

Vorlesung Algorithmische Kartografie

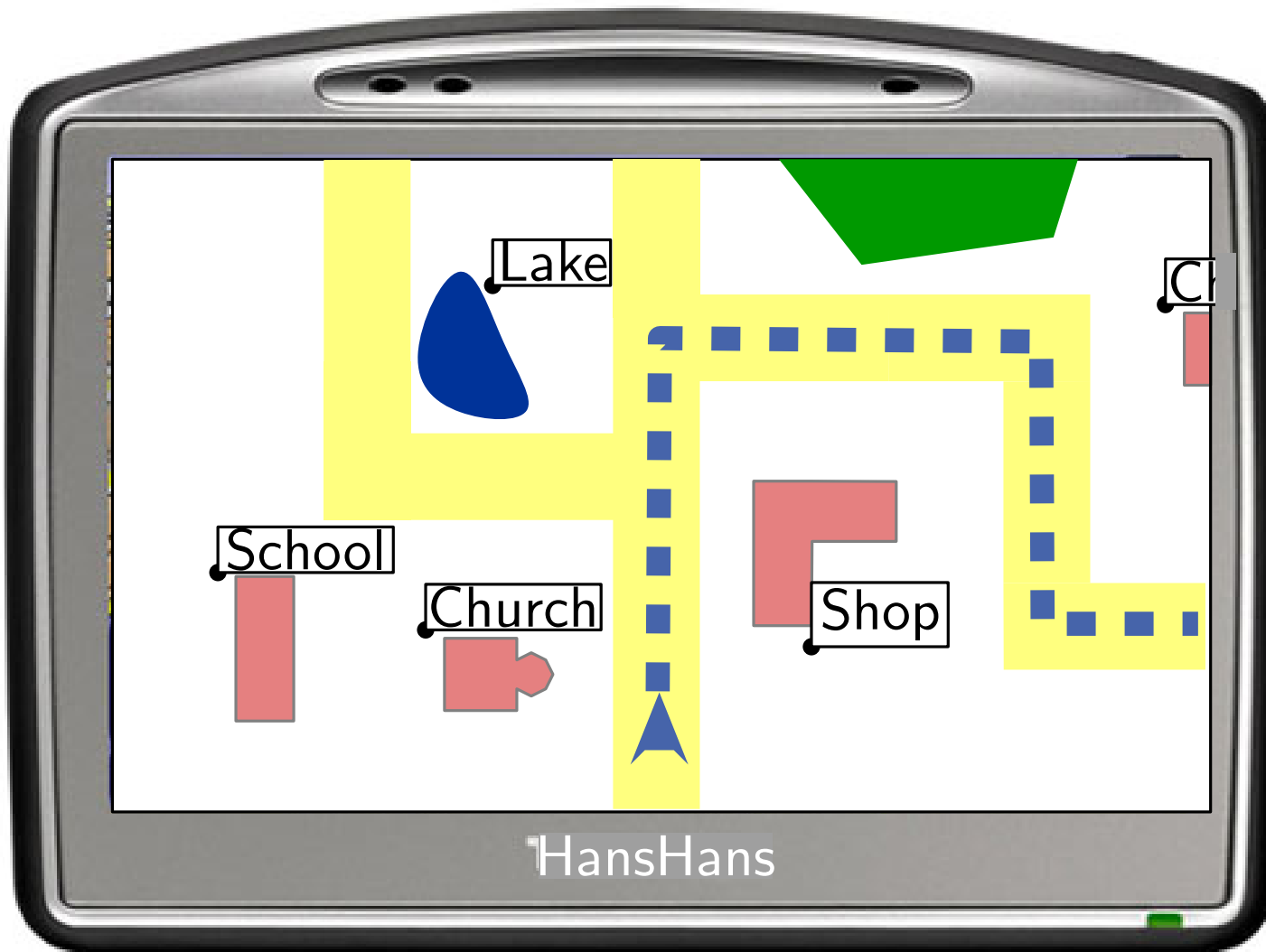
Beschriftung in dynamischen Karten: Rotieren

LEHRSTUHL FÜR ALGORITHMIK I · INSTITUT FÜR THEORETISCHE INFORMATIK · FAKULTÄT FÜR INFORMATIK

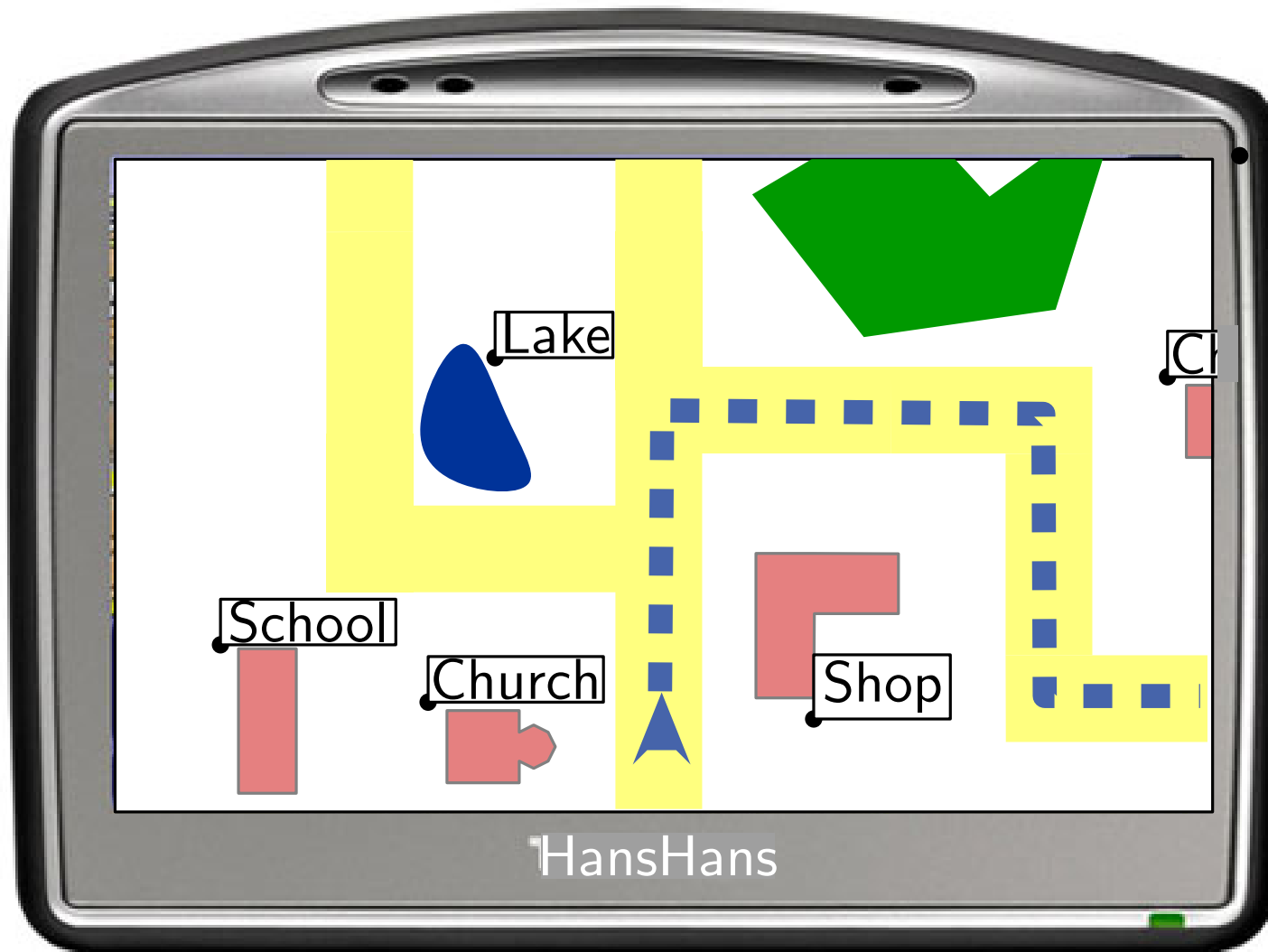
Andreas Gemsa
11.06.2013



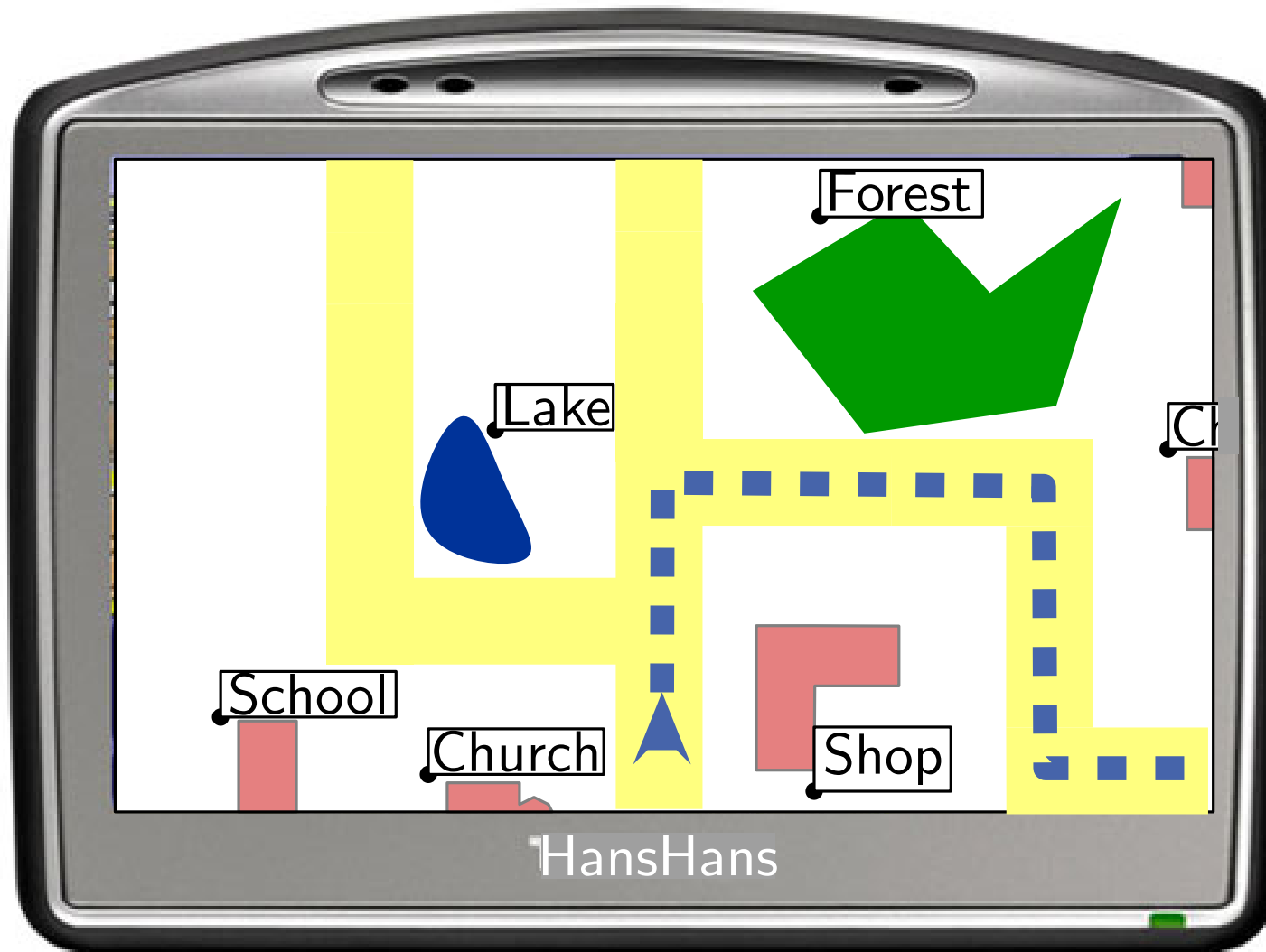
Motivation



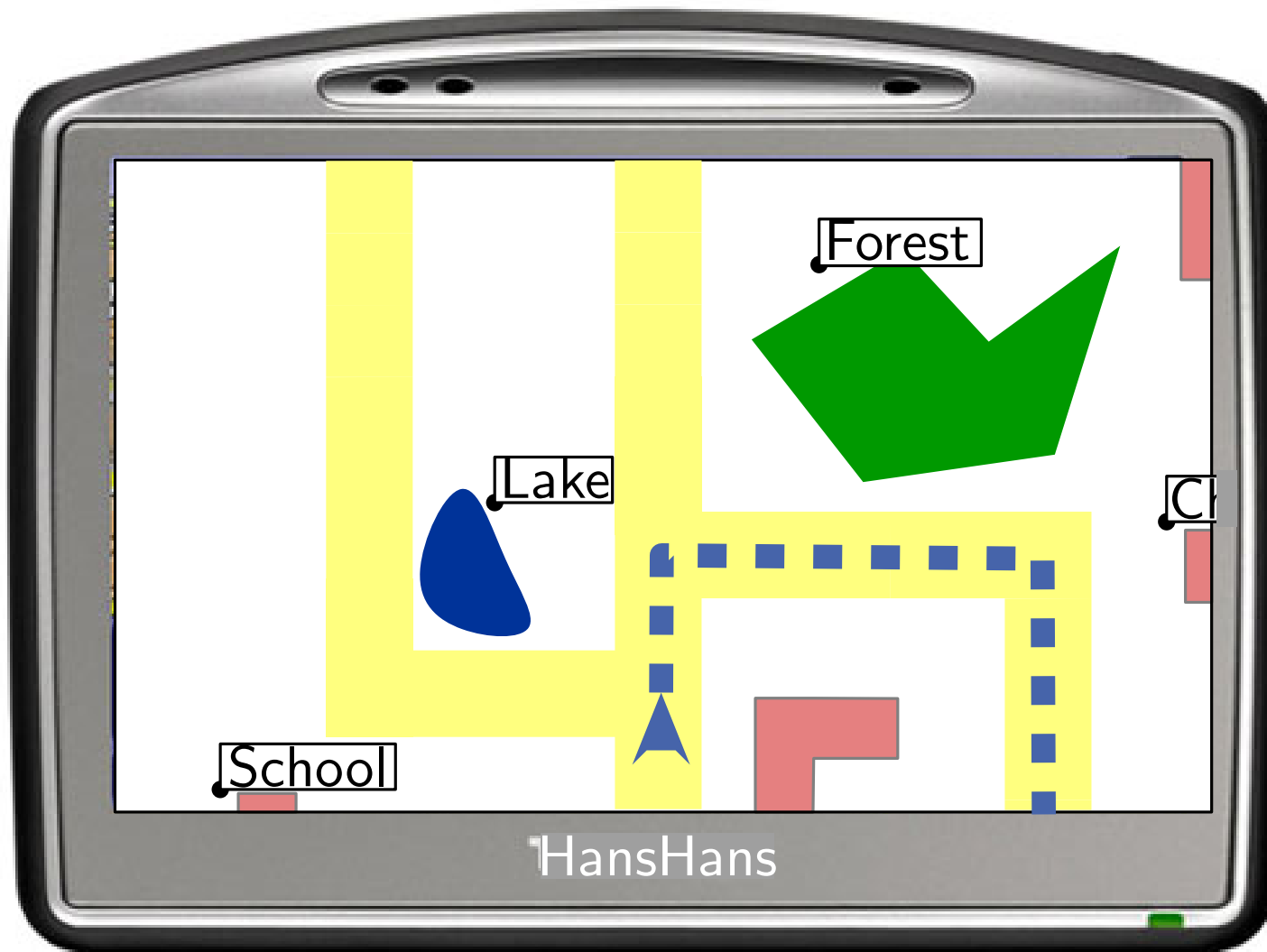
Motivation



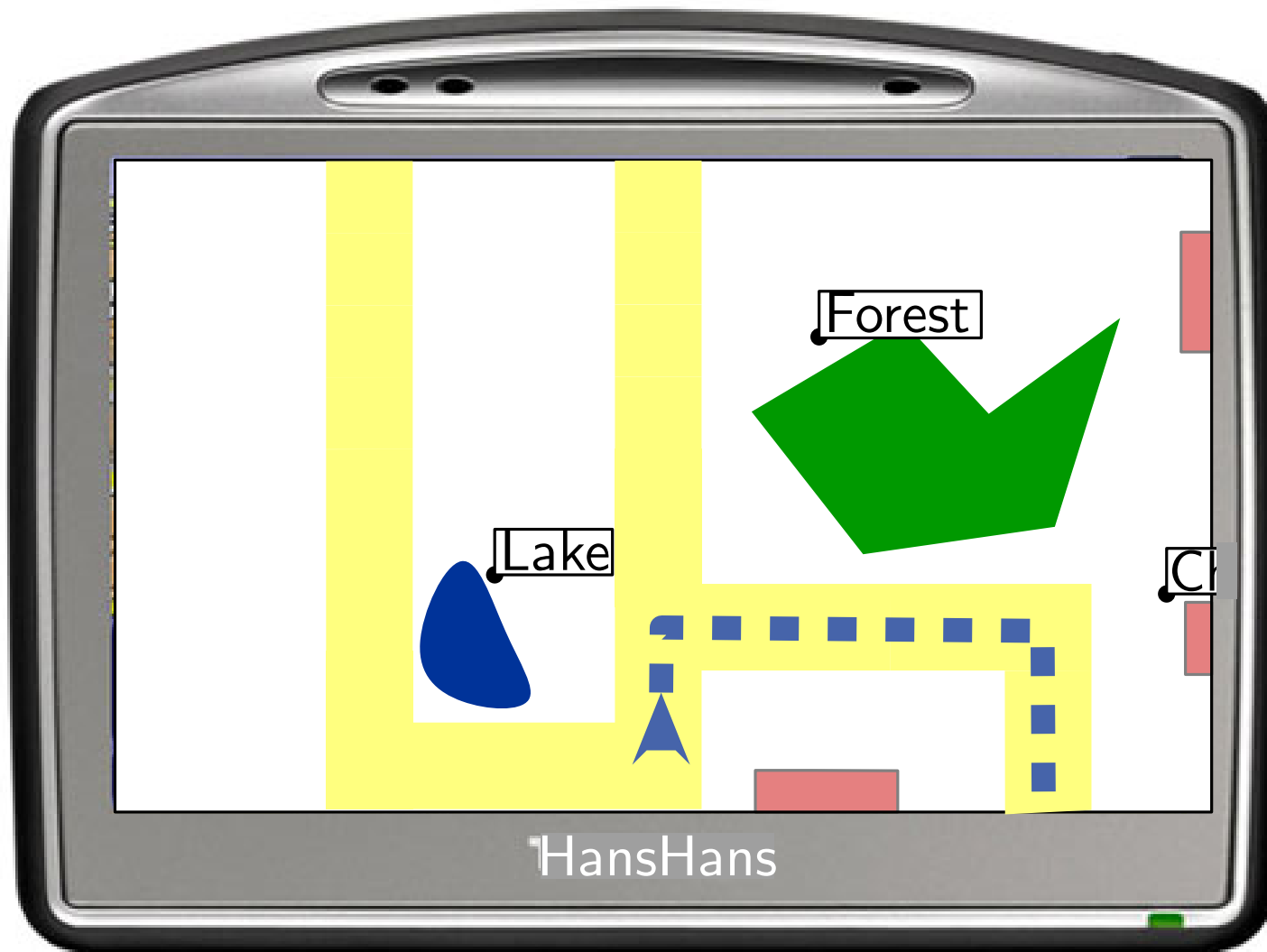
Motivation



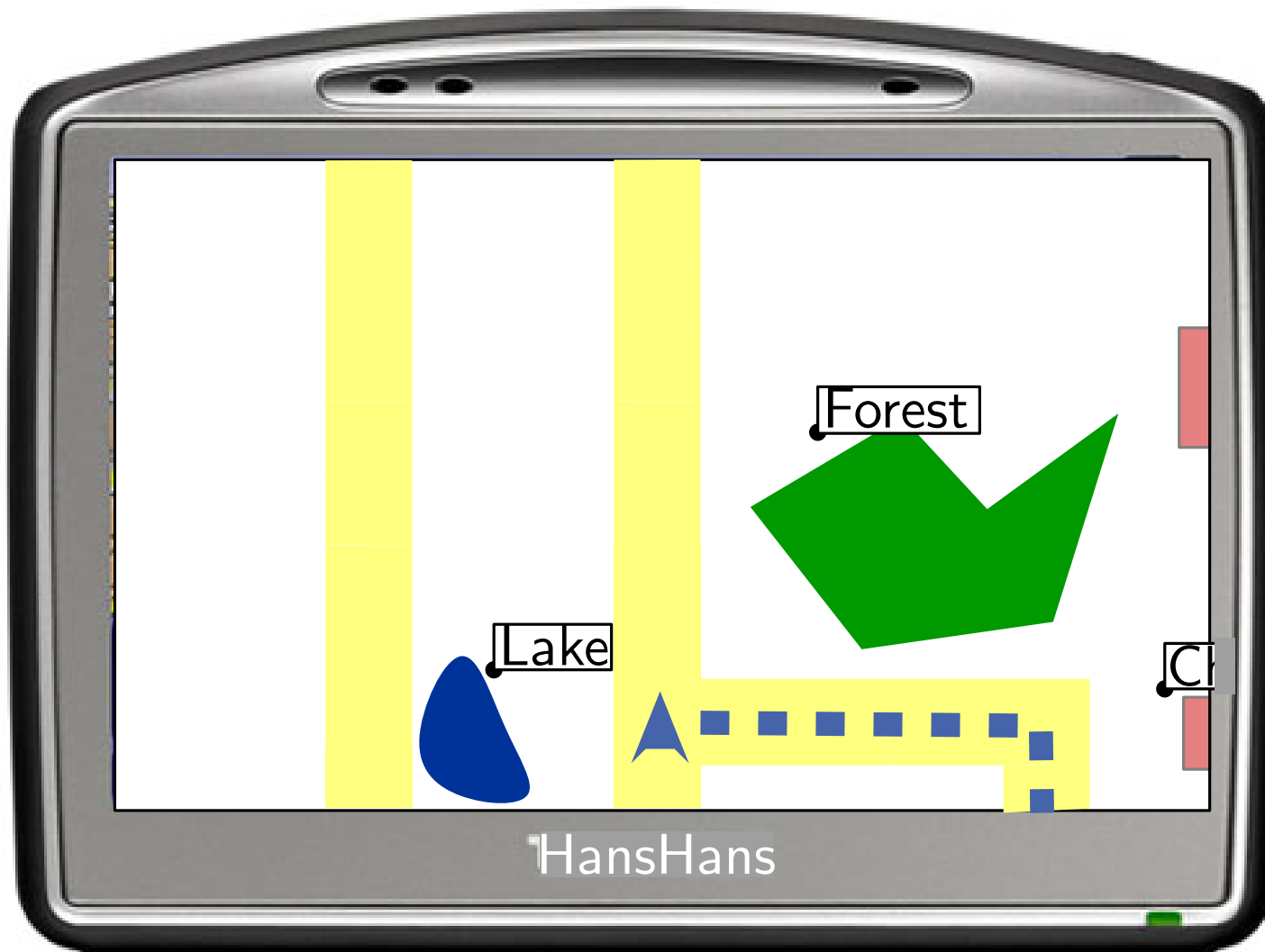
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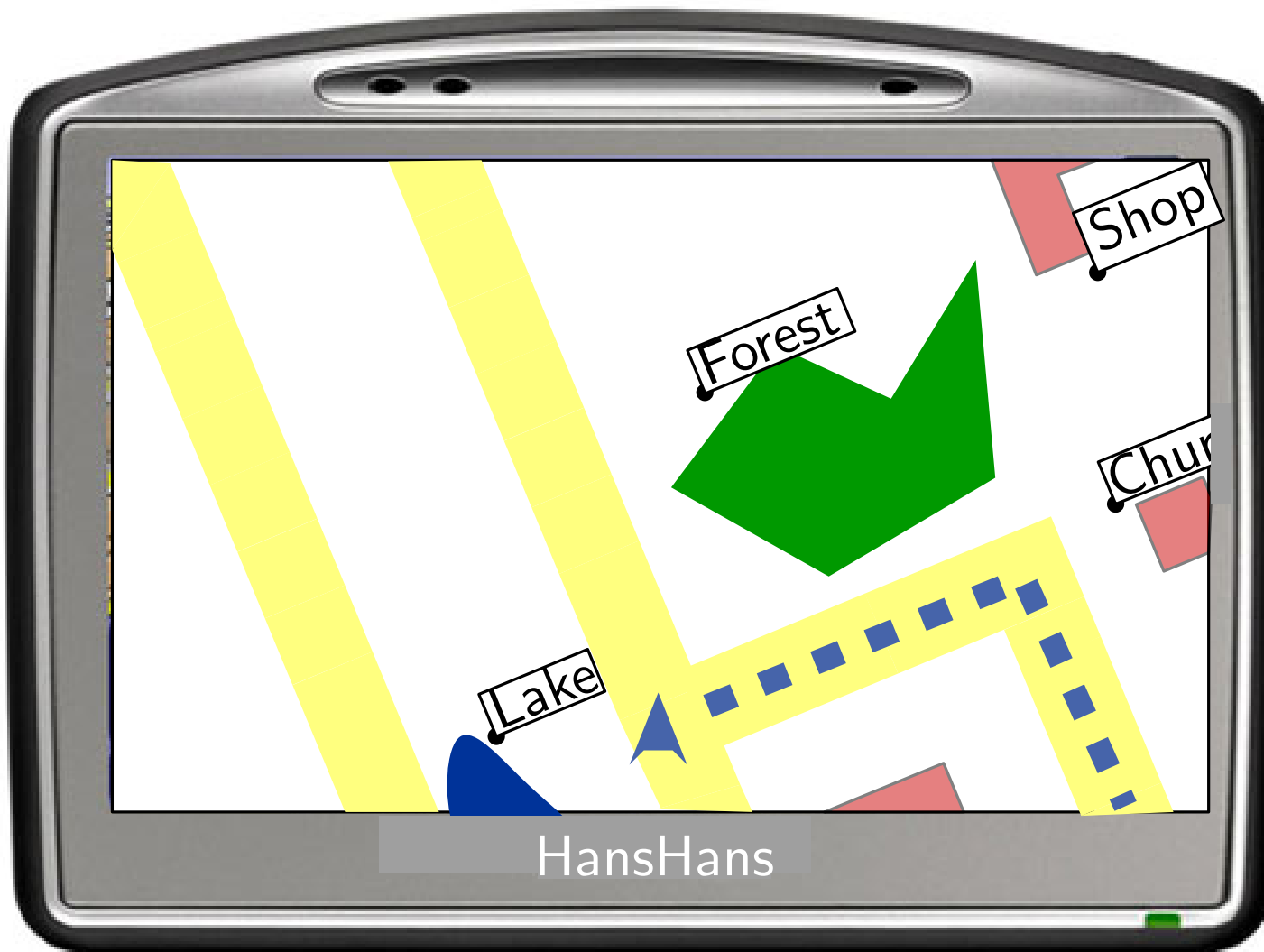
Motivation



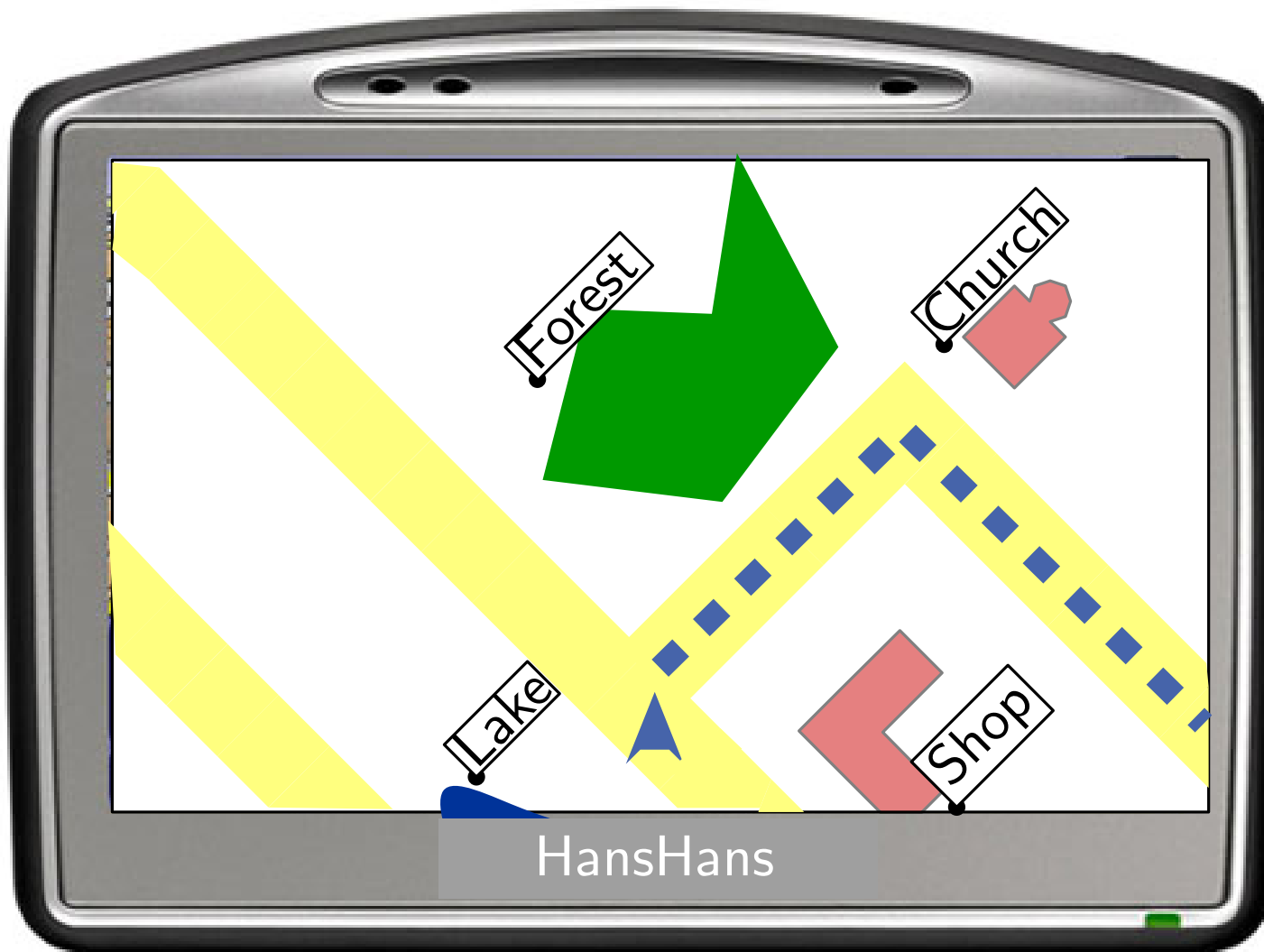
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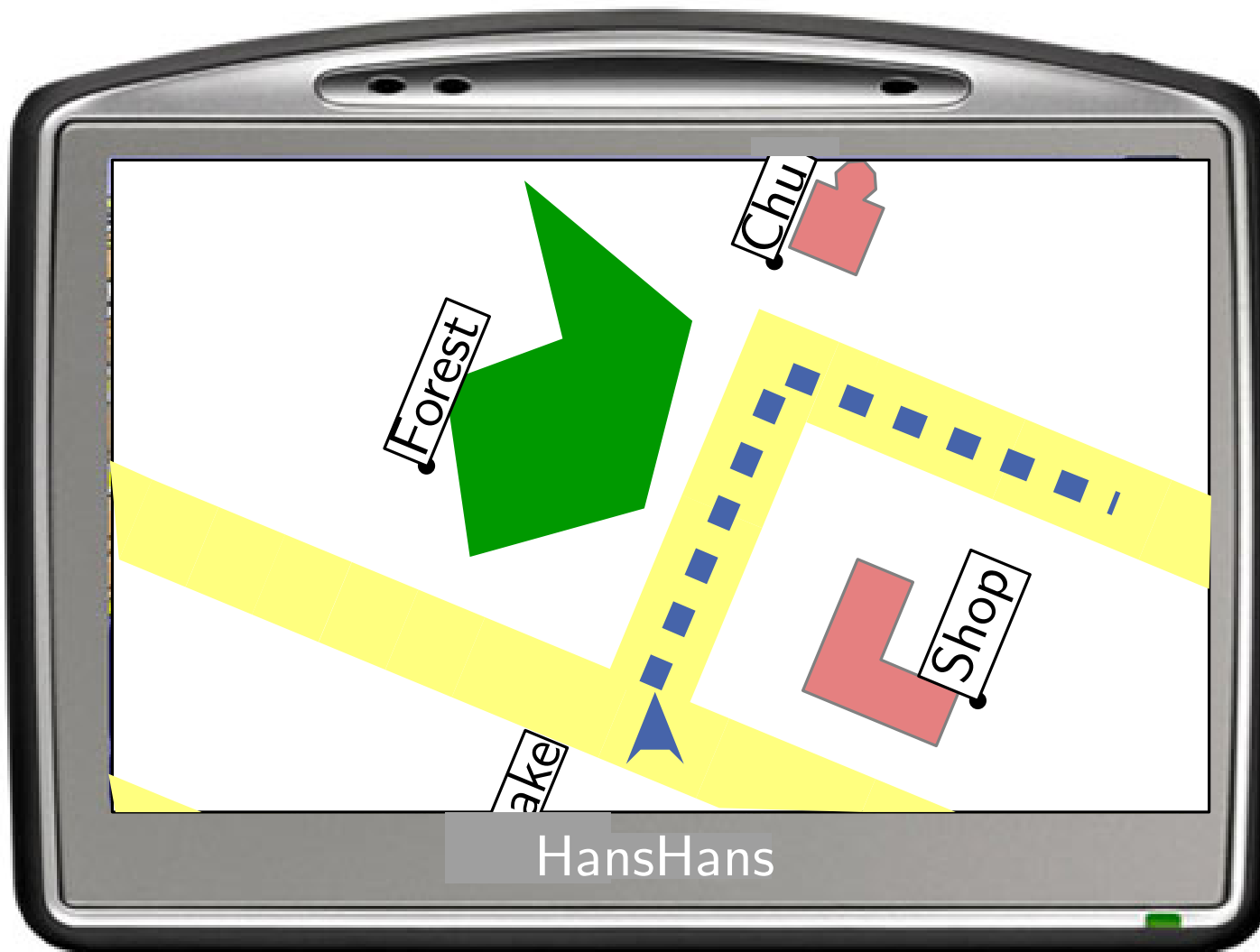
Motivation



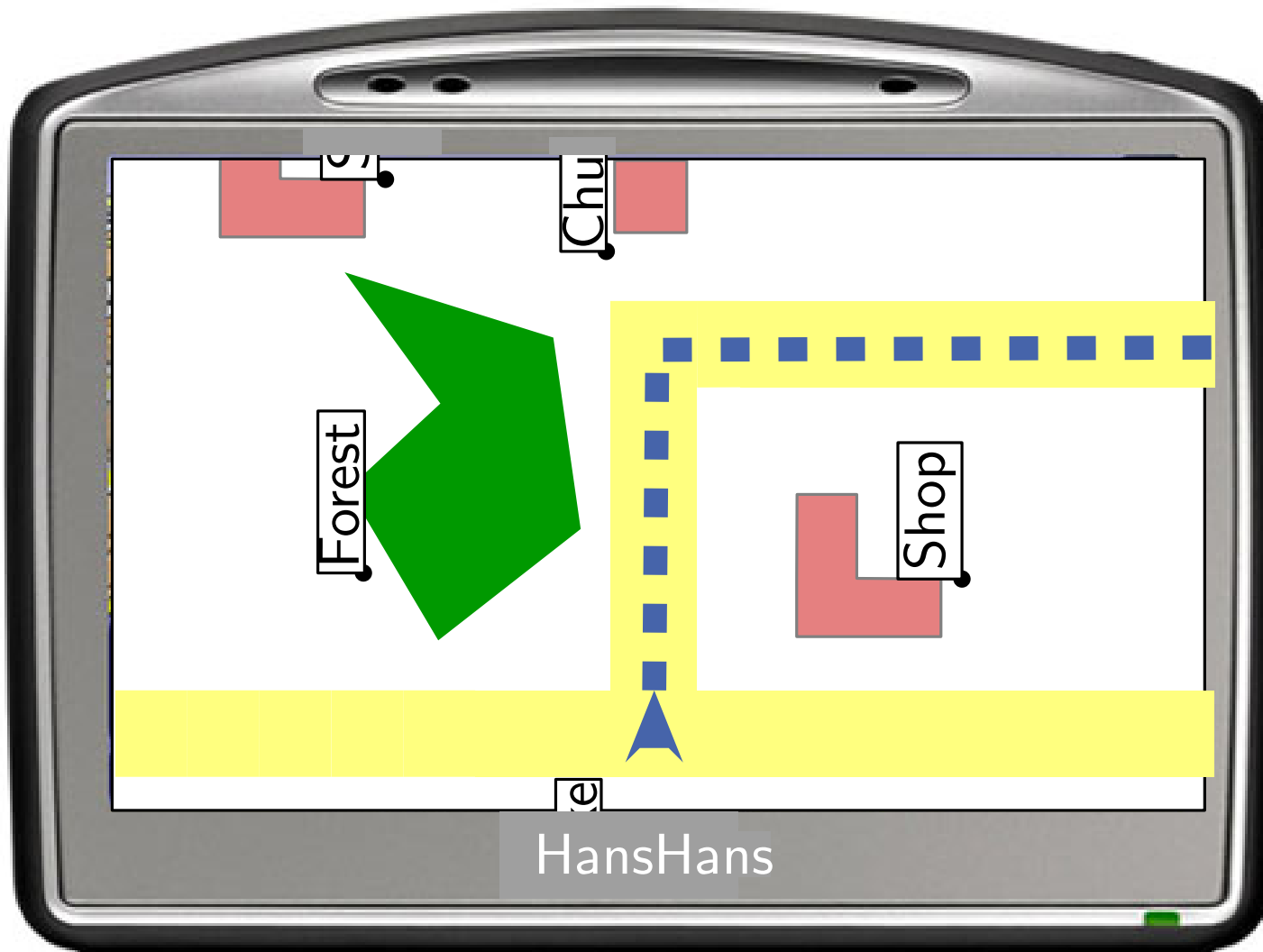
Motivation



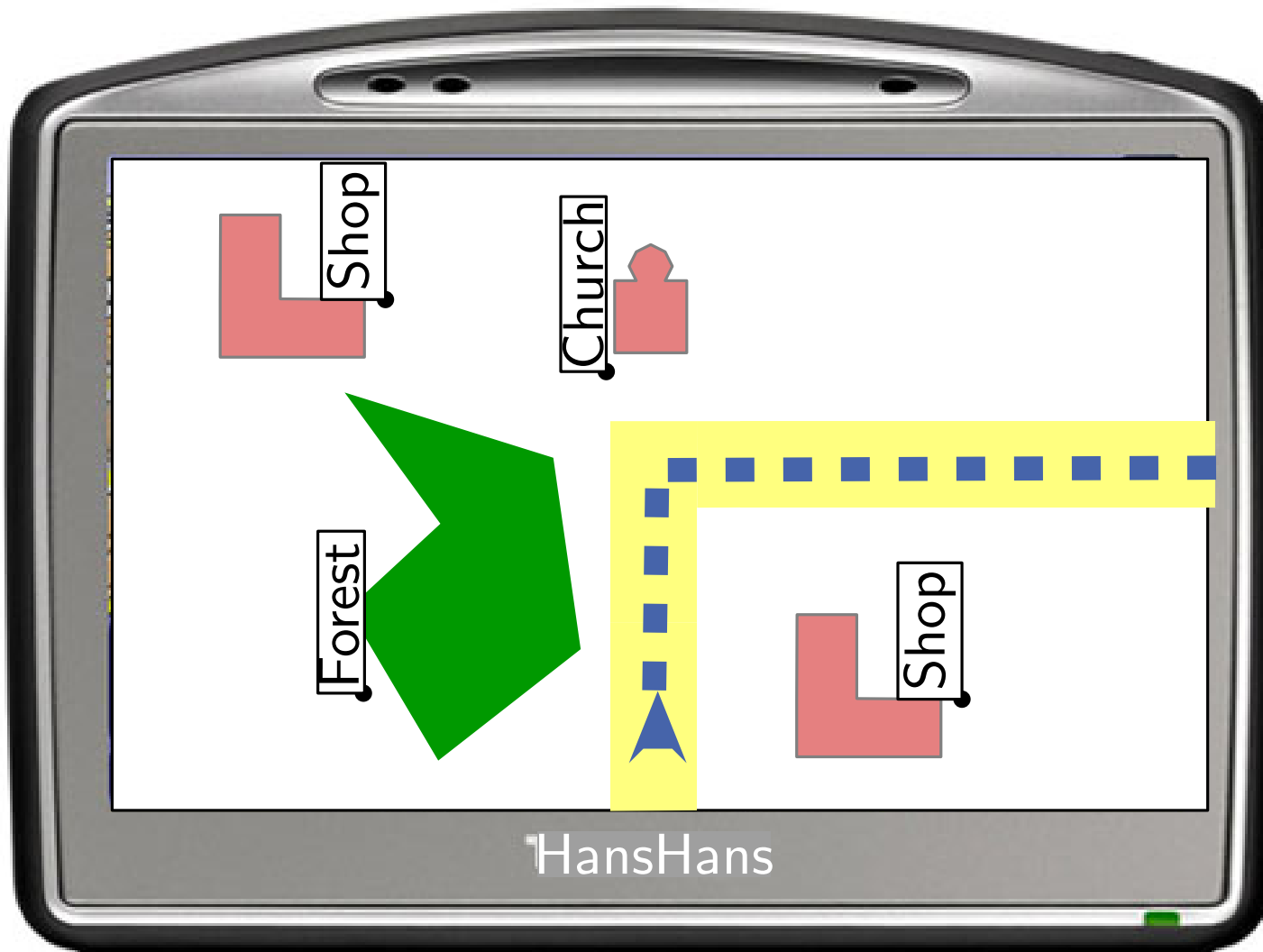
Motivation



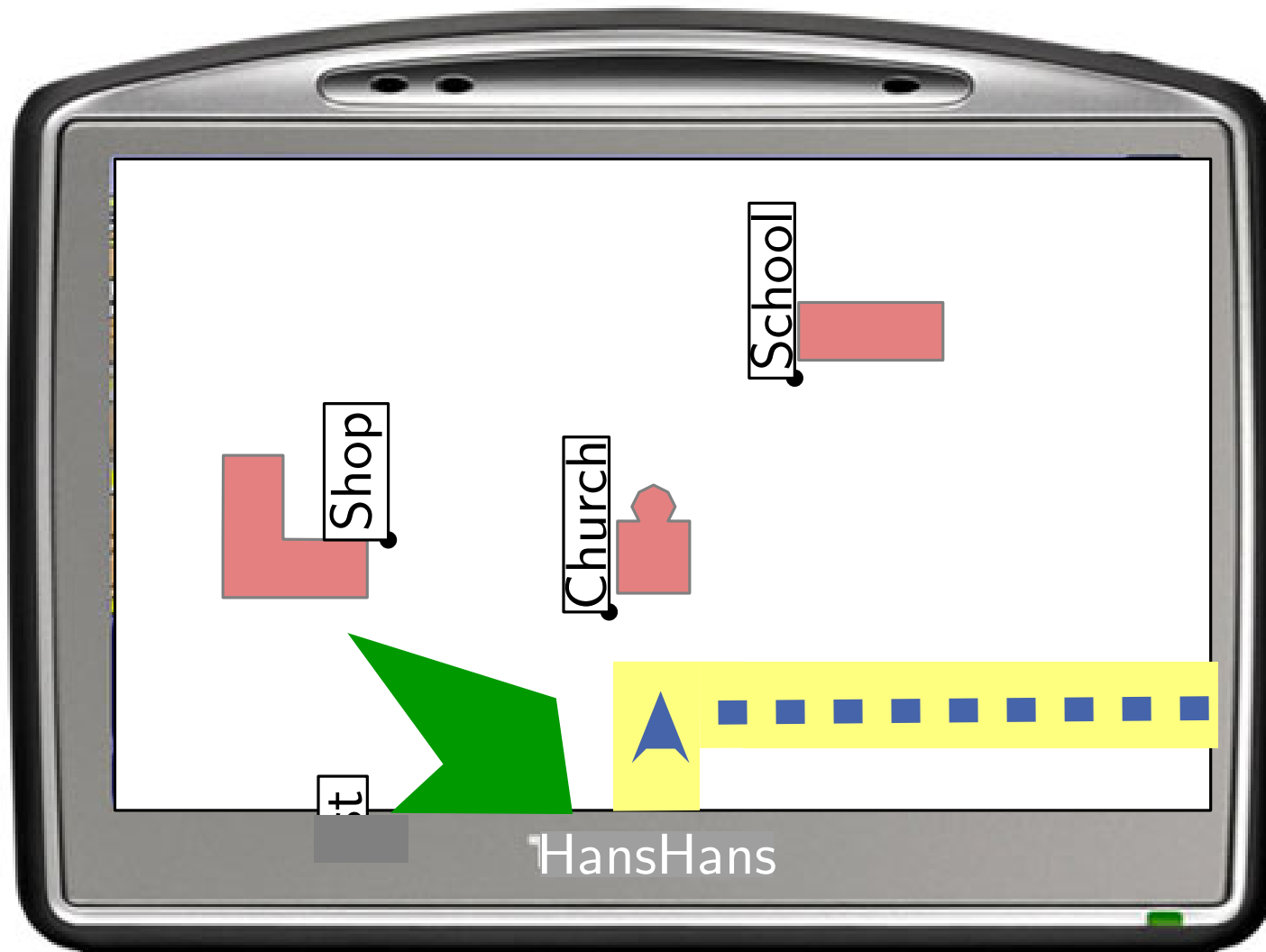
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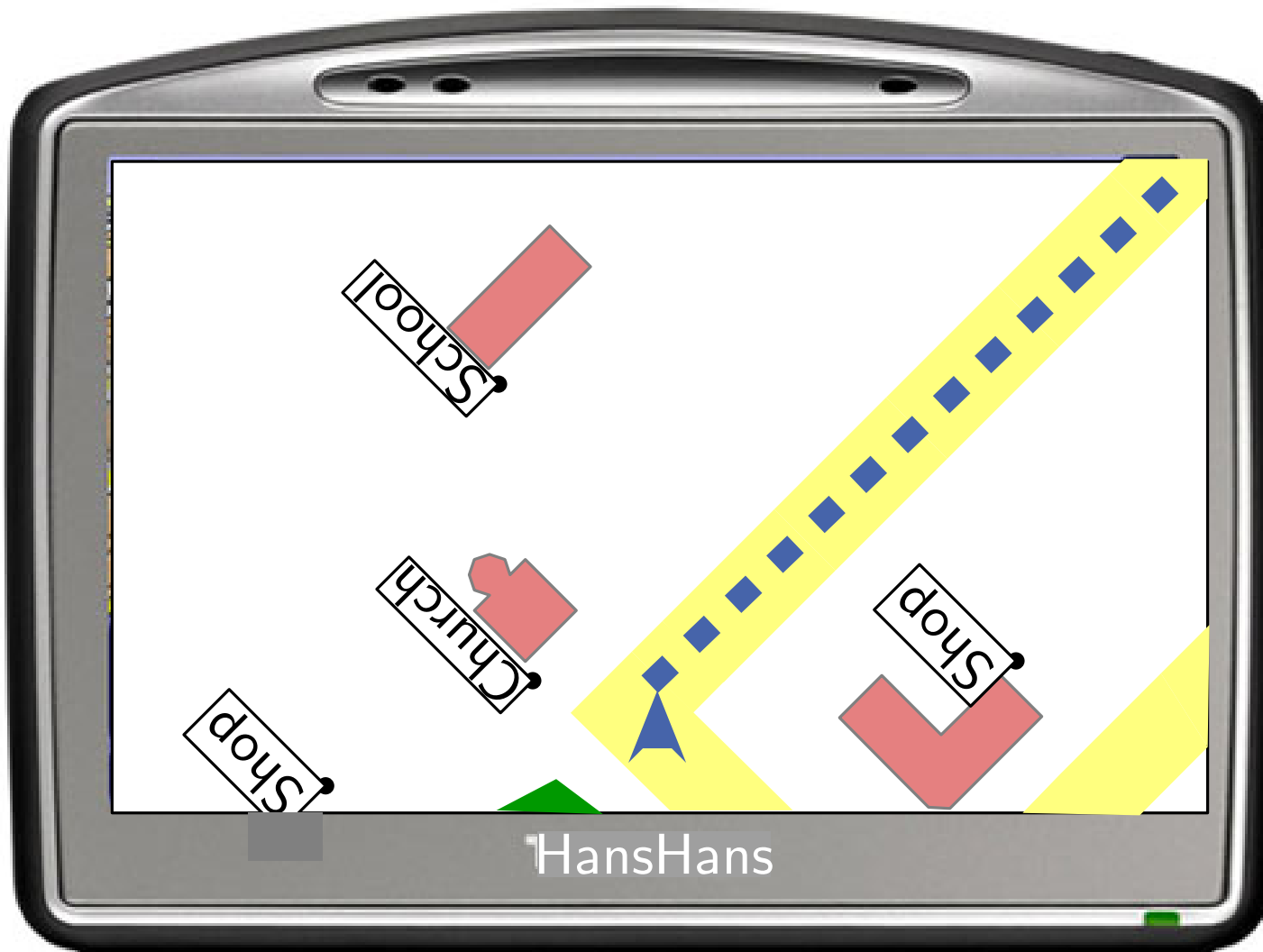
Motivation



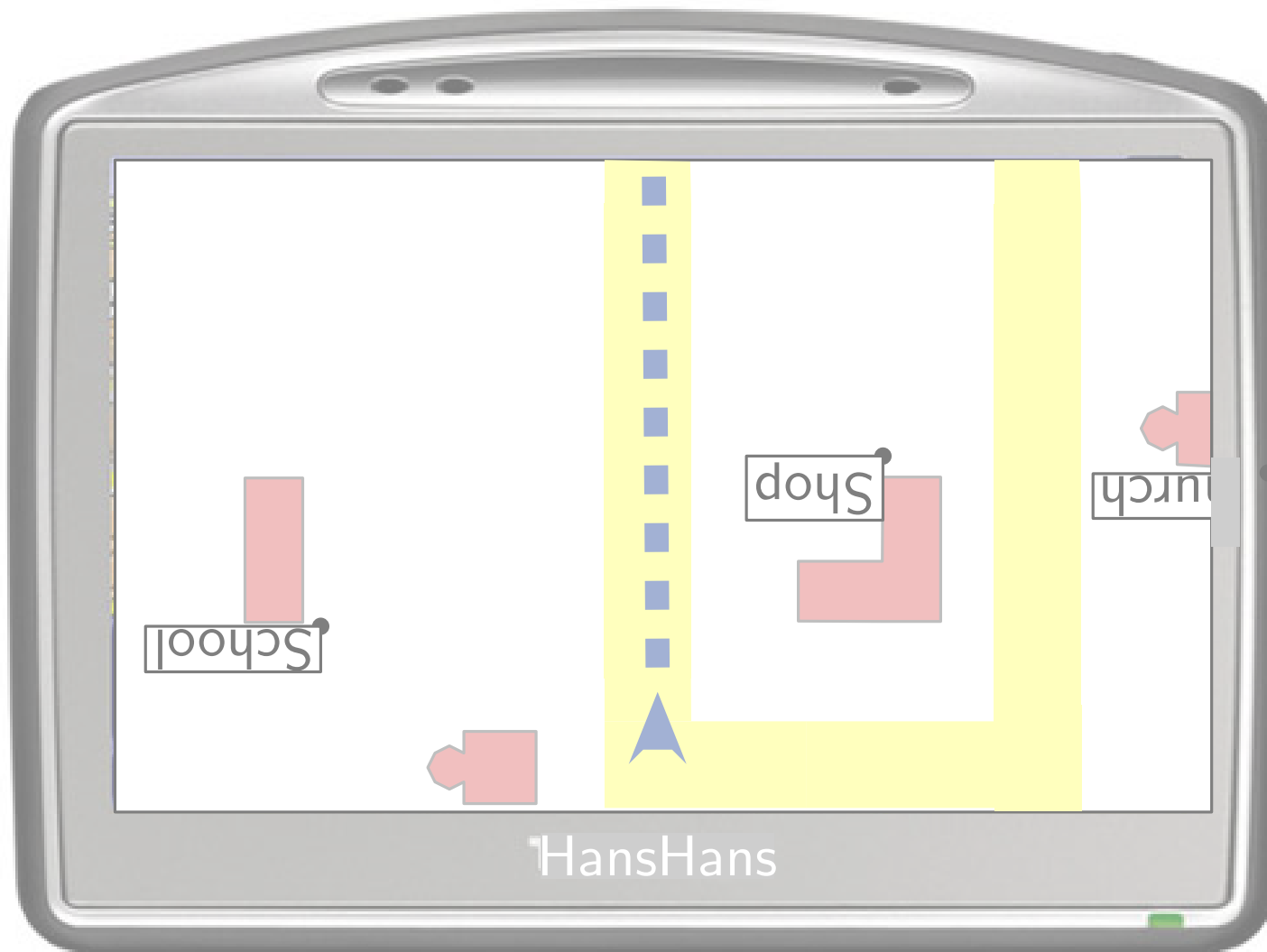
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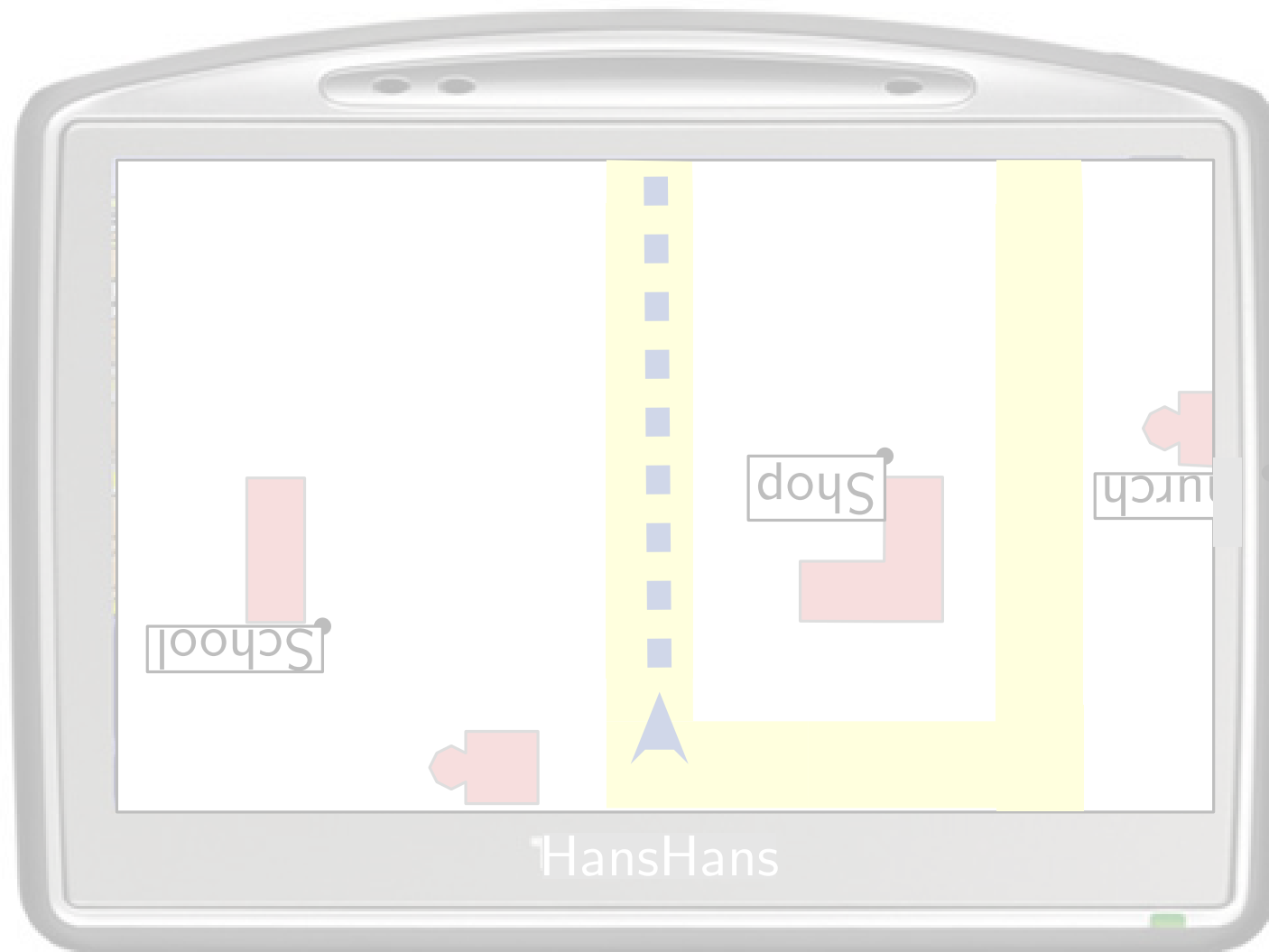
Motivation



Motivation

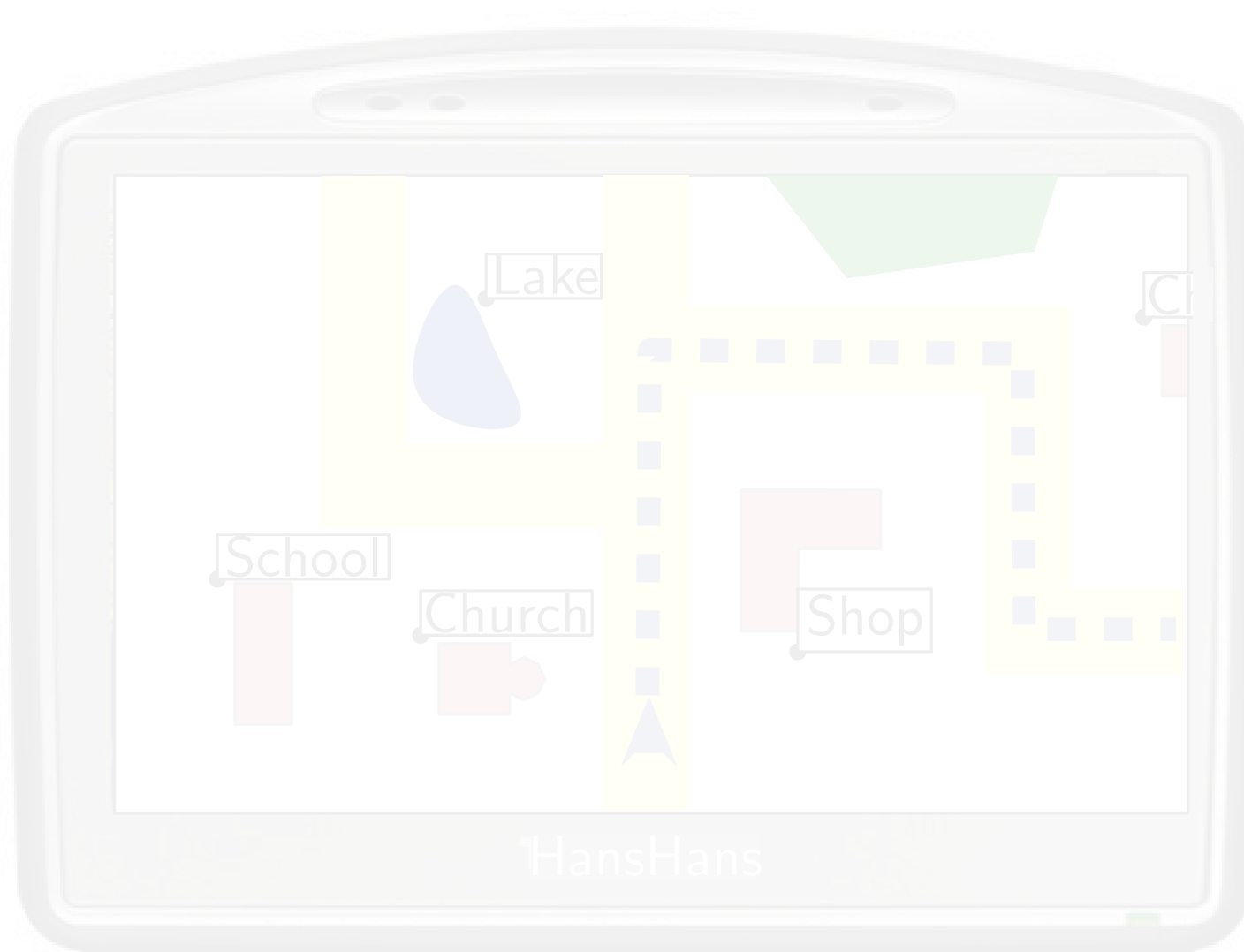


Motivation



Motivation

Motivation



Motivation



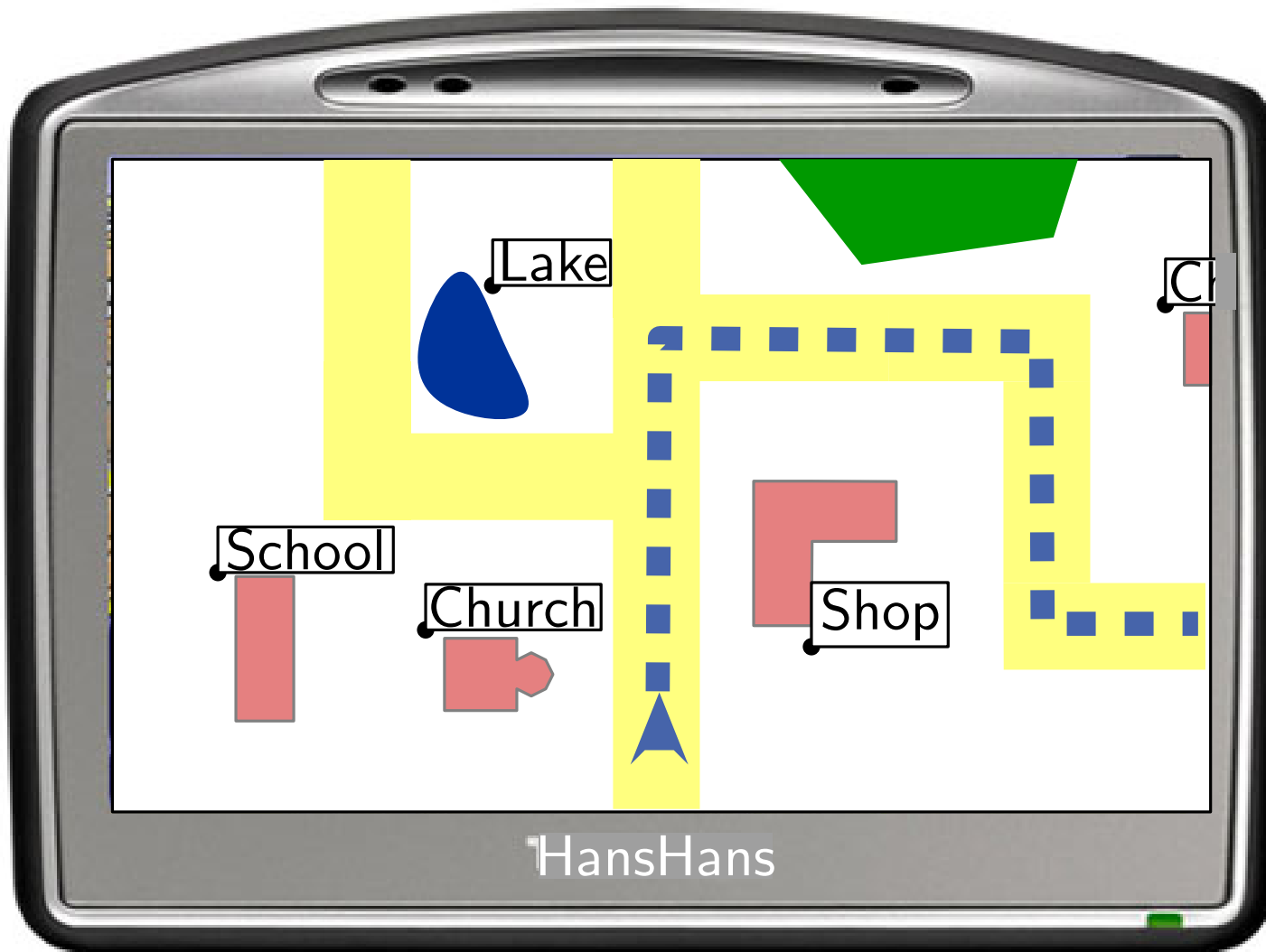
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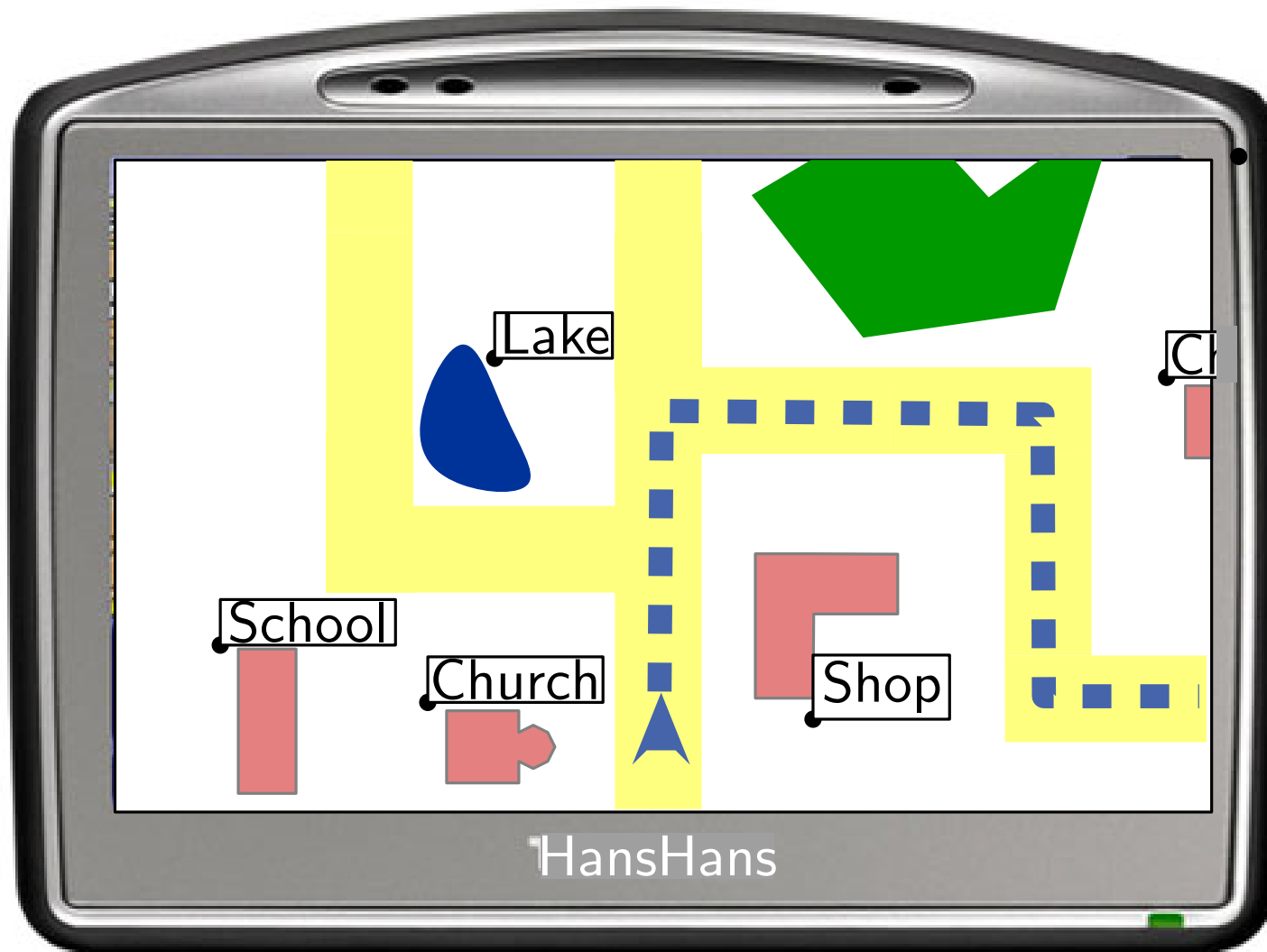
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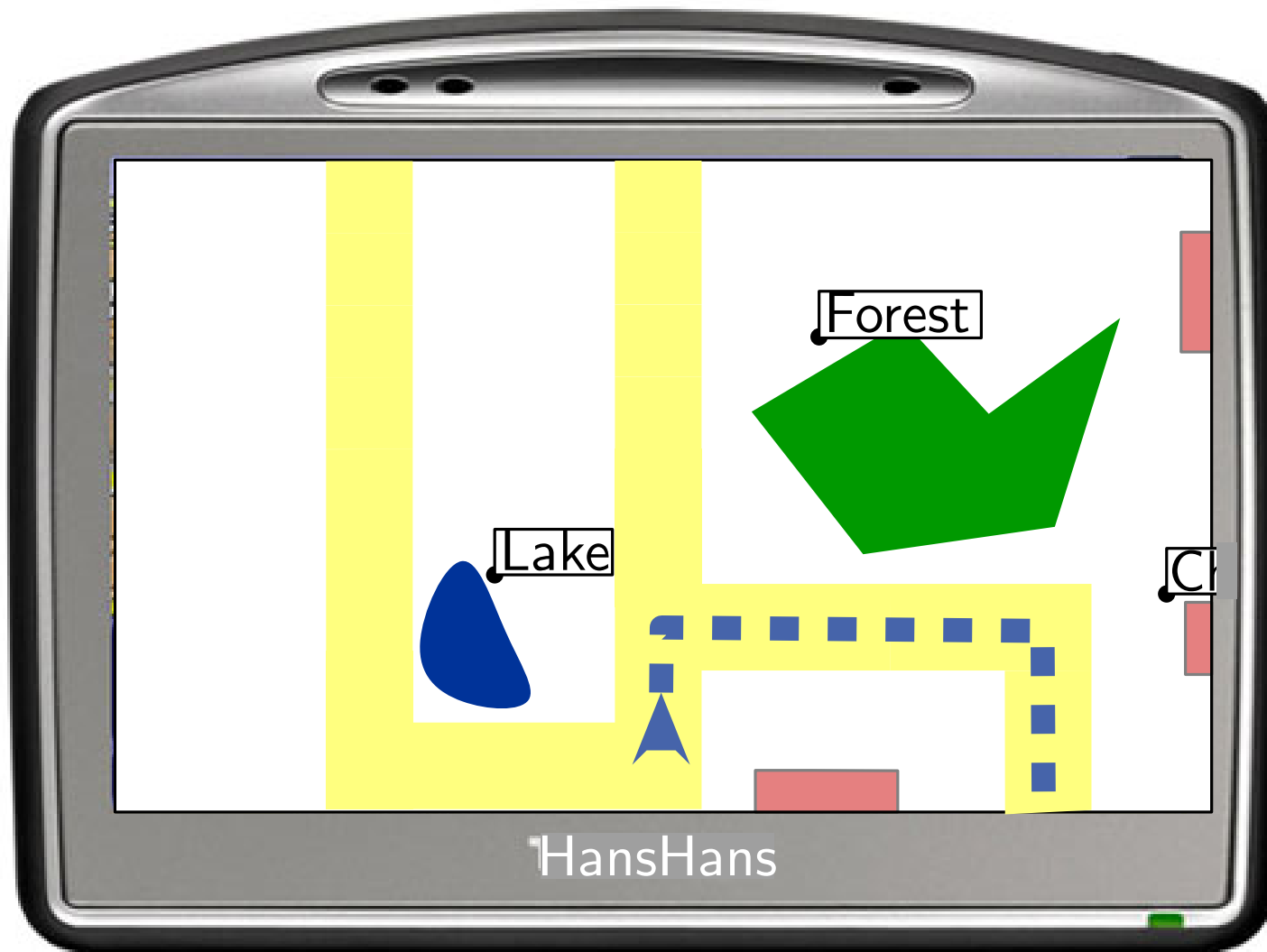
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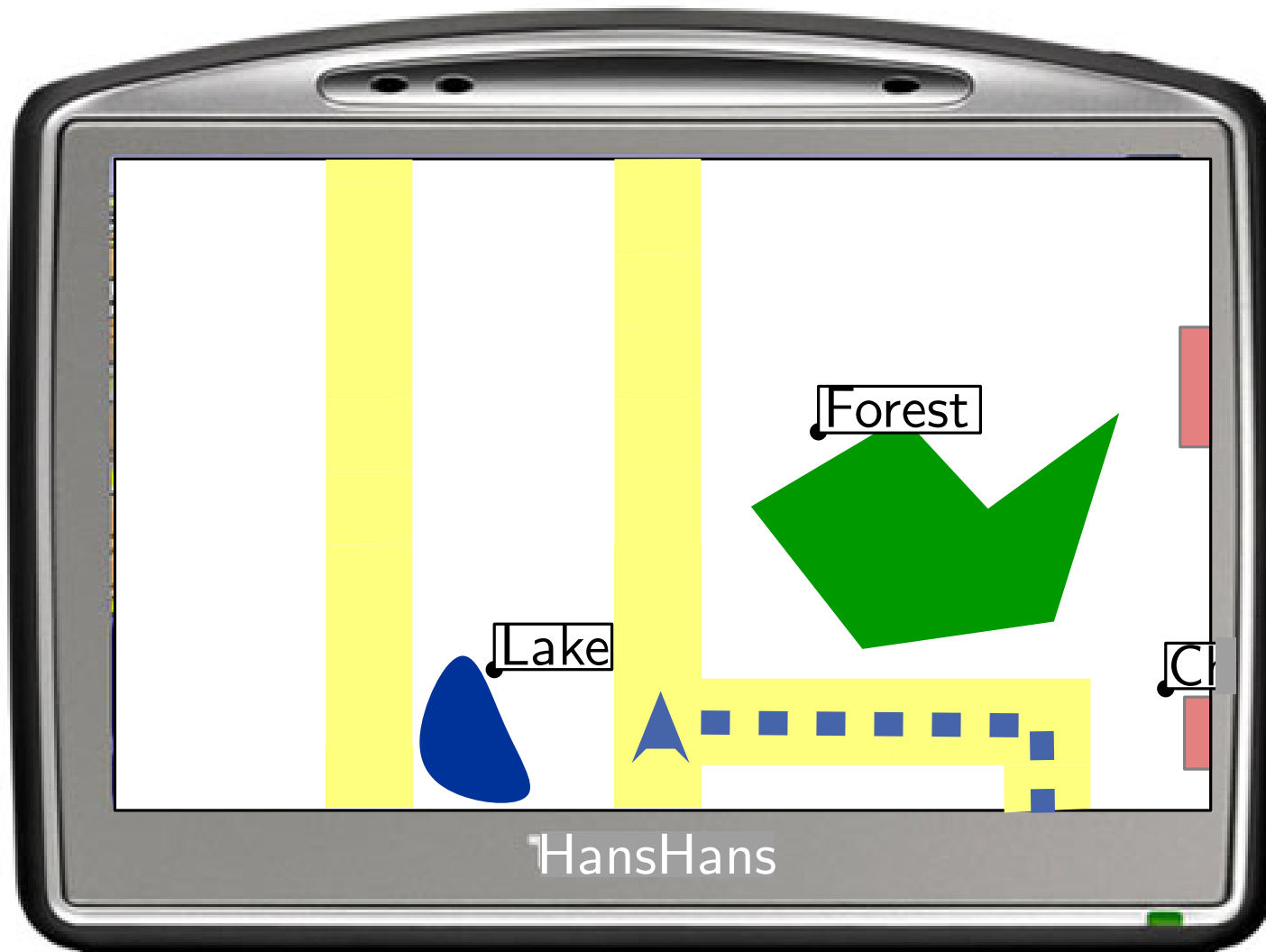
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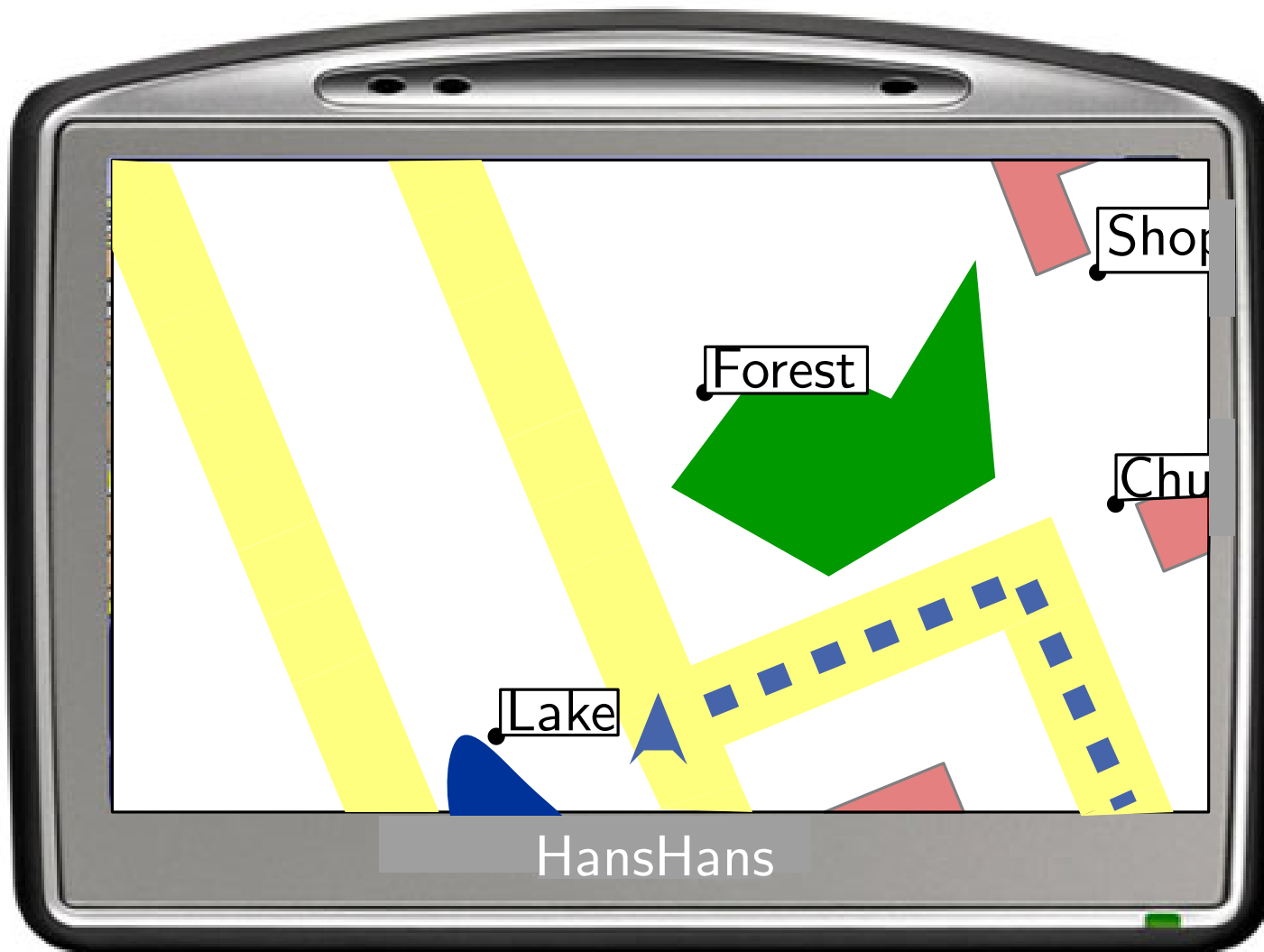
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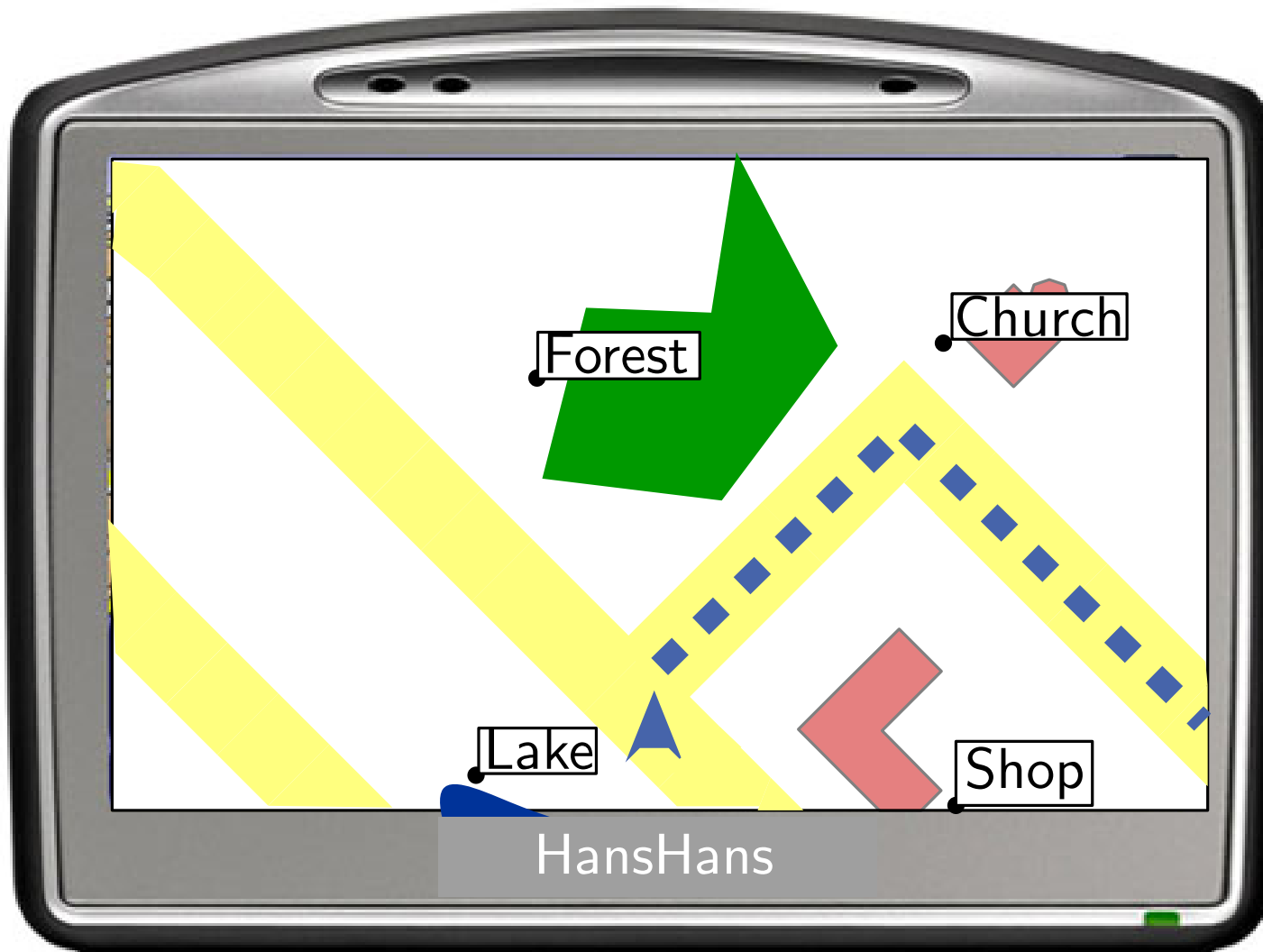
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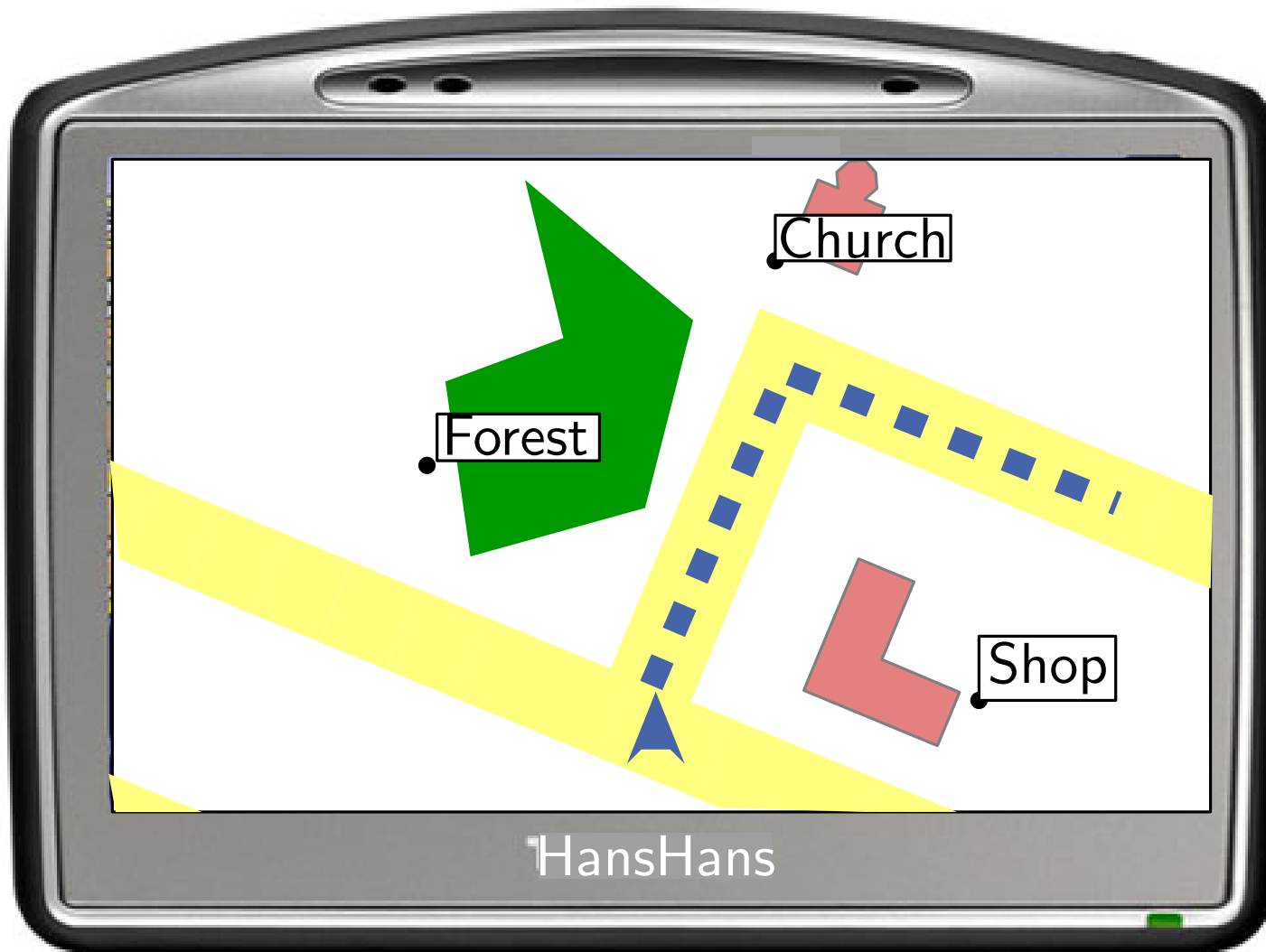
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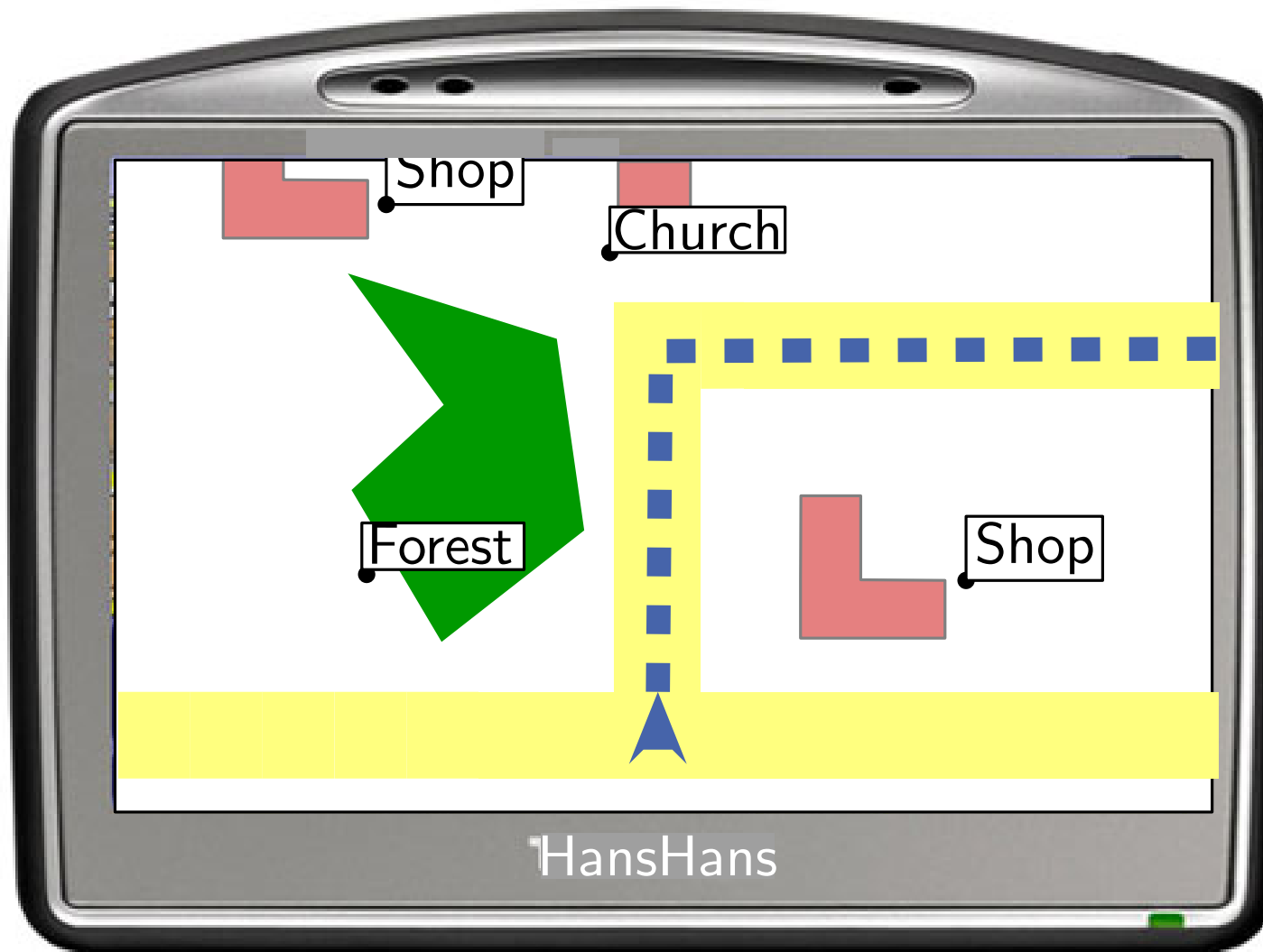
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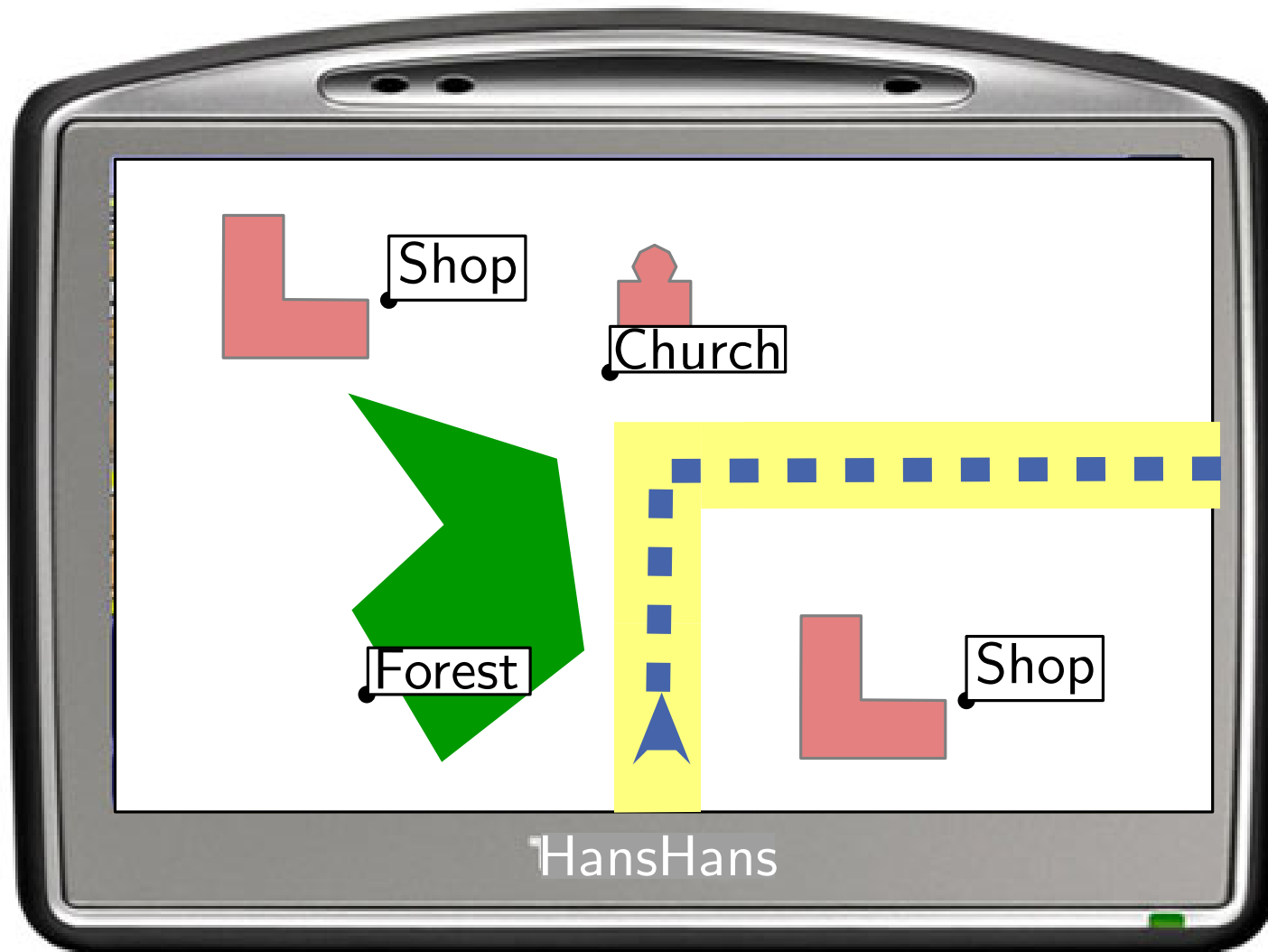
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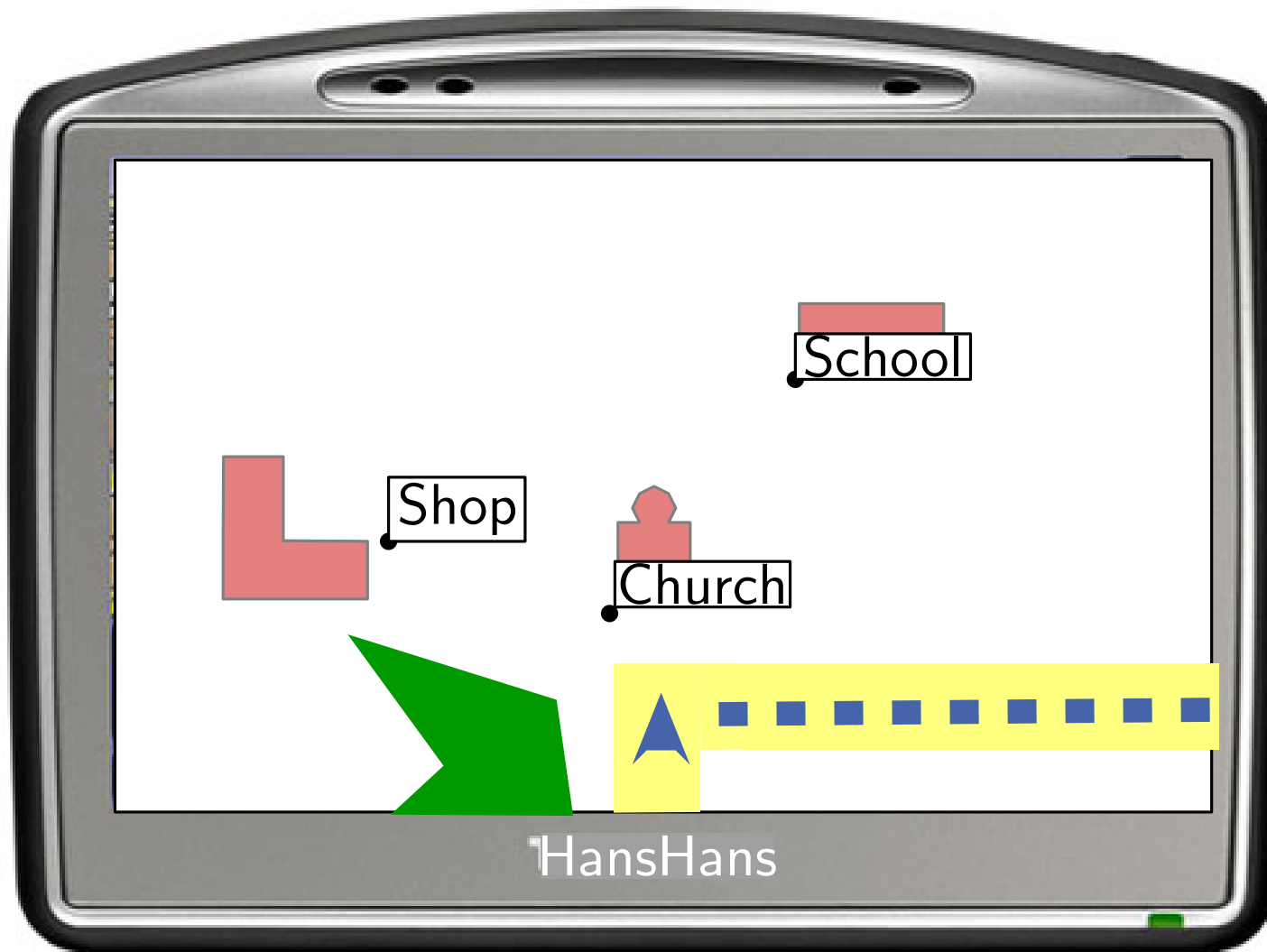
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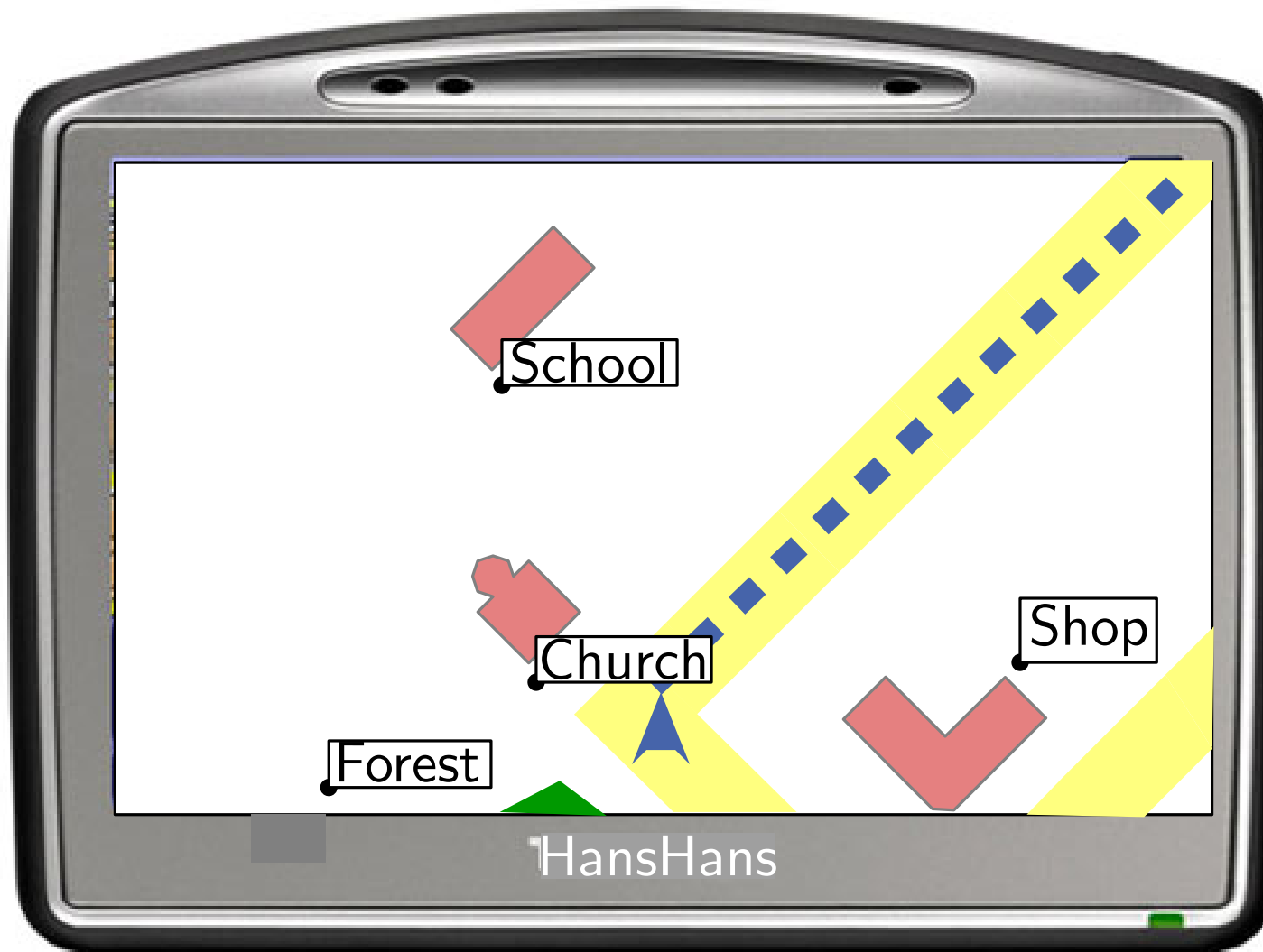
Motivation



