Karlsruher Institut für Technologie
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Algorithmen zur Visualisierung von Graphen
WS 14/15
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Exercise Sheet 2

Assignment: November 28, 2014
Delivery: None, Discussion on December 8, 2014

1 Generalization of Canonical Ordering for Triconnected Planar Graphs

The canonical ordering (which until now has been defined only for maximal planar graphs) can be generalized to triconected planar graphs as follows. Let $G = (V, E)$ be a triconnected planar graph. An ordering $v_1, \ldots, v_n$ of the vertices of G is called *canonical* if vertices v_2, v_n are neighbors of v_1, and v_1, v_2, v_n belong to a common face, and for every k, $k > 3$:

(a) Vertex v_k is on the outer face of G_k and has at least two neighbors in G_{k-1}, which are on the outer face of G_{k-1}. Vertex v_k has at least one neighbor in $G - G_k$. Graph G_k is biconnected,
(b) or there exists an $\ell \geq 1$ such that $v_k, \ldots, v_{k+\ell}$ is a chain on the outer face of $G_{k+\ell}$ and has exactly two neighbors in G_{k-1}, which are on the outer face of G_{k-1}. Every vertex $v_k, \ldots, v_{k+\ell}$ has at least one neighbor in $G - G_{k+\ell}$. Finally, $G_{k+\ell}$ is biconnected.

By G_k we denote the subgraph of G induced by the vertices $v_1, \ldots, v_k$ and by $G - G_k$ the subgraph of G resulting from G by removing the vertices and edges of G_k.

Prove that every triconnected planar graph admits a canonical ordering.

2 Constrained Canonical Ordering

Let G be a maximal plane graph with vertices v_1, v_2, v_n on the outer face. Let P be a simple path in G connecting vertices v_1 and v_2 and not containing v_n. Let G_P be the subgraph of G bounded by path P and edge (v_1, v_2). Prove that there exists a canonical ordering of G such that all the vertices of G_P appear as initial subsequnce of this ordering.

3 Visibility Representation of Maximal Planar Graphs

Recall the definition of *visibility representation* from the previous exercise set. Prove that every maximal planar graph admits a visibility representation.

Hint: Use canonical ordering.

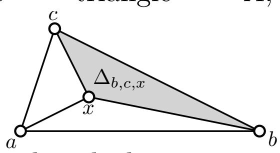
4 Barycentric Coordinates

Let $\Delta_{a,b,c}$ be a triangle on the plane on vertices a, b and c. For each point x laying inside triangle $\Delta_{a,b,c}$ there exists a triple (x_a, x_b, x_c) such that $x_a \cdot a + x_b \cdot b + x_c \cdot c = x$ and $x_a + x_b + x_c = 1$. The triple (x_a, x_b, x_c) is called *barycentric coordinates* of x with respect to $\Delta_{a,b,c}$.

Prove that:

(a) If $A(\Delta)$ denotes the area of the triangle A, then

$$x_a = \frac{A(\Delta_{b,c,x})}{A(\Delta_{a,b,c})},\ x_b = \frac{A(\Delta_{a,c,x})}{A(\Delta_{a,b,c})},\ x_c = \frac{A(\Delta_{a,b,x})}{A(\Delta_{a,b,c})}$$



(b) Equations $x_a = 0$, $x_b = 0$, $x_c = 0$ represent lines through bc, ab and ab, respectively.

(c) Let (x_a, x_b, x_c) be barycentric coordinates of point x in triangle Δ_{abc}. The set of points $\{(x_a, x'_b, x'_c) : x'_b, x'_c \in \mathbb{R}\}$ represents a line parallel to edge bc passing through point x. Similarly, sets of points $\{(x'_a, x_b, x'_c) : x'_a, x'_c \in \mathbb{R}\}$, $\{(x'_a, x'_b, x_c) : x'_a, x'_b \in \mathbb{R}\}$ represent lines parallel to edges ac, ab, respectively, passing through point x.

5 Linear time construction of a Schnyder realizer.

Let G be a maximal planar graph with n vertices. Can a Schnyder labeling and a Schnyder realizer be constructed in time $O(n)$?

Hint: Find a Connection between a canonical ordering and the ordering in which the edge contraction for the construction of a Schnyder labeling is executed.

6 Induced path in a Schnyder realizer.

A path of a graph G is called *induced* if the vertices of this path are connected only by the edges of the path, i.e. path on vertices $v_1, \ldots v_k$ is *induced* if for any $1 \leq i, j \leq n$ such that $|i - j| > 1$, edge (v_i, v_j) does not belong to G. Let G be a maximal planar graph and let T_a, T_b, T_c be a Schnyder realizer of G. Assume that the edges of T_a, T_b, T_c are colored red, blue and green, respectively. Show that a directed monochromatic path in T_a, T_b, T_c is an *induced path* of G.