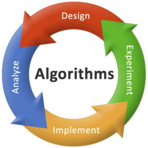
Algorithms for Route Planning

KIT (SS 2016)

Lecture: **Time Dependent Route Planning – I**



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University of Patras, Greece

CTI "DIOPHANTUS" computer technology institute & press

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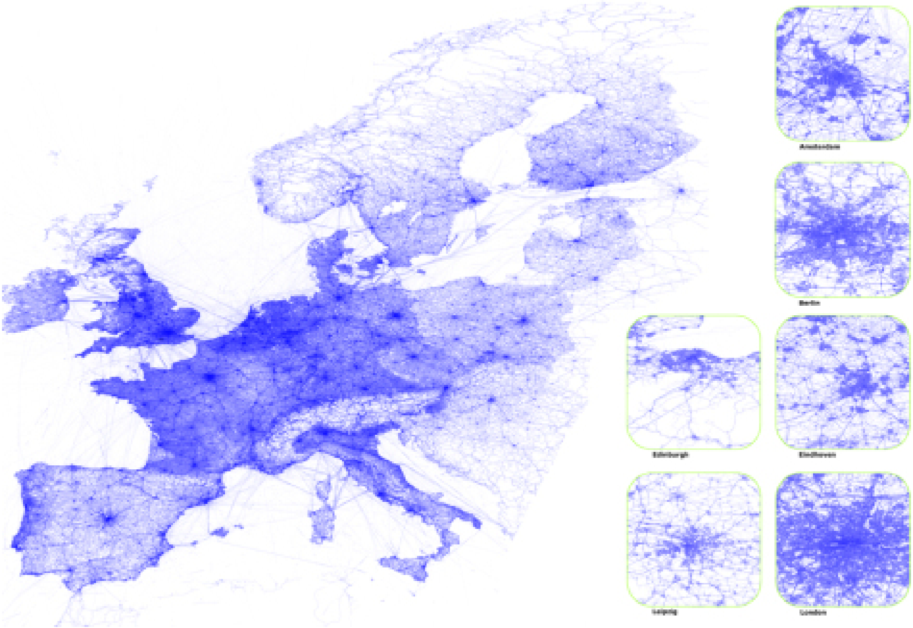
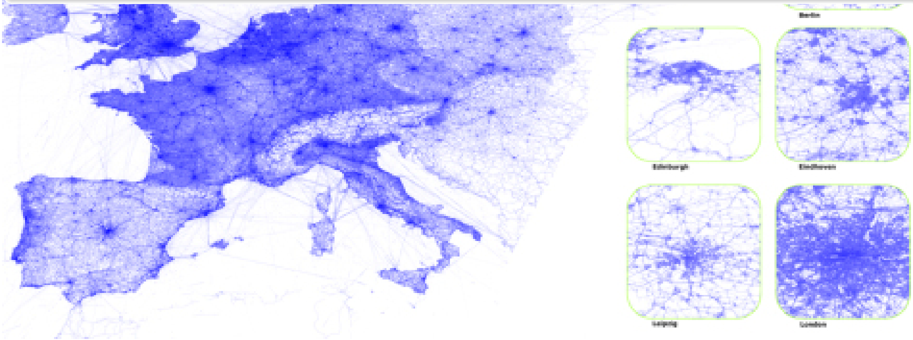


Raw traffic (speed probe) data



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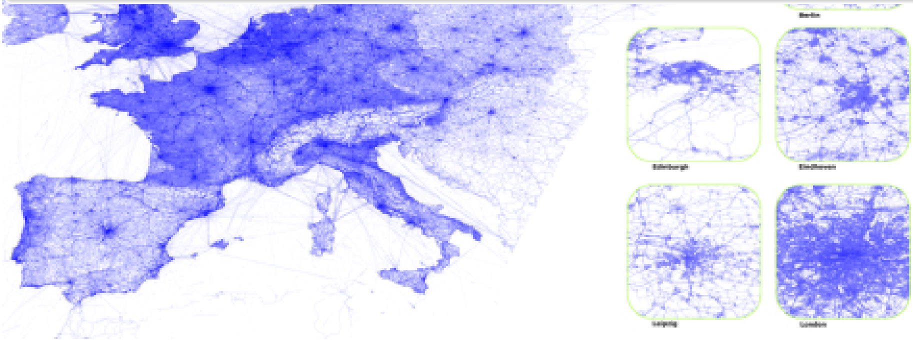
- **70 Million** contributing users



Raw traffic (speed probe) data

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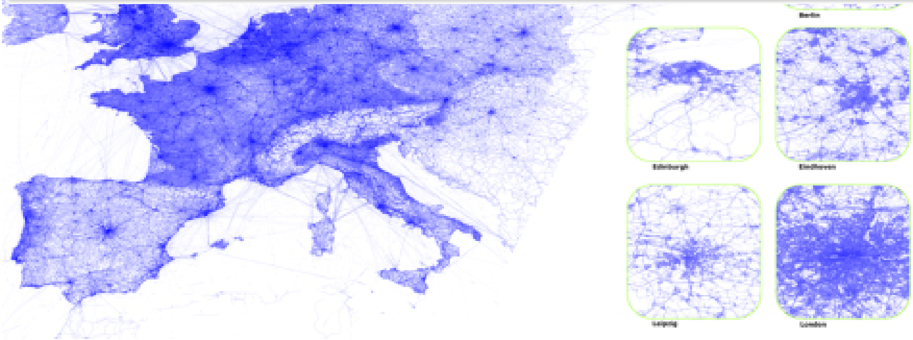
- **4 Billion** measurements per day



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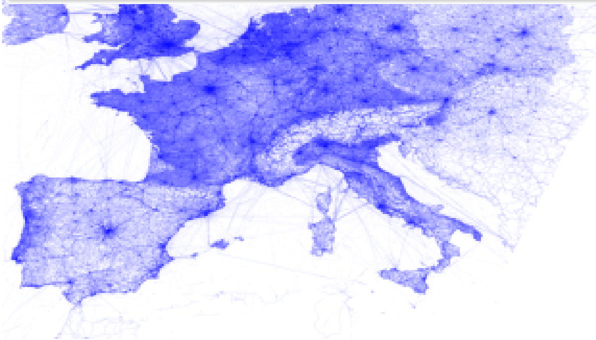