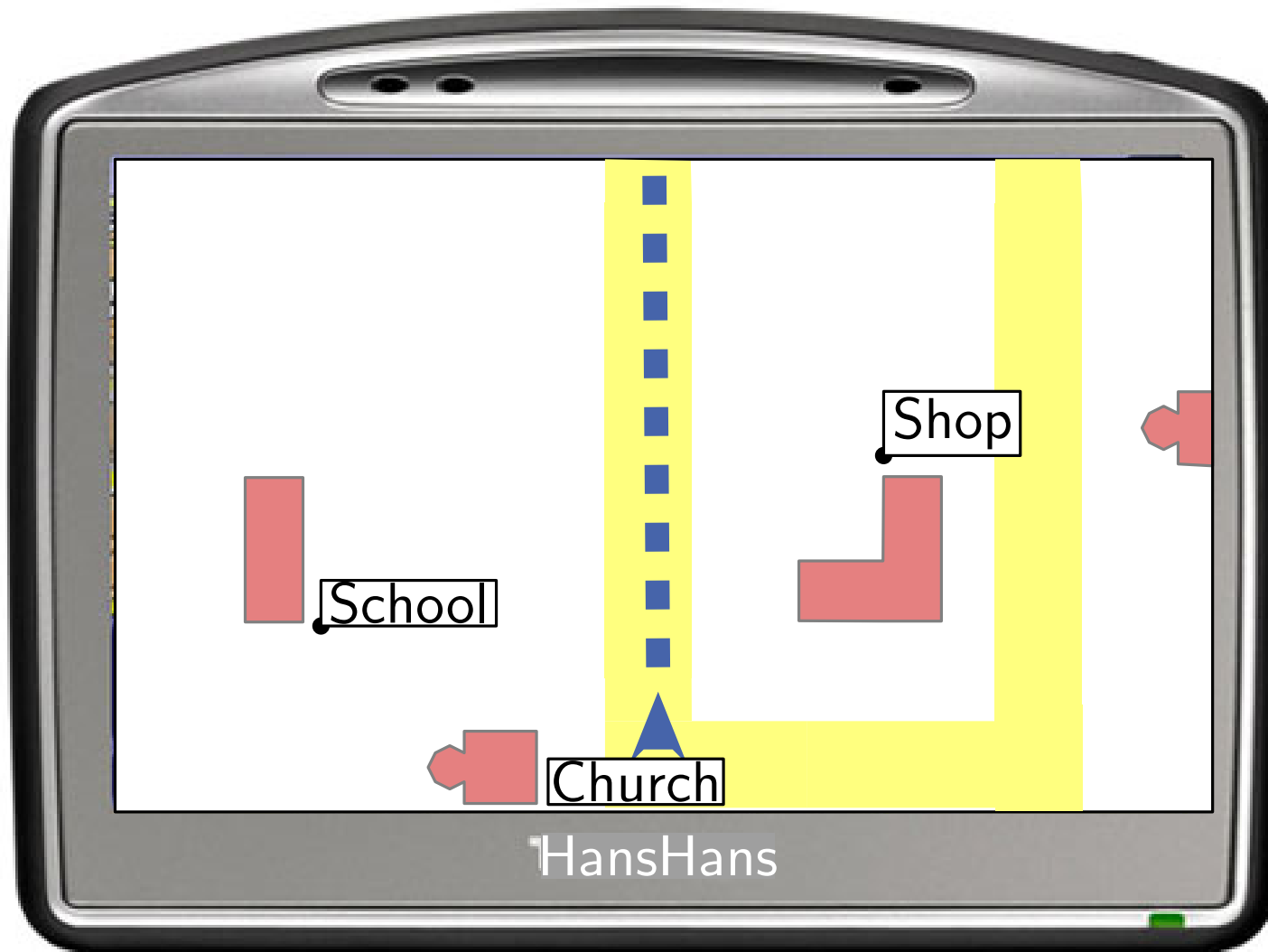
Motivation



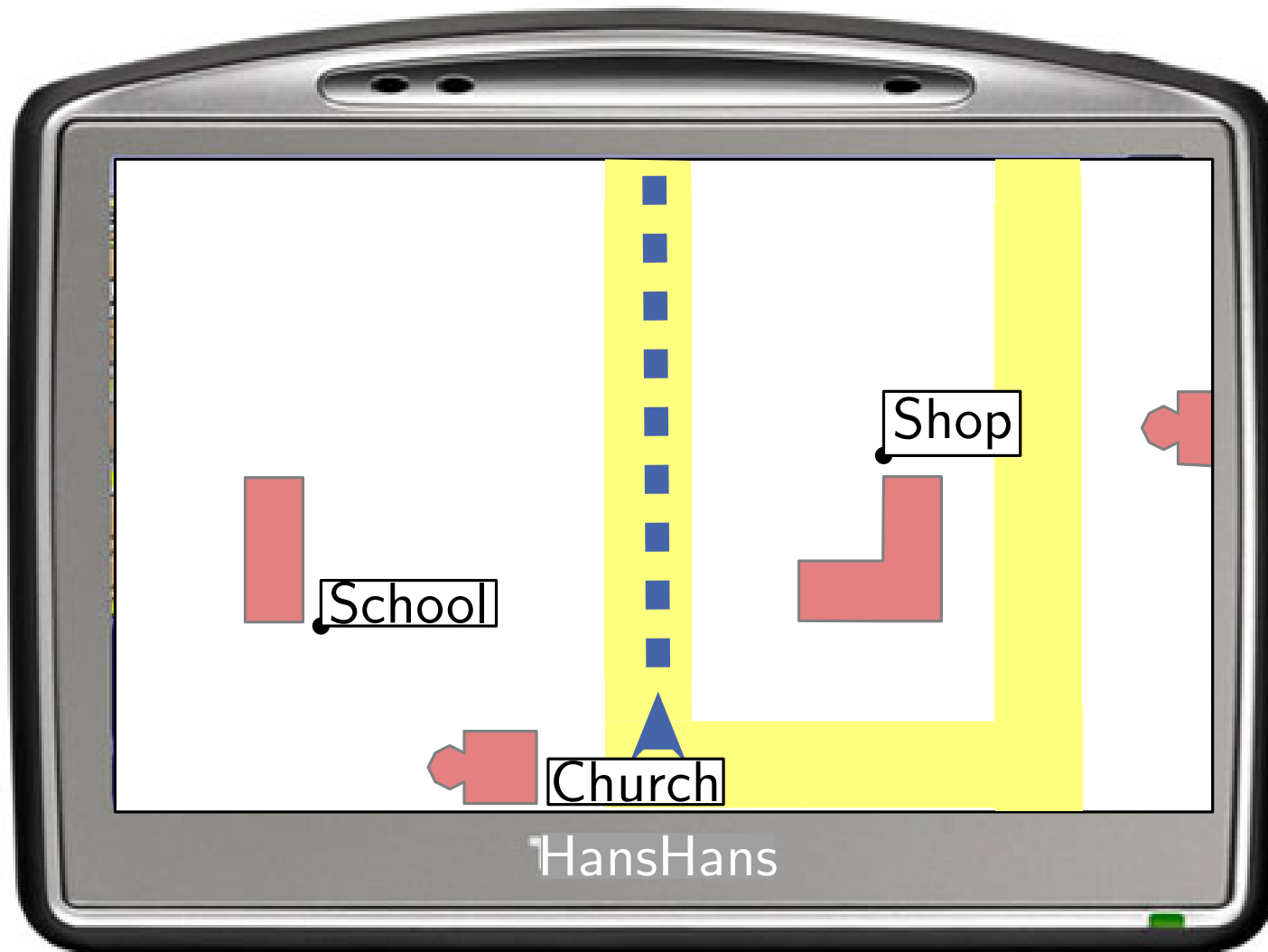
Motivation



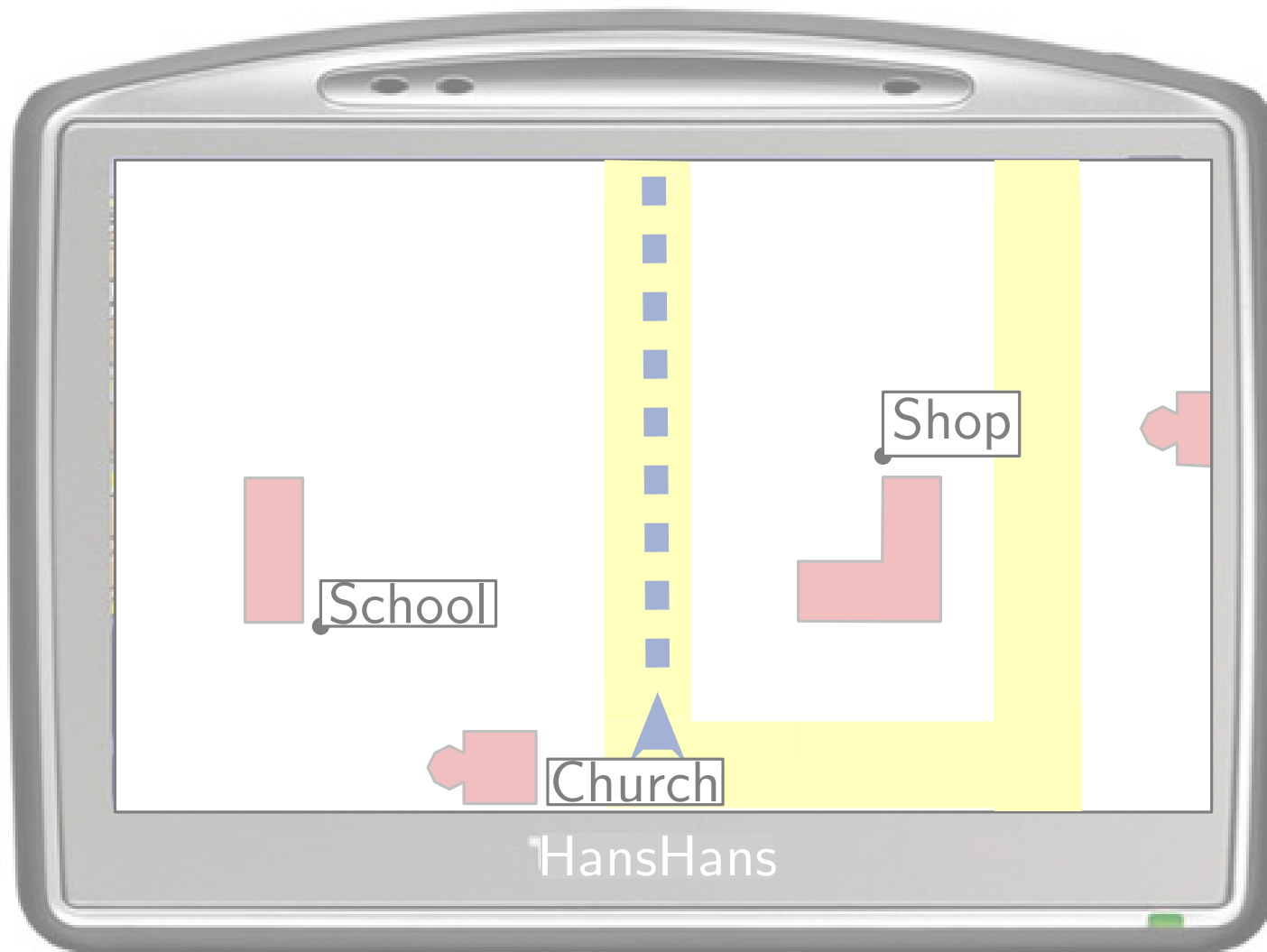
Motivation



Motivation



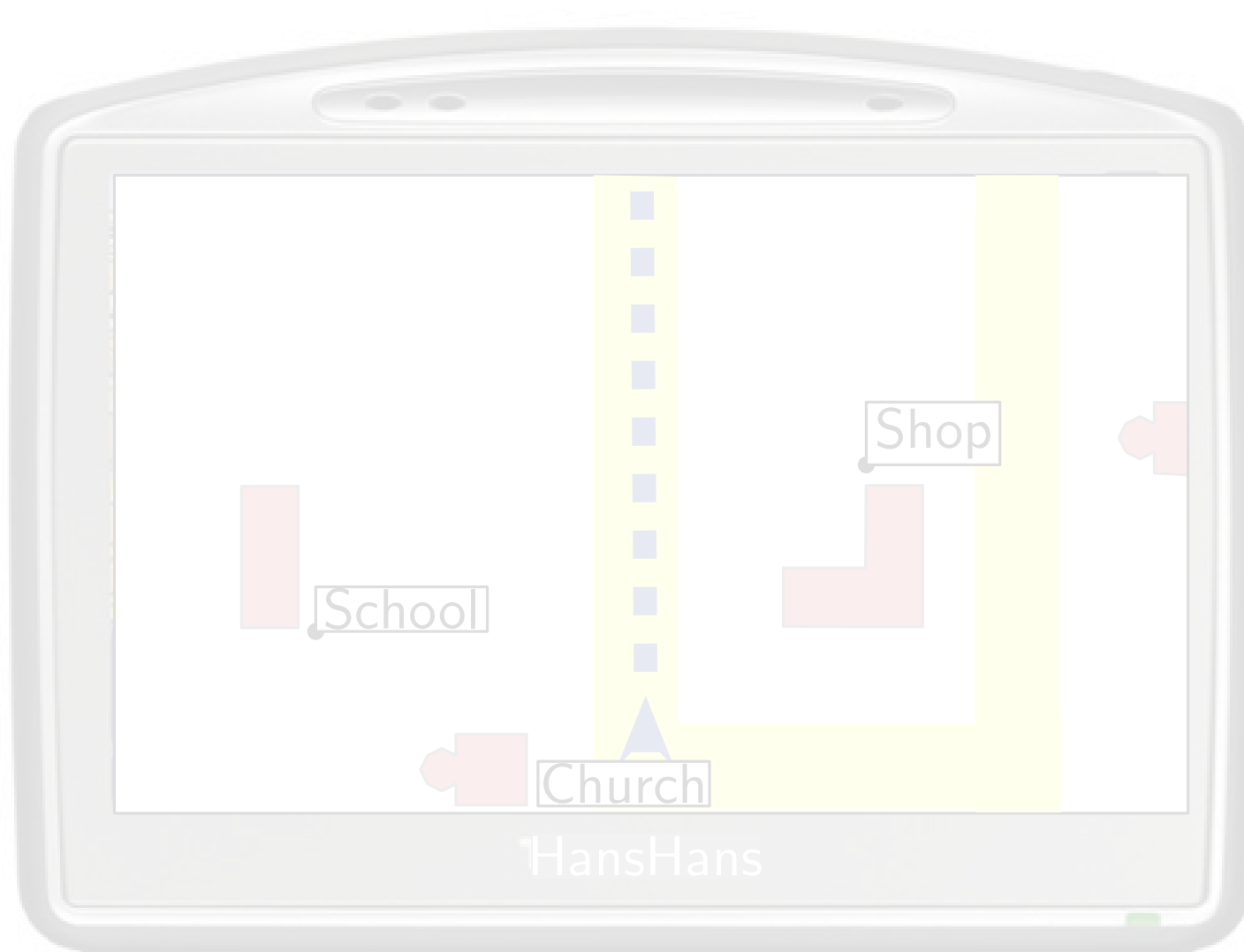
Motivation



Motivation



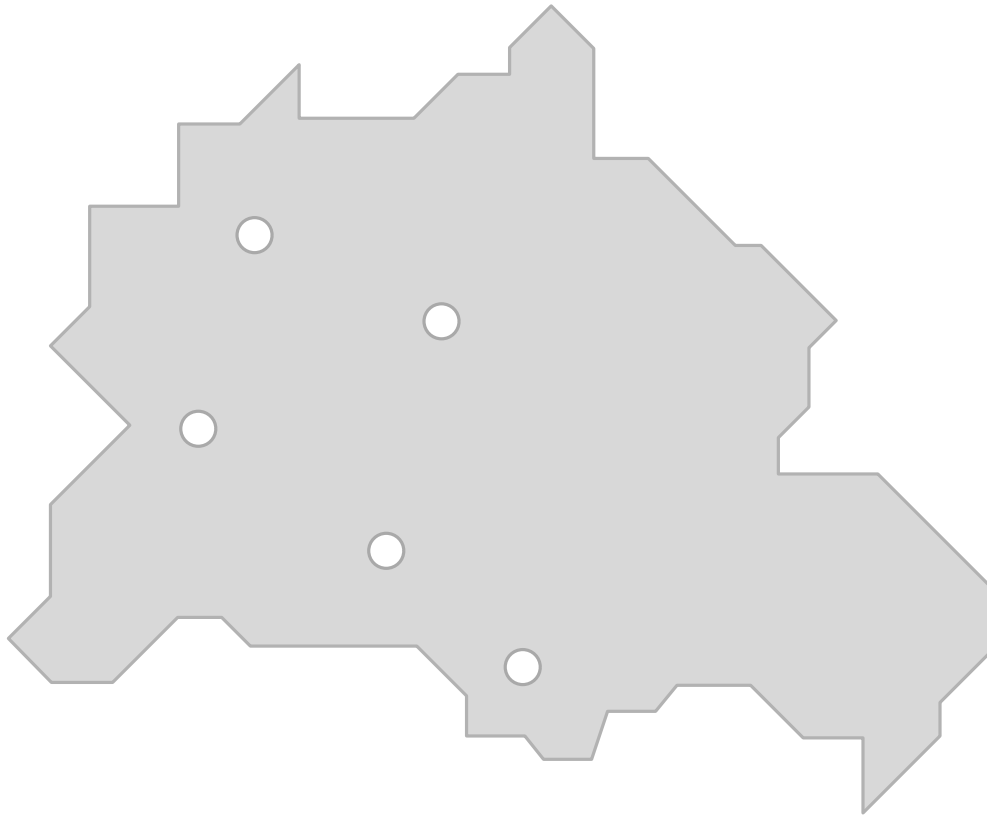
Motivation



Problem Definition

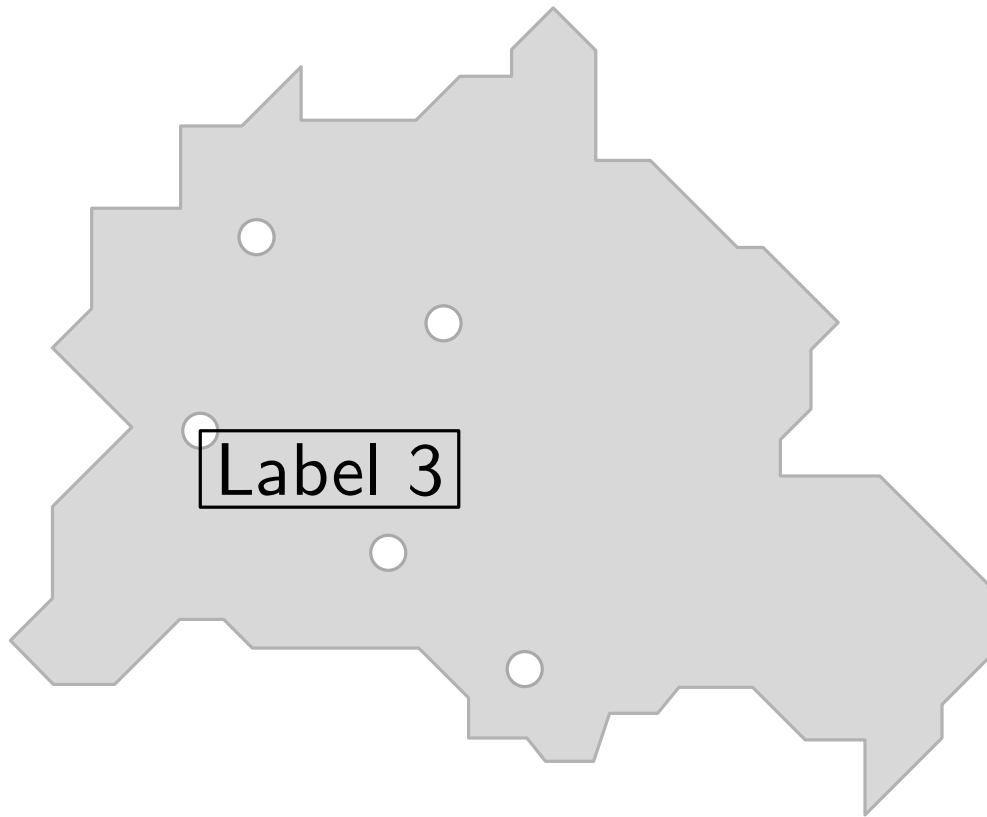
Static Labeling

Input: Map with points



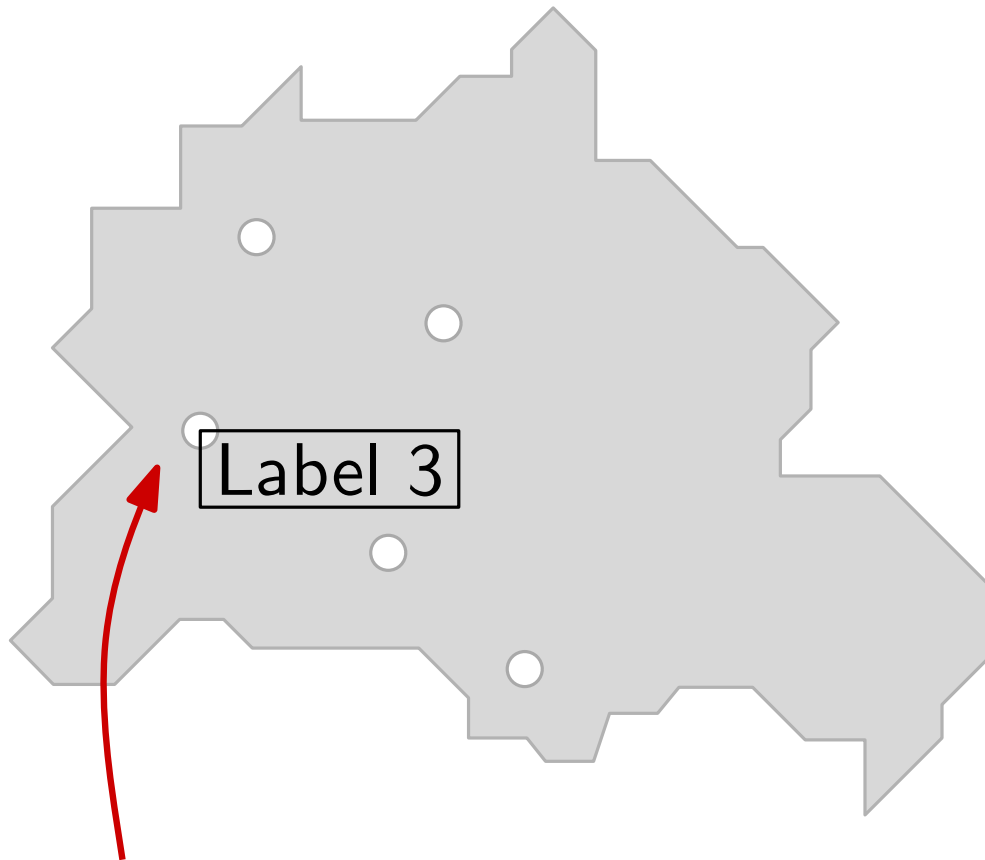
Static Labeling

Input: Map with points



Static Labeling

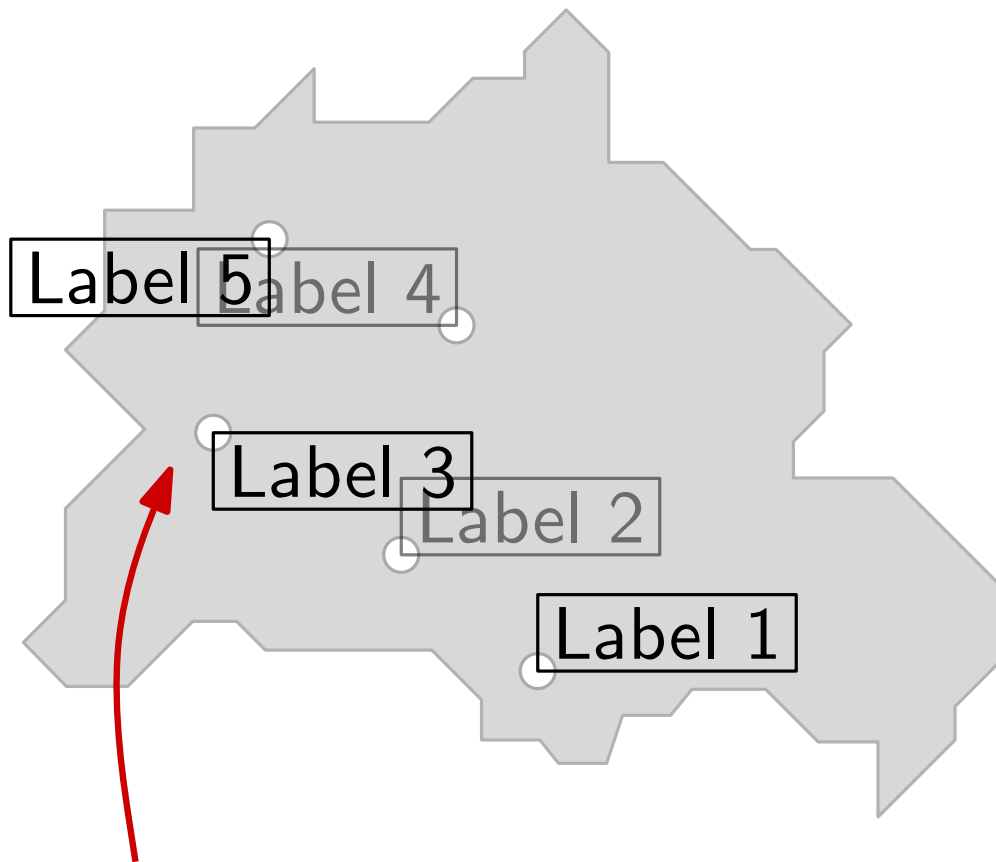
Input: Map with points



anchor point

Static Labeling

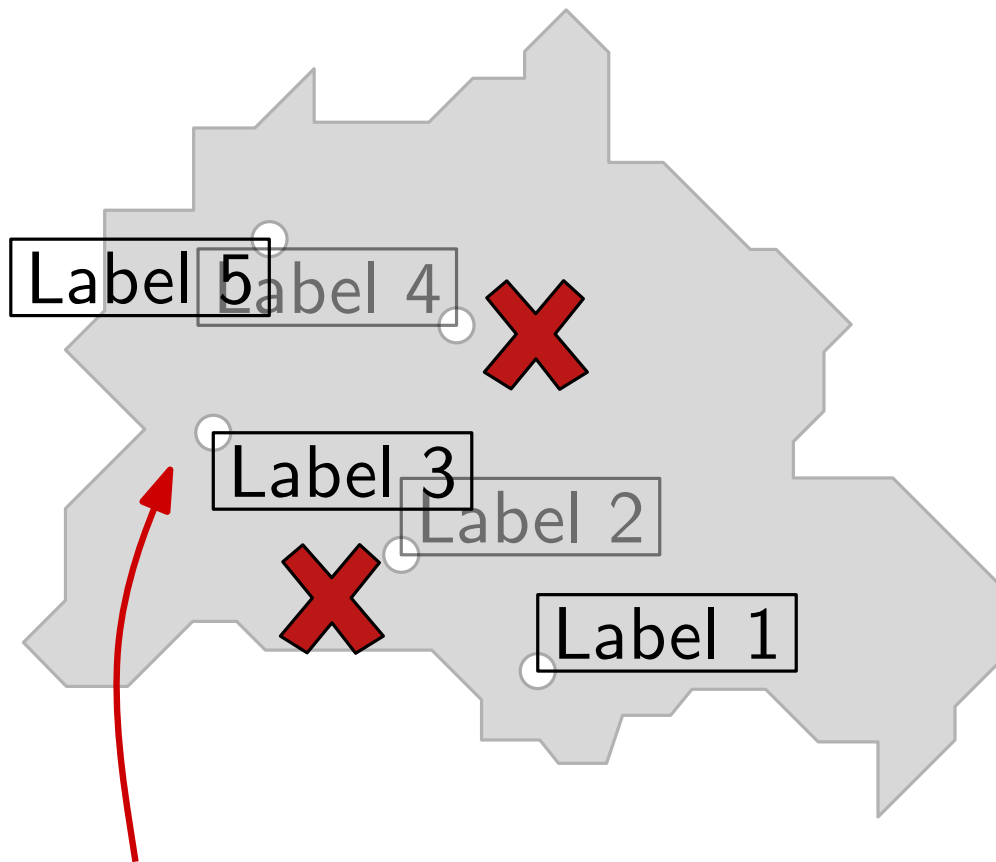
Input: Map with points



anchor point

Static Labeling

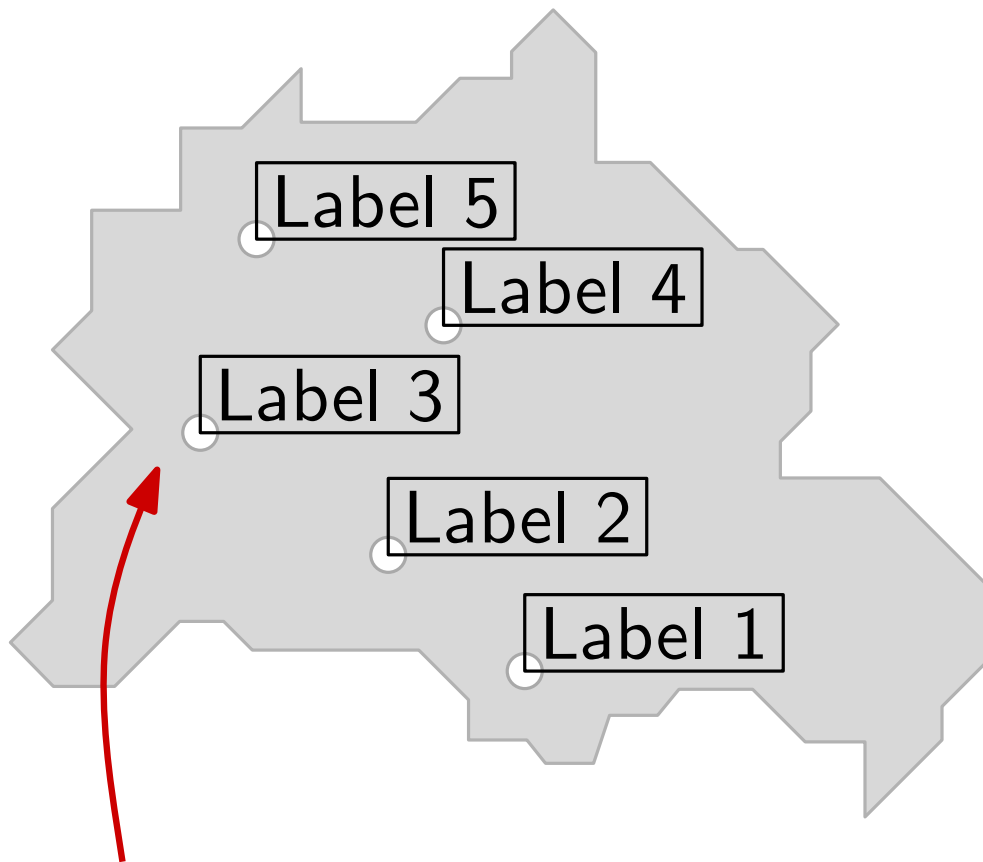
Input: Map with points



anchor point

Static Labeling

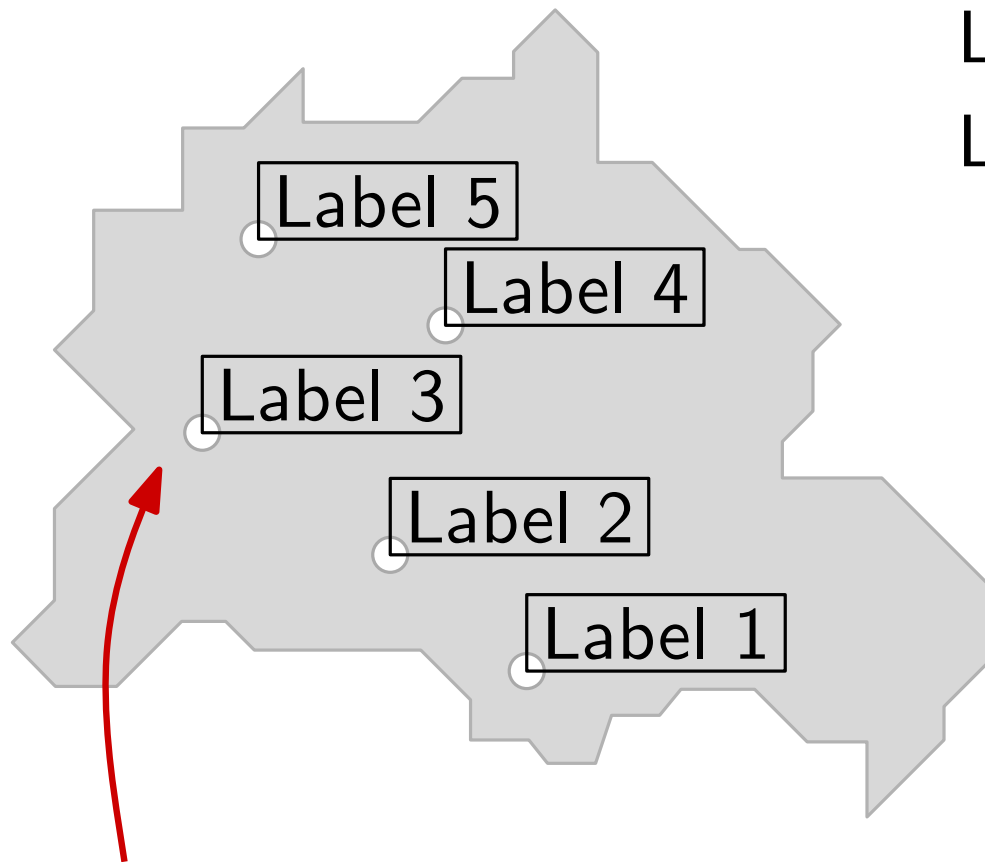
Input: Map with points



anchor point

Static Labeling

Input: Map with points



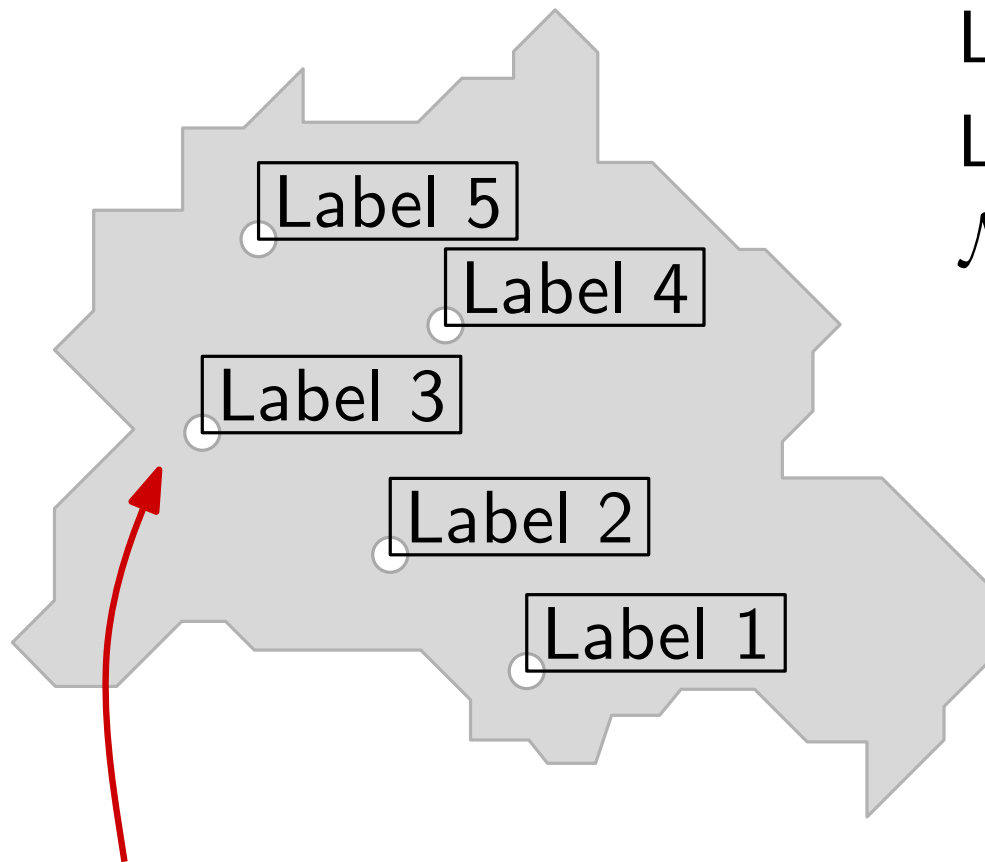
anchor point

Label Size Maximization

Label Number Maximization

Static Labeling

Input: Map with points

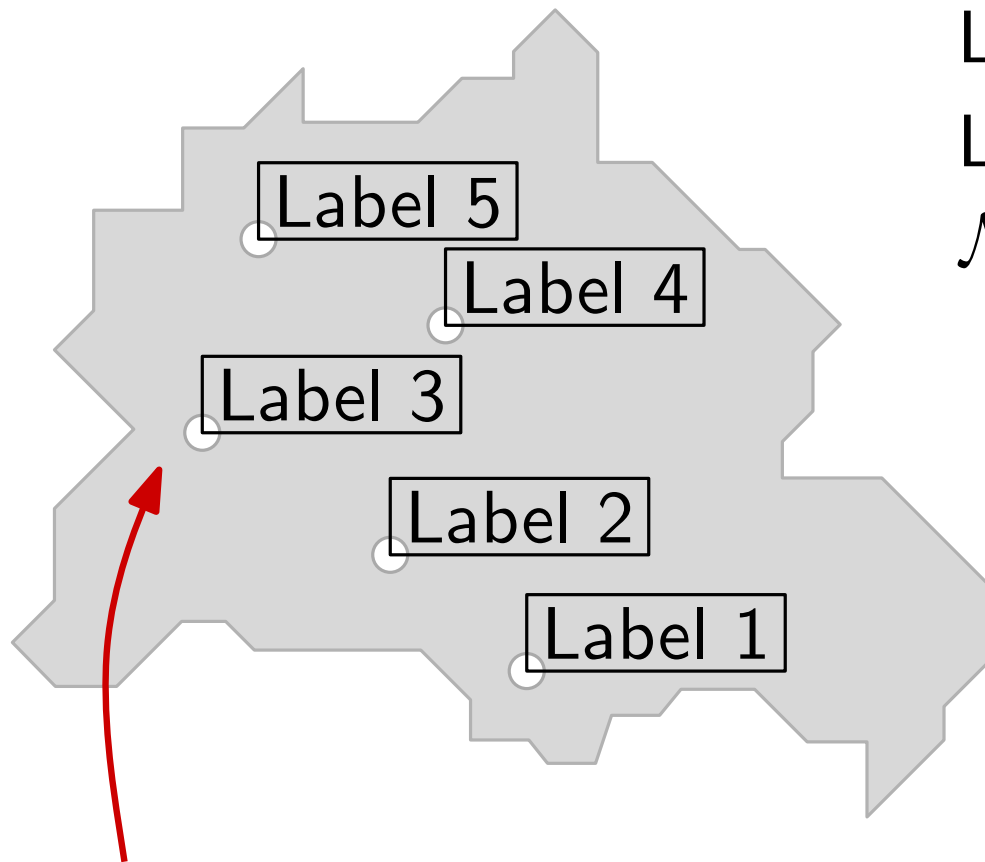


anchor point

Label Size Maximization
Label Number Maximization
 \mathcal{NP} -hard

Static Labeling

Input: Map with points

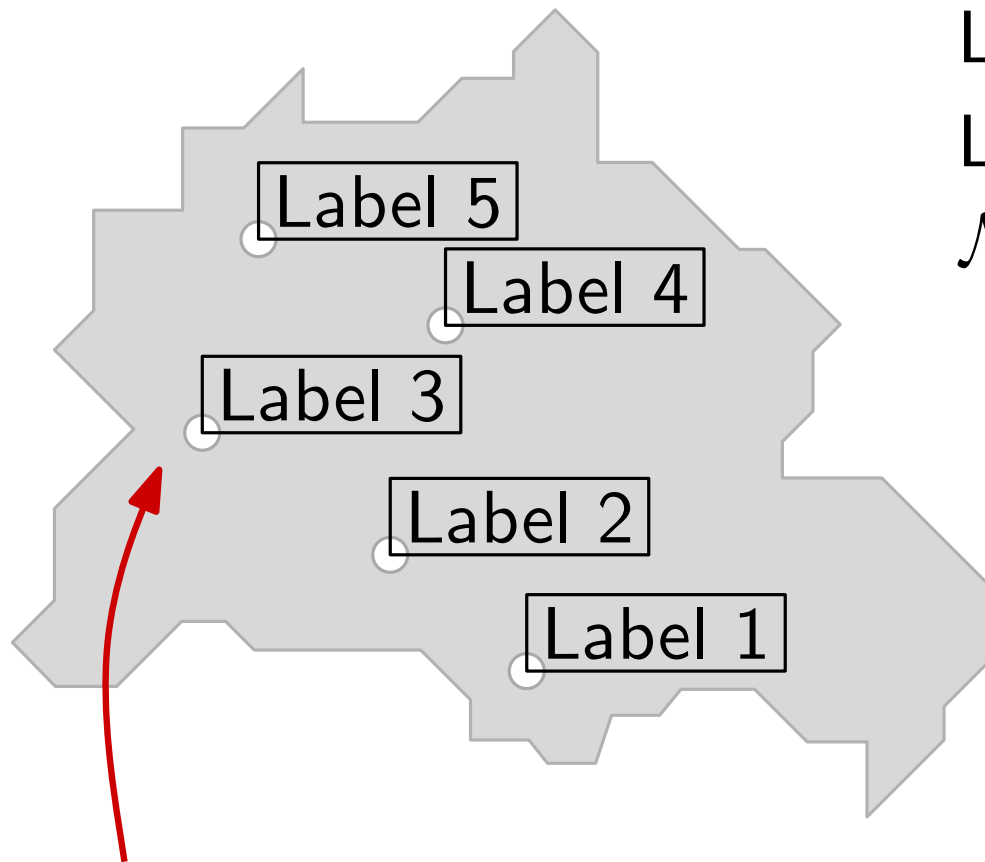


anchor point

Label Size Maximization
Label Number Maximization
 \mathcal{NP} -hard

Static Labeling

Input: Map with points

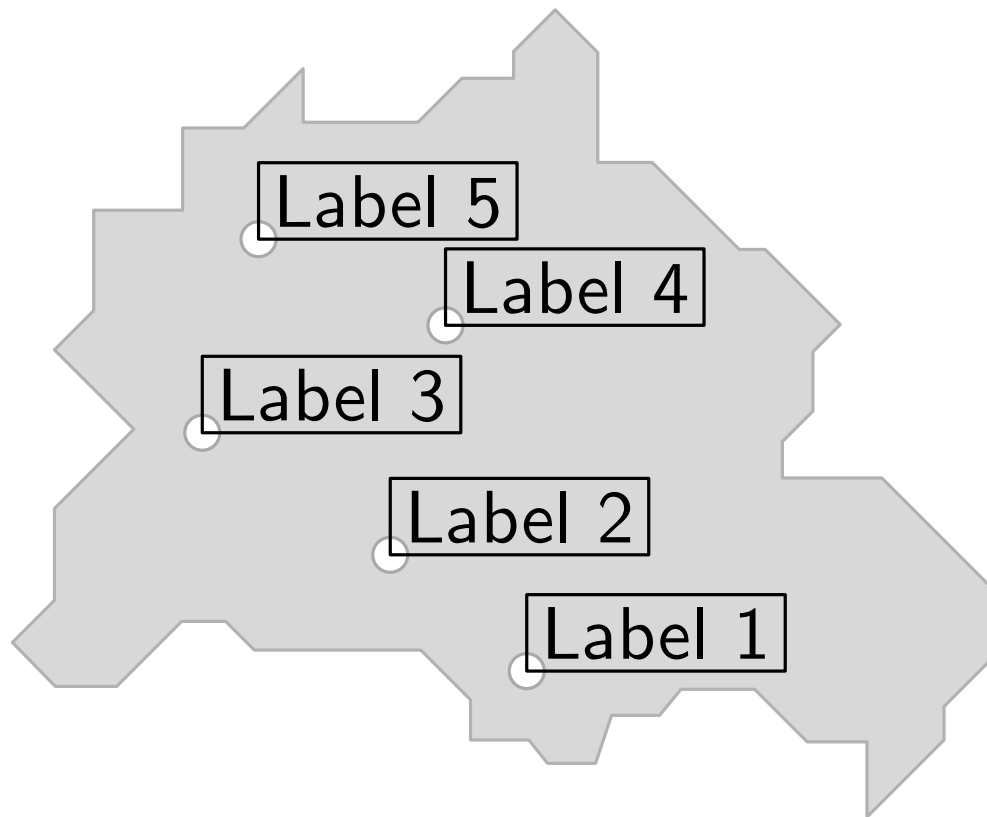


anchor point

Label Size Maximization
Label Number Maximization
 \mathcal{NP} -hard

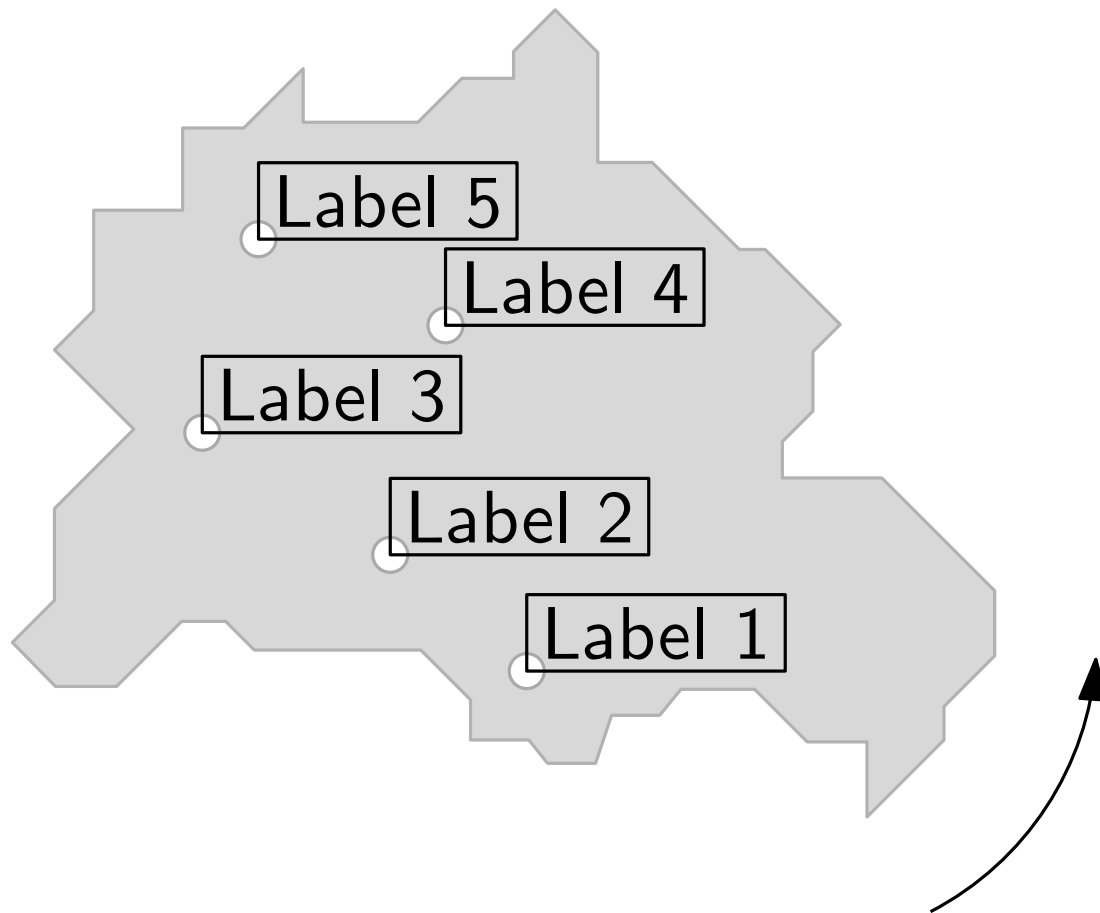
Our Problem

Input: Labeled map



Our Problem

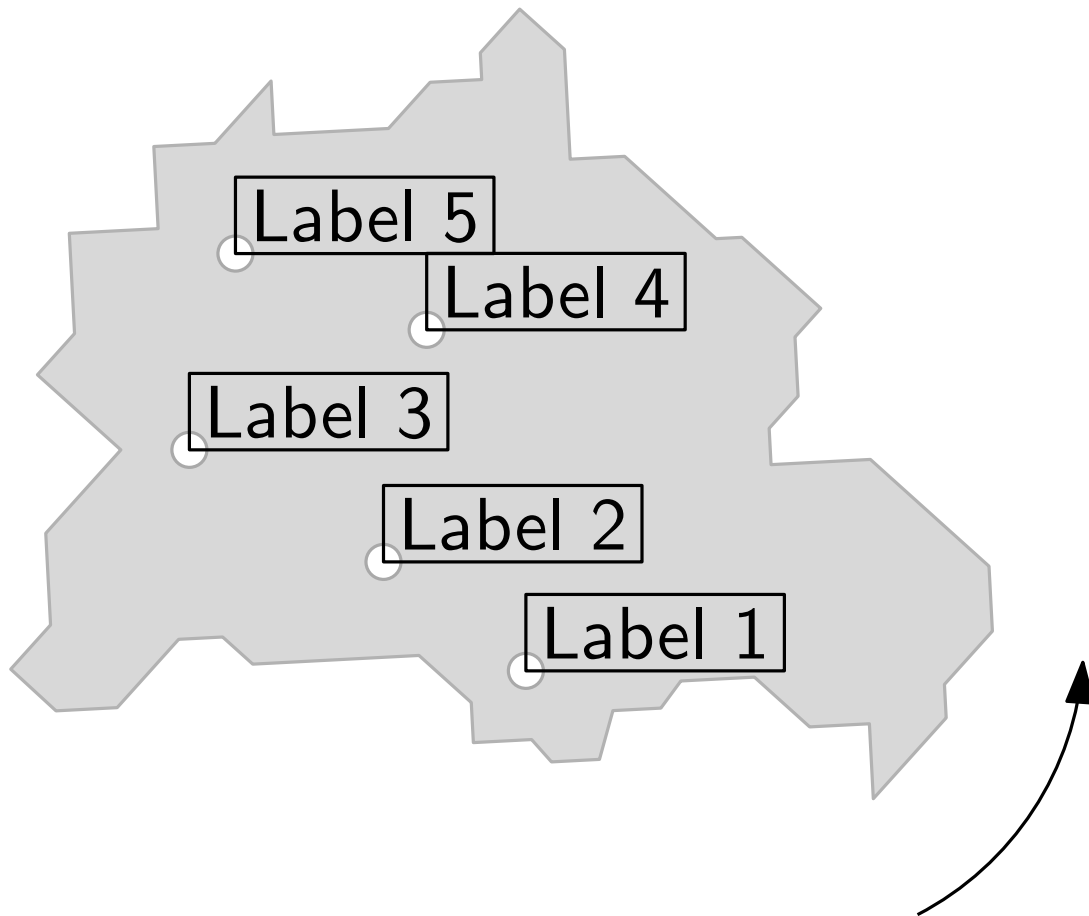
Input: Labeled map



monotone rotation

Our Problem

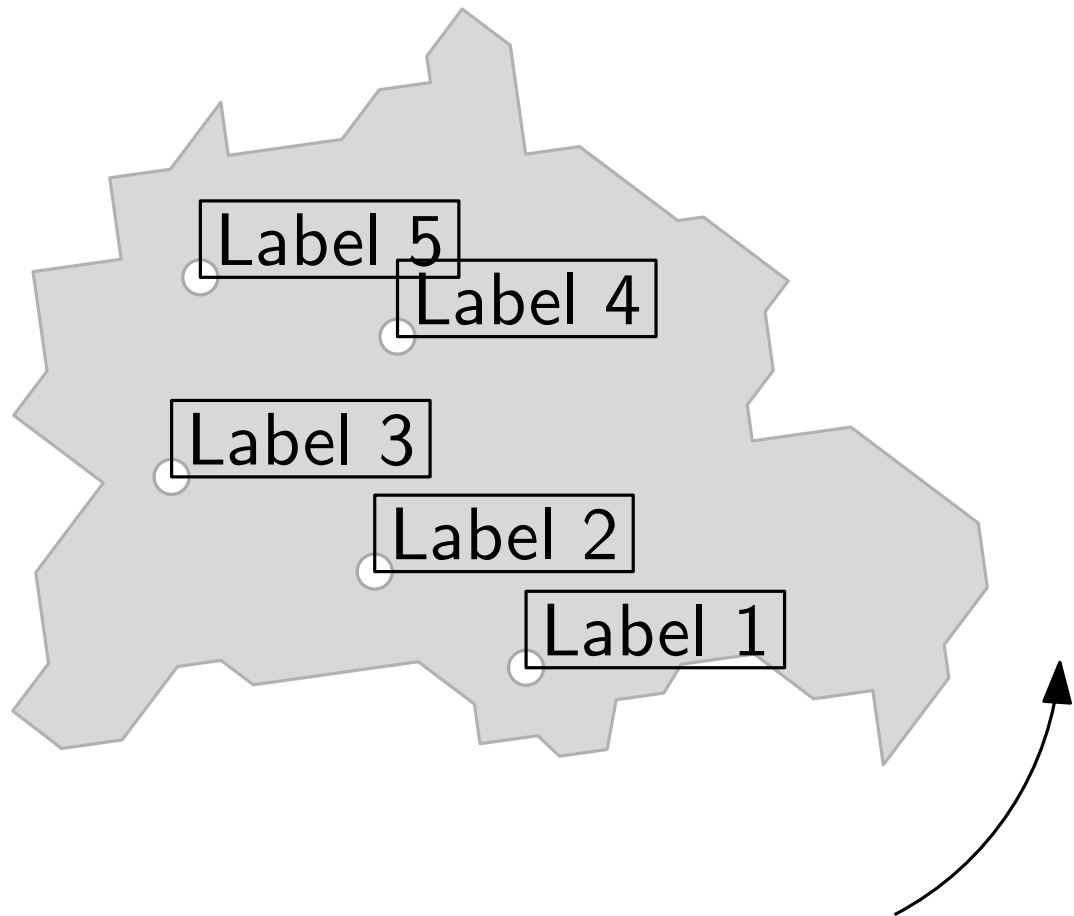
Input: Labeled map



monotone rotation

Our Problem

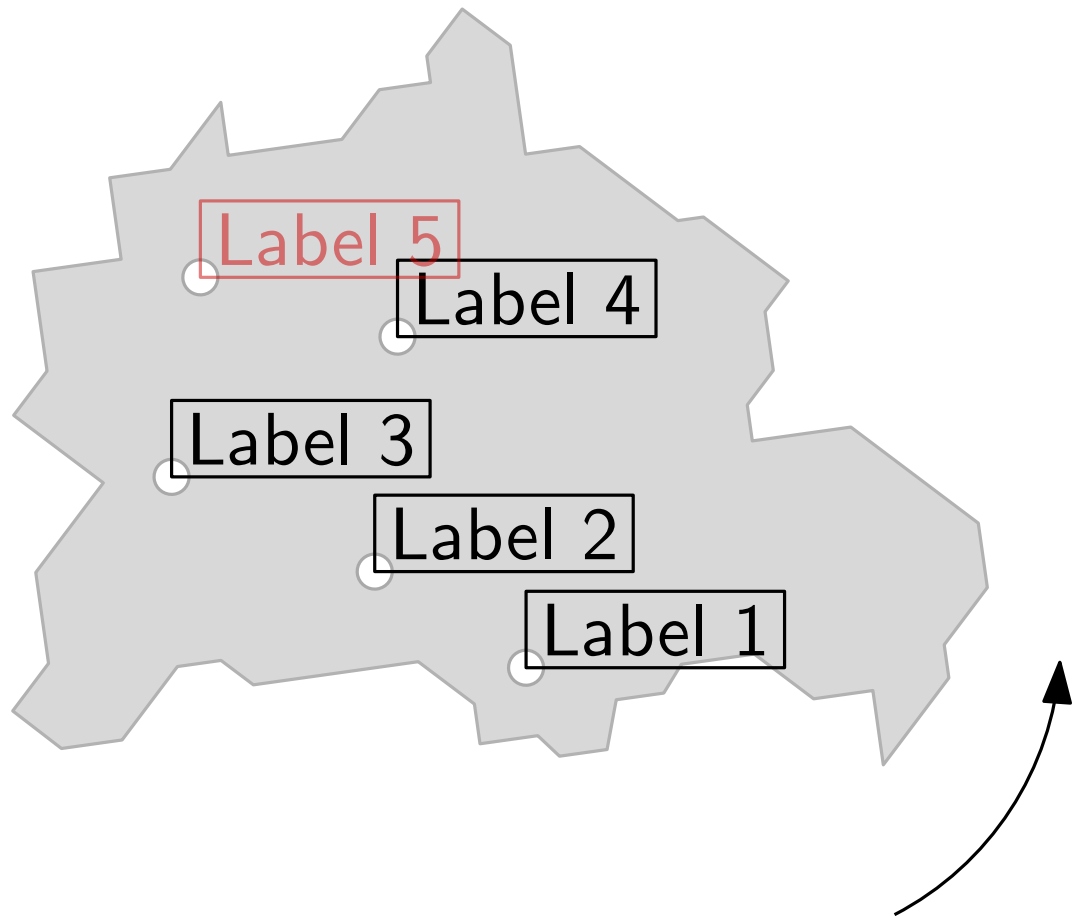
Input: Labeled map



monotone rotation

Our Problem

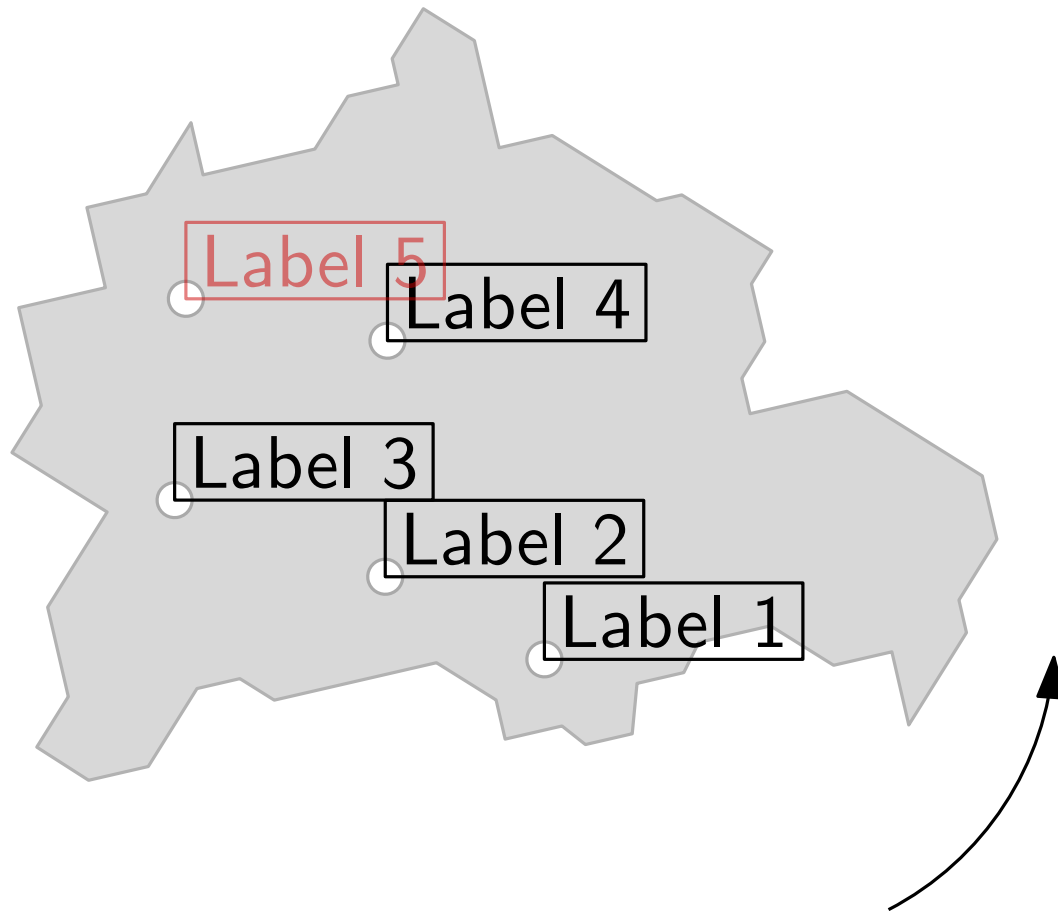
Input: Labeled map



monotone rotation

Our Problem

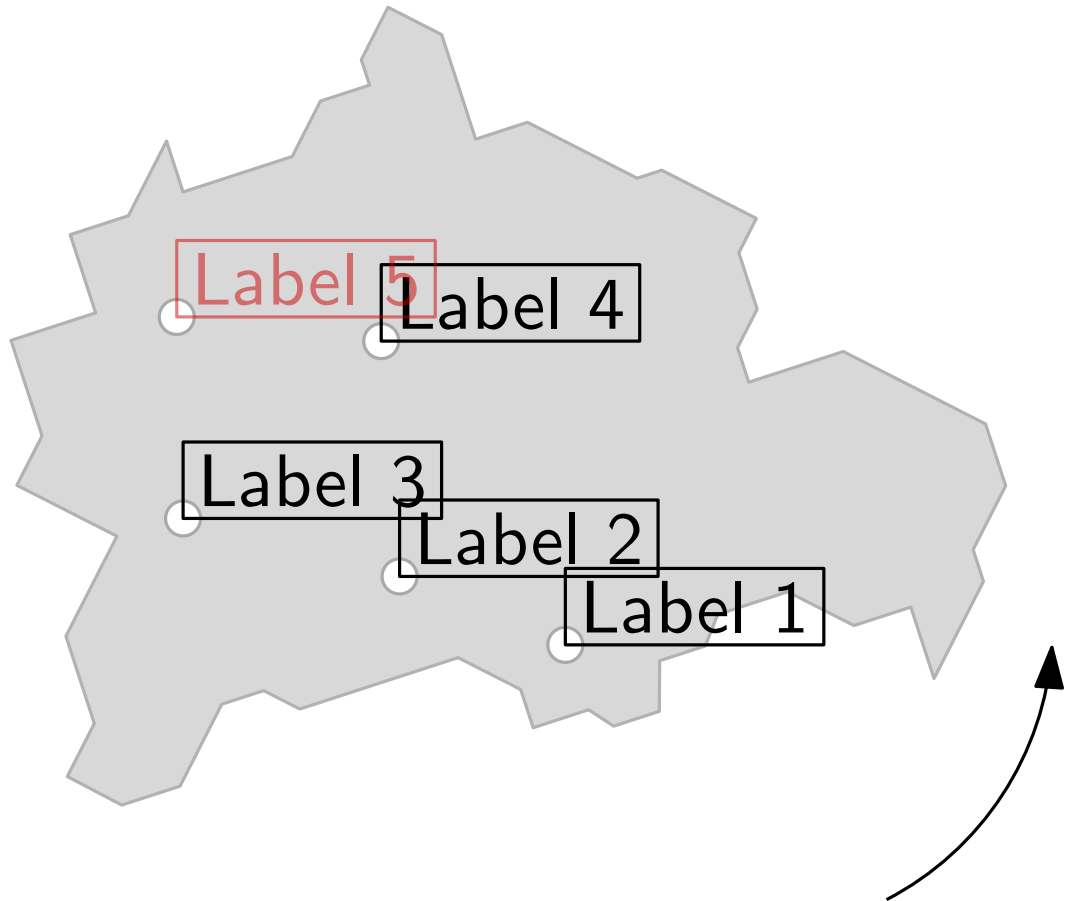
Input: Labeled map



monotone rotation

Our Problem

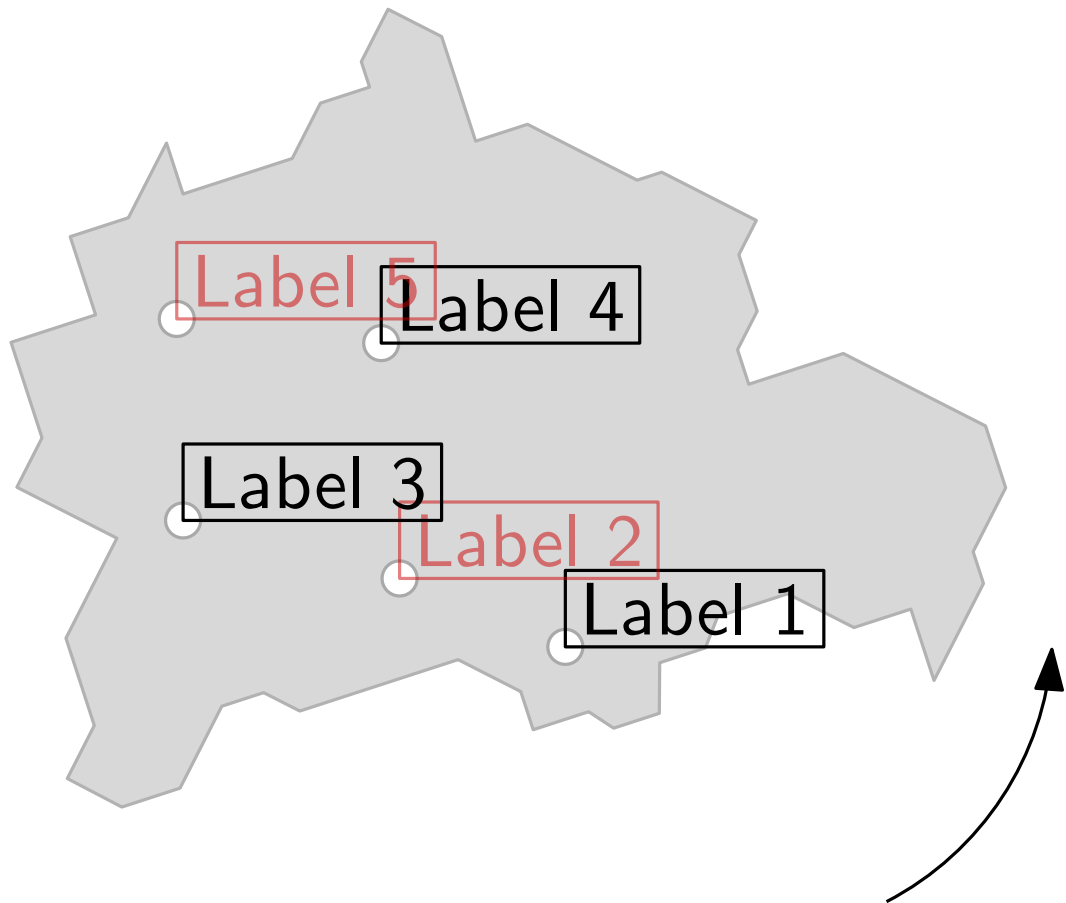
Input: Labeled map



monotone rotation

Our Problem

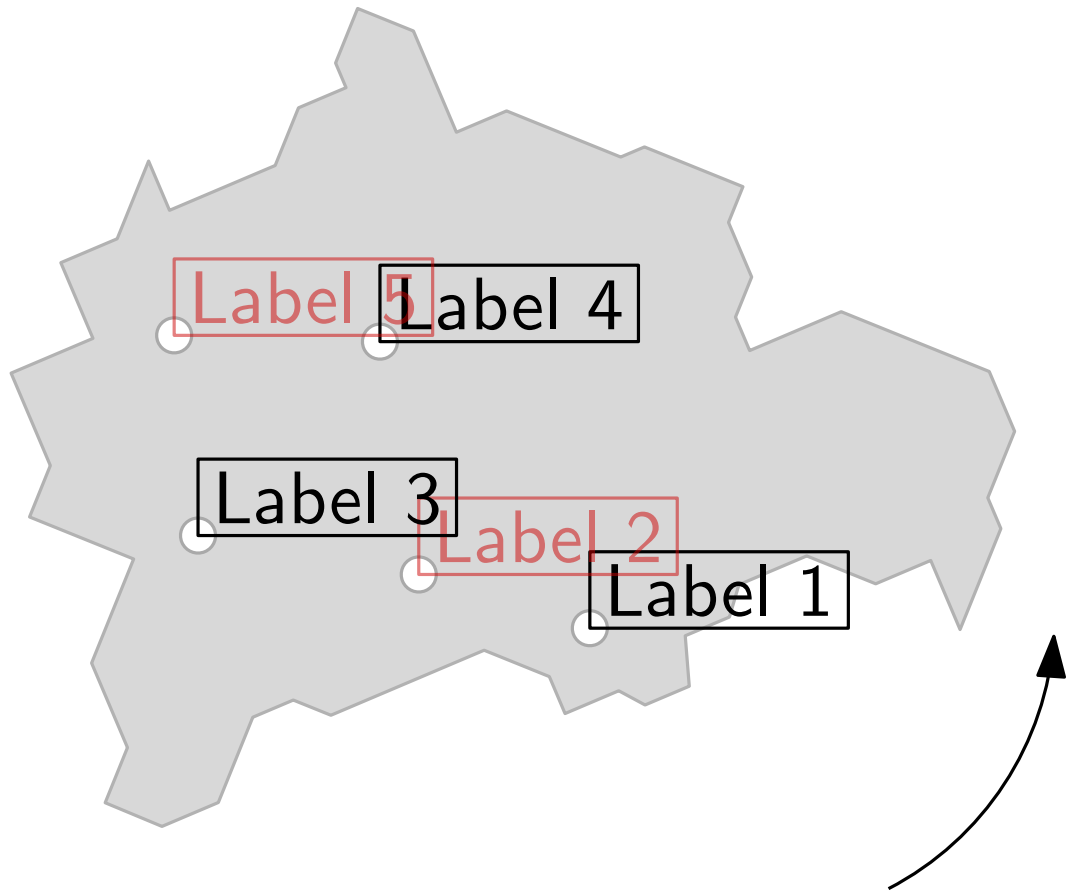
Input: Labeled map



monotone rotation

Our Problem

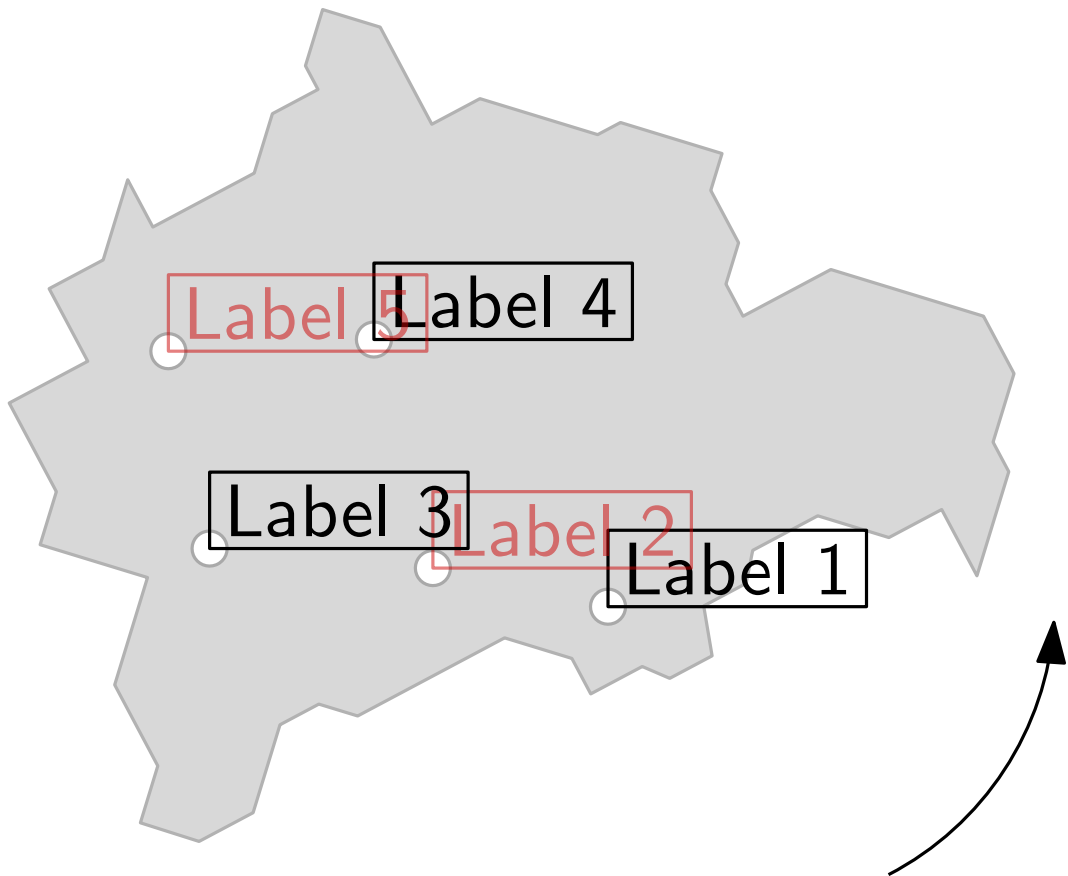
Input: Labeled map



monotone rotation

Our Problem

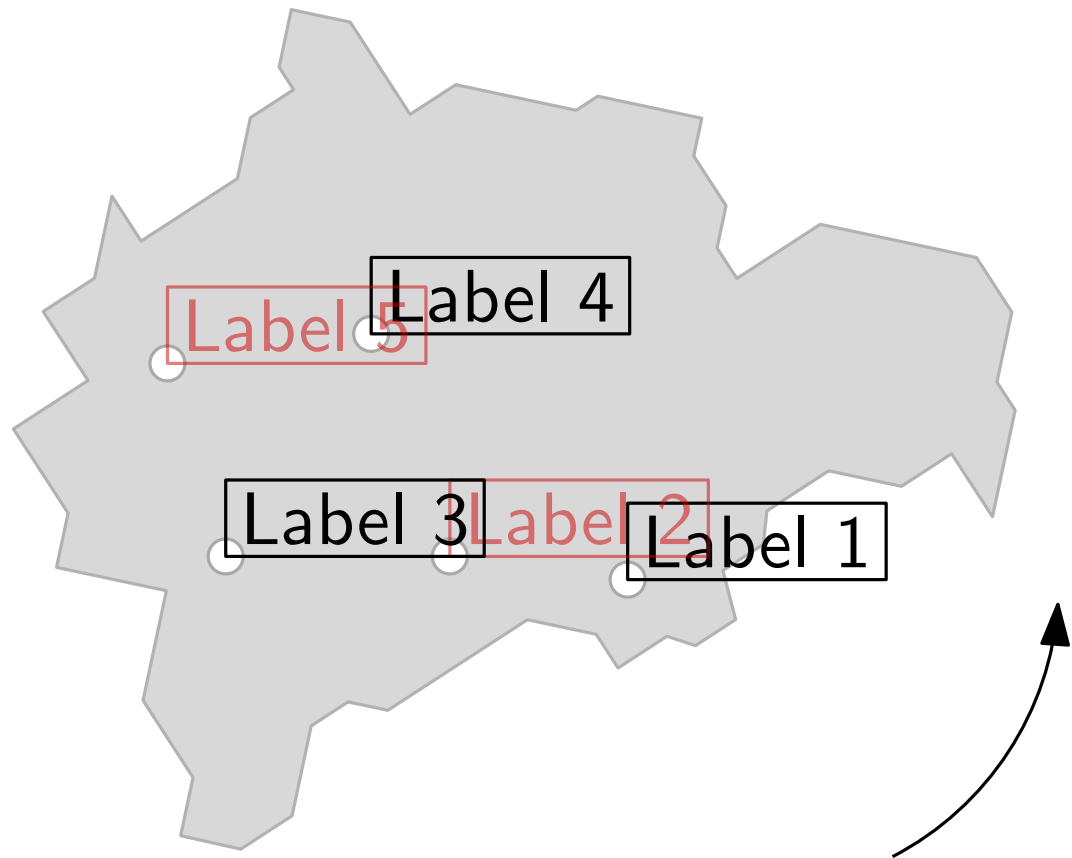
Input: Labeled map



monotone rotation

Our Problem

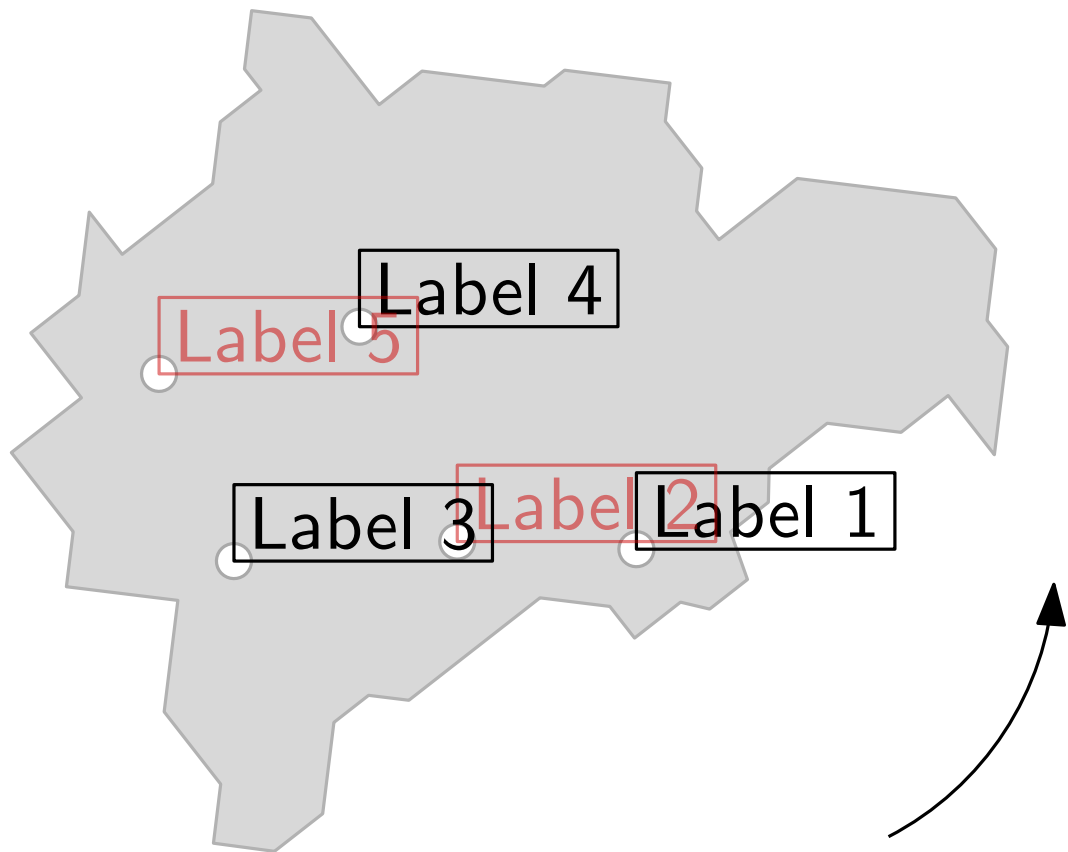
Input: Labeled map



monotone rotation

Our Problem

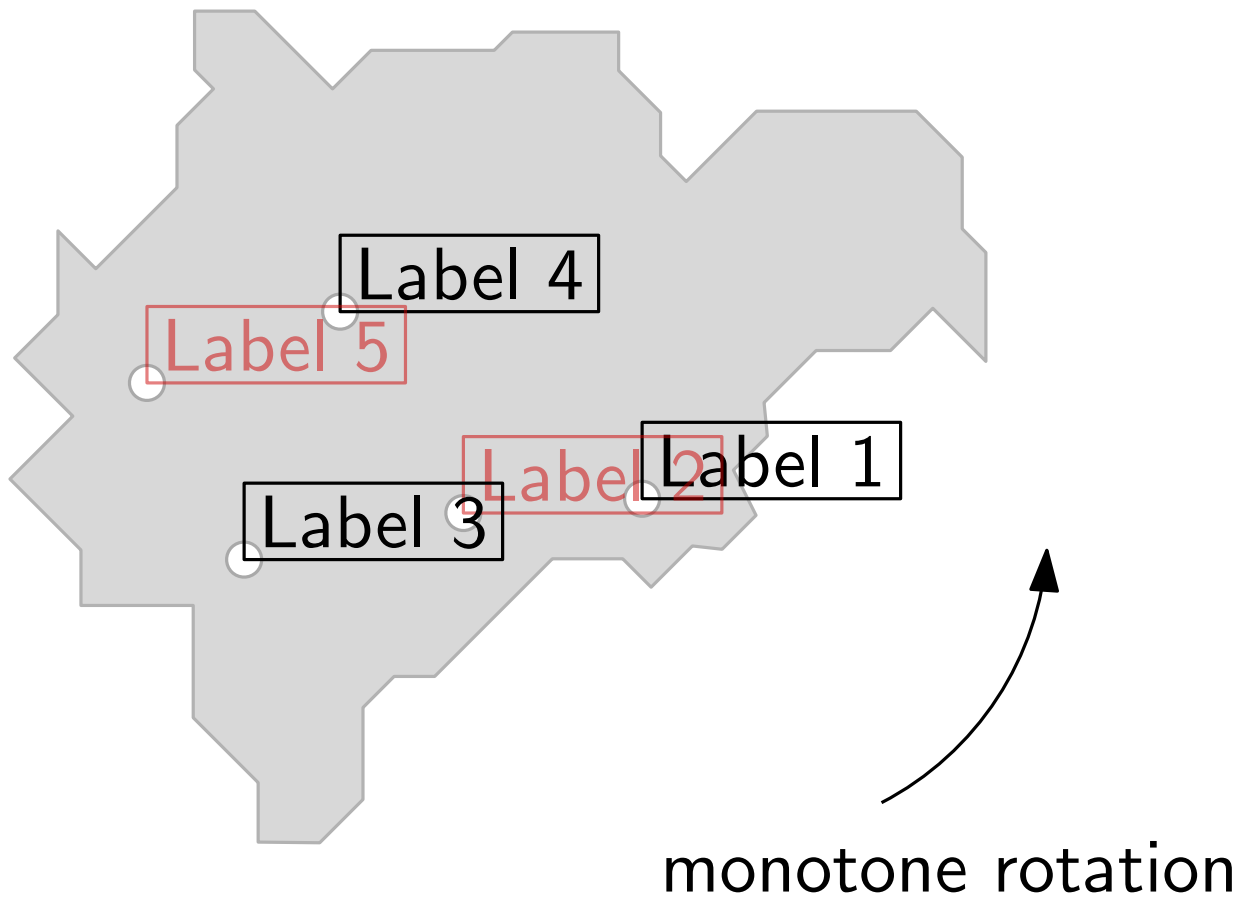
Input: Labeled map



monotone rotation

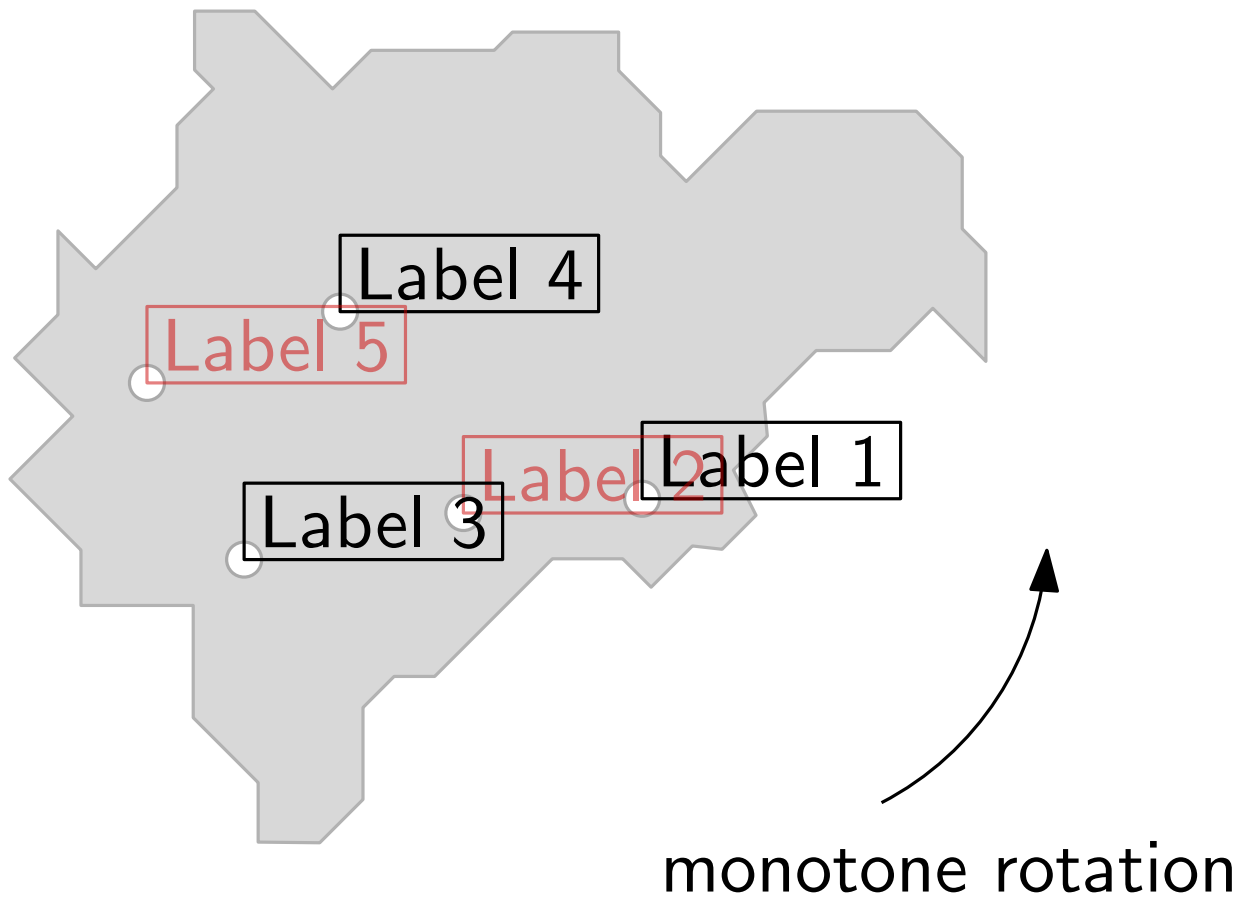
Our Problem

Input: Labeled map



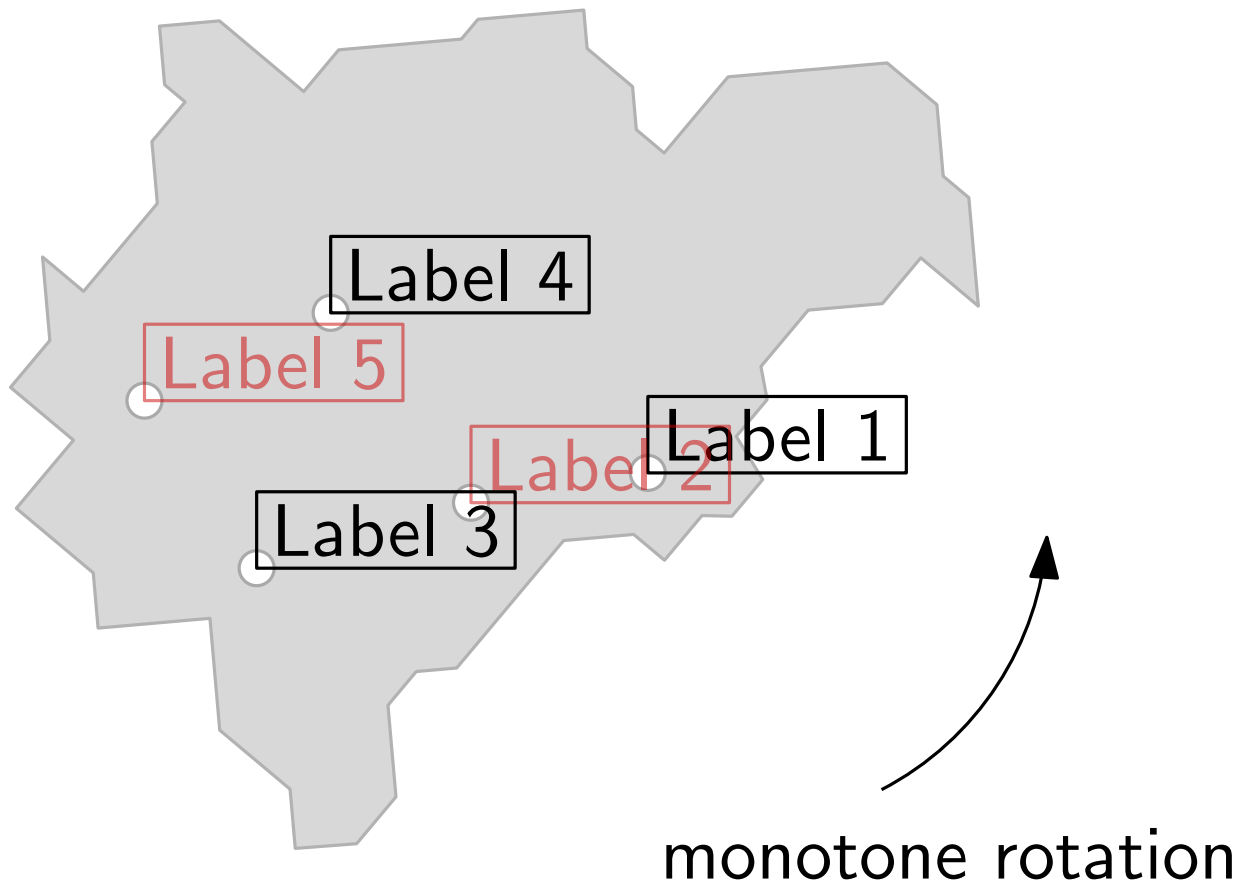
Our Problem

Input: Labeled map



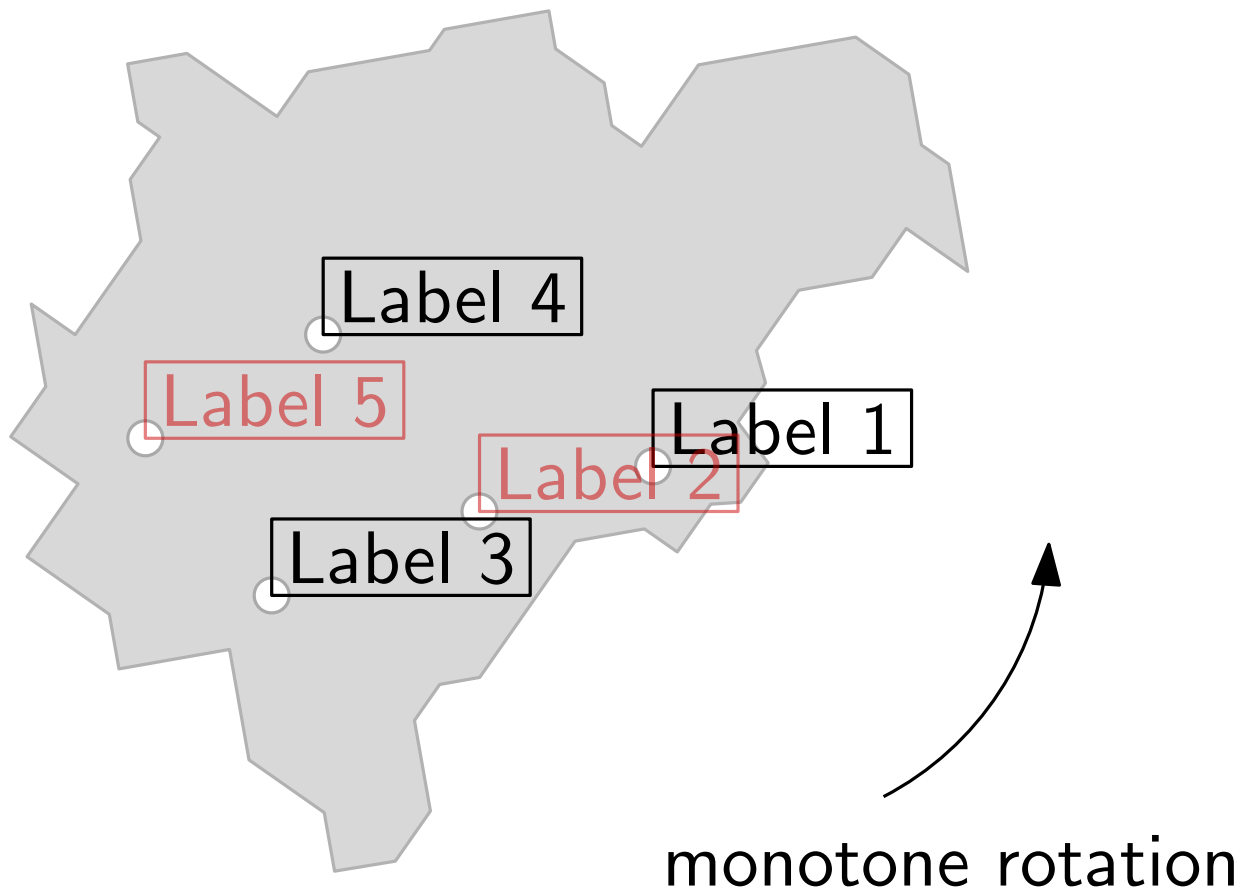
Our Problem

Input: Labeled map



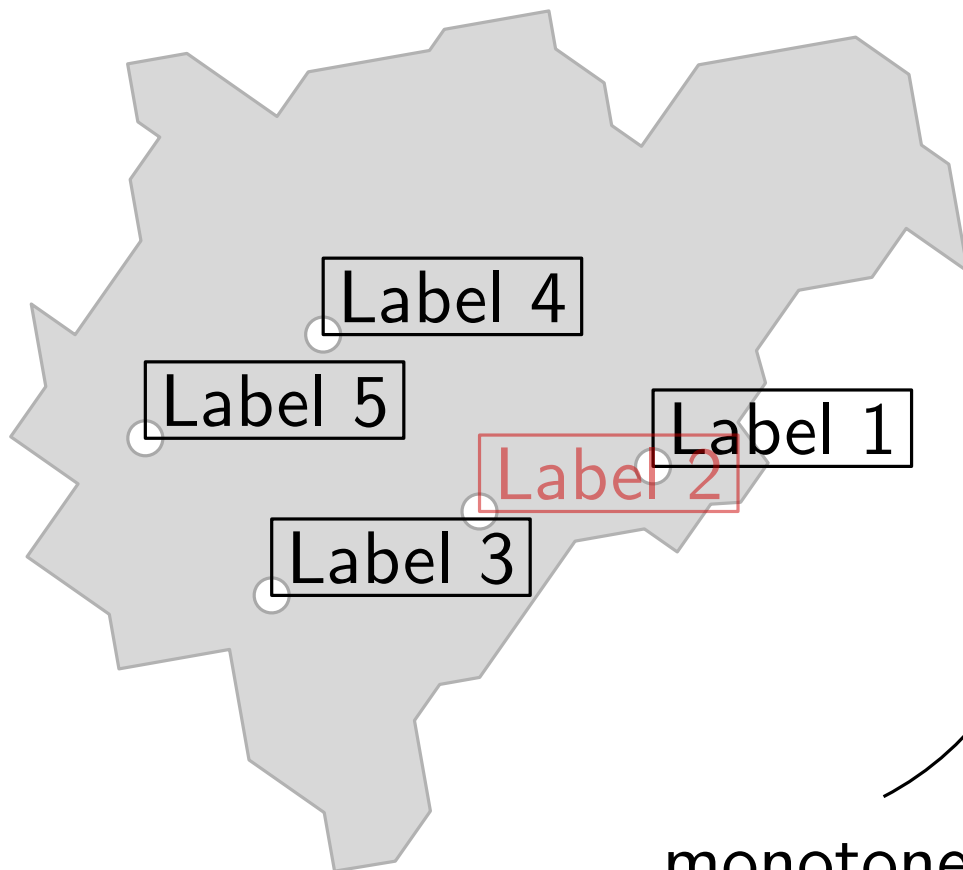
Our Problem

Input: Labeled map



Our Problem

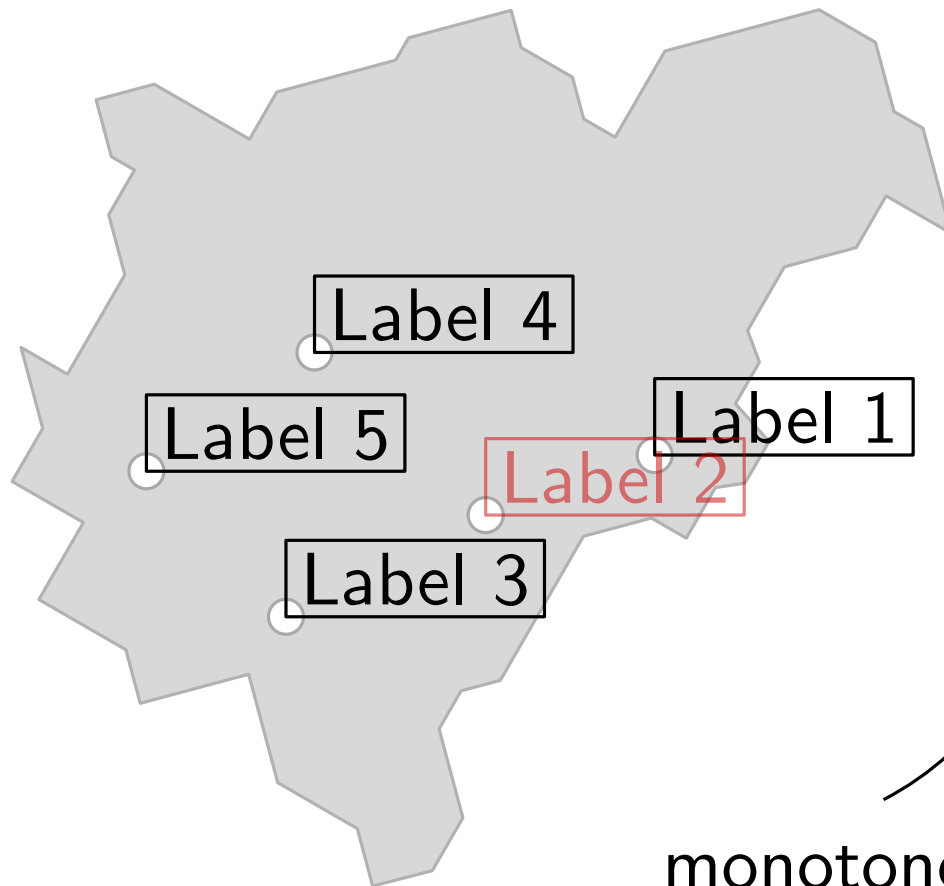
Input: Labeled map



monotone rotation

Our Problem

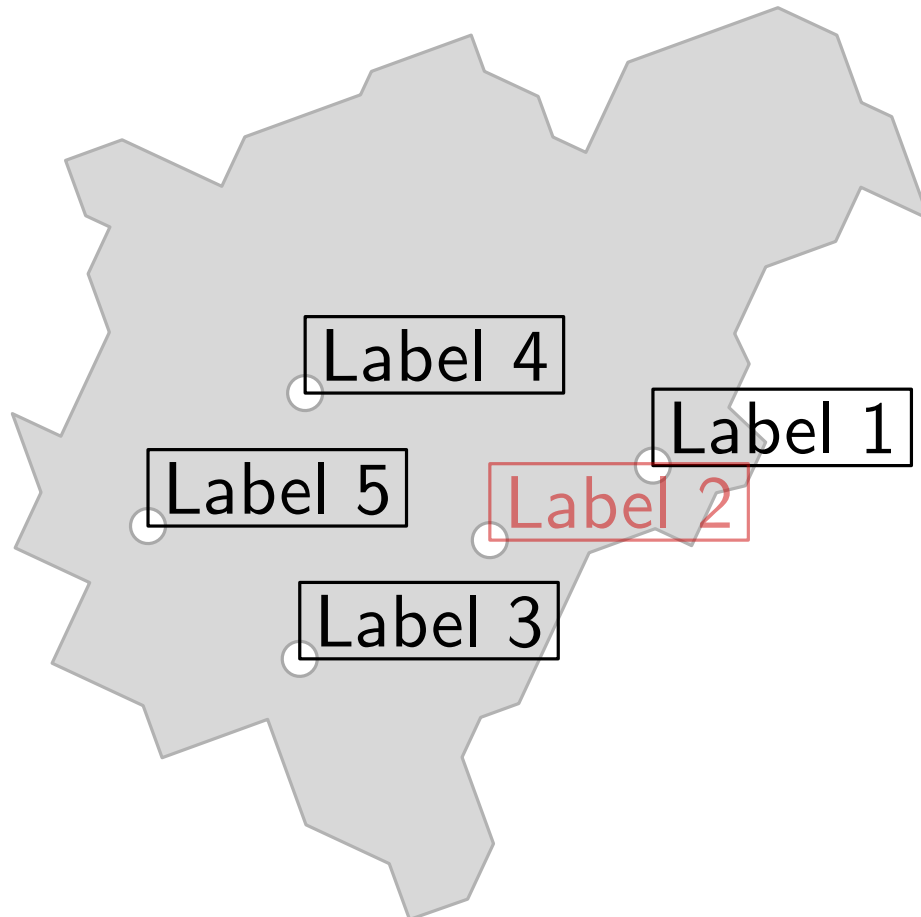
Input: Labeled map



monotone rotation

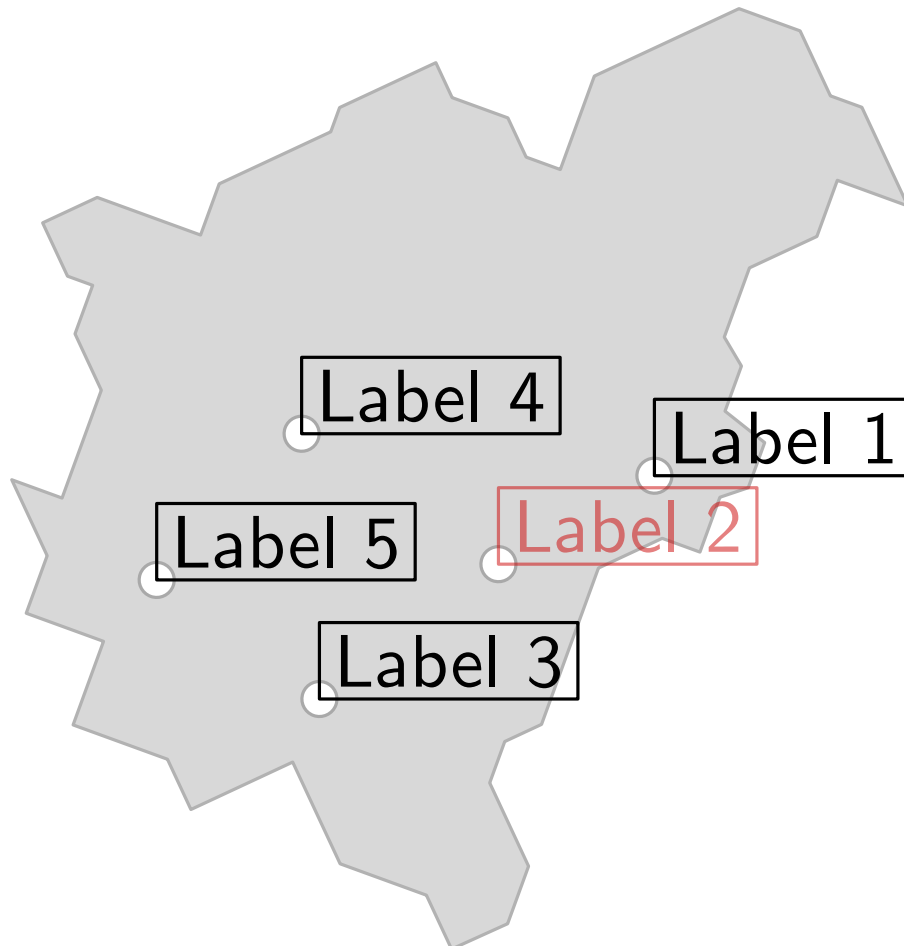
Our Problem

Input: Labeled map



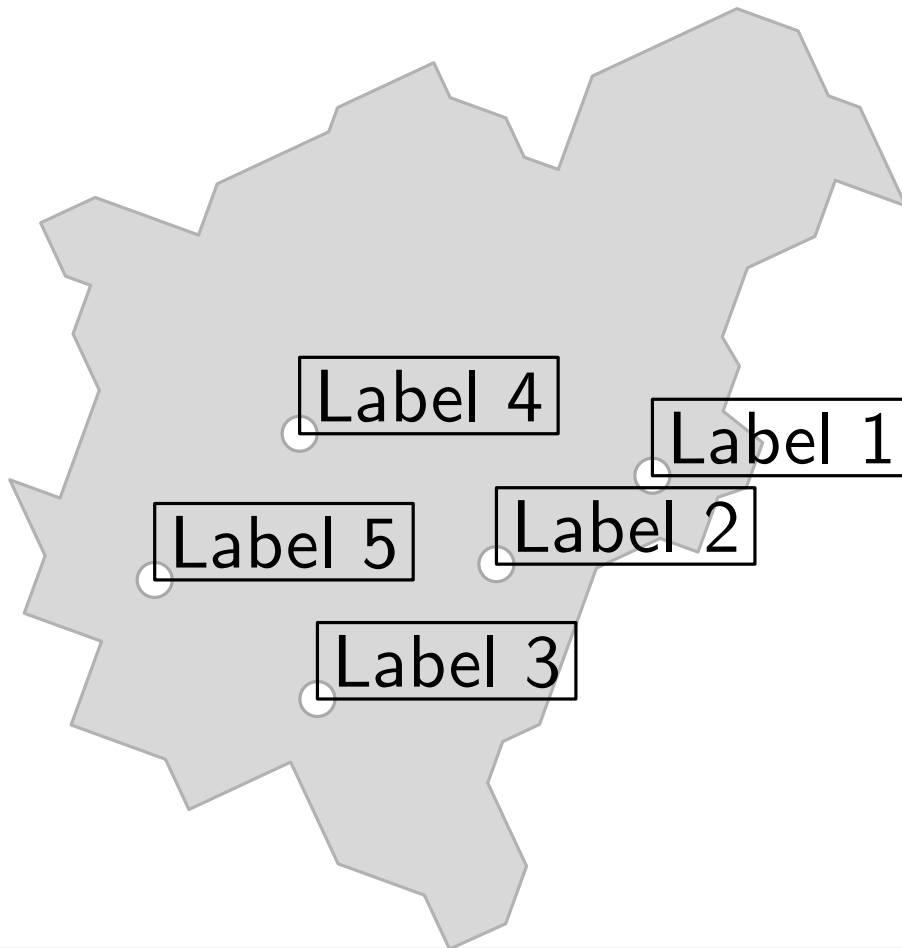
Our Problem

Input: Labeled map



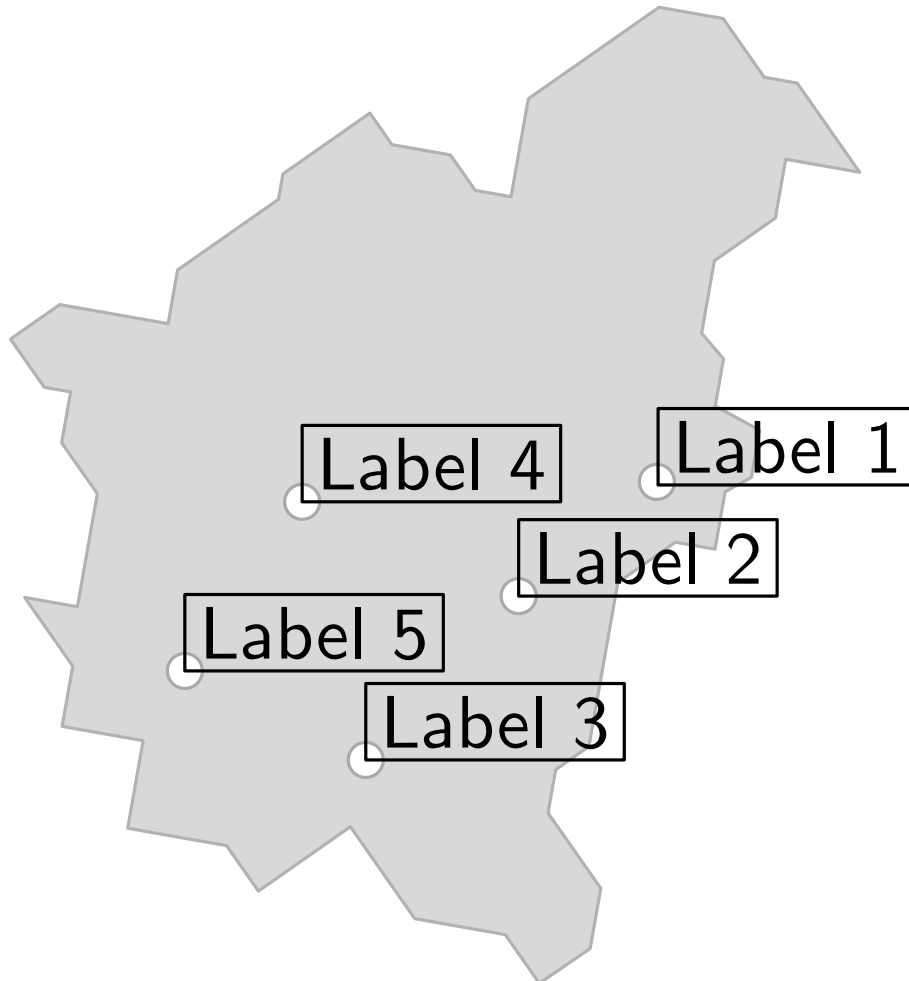
Our Problem

Input: Labeled map



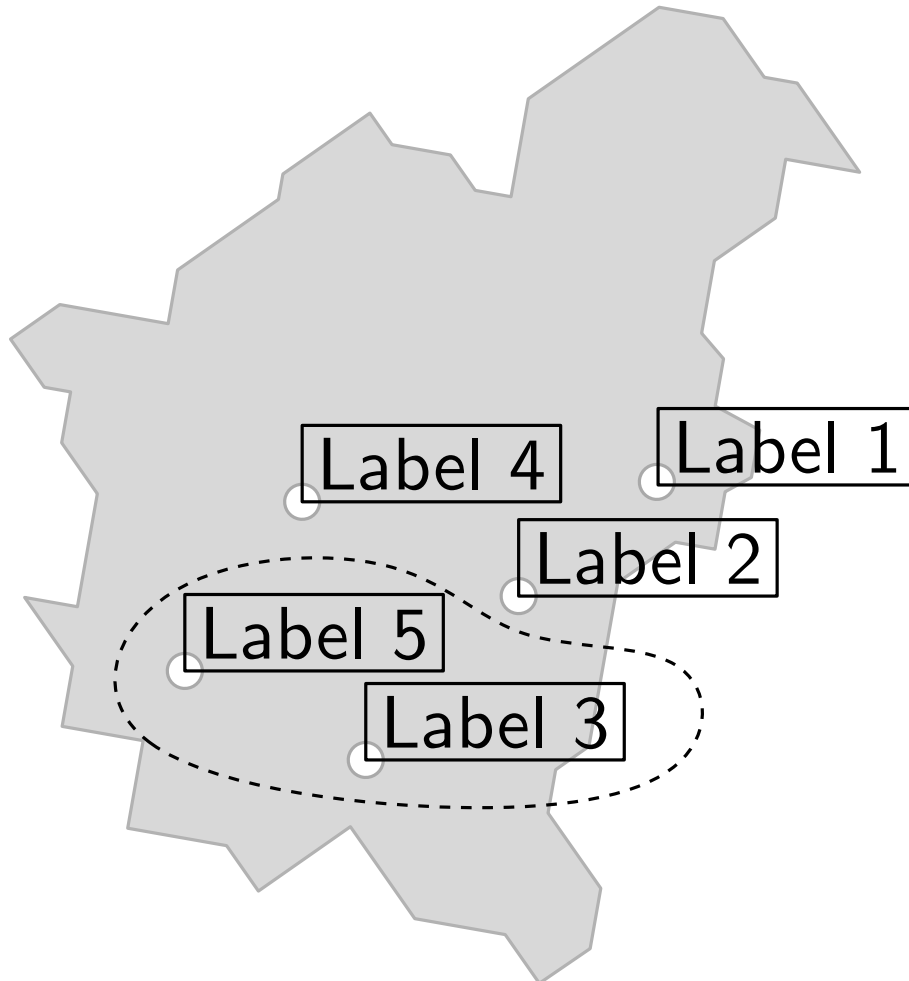
Our Problem

Input: Labeled map



Our Problem

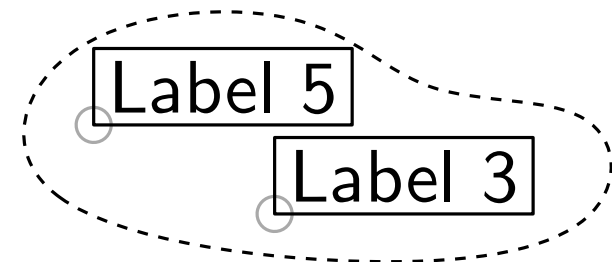
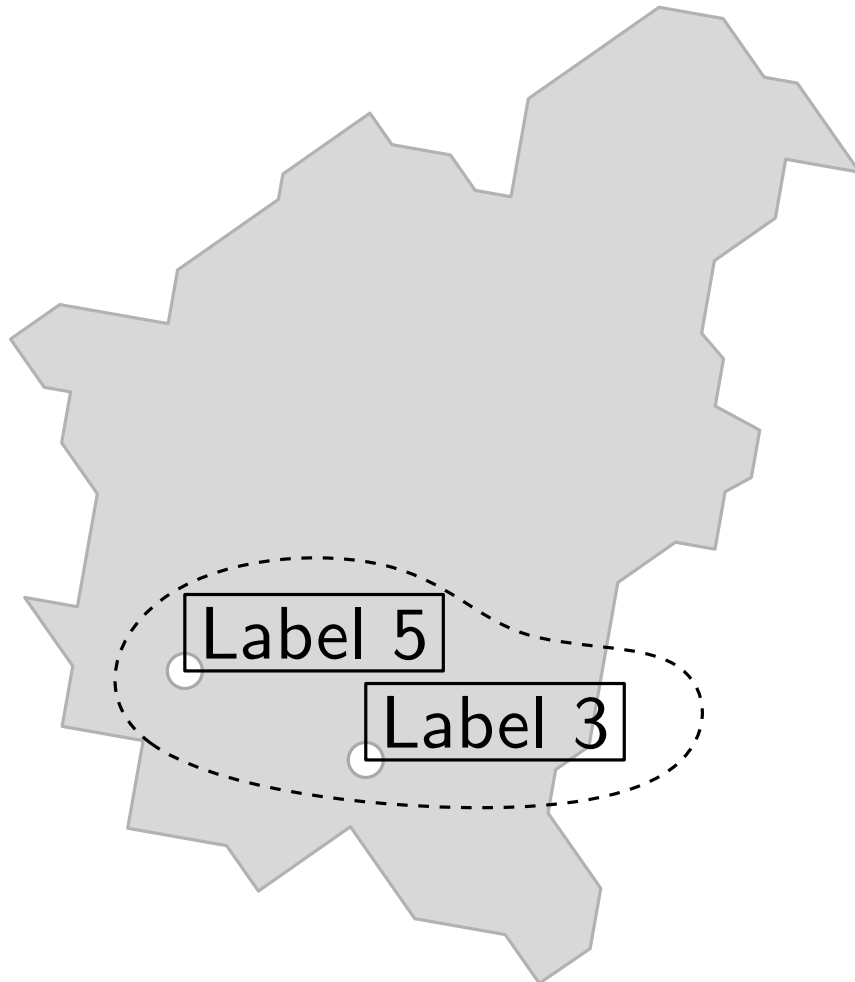
Input: Labeled map



Our Problem

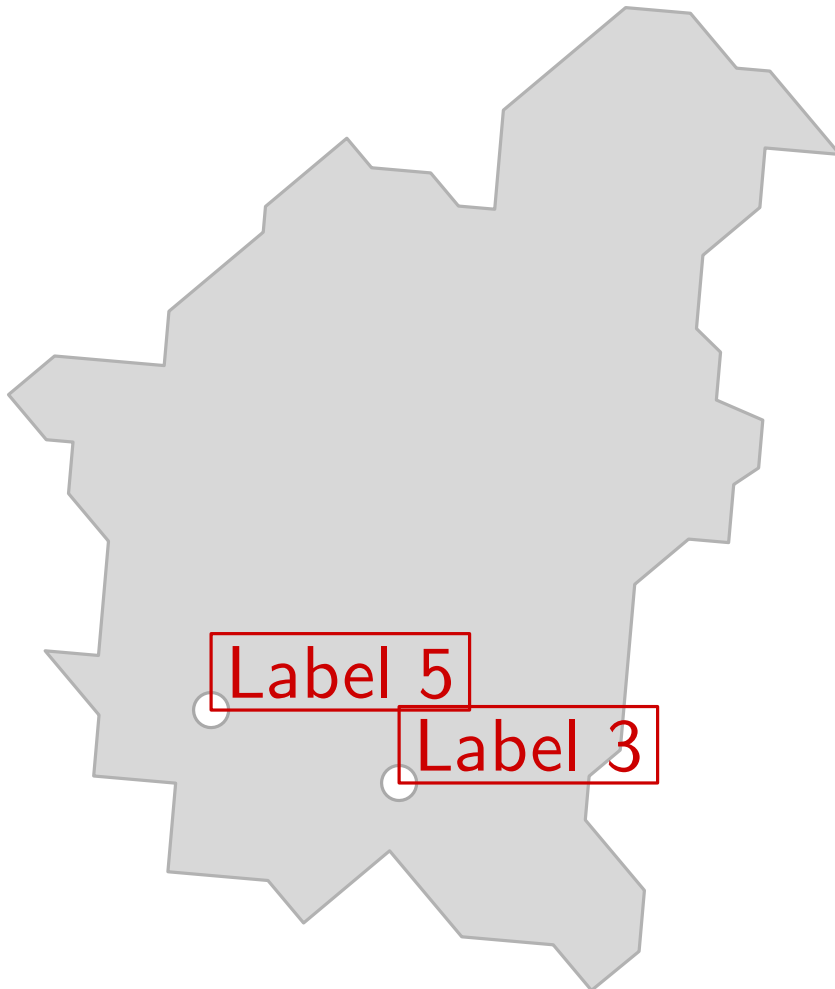
Input: Labeled map

Transformation of the problem

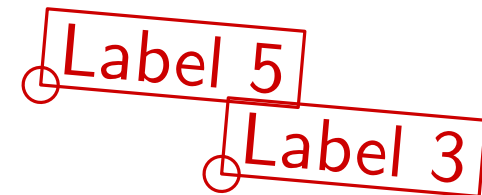


Our Problem

Input: Labeled map

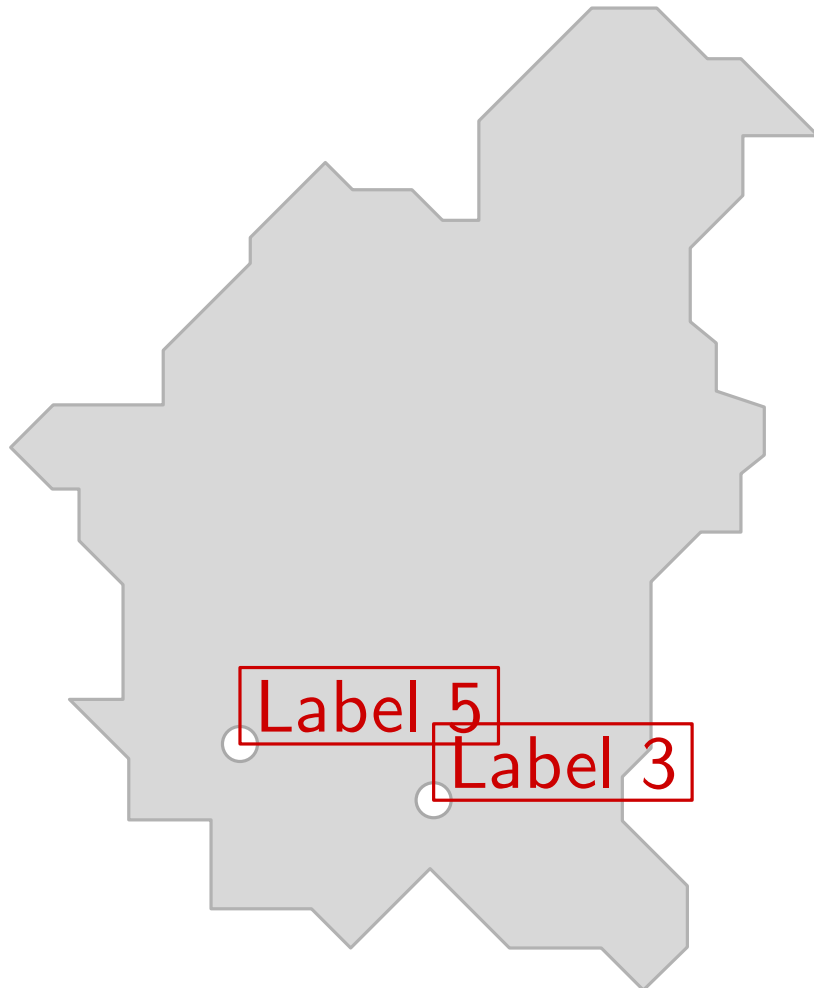


Transformation of the problem

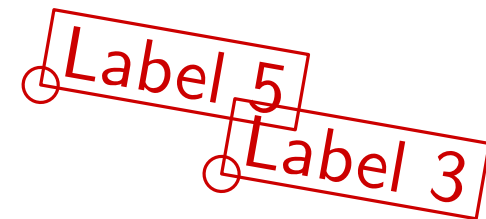


Our Problem

Input: Labeled map

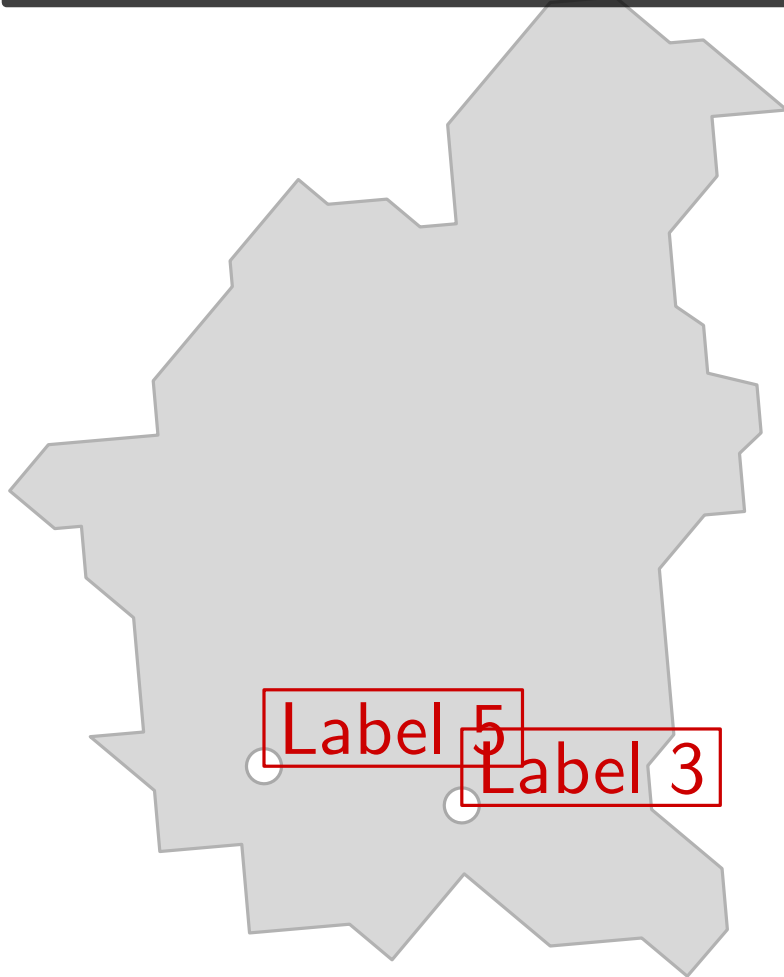


Transformation of the problem

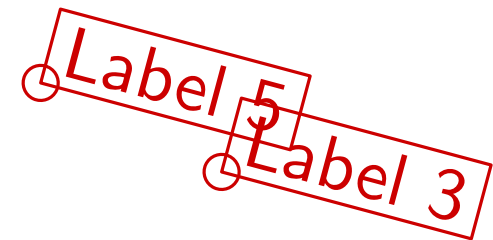


Our Problem

Input: Labeled map

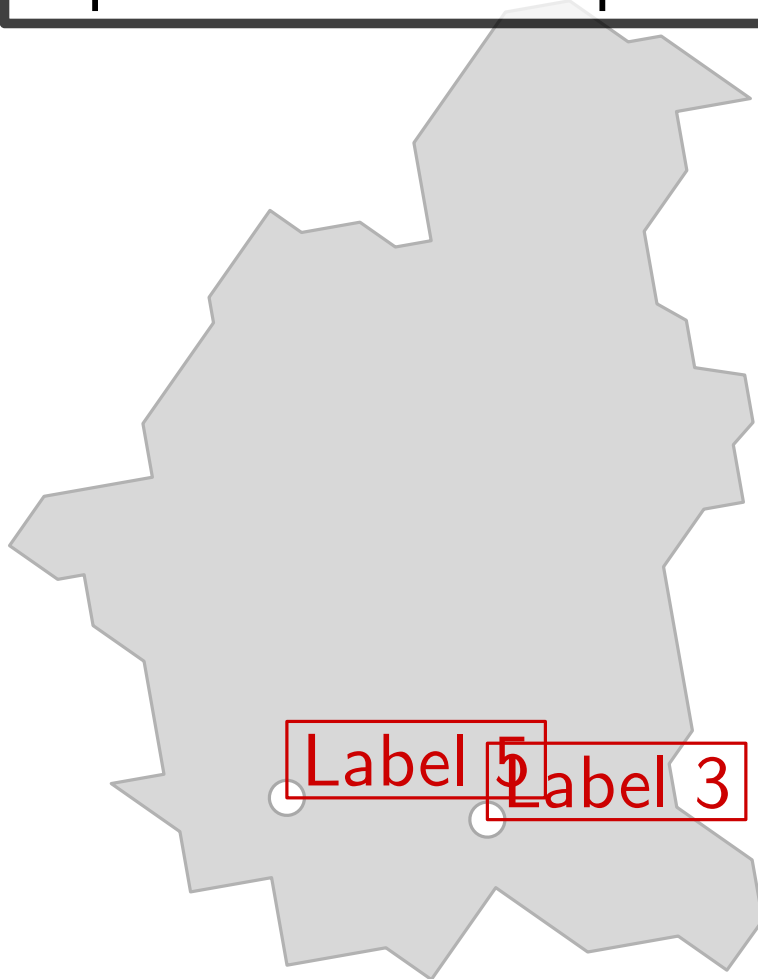


Transformation of the problem



Our Problem

Input: Labeled map

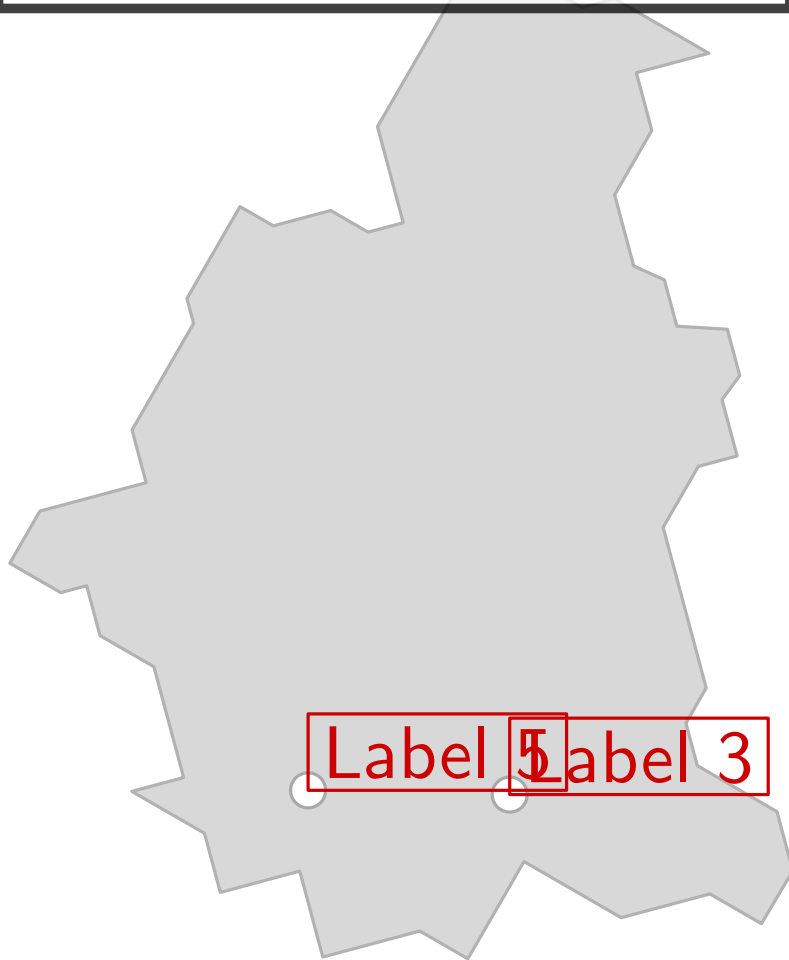


Transformation of the problem



Our Problem

Input: Labeled map

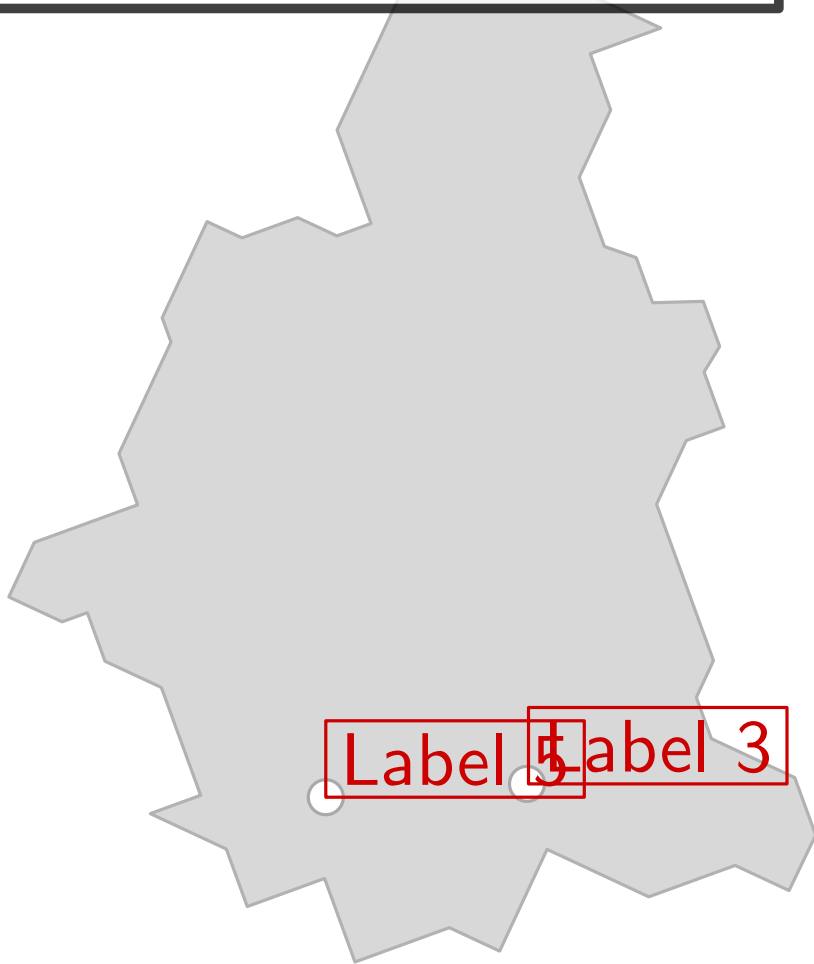


Transformation of the problem



Our Problem

Input: Labeled map

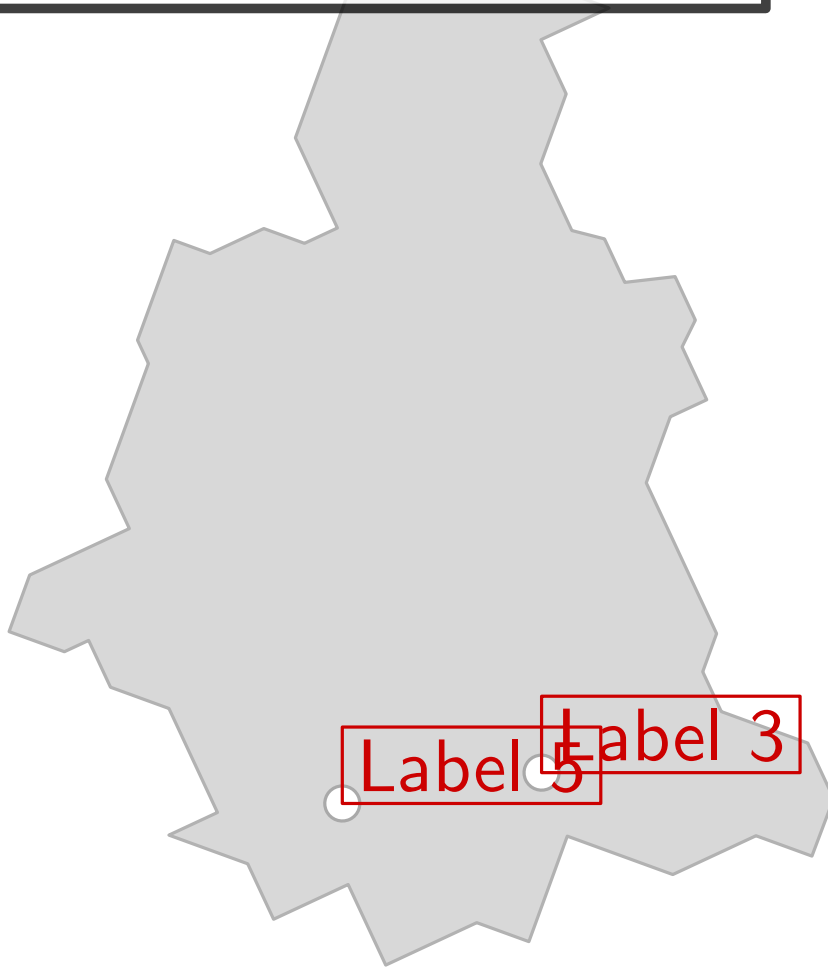


Transformation of the problem



Our Problem

Input: Labeled map

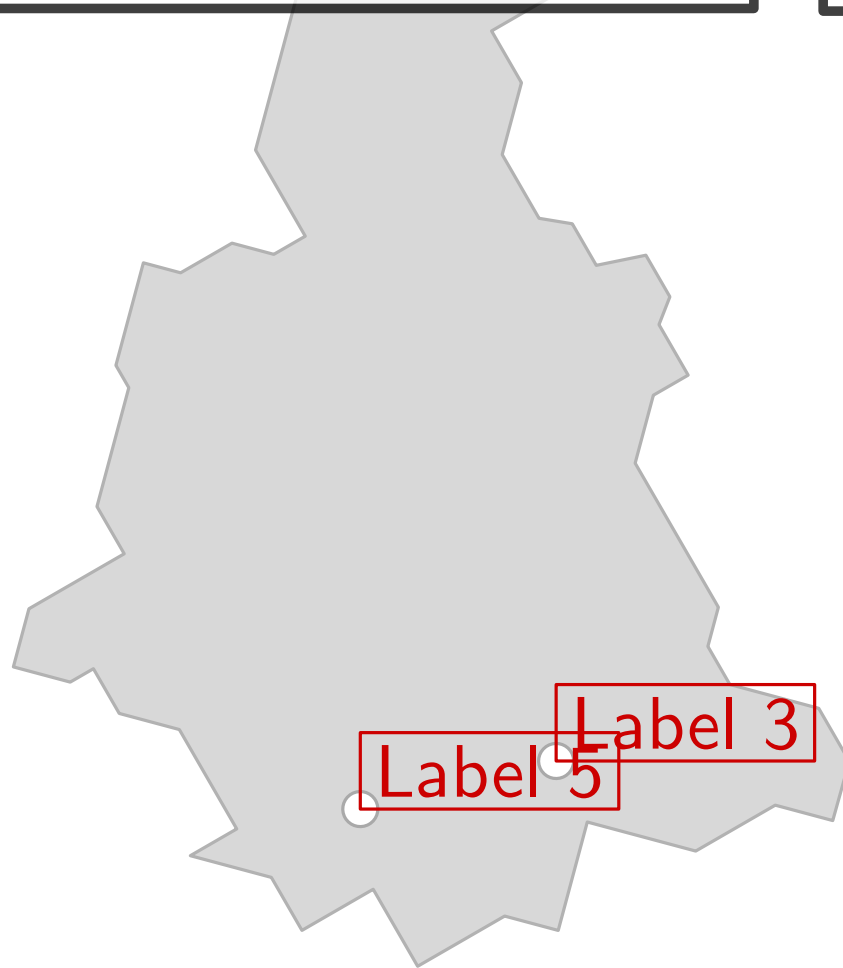


Transformation of the problem

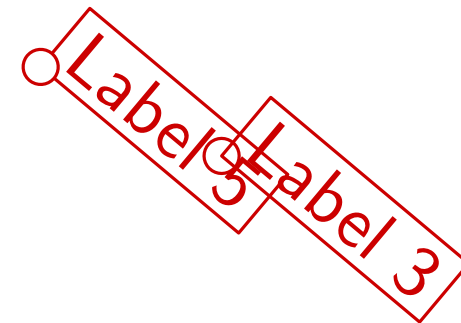


Our Problem

Input: Labeled map

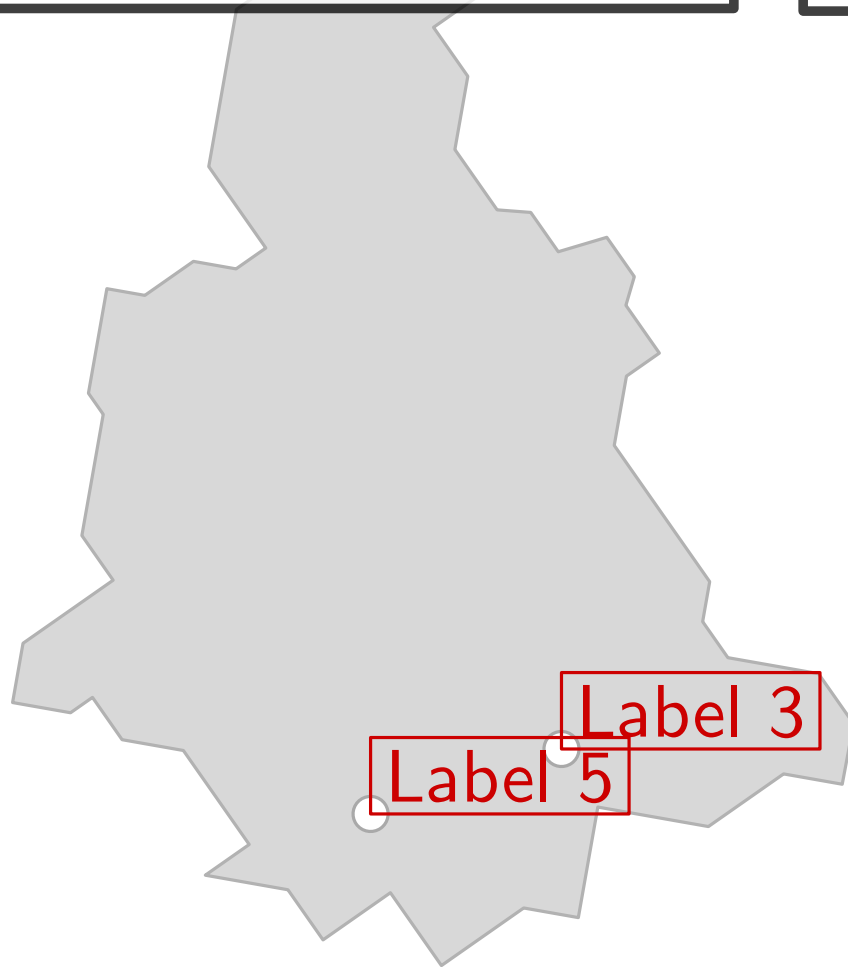


Transformation of the problem

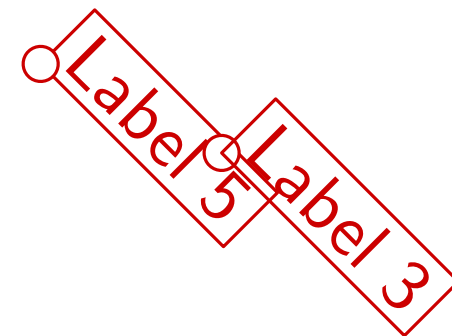


Our Problem

Input: Labeled map

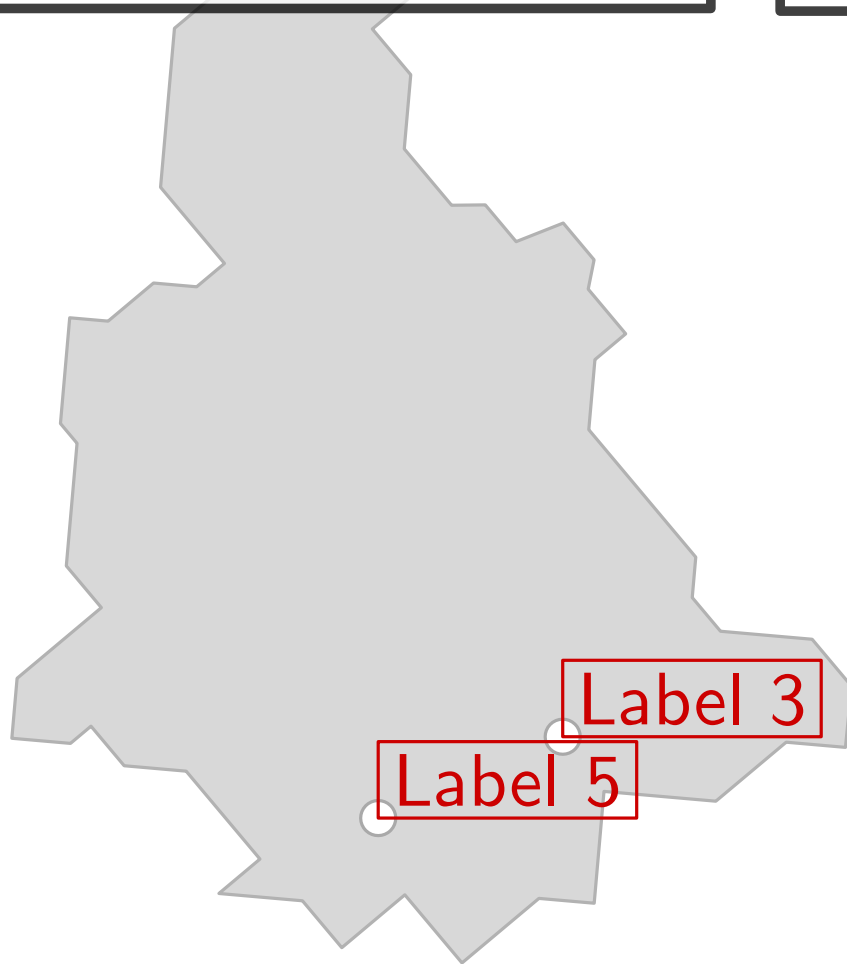


