Exercise Sheet 3

**Assignment:** November 26, 2013  
**Delivery:** None, Discussion on December 3, 2013

1 **Linear time construction of a Schnyder realizer.**

Let $G$ be a maximal planar graph with $n$ vertices. Can a Schnyder labeling and a Schnyder realizer be constructed in time $O(n)$?

**Hint:** Find a Connection between a canonical ordering and the ordering in which the edge contraction for the construction of a Schnyder labeling is executed.

2 **Property of a Schnyder realizer.**

Let $G$ be a maximal planar graph with vertices $a, b, c$ on the outer face. Let $T_a, T_b, T_c$ be the red, the blue and the green trees of a Schnyder realizer, with sinks at vertices $a, b, c$, respectively. Let $v$ be an internal vertex of $G$ and denote by $P_a(v), P_b(v), P_c(v)$ the paths connecting $v$ with $a, b, c$ in $T_a, T_b, T_c$, respectively. Show that paths $P_a(v), P_b(v)$ and $P_c(v)$ do not have common vertices, except for $v$.

3 **Induced path in a Schnyder realizer.**

A path of a graph $G$ is called *induced* if the vertices of this path are connected only by the edges of the path, i.e. path on vertices $v_1, \ldots, v_k$ is *induced* if for any $1 \leq i, j \leq n$ such that $|i - j| > 1$, edge $(v_i, v_j)$ does not belong to $G$. Let $G$ be a maximal planar graph and let $T_a, T_b, T_c$ be a Schnyder realizer of $G$. Assume that the edges of $T_a, T_b, T_c$ are colored red, blue and green, respectively. Show that a directed monochromatic path in $T_a, T_b, T_c$ is an induced path of $G$.

4 **Property of $st$-Ordering**

Let $G = (V, E)$ be a biconnected planar graph with a given embedding and let $v_1, \ldots, v_n$ be an $st$-ordering of $G$ such that $v_1, v_n$ belong to the outer face of $G$. Let $G_i$ denote the plane subgraph of $G$ induced by the vertices $v_1, \ldots, v_i$. Prove that $v_{i+1}$ belongs to the outer face of $G_i$.

5 **Ear decomposition.**

Let $G = (V, E)$ such that for each edge $\{s, t\} \in E$, $G$ has an open ear decomposition that starts with $\{s, t\}$. Show that $G$ is 2-connected. (Recall that the reverse statement was proven in the lecture.)