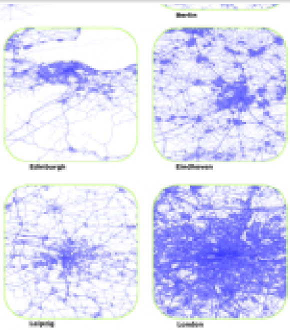

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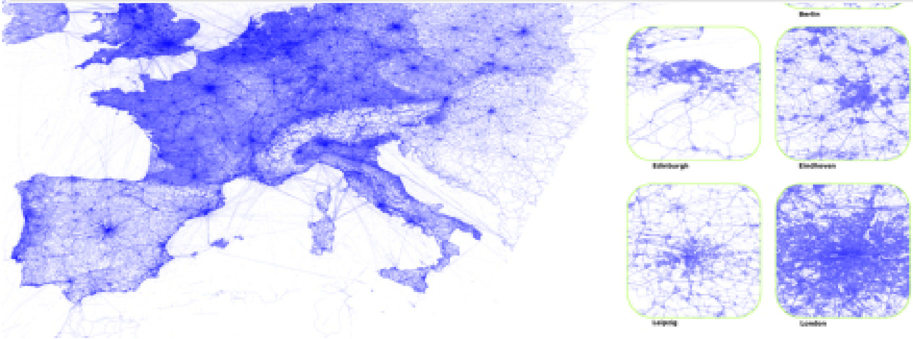

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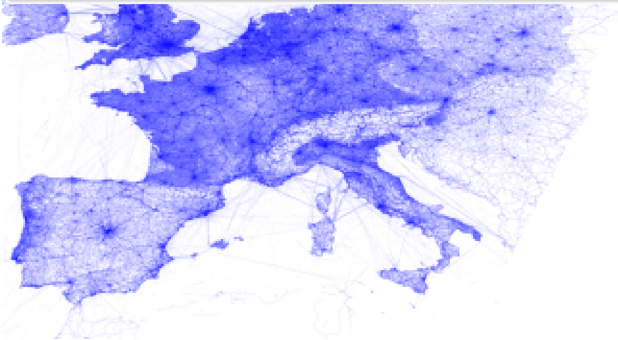

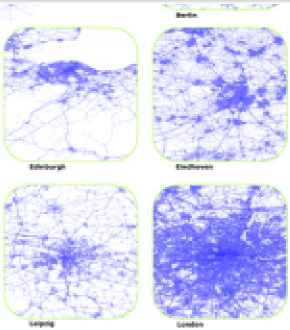

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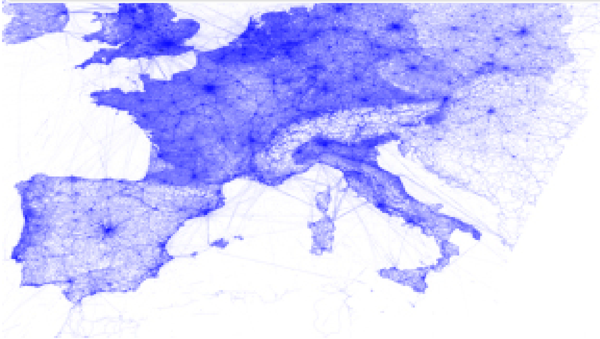

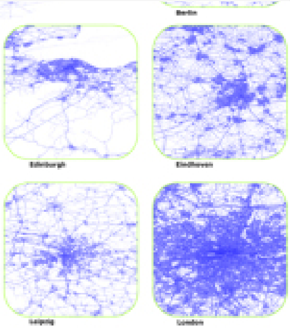

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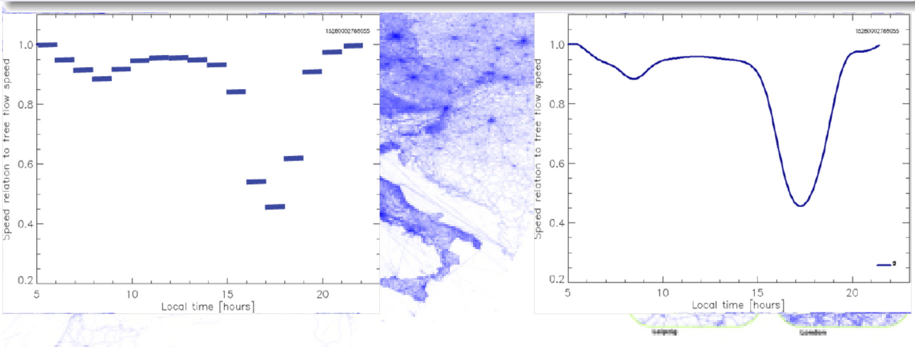

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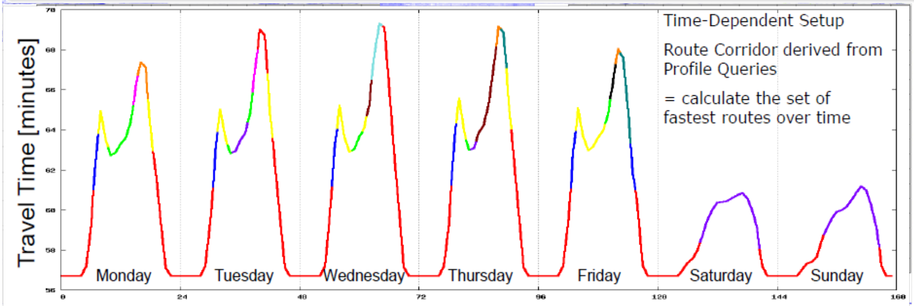

Main Issue: time-dependence

Time Dependent Shortest Paths – I

a more realistic and more involved problem

Why Time-Dependent Shortest Paths?

Real-life networks: Elements demonstrate temporal behavior

Why Time-Dependent Shortest Paths?