Transformation of the problem

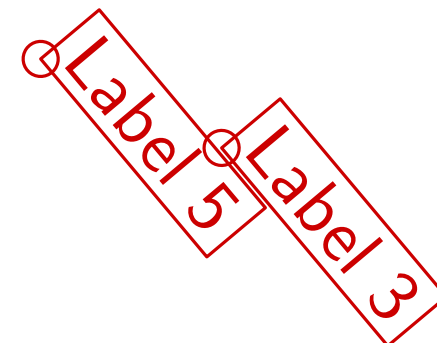


Our Problem

Input: Labeled map

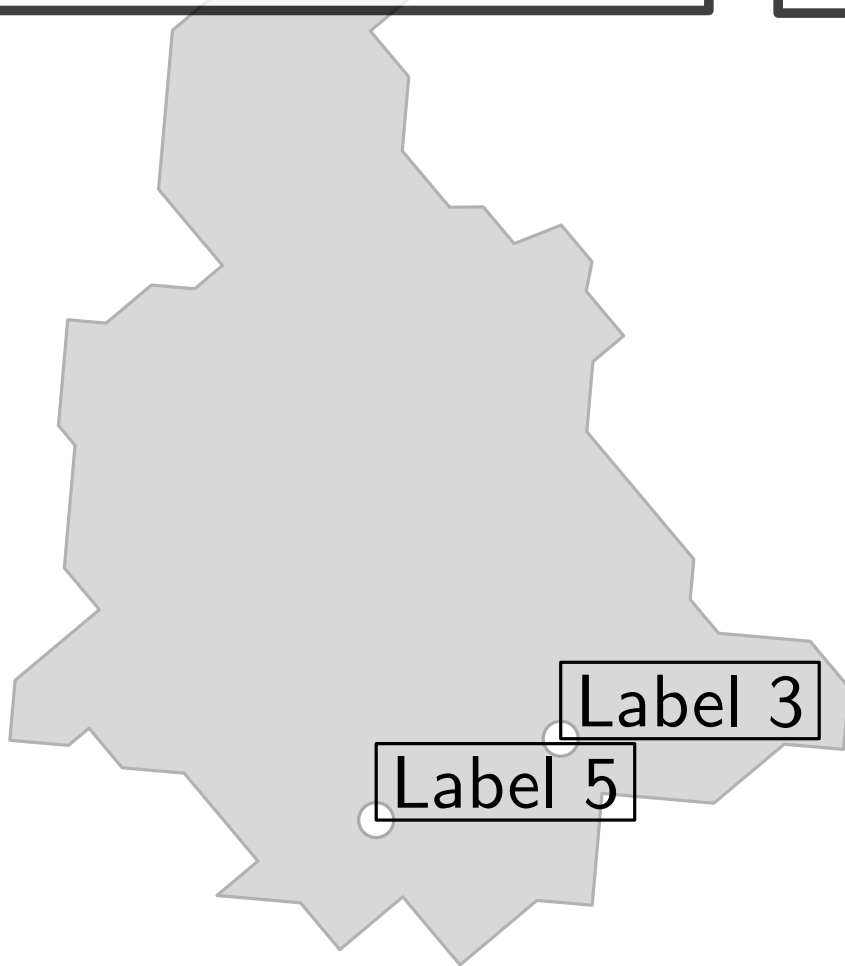


Transformation of the problem

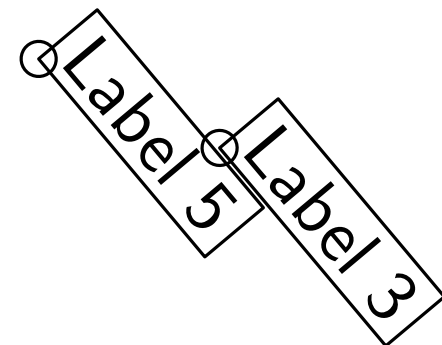


Our Problem

Input: Labeled map



Transformation of the problem

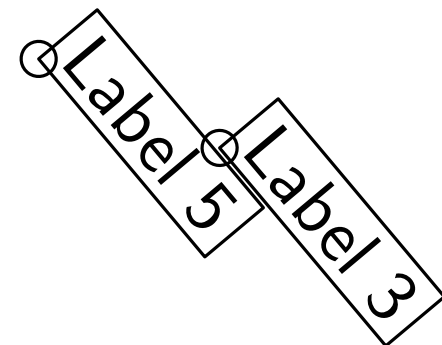
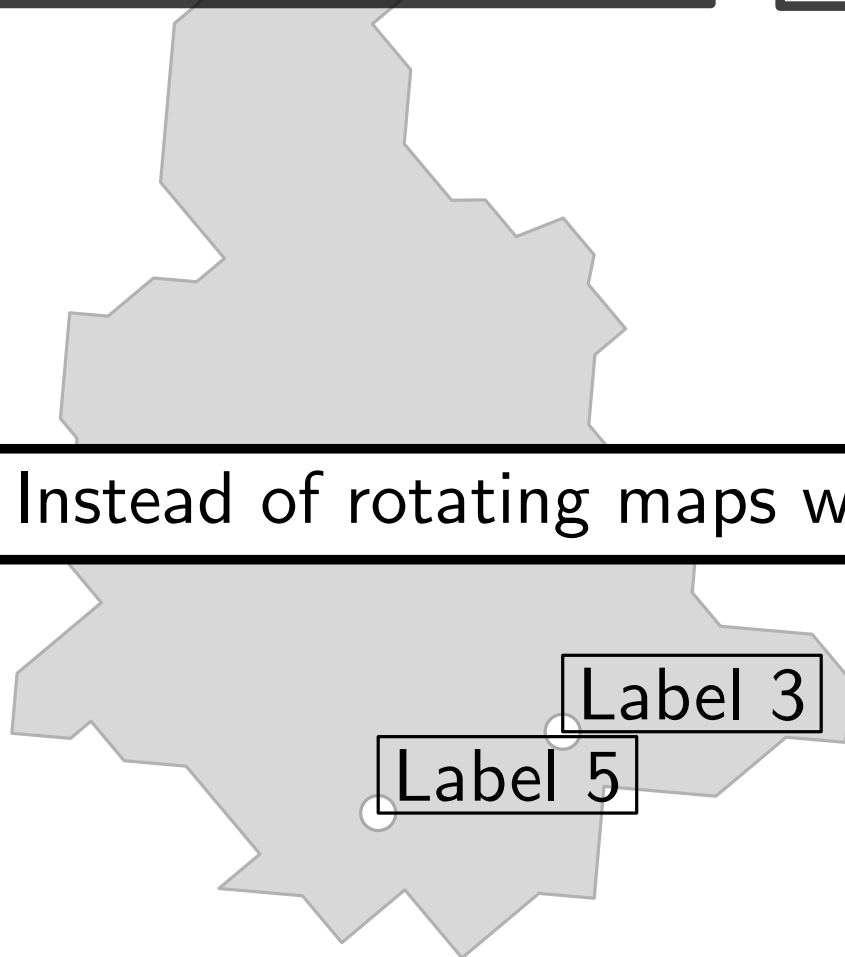


Our Problem

Input: Labeled map

Transformation of the problem

Instead of rotating maps we consider **rotating labels**



Desiderata

- no label occludes a point
- no two labels overlap

consistency:

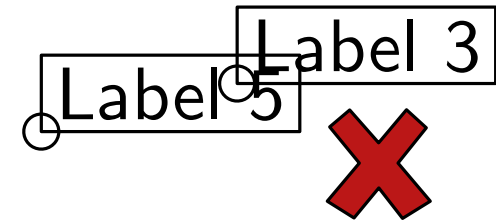
- no **“jumping”**
- no **“flickering”**

Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap

consistency:

- no “**jumping**”
- no “**flickering**”

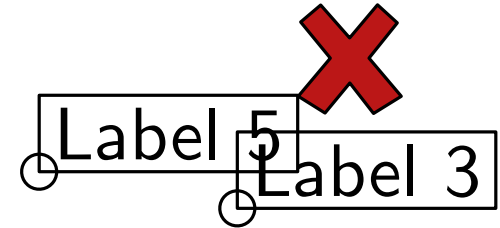


Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

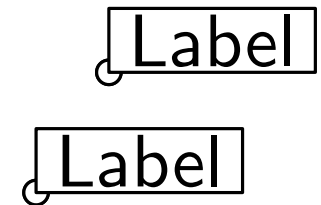


Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

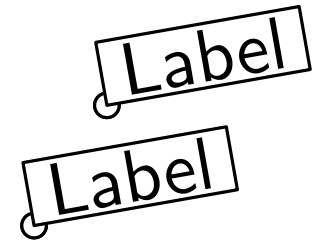


Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

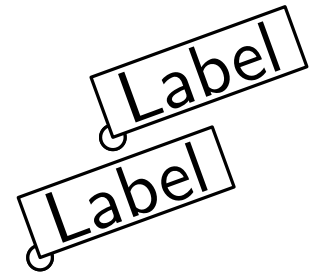


Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

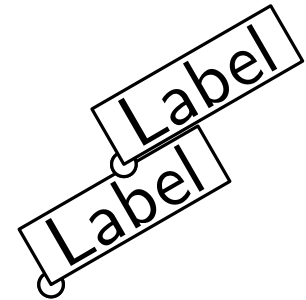


Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

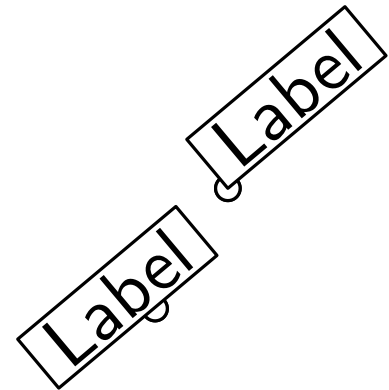


Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

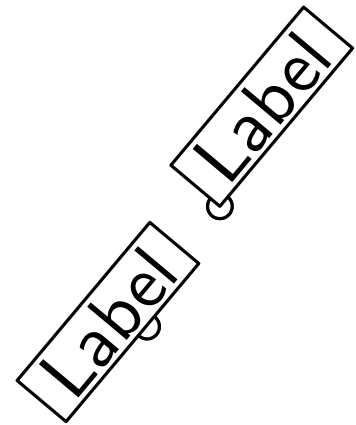


Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

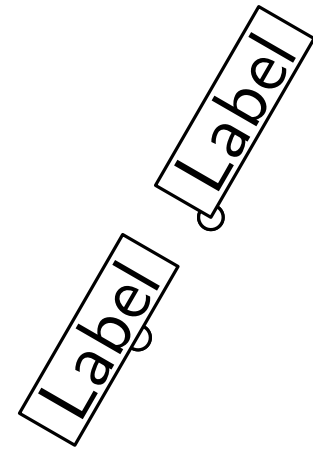


Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”



Desiderata

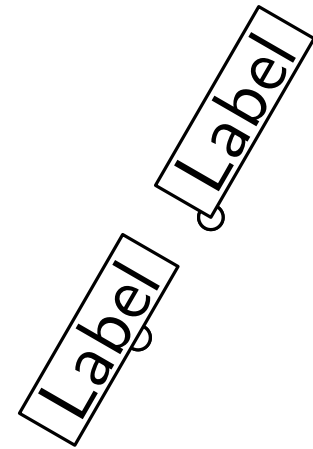
- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor



Desiderata

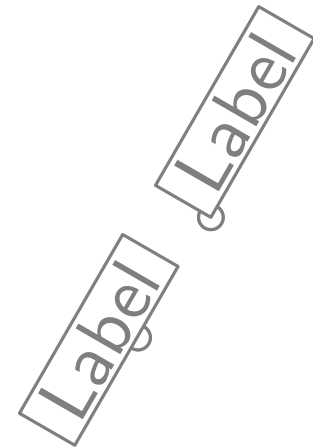
- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

Desiderata

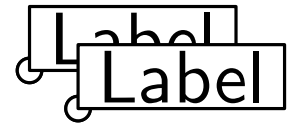
- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor



Desiderata

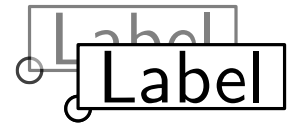
- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor



Desiderata

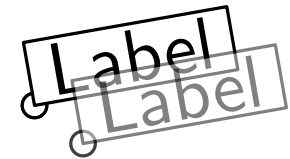
- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

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fix relative position of the anchor



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Desiderata

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- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor



Desiderata

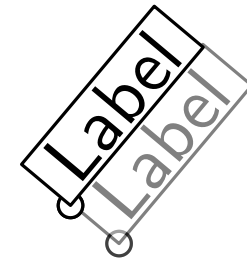
- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

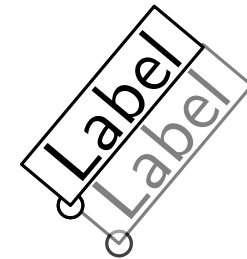
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

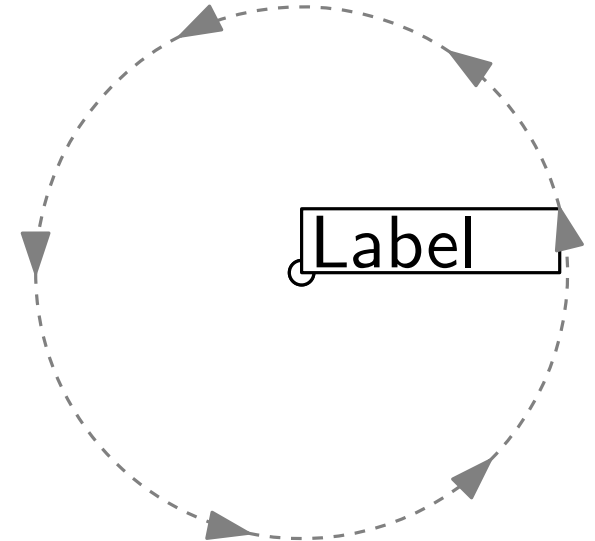
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

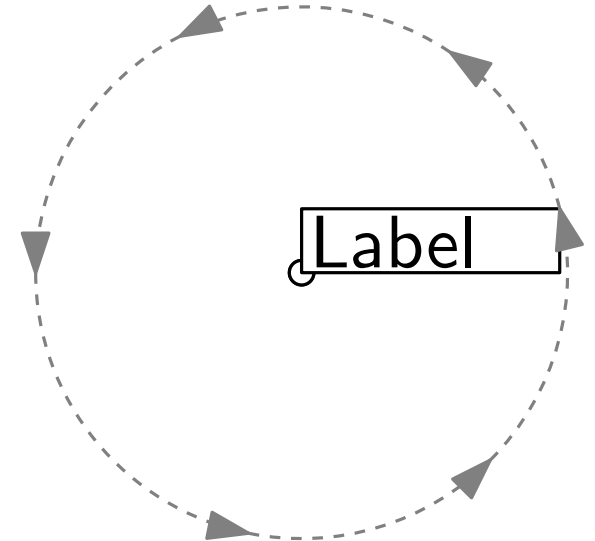
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

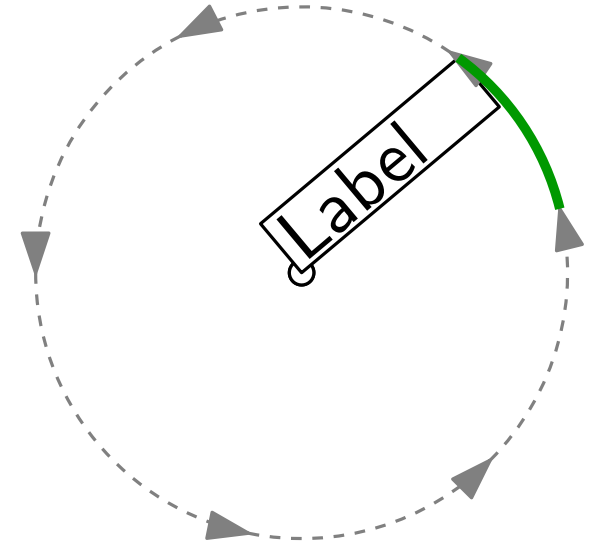
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

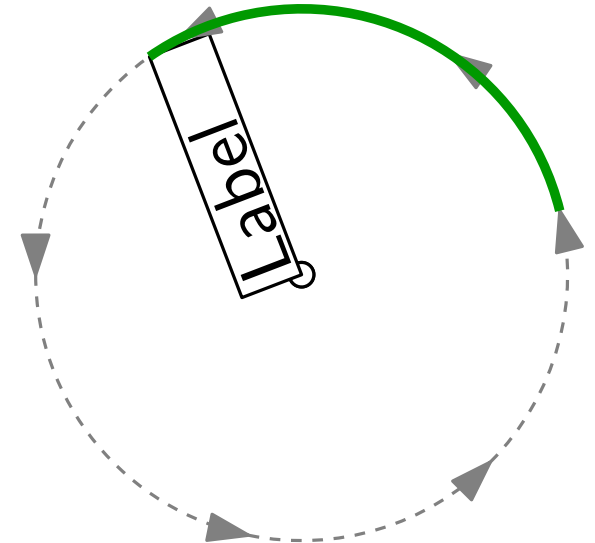
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

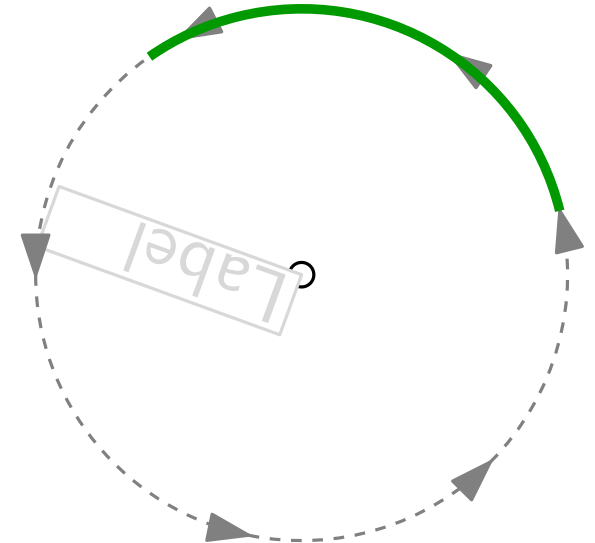
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

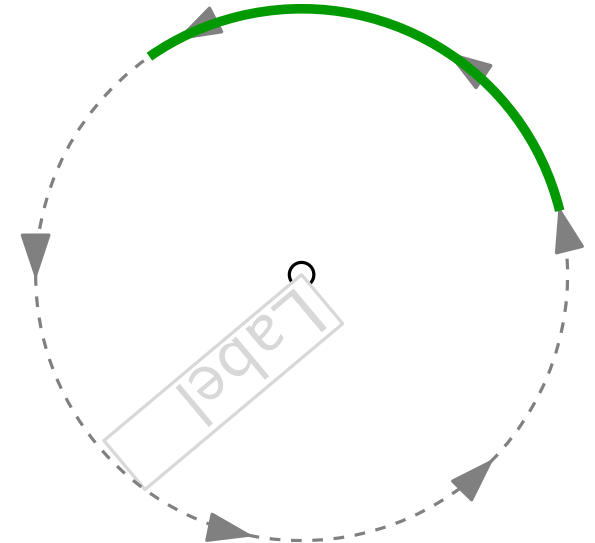
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

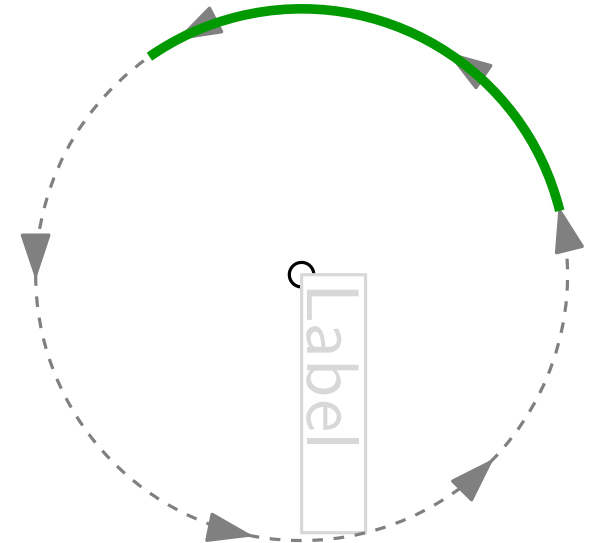
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

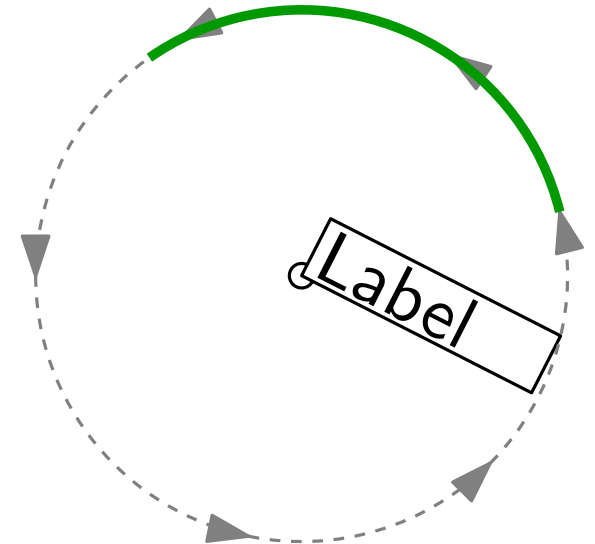
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

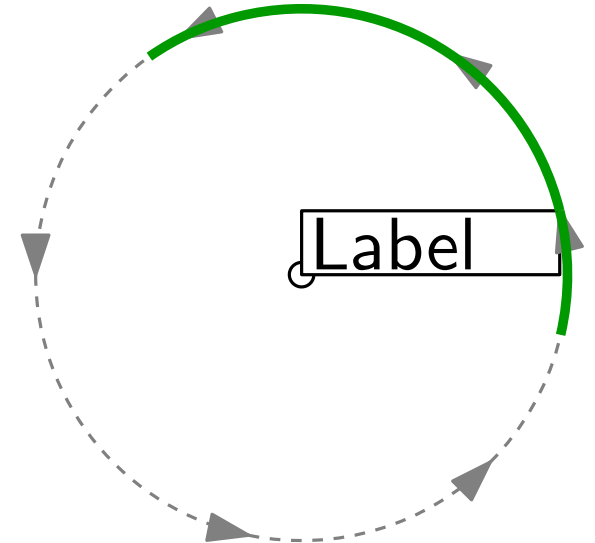
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

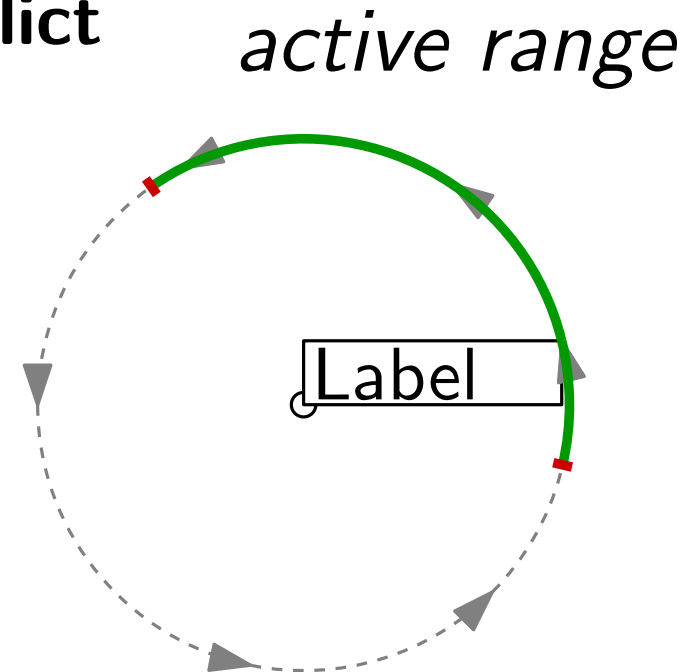
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

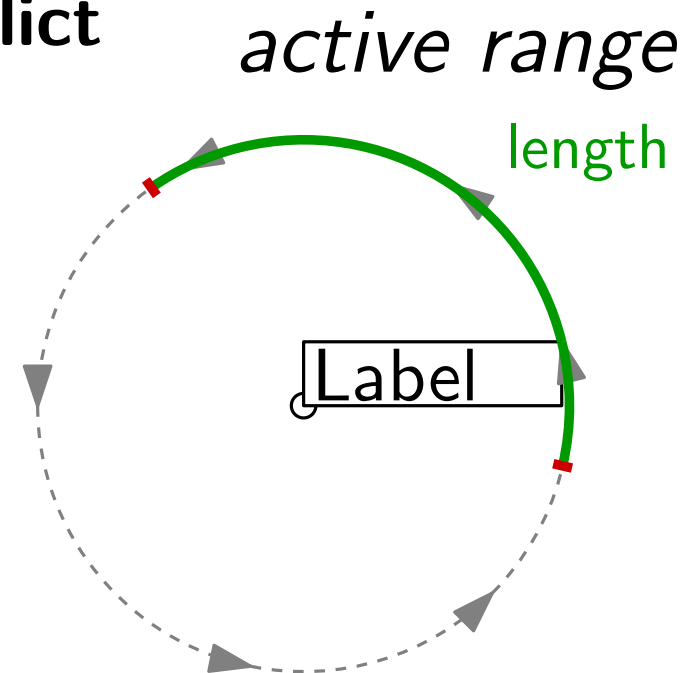
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

active in single contiguous range



Desiderata

- no label occludes a point: **hard conflict**
- no two labels overlap: **soft conflict**

consistency:

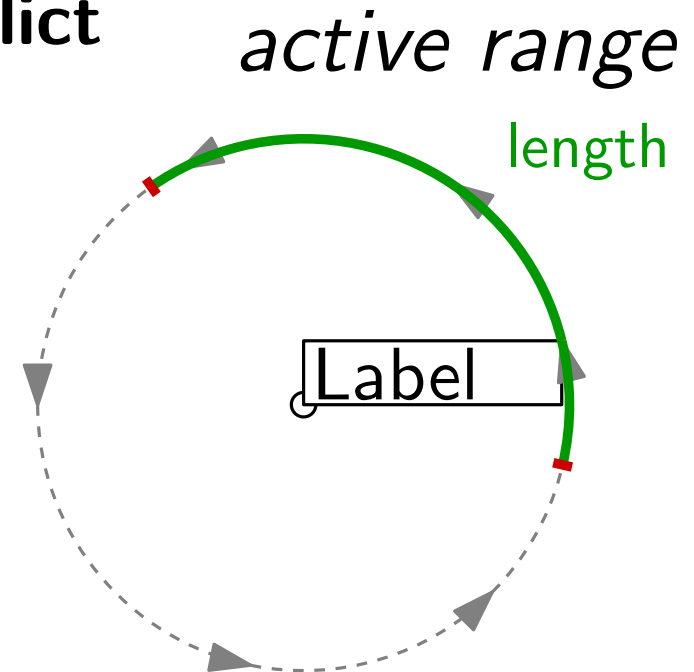
- no “**jumping**”
- no “**flickering**”

no jumping:

fix relative position of the anchor

no flickering:

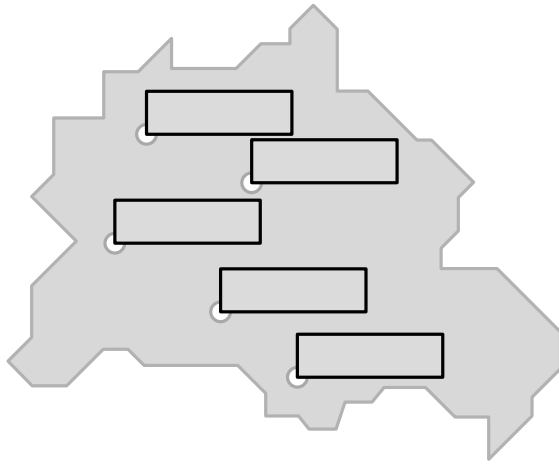
active in single contiguous range



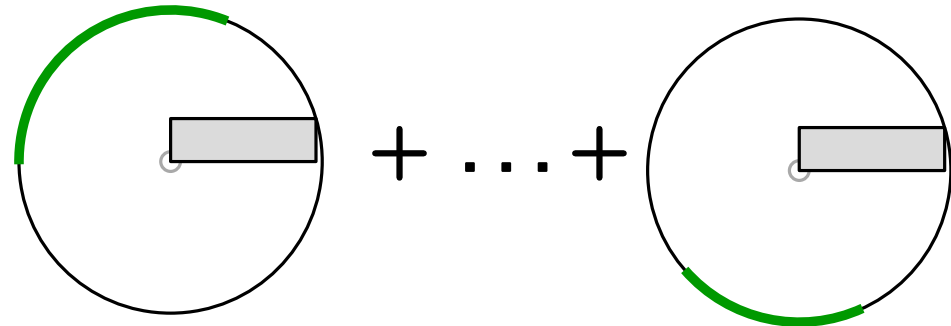
consistent rotation labeling

Problem Statement

Input: A map M , points P , **valid** labeling $L(P)$



MaxTotal

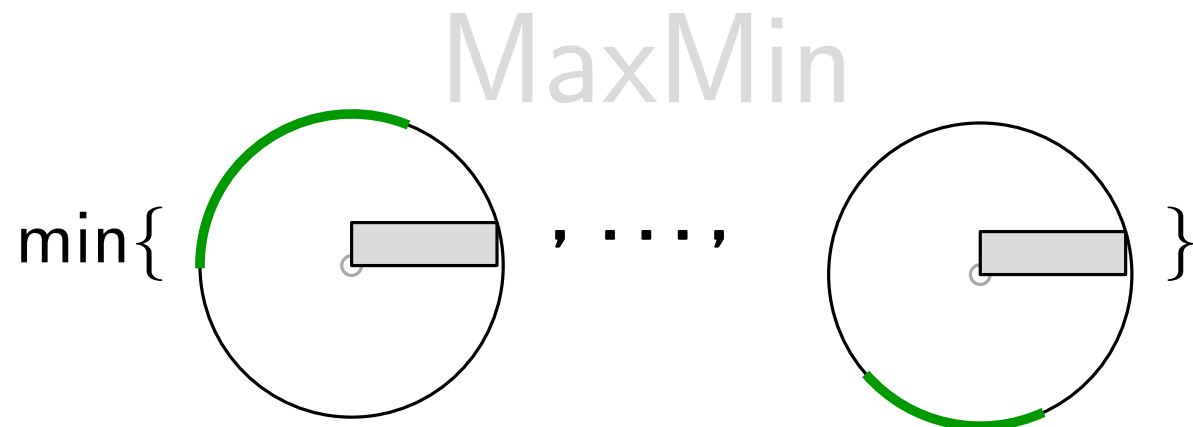
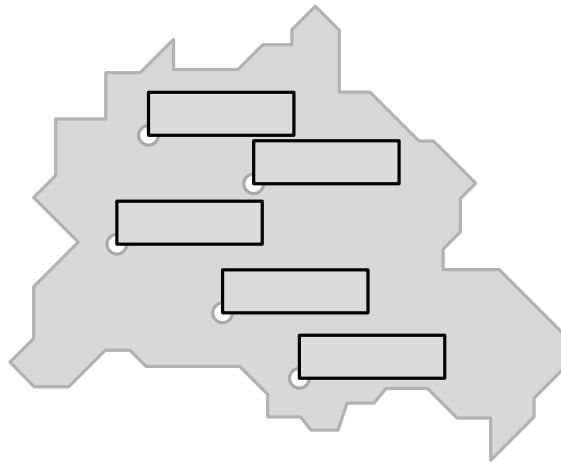


MaxTotal

Output: consistent rotation labeling that maximizes
the sum of all active ranges

Problem Statement

Input: A map M , points P , **valid** labeling $L(P)$



MaxTotal

Output: consistent rotation labeling that maximizes **the sum of all active ranges**

MaxMin

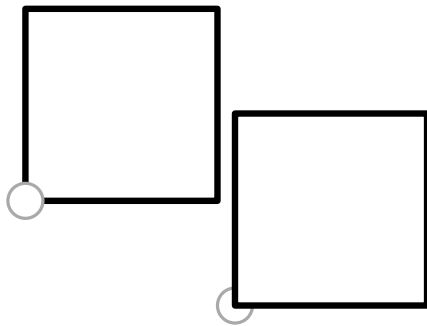
Output: consistent rotation labeling that maximizes **the smallest active range length**

Some Observations

Determining Conflicts

Conflict Lemma:

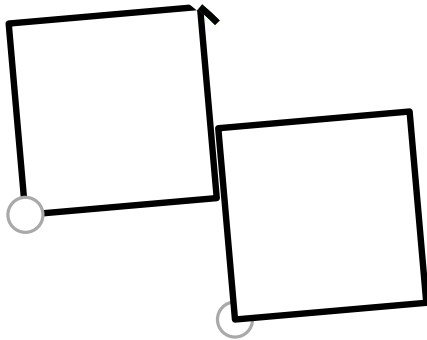
For two labels the set of conflicts consists of at most four contiguous conflict regions.



