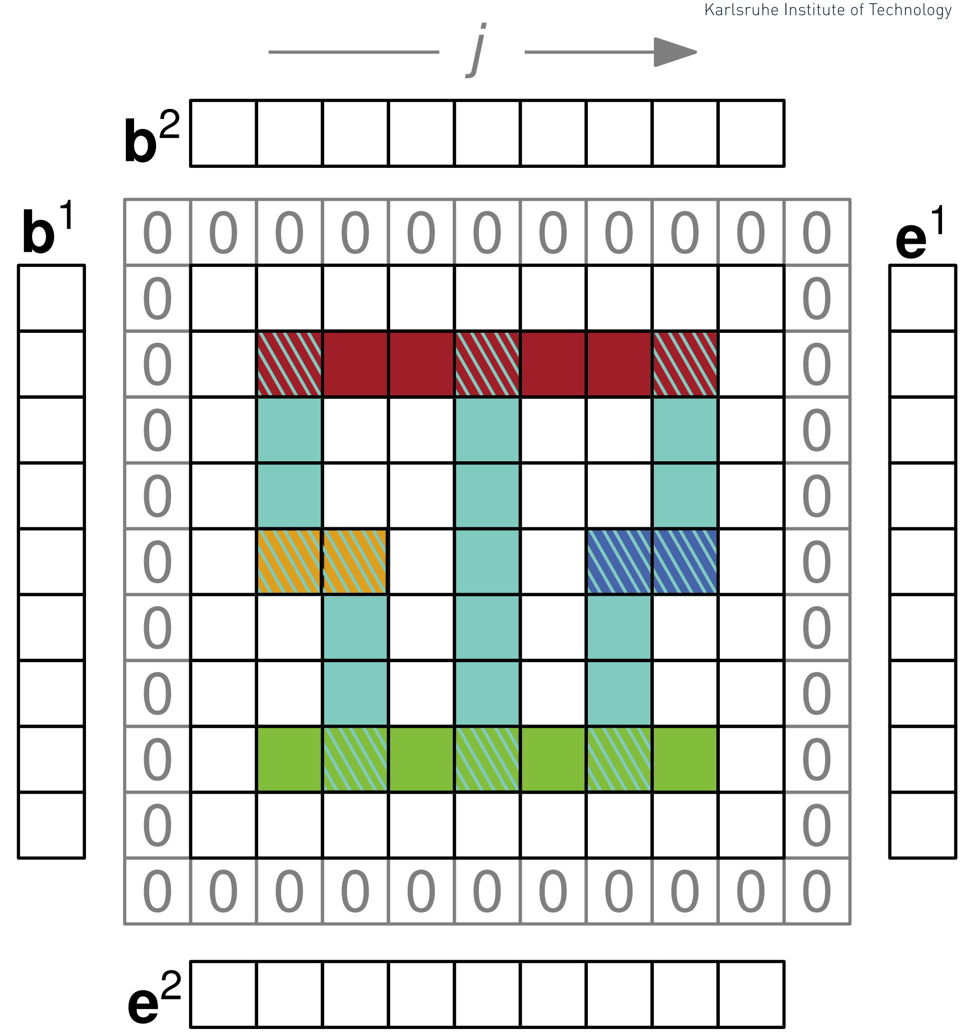
c.) Existence of common point.

$\sum_{i,j} x_{i,j}(e, u) \geq 1$ $\quad \sum_{i,j} x_{i,j}(e, v) \geq 1$



Optimum *st*-orientation

Recall: An *st***-orientation** of an undirected graph $G = (V, E)$ and two vertices $s, t \in V$ with $(s, t) \in E$ is an orientation of the edges in E s.t.

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- ingredient for orthogonal layout algorithm (see lecture 14)
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Find minimum-length *st*-orientation using 1D grid-based model.

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- flexible framework for modeling many grid-based graph layout problems
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But is it easy to use in practice?

PIGRA Demo (Fabian Klute)