Real-life networks: Elements demonstrate temporal behavior

- Graph elements **added/removed** in real-time /∗ Dynamic Shortest Path ∗/
- Metric demonstrates **stochastic behavior** /∗ Stochastic Shortest Path ∗/
- Graph is **fixed**, metric **changes with the value of a parameter** $\gamma \in [0, 1]$ in a predetermined fashion /∗ Parametric Shortest Path ∗/
- Graph is **fixed**, metric **changes over time** in a predetermined fashion

  /∗ Time-Dependent Shortest Path ∗/

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Real-life networks: Elements demonstrate temporal behavior

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  /* Time-Dependent Shortest Path */

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Real-life networks: Elements demonstrate temporal behavior

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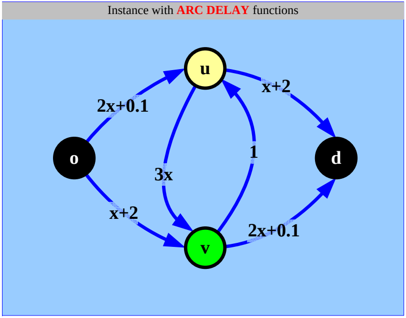
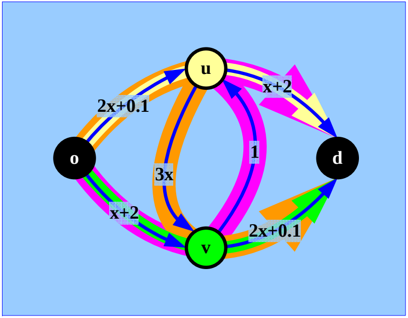
  /∗ Time-Dependent Shortest Path ∗/

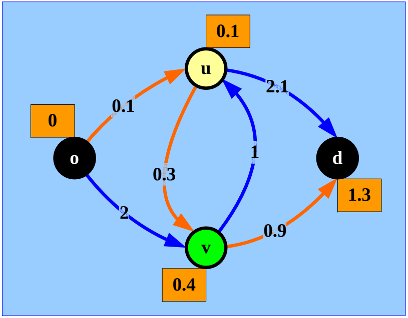
  - Arcs are allowed to become **occasionally unavailable** (e.g., due to periodic maintenance, saving consumption of resources, etc), for predetermined unavailability time-intervals (discrete domain)
  - Arc lengths (e.g., traversal-time / consumption) **change with departure-time from tail** which is treated as a real-valued variable (functions with continuous domain, but not necessarily continuous range)

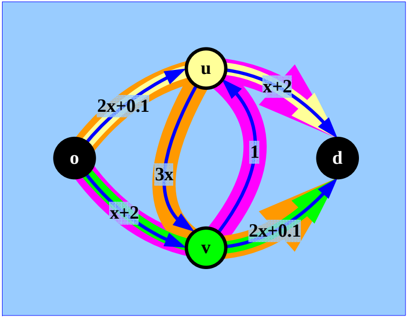




Q1 How would you commute **as fast as possible** from *o* to *d*, for a given departure time (from *o*)?

TDSP :: EXAMPLE 1 (Earliest Arrivals)





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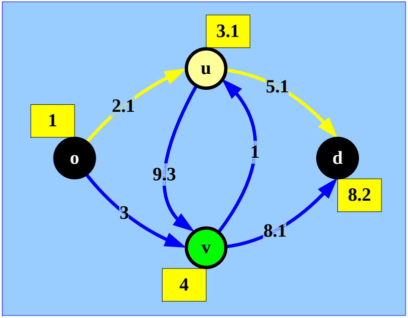

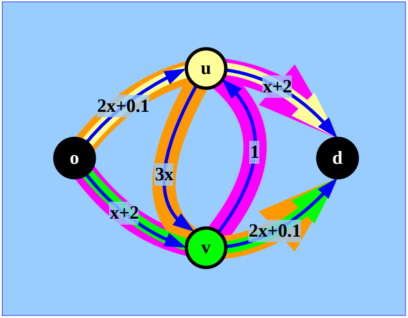

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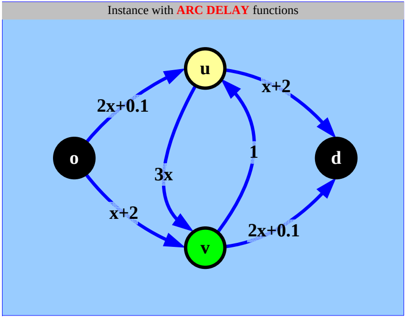

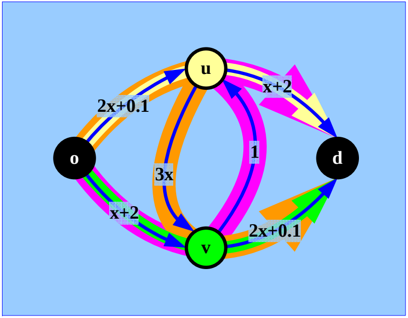

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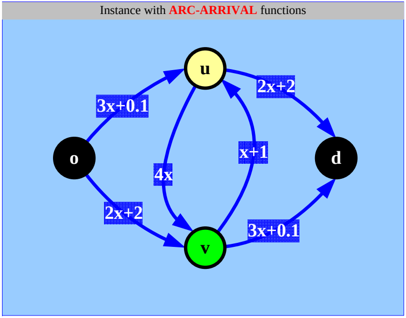

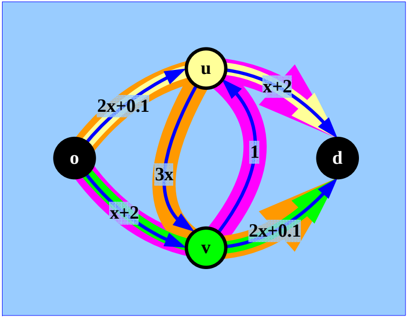

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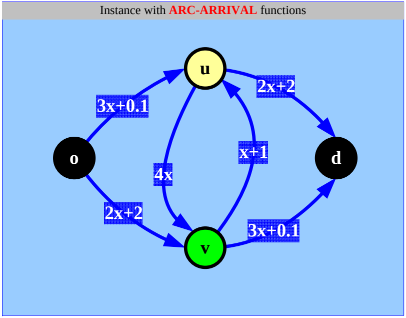