Determining Conflicts

Conflict Lemma:

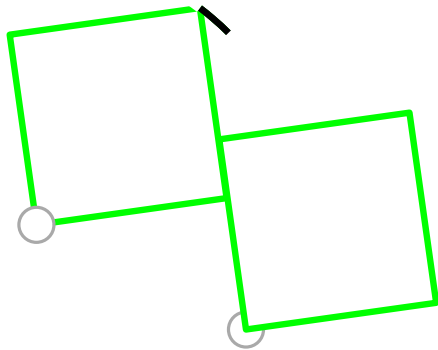
For two labels the set of conflicts consists of at most four contiguous conflict regions.



Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

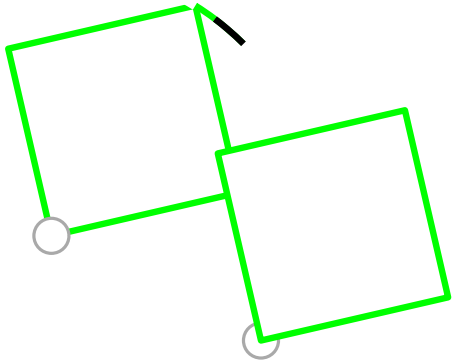


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

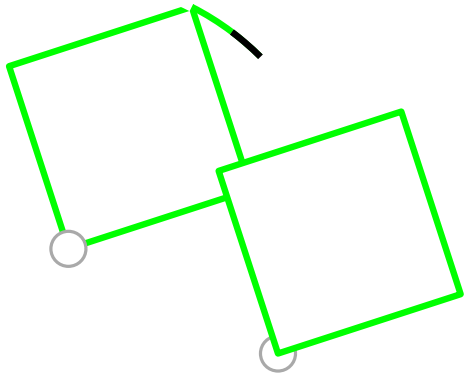


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

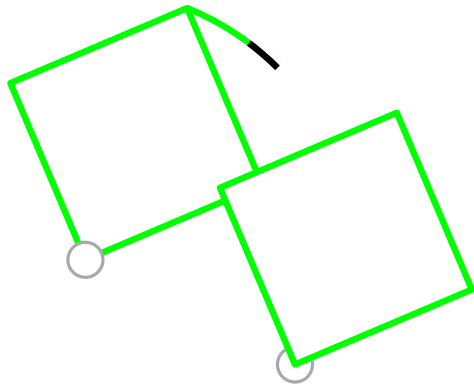


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

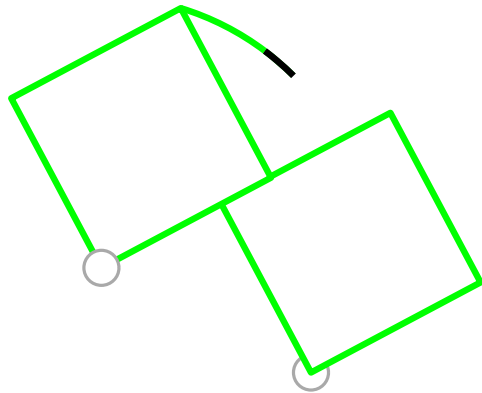


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

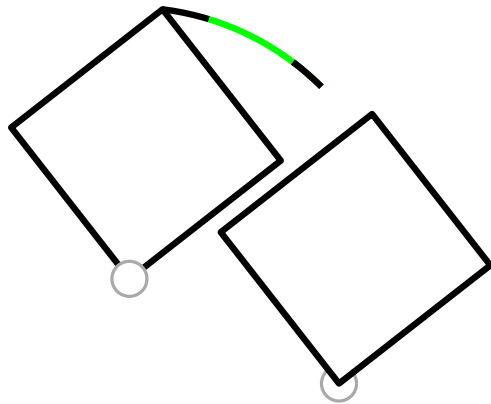


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

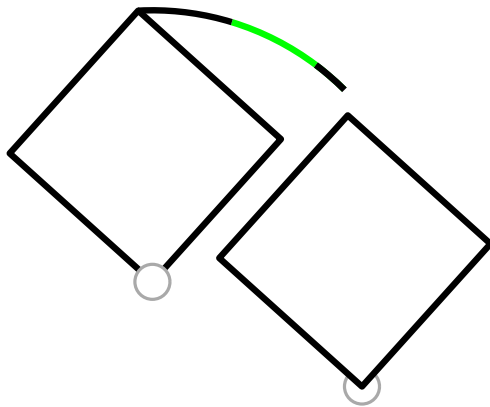


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

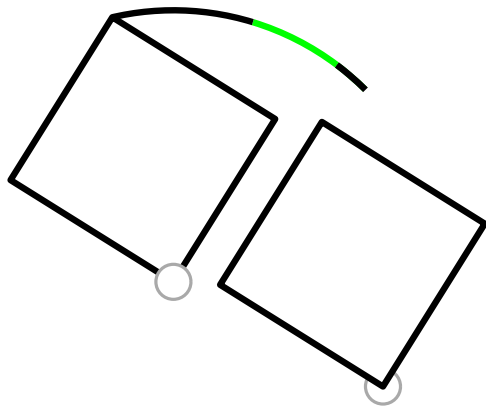


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

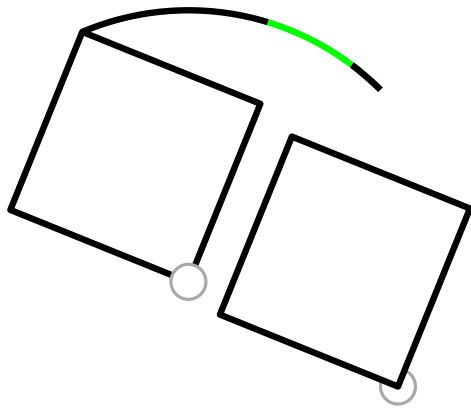


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

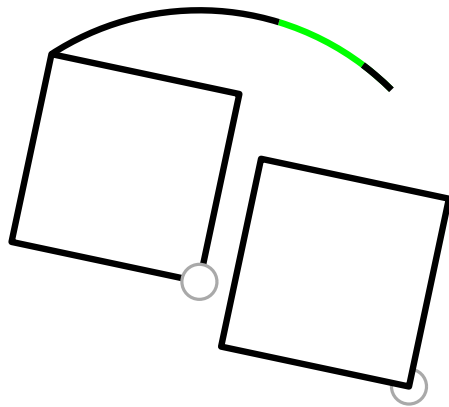


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

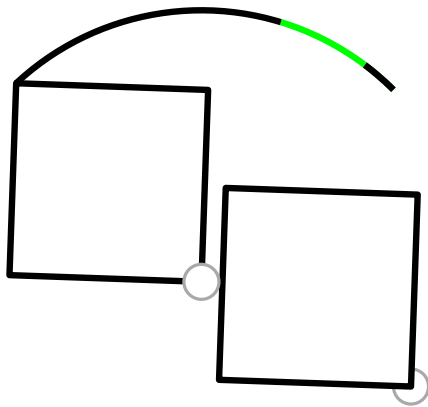


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

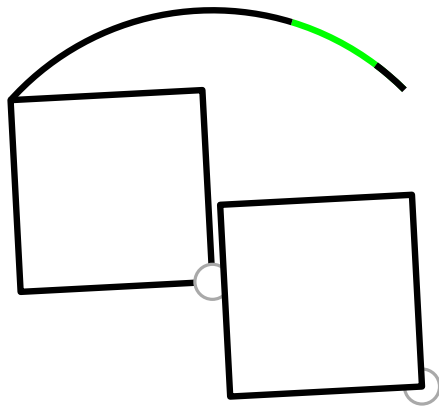


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.

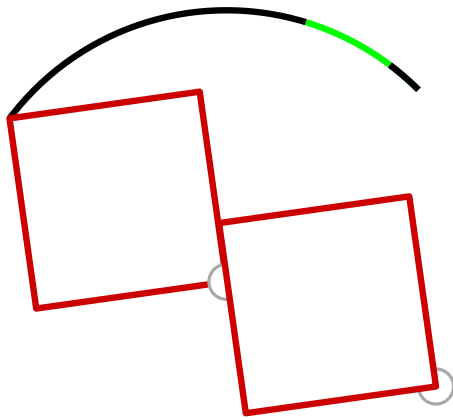


soft conflict: labels overlap

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



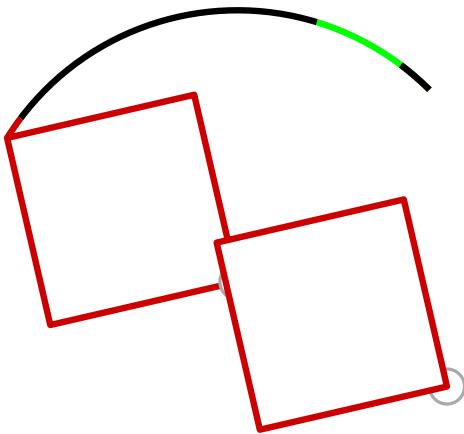
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



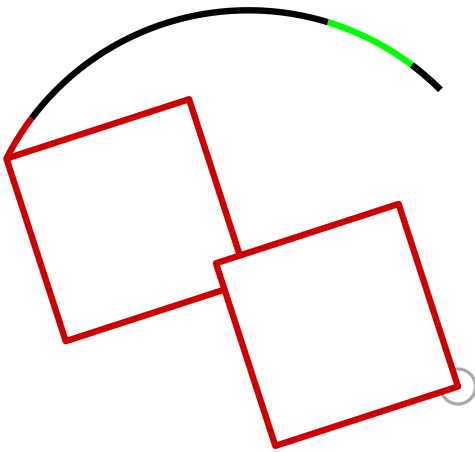
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



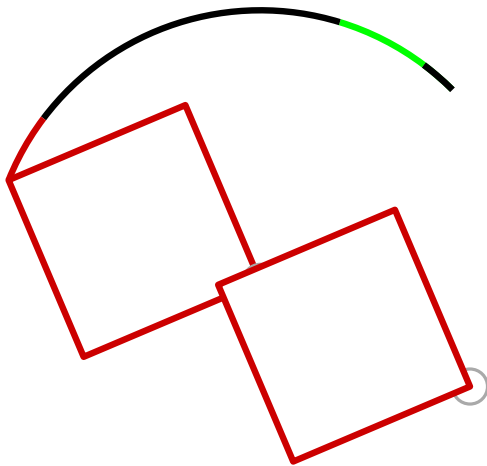
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



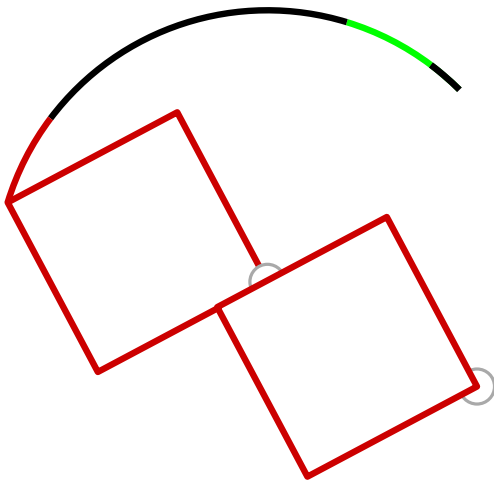
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



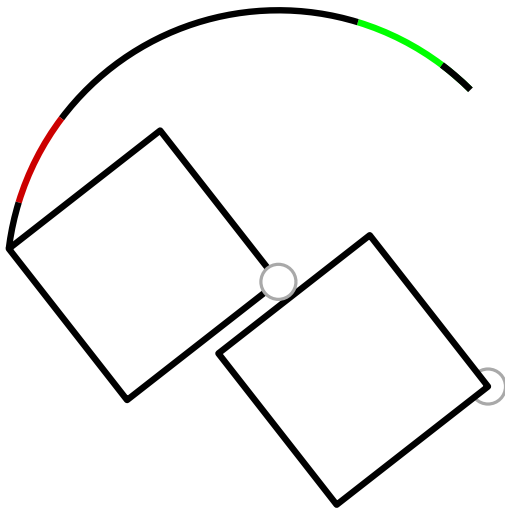
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



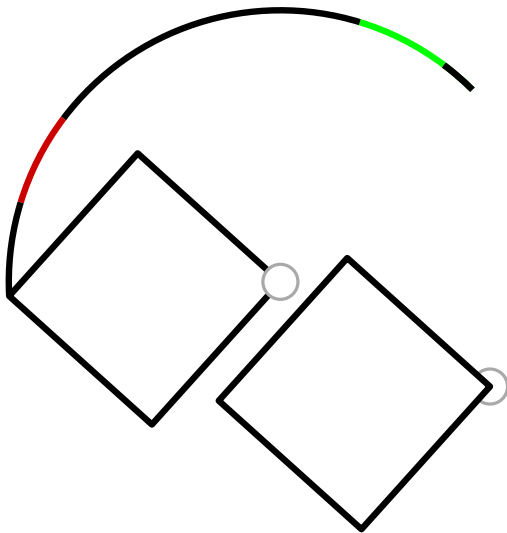
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



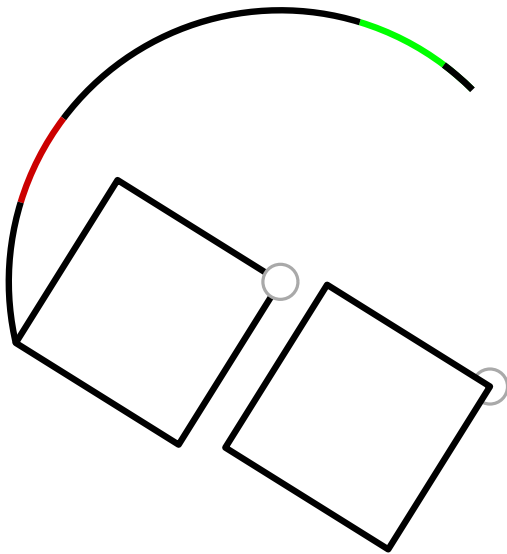
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



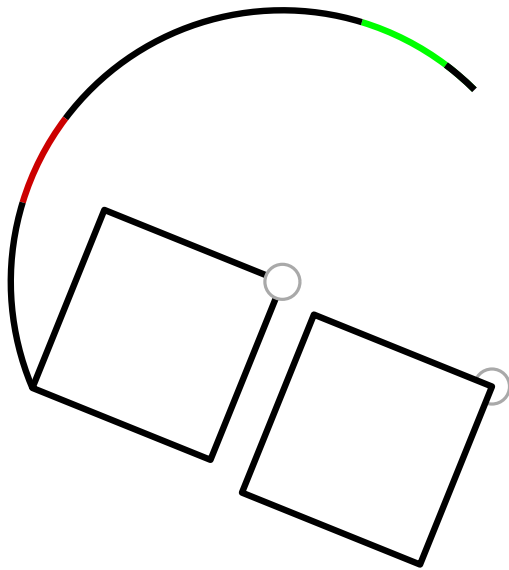
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



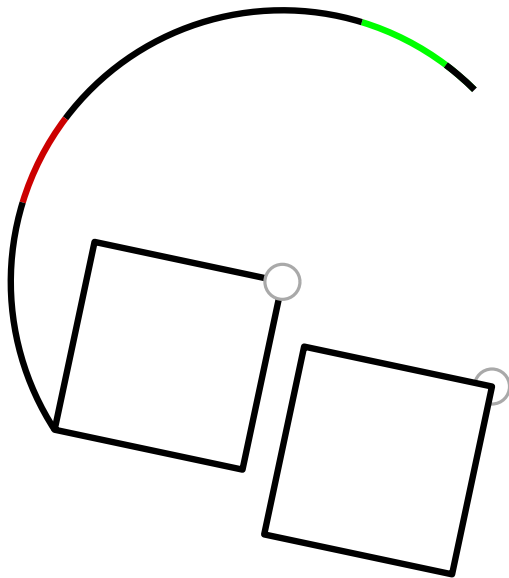
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



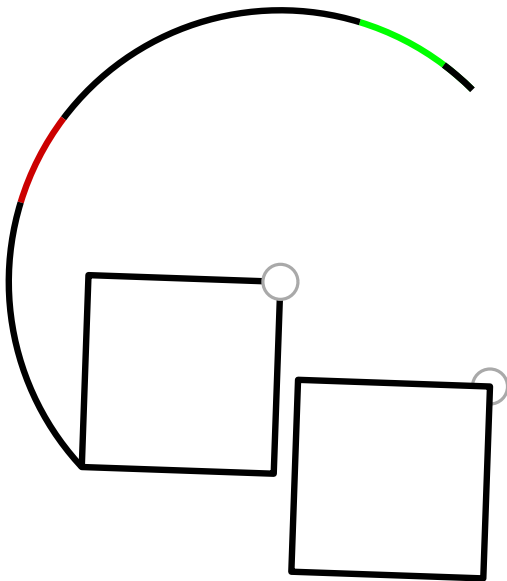
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



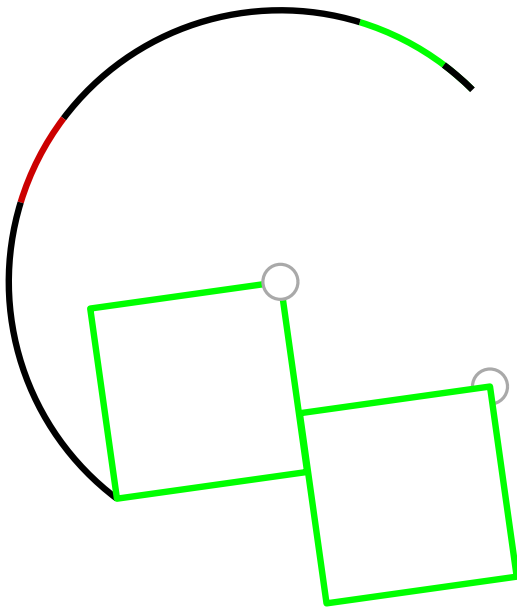
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



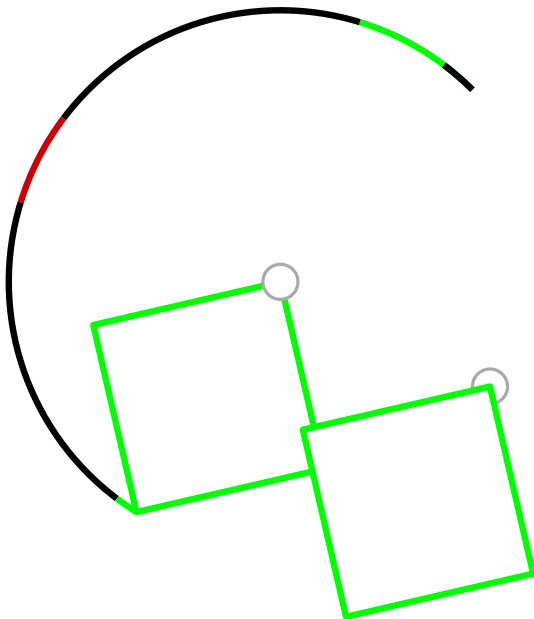
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



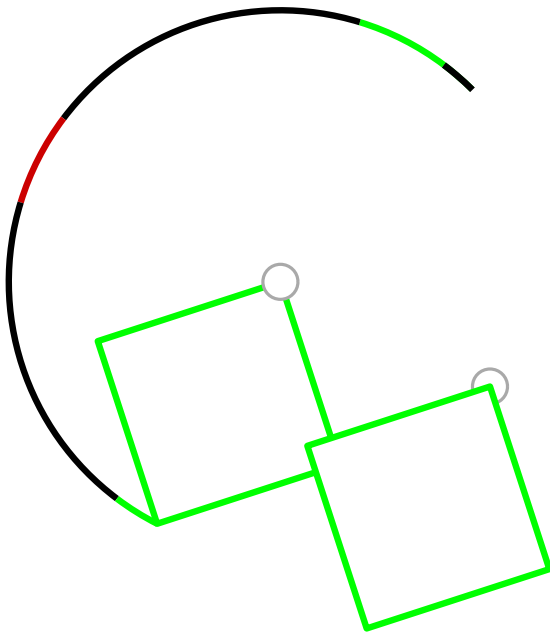
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



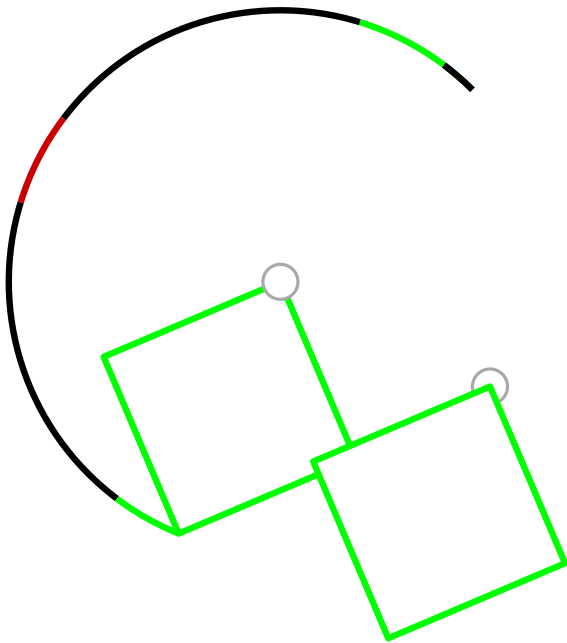
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



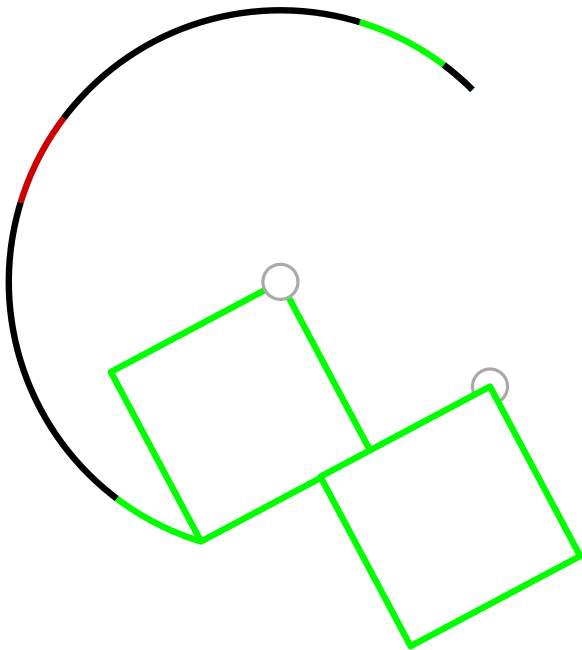
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



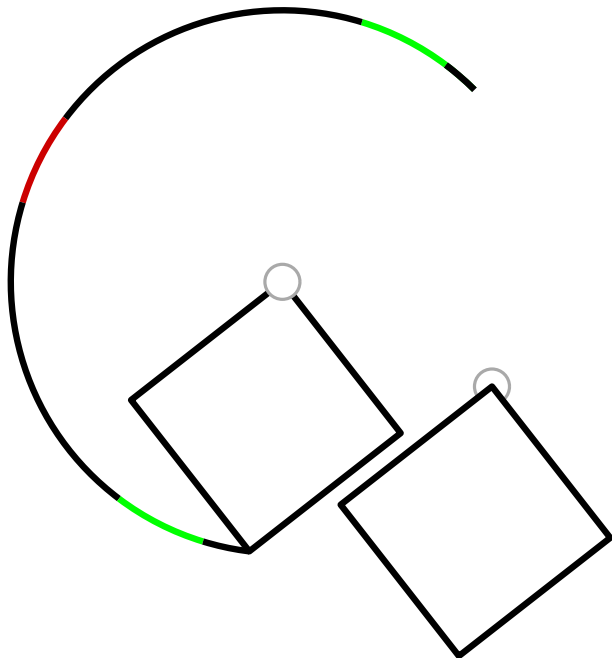
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



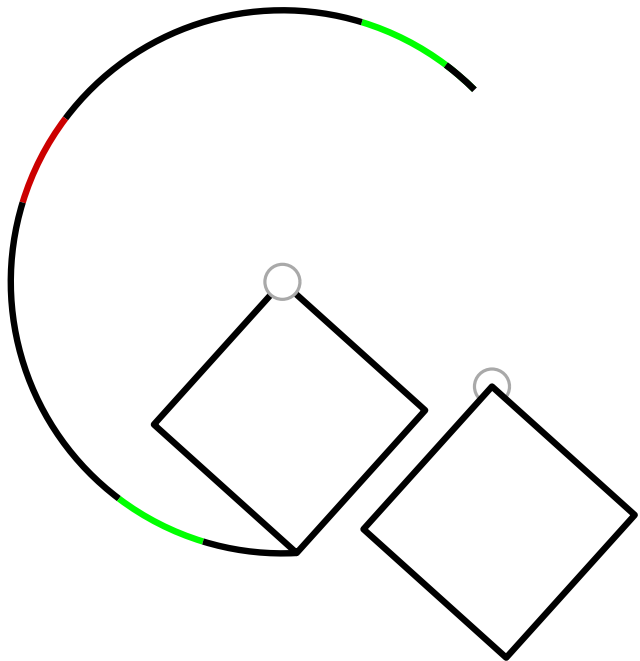
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



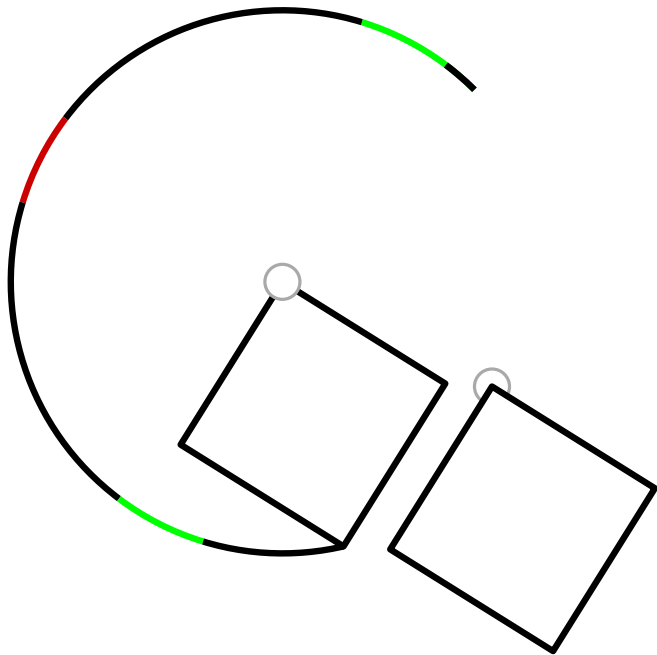
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



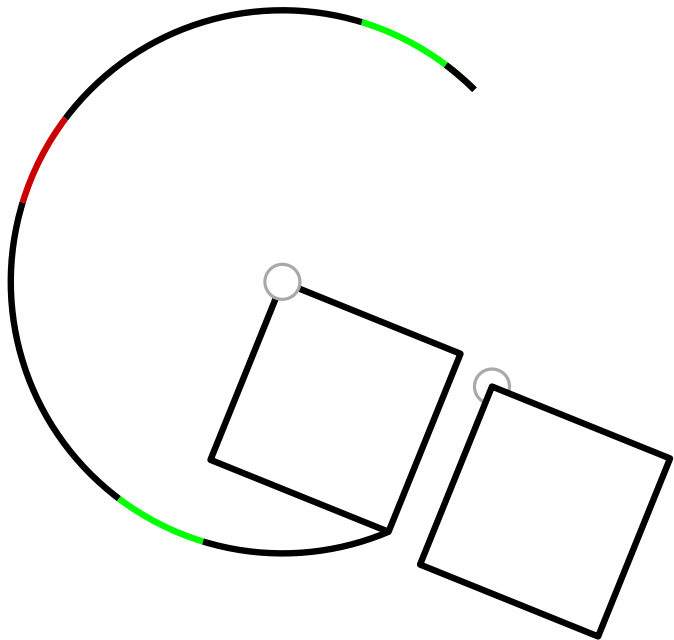
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



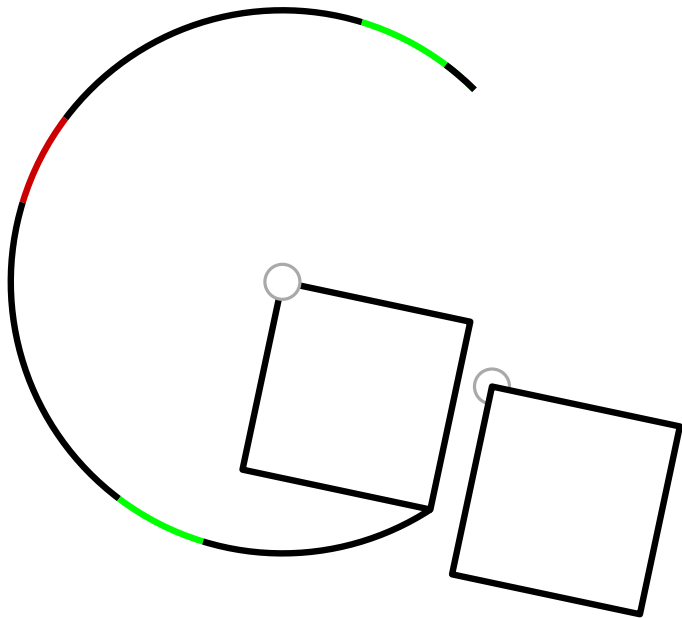
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



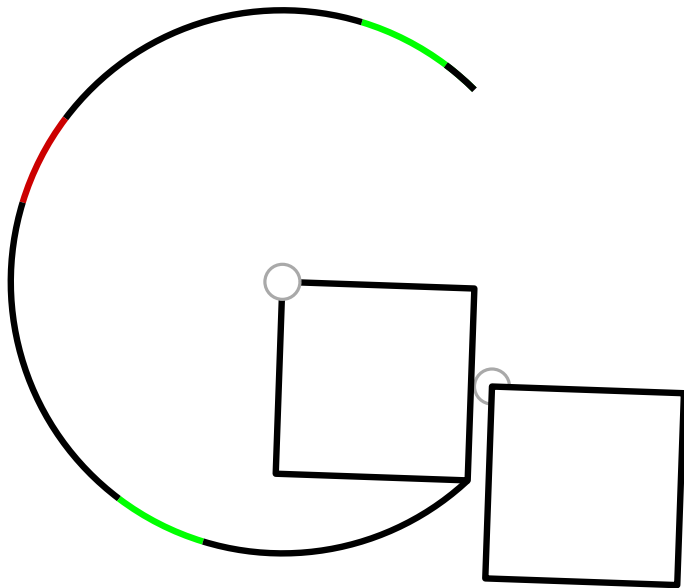
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



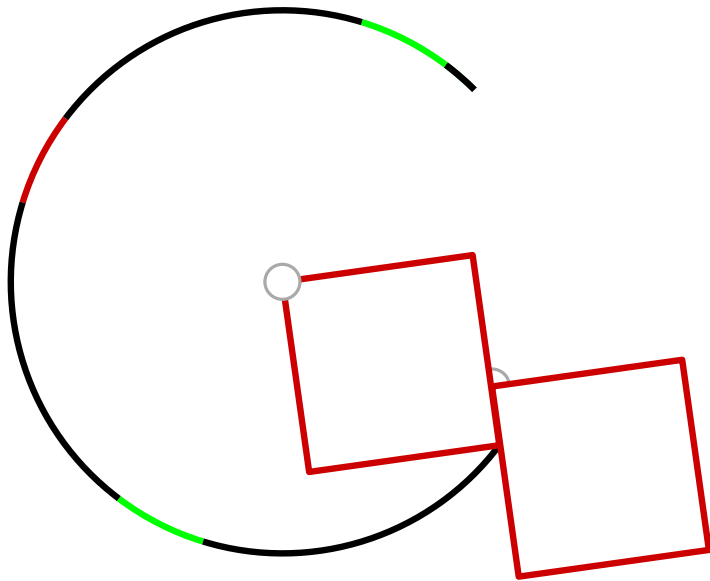
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



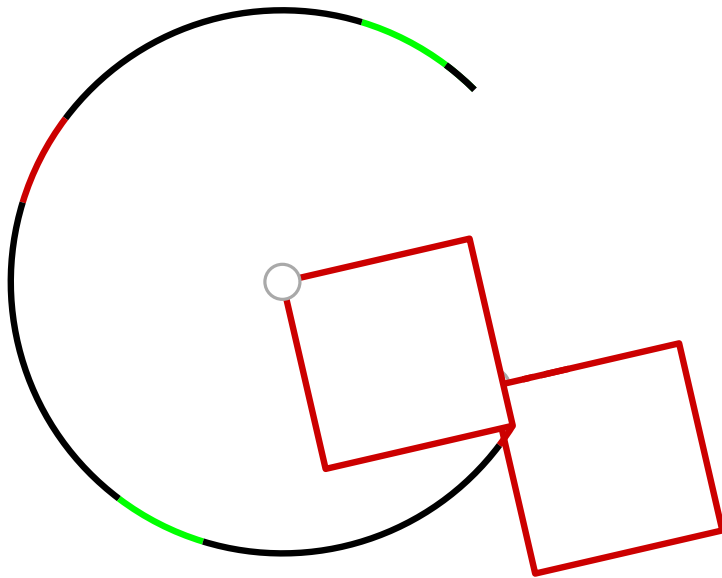
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



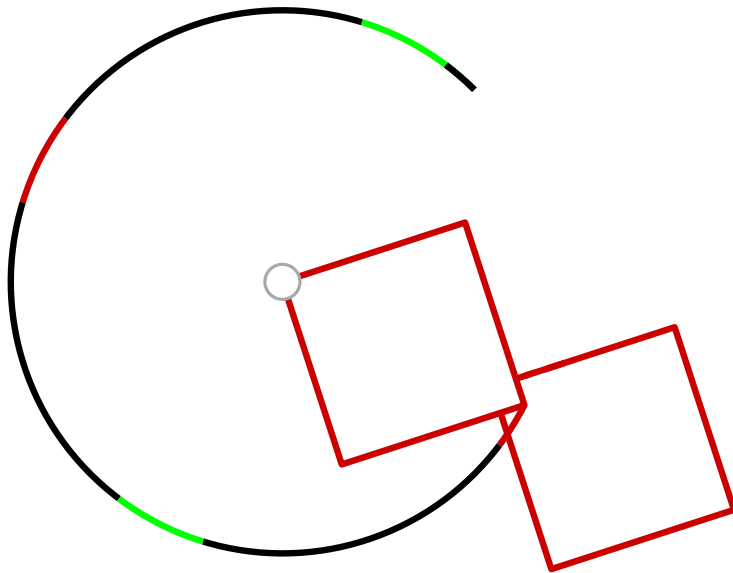
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



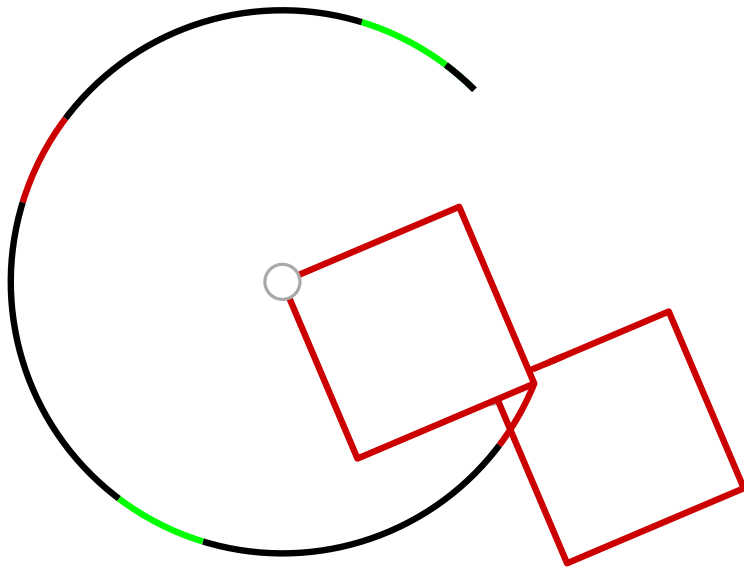
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



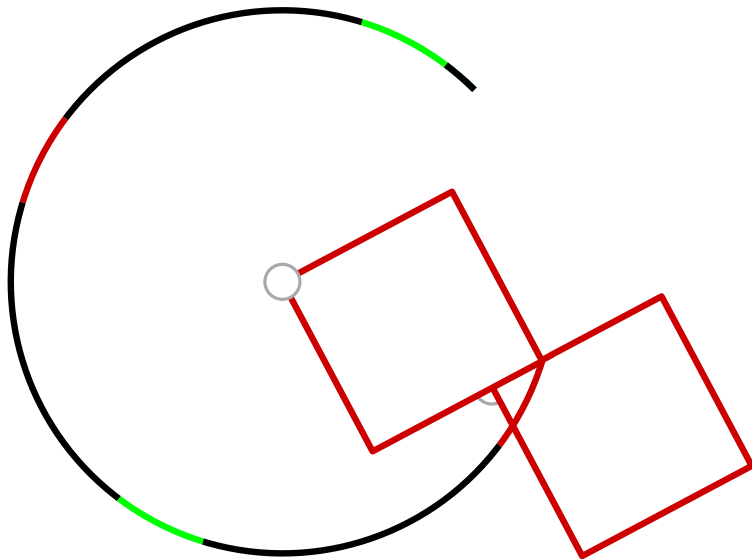
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



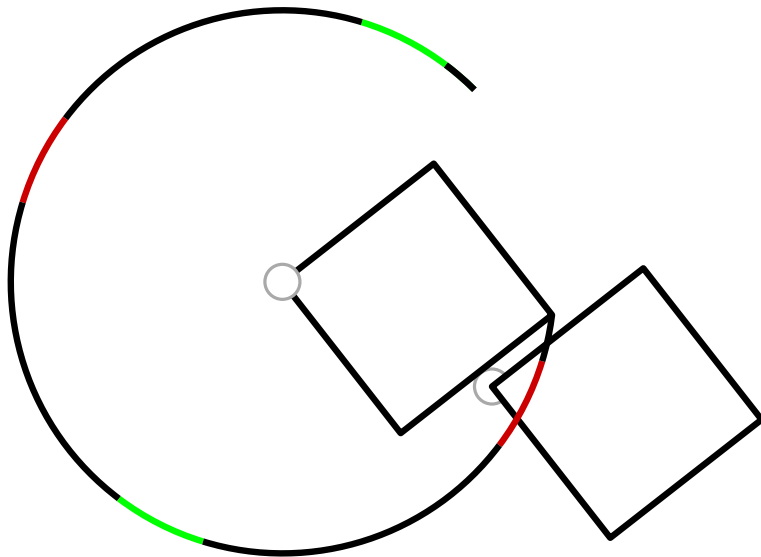
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



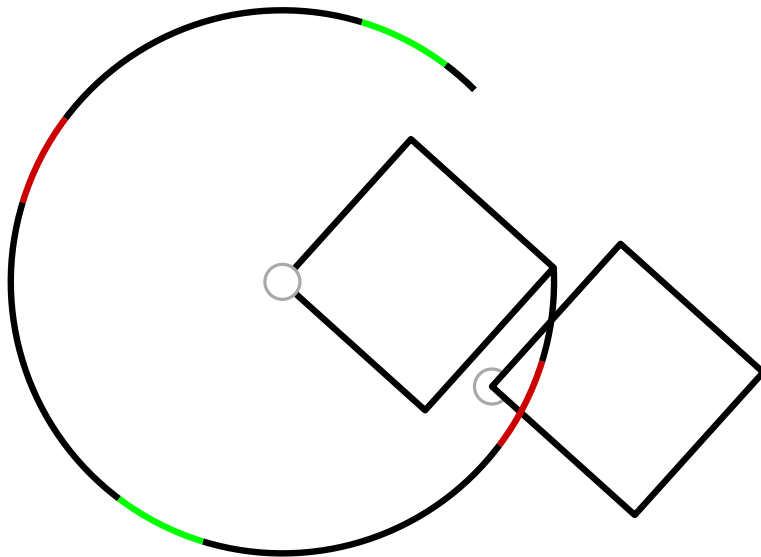
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



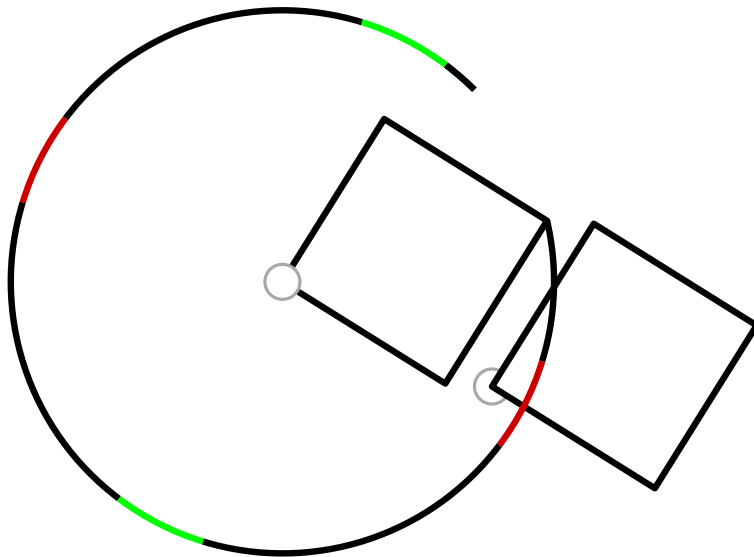
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



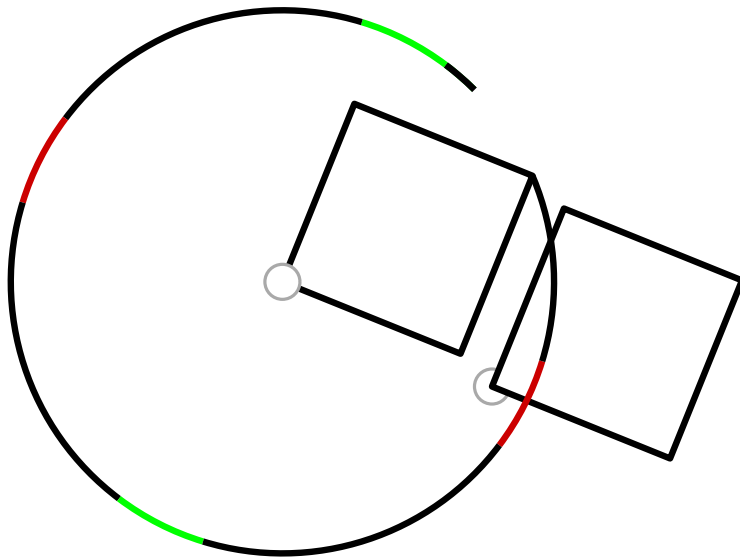
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



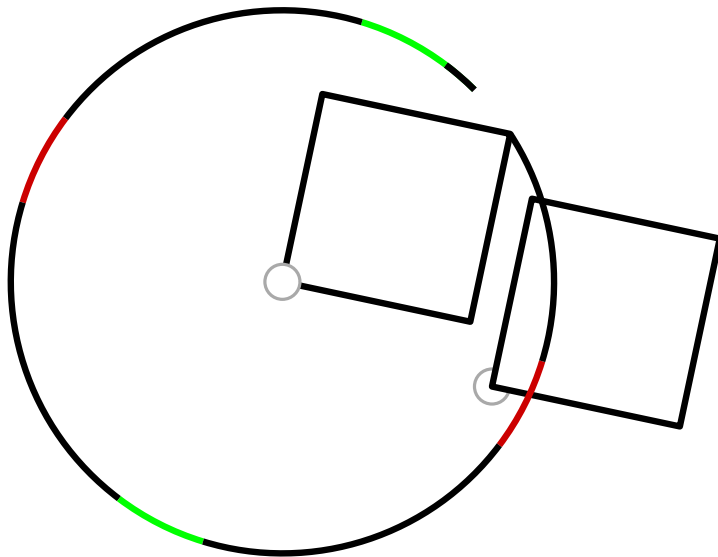
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



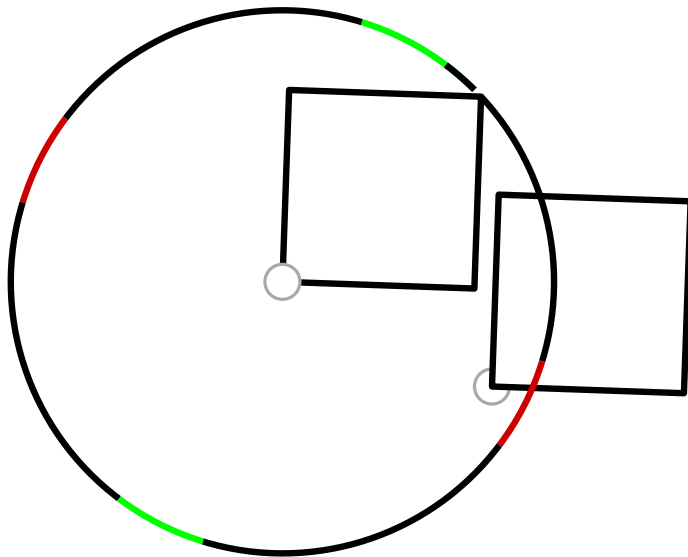
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



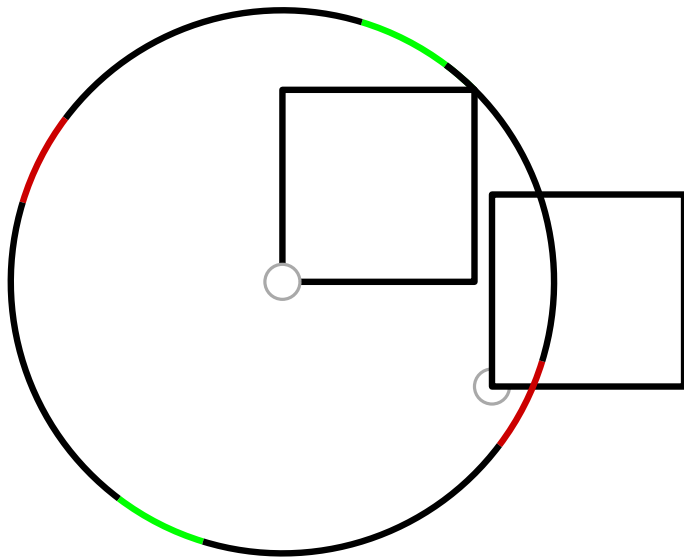
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



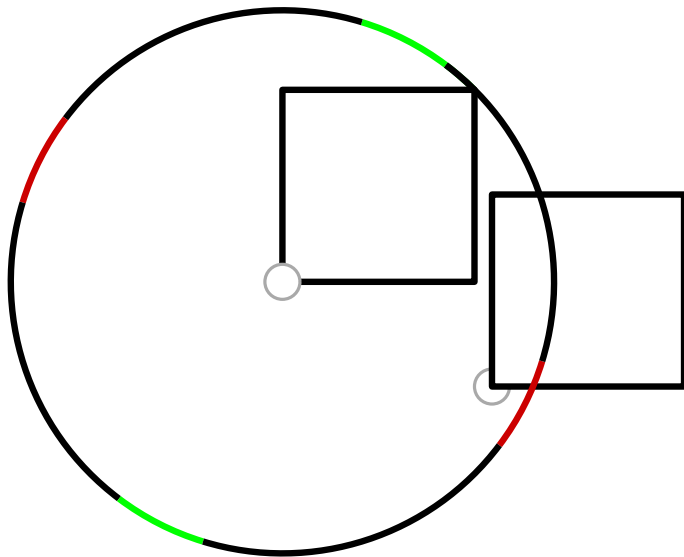
soft conflict: labels overlap

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Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



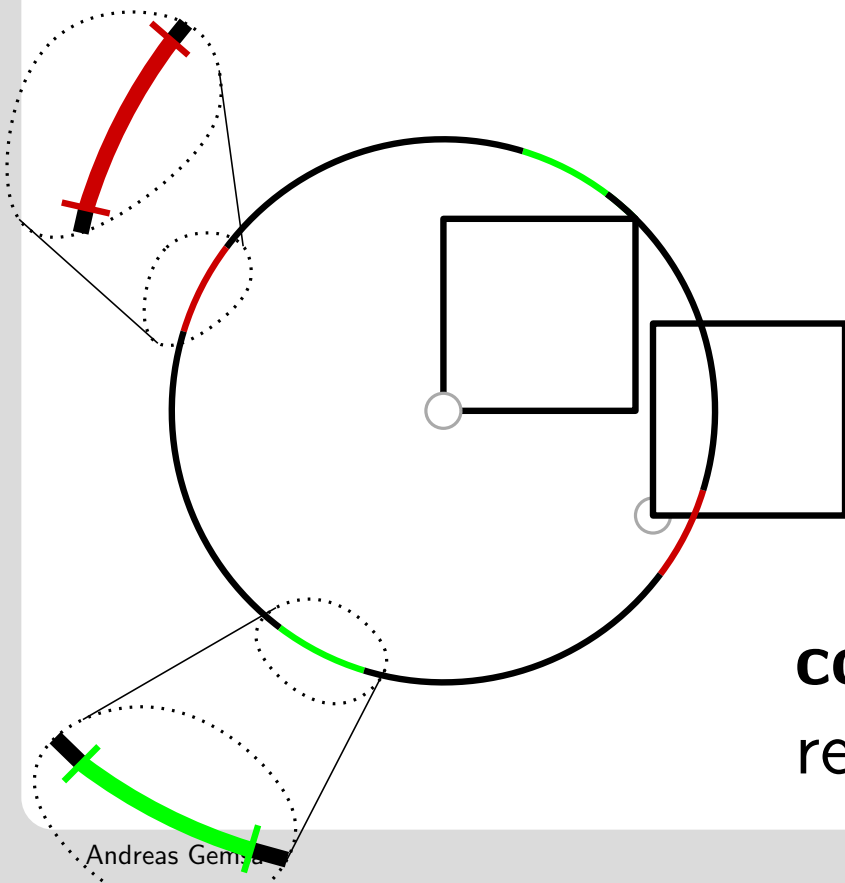
soft conflict: labels overlap

hard conflict: label overlaps anchor point

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



soft conflict: labels overlap

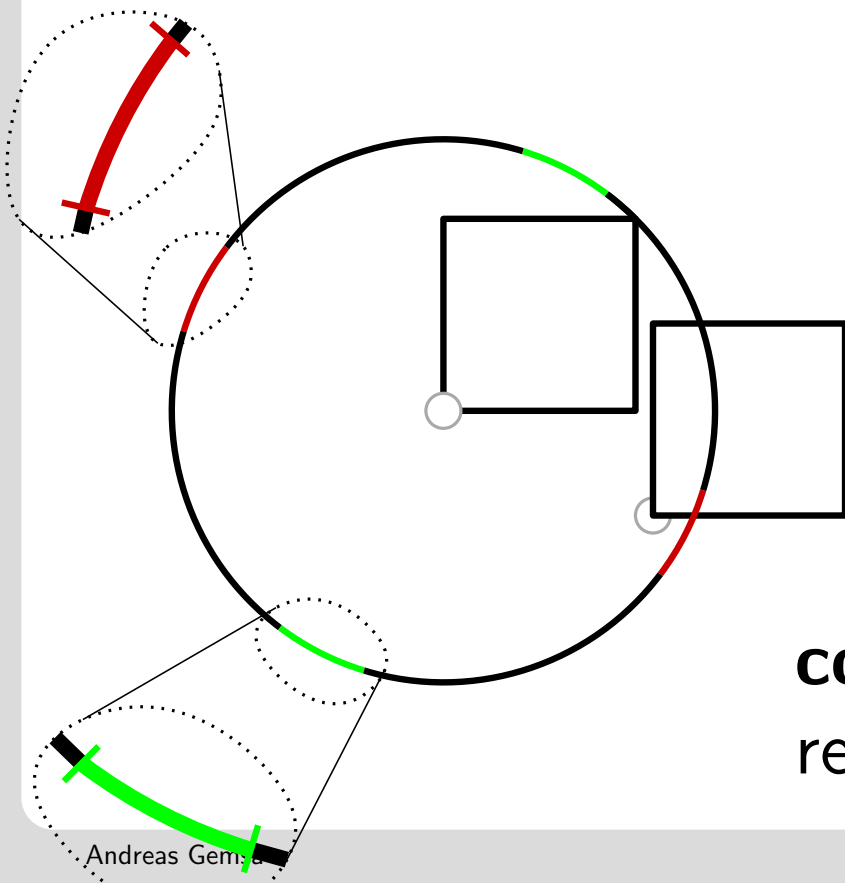
hard conflict: label overlaps anchor point

conflict events: begin/end of a conflict region (borders of labels intersect)

Determining Conflicts

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



soft conflict: labels overlap

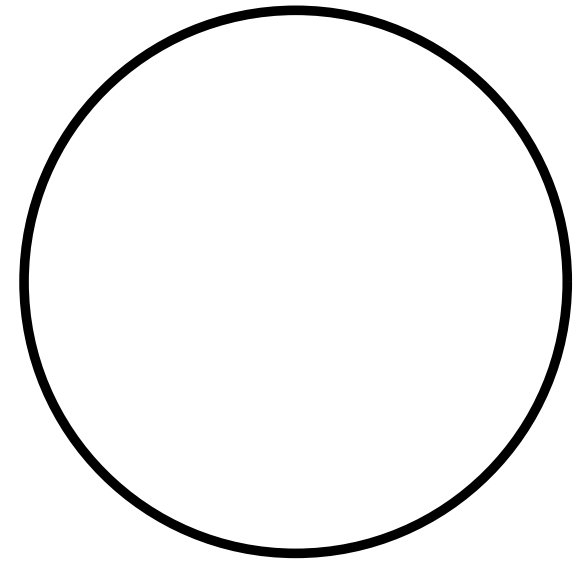
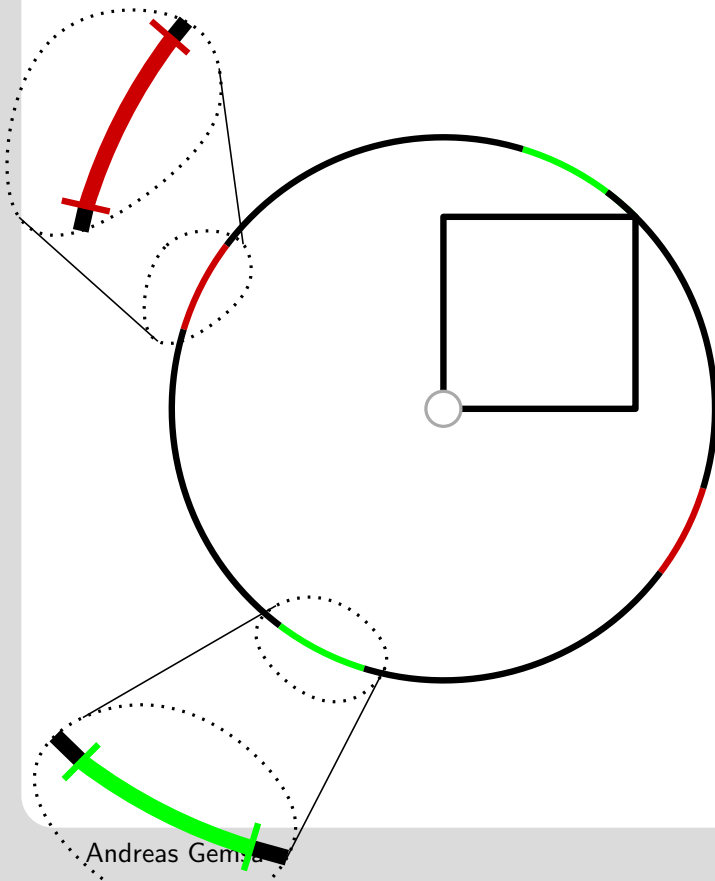
hard conflict: label overlaps anchor point

at most 8 such events per pair

conflict events: begin/end of a conflict region (borders of labels intersect)

Discretization Lemma

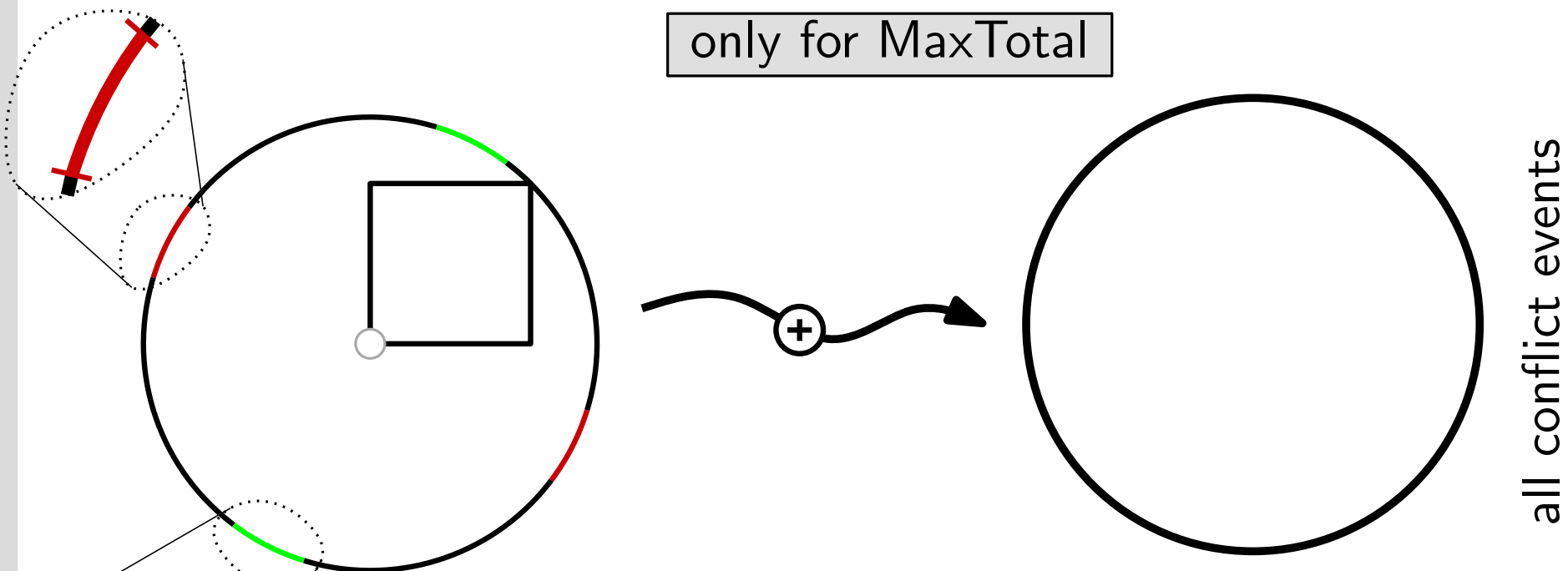
only for MaxTotal



Discretization Lemma

Discretization Lemma:

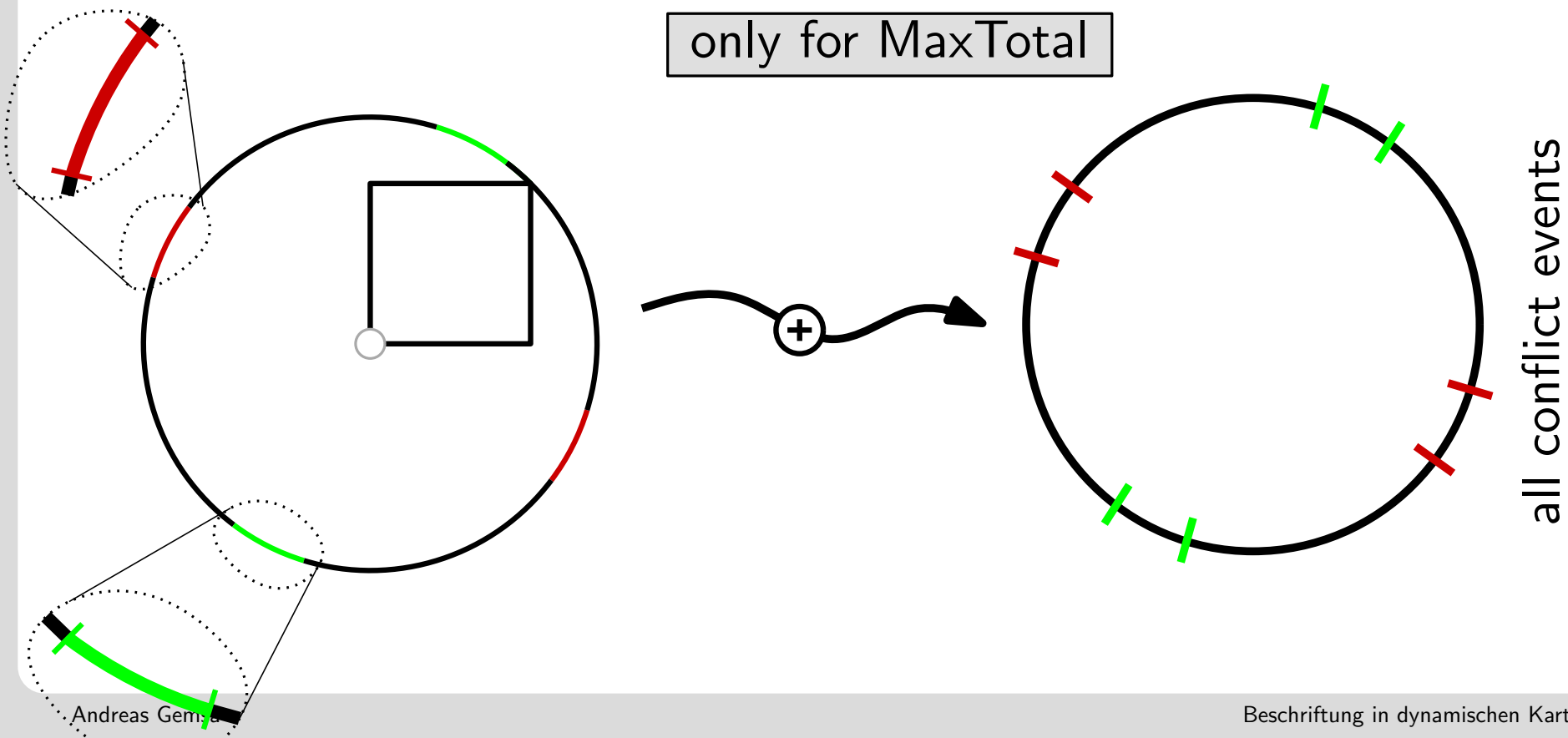
There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.



Discretization Lemma

Discretization Lemma:

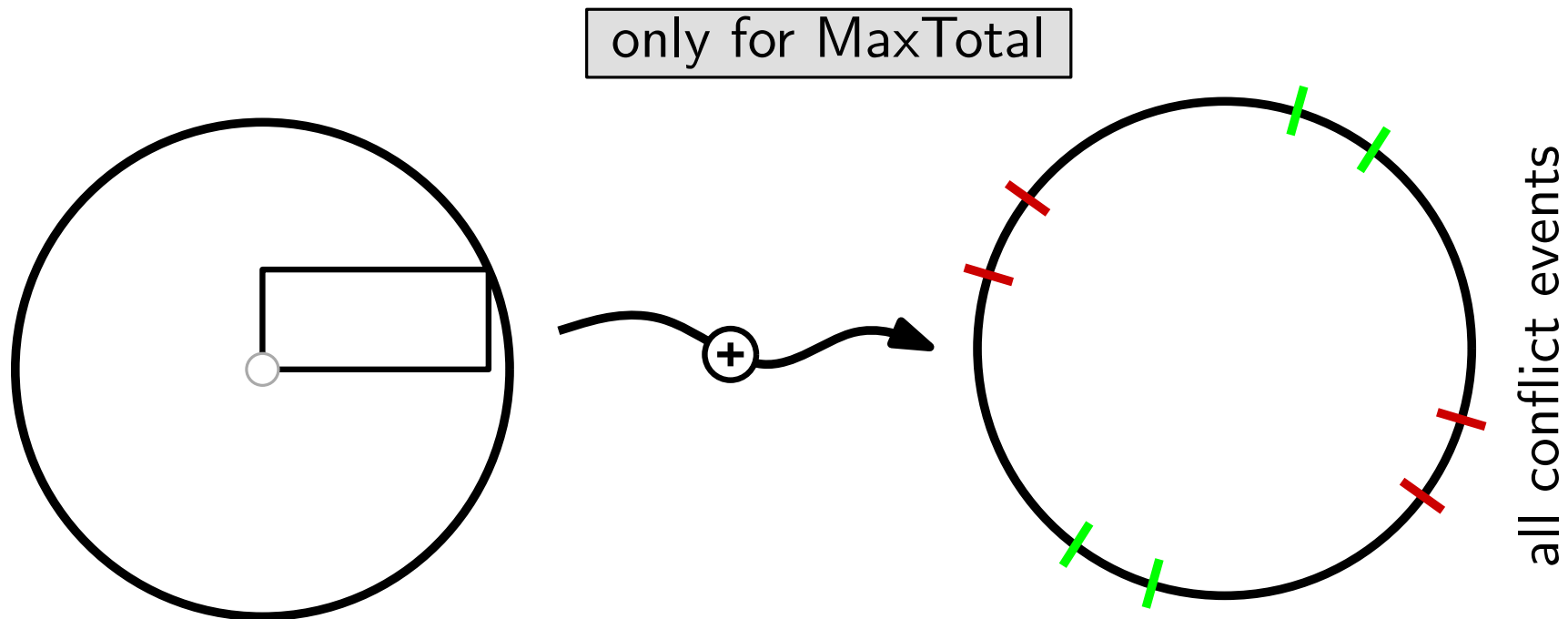
There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.



Discretization Lemma

Discretization Lemma:

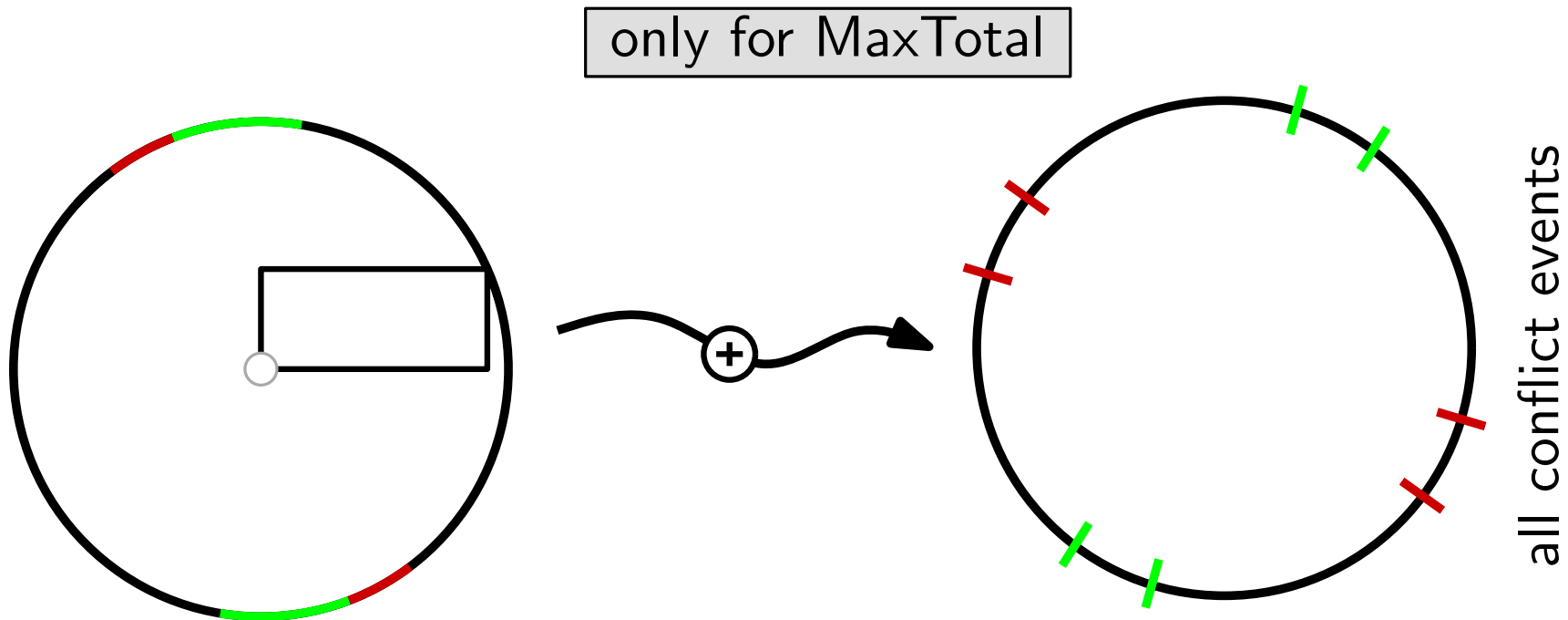
There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.



Discretization Lemma

Discretization Lemma:

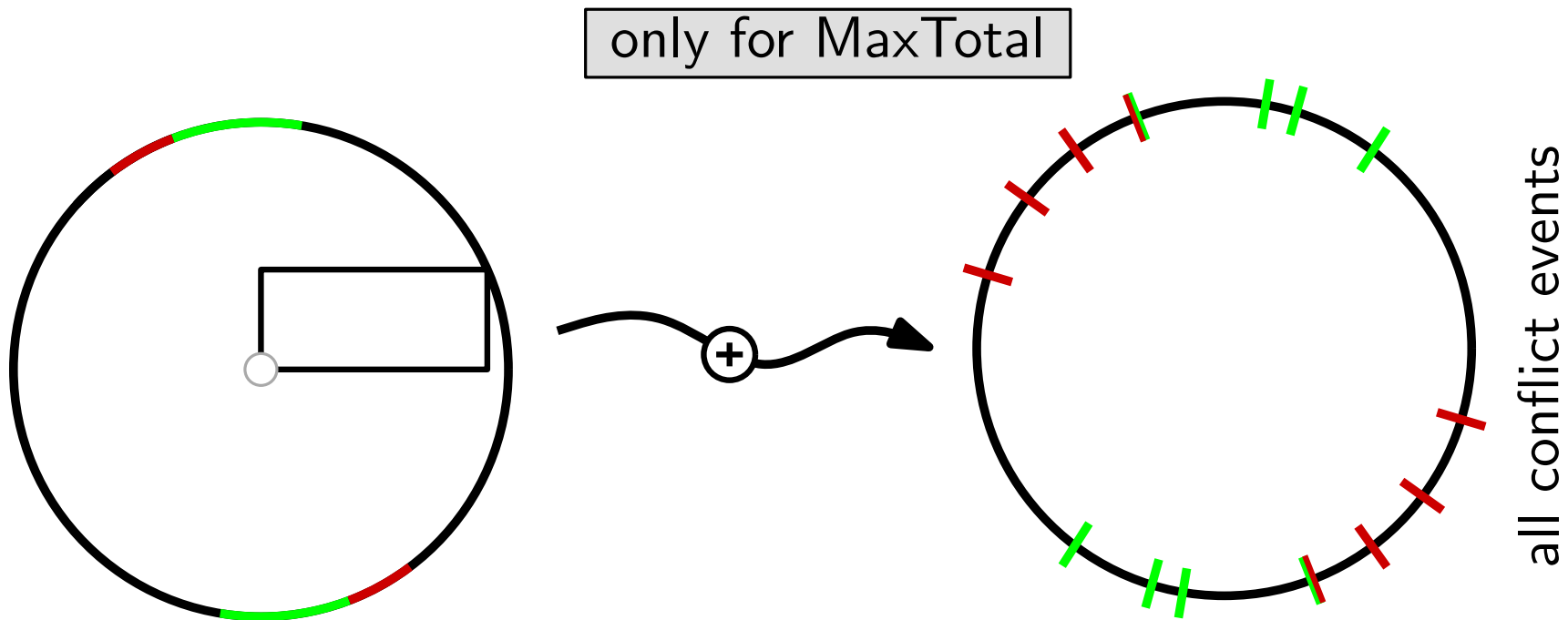
There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.



Discretization Lemma

Discretization Lemma:

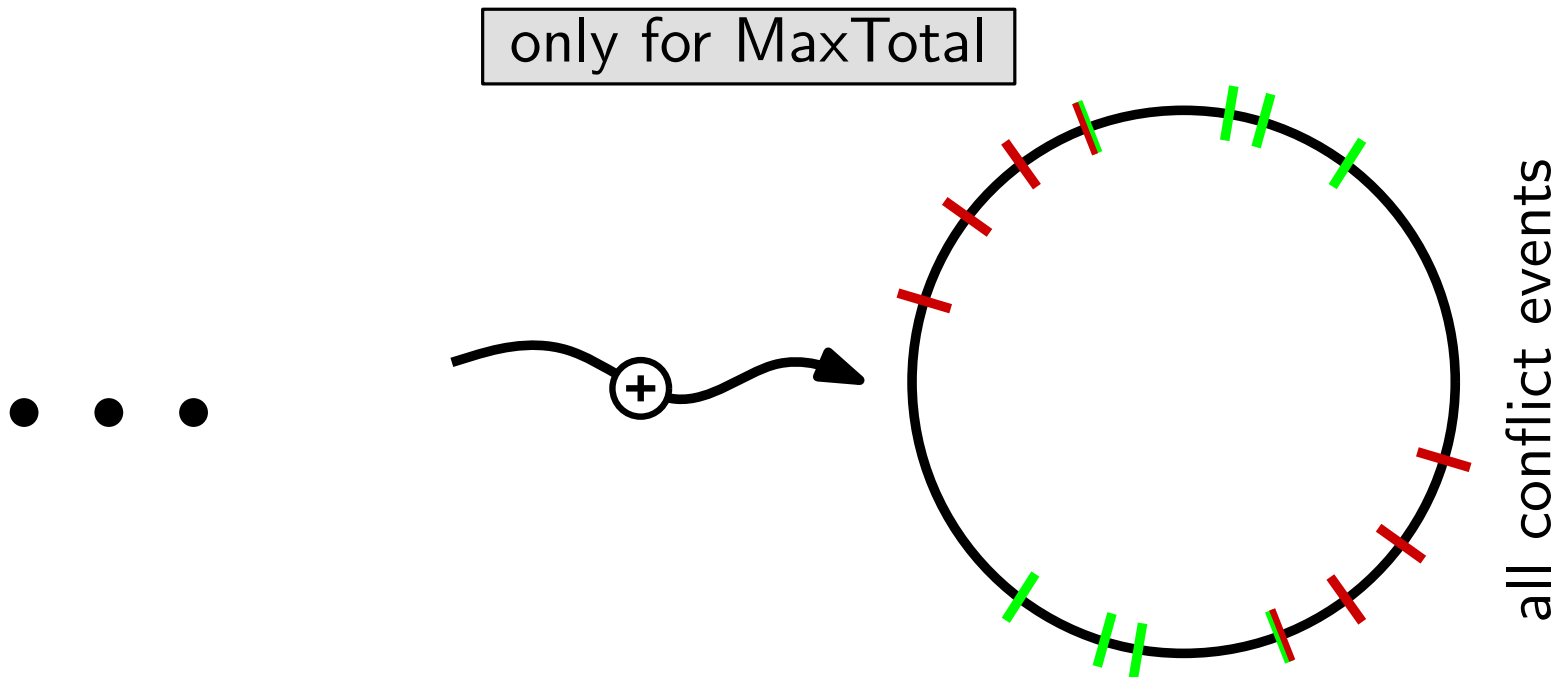
There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.



Discretization Lemma

Discretization Lemma:

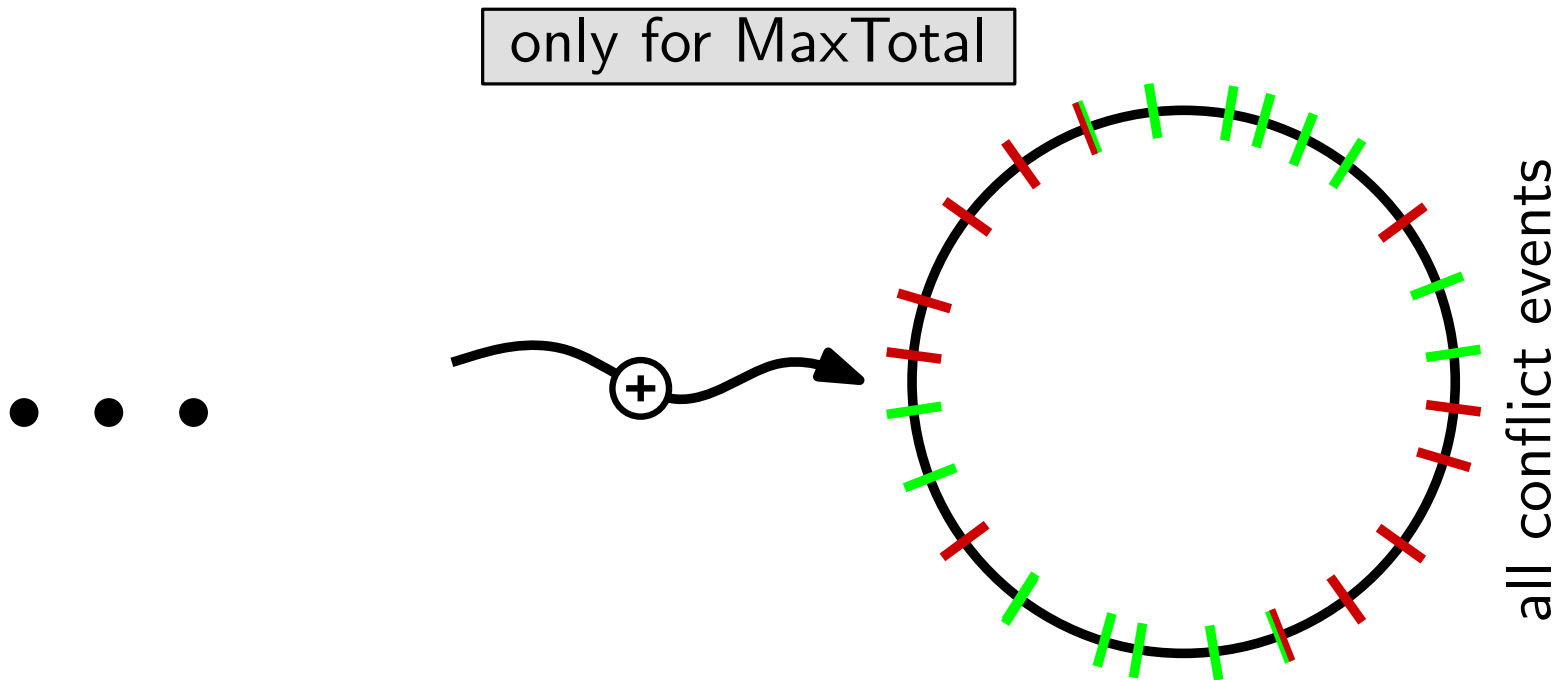
There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.



Discretization Lemma

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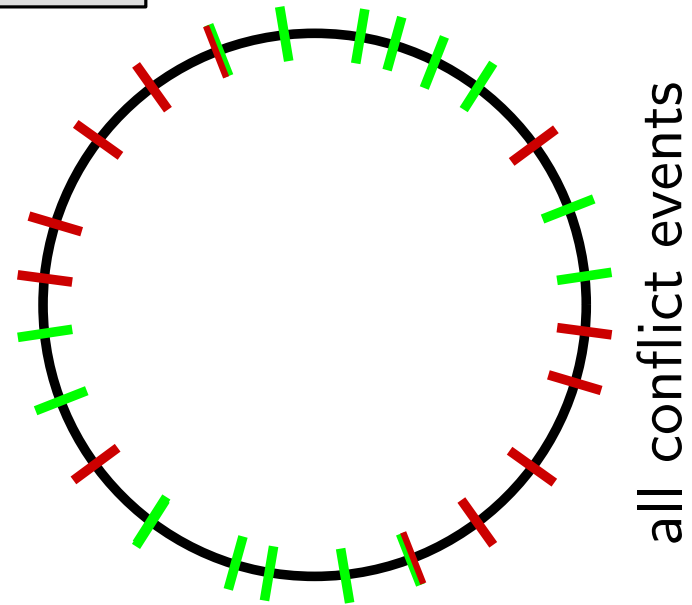


Discretization Lemma

Discretization Lemma:

There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.

only for MaxTotal



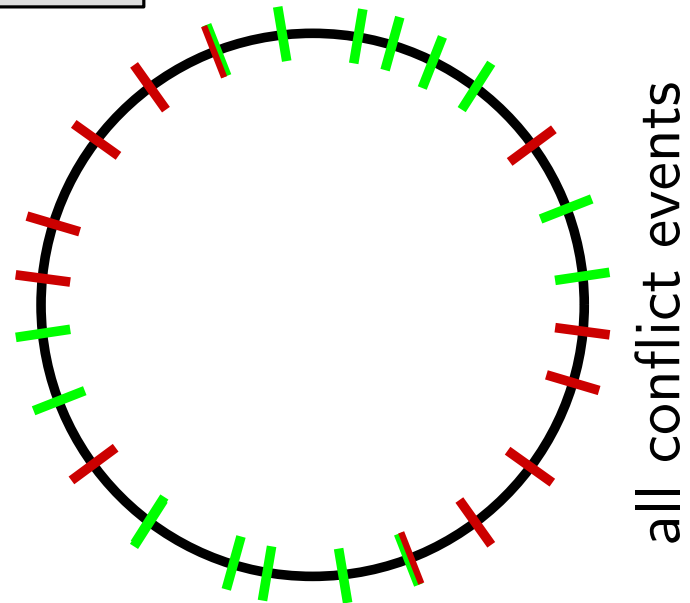
Discretization Lemma

Discretization Lemma:

There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.

only for MaxTotal

Q: (naïve) algorithm for finding optimal solution?



Discretization Lemma

Discretization Lemma:

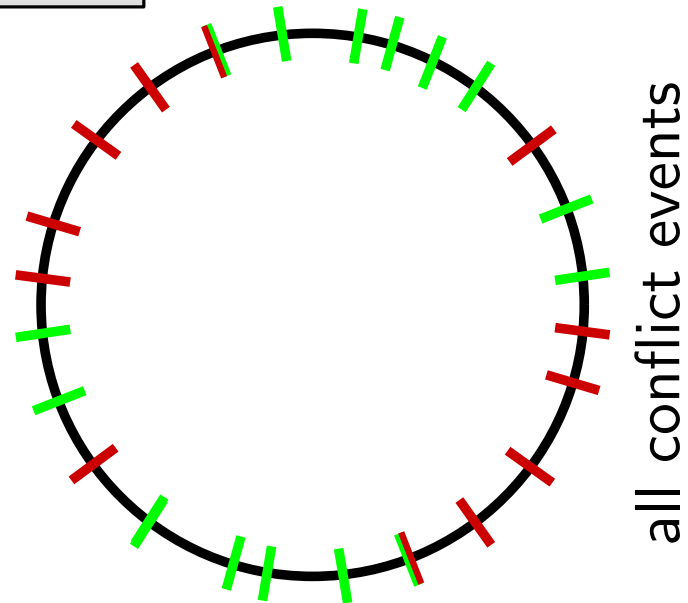
There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.

only for MaxTotal

naïve approach

find optimal solution:

- compute all conflict events
- per label $\mathcal{O}(n^4)$ active range candidates
- determine all possible combinations



Discretization Lemma

Discretization Lemma:

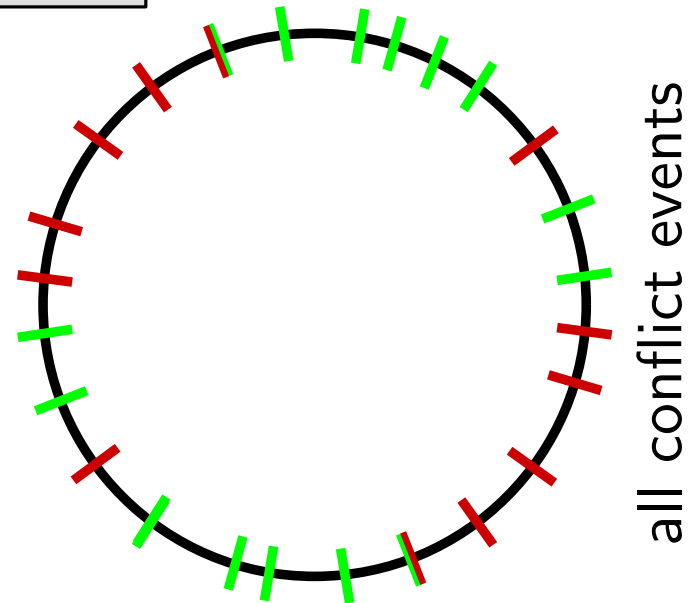
There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.

only for MaxTotal

naïve approach

find optimal solution:

- compute all conflict events
- per label $\mathcal{O}(n^4)$ active range candidates
- determine all possible combinations



Time Complexity?