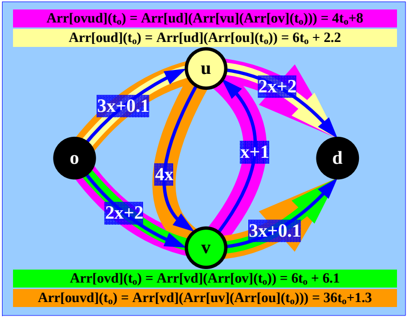

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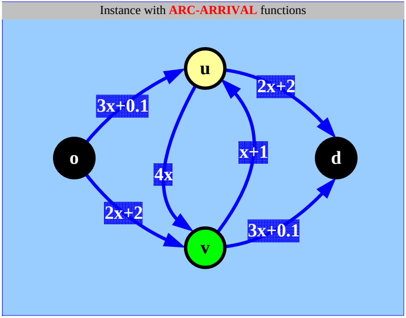

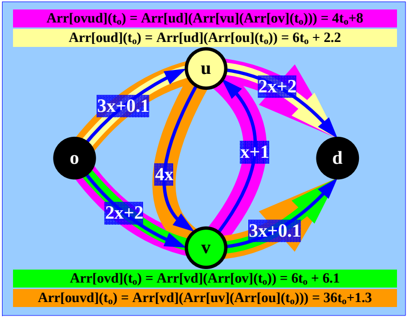

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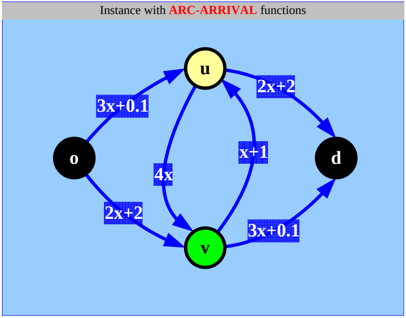
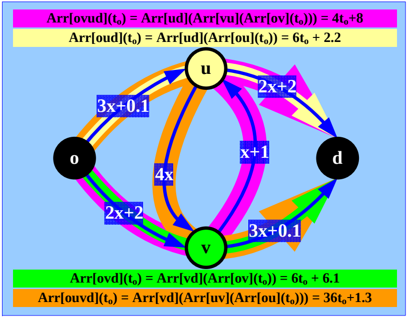
A

$$\text{shortest } od\text{–path} = \begin{cases} \textbf{orange path}, \text{ if} & t_o \in [0, 0.03] \\ \textbf{yellow path}, \text{ if} & t_o \in [0.03, 2.9] \\ \textbf{purple path}, \text{ if} & t_o \in [2.9, +\infty) \end{cases}$$





Q1 Would **waiting-at-nodes** be worth it?

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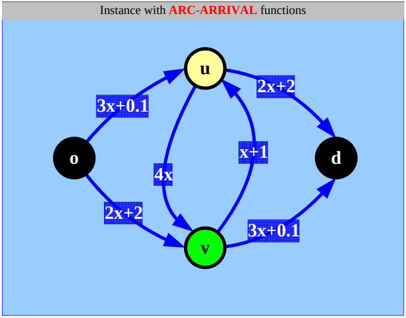

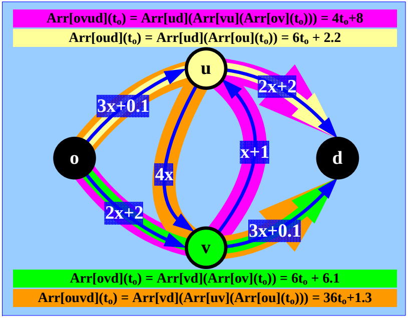

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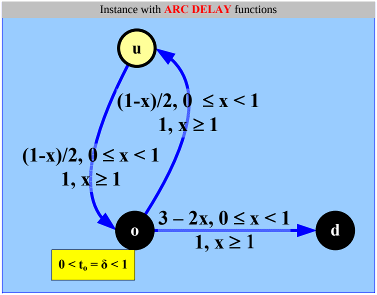

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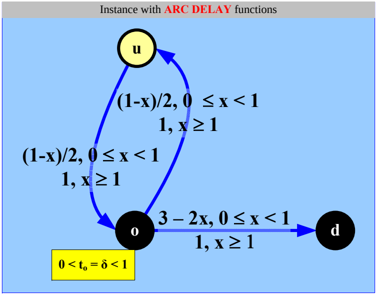

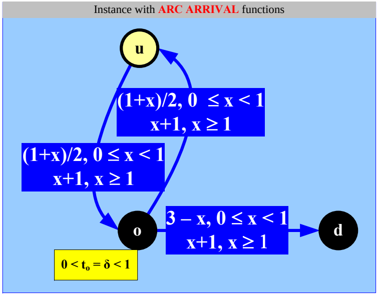

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Q2 Would **waiting-at-nodes** be worth it in this case?

A2 YES, wait until time 1 and then traverse *od*, if already present at *o* at time $t_o < 1$. Otherwise, traverse *od* immediately.

Waiting Policies

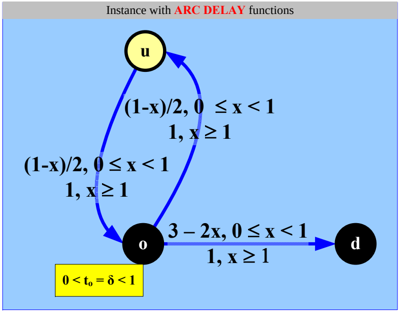
Unrestricted Waiting (UW) Unlimited waiting is allowed at every node along an *od*-path

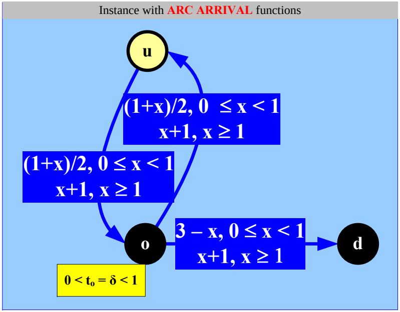
Origin Waiting (OW) Unlimited waiting is only allowed at the origin node of each *od*-path

Forbidden Waiting (FW) No waiting is allowed at any node of each *od*-path

Depending on the *waiting policy*, the scheduler has to decide not only for an optimal connecting path (ensuring earliest arrival at destination), but also for the appropriate optimal waiting times at the nodes along this path

TDSP :: EXAMPLE 2 (Waiting Times) – contd.





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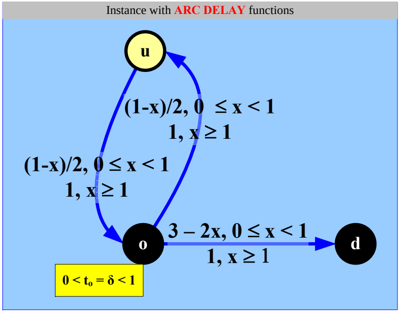
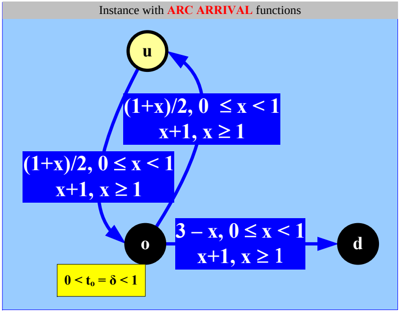


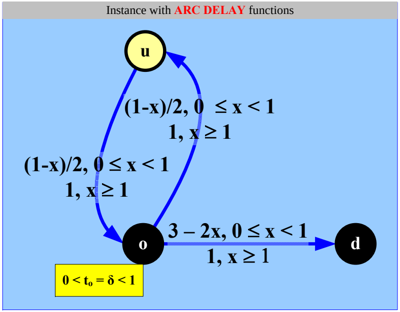


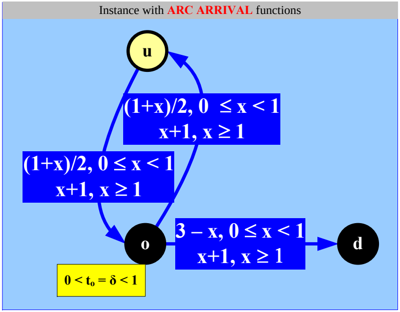
Q3 What if waiting-at-nodes is forbidden?

A3 An infinite, non-simple TD shortest *od*-path with finite delay

o					d
δ					$3 - \delta$

TDSP :: EXAMPLE 2 (Waiting Times) – contd.





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o	u	o			d
δ	$\frac{1+\delta}{2}$	$\frac{3+\delta}{4}$			$3 - \frac{3+\delta}{4} > 2$

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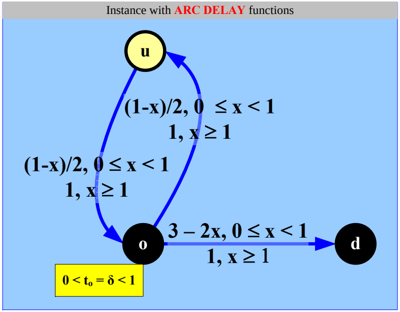

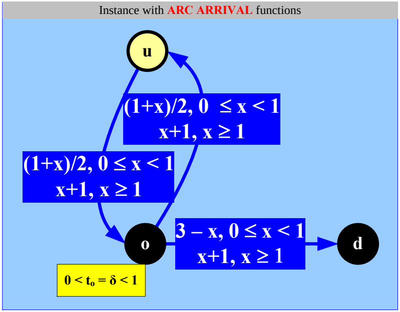

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o	u	o	u	o	d
δ	$\frac{1+\delta}{2}$	$\frac{3+\delta}{4}$	$\frac{7+\delta}{8}$	$\frac{15+\delta}{16}$	$3 - \frac{15+\delta}{16} > 2$

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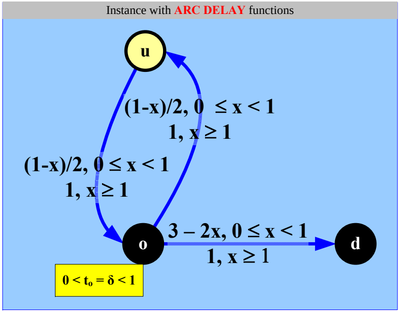

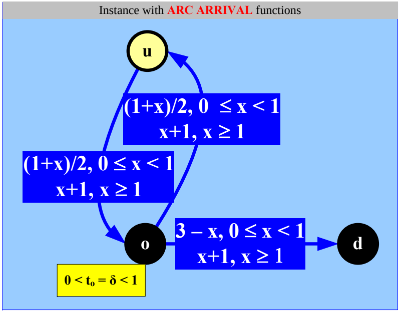

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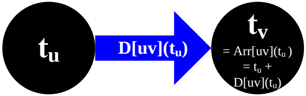
o	u	o	presence at o after $k \uparrow \infty$ visits of u	d
δ	$\frac{1+\delta}{2}$	$\frac{3+\delta}{4}$	$\lim_{k\uparrow\infty} \frac{2^{2k}-1+\delta}{2^{2k}} = 1$	$t_d \downarrow 2$

Subpath optimality and *shortest path simplicity* are **not guaranteed** for TDSP, if waiting-at-nodes is forbidden

FIFO vs non-FIFO Arc Delays

- **(Strict) FIFO Arc-Delays:** The slopes of all the *arc-delay* functions are at least equal to (greater than) -1

Equivalently: *Arc-arrival* functions are **non-decreasing** (aka **no-overtaking** property)



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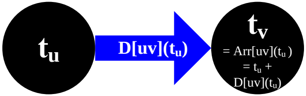

- **Non-FIFO Arc-Delays:** Possibly preferable to wait for some period at the tail of an arc, before trespassing it. E.g.:
  - Wait for the next *(faster) IC train*, than use the (immediately available) *(slower) local train*

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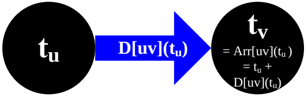

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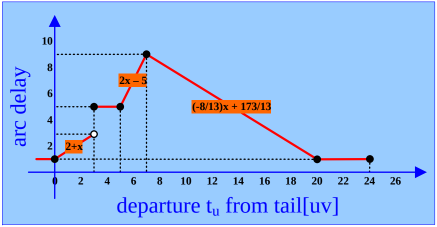