Computational Complexity

Complexity

Theorem 1

MaxTotal is NP-complete

MaxMin is NP-hard

Complexity

Theorem 1

MaxTotal is NP-complete

MaxMin is NP-hard

Reduction from PLANAR 3-SAT:

(NP-hard: [Lichtenstein '82])

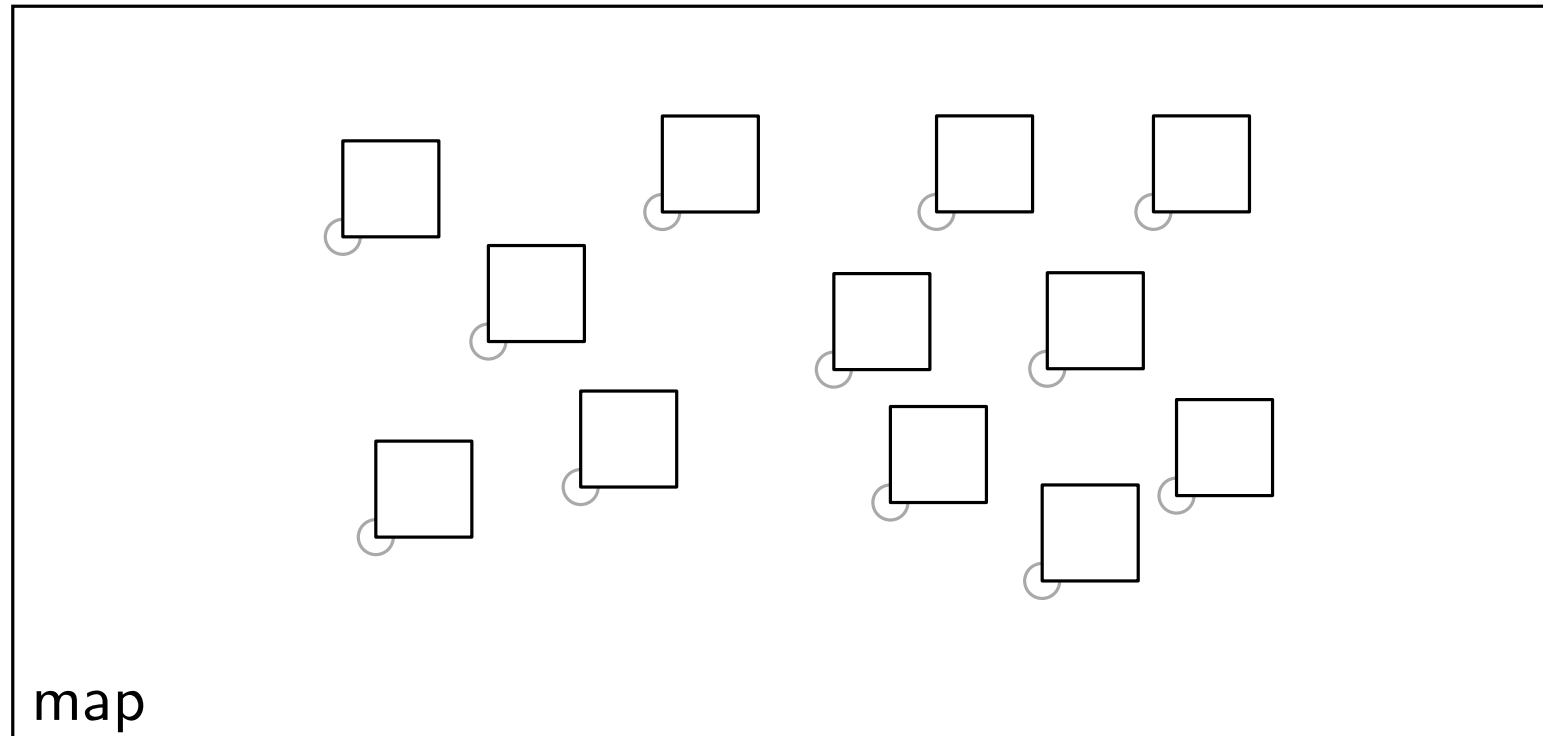
1/4-Approximation for MaxTotal for unit squares

1/4 Approximation of MaxTotal

use **shifting technique** or **line stabbing**
(NP-hard: [Hochbaum and Maass '85])

1/4 Approximation of MaxTotal

Observation 1:

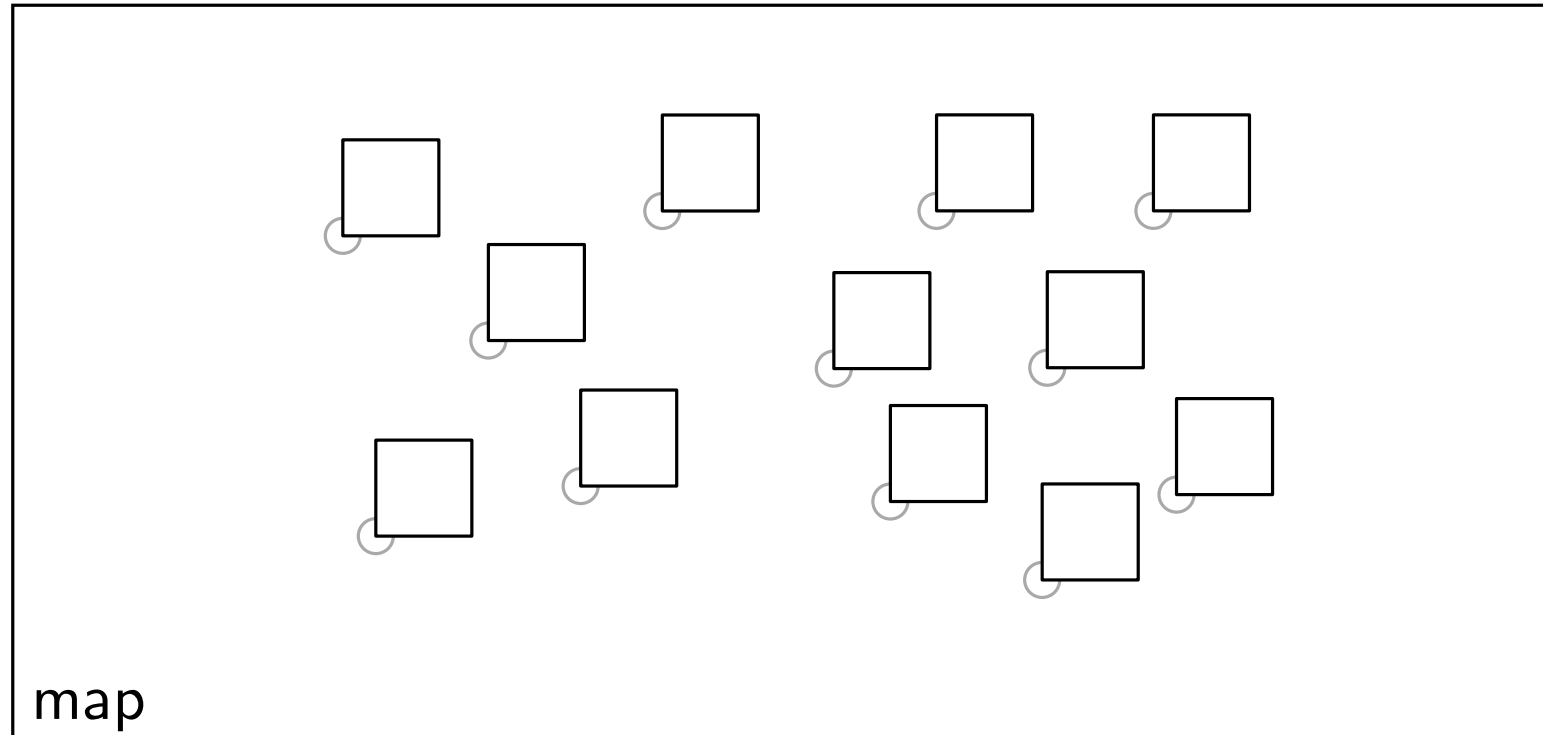


use **shifting technique** or **line stabbing**

(NP-hard: [Hochbaum and Maass '85])

1/4 Approximation of MaxTotal

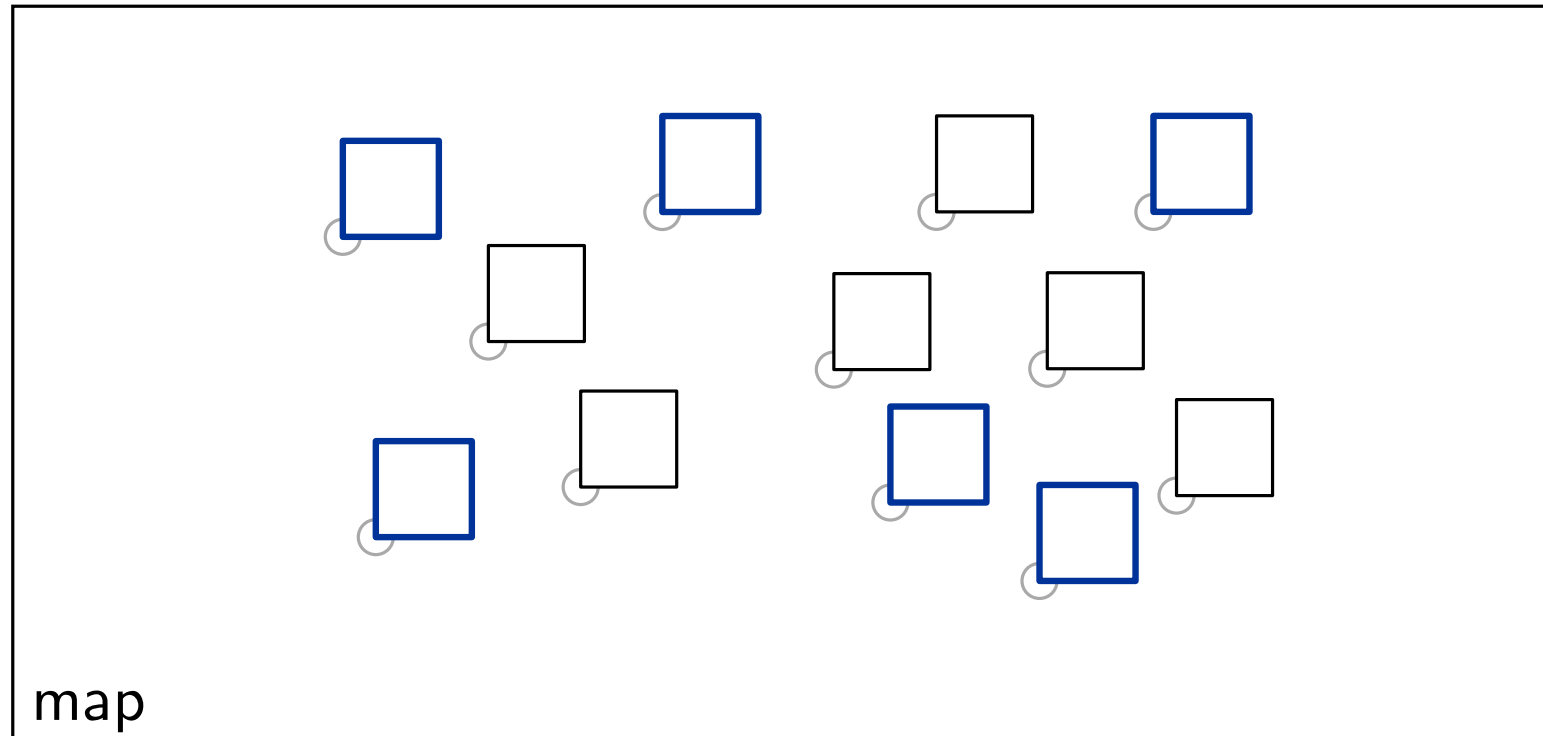
Observation 1:



- split set of labels into two sets

1/4 Approximation of MaxTotal

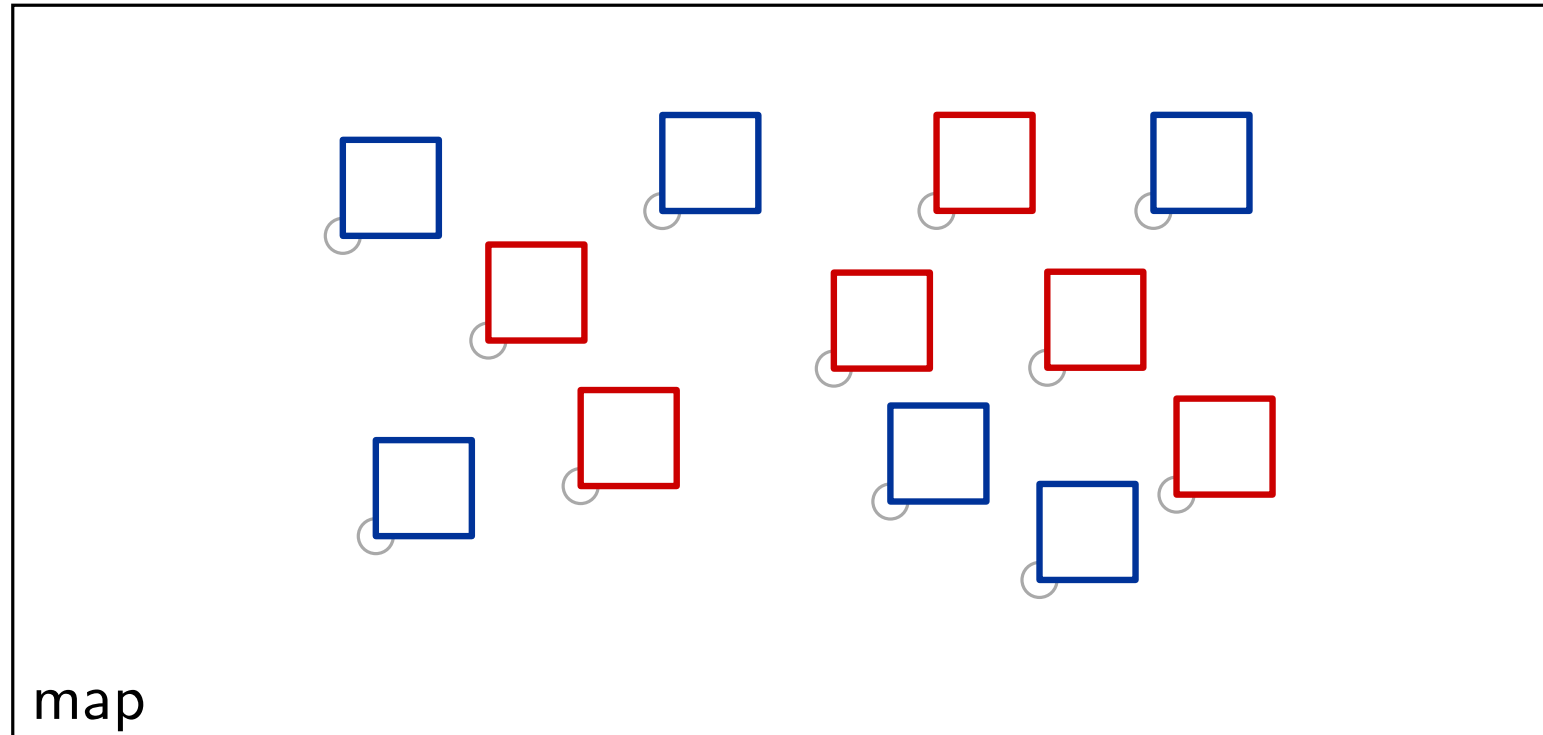
Observation 1:



- split set of labels into two sets

1/4 Approximation of MaxTotal

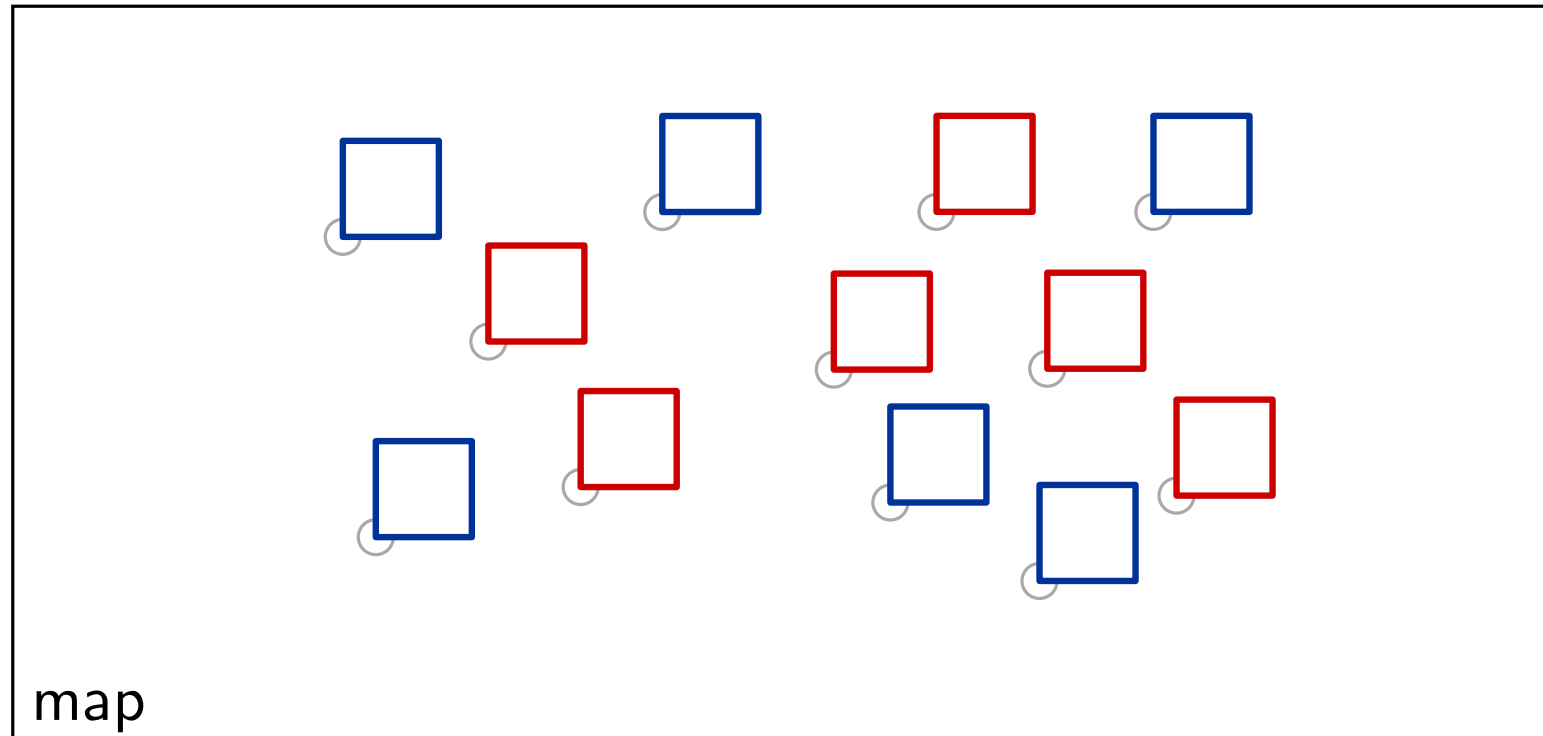
Observation 1:



- split set of labels into two sets

1/4 Approximation of MaxTotal

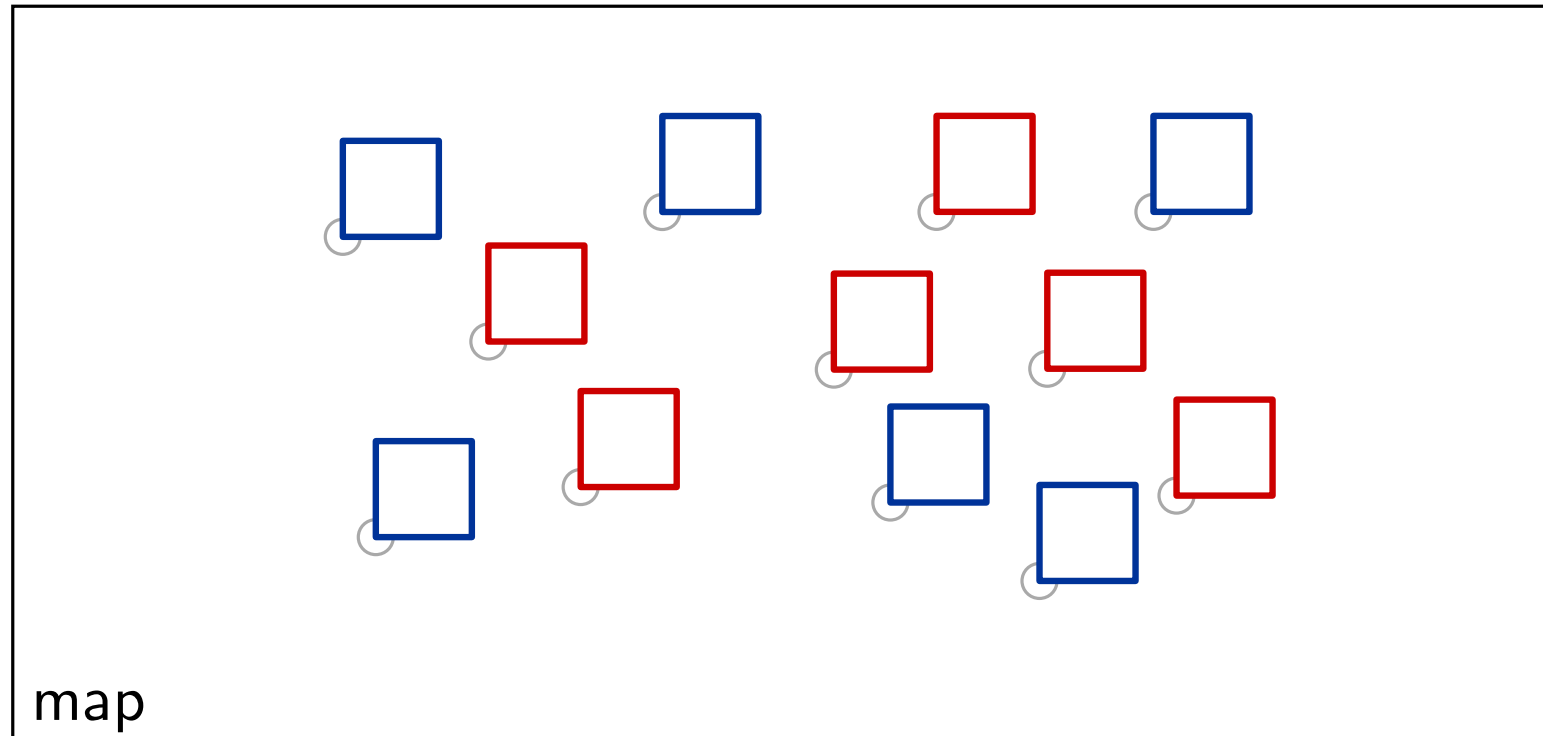
Observation 1:



- split set of labels into two sets
- find optimal solution for each set separately

1/4 Approximation of MaxTotal

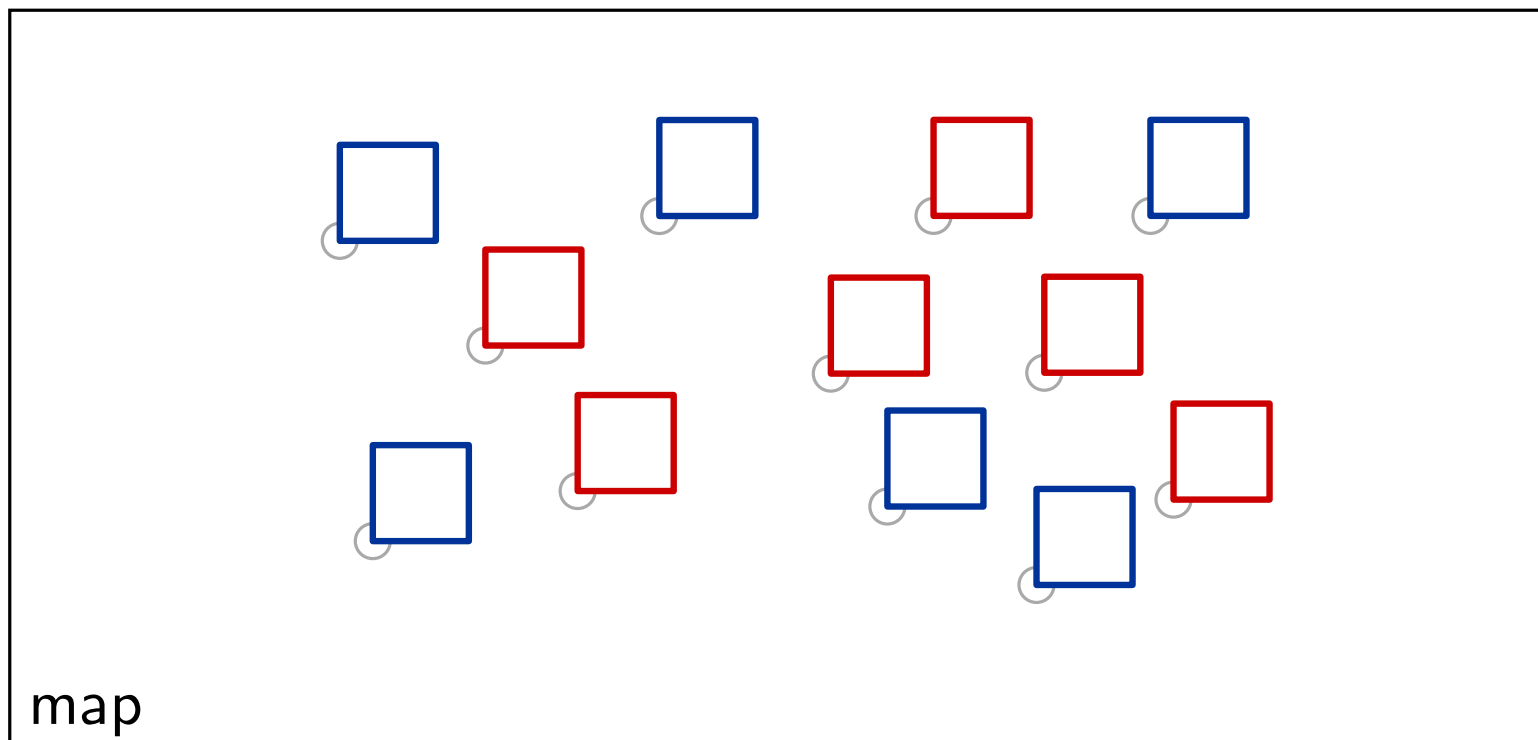
Observation 1:



- split set of labels into two sets
 - find optimal solution for each set separately
- one of those solutions is a $1/2$ -approximation

1/4-Approximation of MaxTotal

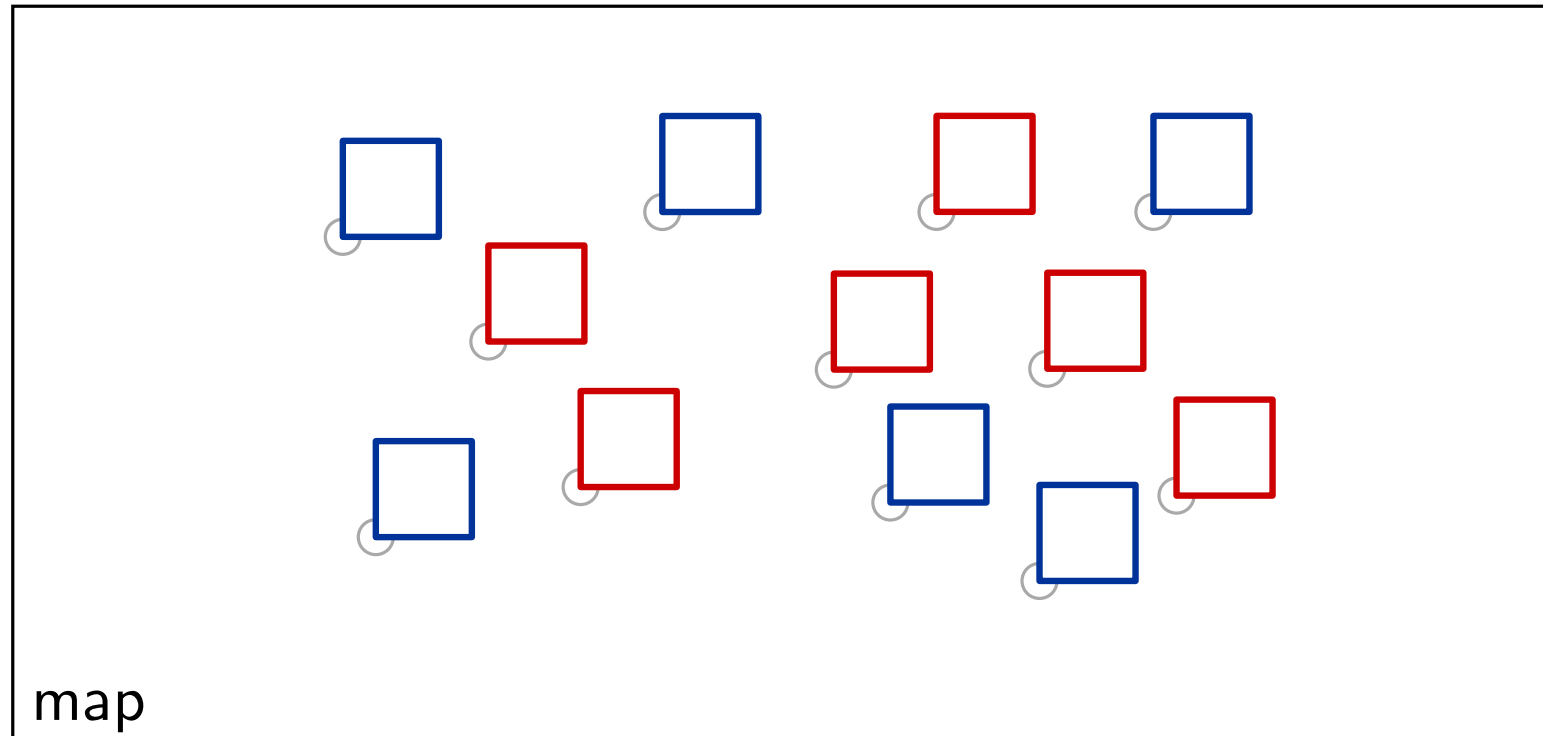
Observation 1:



- split set of labels into **four** sets
 - find optimal solution for each set separately
- one of those solutions is a $1/2$ -approximation

1/4-Approximation of MaxTotal

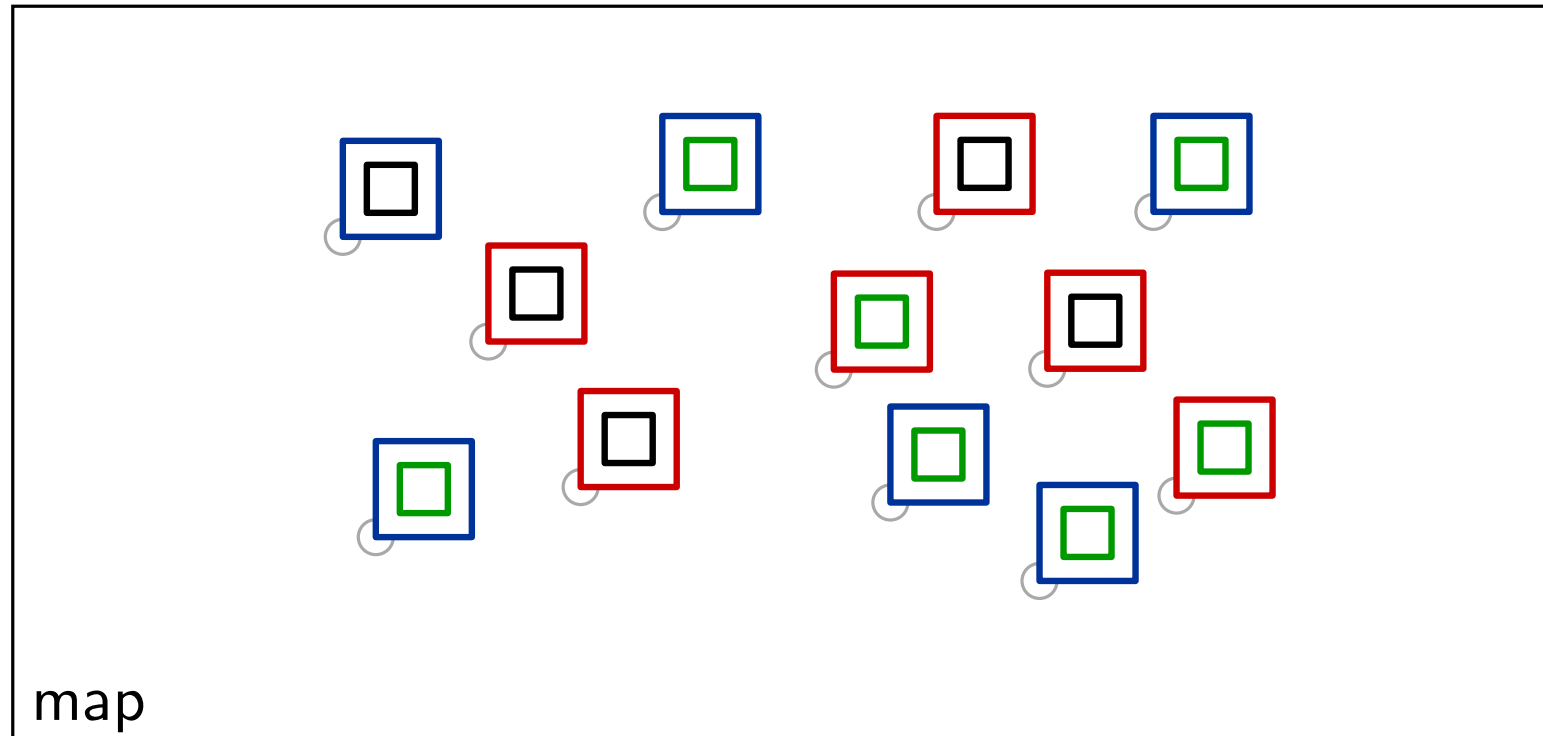
Observation 1:



- split set of labels into **four** sets
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal

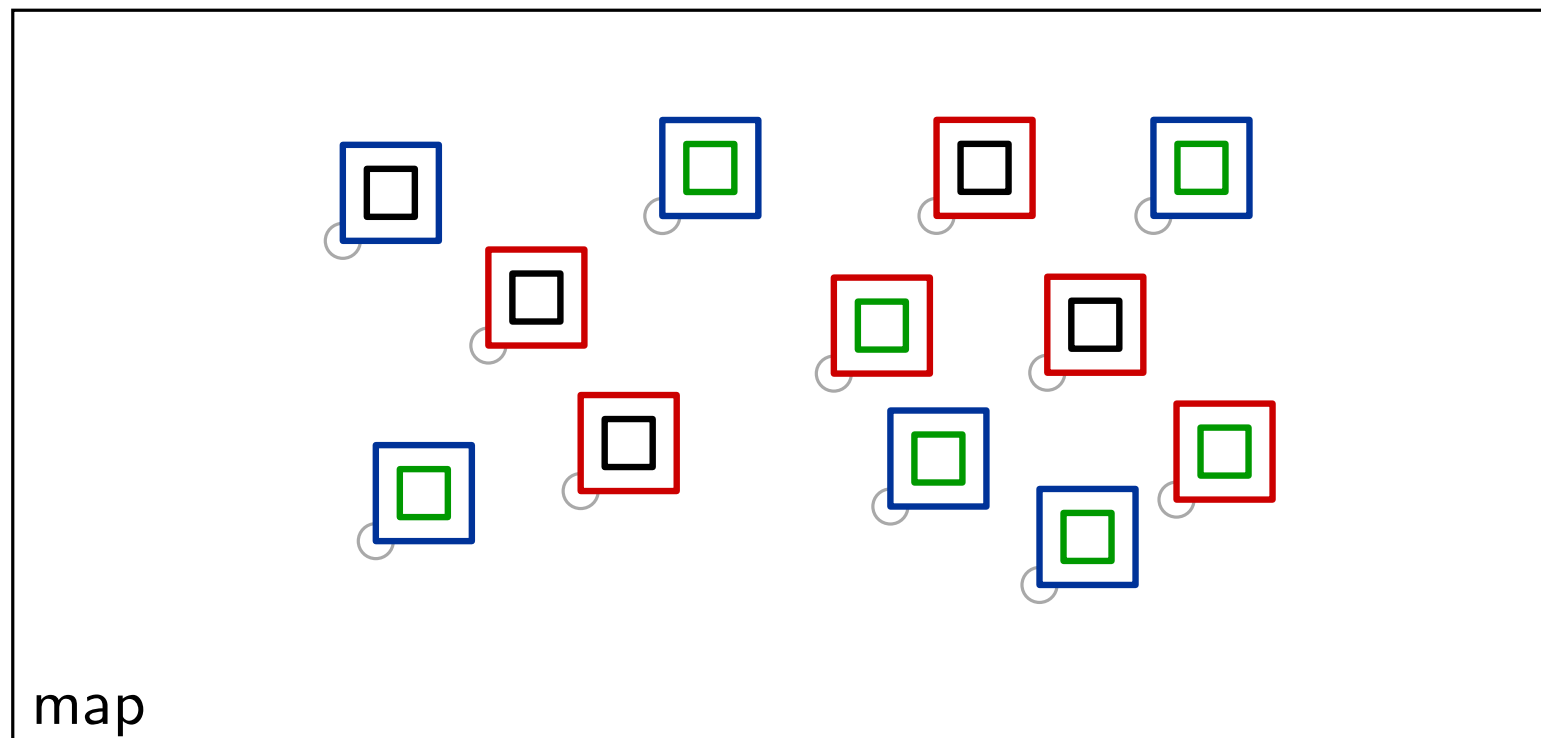
Observation 1:




- split set of labels into **four** sets
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal

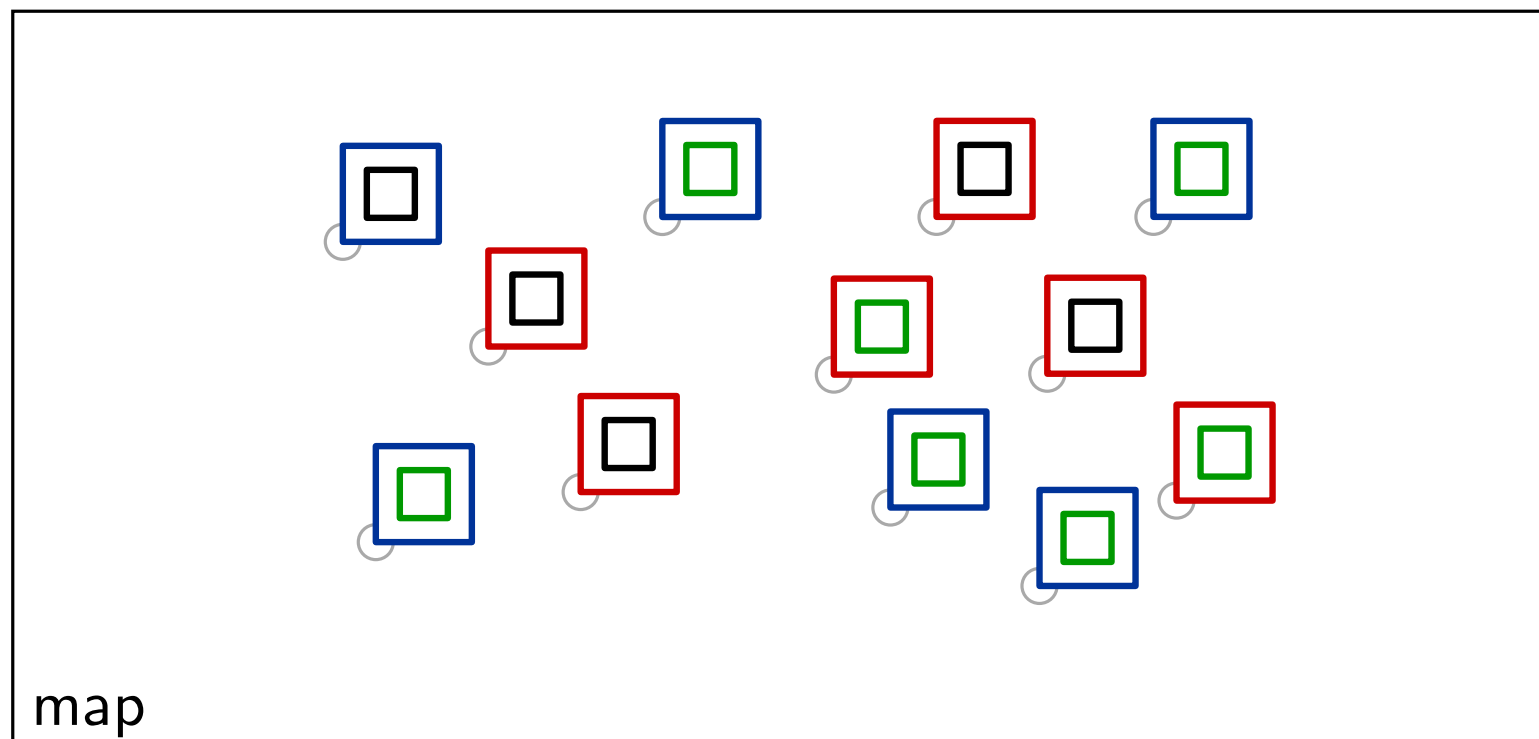
Observation 1:



- split set of labels into **four** sets 
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal

Observation 1:

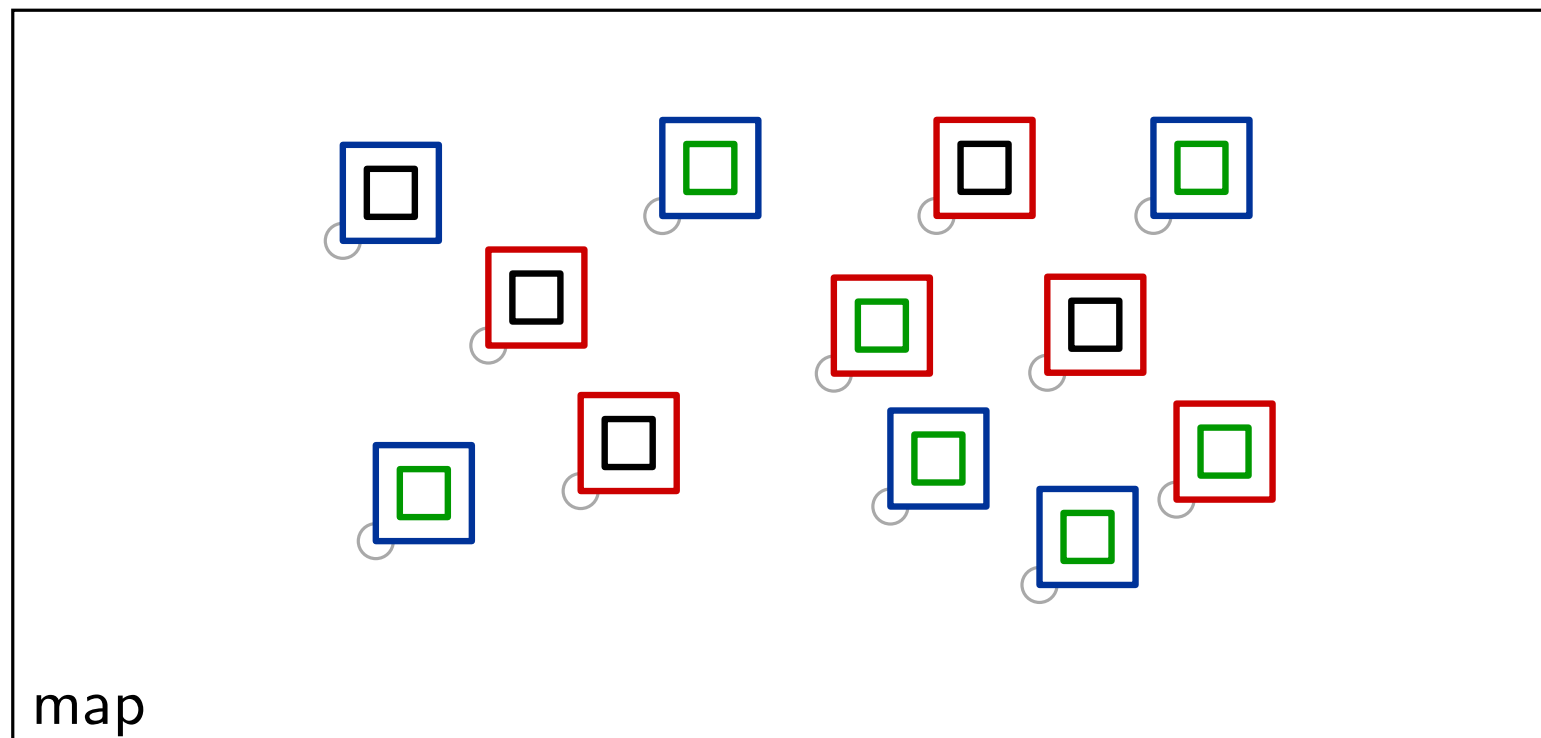


- split set of labels into **four** sets
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation

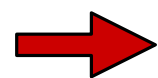


1/4-Approximation of MaxTotal

Observation 1:



■ split set of labels into **four** sets

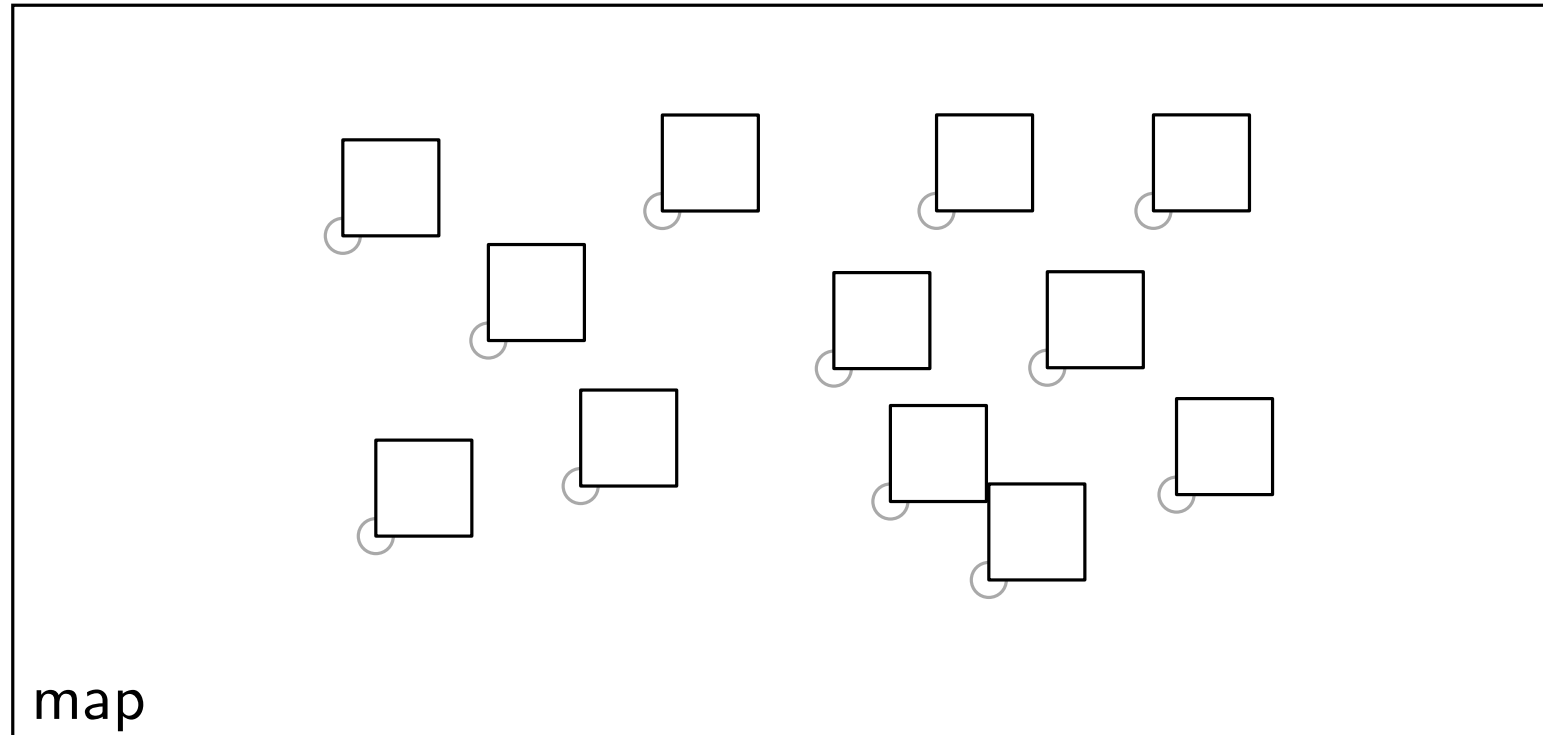


■ find optimal solution for each set separately

one of those solutions is a **1/4**-approximation

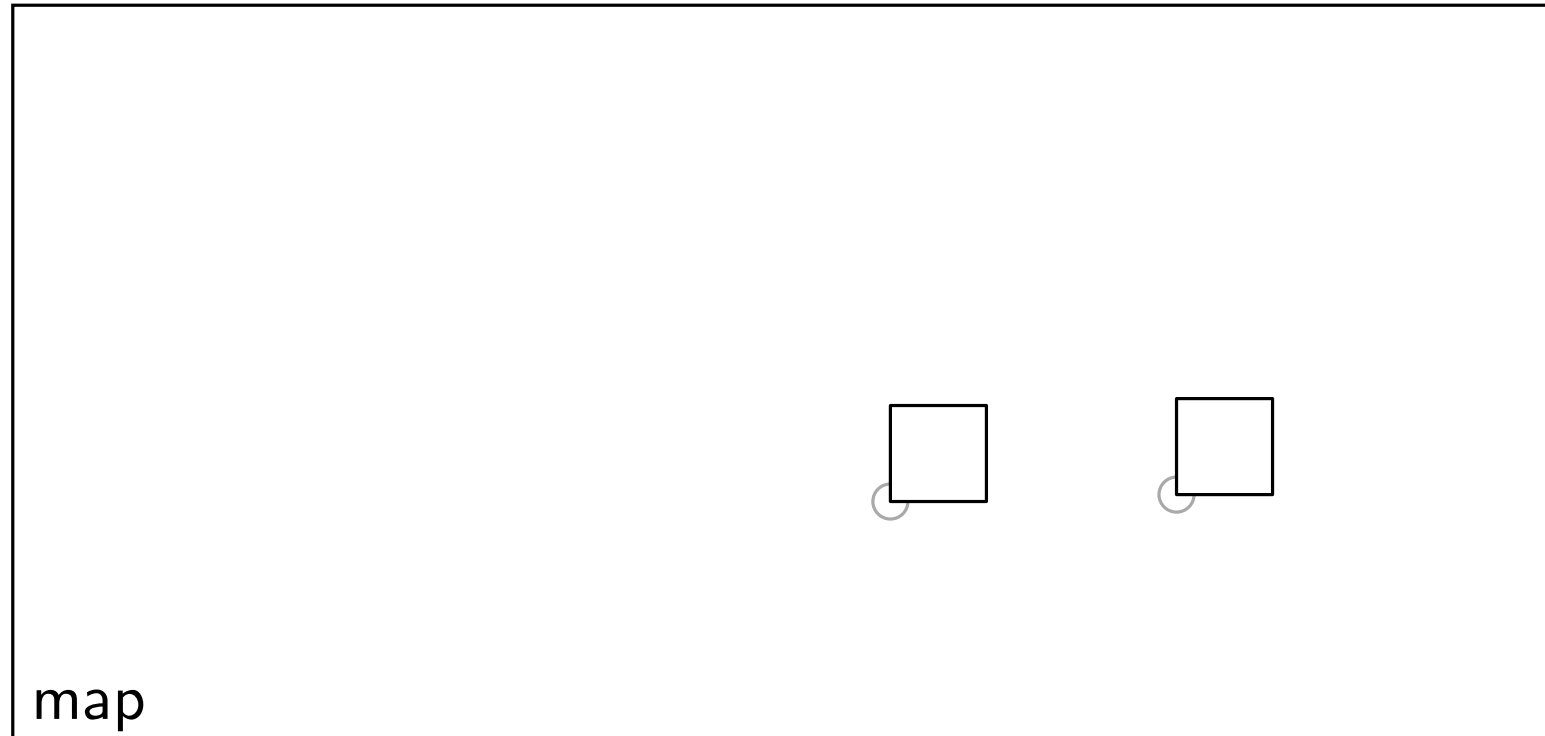
1/4-Approximation of MaxTotal

Observation 2:



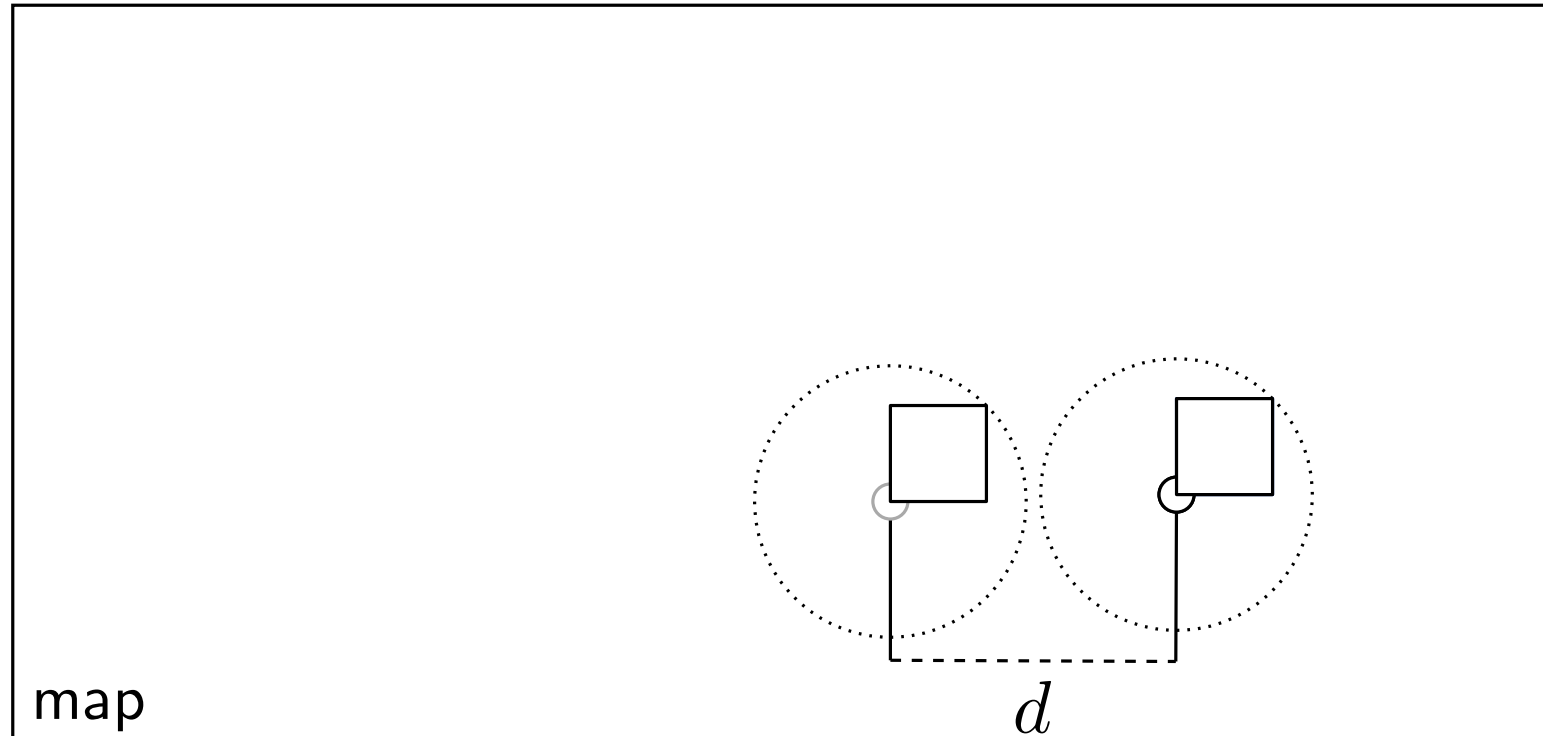
1/4-Approximation of MaxTotal

Observation 2:



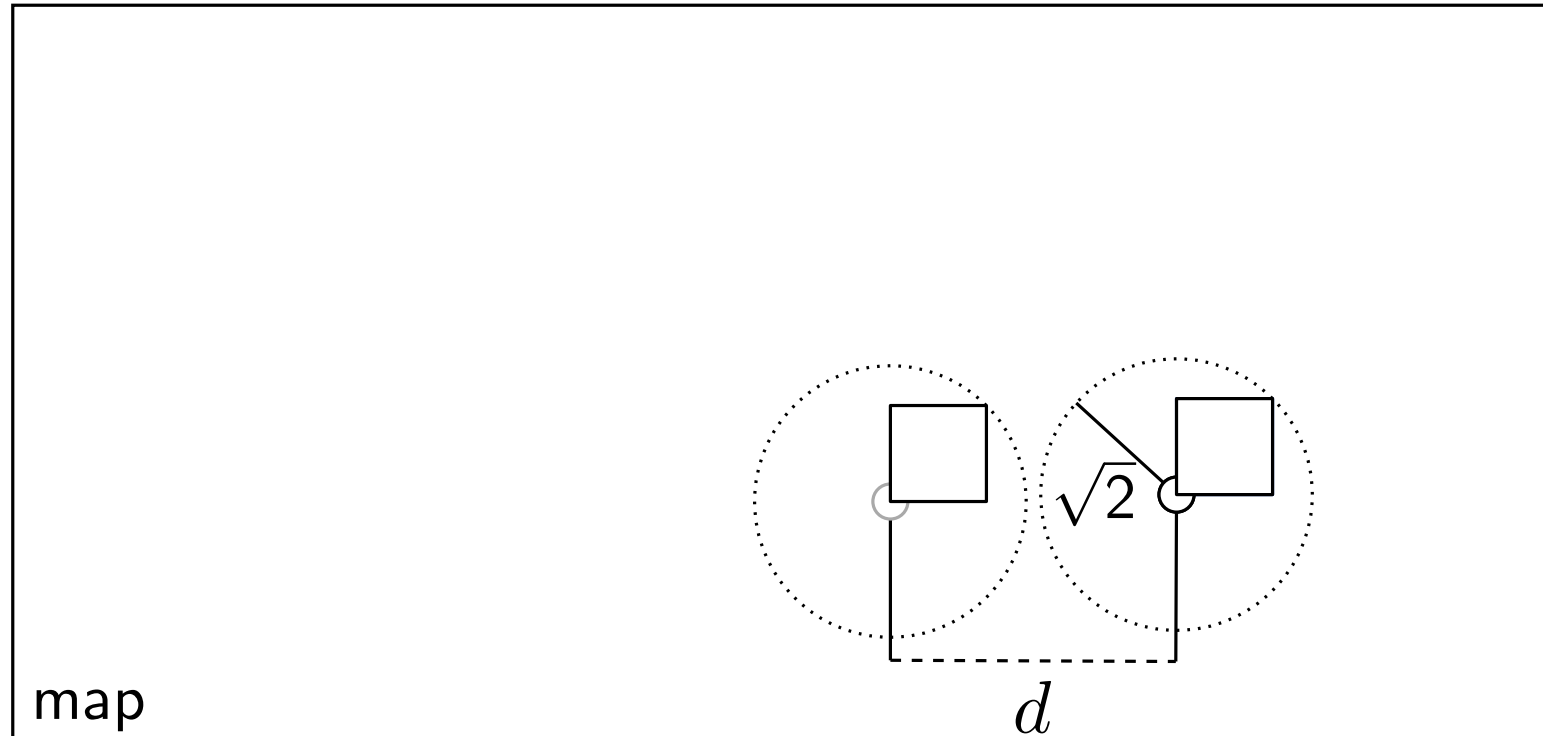
1/4-Approximation of MaxTotal

Observation 2:



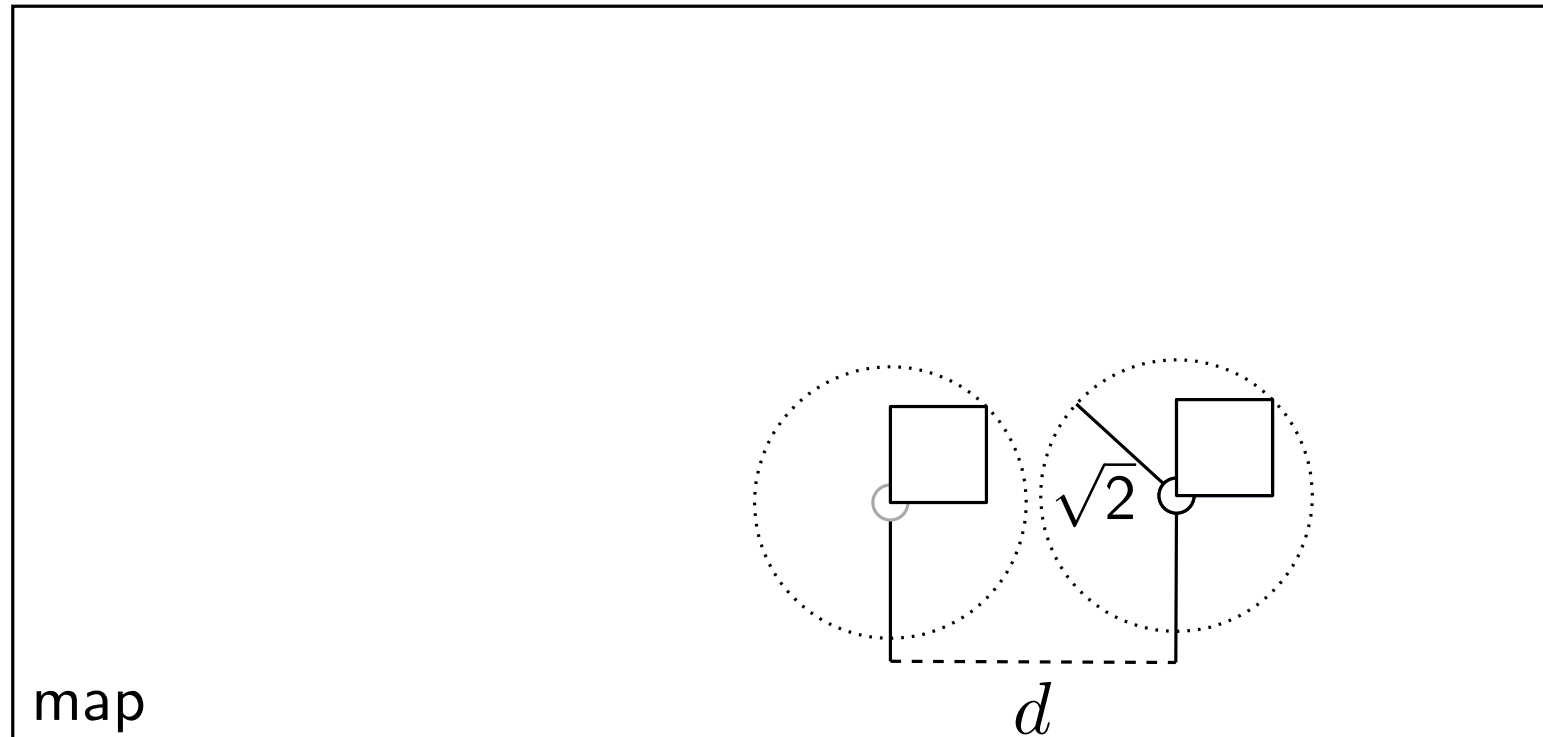
1/4-Approximation of MaxTotal

Observation 2:



1/4-Approximation of MaxTotal

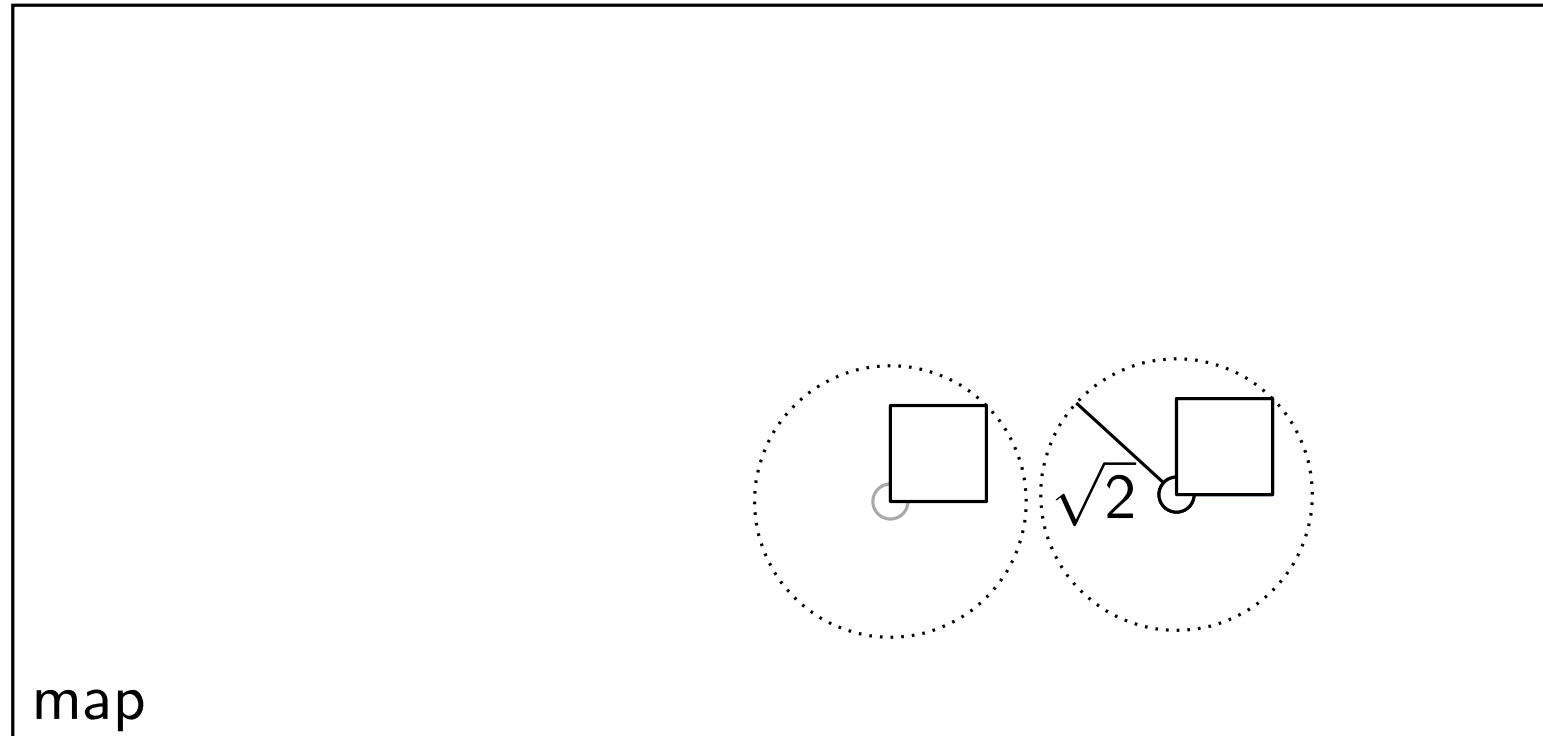
Observation 2:



if $d > 2\sqrt{2}$ then no conflict possible

1/4-Approximation of MaxTotal

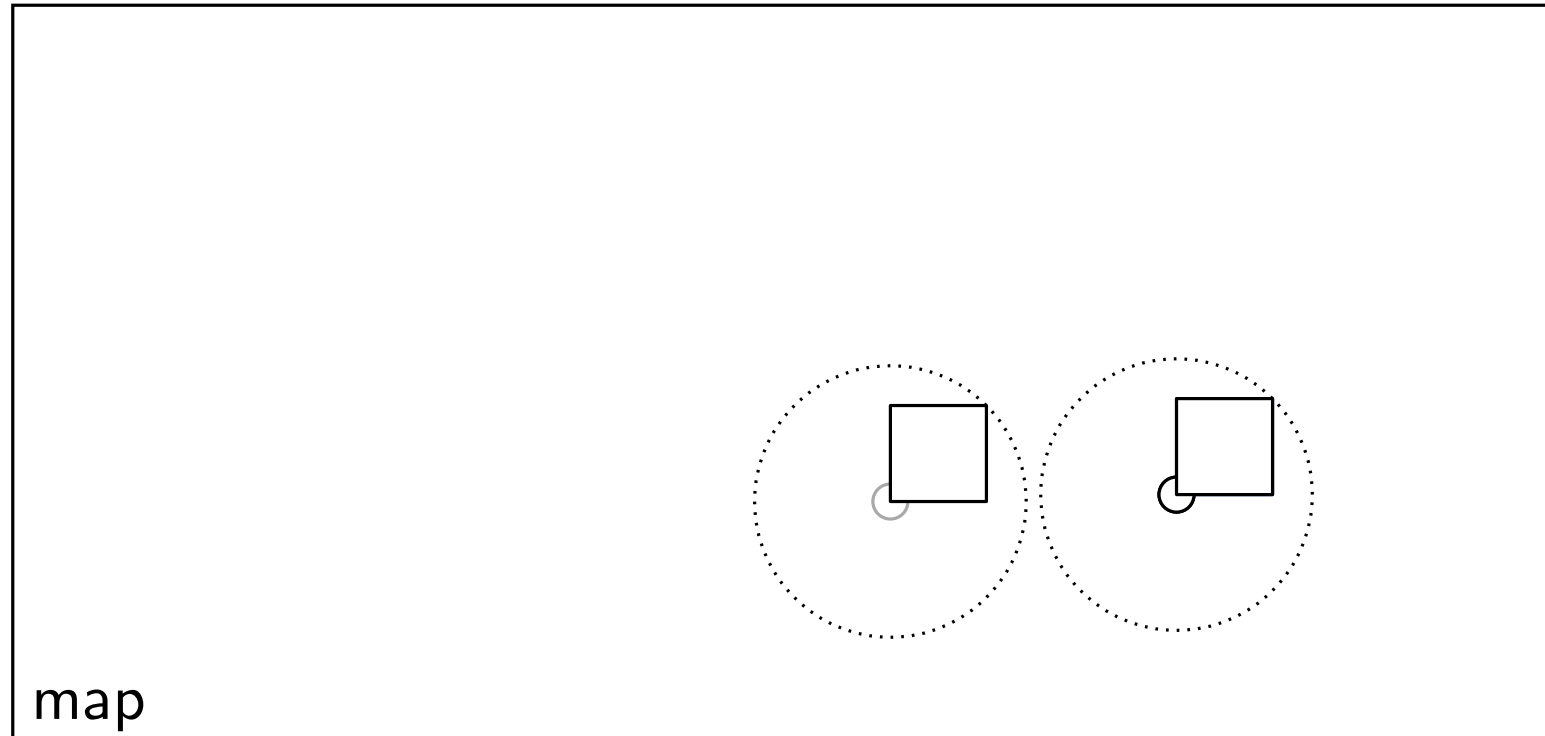
Observation 2:



if $d > 2\sqrt{2}$ then no conflict possible

1/4-Approximation of MaxTotal

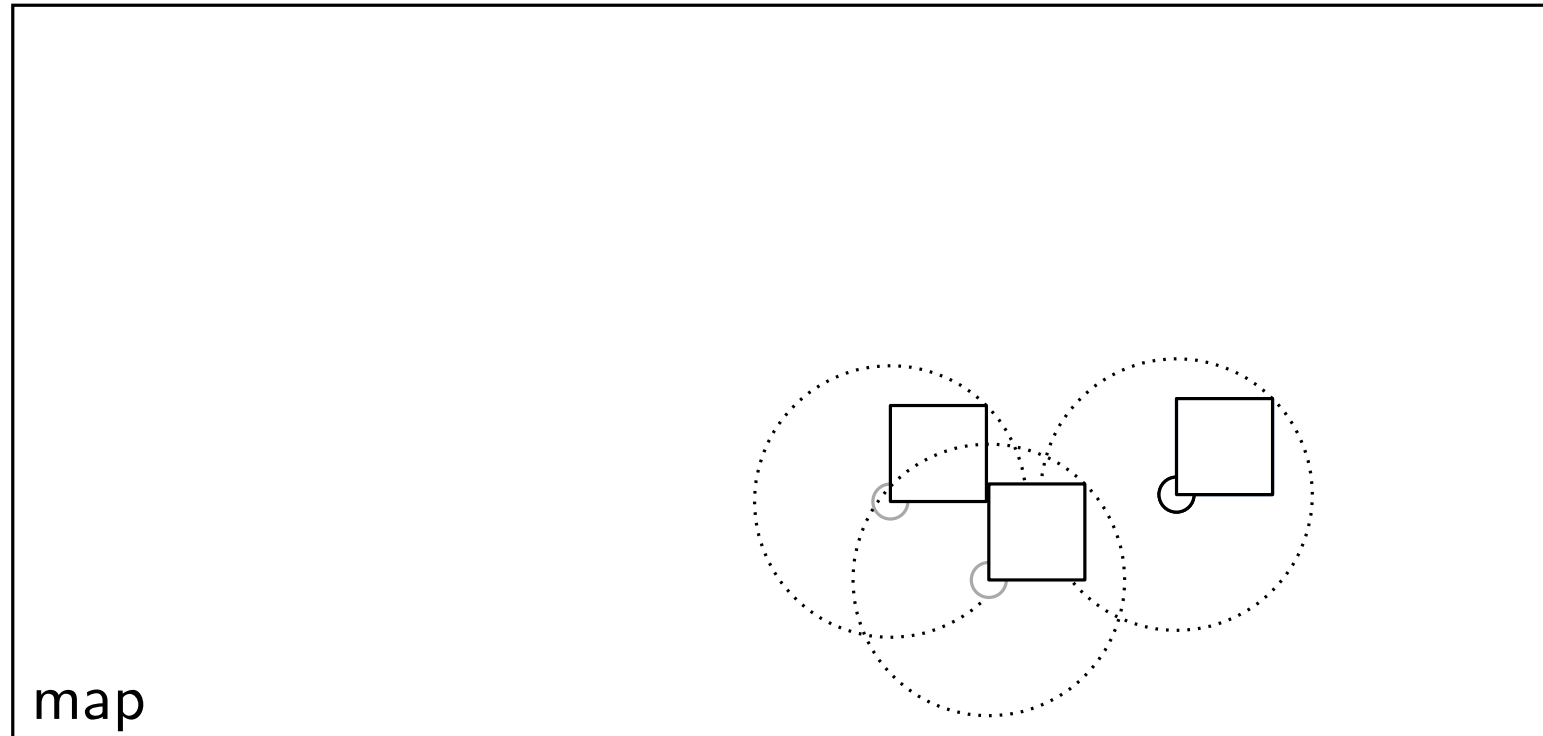
Observation 2:



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1/4-Approximation of MaxTotal

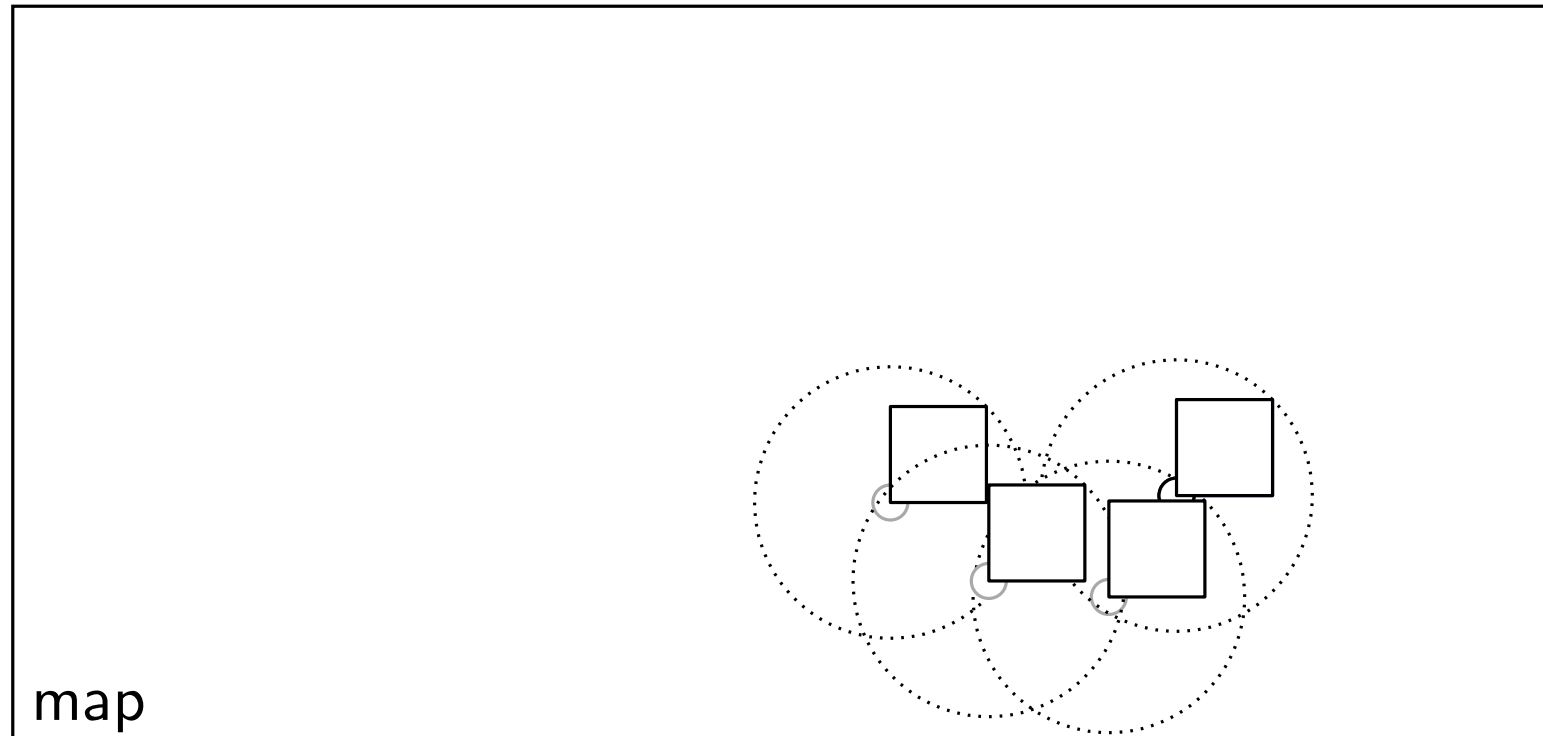
Observation 2:



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1/4-Approximation of MaxTotal

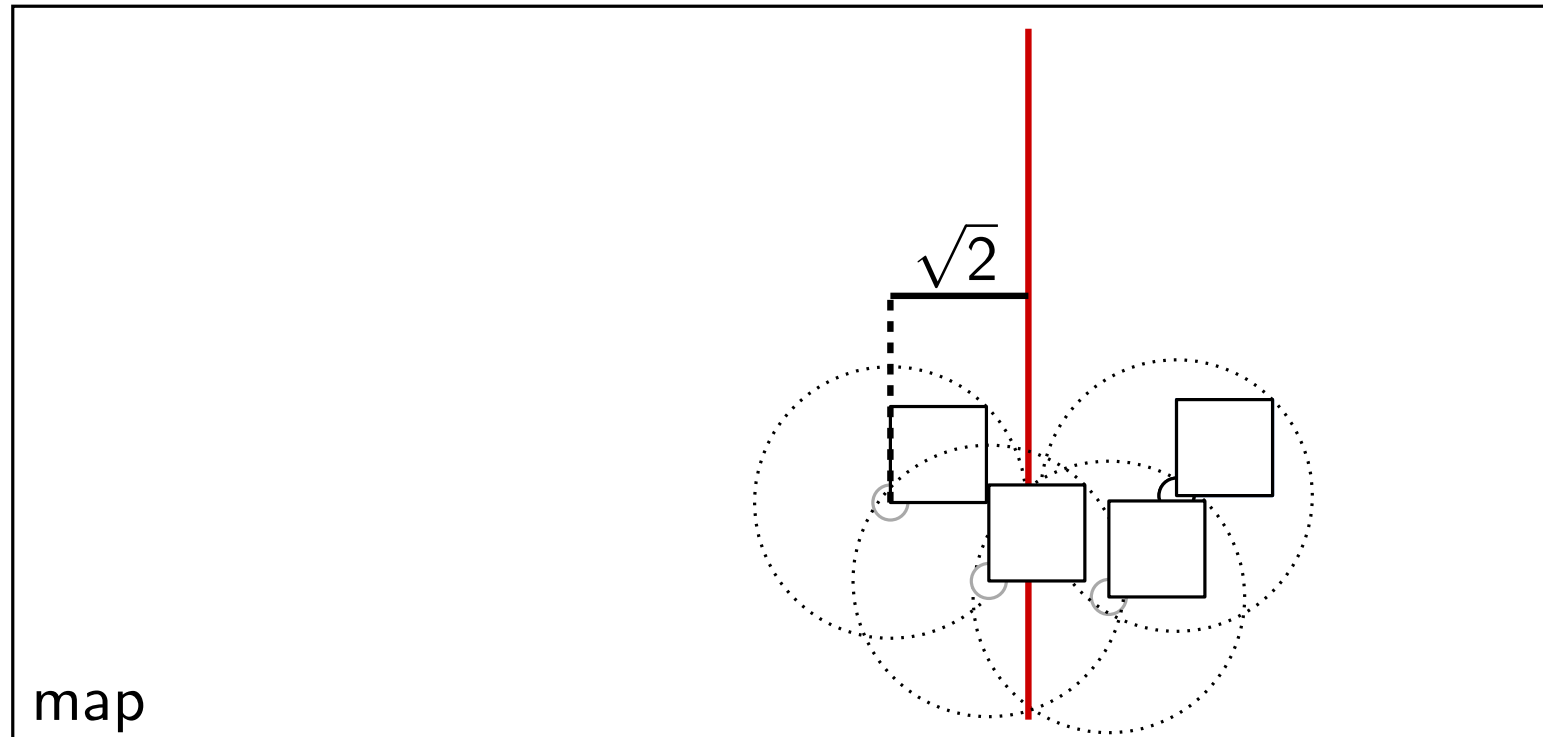
Observation 2:



if $d > 2\sqrt{2}$ then no conflict possible

1/4-Approximation of MaxTotal

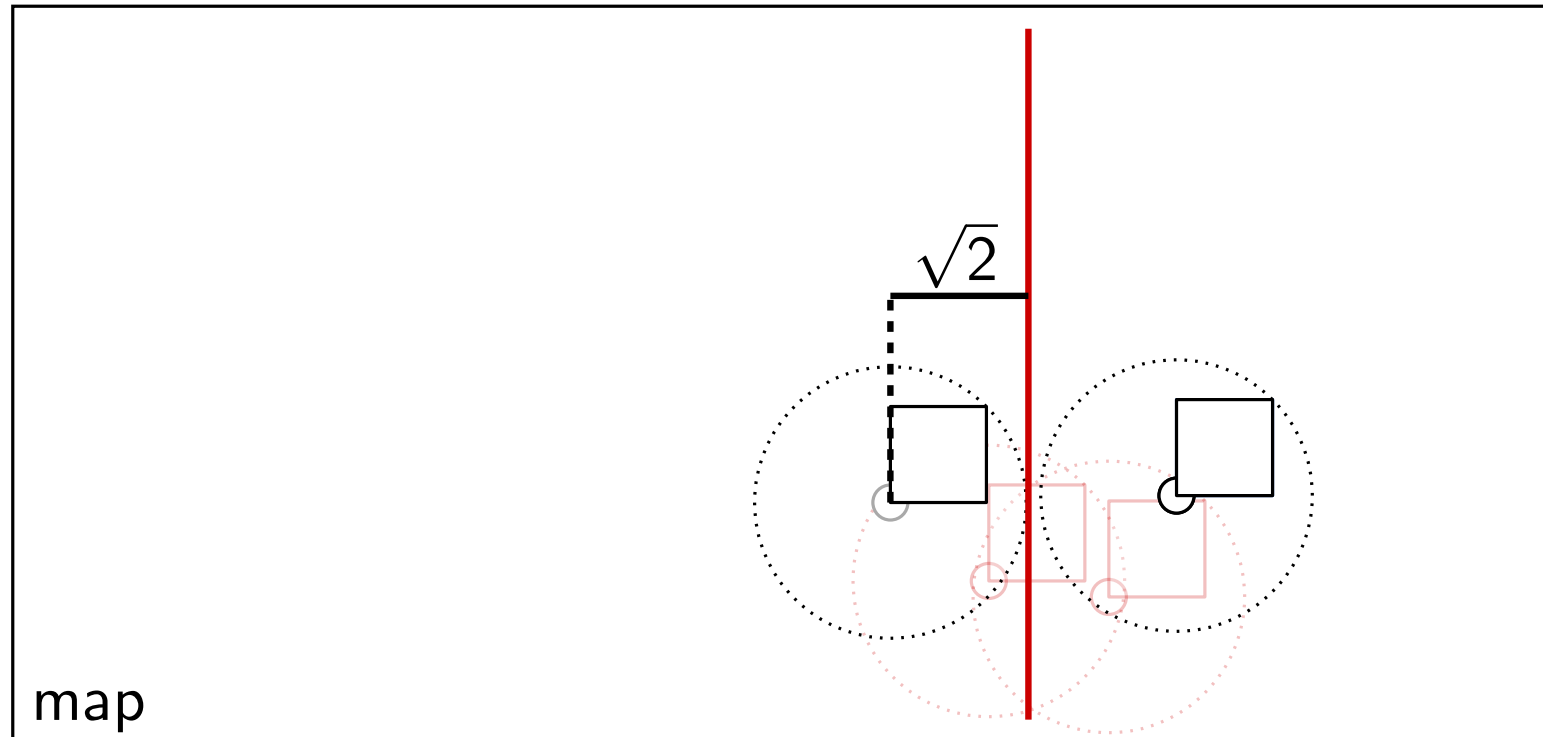
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1/4-Approximation of MaxTotal

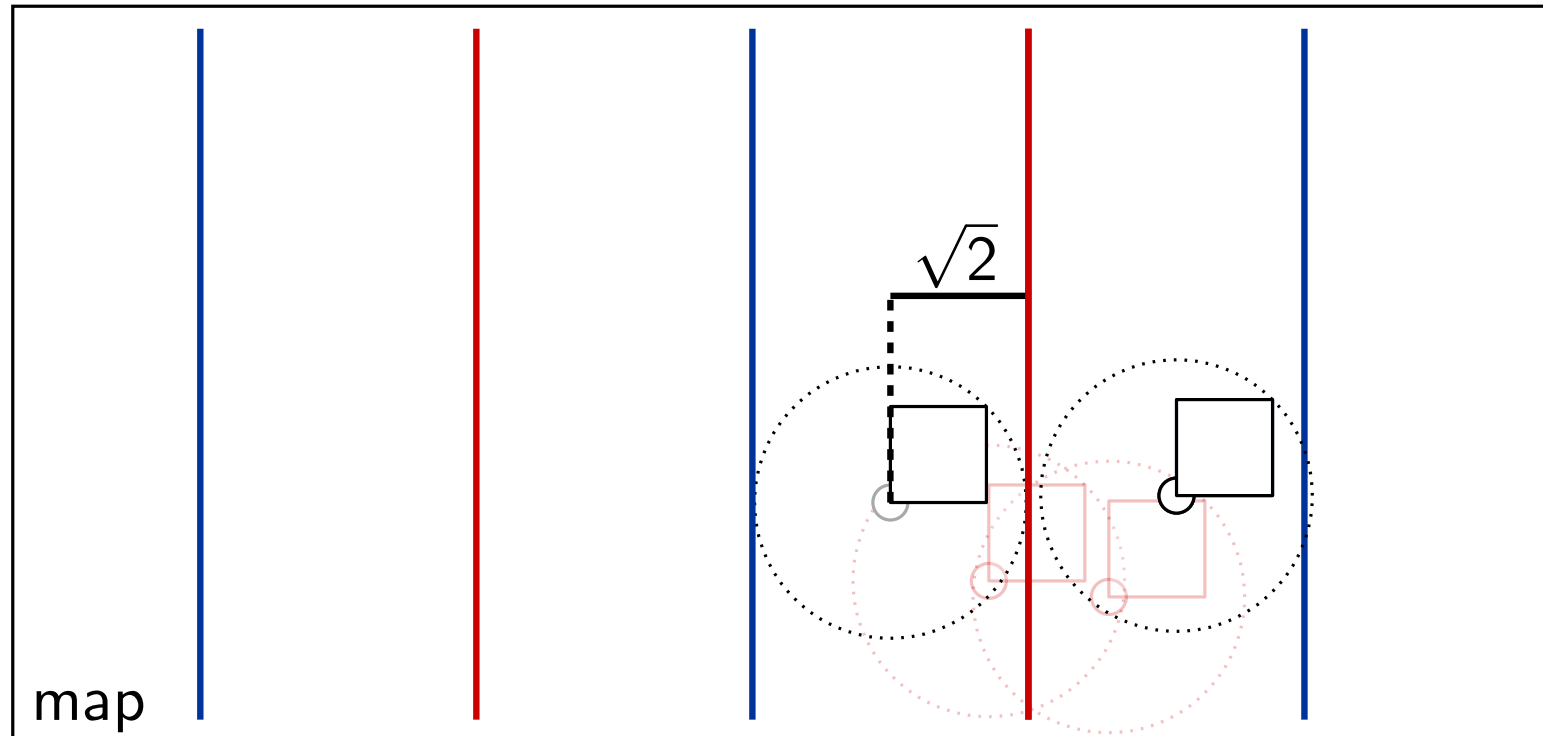
Observation 2:



if $d > 2\sqrt{2}$ then no conflict possible

1/4-Approximation of MaxTotal

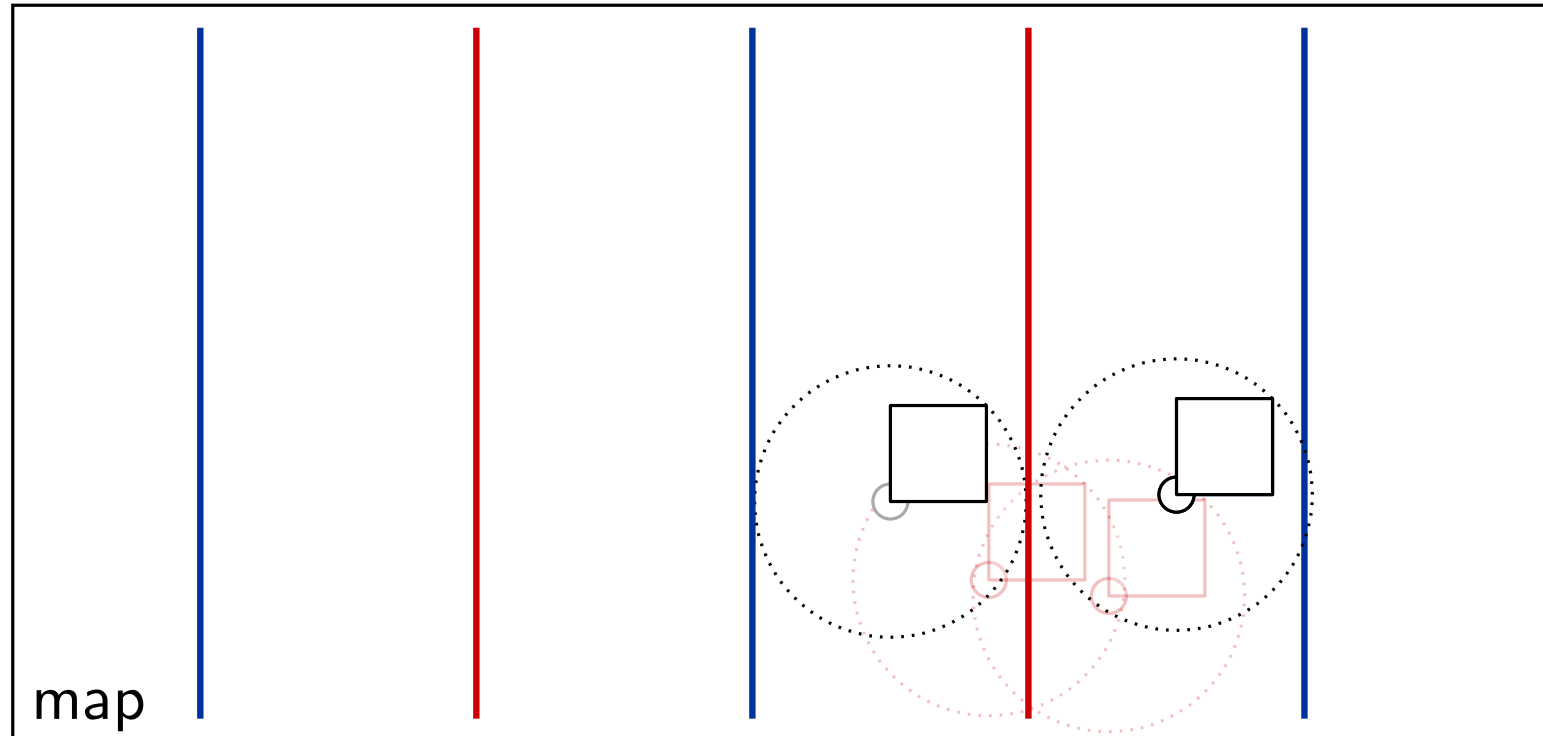
Observation 2:



if $d > 2\sqrt{2}$ then no conflict possible

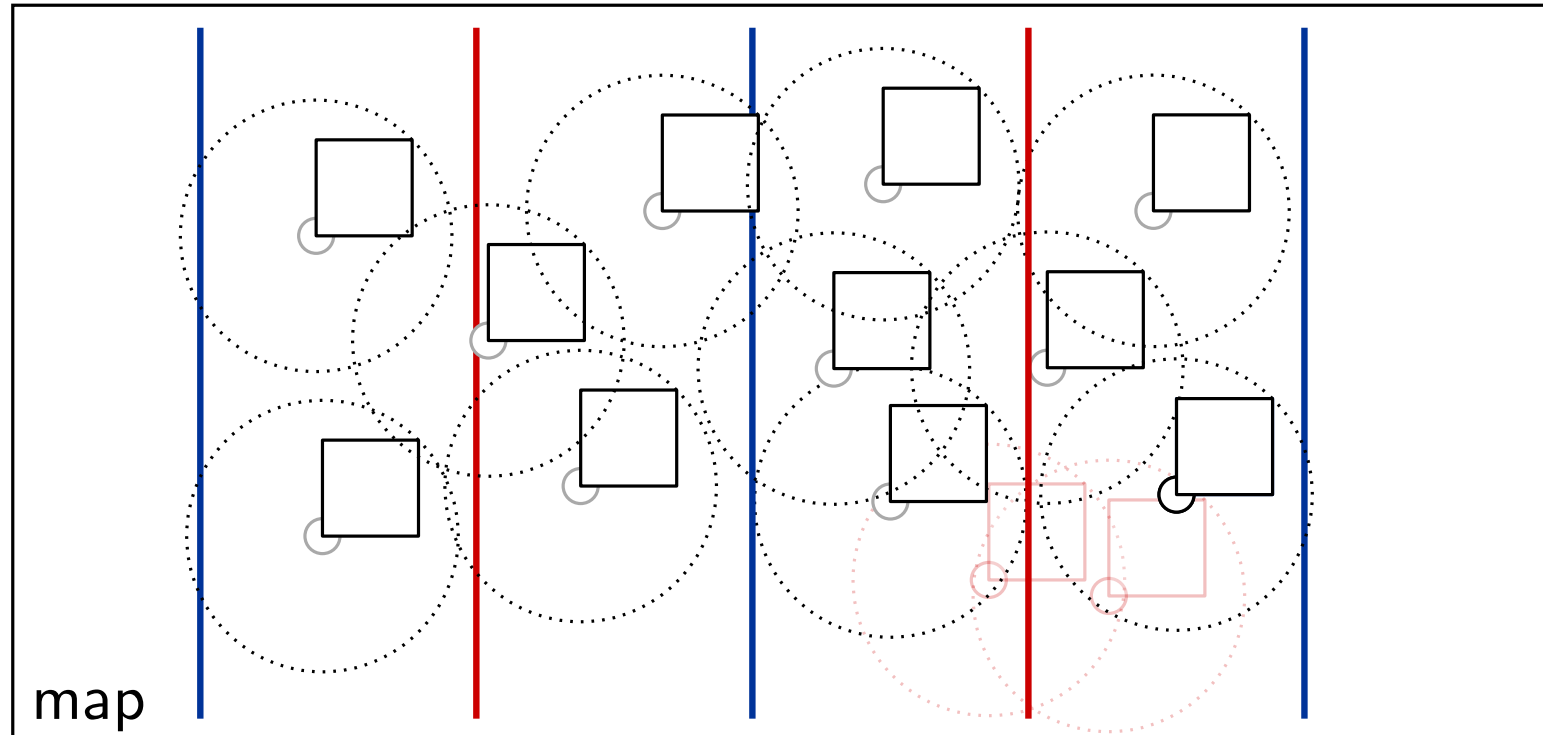
1/4-Approximation of MaxTotal

Observation 2:



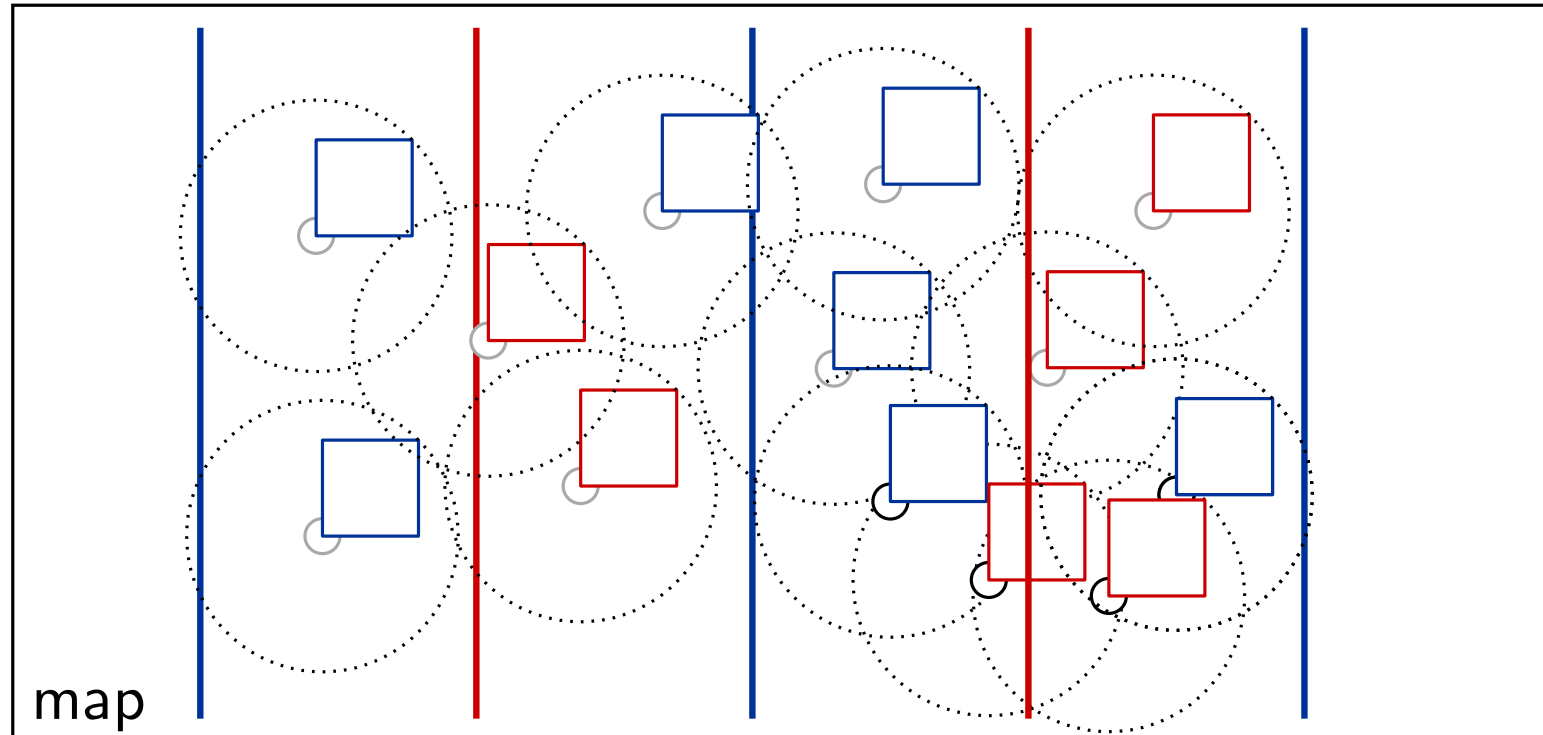
1/4-Approximation of MaxTotal

Observation 2:



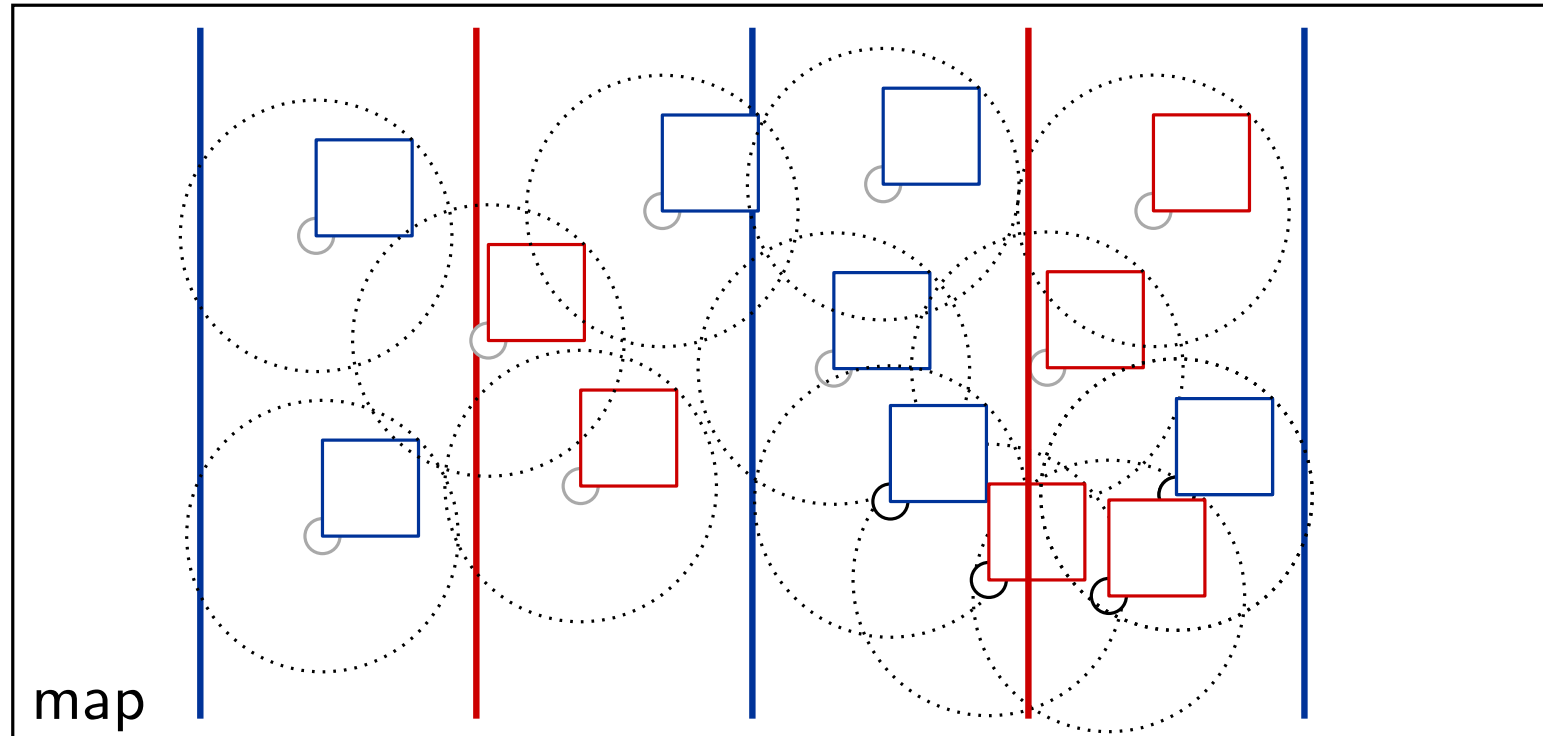
1/4-Approximation of MaxTotal

Observation 2:



1/4-Approximation of MaxTotal

Observation 2:

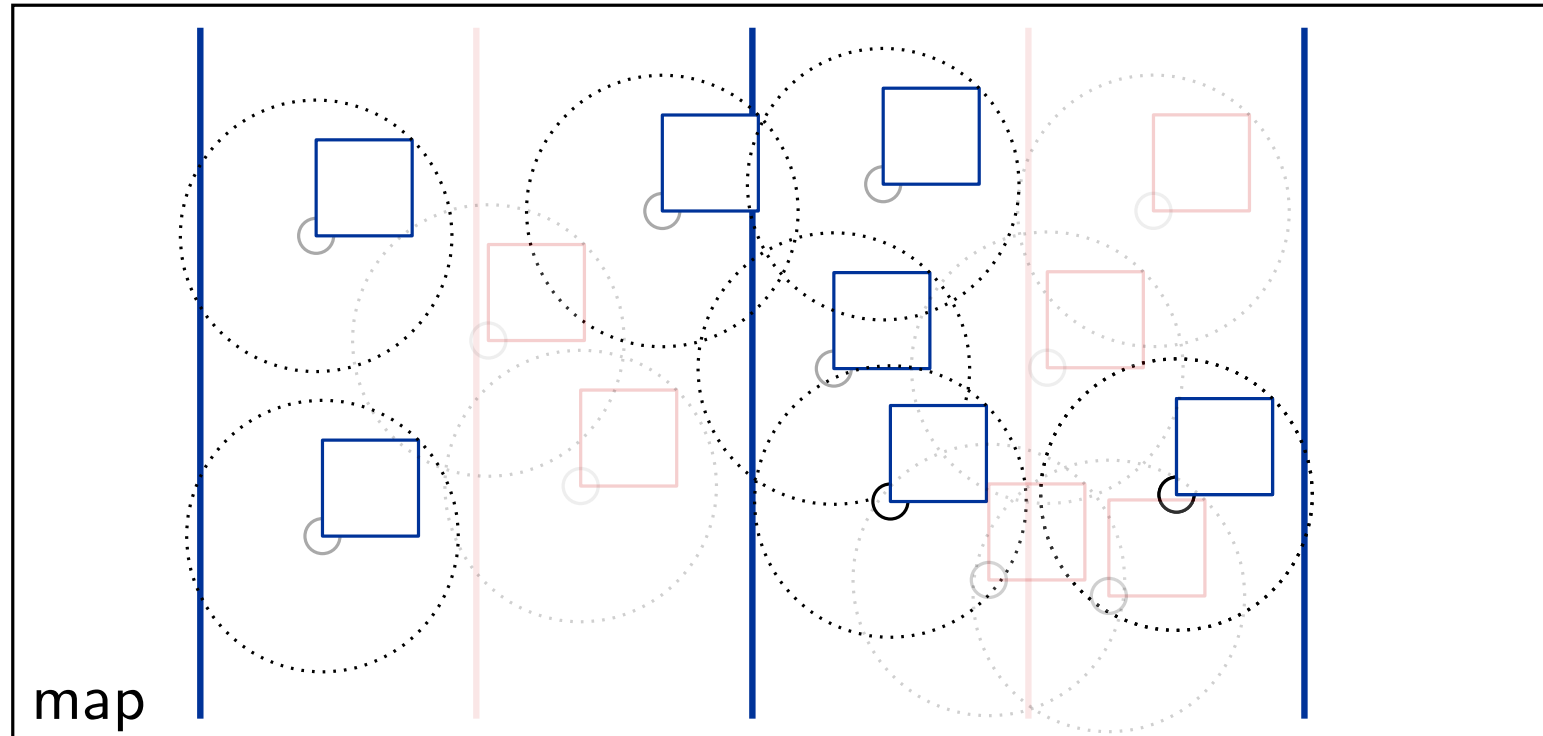


- split set of labels into two sets

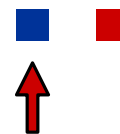


1/4-Approximation of MaxTotal

Observation 2:

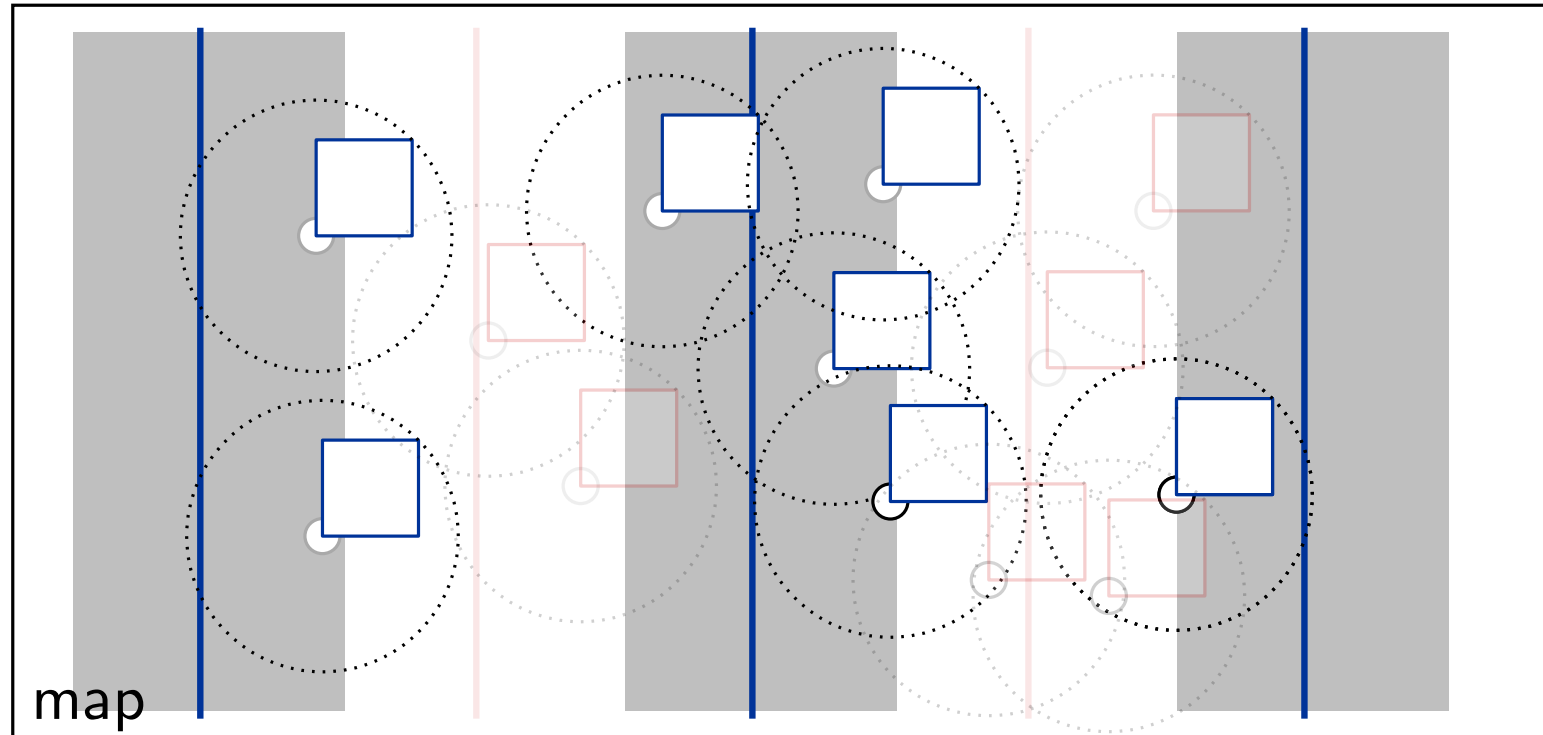


- split set of labels into two sets



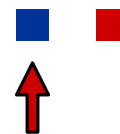
1/4-Approximation of MaxTotal

Observation 2:



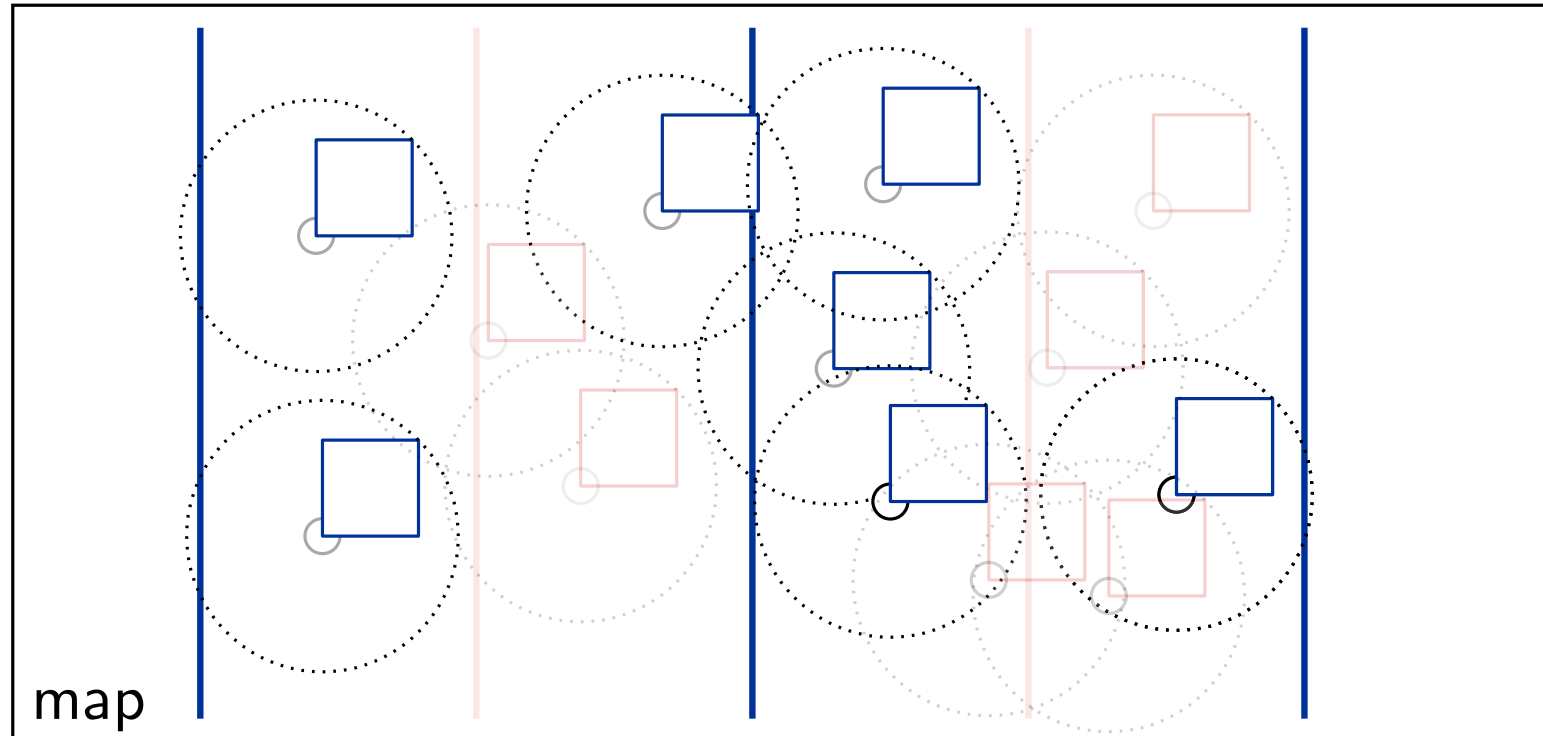
map

- split set of labels into two sets

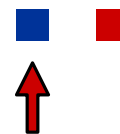


1/4-Approximation of MaxTotal

Observation 2:

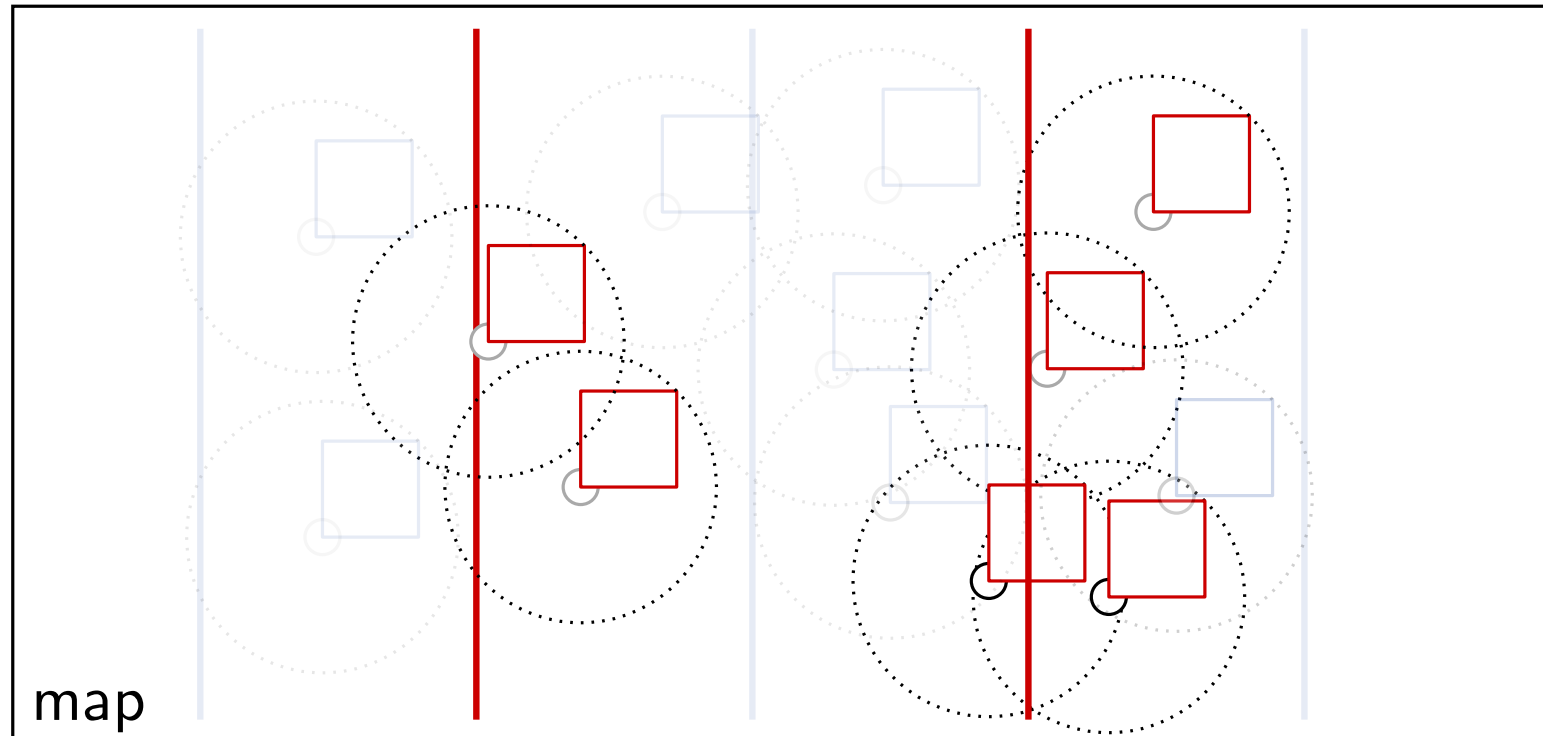


- split set of labels into two sets

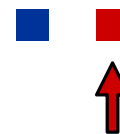


1/4-Approximation of MaxTotal

Observation 2:

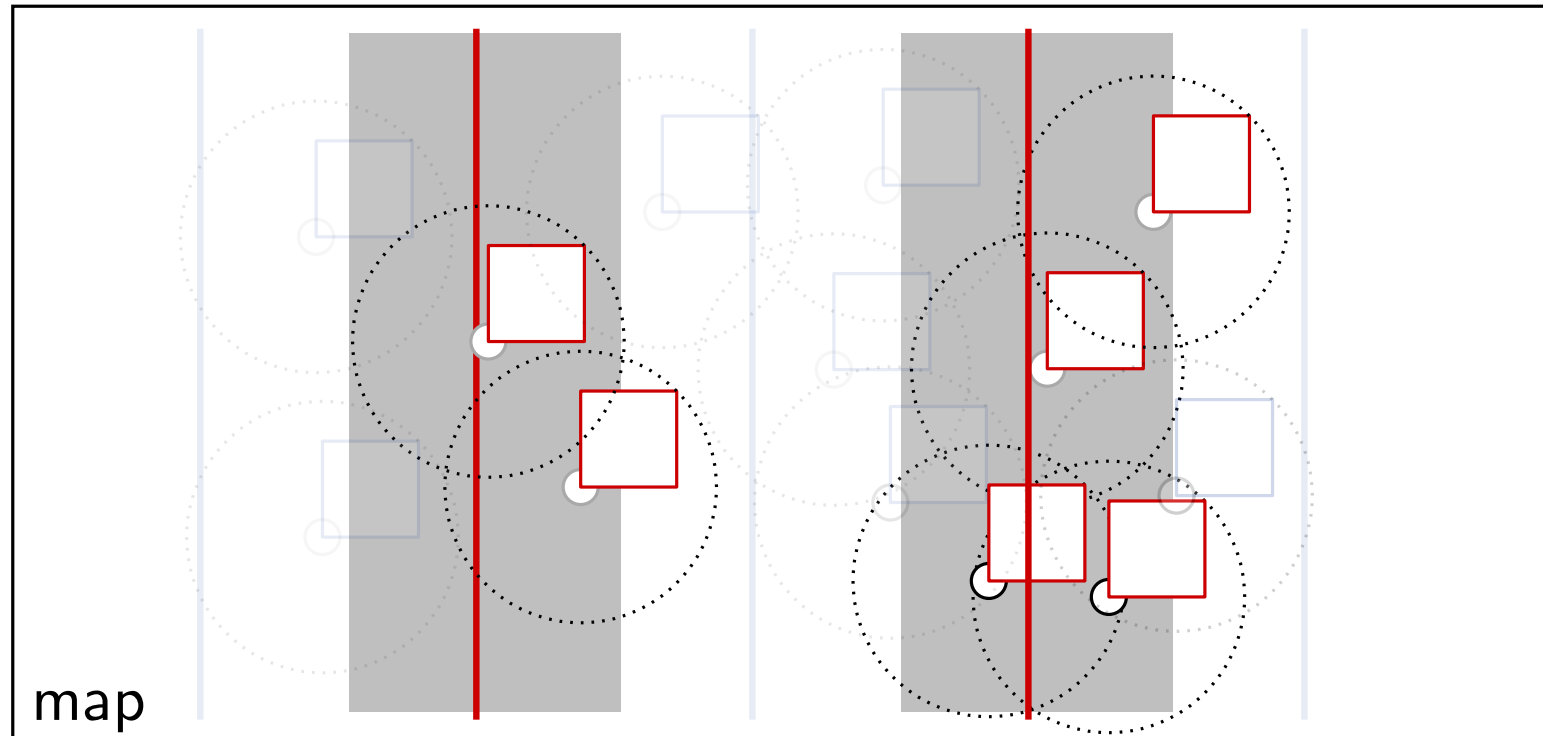


- split set of labels into two sets

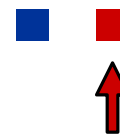


1/4-Approximation of MaxTotal

Observation 2:

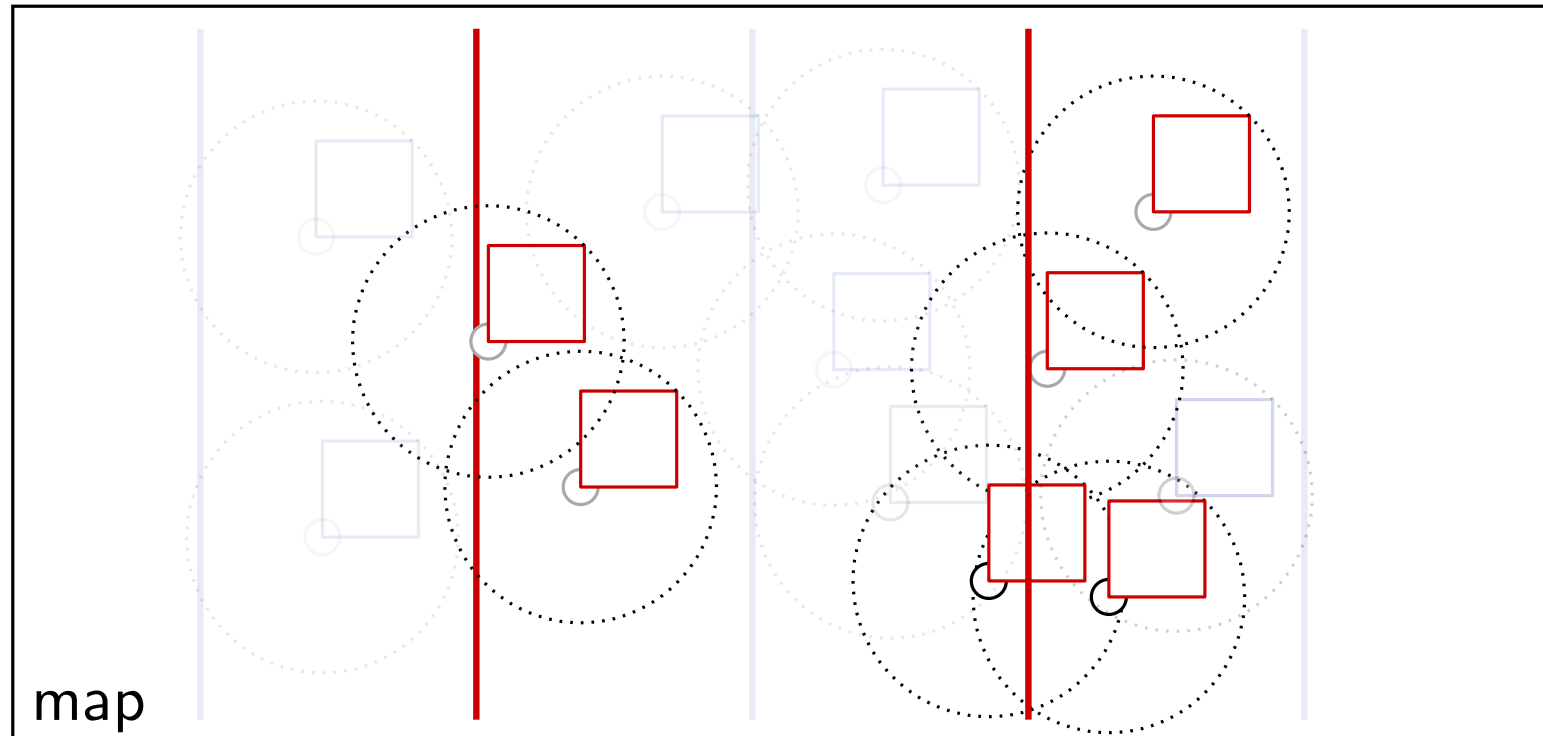


- split set of labels into two sets

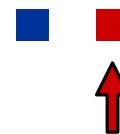


1/4-Approximation of MaxTotal

Observation 2:

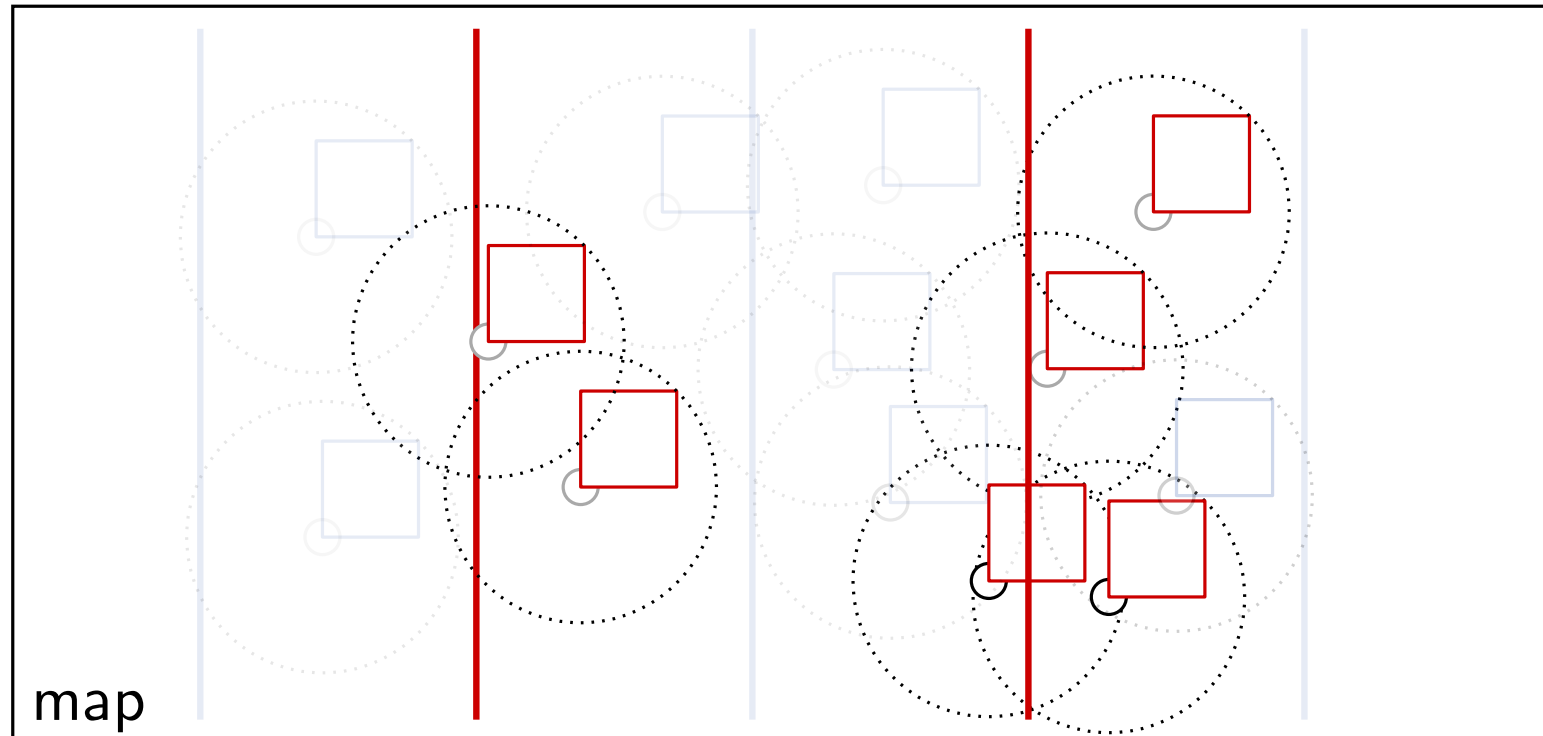


- split set of labels into two sets



1/4-Approximation of MaxTotal

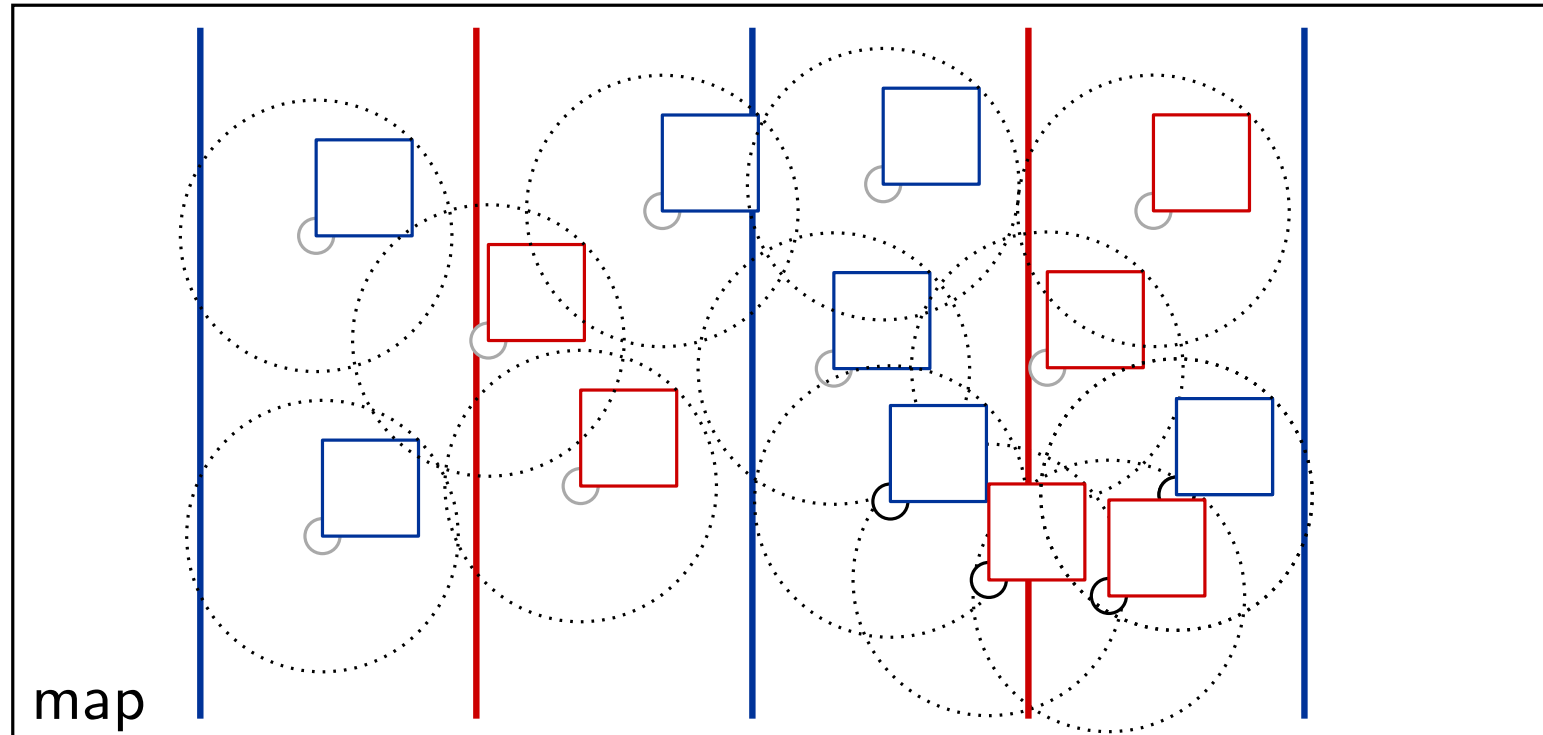
Observation 2:



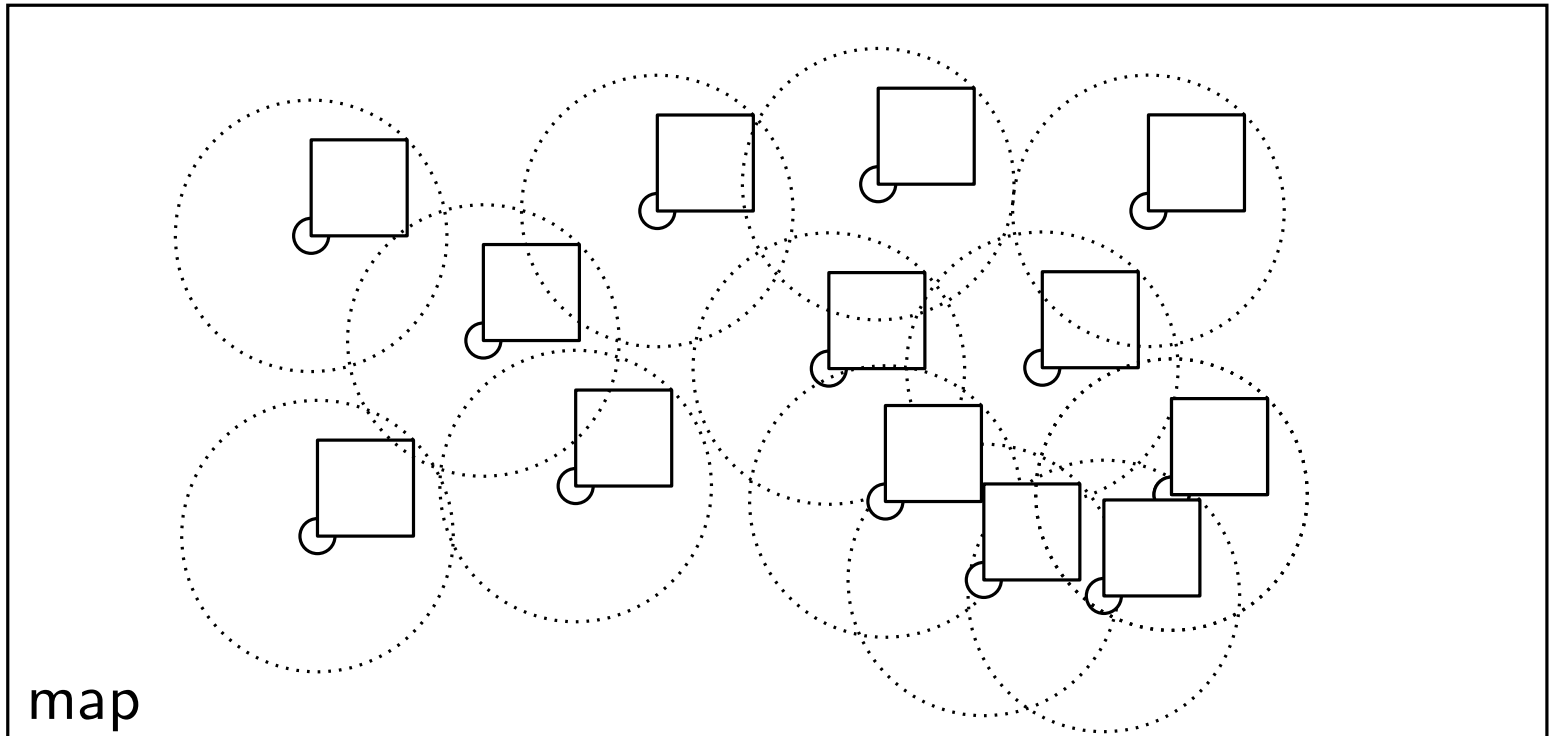
- split set of labels into two sets
 - find optimal solution for each set separately
- one of those solutions is $1/2$ approximation



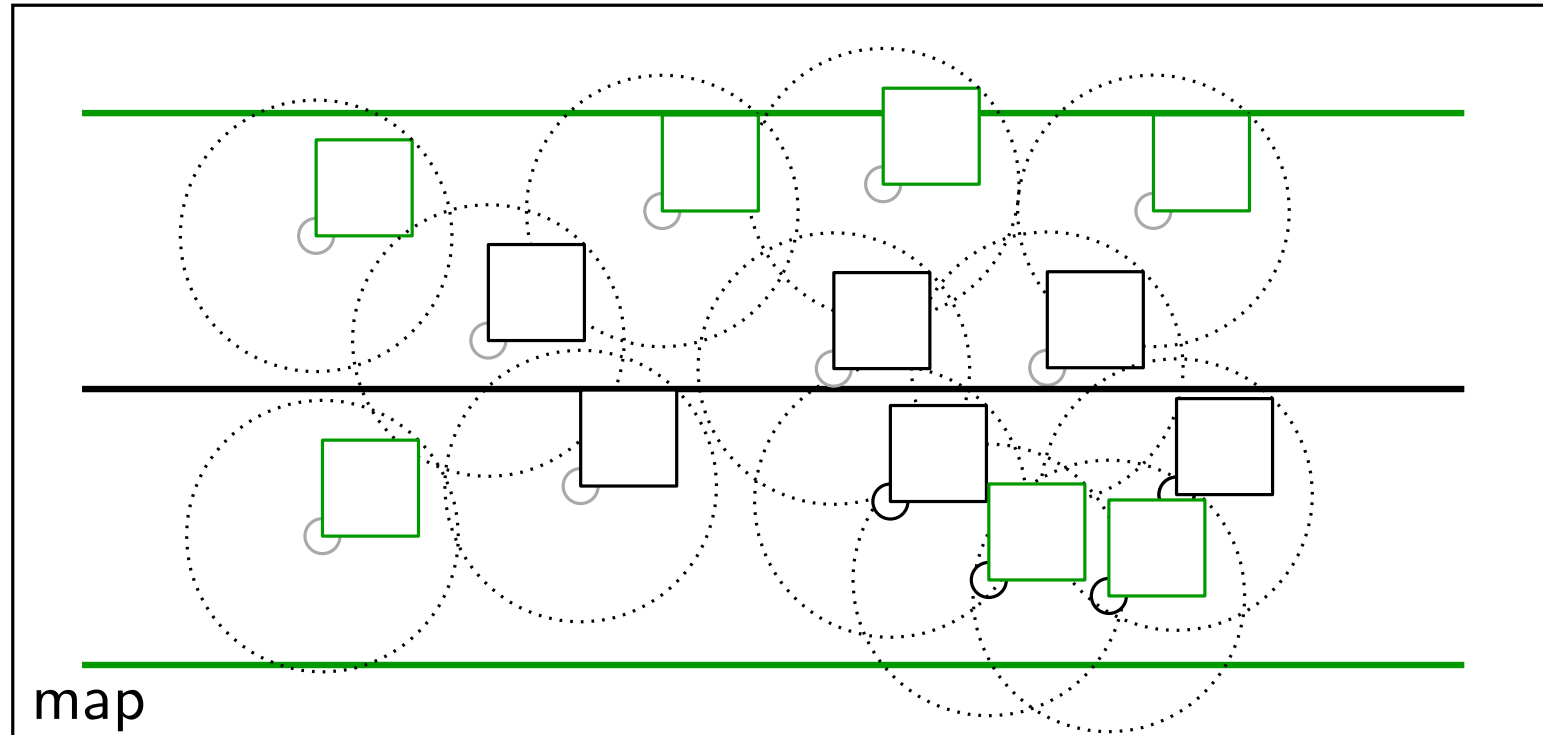
1/4-Approximation of MaxTotal



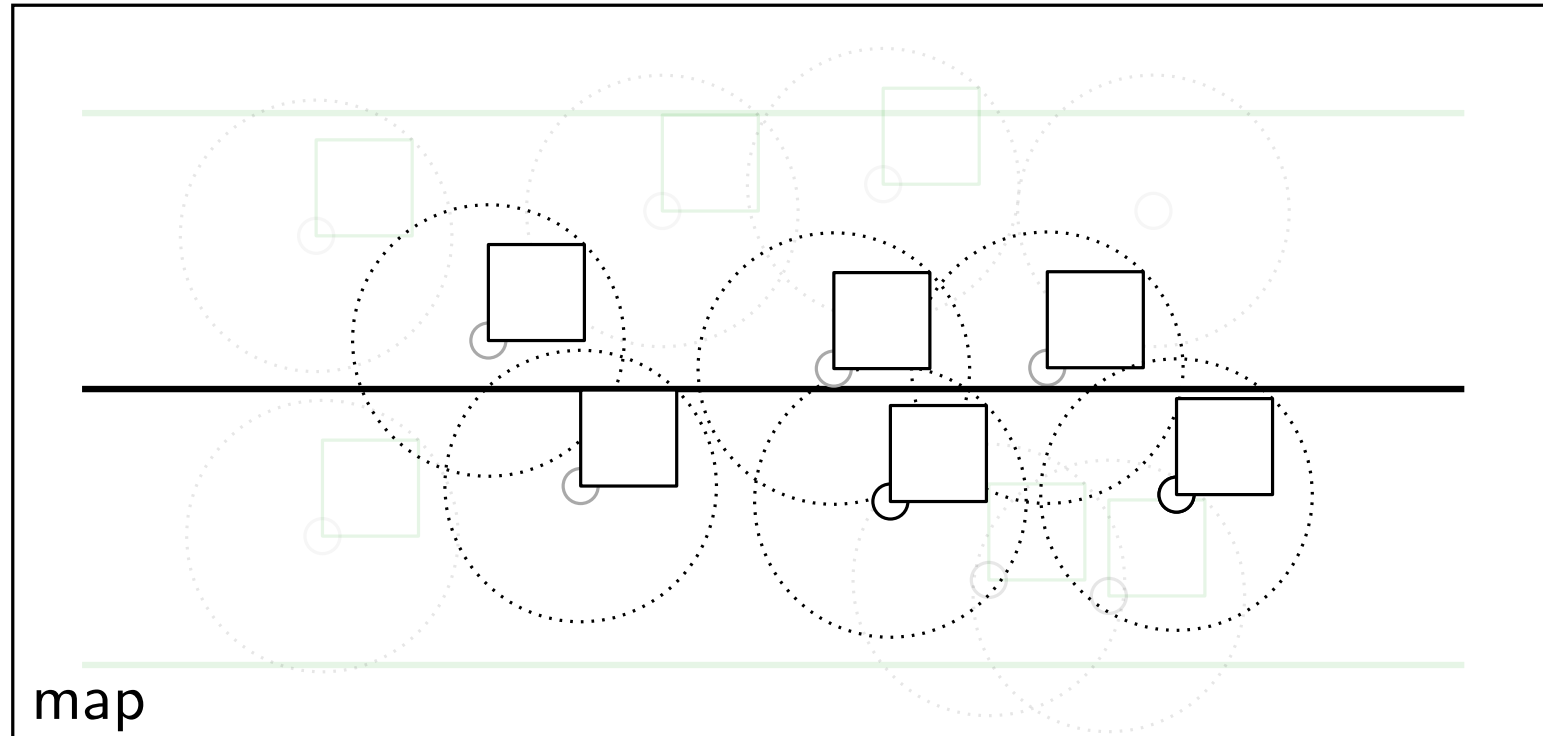
1/4-Approximation of MaxTotal



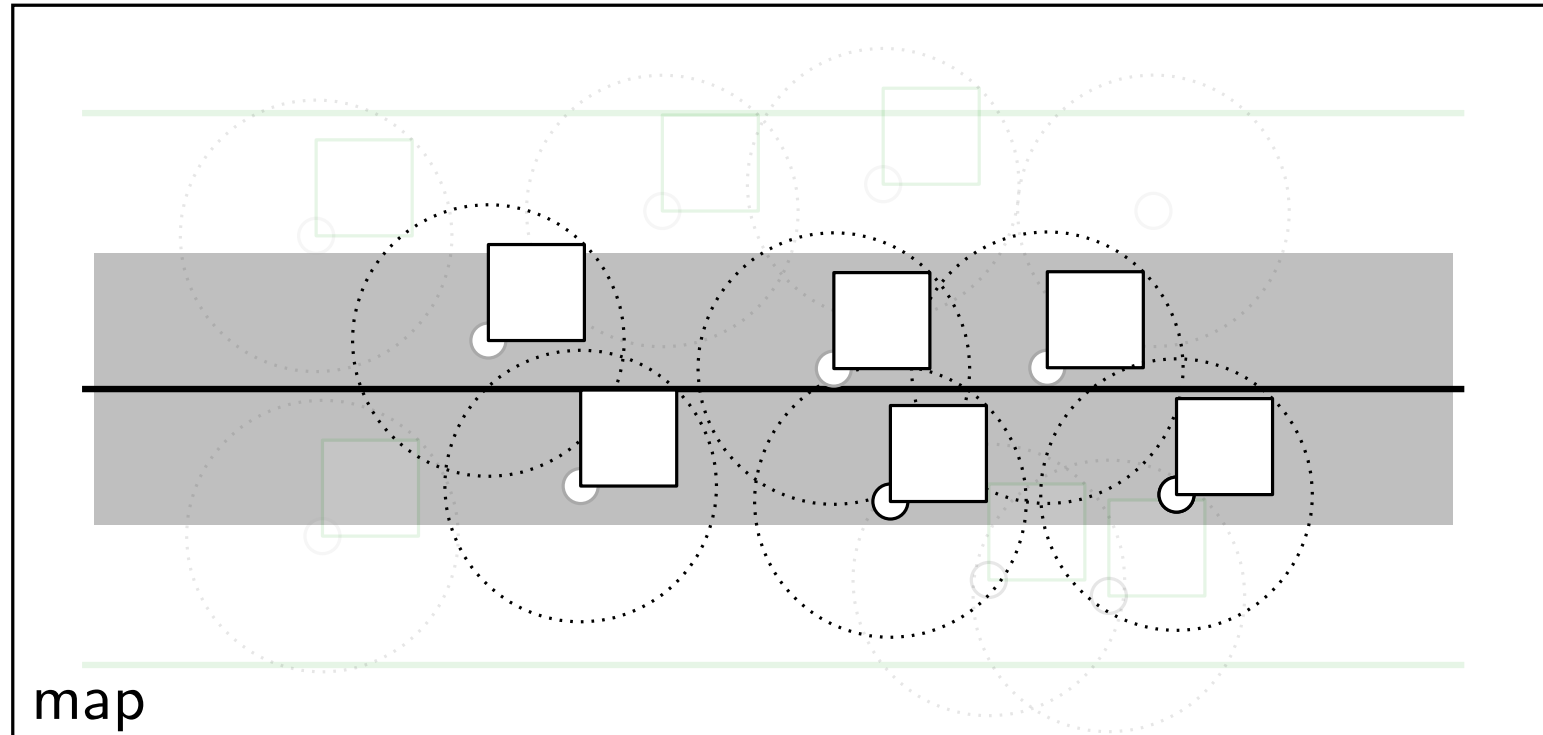
1/4-Approximation of MaxTotal



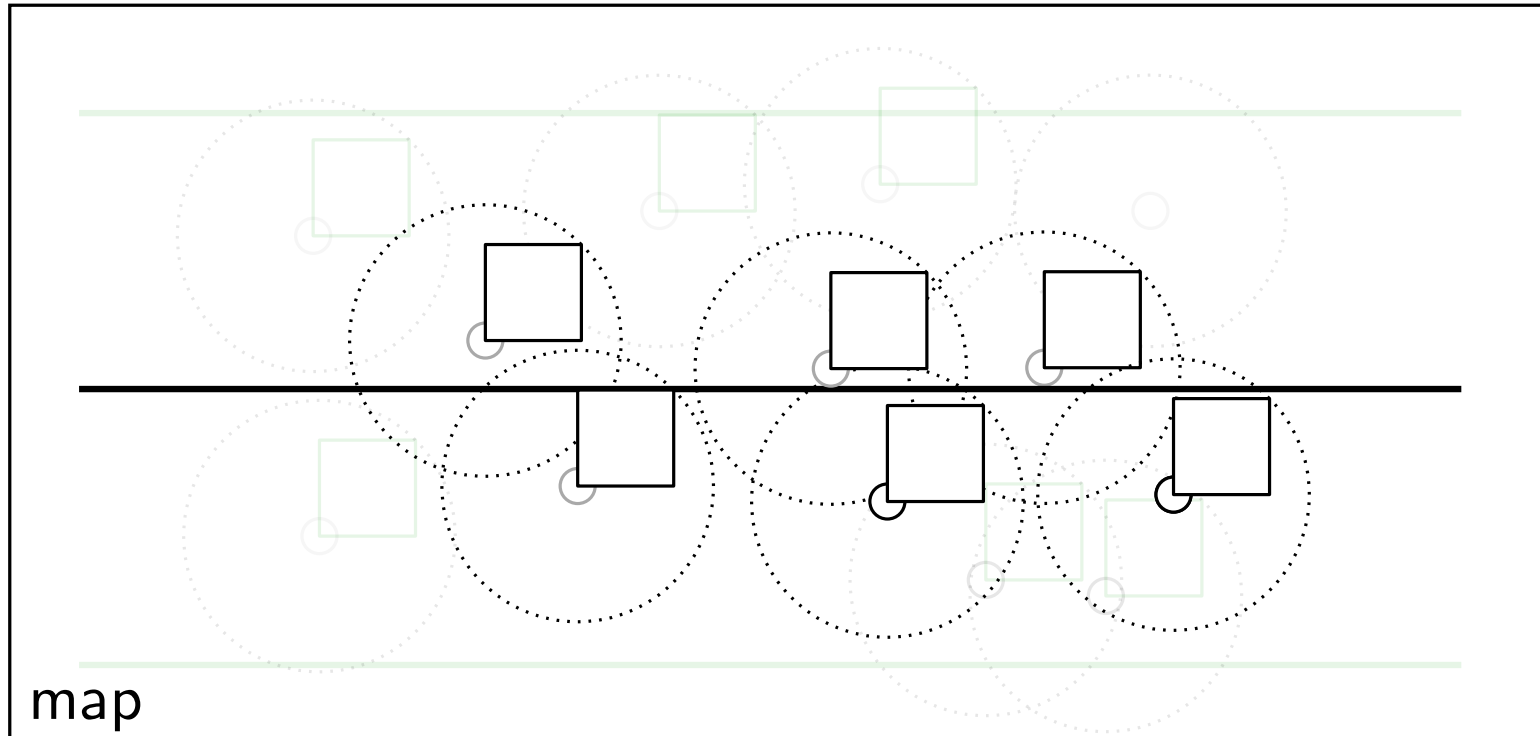
1/4-Approximation of MaxTotal



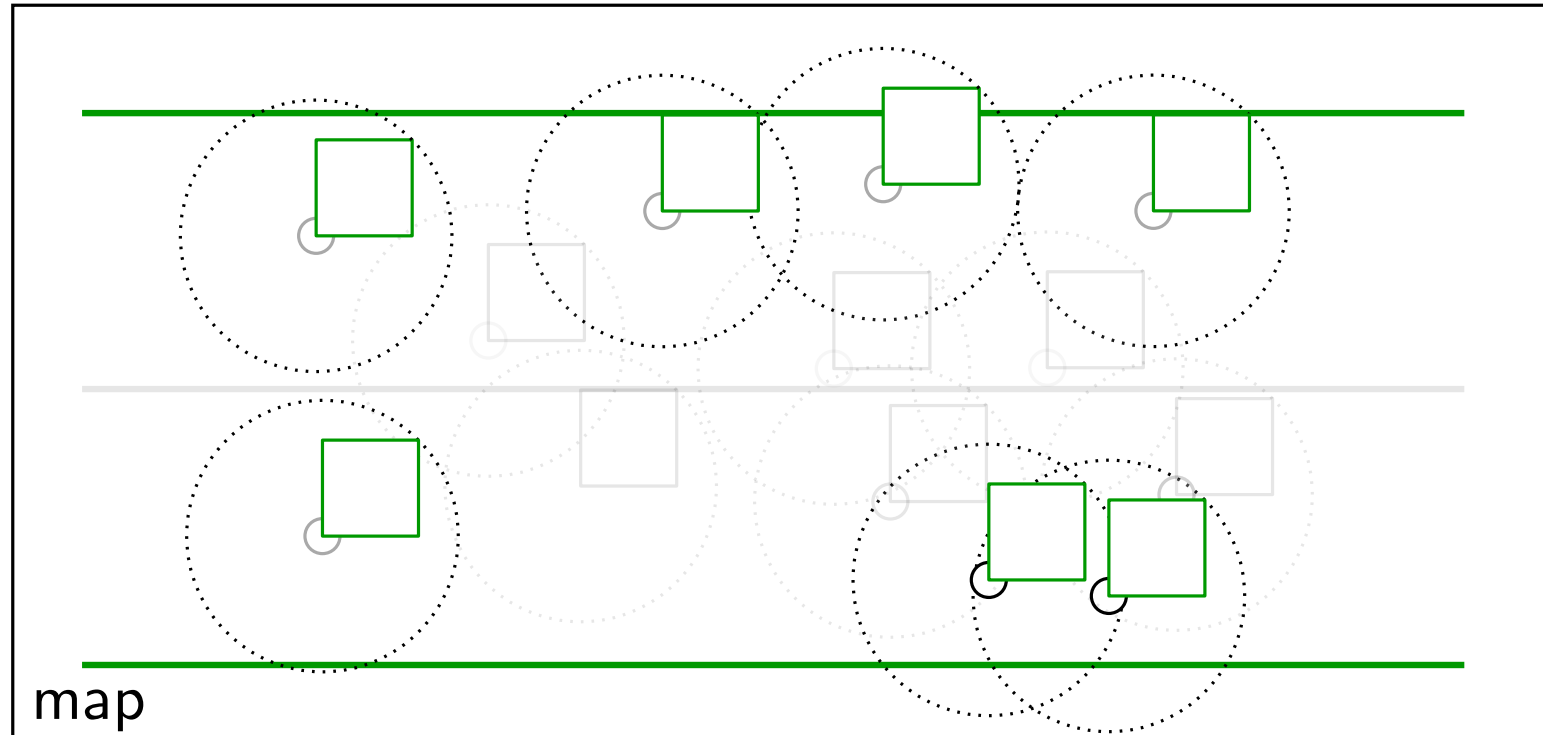
1/4-Approximation of MaxTotal



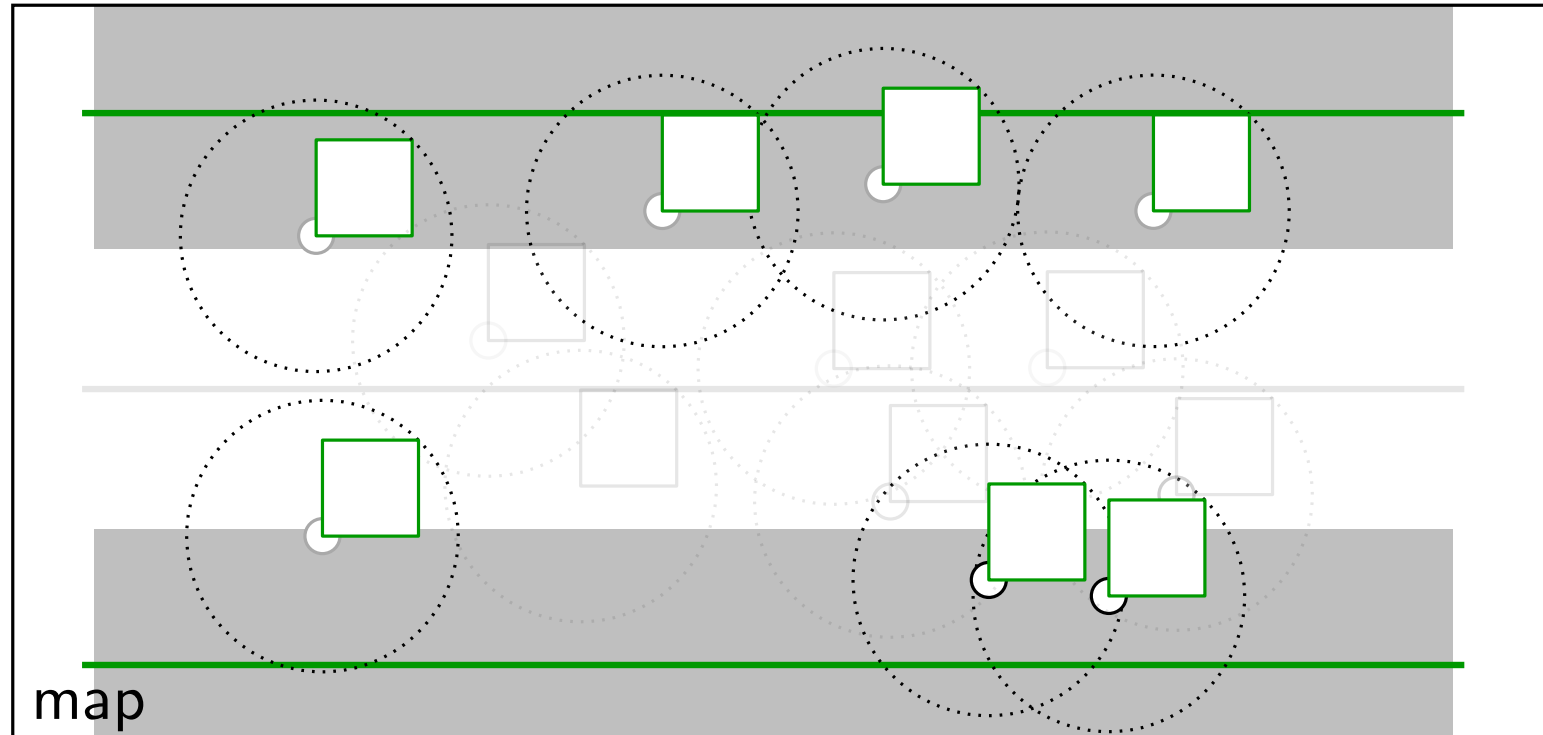
1/4-Approximation of MaxTotal



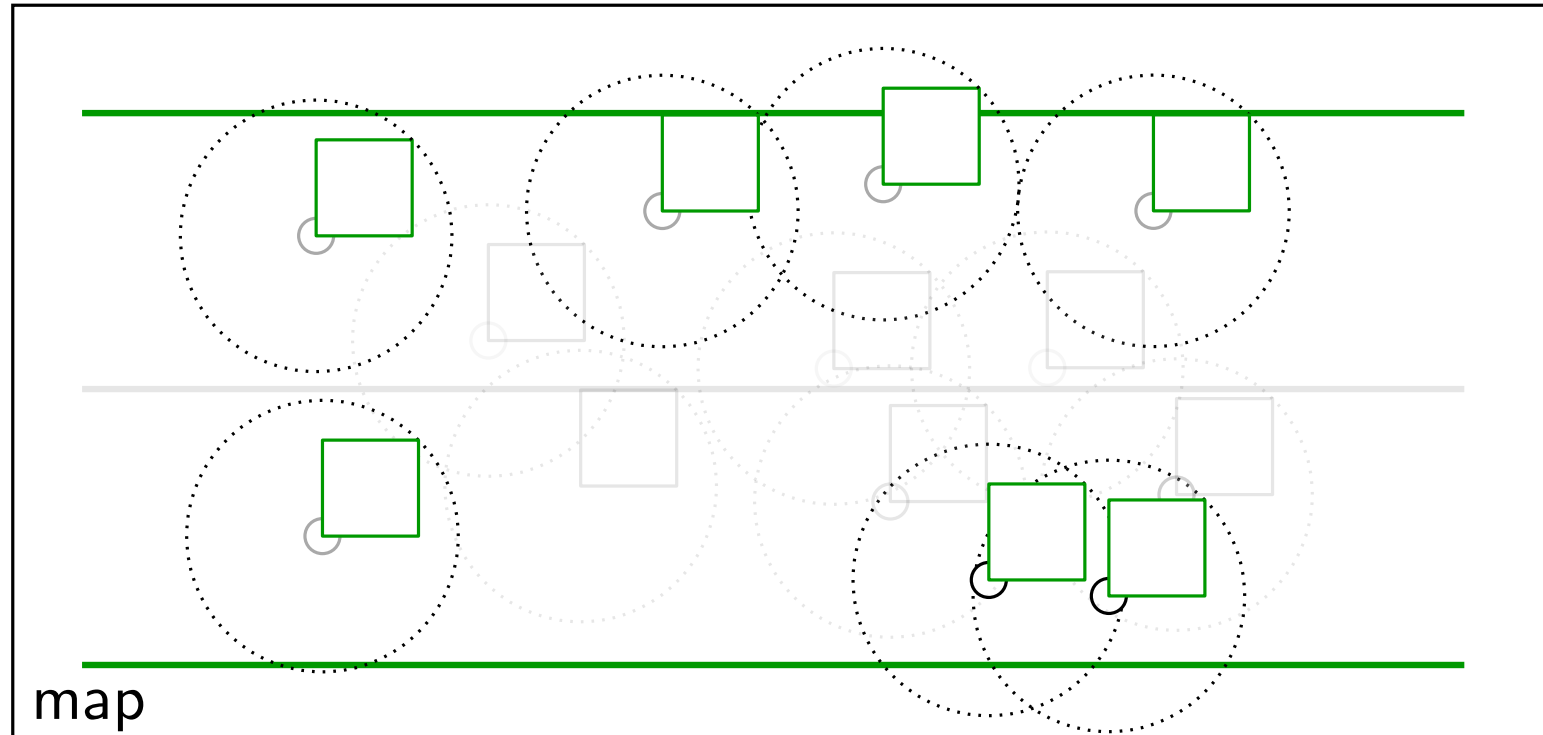
1/4-Approximation of MaxTotal



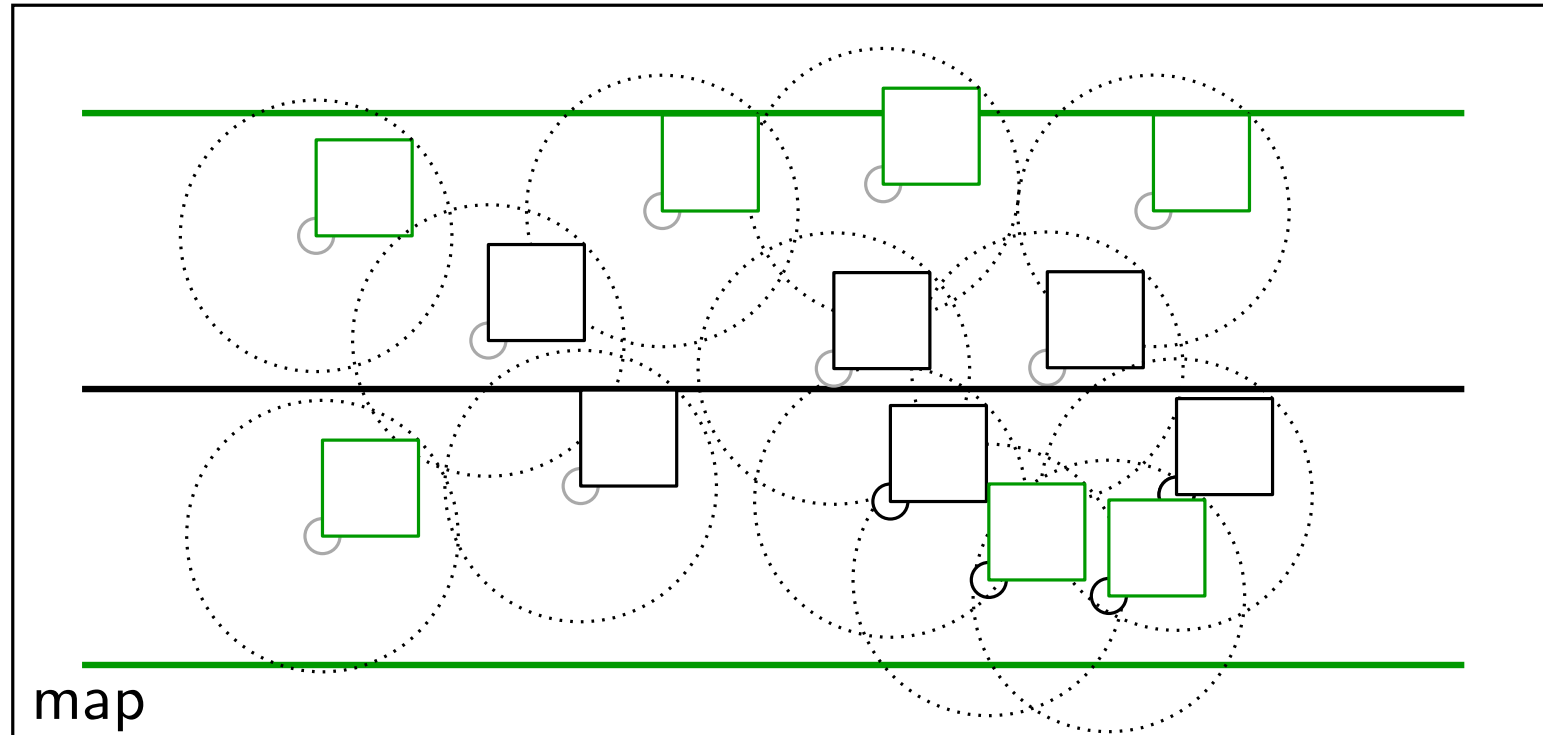
1/4-Approximation of MaxTotal



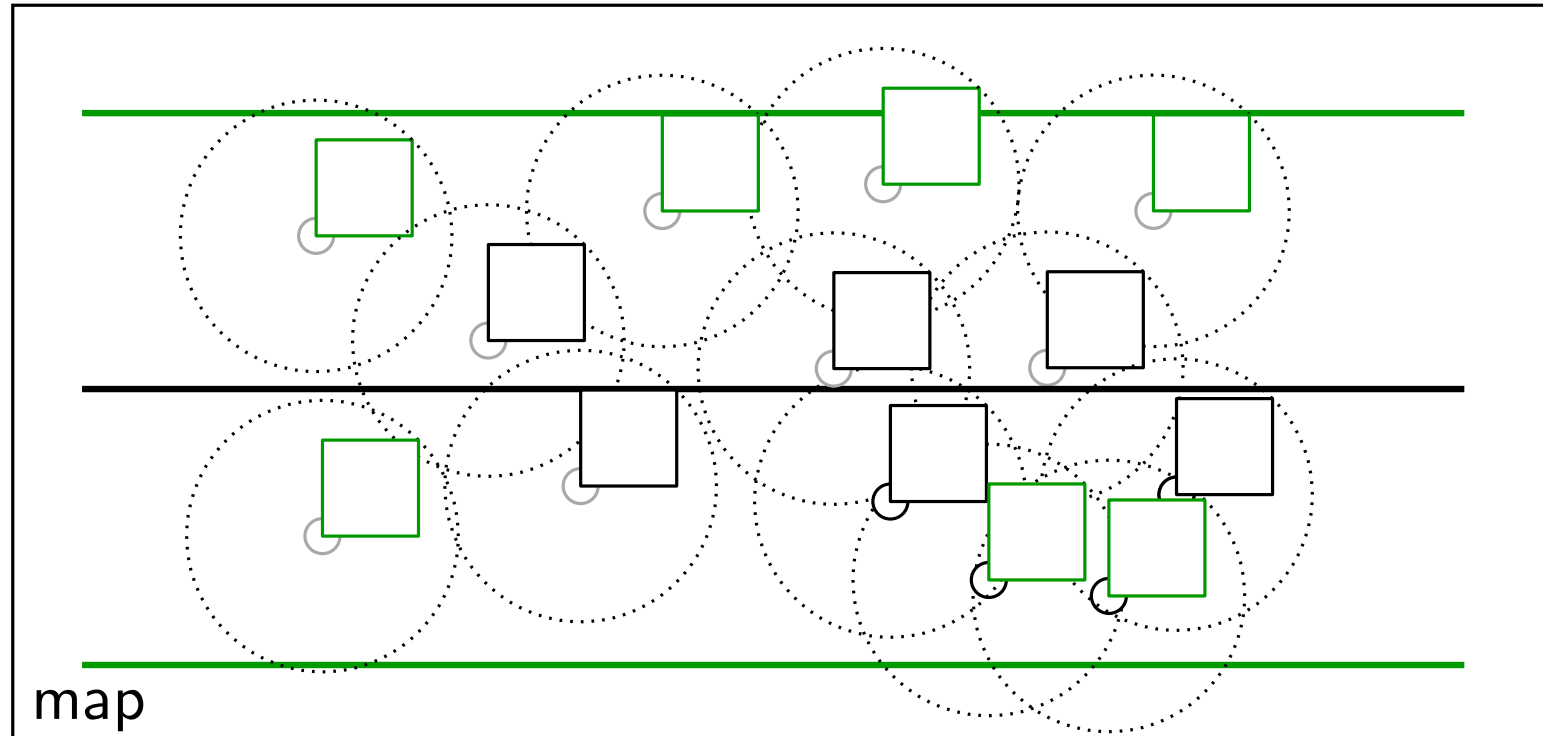
1/4-Approximation of MaxTotal



1/4-Approximation of MaxTotal



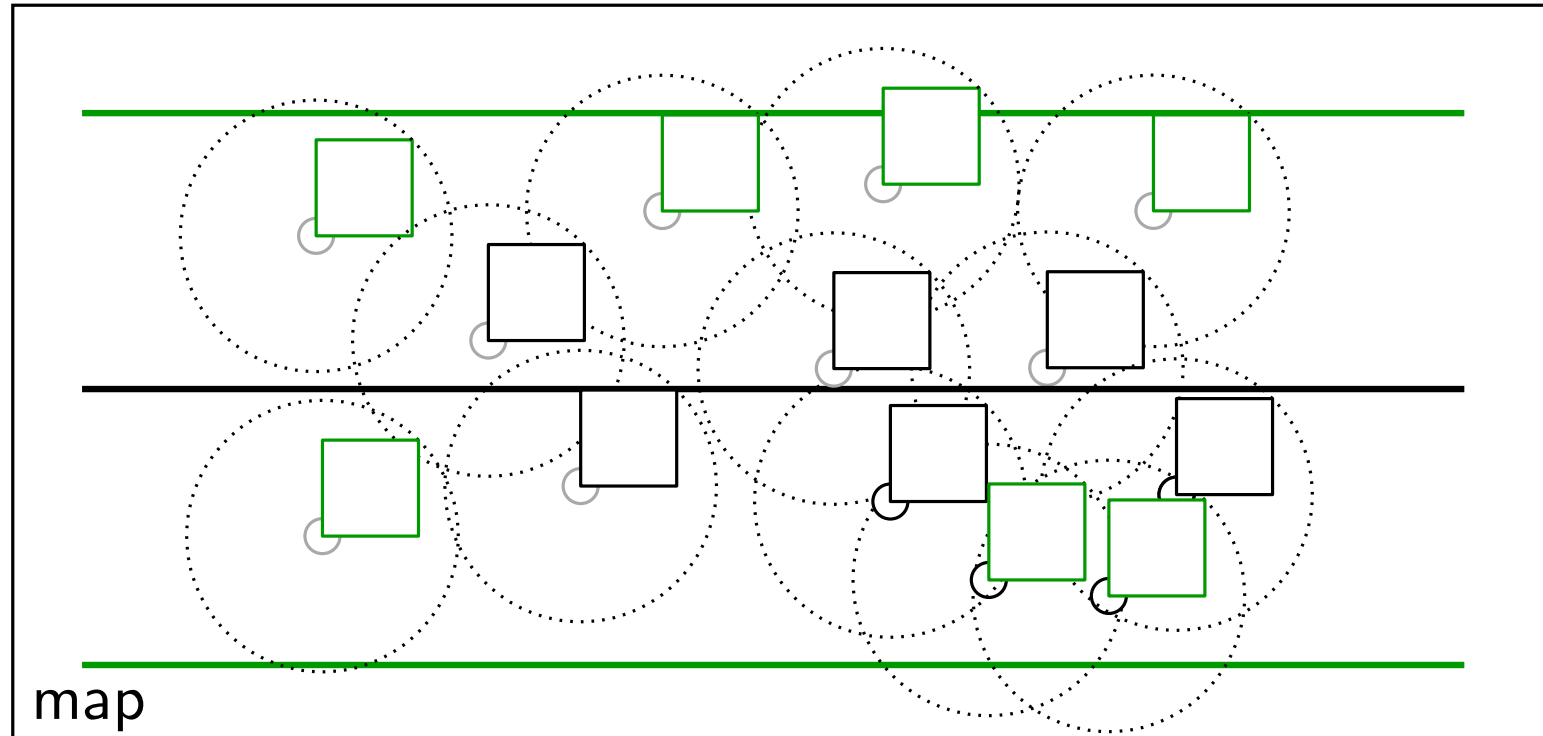
1/4-Approximation of MaxTotal



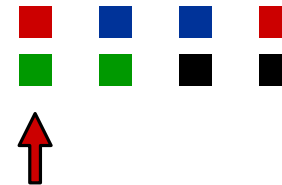
- split set of labels into **four** sets



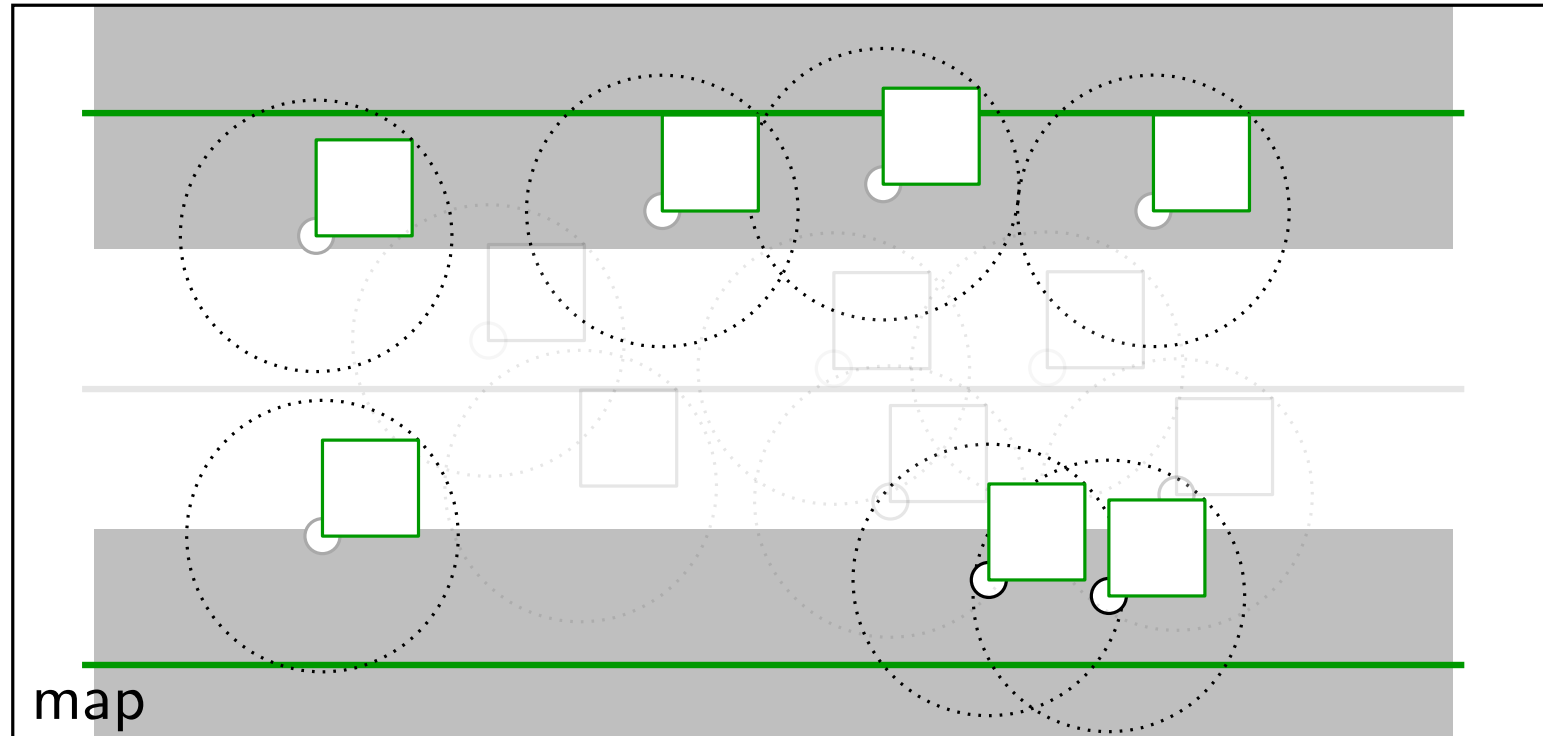
1/4-Approximation of MaxTotal



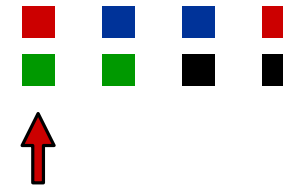
- split set of labels into **four** sets



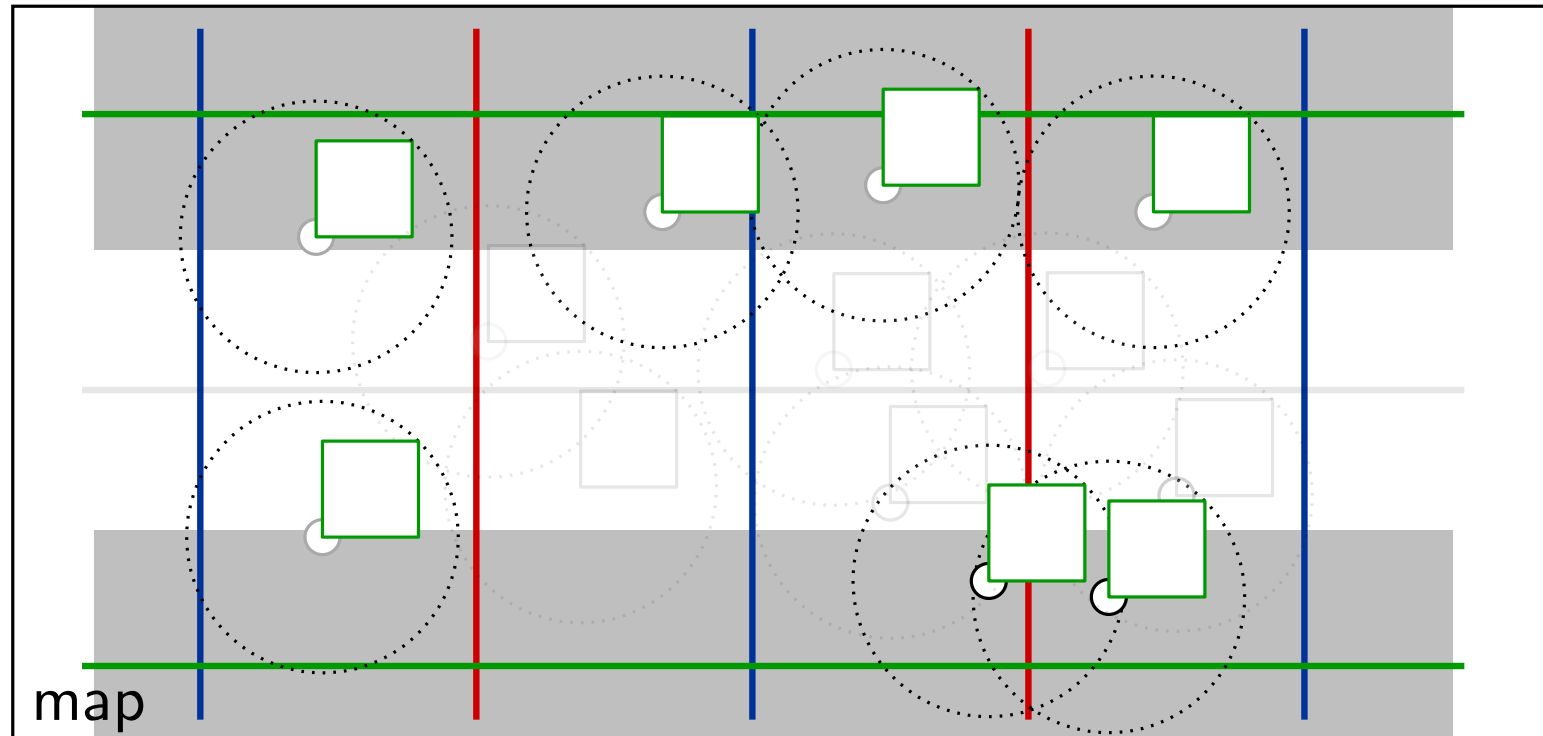
1/4-Approximation of MaxTotal



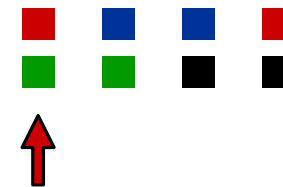
- split set of labels into **four** sets