FIFO arc delay example

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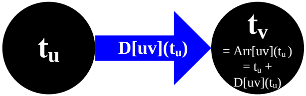

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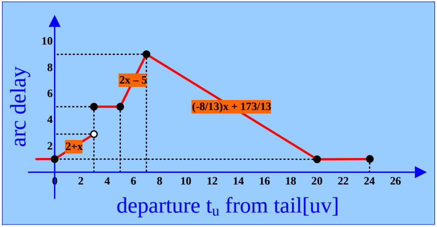

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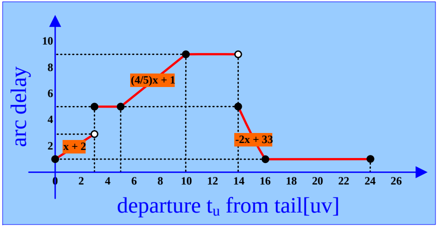
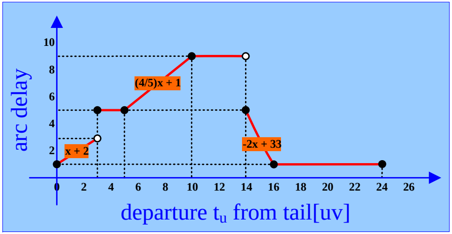


Non-FIFO arc delay example

Non-FIFO+UW Arc ⇔ FIFO Arc



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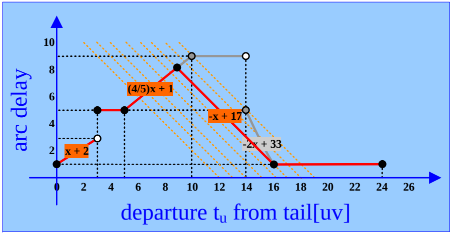

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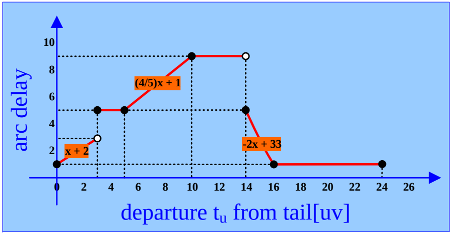

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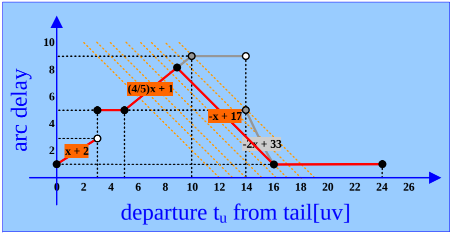

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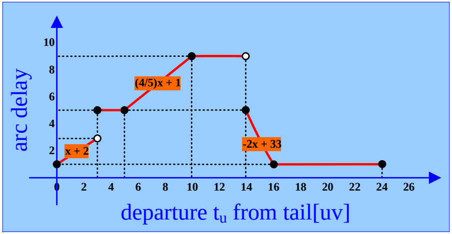

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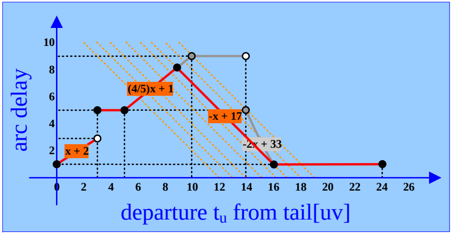

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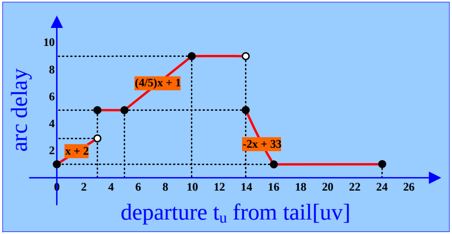

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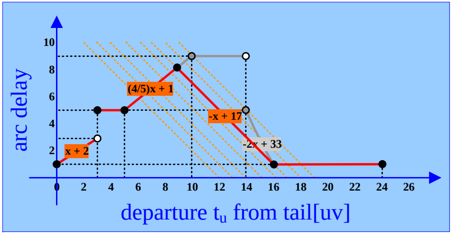

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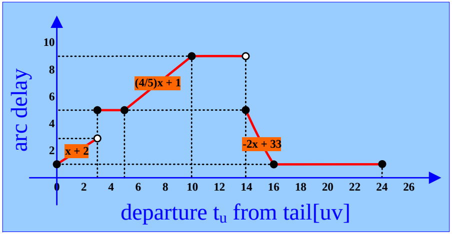

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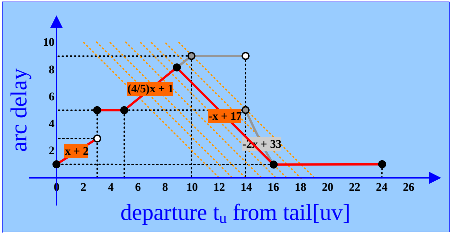

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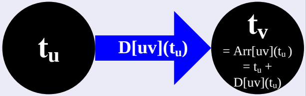
Variants of Time-Dependent Shortest Path

Time-Dependent Shortest Path

- *Directed* graph $G = (V, A)$ with **arc-travel-time** function $D[uv](t)$, $\forall uv \in A$
  $Arr[uv](t) = t + D[uv](t)$



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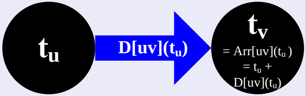

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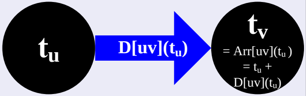

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GOAL1: For departure-time t_o from o, determine $t_d = Arr[o,d](t_o)$
GOAL2: Provide a **succinct representation** of $Arr[o,d]$ (or $D[o,d]$)

Why Care for Both Goals?

1. Not always sure when to depart (still think about it)! Possessing the entire *distance function* $D[o, d]$ allows for easy answers (e.g., via look-ups) in several queries for varying departure times, or even finding the *minimum travel / ealriest-arrival time* within a window of possible departure times.

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- **Preprocessing** of distance summaries (as in static case) requires to precompute functions instead of scalars.

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    - ☹ Complexity is **polynomial** in the number of “elementary” *functional operations*. i.e., (EVAL, LINEAR COMBINATION, MIN, COMPOSITION)
    - ☹ Not so “elementary” operations after all (see next slides)!

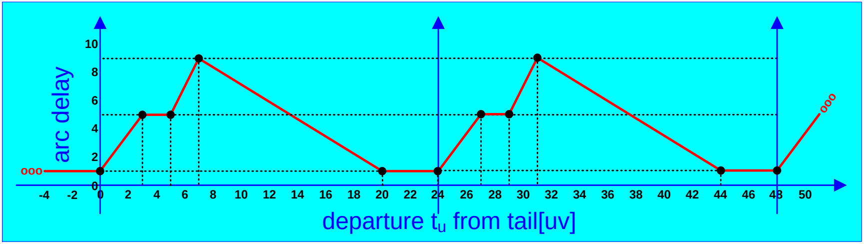
Algorithms for TDSP

... in FIFO, continuous, pwl instances

Input/Output Data

PWL Arc Delays

Forward Description (as function of departure times from origin)



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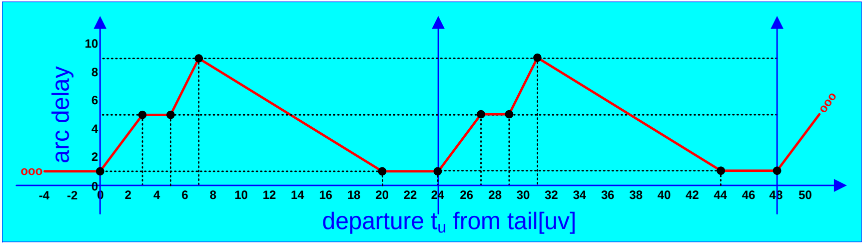
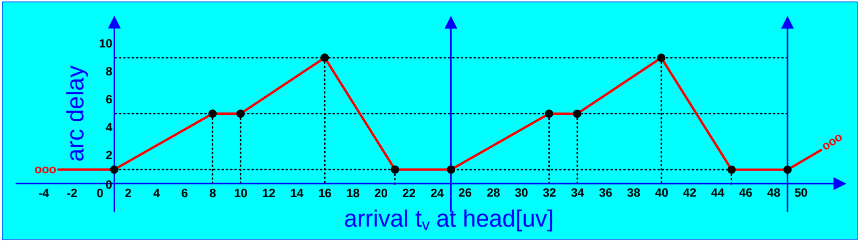
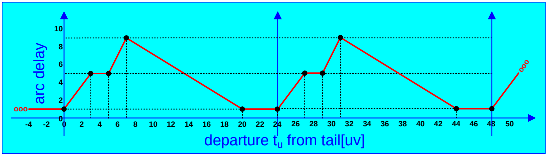


Reverse Description (as function of arrival times at destination)



How to Store/Access PWL Arc Delays



- Exploit *periodicity* and *piecewise-linearity*:

$$\forall t_u \in \mathbb{R},\ \overrightarrow{D}[uv](t_u) = \begin{cases} \frac{4}{3}t_u + 1, & 0 \le t_u \textbf{mod } T \le 3 \\ 5, & 3 \le t_u \textbf{mod } T \le 5 \\ 2t_u - 5, & 5 \le t_u \textbf{mod } T \le 7 \\ -\frac{8}{13}t_u + \frac{173}{13}, & 7 \le t_u \textbf{mod } T \le 20 \\ 1, & 20 \le t_u \textbf{mod } T \le 24 \end{cases}$$

- Representation: Array of **(slope, constant, dep.time UB) triples** equipped with advanced (binary/predecessor) *search capabilities*

$\left(\frac{4}{3}, 1, 3\right)$	$(0, 5, 5)$	$(2, -5, 7)$	$\left(-\frac{8}{13}, \frac{173}{13}, 20\right)$	$(0, 1, 24)$

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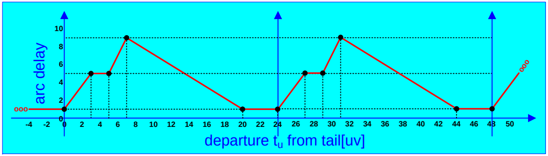

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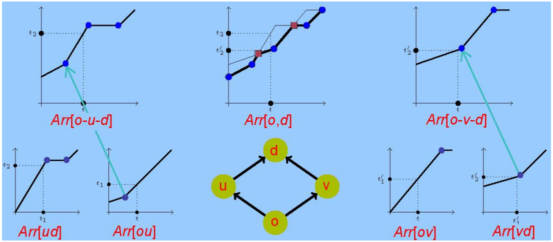
$(0,1)$	$(3,5)$	$(5,5)$	$(7,9)$	$(20,1)$

Piecewise Linearity of Path / Earliest Arrivals



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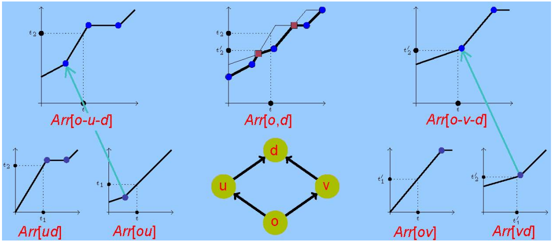

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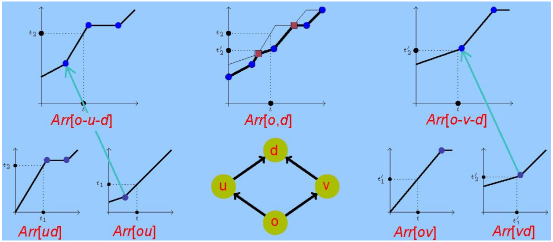

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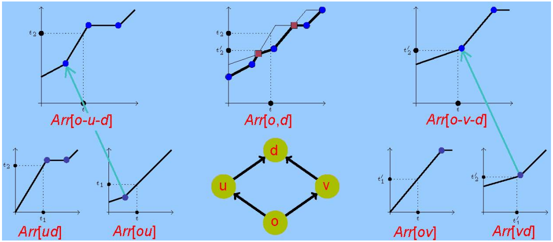

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- **Minimization Breakpoint (MB):** Departure-time b_v from origin o s.t. $Arr[o, v]$ **changes slope** due to application of *MIN*
- Periodicity of arc-delays implies periodicity of earliest-arrival function $Arr[o, d]$