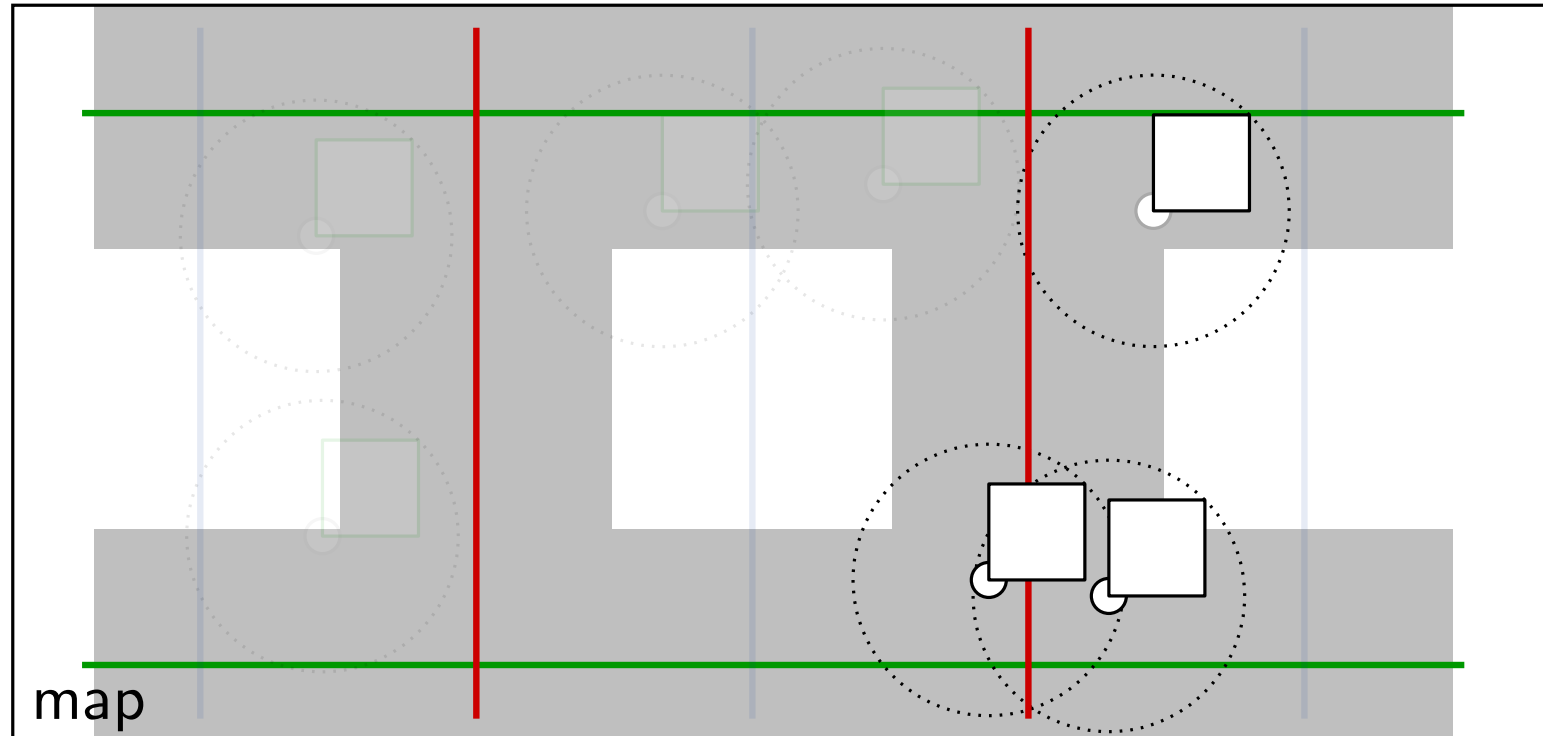
1/4-Approximation of MaxTotal



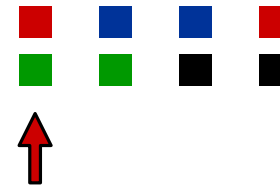
- split set of labels into **four** sets



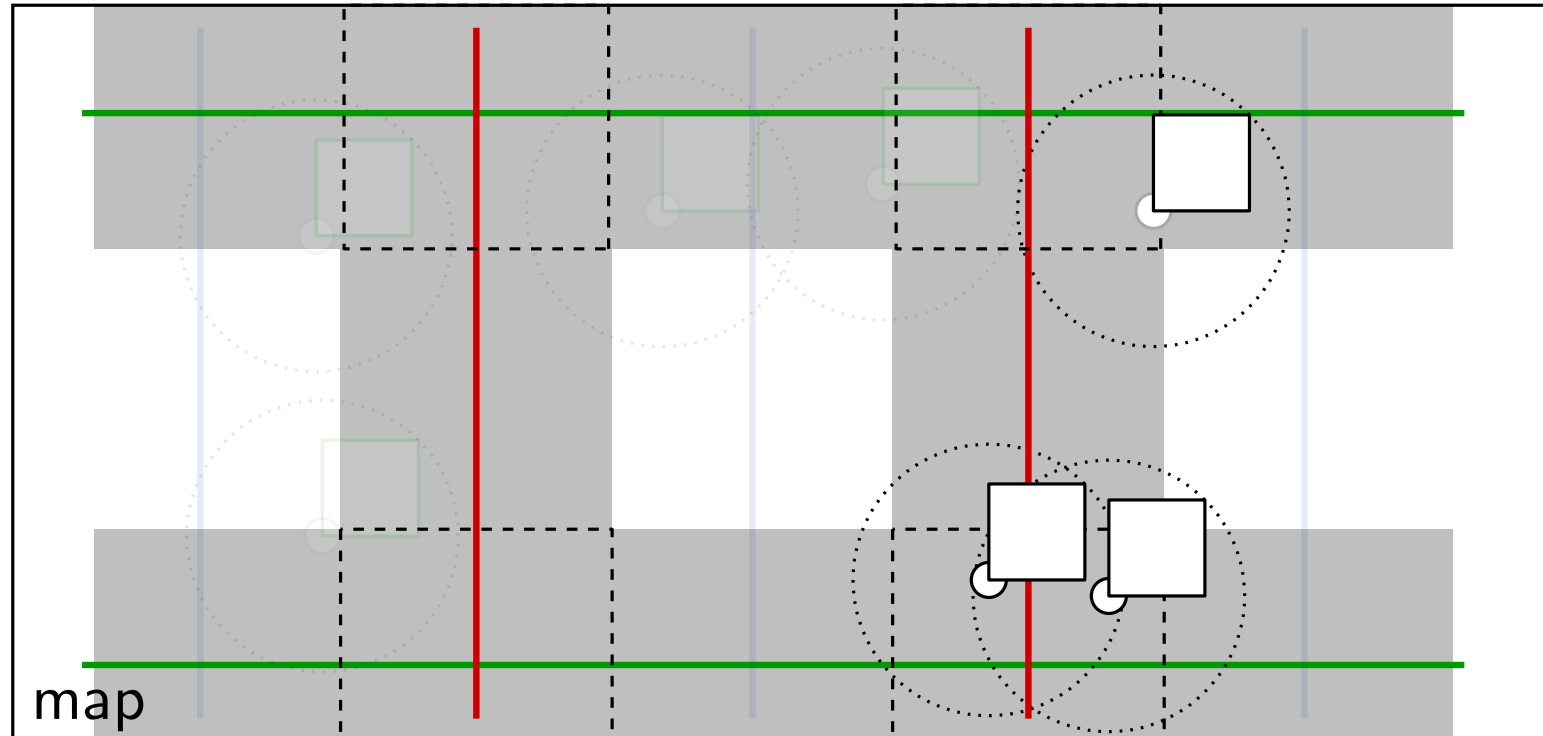
1/4-Approximation of MaxTotal



- split set of labels into **four** sets

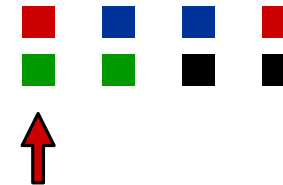


1/4-Approximation of MaxTotal

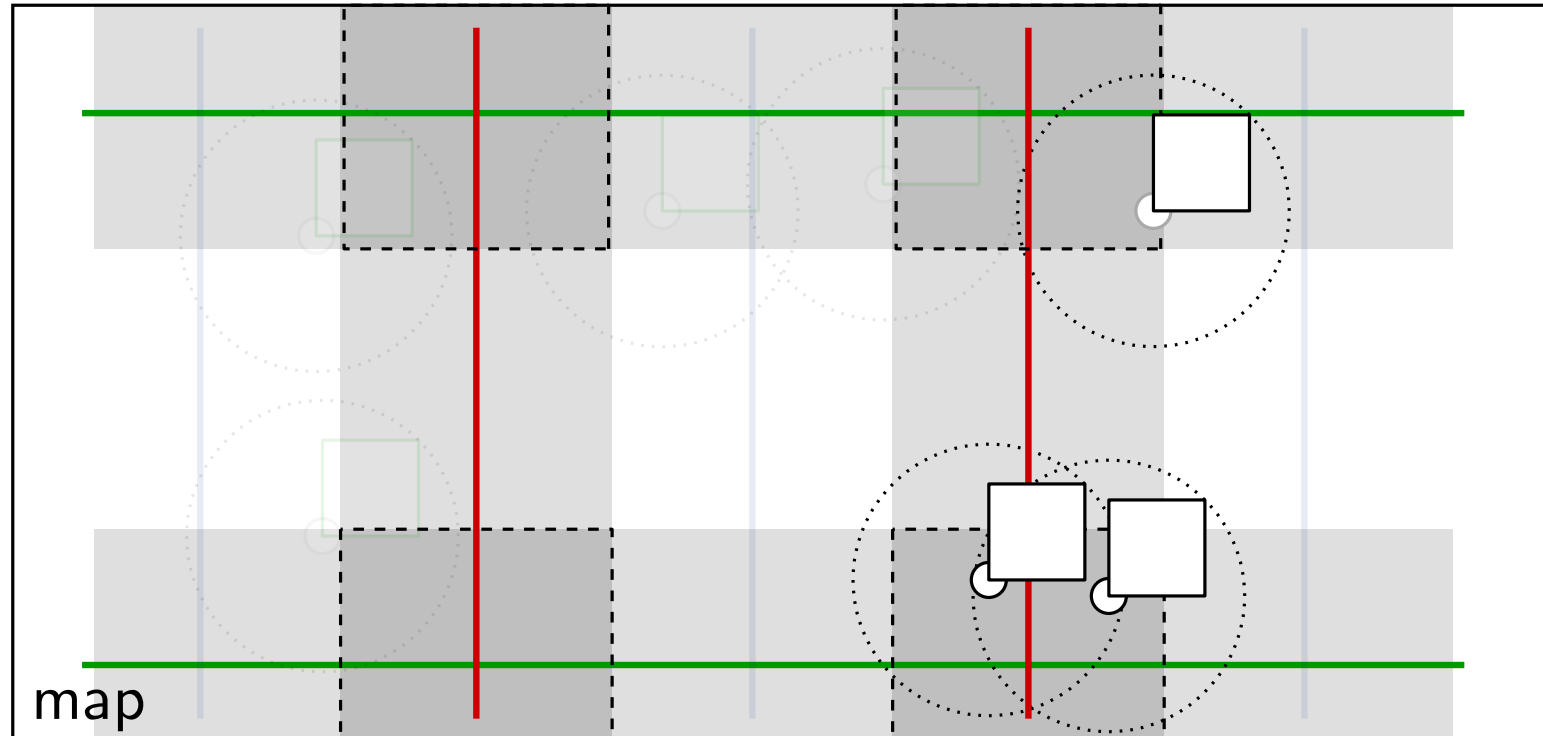


map

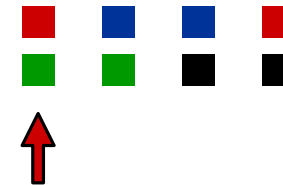
- split set of labels into **four** sets



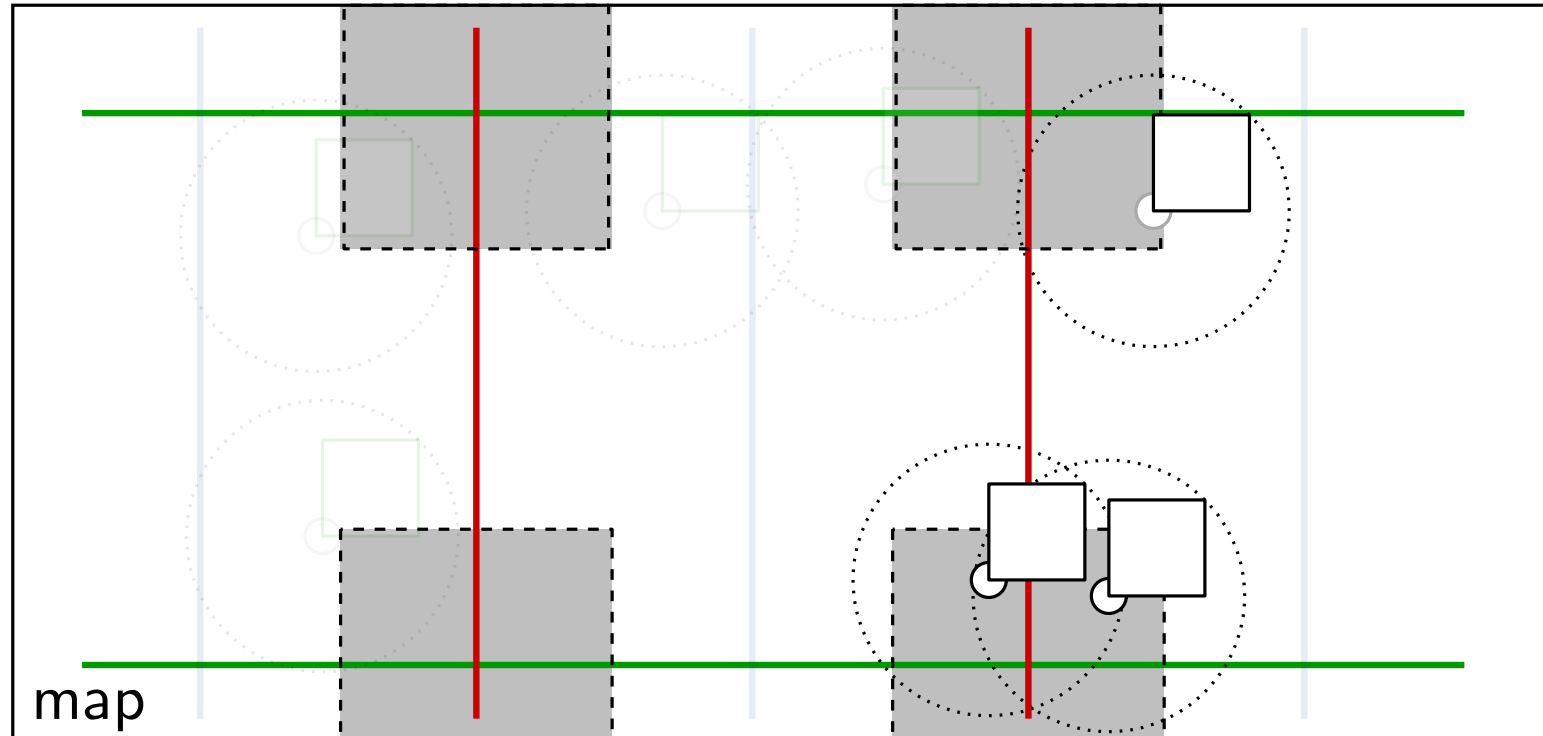
1/4-Approximation of MaxTotal



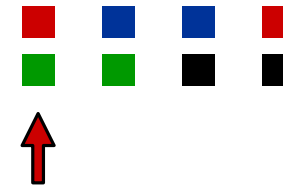
- split set of labels into **four** sets



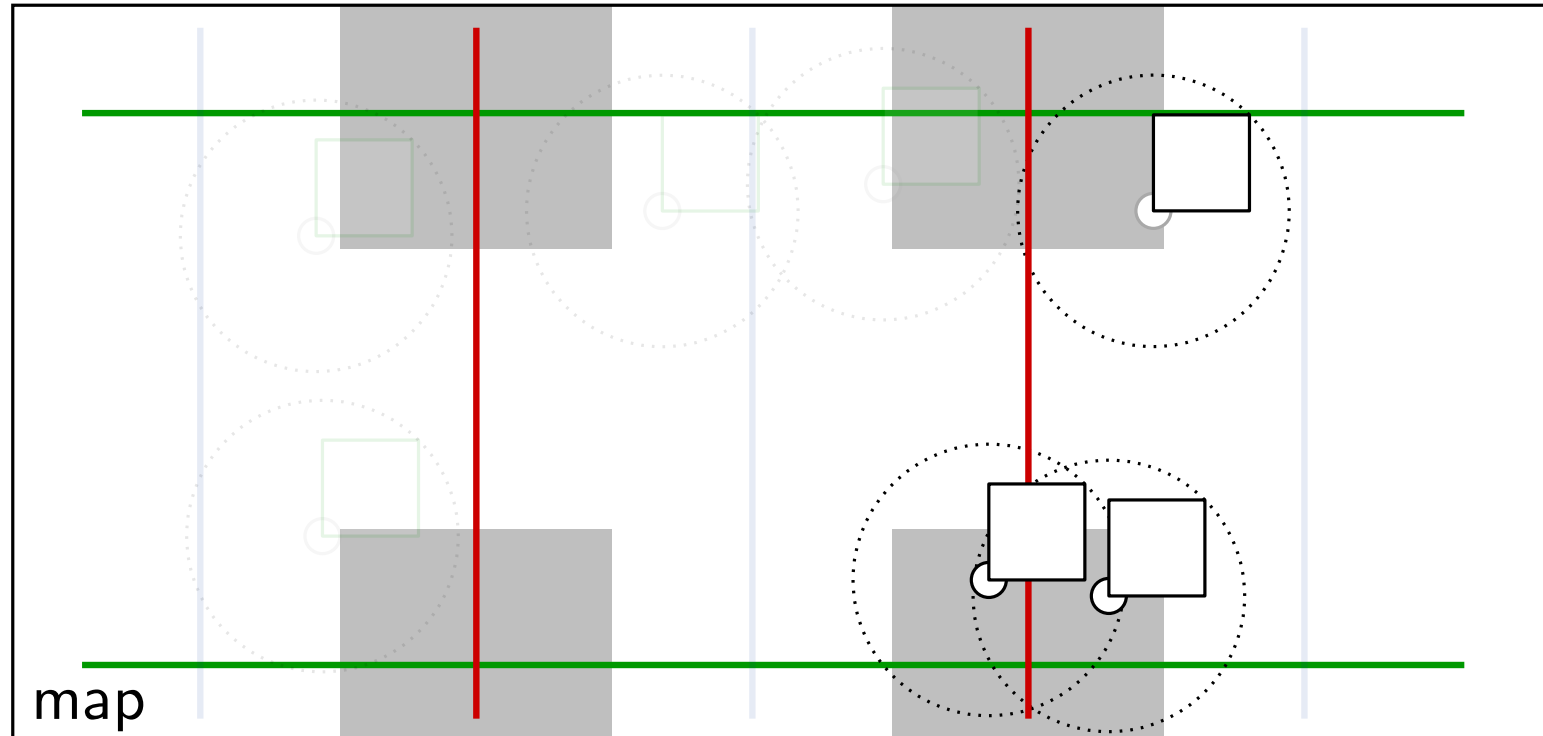
1/4-Approximation of MaxTotal



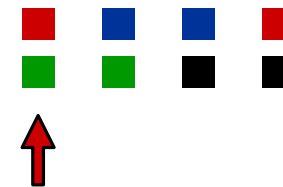
- split set of labels into **four** sets



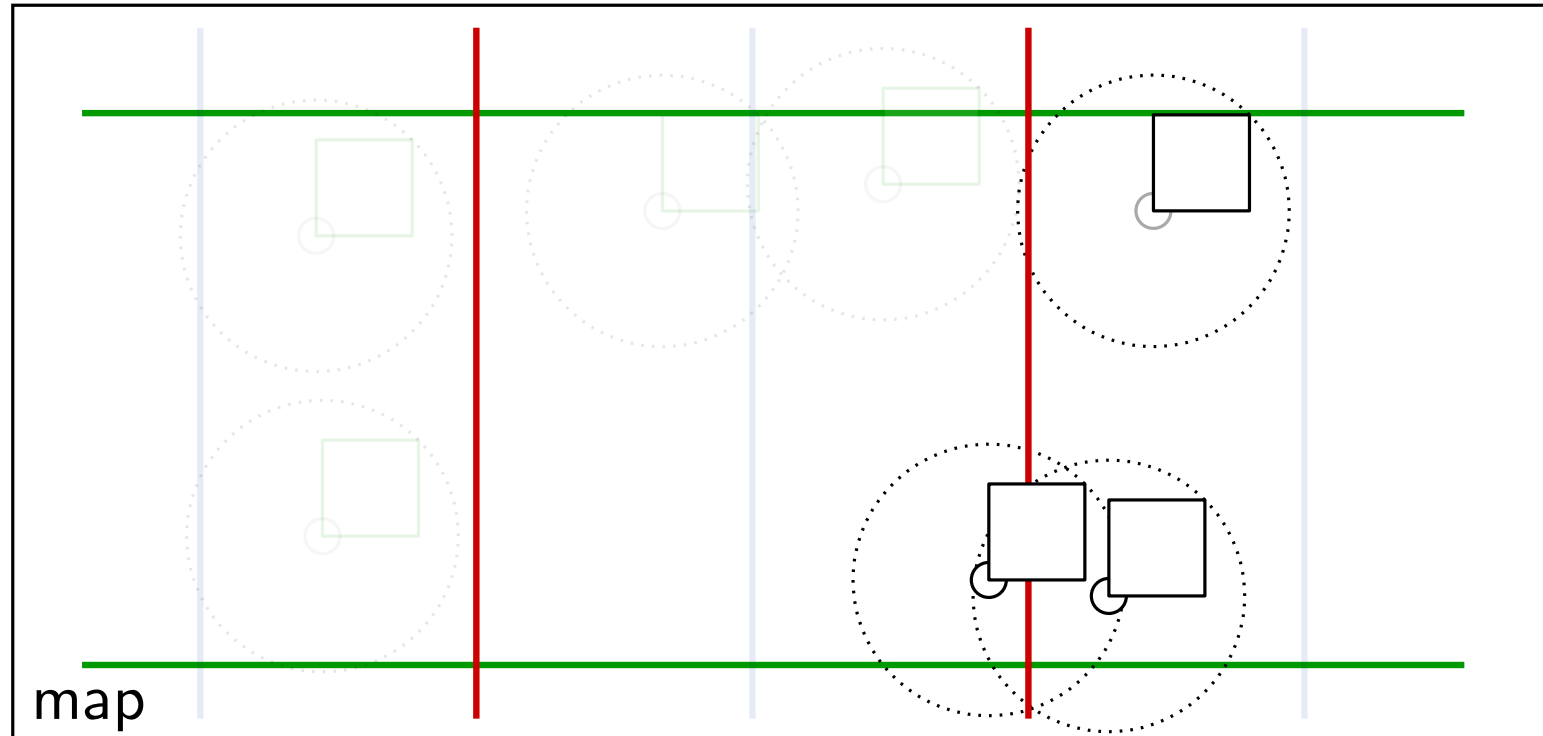
1/4-Approximation of MaxTotal



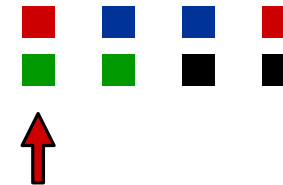
- split set of labels into **four** sets



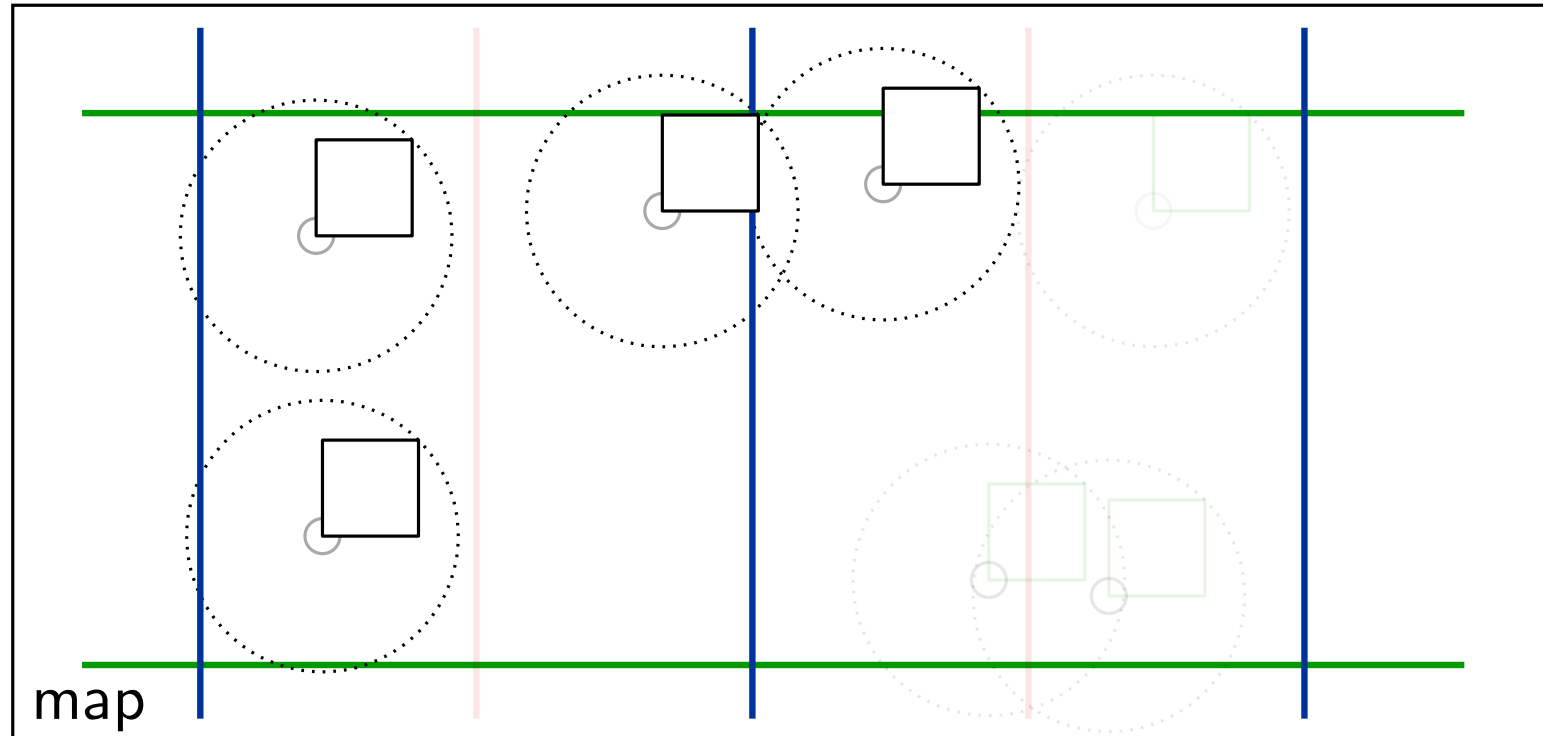
1/4-Approximation of MaxTotal



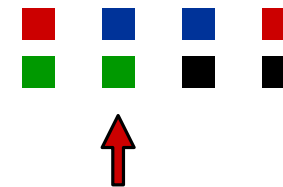
- split set of labels into **four** sets



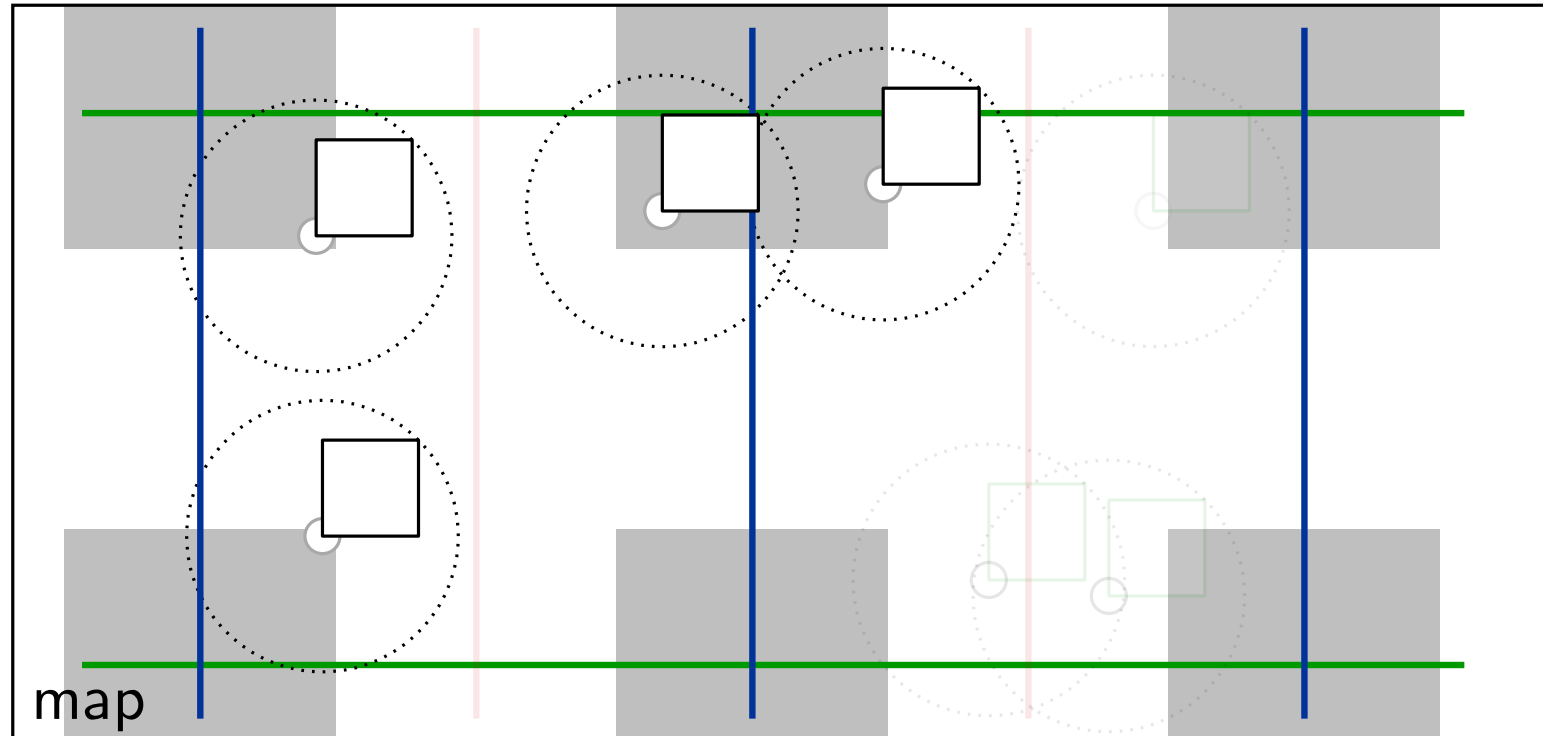
1/4-Approximation of MaxTotal



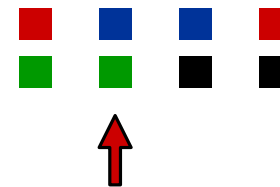
- split set of labels into **four** sets



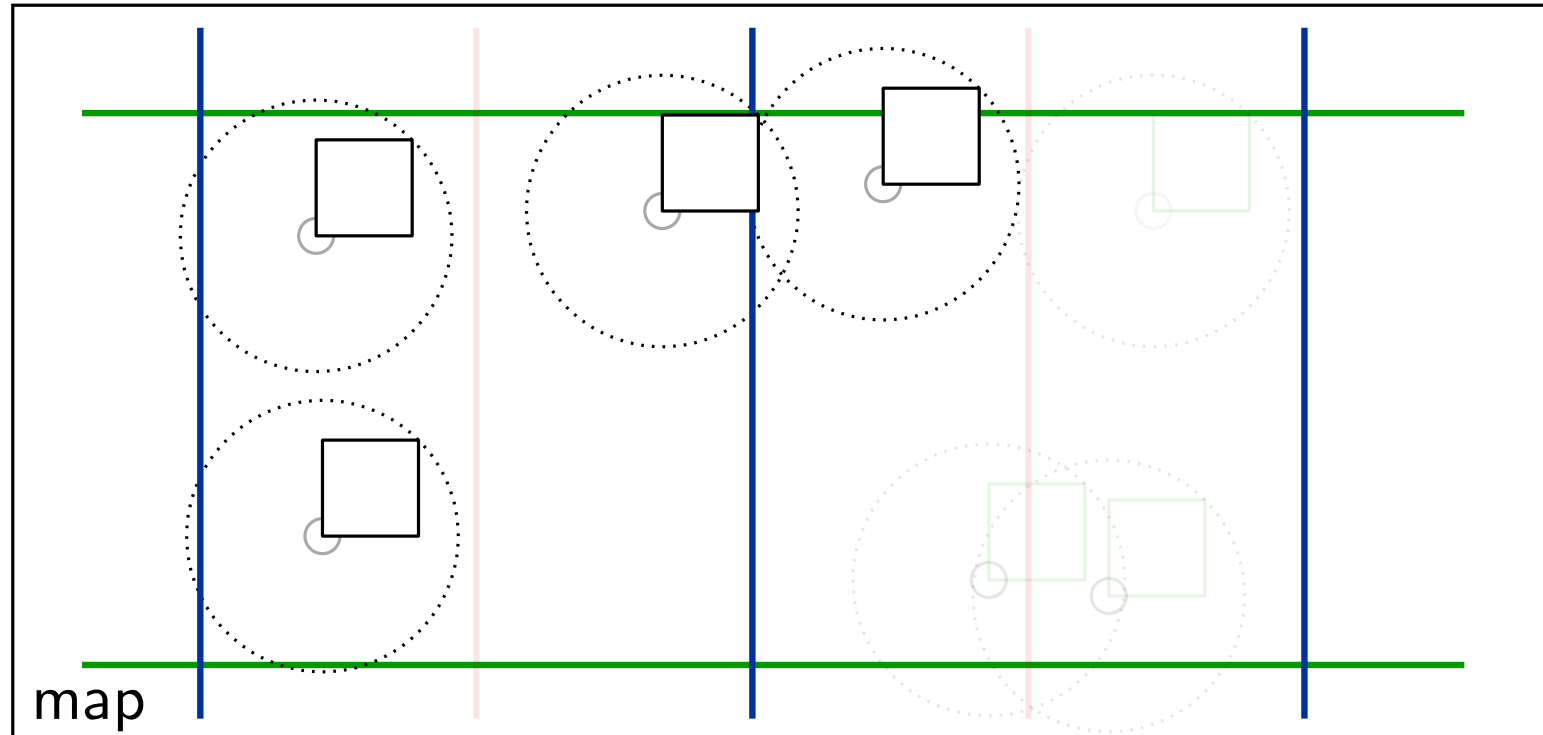
1/4-Approximation of MaxTotal



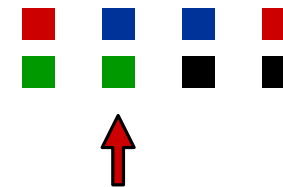
- split set of labels into **four** sets



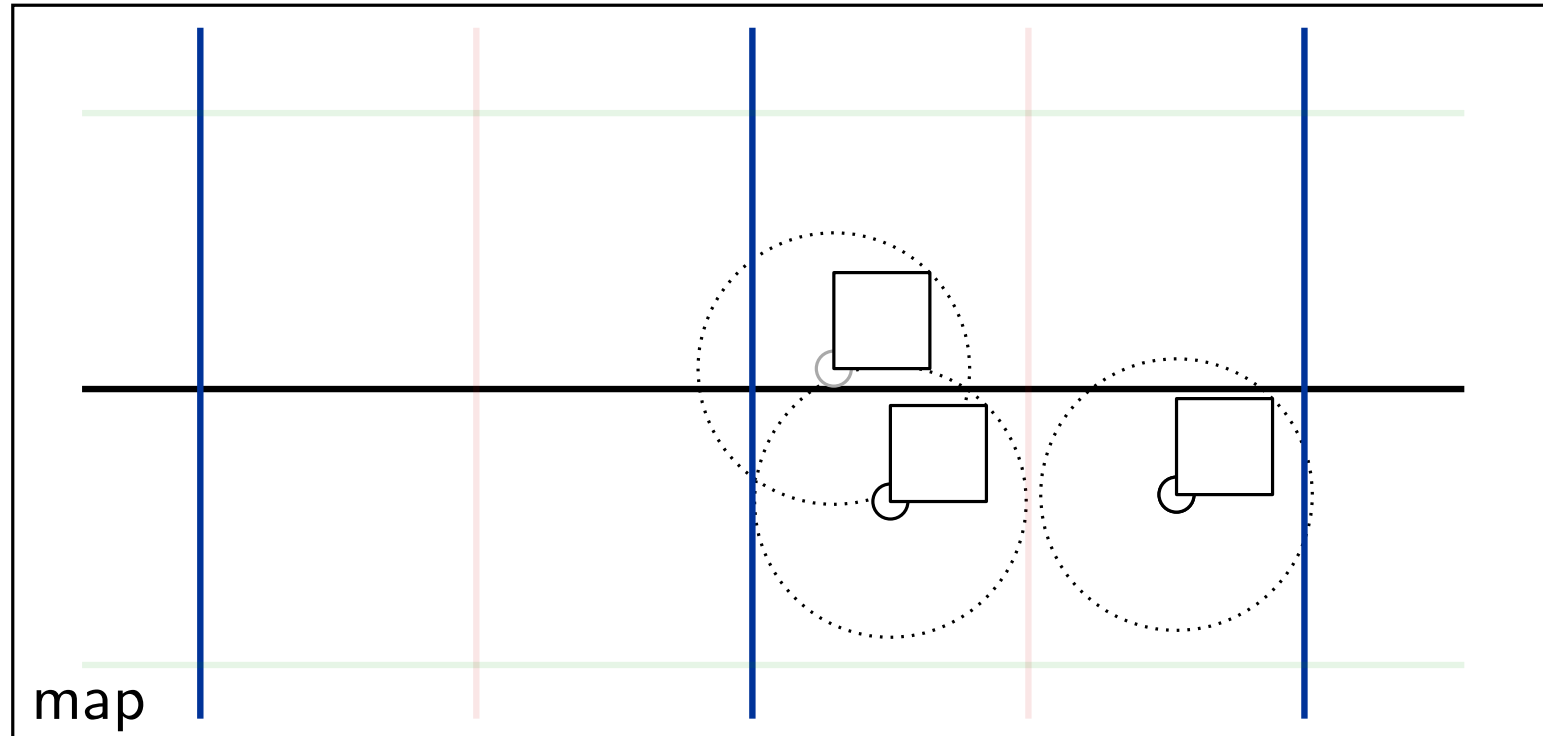
1/4-Approximation of MaxTotal



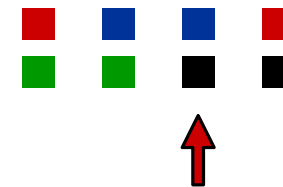
- split set of labels into **four** sets



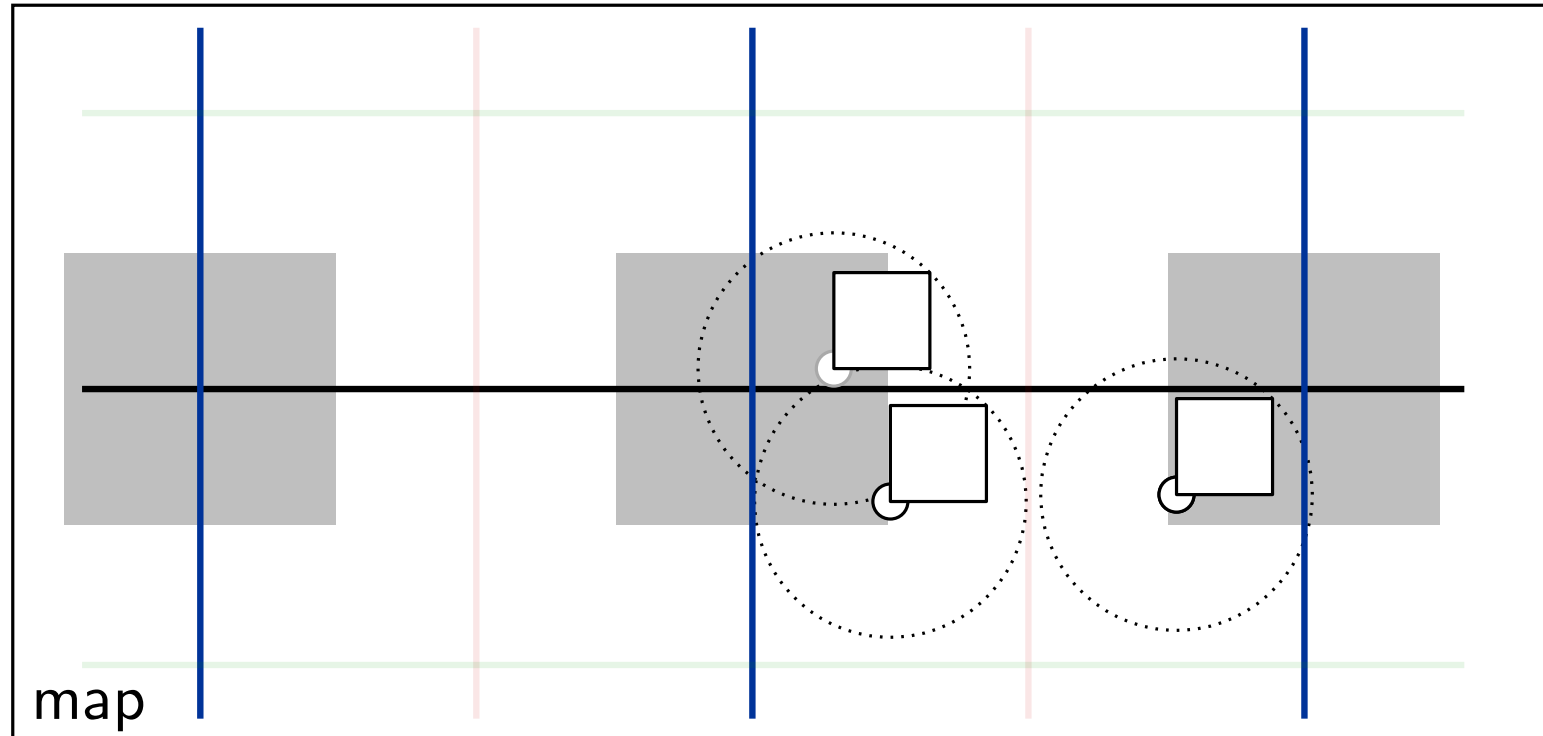
1/4-Approximation of MaxTotal



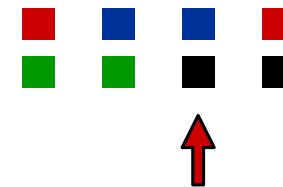
- split set of labels into **four** sets



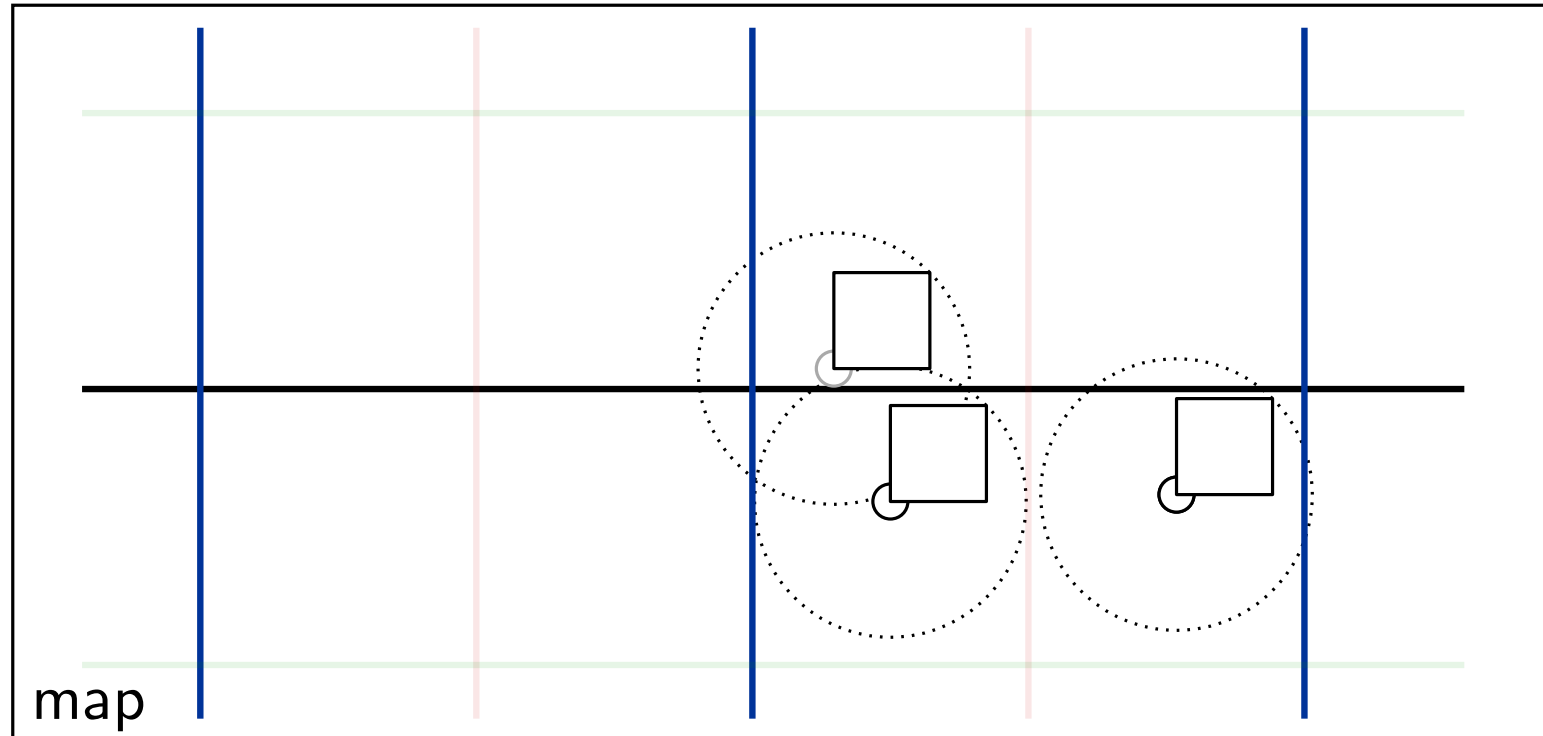
1/4-Approximation of MaxTotal



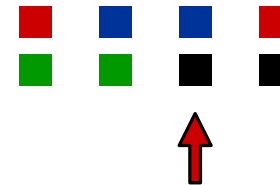
- split set of labels into **four** sets



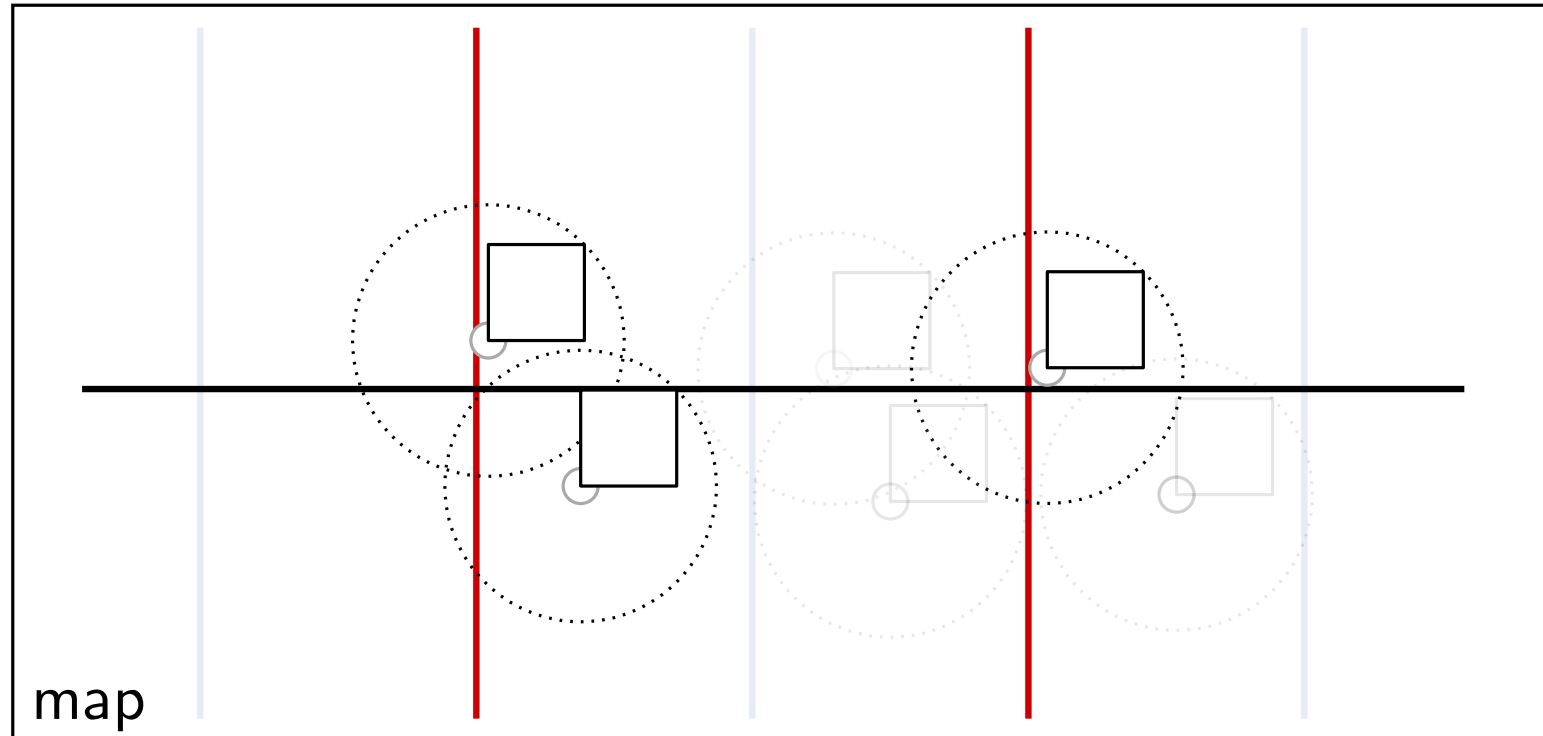
1/4-Approximation of MaxTotal



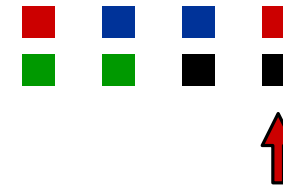
- split set of labels into **four** sets



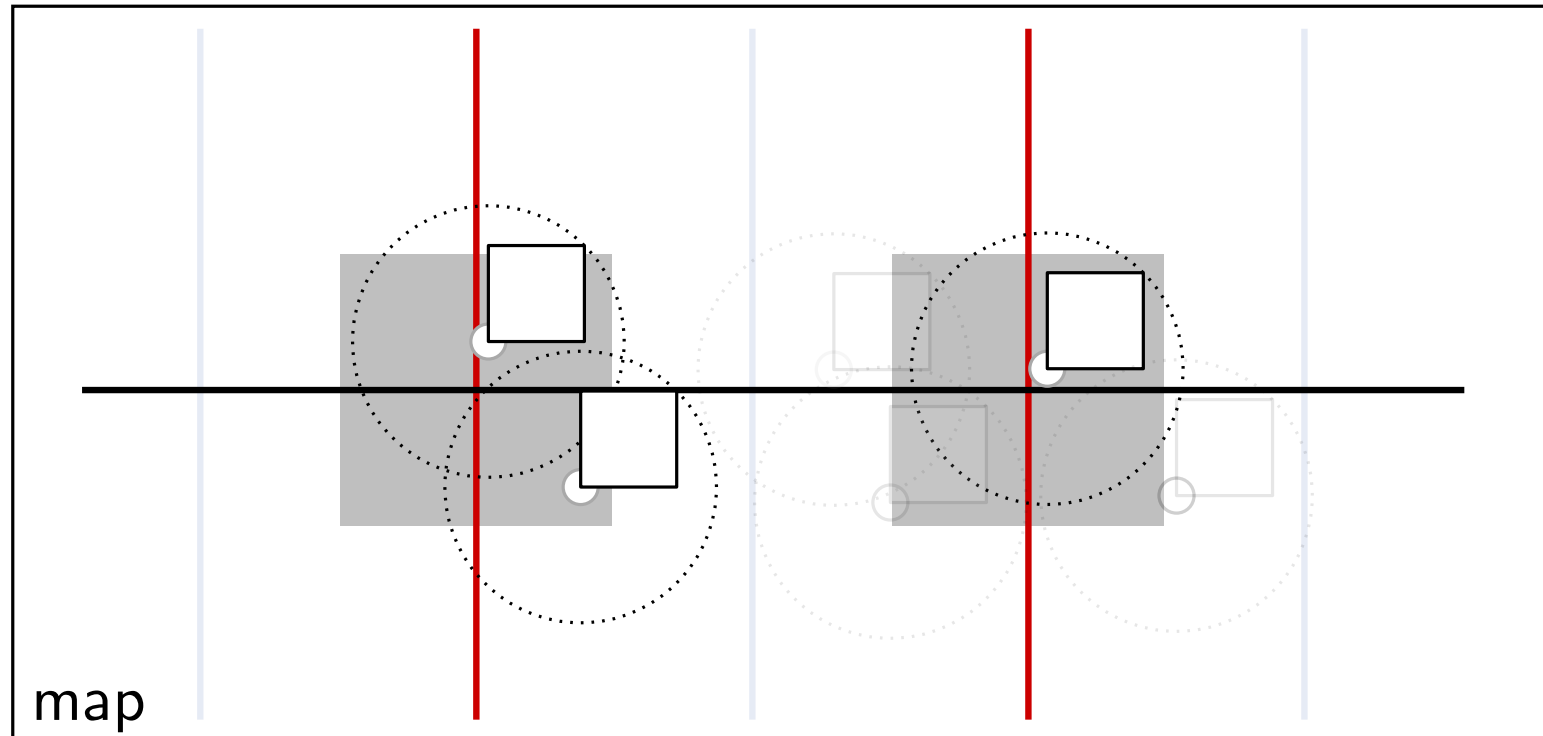
1/4-Approximation of MaxTotal



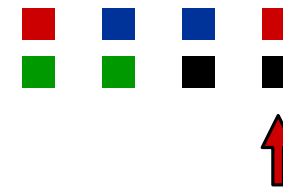
- split set of labels into **four** sets



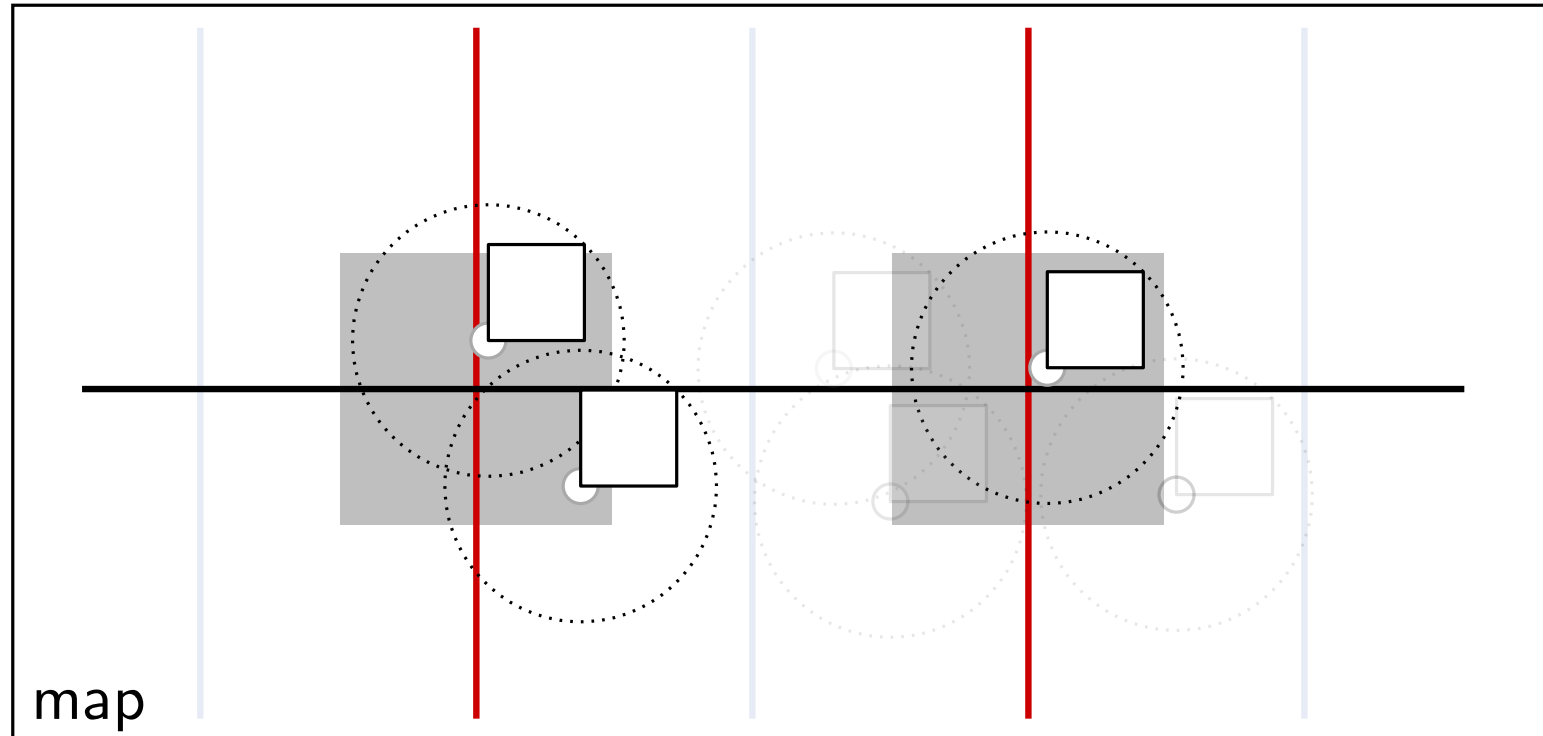
1/4-Approximation of MaxTotal



- split set of labels into **four** sets



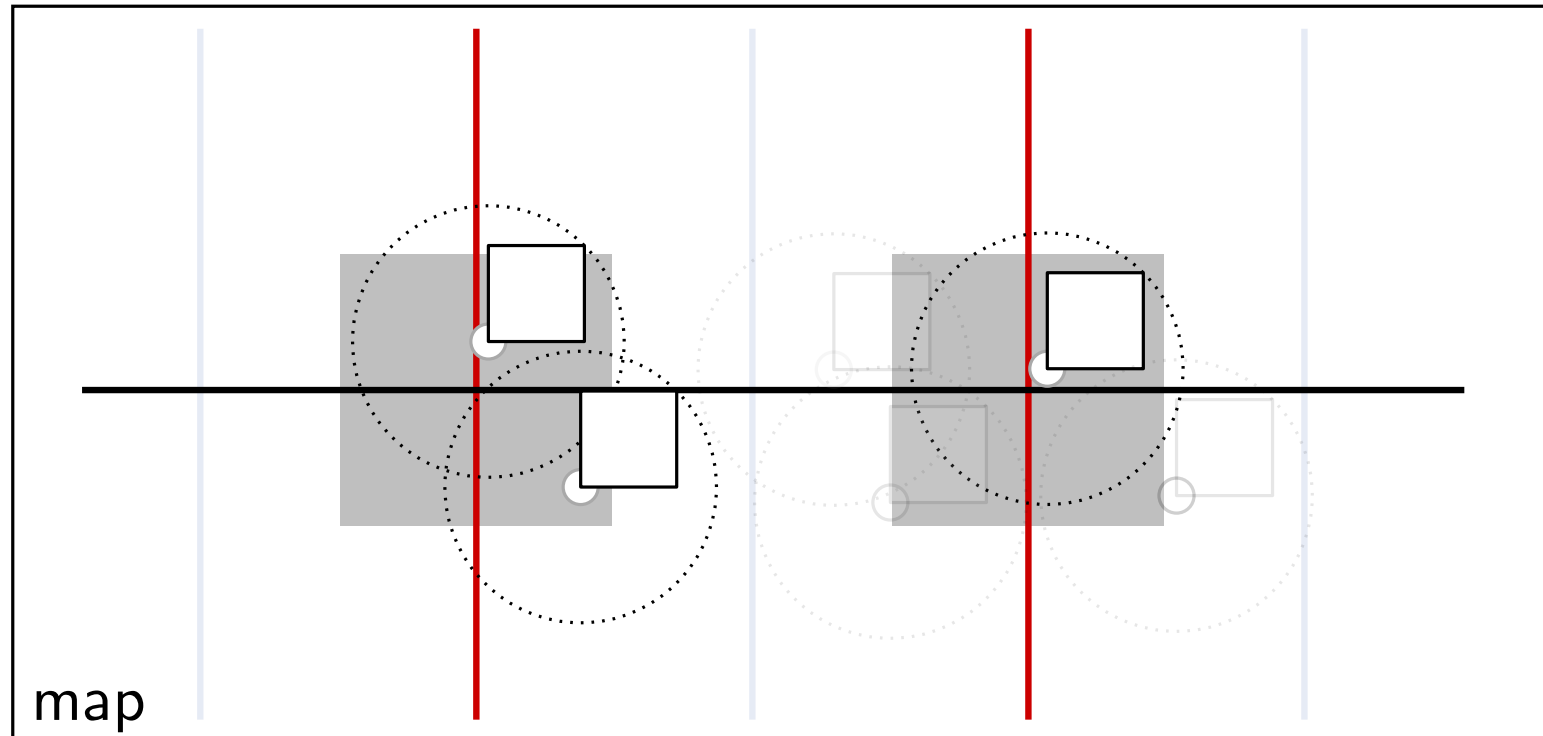
1/4-Approximation of MaxTotal



- split set of labels into **four** sets



1/4-Approximation of MaxTotal

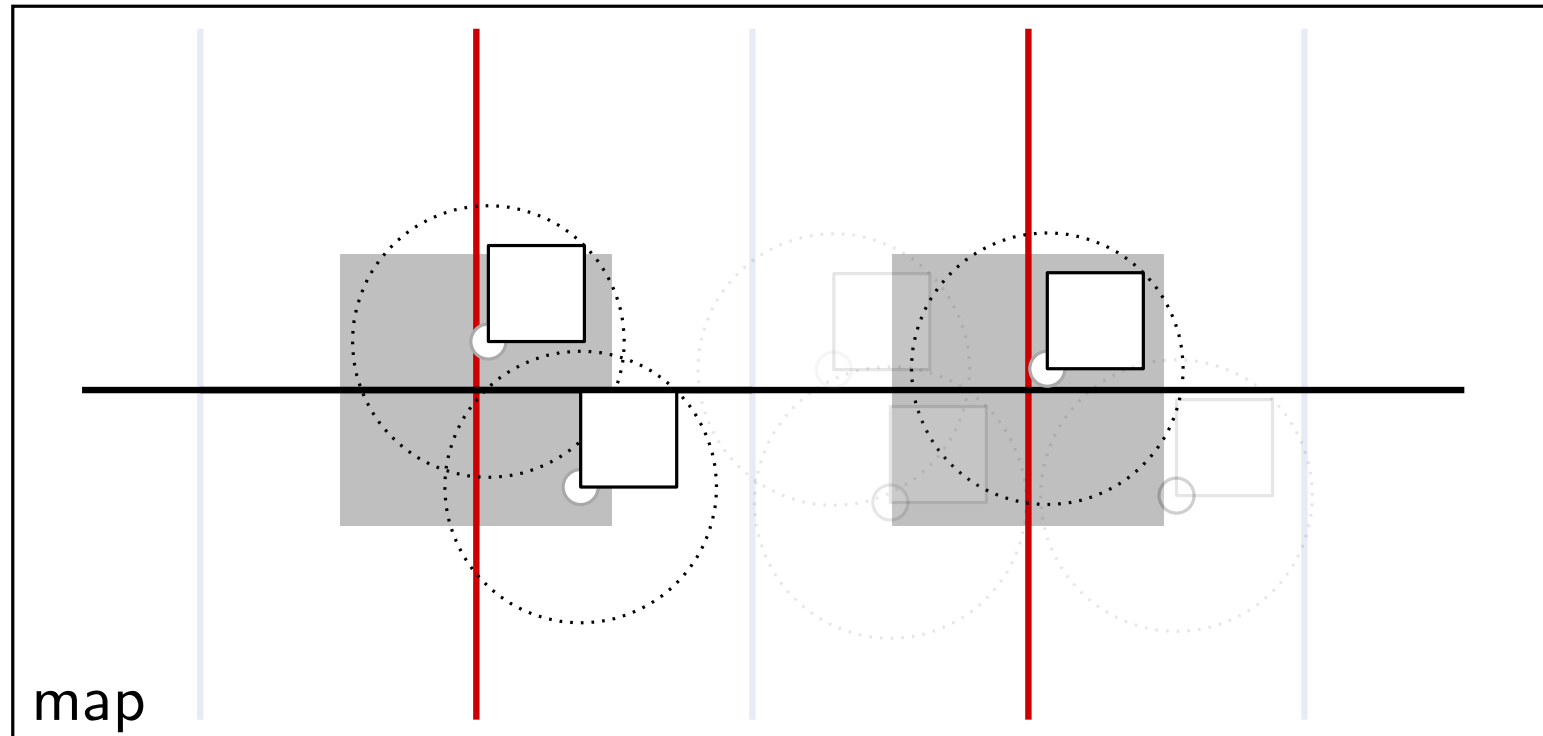


- split set of labels into **four** sets
- find optimal solution for each set separately



one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal

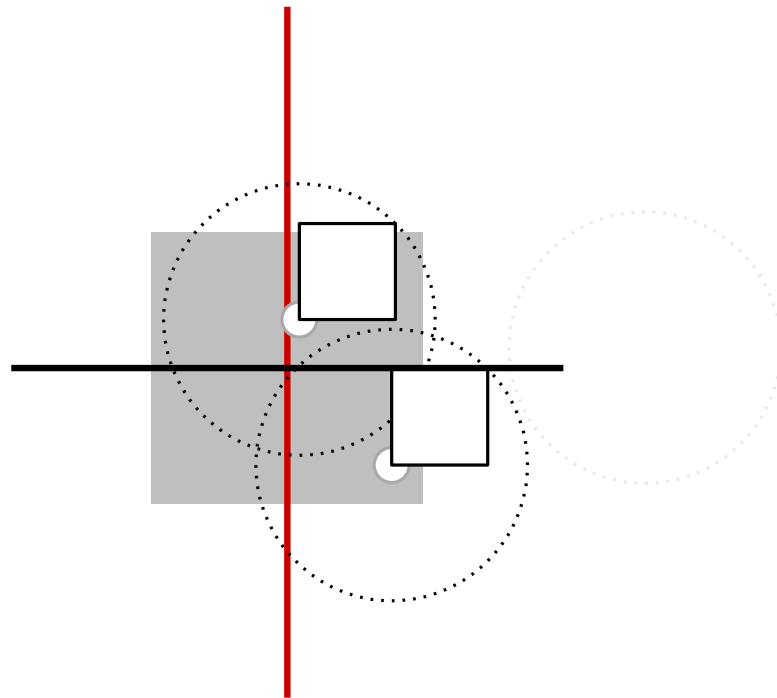




- split set of labels into **four** sets
- find optimal solution for each set separately



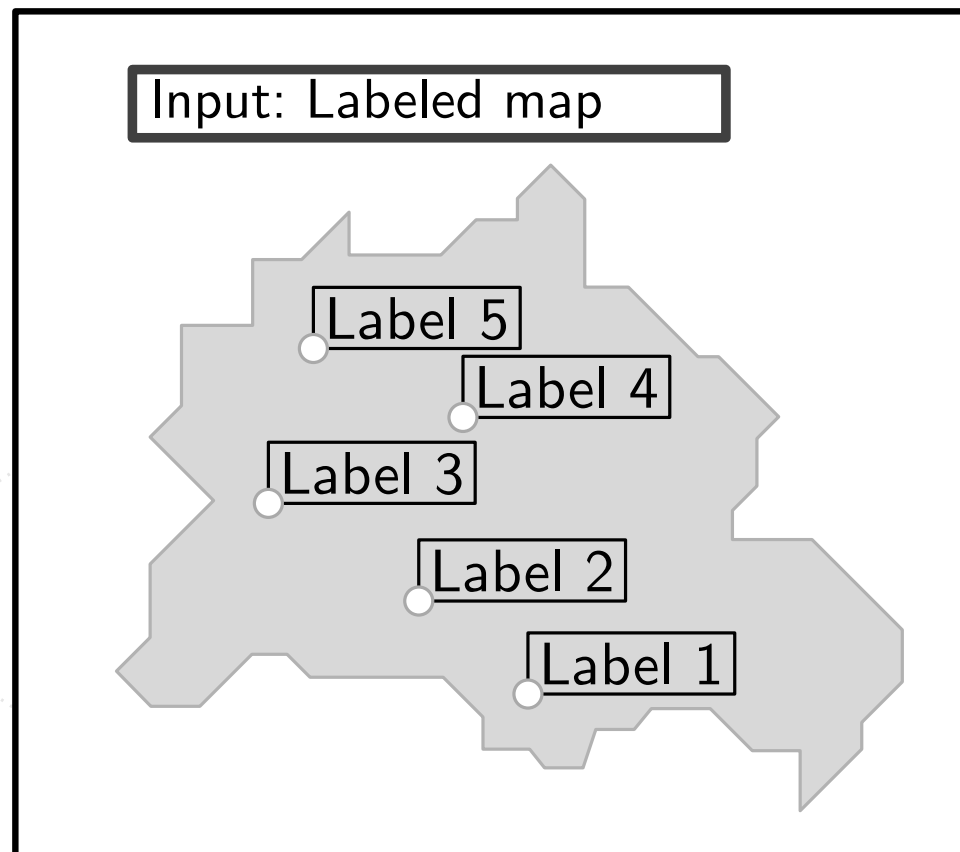
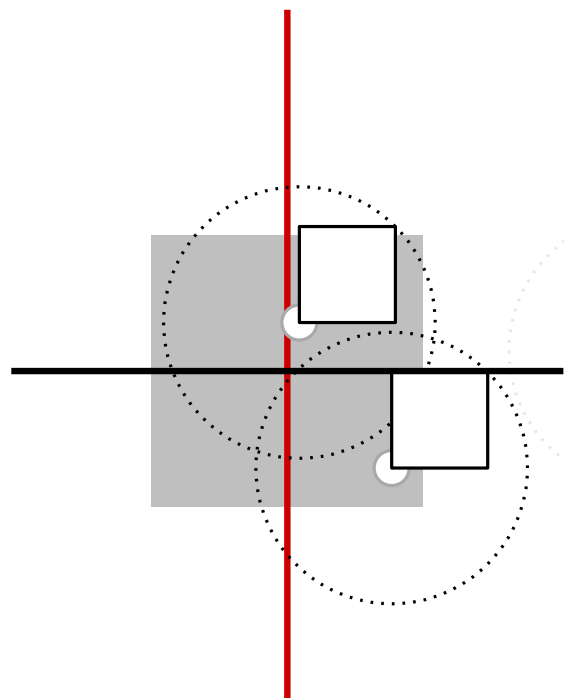
one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal



- split set of labels into **four** sets 
 - find optimal solution for each set separately 
- one of those solutions is a **1/4**-approximation

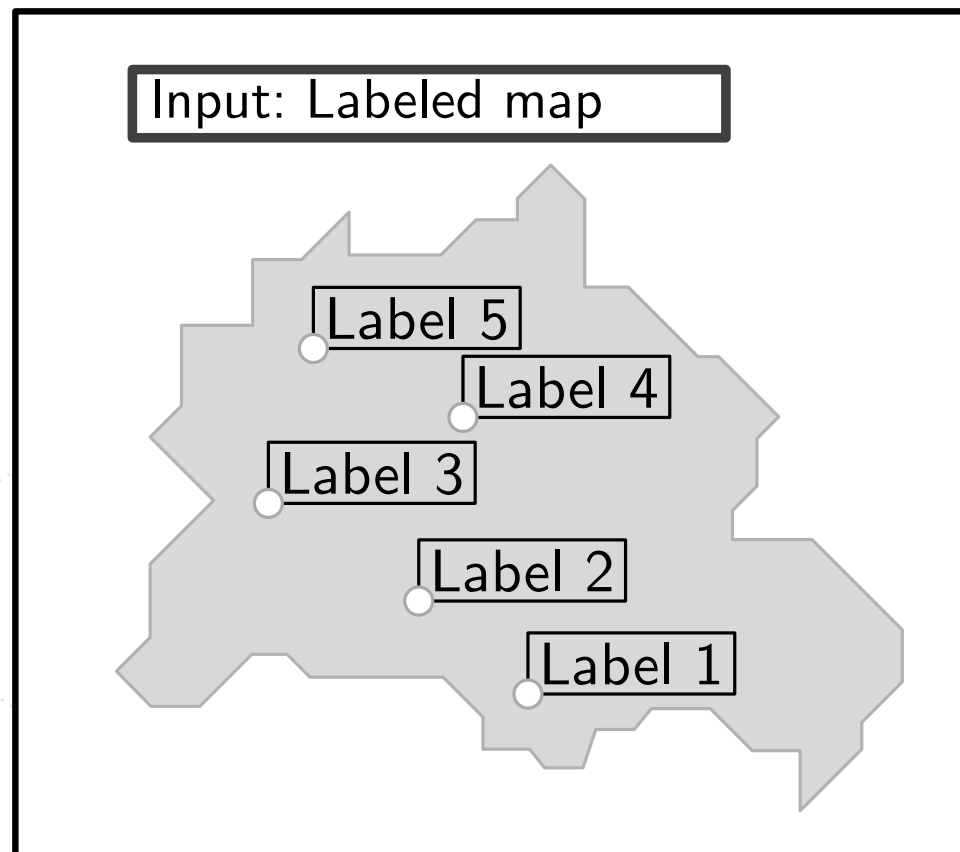
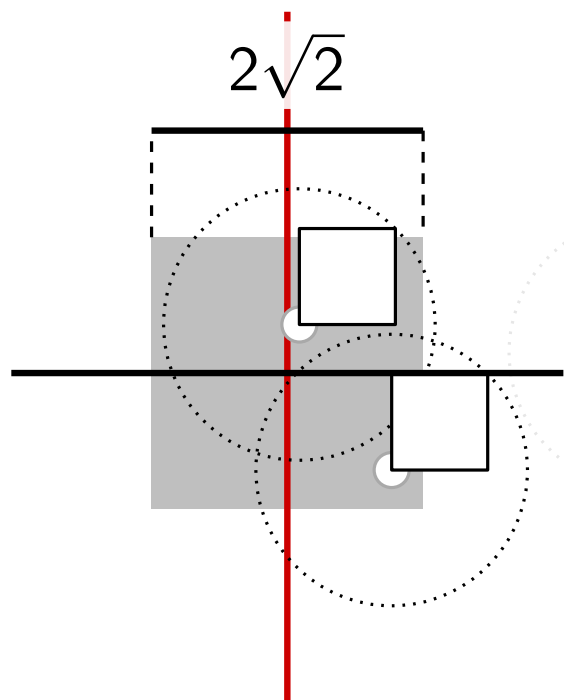
1/4-Approximation of MaxTotal



- split set of labels into **four** sets ■ ■ ■ ■
■ ■ ■ ■
- find optimal solution for each set separately

one of those solutions is a **1/4**-approximation

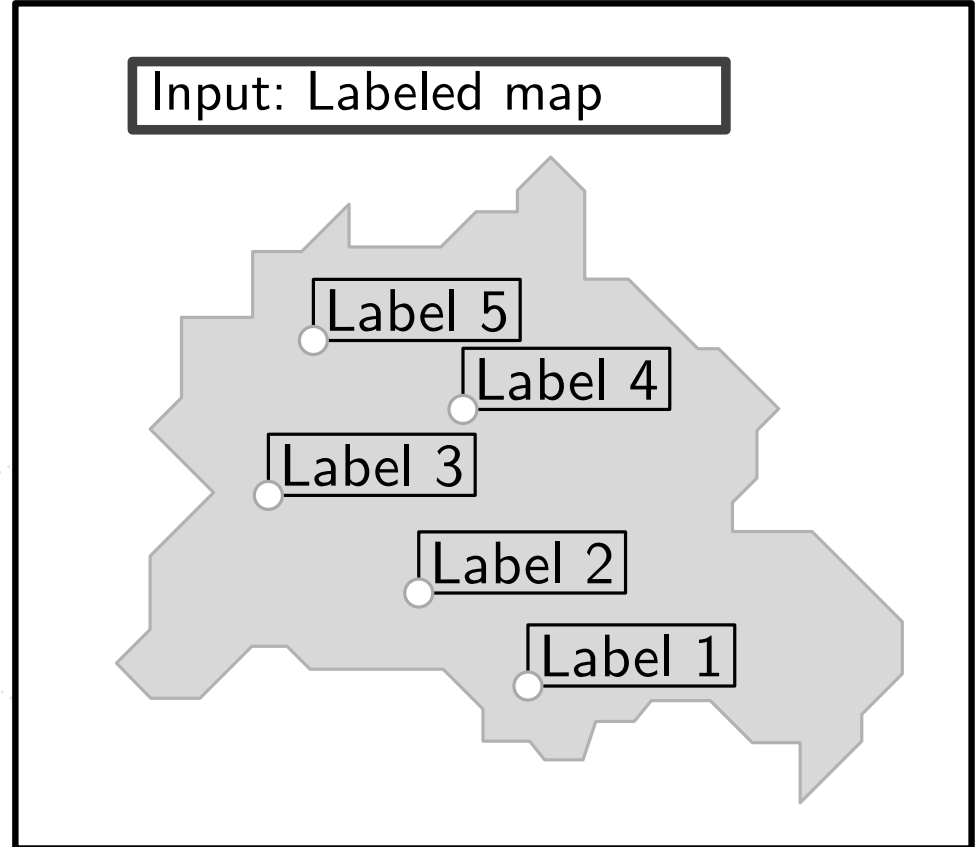
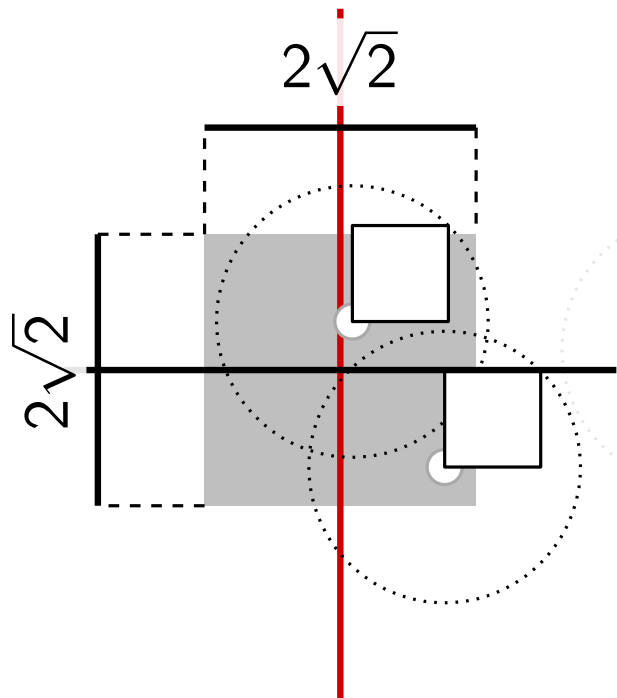
1/4-Approximation of MaxTotal



- split set of labels into **four** sets ■ ■ ■ ■
■ ■ ■ ■
- find optimal solution for each set separately

one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal



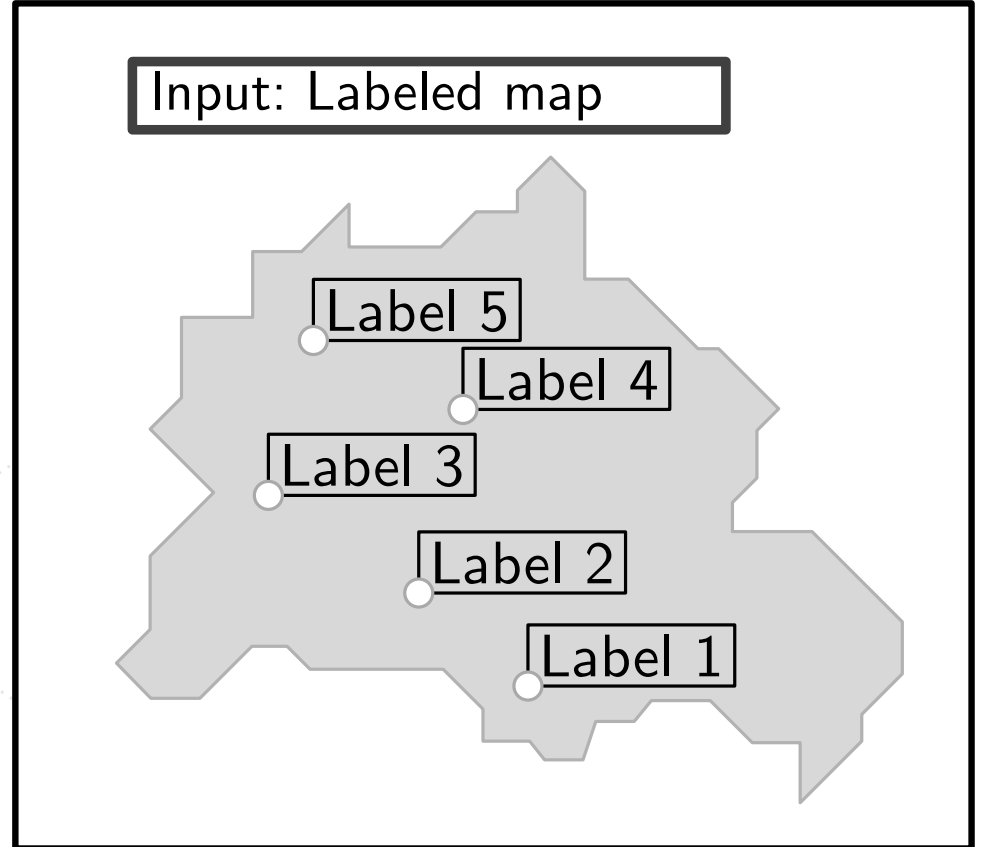
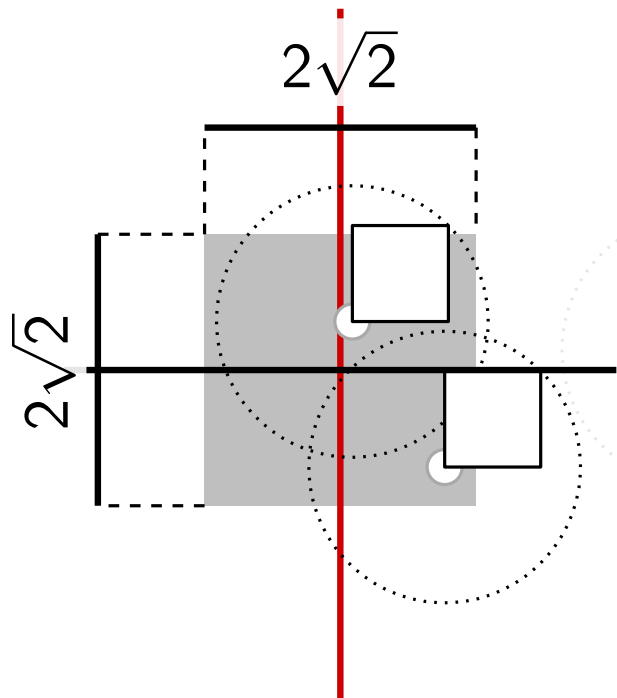
- split set of labels into **four** sets



- find optimal solution for each set separately

one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal



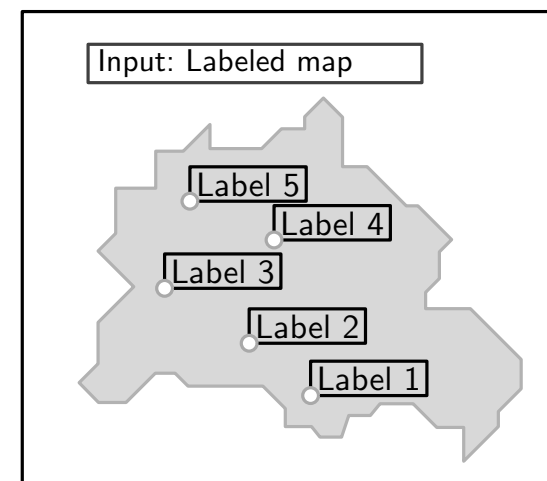
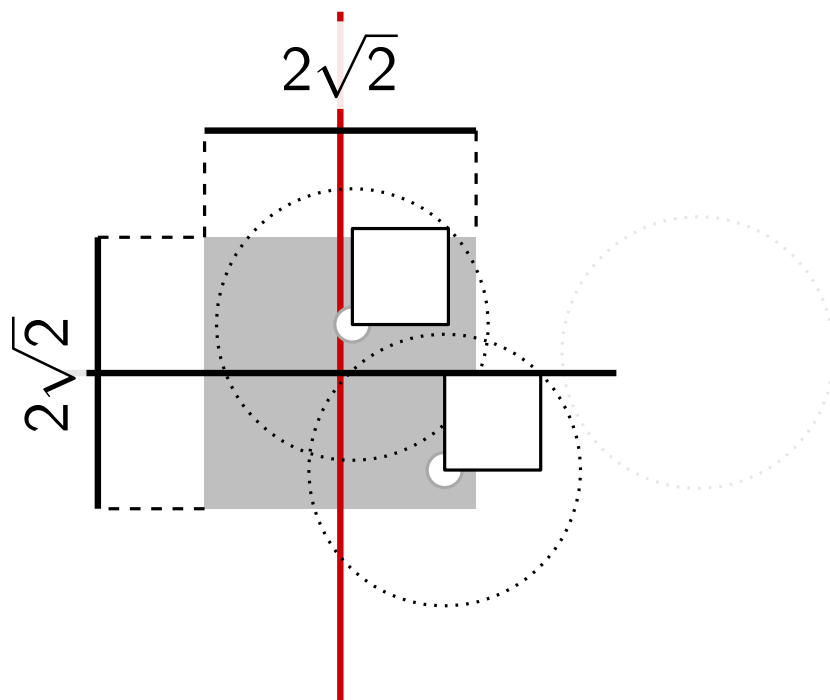
■ split set of labels into **four** sets



■ find optimal solution for each set separately

one of those solutions is a **1/4**-approximation

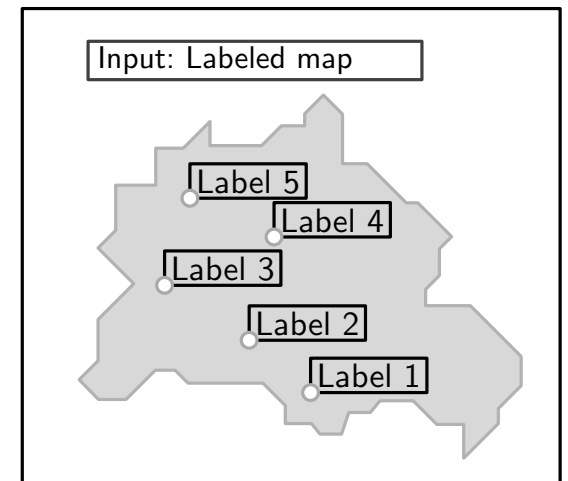
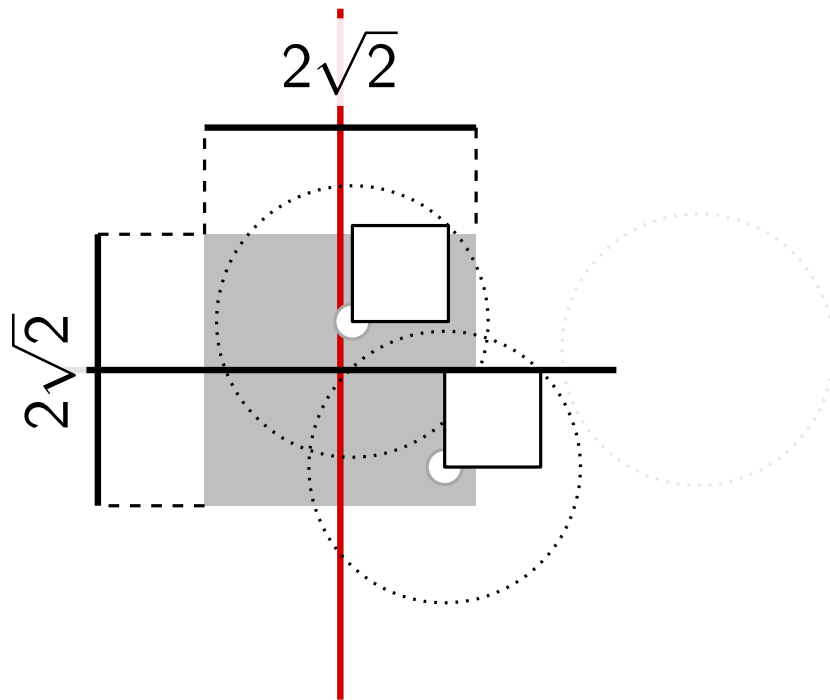
1/4-Approximation of MaxTotal



- split set of labels into **four** sets ■ ■ ■ ■
■ ■ ■ ■
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal

(i) constant number of labels inside square



- split set of labels into **four** sets
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation



1/4-Approximation of MaxTotal

(i) constant number

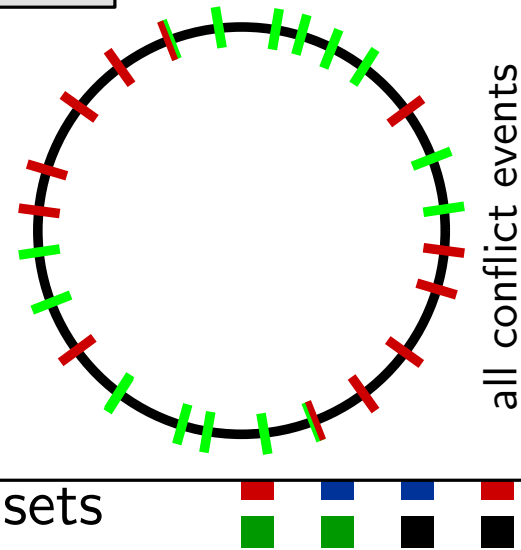
Discretization Lemma:

There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.

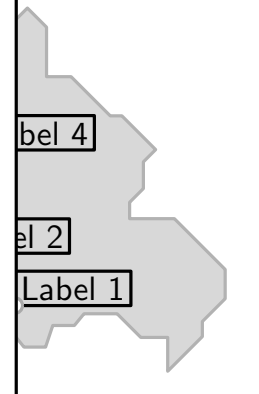
only for MaxTotal

find optimal solution:

- compute all conflict events
- per label $\mathcal{O}(n^4)$ active range candidates
- determine all possible combinations



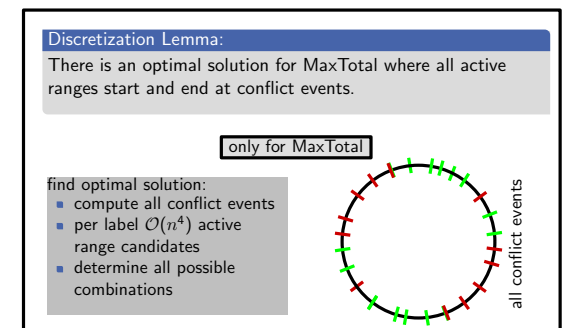
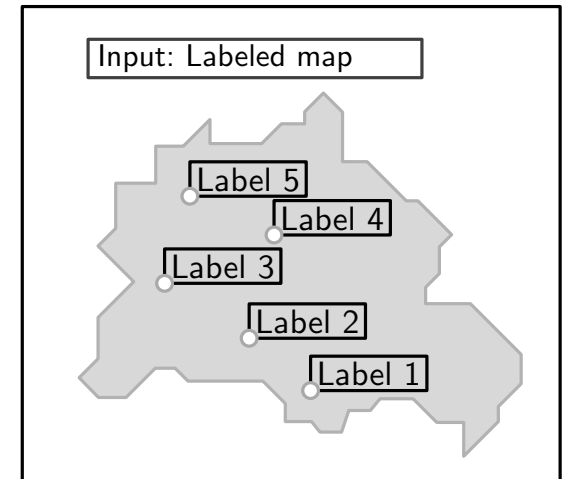
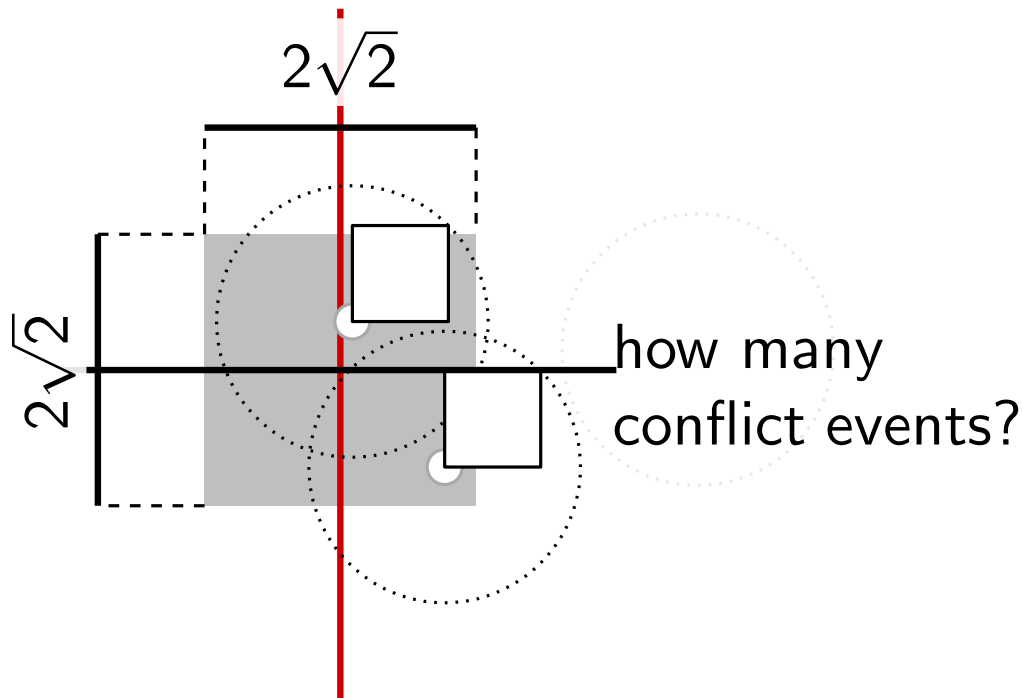
Input: Labeled map



- split set of labels into **four** sets
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation

1/4-Approximation of MaxTotal

(i) constant number of labels inside square

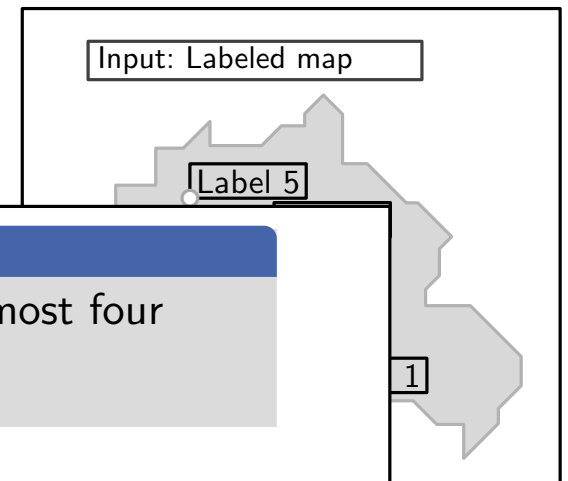


- split set of labels into **four** sets
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation



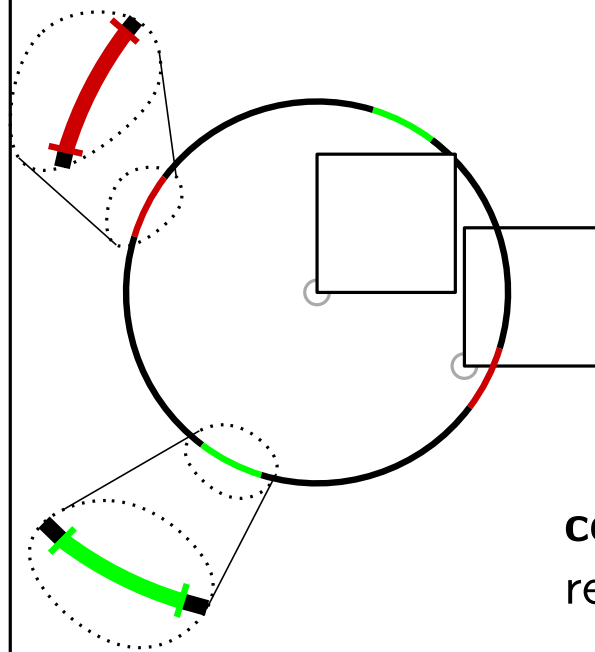
1/4-Approximation of MaxTotal

(i) constant number of labels inside square



Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



soft conflict: labels overlap

hard conflict: label overlaps anchor point

at most 8 such events per pair

conflict events: begin/end of a conflict region (borders of labels intersect)

one

1/4-Approximation of MaxTotal

(i) constant number of labels inside square

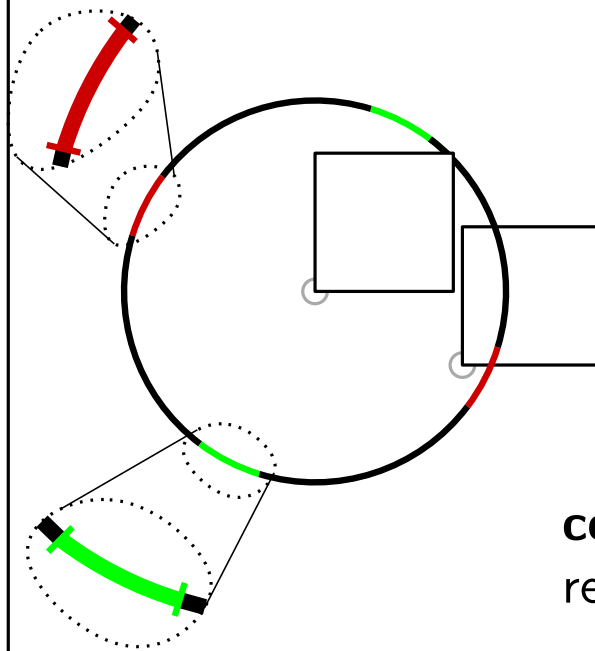
Input: Labeled map

Label 5

1

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict regions.



soft conflict: labels overlap

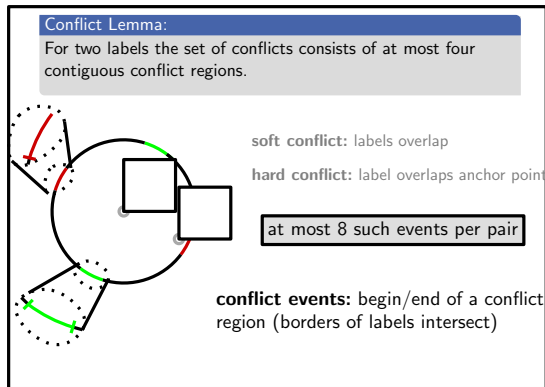
hard conflict: label overlaps anchor point

at most 8 such events per pair

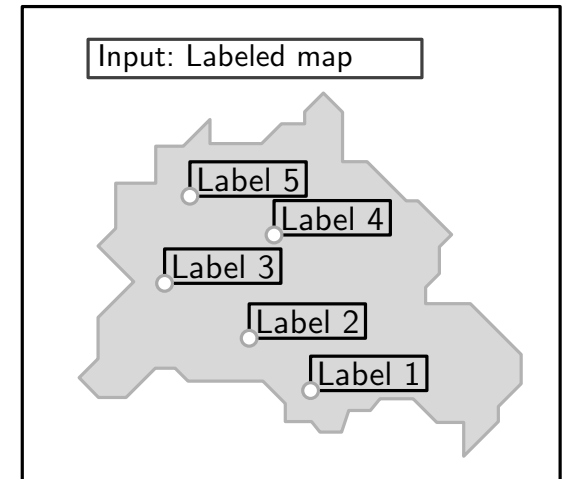
conflict events: begin/end of a conflict region (borders of labels intersect)

one

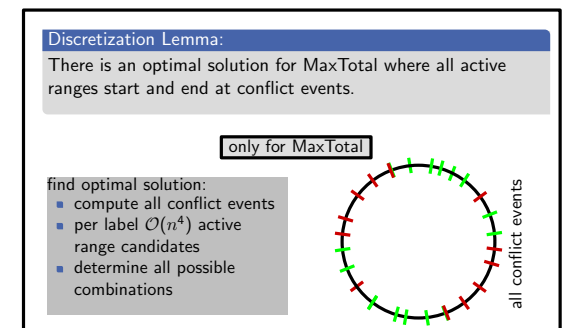
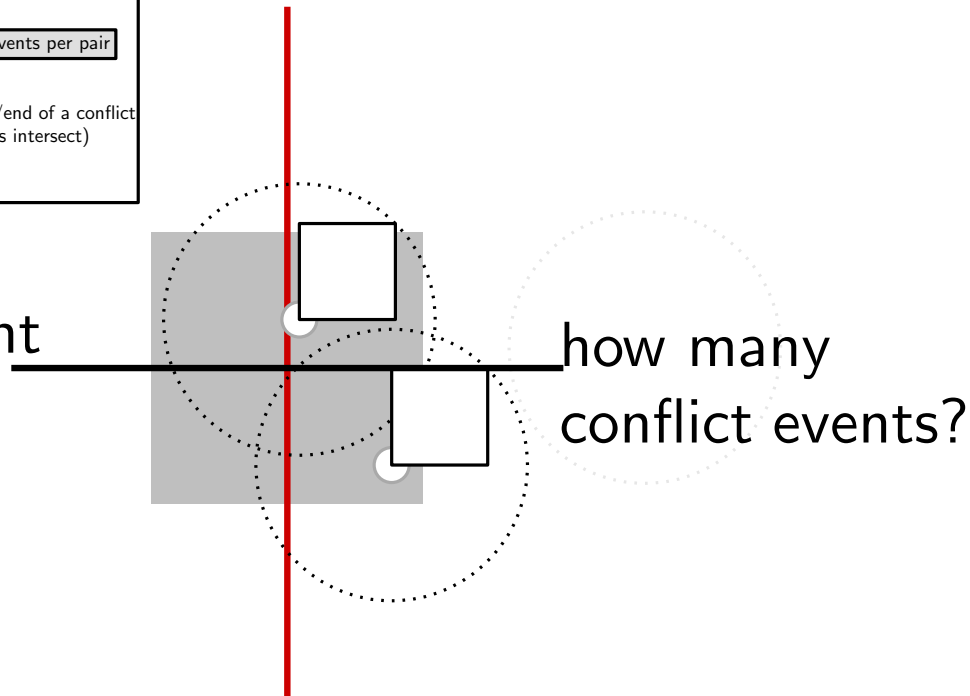
1/4-Approximation of MaxTotal



(i) constant number of labels inside square



(ii) #conflict events: constant



- split set of labels into **four** sets
 - find optimal solution for each set separately
- one of those solutions is a **1/4**-approximation

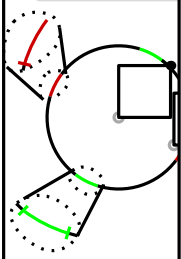


1/4-Approximation of MaxTotal

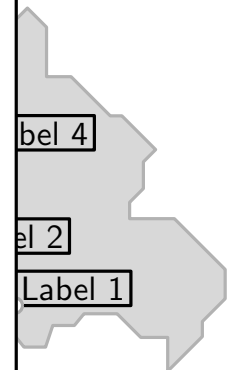
(i) constant number

Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflicts



Input: Labeled map



Discretization Lemma:

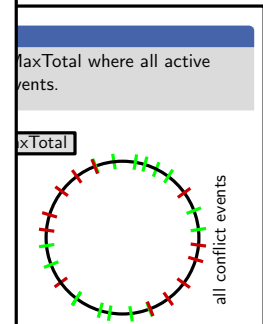
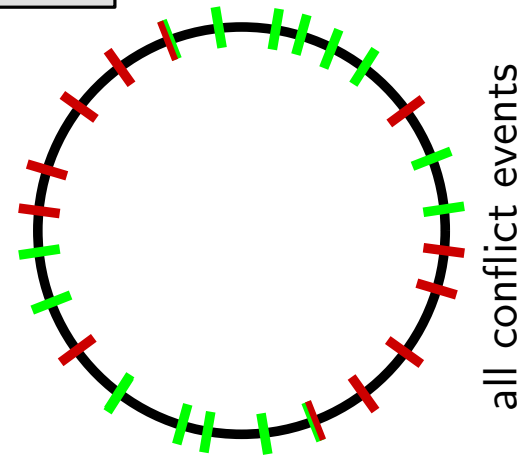
- There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.

only for MaxTotal

(ii) # of conflict events

find optimal solution:

- compute all conflict events
- per label $\mathcal{O}(n^4)$ active range candidates
- determine all possible combinations



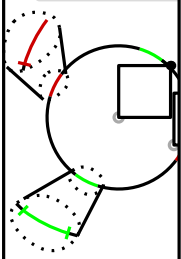
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1/4-Approximation of MaxTotal

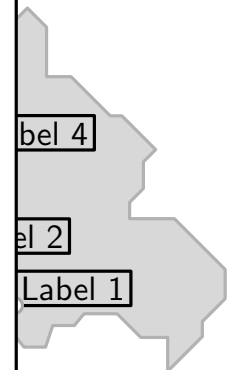
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Input: Labeled map



Discretization Lemma:

- There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.

only for MaxTotal

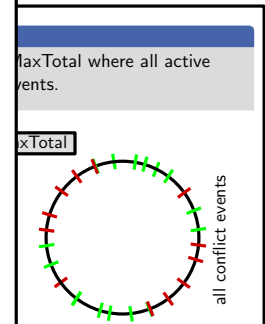
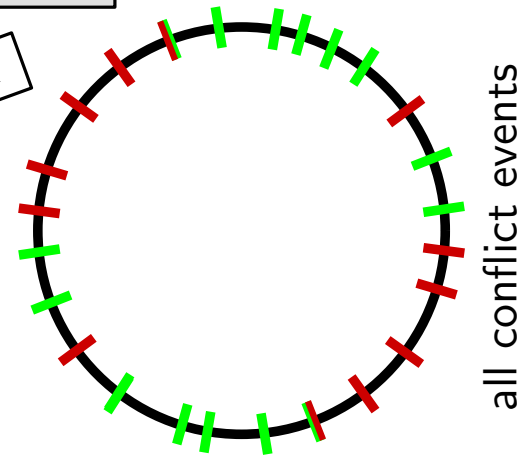
(ii) # of conflict events

find optimal solution:

- compute all conflict events
- per label $\mathcal{O}(1)$ active range candidates
- determine all possible combinations

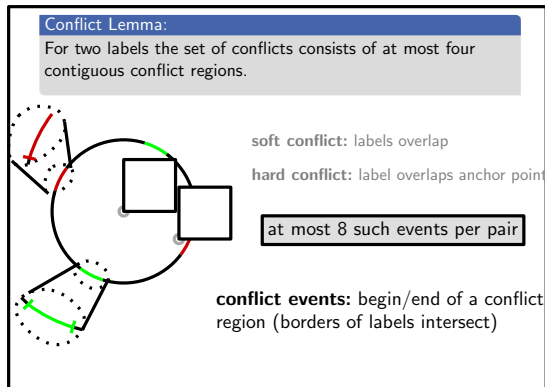
constant

constant

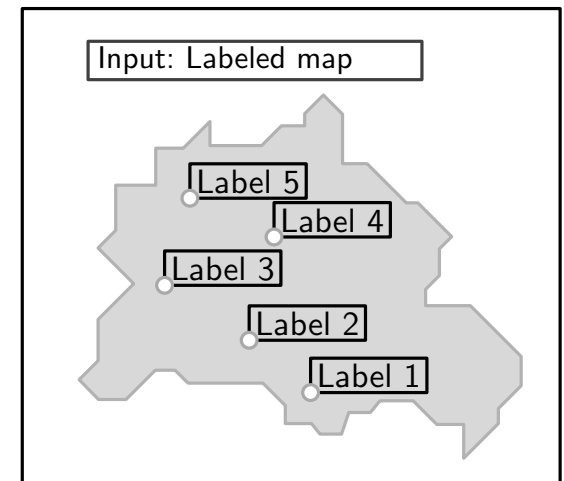


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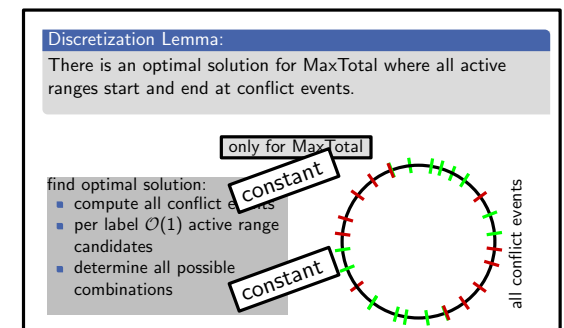
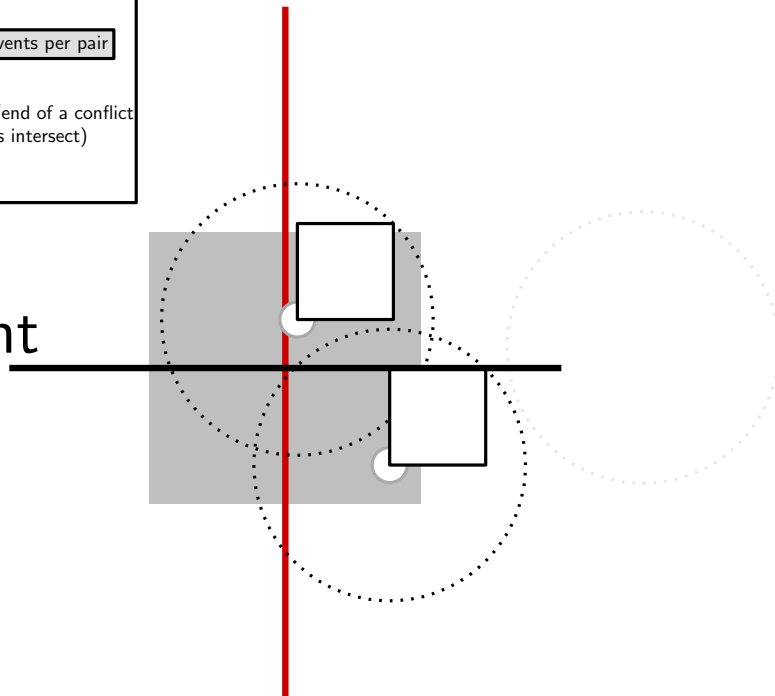
1/4-Approximation of MaxTotal



(i) constant number of labels inside square



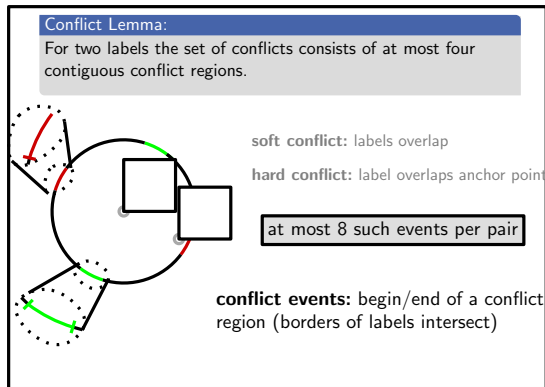
(ii) #conflict events: constant



- split set of labels into **four** sets
 - find optimal solution for each set separately
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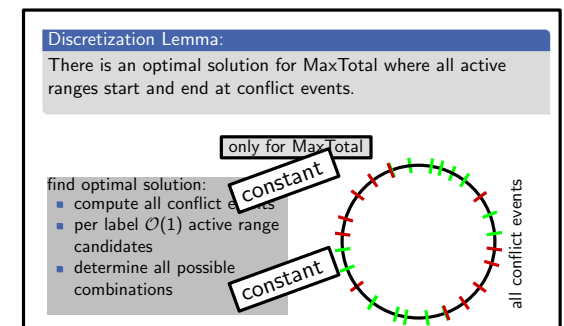
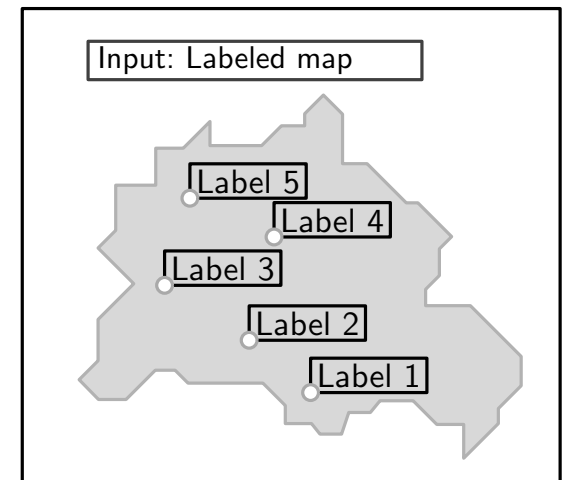
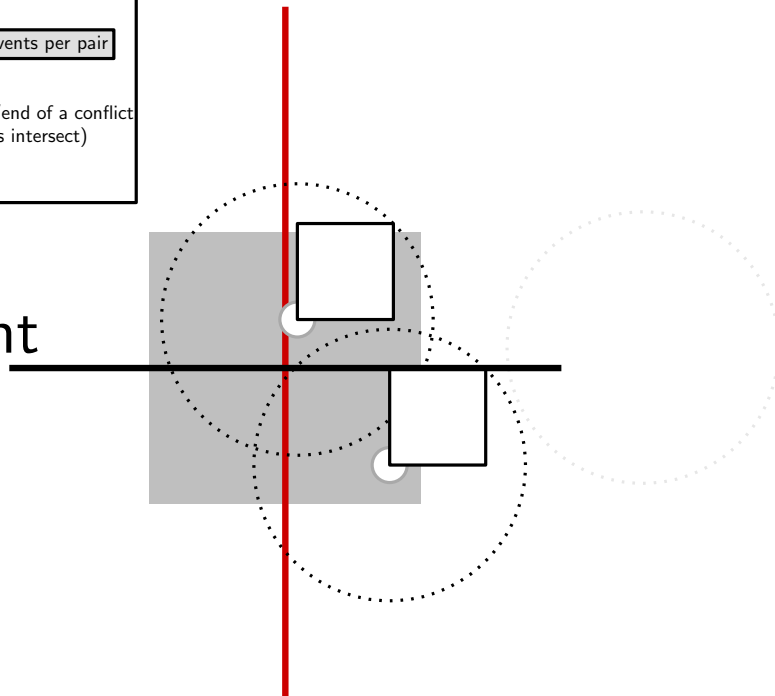


1/4-Approximation of MaxTotal



(i) constant number of labels inside square

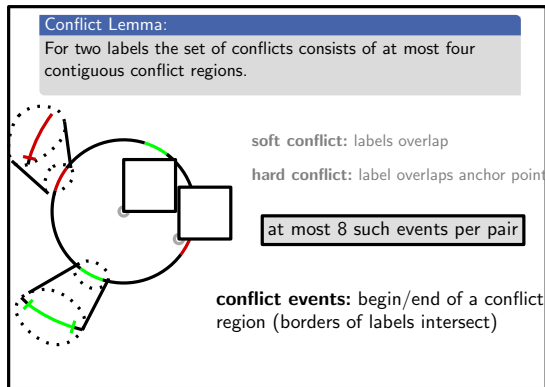
(ii) #conflict events: constant



Theorem 2

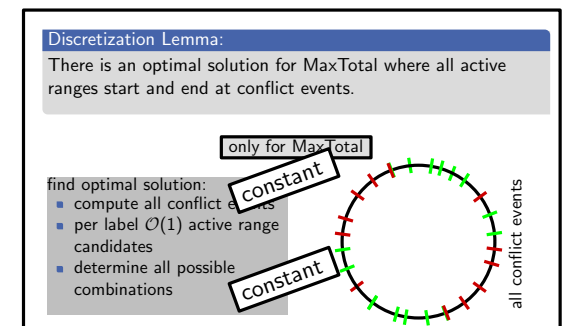
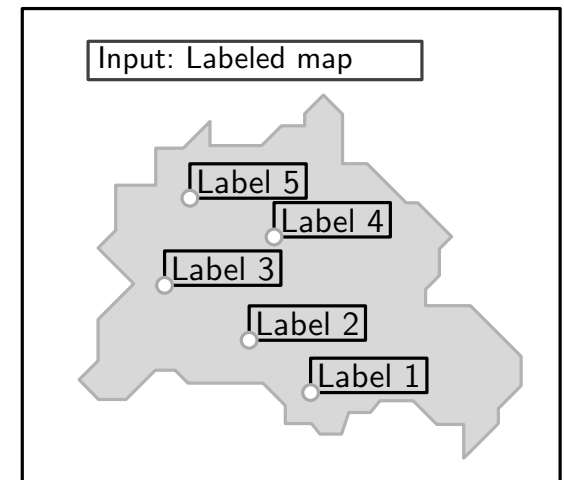
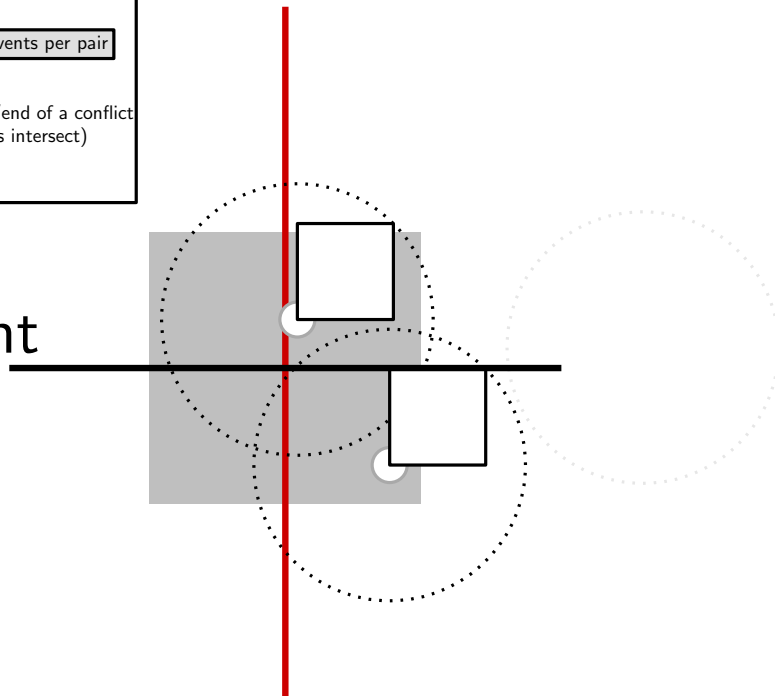
There exists a 1/4-approximation algorithm for MaxTotal with time complexity $O(n \log n)$

1/4-Approximation of MaxTotal



(i) constant number of labels inside square

(ii) #conflict events: constant



Theorem 2

There exists a 1/4-approximation algorithm for MaxTotal with time complexity $O(n \log n)$

An EPTAS for MaxTotal for unit squares*

(E)PTAS?

polynomial-**t**ime **a**pproximation **s**cheme

- approximation algorithm that takes the **instance** and a parameter $\varepsilon > 0$ as input
- produces a solution that is $(1 - \varepsilon)\text{OPT}$ within the optimal solution
- running time is polynomial in n (but typically not in ε)

$$O(n^{1/\varepsilon}), O(n^{1/\varepsilon^{100}}), O(1/\varepsilon^{1/\varepsilon} \cdot n^2 \log n)$$

(E)PTAS?

polynomial-**t**ime **a**pproximation **s**cheme

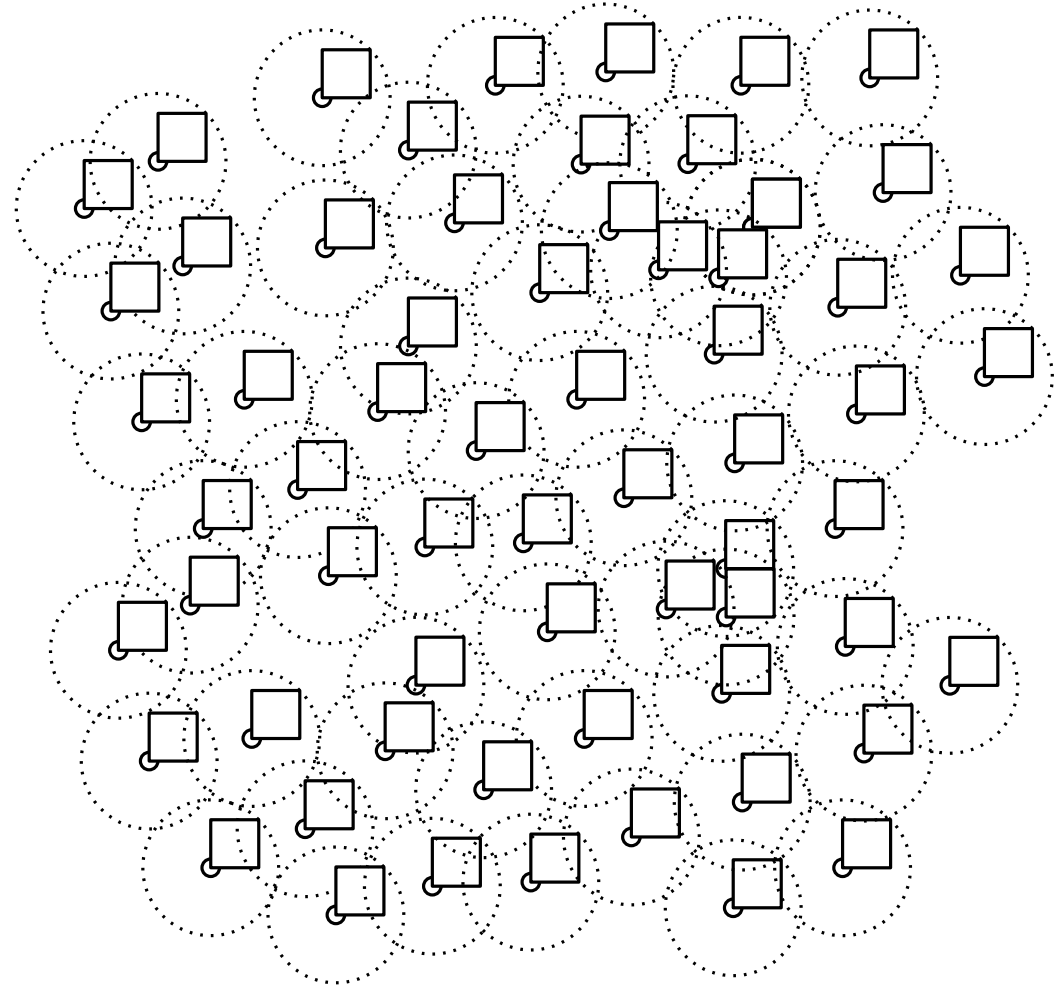
- approximation algorithm that takes the **instance** and a parameter $\varepsilon > 0$ as input
- produces a solution that is $(1 - \varepsilon)\text{OPT}$ within the optimal solution
- running time is polynomial in n (but typically not in ε)
 $O(n^{1/\varepsilon})$, $O(n^{1/\varepsilon^{100}})$, $O(1/\varepsilon^{1/\varepsilon} \cdot n^2 \log n)$

efficient **p**olynomial-**t**ime **a**pproximation **s**cheme

- every EPTAS is a PTAS with...
- running time is in n^c for some constant c
 $O(1/\varepsilon^{1/\varepsilon} \cdot n^2 \log n)$

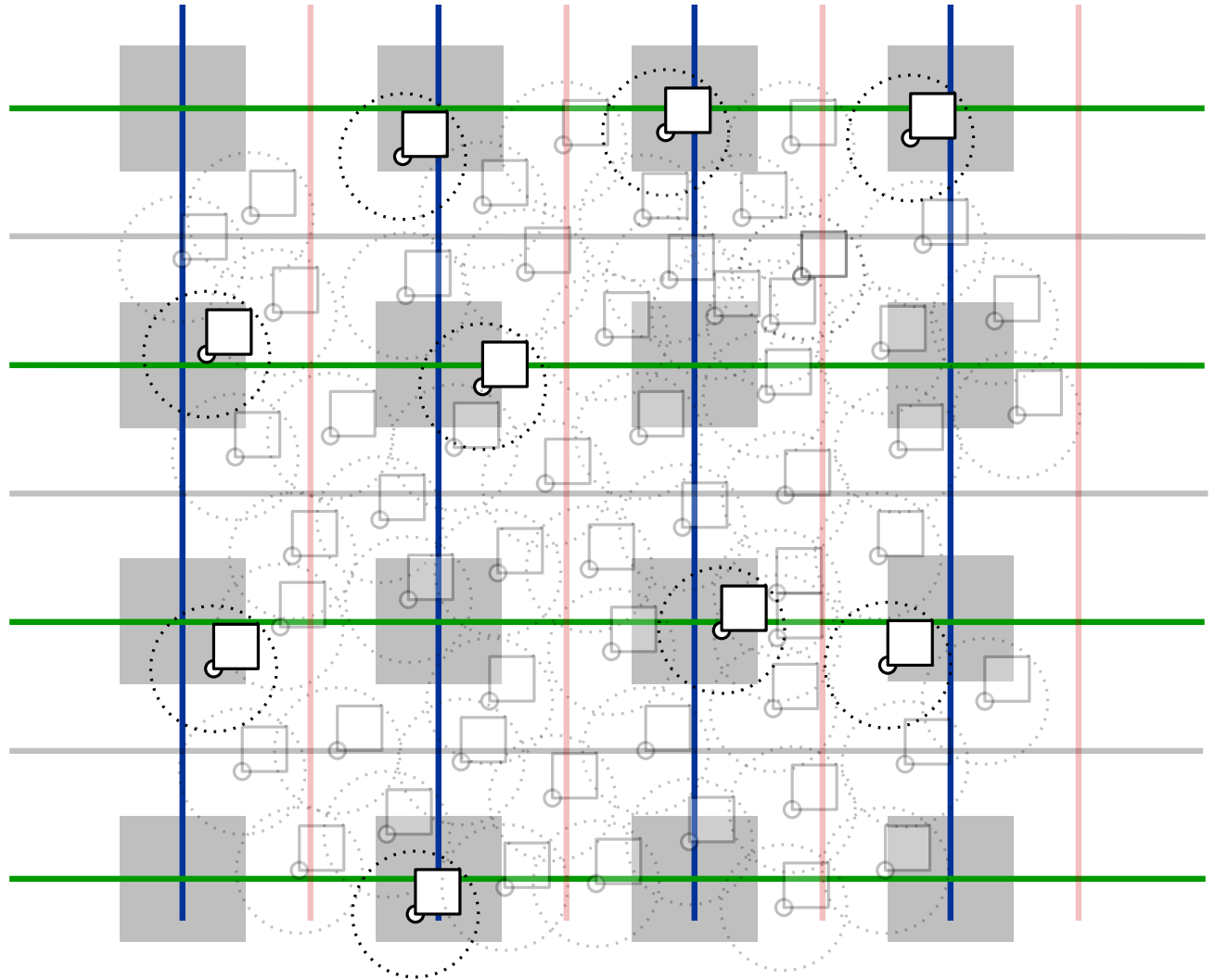
An EPTAS for MaxTotal

use the same principle as before



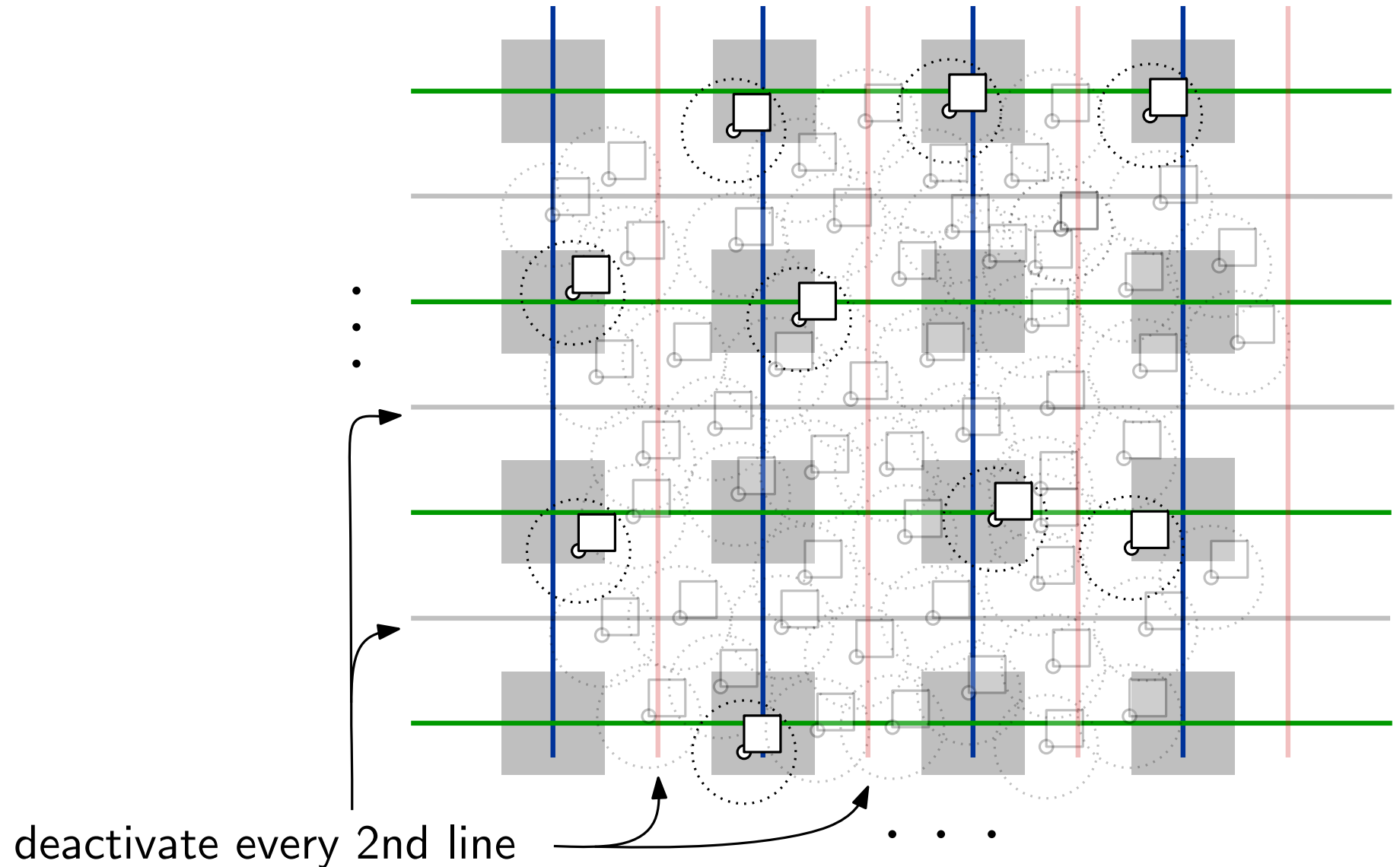
An EPTAS for MaxTotal

use the same principle as before



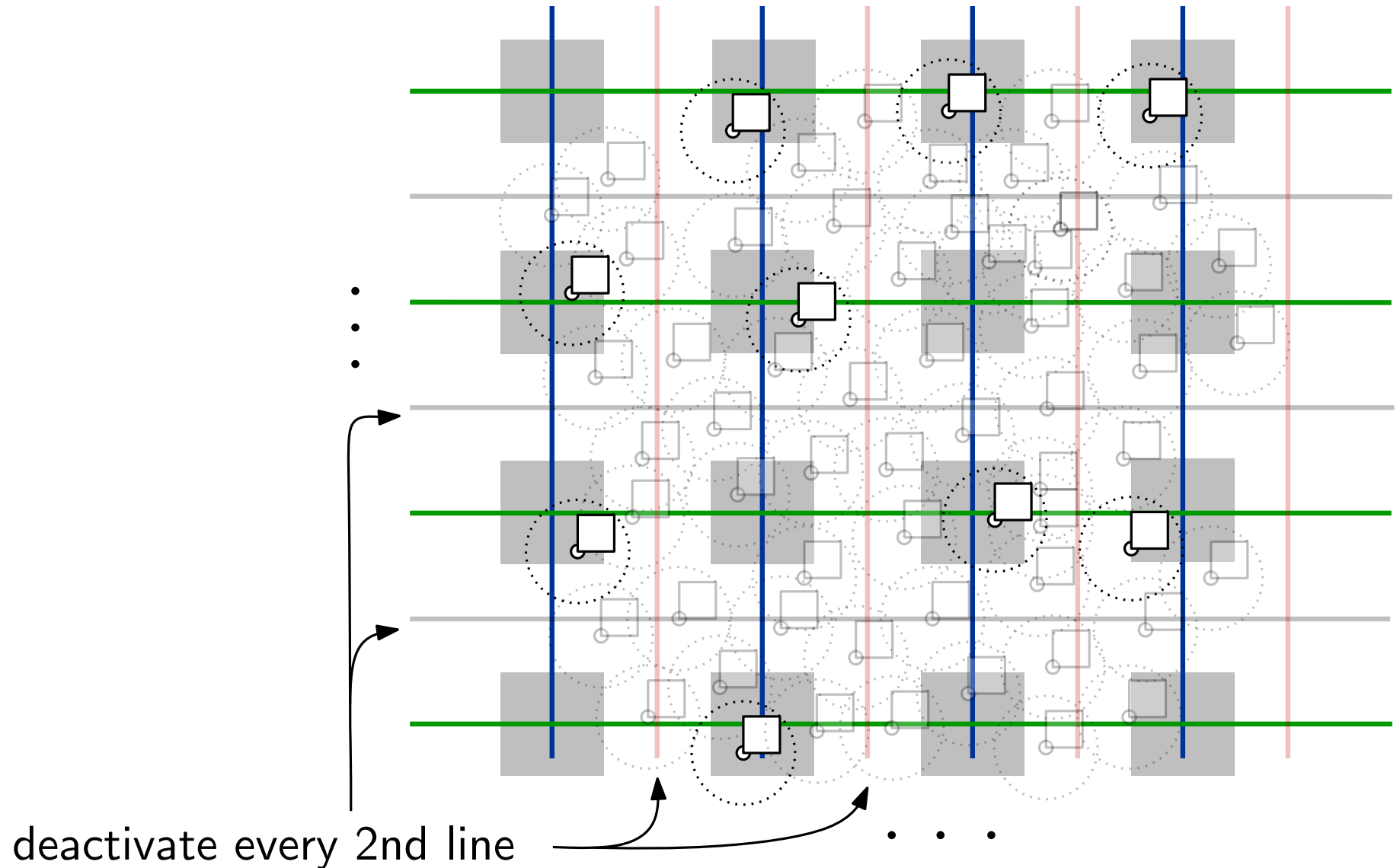
An EPTAS for MaxTotal

use the same principle as before



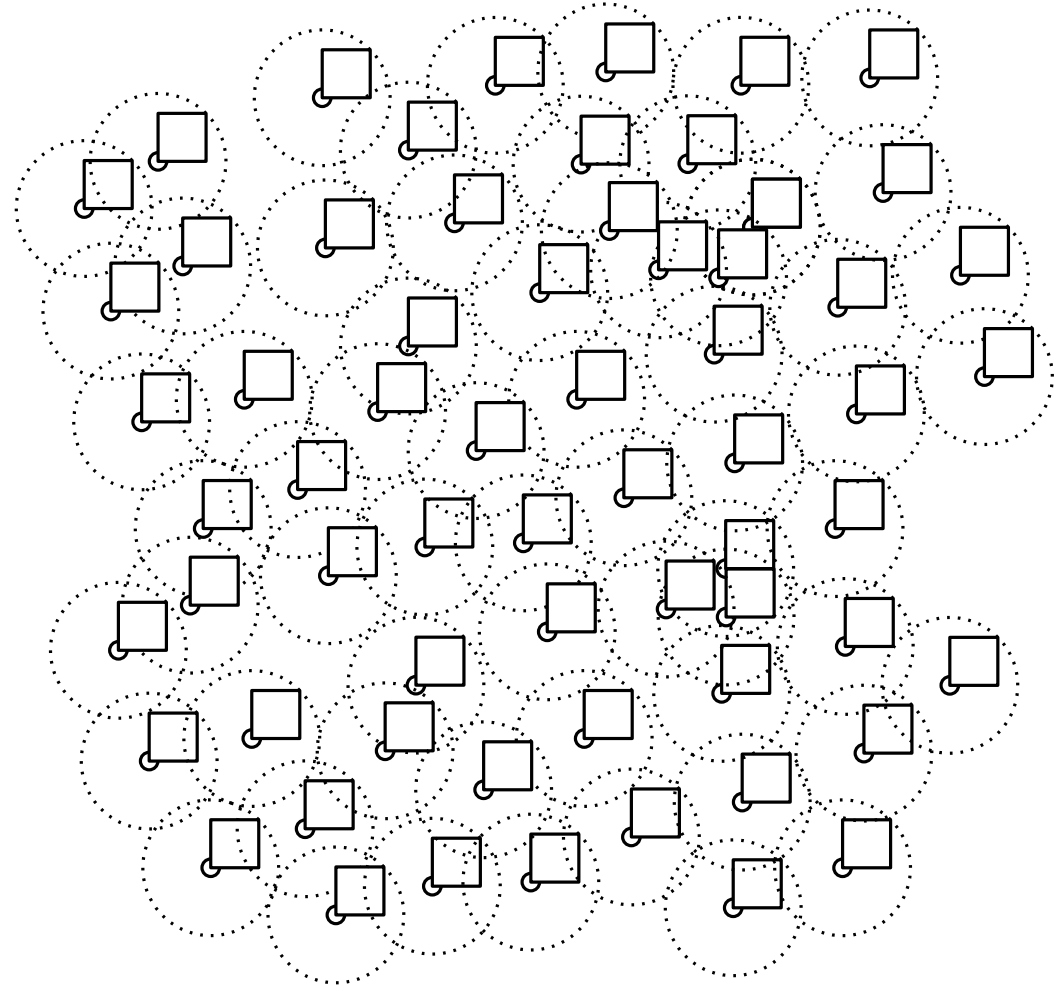
An EPTAS for MaxTotal

use the same principle as before



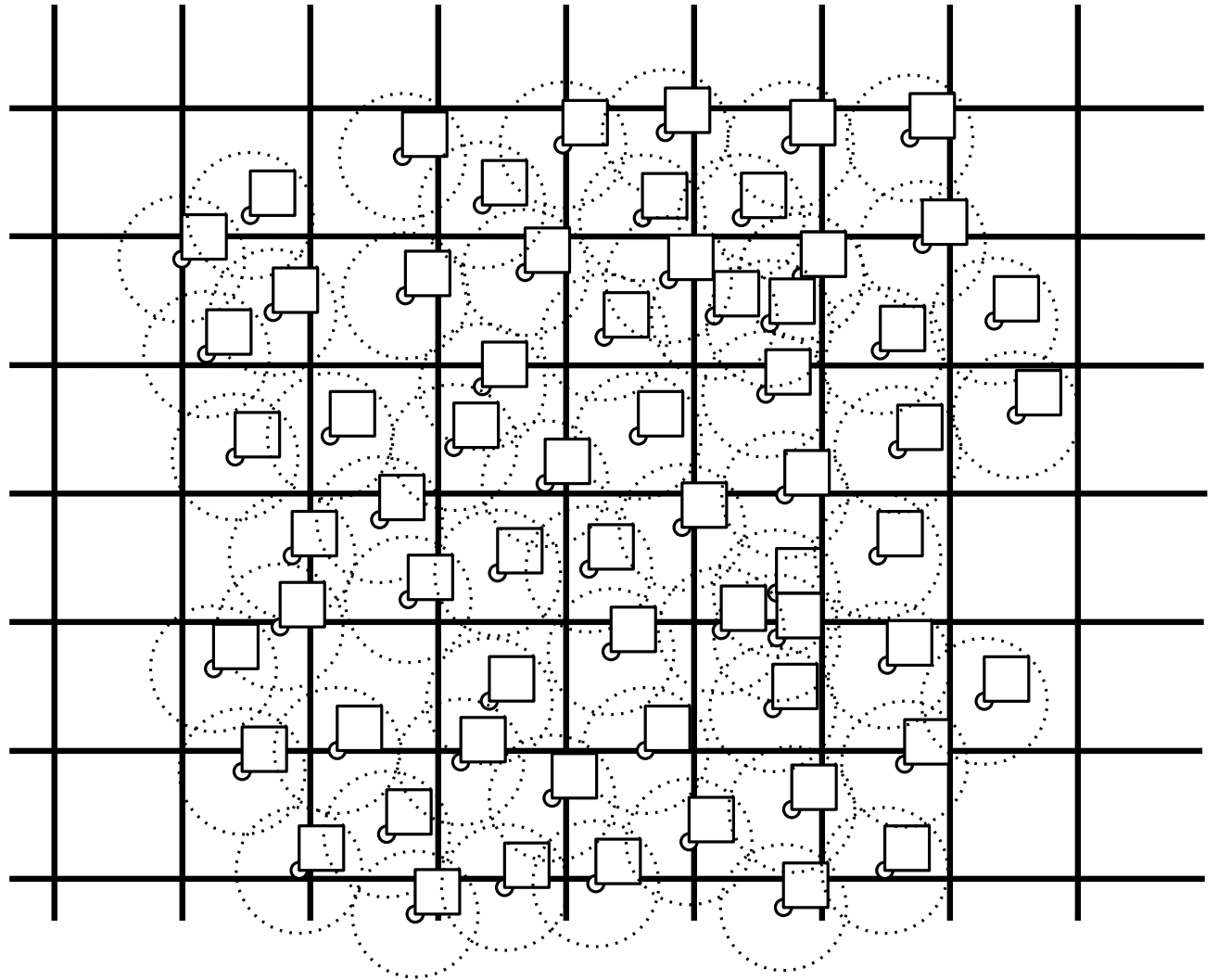
An EPTAS for MaxTotal

use the same principle as before



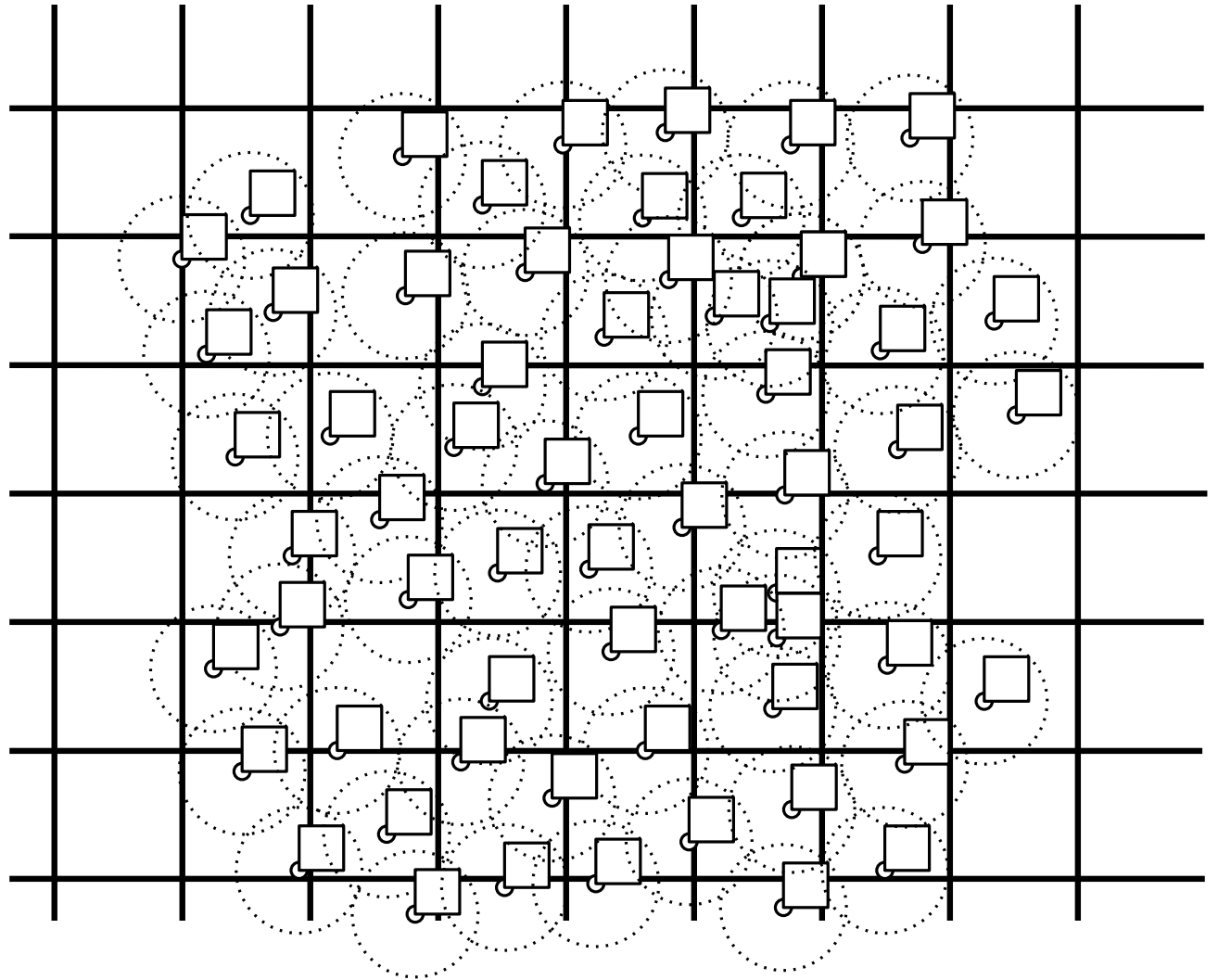
An EPTAS for MaxTotal

use the same principle as before



An EPTAS for MaxTotal

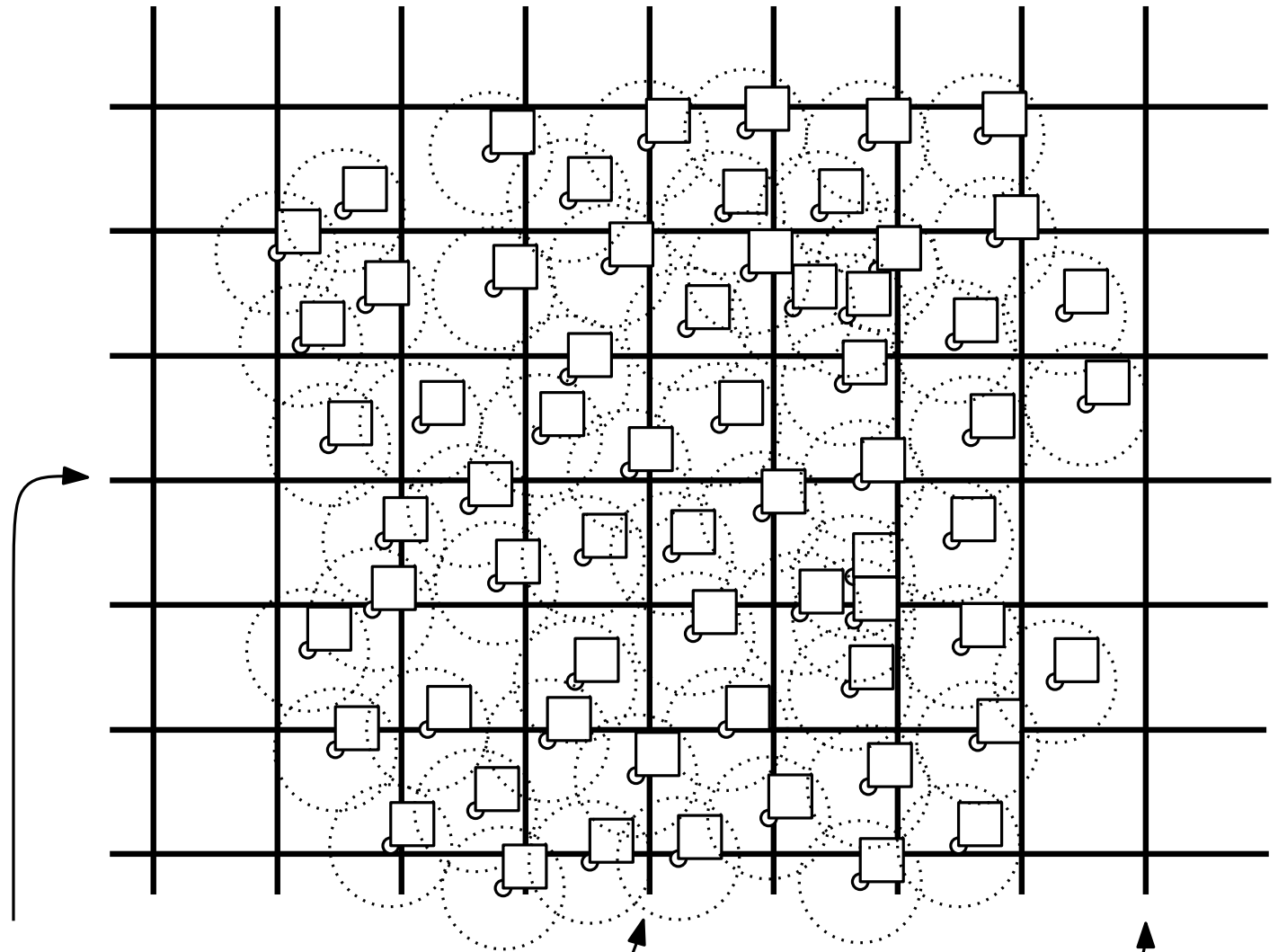
use the same principle as before



deactivate every k -th line

An EPTAS for MaxTotal

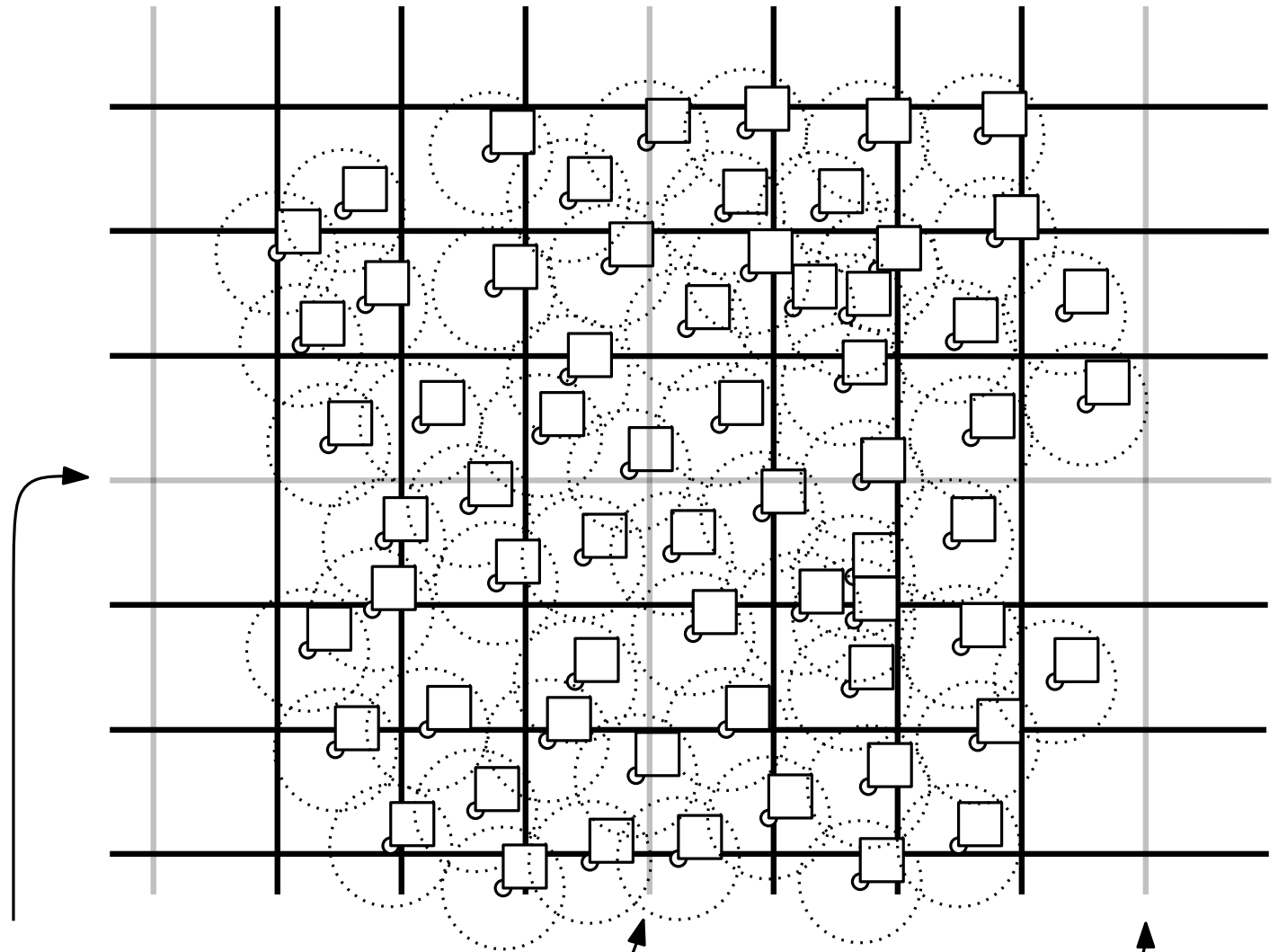
use the same principle as before



deactivate every k -th line

An EPTAS for MaxTotal

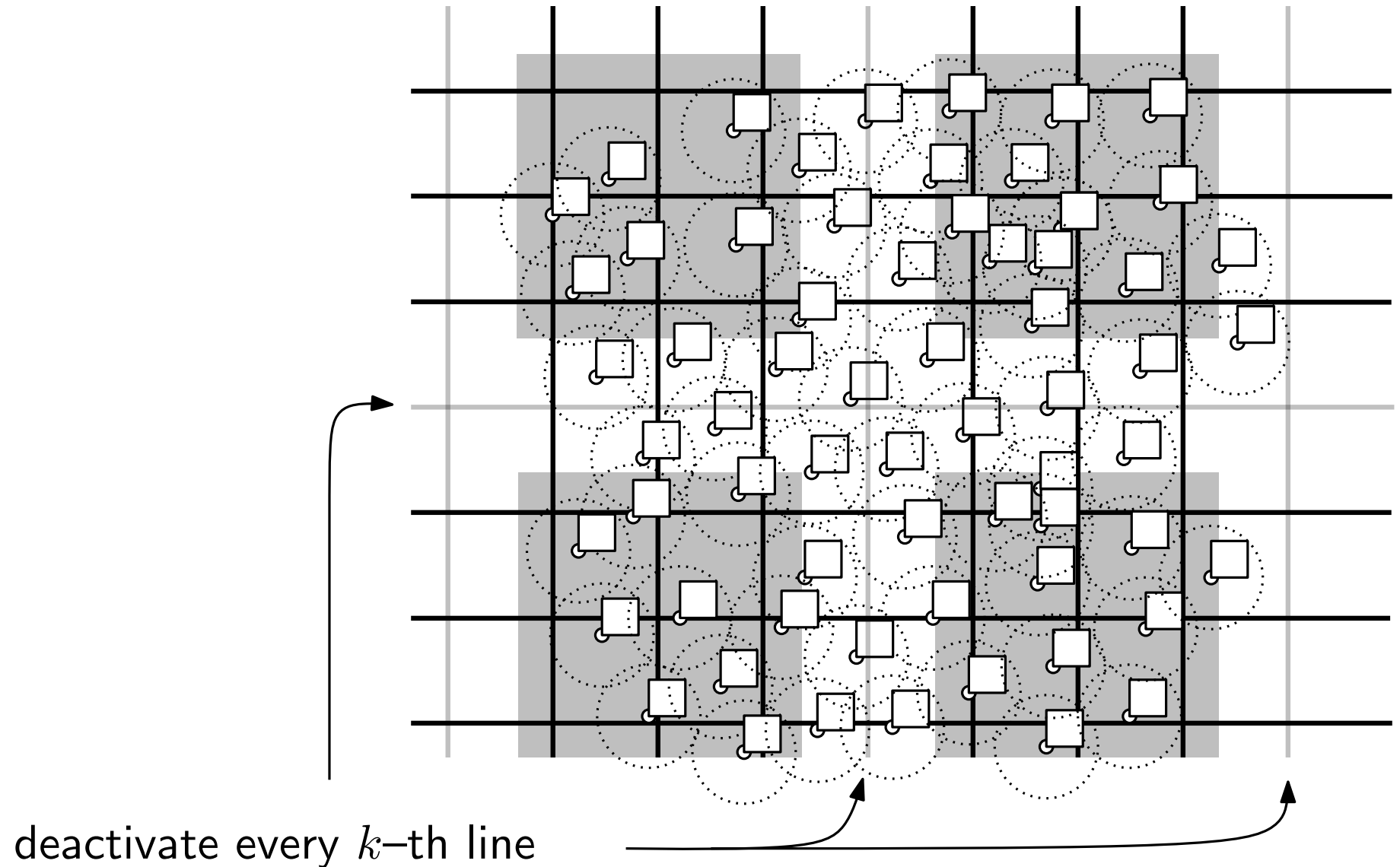
use the same principle as before



deactivate every k -th line

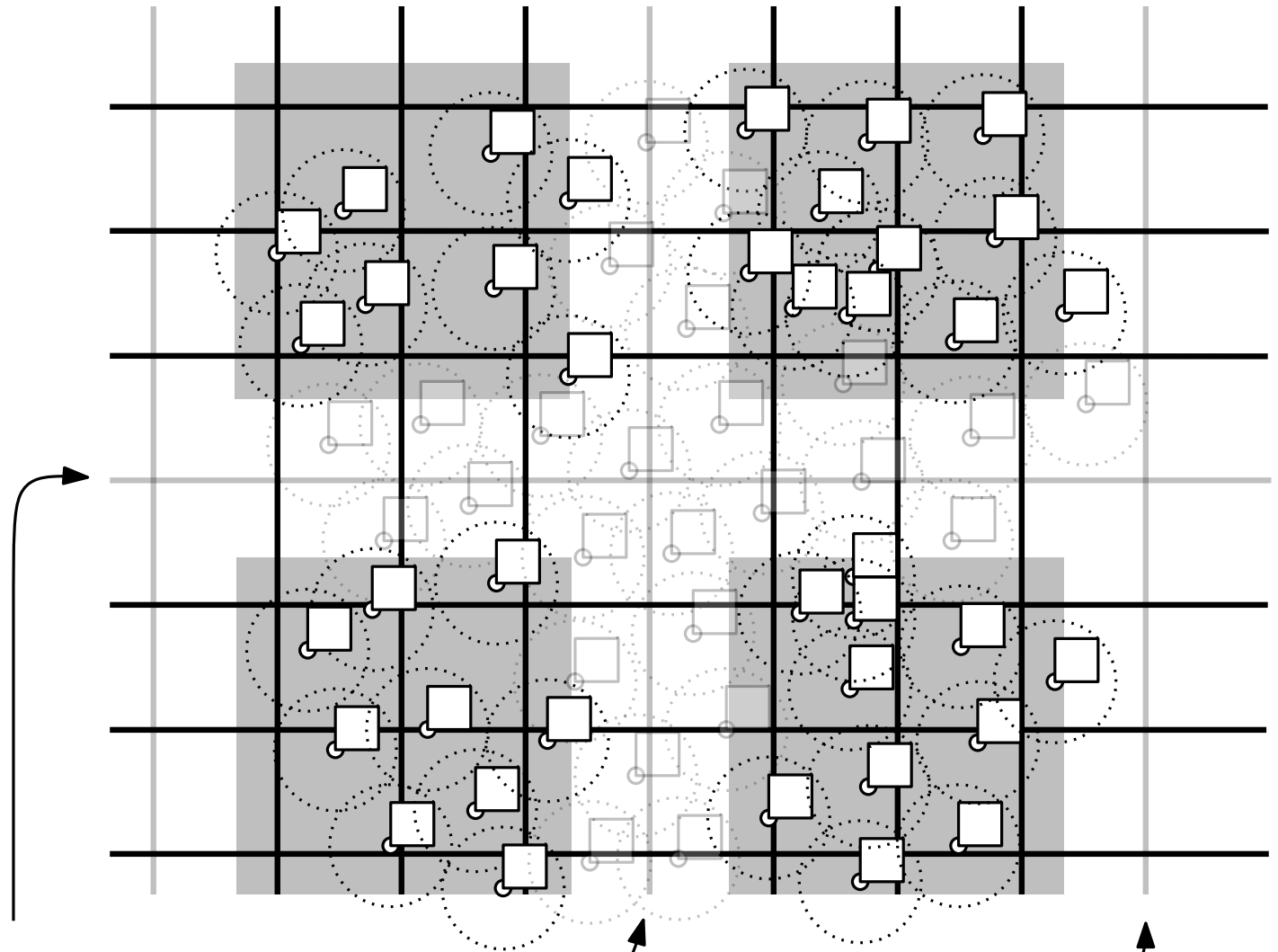
An EPTAS for MaxTotal

use the same principle as before



An EPTAS for MaxTotal

use the same principle as before

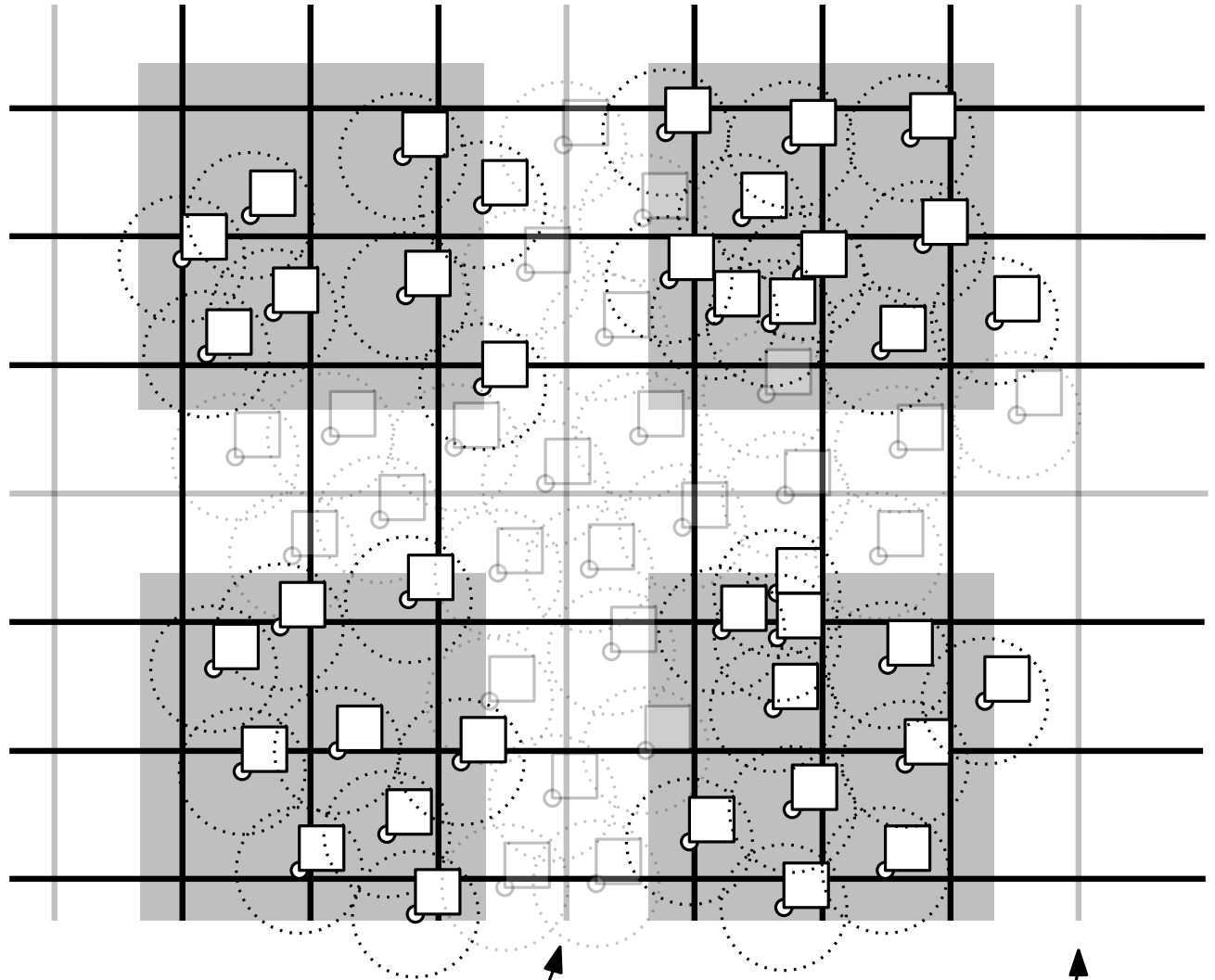


deactivate every k -th line

An EPTAS for MaxTotal

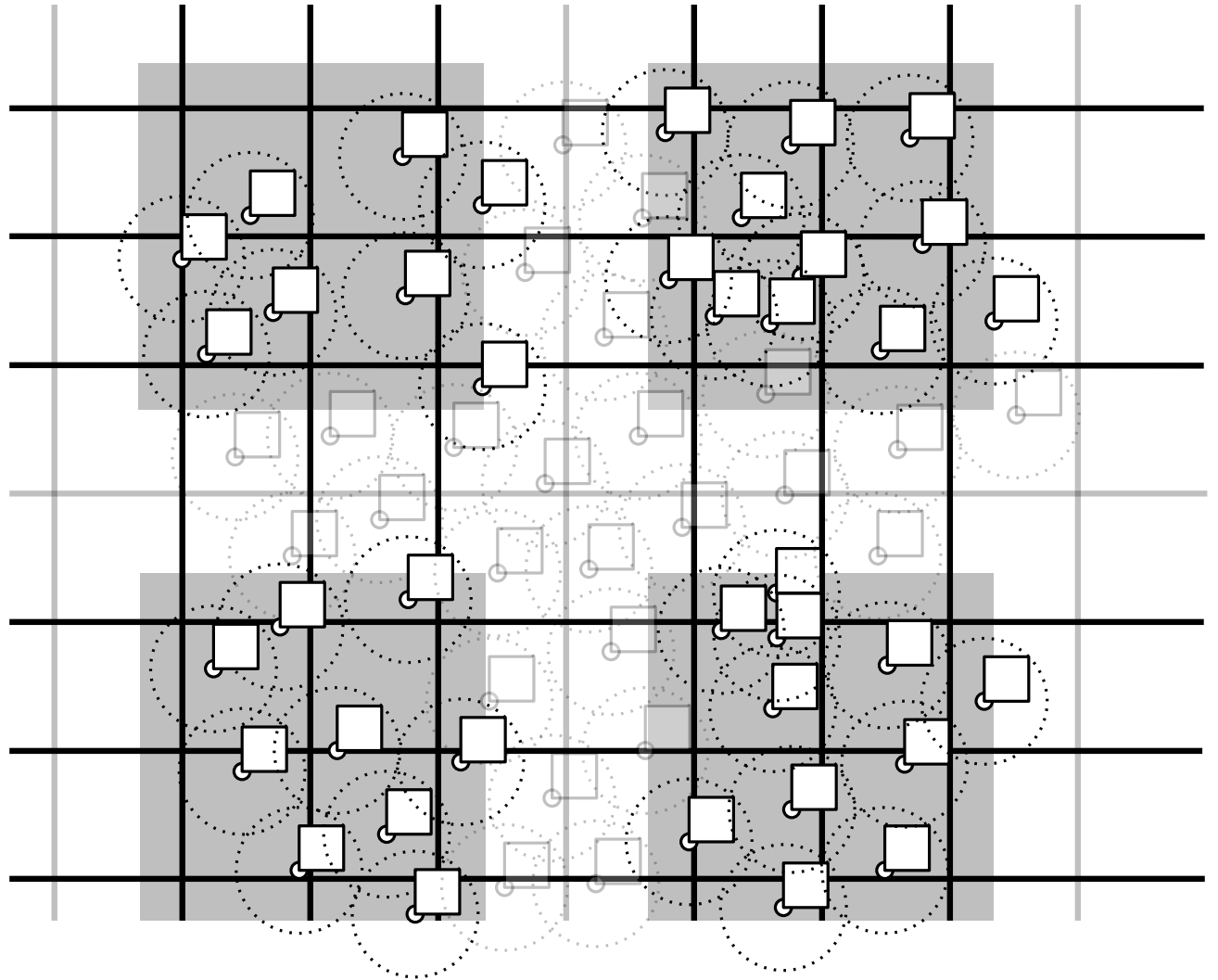
use the same principle as before

- there are k^2 possible combinations
- yielding k^2 sets S_1, \dots, S_{k^2}
- for $k = \lceil 2/\varepsilon \rceil$ one combination yields a $(1 - \varepsilon)$ -approximation

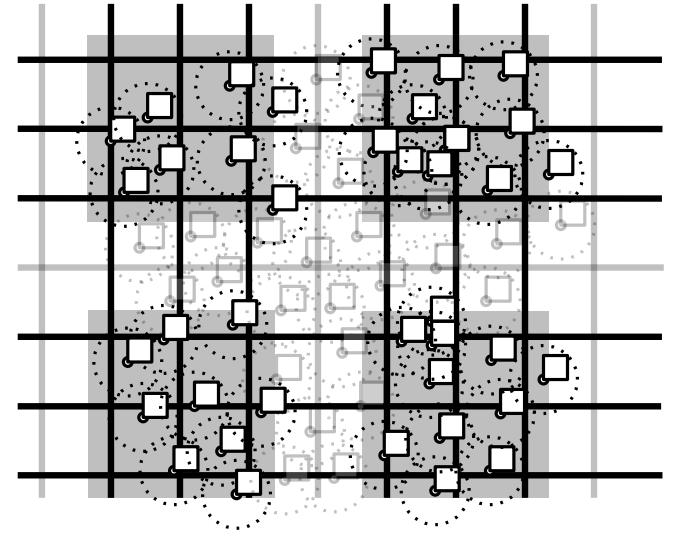


deactivate every k -th line

$$k = \lceil 2/\varepsilon \rceil$$



$$k = \lceil 2/\varepsilon \rceil$$



Lemma

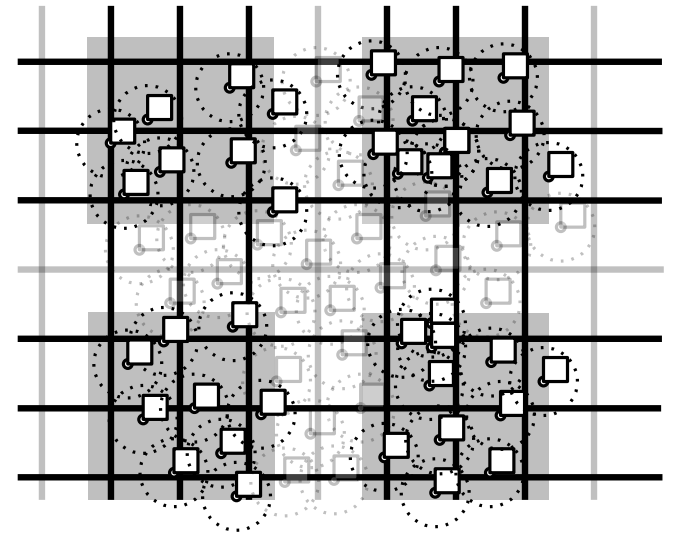
There exists a set S_i , $1 \leq i \leq k^2$ for which $\text{OPT}(S_i) \leq (1 - 1/k)^2 \cdot \text{OPT}$

$$k = \lceil 2/\varepsilon \rceil$$

OPT: value of optimal solution

OPT(S): optimal solution for set S

L : set of all labels of the instance



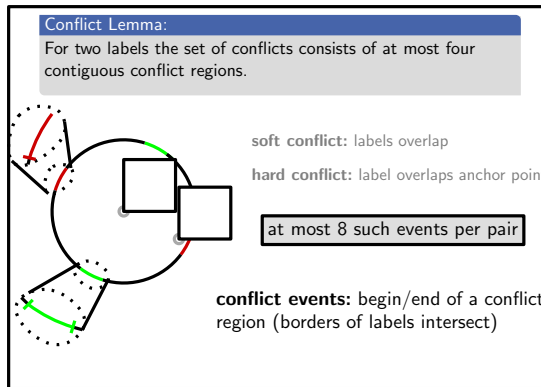
V_r : Labels hit by a **vertical** line with line number mod $k = r$

H_r : Labels hit by a **horizontal** line with line number mod $k = r$

Lemma

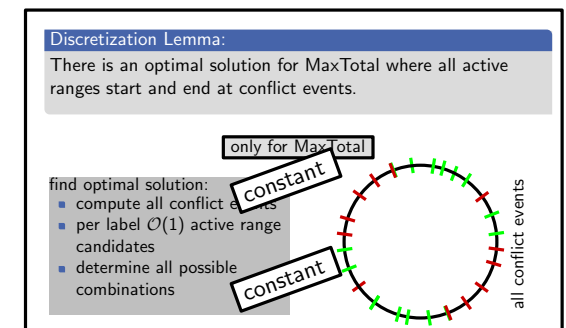
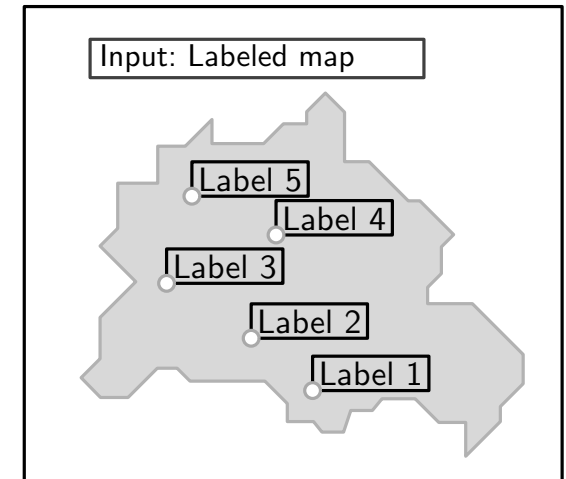
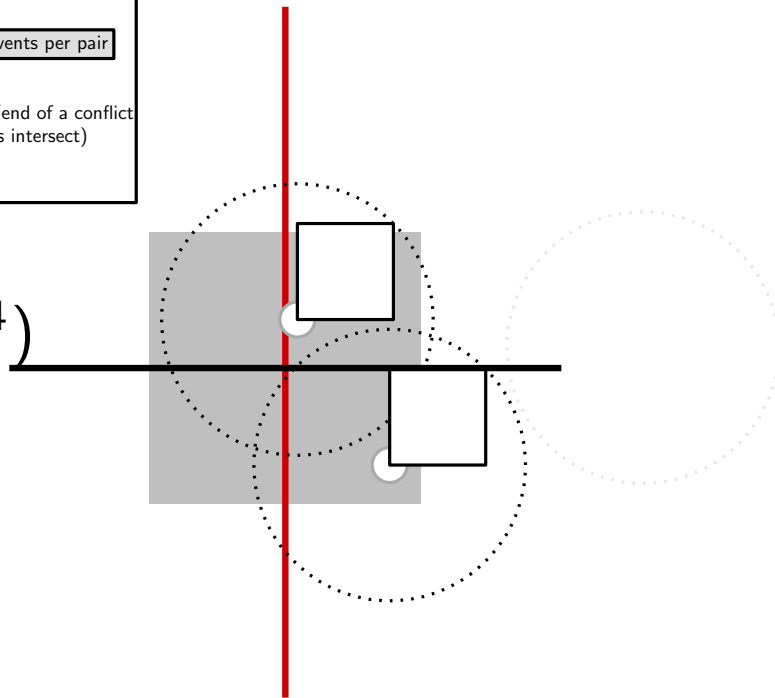
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 $\text{OPT}(S_i) \leq (1 - 1/k)^2 \cdot \text{OPT}$

An EPTAS for MaxTotal



(i) number of labels inside square: $\mathcal{O}(1/\varepsilon^2)$

(ii) #conflict events: $\mathcal{O}(1/\varepsilon^4)$



Theorem 3

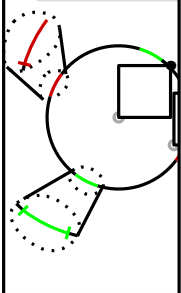
There exists a $(1 - \varepsilon)$ -approximation for MaxTotal.

Time complexity $\mathcal{O}((n \cdot 2^{\mathcal{O}(1/\varepsilon^2 \log 1/\varepsilon^8)} + n \log n) / \varepsilon^2)$.

An EPTAS for MaxTotal

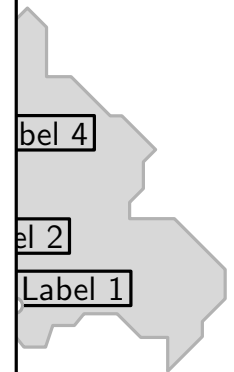
Conflict Lemma:

For two labels the set of conflicts consists of at most four contiguous conflict events.



(i) number of labels

Input: Labeled map



Discretization Lemma:

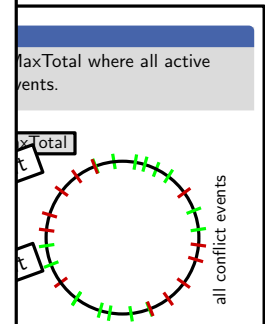
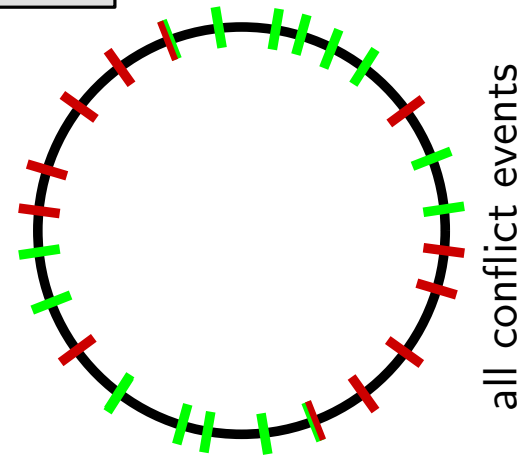
- There is an optimal solution for MaxTotal where all active ranges start and end at conflict events.

(ii) # of conflict events

only for MaxTotal

find optimal solution:

- compute all conflict events
- per label $\mathcal{O}(1/\varepsilon^8)$ active range candidates
- determine all possible combinations



Theorem 5

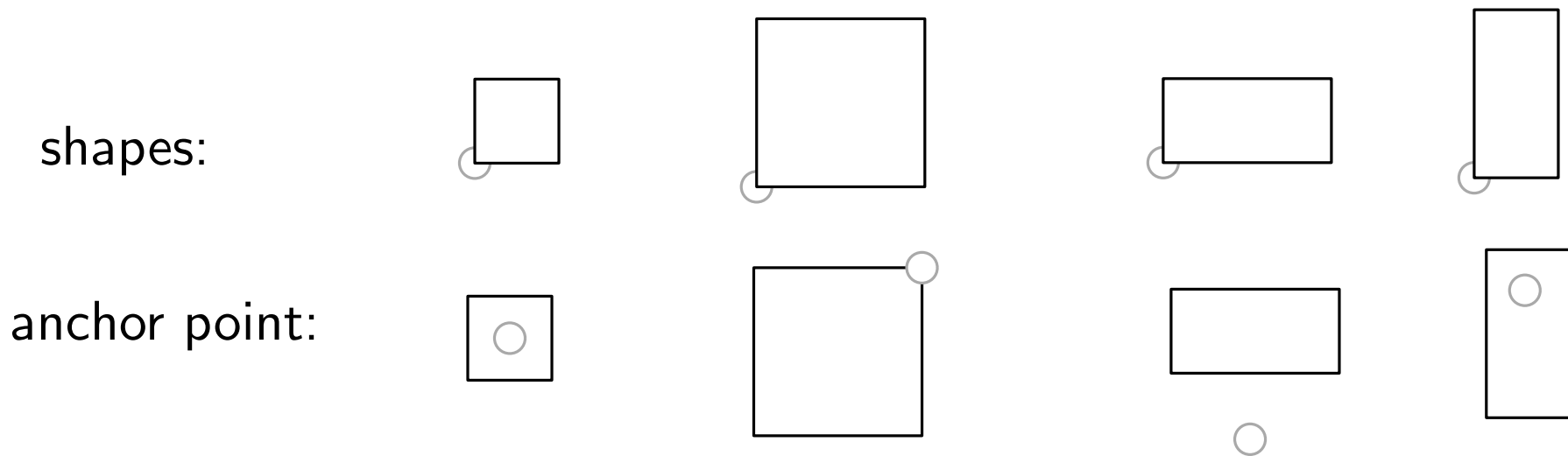
There exists a $(1 - \varepsilon)$ -approximation for MaxTotal.

Time complexity $O((n \cdot 2^{O(1/\varepsilon^2 \log 1/\varepsilon^8)} + n \log n)/\varepsilon^2)$.

An EPTAS for MaxTotal

Our EPTAS can be extended to arbitrary* **label dimensions**
and arbitrary relative **anchor positions**

- Key components Discretization and Conflict Lemma remain valid



Theorem 3

There exists a $(1 - \varepsilon)$ -approximation for MaxTotal.

Time complexity $O((n \cdot 2^{O(1/\varepsilon^2 \log 1/\varepsilon^8)} + n \log n)/\varepsilon^2)$.

Summary I

Results:

- MaxTotal is NP-complete, MaxMin is NP-hard
- There is an EPTAS for MaxTotal
- EPTAS uses **line stabbing** (geometric problems)
- MaxMin is hard to approximate (within $3/4$ opt)

Open Problems:

- Approximation algorithm for MaxMin?
- Simpler/Better algorithms for MaxTotal?

Did we actually solve the problem?