Known Issues wrt Representations

- **Same representation** both for arc-arrival (or delay) functions and earliest-arrival (or shortest-travel-time) functions
  - Convenient for handling artificial arcs (representing shortest-travel-time functions) in *overlay abstractions* of the road network

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- We need only $\mathrm{O}\left(\frac{1}{\varepsilon} \cdot \log\left(\frac{D_{\max}[o,d]}{D_{\min}[o,d]}\right)\right)$ breakpoints for a $(1+\varepsilon)$ **upper approximation** $\overline{D}[o, d]$ of $D[o, d]$, for the case of continuous, piecewise-linear arc-delays

(Exact) Output Sensitive Algorithm for Earliest-Arrival Functions

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3. Interesting to discover whether the complexity of the earliest-arrival functions is indeed so bad **in real (e.g., road) networks**

The Output-Sensitive Algorithm (I)

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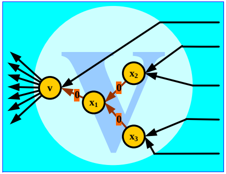
- ASSUMPTION: The in-degree of every node in the graph is at most 2



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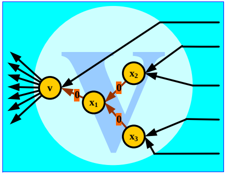

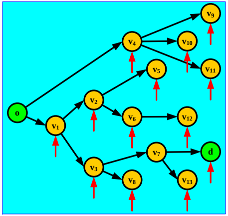

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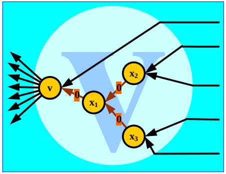

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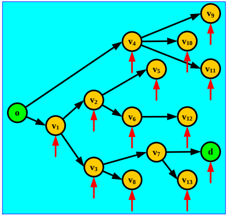
- Discover *until when* the TDSP tree is **valid**:
  - $\forall v \in V$, two short alternatives when departing from o at time t_0: Earliest-arrival to each parent, plus delay of corresponding incoming arc
  - **Minimization (vertex) Certificate** $t_{fail}[v]$: Earliest departure time from o at which the two alternatives of v become **equivalent**
  - **Primitive (arc) Certificate** $t_{fail}[e]$: Primitive image of the next (ie, after t_0) breakpoint of the arc to come



The Output-Sensitive Algorithm (I)

- ASSUMPTION: The in-degree of every node in the graph is at most 2
- Given an arbitrary point in time (*“current time”*) $t_0 \geq 0$ as departure time from origin o, compute a **TDSP tree**
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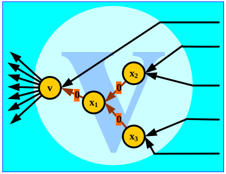
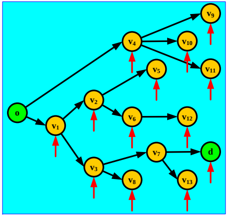
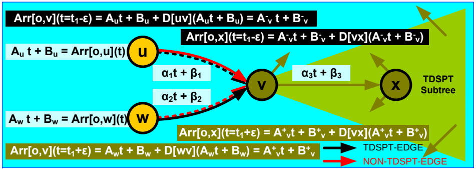
- All $(m + n)$ certificates temporarily stored in a *priority queue*





The Output-Sensitive Algorithm (II)

When current time $t_1 > t_0$ matches the earliest failure-time of a certificate in the priority queue:



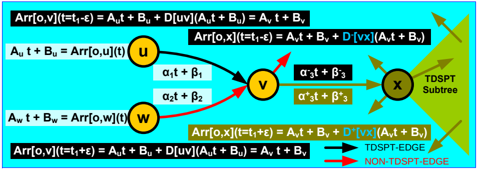
if minimization-certificate failure, at node $v \in V$:

then (1) Update shortest ov–path

/∗ **ONE-BIT** change in combinatorial structure ∗/

(2) Update $Arr[o, x]$ and $t_{fail}[x]$, $\forall x \in T_v$

(3) Update $t_{fail}[e]$, $\forall e \in E : x = tail[e] \in T_v$



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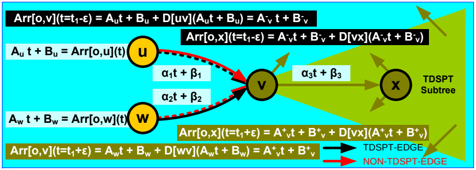

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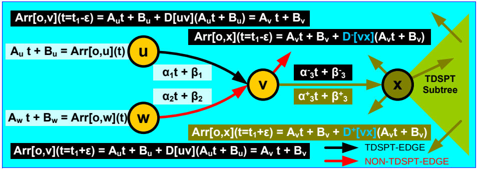

else /* primitive-certificate failure, at arc $e = vx \in E$ */

(1) Update $Arr[o, y]$ and $t_{fail}[y]$, $\forall y \in T_x$

(2) Update $t_{fail}[e']$, $\forall e' \in E : tail[e'] \in T_x$

The Output-Sensitive Algorithm (III)

- What to keep in memory:
  - Breakpoint triples for earliest-arrival functions, plus ONE bit (indicating the parent)
  - Advanced search structures, if number of BPs is large
  - Only temporarily store certificates in a priority queue

The Output-Sensitive Algorithm (III)

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- *Response-time* per certificate failure at $c \in V \cup E$:
  - In the *in-degrees-2 graph* (or any constant-in-degree graph): $\mathrm{O}(|E_c| \cdot \log n)$. E_c is the set of arcs whose tails are in T_c, or $T_{head[c]}$. Logarithmic factor is due to **priority-queue operations**
  - In the *original graph* (in worst-case): $\mathrm{O}\left(m \times \log^2 n\right)$. Second logarithmic factor is due to **updates of tournament trees** implementing the MIN operator at a particular node, upon emergence of a single certificate failure

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- *Worst-case time-complexity* of output-sensitive algorithm:

$$\mathrm{O}\left(m \times \log^2 n \times (\mathrm{PRIMBPs} + \mathrm{MINBPs})\right)$$